Computational Study of Quantum Coupled Chaotic Systems

Project Report

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PHY F376 Design Project

By

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Contents

Chapter 1

Introduction

1.1 Classical Kicked Rotor

In the classical picture, we can write the Hamiltonian of single kicked top as

$$H = \frac{J^2}{2I} + \sum_n V\delta(t - nT) \tag{1.1}$$

where J is the angular momentum of the top, I is the moment of inertia, and V is the kicked potential.

If we try to construct a system having two kicked tops coupled to each other, the corresponding Hamiltonian of the system can be written as

$$H = \frac{{J_1}^2}{2I_1} + \frac{{J_2}^2}{2I_2} + \sum_n (V_1 + V_2 + V_{12})\delta(t - nT) \tag{1.2}$$

where V_{12} is an interaction potential that couples the dynamics of the system.

1.2 Quantum Kicked Rotor

In the quantum picture, we replace the angular momentum J's by the corresponding angular momentum operators $\hat{J}$, and the Hamiltonian for coupled tops can be written as follows

$$H = \frac{p_1}{2j}{J_1}^2 + \frac{p_2}{2j}{J_2}^2 + \frac{\epsilon}{j}J_{z_1}J_{z_2} \tag{1.3}$$

where ϵ is the coupling strength, and j are the eigenvalues of the J^2 operator. The value of ϵ determines the entaglement properties of this coupled system.

1.3 Measures of Entaglement

Entaglement can be defined as the non classical correlation between two spatially separated subsystems [1]. Entaglement can be measured calculating the von Neumann entropy of the Reduced Density Matrix of the Hamiltonian. The von Neumann entropy S_v is given by

$$S_v = -\sum_n \lambda_i \log \lambda_i \tag{1.4}$$

where λ_i's are the eigenvalues of the reduced density matrix.

Chapter 2

Analysis Procedures

2.1 Method by Dalibard et al

Dalibard's method is an a general formalism that captures the essential features ruling the dynamics of quantum periodic systems by construction of an effective Hamiltonian.[2].

For the case of a quantum coupled periodic system, the effective Hamiltonian can be written as

$$H_{eff} = H_0 + V_0 + \sum_n \frac{1}{2n^2\omega^2}([[V_n, H_0], V_{-n}] + h.c.) + \mathcal{O}(\omega^3) \tag{2.1}$$

where

$$H_0 = p_1 J_{y_1} \otimes I_2 + p_2 I_1 \otimes J_{y_2}, V_0 = \frac{k_1}{2j} J_{z_1}^2 \otimes I_2 + \frac{k_2}{2j} I_1 \otimes J_{z_2}^2 \tag{2.2}$$

Using the eigenvectors of H_{eff}, the reduced density matrix can be constructed by first constructing a matrix C which is an $n \times n$ array of the eigenvectors. The RDM ρ can be calculated by the formaula

$$\rho = C^\dagger C \tag{2.3}$$

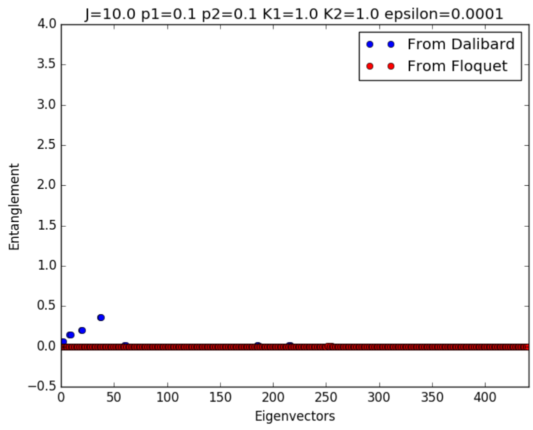
We then use the formula in chapter 1 to calculate the entaglement.

Chapter 3

Results and Conclusions

The results were obtained using Floquet Analysis for a system with $j = 10, p1 = p2 = \frac{1}{j}, K1 = K1 = 1$ and are plotted against that obtained using Dalibard's Analysis. The plots are shown on the following two pages.

