Quantum Mechanics II
Assignment 6

Due Wednesday, 28 November 2018

Problems:

1. Consider Flatland — a two-dimensional world, and where living creatures, including physicists, are confined to a two-dimensional plane.

   - What are the quantum numbers that such physicists would use to describe polar (spherical) waves? Solve the wave-equation in polar coordinates.
   - Find the Green's function for non-relativistic scattering by repeating the procedure that we performed in class.

2. Show that, within the first order Born approximation, the total cross-section for the scattering of a particle with mass m and incoming momentum $\vec{k}$ can be written as

$$\sigma = \frac{m^2}{\pi} \int d^3x d^3x' V(r) V(r') \frac{\sin^2 k|x-x'|}{k^2|x-x'|^2}$$

   with $k = \vec{k}$. Now check that this expression obeys the optical theorem, when the amplitude is calculated to *second order* in the Born approximation.

3. Consider a potential given by $V(r) = 0, r > R$ and $V(r) = V_0, r < R$. Find the phase shifts for spherical waves with angular momentum quantum number, ℓ and use this to find the total cross-section for scattering.

4. The *von Neumann entropy* is defined as $S = -\mathrm{Tr}(\rho \log \rho)$ for a density matrix, ρ.

   - Show that if ρ represents a pure state, then S vanishes.
   - What is the maximum value that S can take, if the dimension of the Hilbert space is d?

5. Consider two identical but non-interacting systems. We denote the energy eigenstates of system 1 by $|E\rangle$, and those of system 2 by $|\tilde{E}\rangle$. Now, consider the following combined state of the two systems

$$|\Psi\rangle = \kappa \sum_E e^{-\frac{\beta E}{2}} |E\rangle |\tilde{E}\rangle$$

   - What is the value that κ must have for the state to be properly normalized?
   - What is the *reduced density matrix* of system 1?
   - If the reduced density matrix you calculated above is denoted by ρ, what is the von Neumann entropy of ρ? Do you recognize the physical significance of this quantity?