

Advanced Quantum Mechanics: Assignment #5

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Problem 1

Problem 2

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Problem 3

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Problem 4

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Problem 5

We first note for $|\psi_I(t)\rangle = \sum_n c_n(t) |\alpha_n\rangle$

$$\begin{aligned} i \frac{\partial |\psi_I\rangle}{\partial t} &= i \frac{\partial (e^{iH_0 t} |\psi_S\rangle)}{\partial t} \\ &= i \left[e^{iH_0 t} \frac{\partial |\psi_S\rangle}{\partial t} + iH_0 e^{iH_0 t} |\psi_S\rangle \right] \\ &= -e^{iH_0 t} (H_0 + V) |\psi_S\rangle - H_0 e^{iH_0 t} |\psi_S\rangle \\ &= e^{iH_0 t} V |\psi_S\rangle \\ i \frac{\partial |\psi_I\rangle}{\partial t} &= V_I |\psi_I\rangle \\ i \frac{\partial \langle \alpha_n | \psi_I \rangle}{\partial t} &= \langle \alpha_n | V_I | \psi_I \rangle \\ \dot{c}_n &= -i \langle \alpha_n | V_I | \psi_I \rangle \\ \dot{c}_n &= -i \langle \alpha_n | V | \alpha_m \rangle e^{i(E_n - E_m)t} c_m \end{aligned}$$

So for the given problem, we have

$$|\psi_I(t)\rangle = c_1(t) |1\rangle + c_2(t)e^{iEt} |2\rangle$$

$$\dot{c}_1 = -iV_{11}c_1 - iV_{12}e^{-iEt}c_2 = -i\gamma e^{i(\omega-E)t}c_2 \quad \text{and} \quad \dot{c}_2 = -iV_{21}e^{iEt}c_1 - iV_{22}c_2 = -i\gamma e^{i(E-\omega)t}c_1$$

To solve the above equations, we make the substitution $c_1 = b_1 e^{i\Delta t}$ and $c_2 = b_2 e^{-i\Delta t}$, where $2\Delta = \omega - E$. We then have the equations in terms of b 's,

$$i\dot{b}_1 = \Delta b_1 + \gamma b_2 \quad \text{and} \quad i\dot{b}_2 = \gamma b_1 - \Delta b_2$$

These are coupled equations, and we can solve these by making the substitution $b_1 = A e^{i\Omega t}$ and $b_2 = B e^{i\Omega t}$. We then have,

$$-A\Omega = \Delta A + \gamma B \quad \text{and} \quad -B\Omega = \gamma A - \Delta B$$

For non-trivial solutions, $-\frac{\gamma}{\Delta + \Omega} = \frac{\Delta - \Omega}{\gamma} \implies \Omega = \pm \sqrt{\gamma^2 + \Delta^2} = \pm \Omega_0$

$$\implies c_1 = A_1 e^{i(\Delta + \Omega_0)t} + A_2 e^{i(\Delta - \Omega_0)t} \quad \text{and} \quad c_2 = B_1 e^{i(-\Delta + \Omega_0)t} + B_2 e^{i(-\Delta - \Omega_0)t}$$

We are told that at $t = 0$, the system is in state $|1\rangle \implies c_1(0) = 1, c_2(0) = 0 \implies A_1 = 1, A_2 = 0, B_1 = 0, B_2 = 0$. We also know for a fact that $|c_1(0)|^2 + |c_2(0)|^2 = 1$