Advanced Quantum Mechanics: Assignment #2

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Aditya Vijaykumar

Problem 1

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Problem 2

We know that the energy eigenstates of the harmonic oscillator form a basis. Hence, the required coherent state $|z\rangle$ can be written in terms of these eigenstates as,

$$|z\rangle = \sum_{n=0}^{\infty} c_n |n\rangle = \sum_{n=0}^{\infty} c_n \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle$$

Substituting this in the equation for coherent state $a|z\rangle = z|z\rangle$,

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n a |n\rangle &= \sum_{n=0}^{\infty} z c_n |n\rangle \\
\sum_{n=1}^{\infty} c_n \sqrt{n} |n-1\rangle &= \sum_{n=0}^{\infty} z c_n |n\rangle \\
\sum_{n=0}^{\infty} c_{n+1} \sqrt{n+1} |n\rangle &= \sum_{n=0}^{\infty} z c_n |n\rangle \\
\therefore c_{n+1}\sqrt{n+1} &= z c_n
\end{aligned}$$

We have effectively derived a recursion relation for the coefficients c_n. If we start off with $c_n = \alpha$,

$$c_1 = z\alpha \quad , \quad c_2 = \frac{z^2\alpha}{\sqrt{2}} \quad , \quad c_3 = \frac{z^3\alpha}{\sqrt{3\cdot 2}} \quad , \quad \dots \quad , \quad c_n = \frac{z^n\alpha}{\sqrt{n!}}$$

So, our coherent state can now be written as,

$$\begin{aligned}
|z\rangle &= \alpha \sum_{n=0}^{\infty} \frac{(za^\dagger)^n}{n!} |0\rangle \\
&= \alpha e^{a^\dagger z} |0\rangle
\end{aligned}$$

Problem 3

We know that,

$$x(0) = \frac{a + a^\dagger}{\sqrt{2m\omega}} \quad , \quad p(0) = \frac{\sqrt{m\omega}(a - a^\dagger)}{\sqrt{2}i} \quad , \quad x(t) = e^{iHt}x(0)e^{-iHt} \quad , \quad p(t) = e^{iHt}p(0)e^{-iHt}$$

From this, we note the following,

$$
\begin{aligned}
x(t)\left|0\right\rangle &= e^{iHt}x(0)e^{-iHt}\left|0\right\rangle \\
&= e^{-i\omega t/2}e^{iHt}x(0)\left|0\right\rangle \\
&= \frac{e^{-i\omega t/2}}{\sqrt{2m\omega}}e^{iHt}\left|1\right\rangle \\
x(t)\left|0\right\rangle &= \frac{e^{i\omega t}}{\sqrt{2m\omega}}\left|1\right\rangle \implies \left\langle 0\right| x(t) = \frac{e^{-i\omega t}}{\sqrt{2m\omega}}\left\langle 1\right|
\end{aligned}
$$

$$
\begin{aligned}
\text{Similarly,}\quad p(t)\left|0\right\rangle &= e^{iHt}p(0)e^{-iHt}\left|0\right\rangle \\
&= e^{-i\omega t/2}e^{iHt}p(0)\left|0\right\rangle \\
&= -\frac{e^{-i\omega t/2}\sqrt{m\omega}}{\sqrt{2}i}e^{iHt}\left|1\right\rangle \\
p(t)\left|0\right\rangle &= -\frac{e^{i\omega t}\sqrt{m\omega}}{\sqrt{2}i}\left|1\right\rangle \implies \left\langle 0\right| p(t) = \frac{e^{-i\omega t}\sqrt{m\omega}}{\sqrt{2}i}\left\langle 1\right|
\end{aligned}
$$

Now consider the quantities to be calculated,

$$
C_1(t) = \langle 0|x(t)x(0)|0\rangle = \frac{e^{-i\omega t}}{\sqrt{2m\omega}}\frac{1}{\sqrt{2m\omega}} = \boxed{\frac{e^{-i\omega t}}{2m\omega}}
$$

$$
C_2(t) = \langle 0|x(t)p(0)|0\rangle - \langle 0|p(0)x(t)|0\rangle = -\frac{e^{-i\omega t}}{2i} + \frac{e^{i\omega t}}{2i} = \boxed{\sin\omega t}
$$

$$
C_3(t) = \langle 0|p(t)x(0)|0\rangle - \langle 0|x(0)p(t)|0\rangle = \frac{e^{-i\omega t}}{2i} - \frac{e^{i\omega t}}{2i} = \boxed{-\sin\omega t}
$$

Problem 4

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