Classical Mechanics - Assignment 3

Due date: Sept 21, 2018

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Note: Submit the assignment to any one of TA's office on/before the due date. For the numerical parts of the questions take print outs of the codes along with the plots, etc. and attach it in the correct place of your solution. If you have any doubt regarding the assignment problems or the topics discussed in the class, feel free to discuss with TAs or the instructor. For numerical parts use any of your favourite programming language and plotting software. Good luck!

Q1 $\boxed{\text{15 marks}}$

Variational principle and fields

(a) Define functional derivative mathematically. As a practice, derive the single variable Euler-Lagrange equation starting from the least action principle by taking the functional derivative. $\boxed{5}$

(b) Take a linear harmonic chain and do the field theory for it, i.e. write down the Lagrangian, take the appropriate limits of the bond lengths, etc., motivate the idea of Lagrangian density, and then take the functional derivative to get the equation of motion. $\boxed{10}$

Q2 $\boxed{\text{15 marks}}$

Small oscillations

(a) Derive all the eigenfrequencies and normal mode coordinates and draw a diagram representing them for a linear tri-atomic molecule. Initially consider all atoms of separate masses and all bonds are harmonic but of different strength. Discuss and draw for various special cases, e.g. all atoms of equal mass, two atoms of same mass, two same spring constants, etc. Choose your favourite linear tri-atomic molecule and look into web for the specific details. Calculate the frequencies for that. $\boxed{15}$

Q3 $\boxed{\text{20 marks}}$

Central Force In Figure 1, point masses m_1 at $\mathbf{r}_1$ and m_2 at $\mathbf{r}_2$ are interacting by the potential, $V(\mathbf{r}_1 - \mathbf{r}_2)$. $\mathbf{R}$ is the position of centre of mass of this two body system.

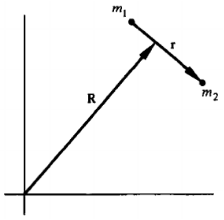

Figure 1: Two body problem

(a) Calculate all the conserved quantities and derive the Kepler law on areas. $\boxed{5}$

(b) Assuming that $m_2 >> m_1$, Calculate r, distance between m_1 and m_2, as a function of time, t. $\boxed{5}$

(c) For $V(r) = -\frac{k}{r}$, for some $k > 0$, Prove that orbit of m_1 around m_2 is a conic section. Moreover, derive conditions for which orbit is hyperbolic, elliptic. $\boxed{10}$