Classical Mechanics - Assignment 2

Due date: Sept 7, 2018

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Note: Submit the assignment to any one of TA's office on/before the due date. For the numerical parts of the questions take print outs of the codes along with the plots, etc. and attach it in the correct place of your solution. If you have any doubt regarding the assignment problems or the topics discussed in the class, feel free to discuss with TAs or the instructor. For numerical parts use any of your favourite programming language and plotting software. Good luck!

Q1 $\boxed{\text{10 marks}}$

The curve $y(x) = k(\frac{x}{l})^{\alpha}$ is rotating around y-axis with fixed angular velocity ω. A bead slides

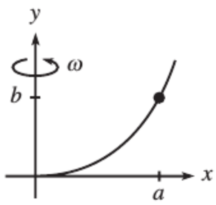

Figure 1: Bead on a curve

along the curve without friction (see above).

(a) Calculate the Equations of motion. $\boxed{2}$

(b) Find values of α, for which the system can have small oscillations about the equilibrium point. $\boxed{5}$

(c) Find the frequency of the oscillations $\boxed{3}$

Q2 $\boxed{\text{20 marks}}$

A solid cylinder (mass M, radius R) rolls without slipping on a cylindrical surface as shown in Figure 2

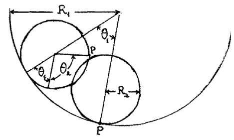

Figure 2: Cylinder rolling inside cylinder

(a) Write the equations of motion in terms of θ_1 and θ_2. $\boxed{5}$

(b) Derive the constraints equation for θ_1 and θ_2. $\boxed{5}$

(c) Calculate the constraint forces applied by the surface on the rolling cylinder. $\boxed{10}$

Q3 $\boxed{\text{10 marks}}$

Degrees of freedom and Lagrangian

(a) *(Bicycle)* Make a simplified model of a bicycle. How many degrees of freedom are there? Restrict your model to the most important degrees of freedom. $\boxed{2}$

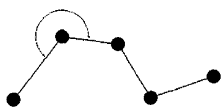

Figure 3: Flexible chain

(b) *(Flexible Chain)* A flexible chain of mass M massive point particles has rigid weightless rods of $M-1$ links as shown in Figure 3. Each joint is free to move in any direction. How many degrees of freedom does the chain have? Finally, suppose the chain is lifted off the table and is closed by one more link. How many degrees of freedom are there then? $\boxed{2}$

(c) *(Guessing the Lagrangian for a free particle)* Assume that you do not know about kinetic energy and Newton's laws of motion. Suppose instead of deriving Euler-Lagrange equations, we postulate them. We define the basic law of mechanics to be these equations and ask ourselves the question: What is the Lagrangian for a free particle? (This is a particle in empty space with no force acting on it. Be sure to set up an inertial reference system) Explain why, on very general grounds, L cannot be function of x, y, or z. It also cannot depend on the individual coordinates of velocity in any way except as a function of the magnitude of the velocity: $v^2 = v_x^2 + v_y^2 + v_z^2$. On what assumption about the properties of space does this depend? $\boxed{2}$

(d) The simplest choice might be to guess it must be proportional to v^2, where $\vec{v}$ is the particle velocity in an inertial frame K. Take $L = v^2$. A second inertial frame K' moves at a constant velocity $-\vec{V}_0$ with respect to K, so that the transformation law of velocities is $\vec{v}' = \vec{v} + \vec{V}_0$. prove that $L' = v'^2$ is a possible choice for the Lagrangian in the frame K'. Explain how this proves that all inertial frames are equivalent. $\boxed{2}$

(e) Instead of proving it, adopt the equivalence of inertial frames as postulate, in addition to the Euler-Lagrange equations. Explain why this means that $\boxed{2}$

$$L'(v + V_0) = L(v) + dF(x,t)/dt. \tag{1}$$

$L(v)$ is an unknown function for the free particle that we are trying to determine from these principles. (Work in 1d to make things easier.) Let V_0 be an infinitesimal quantity. Expand the left side of the Equation (1) in Taylor series and keep only the first two terms. From this prove $L(v) \sim v^2$.

Q4 $\boxed{\text{10 marks}}$

Variational principle

(a) *(Variational principle for quantum mechanics)* The quantum mechanics of a one-dimensional system is described by the Schroedinger equation for the complex wave function $\psi(x,t)$: $\boxed{4}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar\frac{\partial \psi}{\partial t} \tag{2}$$

where $\hbar$ is Plank's constant, m is mass and $V(x)$ is the potential energy. Find a variational principle for quantum mechanics using the two dependent variables ψ, ψ^* and the two independent variables x, t. you can treat ψ, ψ^* as two independent generalized coordinates, since real and imaginary parts are independent variables.

(b) *(Shape of a hanging wire)* Find the shape of a wire of mass density ρ hanging from two horizontal points in gravity. $\boxed{3}$

(c) *(Geodesics on a sphere)* Find the shortest distance curve between two points on a sphere. $\boxed{3}$

Q5 $\boxed{\text{20 marks}}$

Conservations, Noether theorem, Gauge

In class we discussed about charged particle moving in magnetic fields. Let's consider a charged particle of charge q is moving in the x-y plane and there is a constant magnetic field B applied along z-direction.

(a) Discuss briefly about your understanding of non-uniqueness of the magnetic vector potential $\vec{A}$ and Gauges. $\boxed{3}$

(b) *(Landau Gauge)* Consider $\vec{A} = \vec{A}(x \text{ only})$. Find a form of $\vec{A}$. What are the symmetries of the Lagrangian and figure out the conserved quantities/Noether charges. $\boxed{8}$

(c) *(Symmetric Gauge)* Consider $\vec{A} = \vec{A}(x,y)$. Find a form of $\vec{A}$. What are the symmetries of the Lagrangian and figure out the conserved quantities/Noether charges. $\boxed{8}$

(d) Comment on whether the symmetries and conserved quantities should depend on the choice of the Gauge. $\boxed{1}$