Binaries and Tides

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Contents

1 Tanja's Lectures at Gravitational Wave School 2017

Non-black hole objects differ from black holes in that :-

- They can deform due to their rotational motion
- They can have effects due to no presence of horizon
- They can get tidally deformed

Tanja's lectures focus on tidal effects, which are the most promising candidates (as of 2017) to detect parameters of neutron stars.

We start off with tides in Newtonian physics.

1.1 Newtonian Physics

Notation - $\mathbf{a}$ is 3-vector and $\vec{\mathbf{a}}$ is 4-vector.

The force between two bodies of masses m and M is,

$$\mathbf{F} = -\frac{GmM}{r^2}\hat{\mathbf{n}} \quad , \quad U = -\frac{GM}{r} \quad , \quad \mathbf{a} = \boldsymbol{\nabla} U \tag{1.1}$$

where U and $\mathbf{a}$ are the gravitational potential due to mass M and the acceleration respectively.

To make things more specific, consider a body of mass m_A with position vector $\mathbf{z}_A$. The potential at $\mathbf{x}$ due to m_A is,

$$U_A(\mathbf{x}) = \frac{Gm_A}{|\mathbf{x} - \mathbf{z}_A|} \tag{1.2}$$

Alternatively, if we consider extended bodies with density $\rho(x)$, the potential can be written as,

$$U_A(\mathbf{x}) = G \int \mathrm{d}^3\mathbf{x}' \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \tag{1.3}$$

$$\implies \nabla^2 U_A = G \int \mathrm{d}^3\mathbf{x}' \rho(\mathbf{x}') \nabla^2 \frac{1}{|\mathbf{x} - \mathbf{x}'|} \tag{1.4}$$

$$= G \int \mathrm{d}^3\mathbf{x}' \rho(\mathbf{x}') \big(-4\pi\delta(\mathbf{x} - \mathbf{x}')\big) \tag{1.5}$$

$$\nabla^2 U_A = -4\pi G \rho(\mathbf{x}) \tag{1.6}$$