Advanced Quantum Mechanics: Assignment #2

Due on 20th September, 2018

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Problem 1

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Problem 2

We know that the energy eigenstates of the harmonic oscillator form a basis. Hence, the required coherent state $|z\rangle$ can be written in terms of these eigenstates as,

$$|z\rangle = \sum_{n=0}^{\infty} c_n |n\rangle = \sum_{n=0}^{\infty} c_n \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle$$

Substituting this in the equation for coherent state $a|z\rangle = z|z\rangle$,

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n a |n\rangle &= \sum_{n=0}^{\infty} z c_n |n\rangle \\
\sum_{n=1}^{\infty} c_n \sqrt{n} |n-1\rangle &= \sum_{n=0}^{\infty} z c_n |n\rangle \\
\sum_{n=0}^{\infty} c_{n+1} \sqrt{n+1} |n\rangle &= \sum_{n=0}^{\infty} z c_n |n\rangle \\
\therefore c_{n+1}\sqrt{n+1} &= z c_n
\end{aligned}$$

We have effectively derived a recursion relation for the coefficients c_n. If we start off with $c_n = \alpha$,

$$c_1 = z\alpha \quad , \quad c_2 = \frac{z^2\alpha}{\sqrt{2}} \quad , \quad c_3 = \frac{z^3\alpha}{\sqrt{3\cdot 2}} \quad , \quad \ldots \quad , \quad c_n = \frac{z^n\alpha}{\sqrt{n!}}$$

So, our coherent state can now be written as,

$$\begin{aligned}
|z\rangle &= \alpha \sum_{n=0}^{\infty} \frac{(za^\dagger)^n}{n!} |0\rangle \\
&= \alpha e^{a^\dagger z} |0\rangle
\end{aligned}$$

Problem 3

We know that,

$$x(0) = \frac{a + a^\dagger}{\sqrt{2m\omega}} \quad , \quad p(0) = \frac{\sqrt{m\omega}(a - a^\dagger)}{\sqrt{2}i} \quad , \quad x(t) = e^{iHt}x(0)e^{-iHt} \quad , \quad p(t) = e^{iHt}p(0)e^{-iHt}$$

Consider $C_1(t)$,

$$
\begin{aligned}
C_1(t) &= \langle 0|e^{iHt}x(0)e^{-iHt}x(0)|0\rangle \\
&= \frac{e^{i\omega t/2}}{\sqrt{2m\omega}} \langle 0|\, x(0)e^{-iHt}\, |1\rangle \impliedby \{a^\dagger\, |0\rangle = |1\rangle \quad , \quad \langle 0|\, e^{iHt} = e^{i\omega t/2}\, \langle 0|\} \\
&= \frac{e^{i\omega t/2}}{\sqrt{2m\omega}} \frac{e^{-3i\omega t/2}}{\sqrt{2m\omega}} \langle 1|1\rangle \impliedby \{\langle 0|\, a = \langle 1| \quad , \quad e^{-iHt}\, |1\rangle = e^{-3i\omega t/2}\, |1\rangle\} \\
&= \frac{e^{-i\omega t}}{2m\omega}
\end{aligned}
$$

Problem 4

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