Quantum Mechanics II
Assignment 4

Due Thursday, 8 November 2018

Problems:

1. In non-degenerate time-independent perturbation theory, find the probability of finding the unperturbed eigenstate, $|E_0\rangle$, in the perturbed eigenstate $|E\rangle$ up to *second order* in the coupling constant.

2. Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by
$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + m\frac{\omega^2}{2}(x^2 + y^2)$$
   (a) What are the energies of the three lowest-lying states?
   (b) Now we apply a perturbation, $V = \lambda m\omega^2 xy$. Calculate the zeroth-order energy eigenstates and the first-order energy shift for all of the three lowest states.
   (c) Solve for the three *exact* lowest eigenvalues of $H = H_0 + V$. Compare this with the answer you obtained in perturbation theory above.

3. Consider the three states of a hydrogen atom with quantum numbers $n = 2, l = 1$ and $m = -1, 0, 1$. We now apply a potential $V = \lambda(x^2 - y^2)$. Find the correct zeroth-order energy eigenstates and the first order energy shifts for all these three states.

4. In a three-dimensional Hilbert space, consider the Hamiltonian
$$H = \begin{pmatrix} E_1 & 0 & \lambda a \\ 0 & E_1 & \lambda b \\ \lambda a^* & \lambda b^* & E_2 \end{pmatrix}$$
Here, $\lambda \ll 1$. In perturbation theory, calculate the perturbed eigenvalues to *second order* in λ. (Note that the degeneracy between the energy levels is *not* removed at first order!) Then diagonalize the matrix to find the exact eigenvalues. Compare the two results obtained.

5. Suppose the Hamiltonian of a system is
$$H = A(L_x^2 + L_y^2 + L_z^2) + BL_z + \lambda CL_y,$$
where L_i are the components of the angular momentum operator. Find the energy eigenvalues for this system to lowest order in λ.

6. Estimate the ground state energy of a one-dimensional simple harmonic oscillator using $\psi_\beta(x) = e^{-\beta|x|}$ as a trial wave-function and by varying β to find the minimum possible energy.