Mukhanov Cosmology: Chapter #1

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Problem 1

Solution

The following condition should be satisfied for a law to be the same for all observers, and hence, physical.

$$f(\vec{\mathbf{r}}_{CA} - \vec{\mathbf{r}}_{BA}, t) = f(\vec{\mathbf{r}}_{CA}, t) - f(\vec{\mathbf{r}}_{BA}, t)$$

One can recall that this is an example of a linear transformation/operation. A general transformation linear in x can be written as $f(\vec{\mathbf{r}}, t) = H(t)\vec{\mathbf{r}}$, which is exactly the form of the Hubble law.

Problem 2

Let v_H be the Hubble velocity, and v_P be the peculiar velocity of the galaxy. For peculiar velocity to be neglected, we assume that v_H should be at least one order of magnitude larger than v_P ie $v_H \approx 1000$ km/s

$$v_H = Hr = 75r \ ; \ r = 1000/75 \approx \boxed{13.33\text{MPc}}$$

Problem 3

We know that,

$$\frac{\dot{a}^2}{2} = \frac{4\pi G\epsilon_0}{3}\left(\frac{a_0^3}{a}\right) + \text{constant}$$

Hence when $a \to \infty$, $\dot{a} \to \infty$ and $H = \frac{\dot{a}}{a} \to \infty$

We also know that,

$$\epsilon^{cr}(1 - \Omega(t)) = \frac{3E}{4\pi G}\frac{\epsilon}{a^2}$$

As the RHS is a finite value, $E \to \infty$ when $a \to \infty$.

Problem 4

TO BE DONE

$$\epsilon^{cr}(1 - \Omega(t)) = \frac{3E}{4\pi G}\frac{\epsilon}{a^2}$$

$$\Omega(t) = 1 - \frac{3E}{4\pi G}\frac{\epsilon}{\epsilon^{cr}}\frac{1}{a^2}$$

$$PE = -\frac{GM^2}{R} \ ; \ KE = \frac{M}{den}$$

Problem 5

Recall that Newton's second law applied to an expanding spherical ball of dust gives us

$$\ddot{a} = -\frac{4\pi G}{3}\epsilon a$$
$$\therefore \; q = -\frac{\ddot{a}}{aH^2} = \frac{4\pi G}{3}\frac{\epsilon}{H^2}$$

We know that $\epsilon^{cr} = \frac{3H^2}{8\pi G}$ and $\Omega(t) = \frac{\epsilon}{\epsilon^{cr}}$. Hence,

$$\boxed{q = \frac{1}{2}\Omega(t)}$$

For a spatially flat universe, $\Omega(t) = 1$. Hence $\boxed{q = \frac{1}{2}}$.

Problem 6

Restoring units of c, the expression for energy density $\epsilon(t)$ is

$$\epsilon(t) = \frac{c^2}{6\pi G t^2} = \frac{7.162}{t^2} \times 10^{14} \text{ J/m}^3$$

Substituting values, one gets

$$\epsilon(t = 10^{-43}s) = 7.162 \times 10^{100} \text{ J/m}^3$$
$$\epsilon(t = 1s) = 7.162 \times 10^{14} \text{ J/m}^3$$
$$\epsilon(t = 1yr) = 0.72 \text{ J/m}^3$$

Problem 7

$$H^2 - \frac{2E}{a(t)^2} = \frac{8\pi G a_0^3}{3a^3}\epsilon_0$$
$$\dot{a}^2 - 2E = \frac{8\pi G a_0^3}{3a}\epsilon_0$$

For $t \to \infty$, $\dot{a} \sim 1$ and hence $a \sim t$.

Problem 8