Dynamical Systems: Homework #3

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Problem 1

Given $\overline{\lim_{t\to\infty}} f(t) = a \neq \pm\infty$, we can think of two broad cases :-

Case I - $\lim_{t\to\infty} f(t)$ exists $\implies \lim_{t\to\infty} f(t) = a$.

(a) $\lim_{t\to\infty} [f(t) - a - \epsilon] = -\epsilon$

(b) $\overline{\lim_{t\to\infty}} [f(t) - a - \epsilon] = -\epsilon$

(c) $\lim_{t\to\infty} [f(t) - a + \epsilon] = \epsilon$

(d) $\overline{\lim_{t\to\infty}} [f(t) - a + \epsilon] = \epsilon$

Case II - $\lim_{t\to\infty} f(t)$ does not exist $\implies f(t) \leq a$ as $t \to \infty$.

(a) $\lim_{t\to\infty} [f(t) - a - \epsilon]$ does not exist.

(b) $\overline{\lim_{t\to\infty}} [f(t) - a - \epsilon] = -\epsilon$

(c) $\lim_{t\to\infty} [f(t) - a + \epsilon]$ does not exist

(d) $\overline{\lim_{t\to\infty}} [f(t) - a + \epsilon] = \epsilon$

Problem 2

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Problem 3

The most general quadratic time-independent Hamiltonian for a system with n degrees of freedom is,

$$H = \frac{1}{2} \sum_{i=1,j=1}^{n} A_{ij} q_i q_j + B_{ij} p_i p_j + C_{ij} q_i p_j$$

where $A_{ij} = A_{ji}, B_{ij} = B_{ji}, C_{ij} = C_{ji}$, and q_i and p_i are the generalized positions and momenta respectively. Using Hamilton's equations of motion, one can write,

$$\dot{q}_i = \sum_{j=1}^{n} B_{ij} p_j + C_{ij} q_j (2 - \delta_{ij}) \quad \text{and} \quad \dot{p}_i = -\sum_{j=1}^{n} A_{ij} q_j - C_{ij} p_j (2 - \delta_{ij})$$