Theory and Numerics of PDEs: Assignment #2

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Problem 1

Given:

$$\lim_{n\to\infty} a_n = L \quad ; \quad \lim_{n\to\infty} b_n = 1 \quad ; \quad \lim_{n\to\infty} \epsilon_n = 0 \tag{1}$$

Using properties of limits for the sum and products of functions, we can write:

$$\lim_{n\to\infty} a_n b_n + \epsilon_n = \lim_{n\to\infty} a_n \lim_{n\to\infty} b_n + \lim_{n\to\infty} \epsilon_n = L \times 1 + 0 = L \tag{2}$$

Problem 2

Given:

$$\lim_{n\to\infty} a_n = L \quad ; \quad M = \text{constant} > 0 \quad ; \quad S \in [L-M, L+M] \quad ; \quad f_n : \Re \to \Re \tag{3}$$

$$\lim_{n\to\infty} \left(\sup_{x\in S} |f_n(x) - x| \right) = 0 \tag{4}$$

Consider,

$$|f_n(a_n) - L| = |f_n(a_n) - a_n + a_n - L| \tag{5}$$

$$|f_n(a_n) - a_n + a_n - L| \le |f_n(a_n) - a_n| + |a_n - L| \tag{6}$$

$$\therefore |f_n(a_n) - L| \le |f_n(a_n) - a_n| + |a_n - L| \tag{7}$$

$$\therefore |f_n(a_n) - L| \le \sup_{x\in S} |f_n(x) - x| + |a_n - L| \tag{8}$$

$$\therefore \lim_{n\to\infty} |f_n(a_n) - L| \le \lim_{n\to\infty} \sup_{x\in S} |f_n(x) - x| + \lim_{n\to\infty} |a_n - L| \tag{9}$$

$$\therefore \lim_{n\to\infty} |f_n(a_n) - L| \le 0 + 0 \tag{10}$$

$$\therefore \lim_{n\to\infty} |f_n(a_n) - L| = 0 \implies \lim_{n\to\infty} f_n(a_n) = L \tag{11}$$

The sup in the condition for f_n is indeed necessary, since without that we wouldn't have been able to manipulate the inequality to introduce the condition instead of $|f_n(a_n) - a_n|$.