# Classical Mechanics - Assignment 1

# Due date: August 31, 2018

Instructor: Manas Kurkarni (H-204) - `manas.kulkarni@icts.res.in`
TA: Avijit Das (H-107) - `avijit.das@icts.res.in` and
Anugu Sumith Reddy (H-104) - `anugu.reddy@icts.res.in`

August 20, 2018

***Note:*** Submit the assignment to any one of TA's office on/before the due date. For the numerical parts of the questions take print outs of the codes along with the plots, etc. and attach it in the correct place of your solution. If you have any doubt regarding the assignment problems or the topics discussed in the class, feel free to discuss with TAs or the instructor. For numerical parts use any of your favourite programming language and plotting software. Good luck!

**Q1** $\boxed{\text{60 marks}}$

### Duffing Oscillator

(a) Consider the 1d potential $V(x) = \alpha x^2/2 + \beta x^4/4$. Write down the equation of motion introducing a damping term $\delta \dot{x}$. What is the dissipation rate of total mechanical energy $E = T + V$? $\boxed{3}$

(b) Take $\delta = 0$. Plot the potential for a range of values of $\alpha$ and $\beta$ parameters. Describe the motion qualitatively using energy arguments, etc. Different parameter ranges will give different kinds of motion. Note down a set of interesting parameter values which give qualitatively different motion. $\boxed{5}$

(c) Take $\delta = 0$. Energy is one integral of motion. Does that information give you some closed curve in the phase plane $x - \dot{x}$ depending on the value of the energy? Plot those constant energy curves for a set of parameters chosen in the previous part. Solve the equation of motion numerically for each parameter values and plot in the phase plane. Can you see energy dissipation due to numerical artifact? $\boxed{10}$

(d) Take $\delta = 0$. This potential is an approximation of the simple pendulum potential, namely $V(\theta) = 1 - \cos\theta$ with $\theta \in (-\pi, \pi]$. Derive analytical expressions of the time period for both cases. (*Hint*: both will involve elliptic functions.) $\boxed{10}$

(e) Take $\delta$ non-zero. Now energy is no more conserved. Numerically solve the equation of motion for the set of parameters chosen in part-2 and choosing some initial condition do the phase plot. $\boxed{10}$

(f) Simplest driven-dissipative system, i.e. a damped simple harmonic oscillator with a forcing term given by $\gamma\cos(\omega t)$ (Can you think of a system which is even simpler than this?). Discuss transients and steady state. The steady state solution will be $x = A\cos(\omega t + \phi)$. Substitute this solution into the ODE to get expressions of $A$ and $\phi$. Plot these as function of driving frequency. These are response characteristics. $\boxed{7}$

(g) Now let's add some periodic forcing to the dissipative Duffing oscillator. Can you derive an expression(a series expansion maybe) of the amplitude in steady state using similar method as before? $\boxed{5}$

(h) Using the knowledge from previous parts choose the parameters and initial condition such that there is a bounded motion in phase plane. Now choose some $\gamma$ and $\omega$. Numerically solve the equation and do the phase plot up to some time for the chosen initial condition. Now plot the $(x, \dot{x})$ only at times $t = t_0 + nT$ $(n = 0, 1, 2, ..., N)$ where $T = 2\pi/\omega$ and $t_0 \in [0, T)$ by choosing some large $N$ and a few $t_0$'s. This plot is called the Poincare section and the map $P : (x(t_0 + nT), \dot{x}(t_0 + nT)) \to (x(t_0 + (n+1)T), \dot{x}(t_0 + (n+1)T))$ is called the Poincare map. $\boxed{10}$

**Q2** $\boxed{\text{40 marks}}$

## Triple Pendulum (Fig-1)

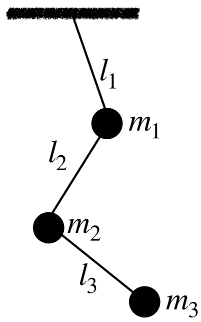


Figure 1: Triple pendulum

(a) Identify the generalized coordinates and write down the Lagrangian of a triple pendulum (see Figure). Connecting rods are rigid. Derive the equation of motion for each generalized coordinate. Choosing some initial condition, mass and length of connecting rods numerically solve the equations and make phase diagrams ($q - \dot{q}$ plane) for each generalized coordinate. $\boxed{20}$

(b) Repeat the last part using small angle approximation, i.e. using $\sin(\theta) \sim \theta$ and $\cos\theta \sim 1 - \theta^2/2$. $\boxed{20}$

**Q3** $\boxed{\text{10 marks}}$

**Earth - Satellite system** Consider a satellite of mass $m$, revolving around Earth of mass $M$ in an orbit of semi-major length R and eccentricity $e$. Earth's motion can be ignored.

(a) Calculate the total energy of the system in the above mentioned situation. $\boxed{2}$

(b) If the satellite can generate some impulse to change orbit, give qualitatively the mechanism to change the orbit to a new orbit (with same major axis) of semi major length 2R and eccentricity 0 (circle). $\boxed{3}$

(c) Calculate the total impulse needed to shift to the new orbit. $\boxed{3}$

(d) Calculate the angular momentum of the system in the new configuration. $\boxed{2}$

**Q4** $\boxed{\text{10 marks}}$

**Van der Pol Oscillator** Consider the differential equation, $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$, $\mu \geq 0$

(a) Qualitatively, give a reason why $|x(t)| + |\dot{x}(t)|$ is bounded in time for any initial condition. $\boxed{2}$

(b) Numerically generate the phase-space (x$-\dot{x}$) diagram of the system for different values of $\mu \in \{0, 1, 1.5, 2\}$. For each value of $\mu$ take about 50 random initial conditions. For e.g., choose uniformly between $[-10, 10]$ for x(0) and $\dot{x}(0)$. $\boxed{4}$

(c) If $\mu < 0$, will $|x(t)| + |\dot{x}(t)|$ be bounded for all initial conditions? Explain. $\boxed{2}$

(d) Repeat part (b) for $\mu \in \{-2, -1.5, -1, -0.5, -0.0001\}$ $\boxed{2}$