We can see from the plots that S_v both the Dalibard and Floquet results match at low ϵ values and both these values tend to zero, as is expected for this ϵ range. The results also match for $\epsilon = 1$, which itself is a pretty high value of coupling. As we increase our ϵ value above 1, we can see that the Dalibard method breaks down. This is majorly because our method is an approximation, and the terms in the H_{eff} of our system break down at higher values of ϵ.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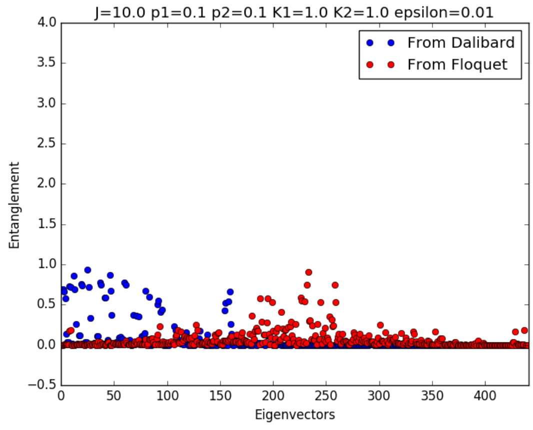
From Dalibard
From Floquet
Entanglement
Eigenvectors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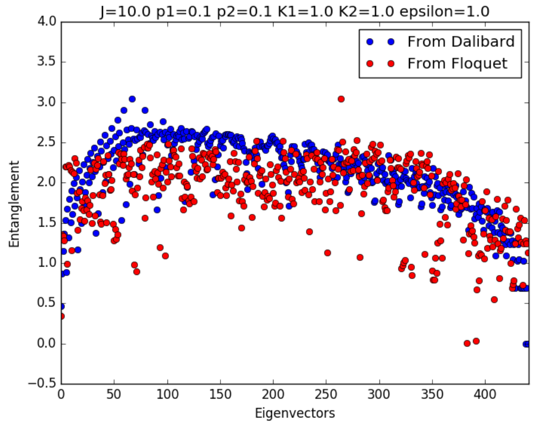
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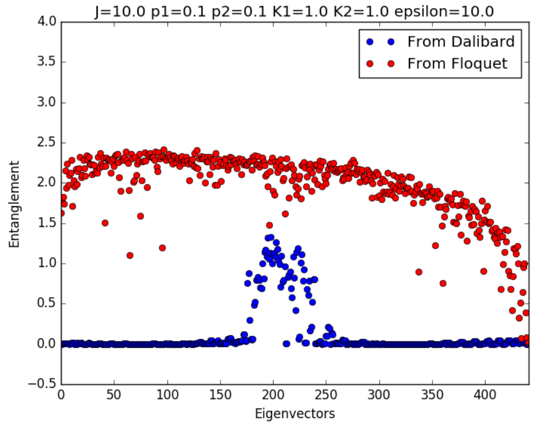
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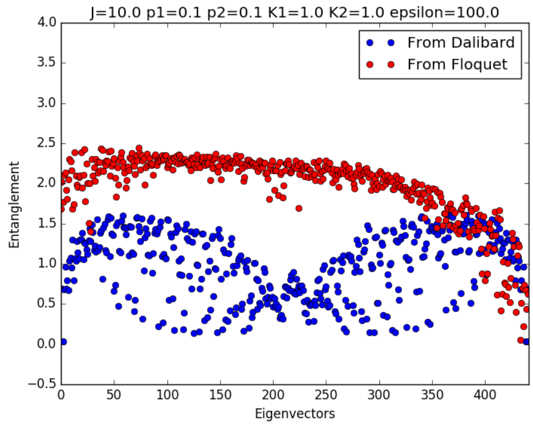
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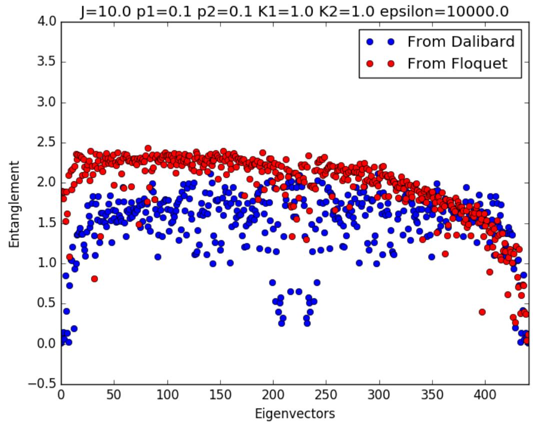
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From Dalibard
From Floquet
Entanglement
Eigenvectors

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[1] Jayendra N. Bandyopadhyay and Arul Lakshminarayan. "Entanglement production in coupled chaotic systems: Case of the kicked tops". In: *Phys. Rev. E* 69 (1 2004), p. 016201. DOI: `10.1103/PhysRevE.69.016201`. URL: `http://link.aps.org/doi/10.1103/PhysRevE.69.016201`.

[2] N. Goldman and J. Dalibard. "Periodically Driven Quantum Systems: Effective Hamiltonians and Engineered Gauge Fields". In: *Phys. Rev. X* 4 (3 2014), p. 031027. DOI: `10.1103/PhysRevX.4.031027`. URL: `http://link.aps.org/doi/10.1103/PhysRevX.4.031027`.