Quantum Mechanics II
Assignment 3

Due Thursday, 4 October 2018

Problems:

1. Consider a spin-2 particle. Write down the 5×5 matrices that represent J_1, J_2, J_3.

2. Consider the sequence of rotations of a spin-1/2 particle given by

$$U = \exp\{-i\frac{\sigma_3\alpha}{2}\}\exp\{-i\frac{\sigma_2\beta}{2}\}\exp\{-i\frac{\sigma_3\gamma}{2}\}$$

   We expect that U can be represented as a single rotation by an angle θ about some axis. What is θ?

3. Consider a state in the representation with highest weight l and J_3 eigenvalue m. Prove that in this state

$$\langle J_1\rangle = \langle J_2\rangle = 0; \quad \langle J_1^2\rangle = \langle J_2^2\rangle = \frac{l(l+1)-m^2}{2}.$$

4. Consider the representation with highest weight 1. Write J_2 as a 3×3 matrix in this representation. Show that

$$e^{-iJ_2\beta} = 1 - iJ_2\sin\beta - J_2^2(1-\cos\beta)$$

   in this representation.

5. Consider two representations with highest weight 1. Explicitly reduce the tensor product of these representations into irreducible representations by writing the states in the irreducible representations as linear combinations of states in the original representations. This procedure is equivalent to finding all the Clebsch Gordon coefficients, and you can use the "sieve" procedure outlined in the class or any other procedure.