Trefethen and Bau: Lecture #6

September 17, 2018

Aditya Vijaykumar

Problem 1

As P is orthogonal, $P^* = P$ and $P^2 = P$. Consider $I - 2P$,

$$\begin{aligned}(I-2P)^*(I-2P) &= I - 2P - 2P^* + 4P^*P \\ &= I - 2P - 2P + 4P \\ &= I\end{aligned}$$

Hence, $I - 2P$ is unitary.

Problem 2

Given, $E = \frac{1}{2}(1 + F)$, where F just reverses the sequence of elements of the vector. F is hence just the anti-diagonal matrix of shape $m \times m$ with anti-diagonal elements 1. Also, $F^* = F \implies E^* = E \implies$ *orthogonality*. If m is odd, then E has all diagonal and anti-diagonal elements equal to $\frac{1}{2}$, except the middle element which will be equal to 1. If m is even, then E has all diagonal and anti-diagonal elements equal to $\frac{1}{2}$. Two examples are given below.

Problem 3