

# Trefethen and Bau: Lecture #2

August 24, 2018

Aditya Vijaykumar

## Problem 1

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}^B \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A is just the product of all matrices to the left of B and C is just the product of all matrices to the right of B.

## Problem 2

Part (a)

$$\begin{aligned} f_1 &= k_{12}(-l_{12} - x_1 + x_2) \\ f_2 &= k_{23}(-l_{32} - x_2 + x_3) - k_{12}(-l_{12} - x_1 + x_2) \\ f_3 &= k_{34}(-l_{34} - x_3 + x_4) - k_{23}(-l_{32} - x_2 + x_3) \\ f_4 &= -k_{34}(-l_{34} - x_3 + x_4) \end{aligned}$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} -k_{12} & k_{12} & 0 & 0 \\ k_{12} & -(k_{12} + k_{23}) & k_{23} & 0 \\ 0 & k_{23} & -(k_{23} + k_{34}) & k_{34} \\ 0 & 0 & k_{34} & -k_{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + (\text{constant matrix})$$

Part (b) The dimensions of K will be that of  $\frac{f}{x}$ , ie.  $\frac{kg}{s^2}$ .

Part (c) The dimensions of  $\det(K)$  will be  $(\frac{kg}{s^2})^4$ .

Part (d)

$$\begin{aligned} 1 \frac{kg}{s^2} &= 1000 \frac{g}{s^2} \\ 1 \left(\frac{kg}{s^2}\right)^4 &= 10^{12} \frac{g}{s^2} \\ \det(K) &= 10^{12} \det(K') \end{aligned}$$

### Problem 3

Let the identity  $I = [e_1, e_2, \dots, e_m]$ ,  $R^{-1} = [a_1, a_2, \dots, a_n]$ . Let us write the expression  $I = R^{-1}R$  as follows,

$$[e_1, e_2, \dots, e_m] = [a_1, a_2, \dots, a_n] \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1m} \\ r_{21} & r_{22} & \dots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{1m} & r_{2m} & \dots & r_{mm} \end{bmatrix}$$