Advanced Quantum Mechanics
Assignment 5

Due Tuesday, 13 November 2018

Problems:

1. Consider a simple harmonic oscillator in its ground state. At $t = 0$, we apply a perturbation
$$V(x,t) = \lambda x^2 e^{\frac{-t}{\tau}}$$
where A, τ are constants.

   (a) If we ask for the probability that the system will be in a given excited state $|n\rangle$ with n units of energy then how does this probability vary as a *power* of λ. More precisely, this probability can be written as $A_n \lambda^{f(n)}$ where A_n is some constant. The question is to find $f(n)$.

   (b) What excited states can be reached if we consider terms up to $\mathrm{O}(\lambda^2)$. Calculate these probabilities up to $\mathrm{O}(\lambda^2)$. (For this subpart, you need to keep track of all numerical factors!)

2. Consider a system of two spin-1/2 particles. Initially, all states are degenerate, so $H = 0$. For $t > 0$ the Hamiltonian suddenly changes to
$$H = \lambda S_1 \cdot S_2$$
If the system is initially in $|+-\rangle$ find the probability for it to be in any of the four states $|+-\rangle, |++\rangle, |-+\rangle, |--\rangle$ as a function of time.

3. Consider a particle moving in one dimension, under the influence of some given potential. Assume that the particle is initially in its ground state, with wave-function, $u_i(x)$. Then, at time $t = 0$, the particle is subject to a perturbation given by
$$V(x,t) = \lambda\delta(x - vt) \tag{1}$$
Find the probability that, at very late times, the system will transition to another energy eigenstate state, with wave-function, $u_f(x)$. Relate this result to the results on periodic perturbations that we derived in class by writing the delta function as
$$\delta(x - vt) = \frac{1}{2\pi v}\int e^{i\omega(x/v-t)}$$

4. Consider a tritium atom, with one electron and a nucleus that has one proton and two neutrons. Imagine that the electron is initially in its ground state. Now imagine that as a result of a nuclear reaction, one of the neutrons gets converted to a proton so that we now have a ^{3}He nucleus with electric charge $+2$. What is the probability that the electron will be in the ground state of this new configuration.

5. In class, we considered the two-state system with unperturbed energy eigenvalues given by $0, E$ with $E > 0$. We denote these states by $|1\rangle, |2\rangle$ respectively. Now, we add a time-dependent potential with matrix elements

$$V_{11} = V_{22} = 0. \quad V_{12} = \gamma e^{i\omega t} \quad V_{21} = \gamma e^{-i\omega t}.$$

Here, γ, ω have to be real by Hermiticity. At $t = 0$, we assume that the system is in the state $|1\rangle$.

(a) Find the *wave-function* of the system after time t by writing down a linear differential equation for the components of the wave-function and solving this differential equation. (We found the probabilities for the system to be in various states in class; here you also need to keep track of phases.)

(b) Second, verify the answer above using time-dependent perturbation theory. (We worked out the leading order answer in class; here you need to work to all orders.)