Fluid Mechanics: Assignment #2

Due on 11th September, 2018

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Problem 1

Part (a)

Given the assumptions, we can effectively consider the two volcanoes as sources/sinks in 2 dimensions. At some height h, this then makes mh and nh the strength of the sources respectively. Consider the volcano at $(0,0)$ to have strength m/h and the one at $(d,0)$ to have n/h. At some point (x,y), the velocity purely due to each of the volcanoes is given by,

$$\mathbf{v}_1 = \frac{m}{2\pi h(x^2+y^2)}(x\hat{\mathbf{x}} + y\hat{\mathbf{y}}) \quad \text{and} \quad \mathbf{v}_2 = \frac{n}{2\pi h((x-d)^2+y^2)}((x-d)\hat{\mathbf{x}} + y\hat{\mathbf{y}})$$

The final velocity field is just the vector addition,

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 = \frac{1}{2\pi h}\left[\left(\frac{mx}{x^2+y^2} + \frac{n(x-d)}{(x-d)^2+y^2}\right)\hat{\mathbf{x}} + \left(\frac{my}{x^2+y^2} + \frac{ny}{(x-d)^2+y^2}\right)\hat{\mathbf{y}}\right]$$

Part (b)

Obviously, h is not truly constant. There will be some additional force generated due to the pressure difference, which will then require us to solve the full Navier-Stokes equation to find the velocity field.

Part (c)

If $n < 0$, the second volcano is basically sucking in ash, and hence will act as a sink. The field sketches for both parts is given below

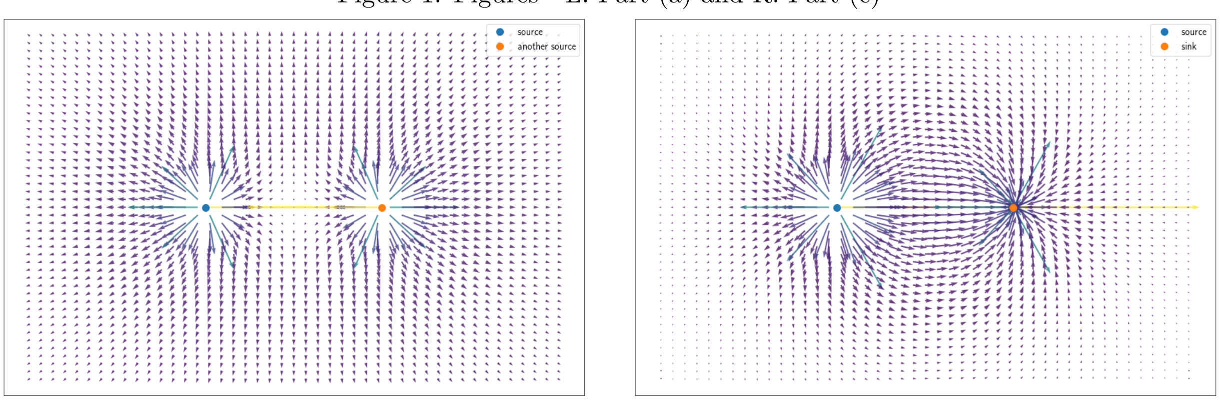
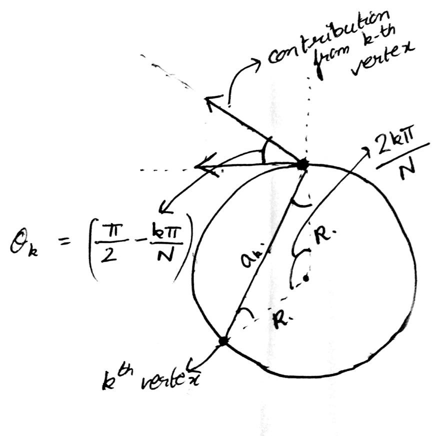


Figure 1: Figures - L: Part (a) and R: Part (c)

Problem 2

Part (a)



For a regular polygon with N sides, each side will subtend an angle $\frac{2\pi}{N}$ at the centre. As the problem is symmetric, it is enough to solve for the motion of one point vortex.

Considering a single vertex of the polygon, we can see that the motion will have contributions from the other $N-1$ vertices. As shown in the figure, only the horizontal components of these contributions will survive. Hence resultant tangential velocity will be given by,

$$v = \sum_{k=1}^{N-1} \frac{\Gamma}{2\pi a_k} \cos\theta_k$$

where k is the serial number of vertices starting anticlockwise from the vertex under consideration. It is evident from the figure that,

$$\theta_k = \frac{\pi}{2} - \frac{k\pi}{N} \quad \text{and} \quad a_k = 2R\sin\frac{k\pi}{N}$$

where R is the distance of each vertex from the centre. Substituting this in the expression for velocity, and noting that $l = 2R\sin\frac{\pi}{N}$ where l is side length

$$v = \frac{(N-1)\Gamma}{4\pi R} = \frac{(N-1)\Gamma\sin\frac{\pi}{N}}{2\pi l}$$

The time period T is given by,

$$\boxed{T = \frac{2\pi R}{v} = \frac{8\pi^2 R^2}{(N-1)\Gamma} = \frac{2\pi^2 l^2}{(N-1)\Gamma\sin^2\frac{\pi}{N}}}$$

Part (b)

As the expression for time period we have got is pretty simple, there is no need to solve this problem on the computer.

$$\boxed{\text{For } N = 4; \quad T = \frac{8\pi^2 R^2}{3\Gamma}}$$

$$\boxed{\text{For } N = 10; \quad T = \frac{8\pi^2 R^2}{9\Gamma}}$$

Part (c)

For a non-identical polygon, the flow will not be circular, and could follow some chaotic trajectory.