Classical Mechanics - Assignment 5

Due date: Nov 4, 2018

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Note: Submit the assignment to any one of TA's office on/before the due date. For the numerical parts of the questions take print outs of the codes along with the plots, etc. and attach it in the correct place of your solution. If you have any doubt regarding the assignment problems or the topics discussed in the class, feel free to discuss with TAs or the instructor. For numerical parts use any of your favourite programming language and plotting software. Good luck!

Q1 20 marks

Rigid body dynamics

(a) A uniform right circular cone of height h, half-angle α, and density ρ rolls on its side without slipping on a uniform horizontal plane in such a manner that it returns to its original position in a time τ. Find expressions for the kinetic energy and the components of the angular momentum of the cone. 10

(b) A uniform bar of mass M and length $2l$ is suspended from one end by a spring of force constant k. The bar can swing freely only in one vertical plane, and the spring is constrained to move only in the vertical direction. Set up the equations of motion in the Lagrangian formulation. 10

Q2 20 marks

Hamiltonian dynamics

(a) It has been previously noted that the total time derivative of a function of q_i and t can be added to the Lagrangian without changing the equations of motion. What does such an addition do to the canonical momenta and the Hamiltonian? Show that the equations of motion in terms of the new Hamiltonian reduce to the original Hamilton's equations of motion. 10

(b) The Lorentz force equation remains valid for relativistic particles, provided that We recognize that the force is $\dot{\vec{p}}$. If the charged particle moves through electric and magnetic fields at velocities close to the velocity of light, the relativistic form of the Lagrangian is given by $L = -mc^2\sqrt{1 - v^2/c^2} - e\Phi + \frac{e}{c}\vec{v}\cdot\vec{A}$. Find the canonical momenta p_x, p_y, p_z and the Hamiltonian H. Is $H = T + V$? Find the Hamilton's wquations of motion. 10

Q3 10 marks

Rigid Body Dynamics

(a) Show that a body whose principal moments of inertia I_1, I_2 and I_3 are all different, can rotate uniformly around one of the principal axis. 10

(b) Discuss the stability of such a motion. 10

Q4 10 marks

Rigid Body Dynamics

(a) If a rectangular parallelopiped with its edges, $2a$, $2a$, $2b$ rotates about its centre of gravity under no forces. Prove that, its angular velocity about one principal axis is constant and about the another axis it is periodic. 10