Trefethen and Bau: Lecture #3

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Aditya Vijaykumar

Problem 1

For something to be a vector norm it should satisfy three conditions. We list down these three conditions and see if $\|\cdot\|_W = \|Wx\|$ does indeed satisfy them,

- $\|x\|_W \geq 0$. This is automatically satisfied by the definition of the norm.
- $\|x\|_W = 0$ iff $x = 0$. As W is a nonsingular matrix by assumption, this property is satisfied as well.
- $\|\alpha x\|_W = |\alpha|\|x\|_W$. As α is just a multiplicative constant and W is indeed a matrix, this property is satisfied as well.

Hence, $\|\cdot\|$ is a vector norm.

Problem 2

By the definition of $\|A\|$, $\|A\| \geq \|Ax\|, \forall$ unit vectors x
DO this problem

Problem 3

Part (a)

$$\|x\|_\infty = \sqrt{x_0^2} \leq \sqrt{x_0^2 + \sum_{i \neq 0} x_i^2} \leq \|x\|_2$$

Equality is achieved for any vector having only one element.

Part (b)

$$\|x\|_2 = \sqrt{\sum_i x_i^2} \leq \sqrt{m(\max x_i)^2} \leq \sqrt{m}\|x\|_\infty$$

Equality is achieved for any vector having elements with the same absolute value.

Part (c)