Classical Mechanics: Assignment #1

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Problem 1

Part (a)

$$V(x) = \alpha x^2/2 + \beta x^4/4$$
$$F(x) = -\frac{\partial V}{\partial x} = -\alpha x - \beta x^3$$

Including the damping term, we write the equation of motion as,

$$m\ddot{x} + \delta\dot{x} = -\alpha x - \beta x^3$$
$$m\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = 0$$

The total energy of the system $E = T + V = m\dot{x}^2/2 + \alpha x^2/2 + \beta x^4/4$. Taking the time derivative, one gets

$$\dot{E} = m\ddot{x}\dot{x} + \alpha x\dot{x} + \beta x^3\dot{x}$$

Substituting from the equation of motion for $m\ddot{x}$,

$$\dot{E} = -(\delta\dot{x} + \alpha x + \beta x^3)\dot{x} + \alpha x\dot{x} + \beta x^3\dot{x}$$
$$\dot{E} = -\delta\dot{x}^2$$

Hence energy is dissipated from the system at a rate $\delta\dot{x}^2$.

Problem 2

Give an appropriate positive constant c such that $f(n) \leq c \cdot g(n)$ for all $n > 1$.

1. $f(n) = n^2 + n + 1$, $g(n) = 2n^3$
2. $f(n) = n\sqrt{n} + n^2$, $g(n) = n^2$
3. $f(n) = n^2 - n + 1$, $g(n) = n^2/2$

Solution
We solve each solution algebraically to determine a possible constant c.

Part One

$$\begin{aligned} n^2 + n + 1 = & \\ & \leq n^2 + n^2 + n^2 \\ & = 3n^2 \\ & \leq c \cdot 2n^3 \end{aligned}$$

Thus a valid c could be when $c = 2$.

Part Two

$$\begin{aligned}
n^2 + n\sqrt{n} = & \\
&= n^2 + n^{3/2} \\
&\leq n^2 + n^{4/2} \\
&= n^2 + n^2 \\
&= 2n^2 \\
&\leq c \cdot n^2
\end{aligned}$$

Thus a valid c is $c = 2$.

Part Three

$$\begin{aligned}
n^2 - n + 1 = & \\
&\leq n^2 \\
&\leq c \cdot n^2/2
\end{aligned}$$

Thus a valid c is $c = 2$.