Classical Mechanics: Assignment #1

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Aditya Vijaykumar

Problem 1

Part (a)

$$V(x) = \alpha x^2/2 + \beta x^4/4$$
$$F(x) = -\frac{\partial V}{\partial x} = -\alpha x - \beta x^3$$

Including the damping term, we write the equation of motion as,

$$m\ddot{x} + \delta\dot{x} = -\alpha x - \beta x^3$$

$$m\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = 0$$

The total energy of the system $E = T + V = m\dot{x}^2/2 + \alpha x^2/2 + \beta x^4/4$. Taking the time derivative, one gets

$$\dot{E} = m\ddot{x}\dot{x} + \alpha x\dot{x} + \beta x^3\dot{x}$$

Substituting from the equation of motion for $m\ddot{x}$,

$$\dot{E} = -(\delta\dot{x} + \alpha x + \beta x^3)\dot{x} + \alpha x\dot{x} + \beta x^3\dot{x}$$

$$\dot{E} = -\delta\dot{x}^2$$

Hence energy is dissipated from the system at a rate $\delta\dot{x}^2$.

Part (b) and (c) - *code files attached*

Part (d)

$$E = m\dot{x}^2/2 + \alpha x^2/2 + \beta x^4/4$$

The turning points will satisfy $E = \alpha x^2/2 + \beta x^4/4$

Part (f)

Substituting $x = A\cos(\omega t + \phi)$ in $m\ddot{x} + \delta\dot{x} + \alpha x = \gamma\cos(\omega t)$

$$(-m\omega^2 + \alpha)\cos(\omega t + \phi) - \delta\omega\sin(\omega t + \phi) = \frac{\gamma}{A}\cos(\omega t)$$

Expanding the RHS and equating coefficients of $\cos(\omega t)$ and $\sin(\omega t)$, one gets

$$(-m\omega^2 + \alpha)\cos(\phi) - \delta\omega\sin(\phi) = \frac{\gamma}{A}$$

$$-(-m\omega^2 + \alpha)\sin(\phi) - \delta\omega\cos(\phi) = 0$$

Solving these, we get

$$A = \frac{\gamma}{(-m\omega^2 + \alpha)\cos(\phi) - \delta\omega\sin(\phi)} \; ; \; \phi = \tan^{-1}\left(\frac{\delta\omega}{\alpha - m\omega^2}\right)$$

[PLOT THESE]

Part (g)
Substituting $x = \sum_{n=0}^{\infty} a_n x^n$ in $m\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma\cos(\omega t)$