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# Notes on Gravity as a Quantum Theory

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## Contents

### 1 Classical Fields

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1

## Contents

### 1 Classical Fields

$\phi(\vec{x}, t)$  gives the value of the classical field at every point in spacetime. The simplest classical field is the *real scalar field*, which is characterized only by real numbers. The Klein-Gordon equation governs a free massive scalar field.

$$\frac{\partial^2 \phi}{\partial t^2} - \sum_{x_j} \frac{\partial^2 \phi}{\partial x_j^2} + m^2 \phi = 0$$

An interesting part about the free scalar field is that one can describe it as an infinite set of decoupled harmonic oscillators. Put this field into a box of length  $L$  and volume  $V = L^3$ , and having periodic boundary conditions. One can Fourier decompose this as,

$$\phi(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \phi_{\vec{k}}(t) \exp(i\vec{k} \cdot \vec{x}) \text{ where } k_x = \frac{2\pi n_x}{L}, \dots$$

Substituting this into the first equation, we find that the harmonic oscillators get nicely decoupled into an infinite set of ODEs of the form,

$$\ddot{\phi}_{\vec{k}} + (k^2 + m^2)\phi_{\vec{k}} = 0$$

which is basically the harmonic oscillator equation with frequency  $\omega_k = \sqrt{k^2 + m^2}$ . The energy of oscillators is simply equal to the sum of individual energies of the oscillators,

$$E = \sum_{\vec{k}} \left[ \frac{1}{2} \dot{\phi}_{\vec{k}}^2 + \frac{1}{2} \omega_k^2 \phi_{\vec{k}}^2 \right]$$