Trefethen and Bau: Lecture #1

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Problem 1

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathrm{B} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A is just the product of all matrices to the left of B and C is just the product of all matrices to the right of B.

Problem 2

Part (a)

$$\begin{aligned} f_1 &= k_{12}\left(-l_{12} - x_1 + x_2\right) \\ f_2 &= k_{23}\left(-l_{32} - x_2 + x_3\right) - k_{12}\left(-l_{12} - x_1 + x_2\right) \\ f_3 &= k_{34}\left(-l_{34} - x_3 + x_4\right) - k_{23}\left(-l_{32} - x_2 + x_3\right) \\ f_4 &= -k_{34}\left(-l_{34} - x_3 + x_4\right) \end{aligned}$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} -k_{12} & k_{12} & 0 & 0 \\ k_{12} & -(k_{12}+k_{23}) & k_{23} & 0 \\ 0 & k_{23} & -(k_{23}+k_{34}) & k_{34} \\ 0 & 0 & k_{34} & -k_{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + (constant\ matrix)$$

Part (b) The dimensions of K will be that of $\frac{f}{x}$, ie. $\frac{kg}{s^2}$.

Part (c) The dimensions of $\det(K)$ will be $(\frac{kg}{s^2})^4$.

Part (d)

$$1\frac{kg}{s^2} = 1000\frac{g}{s^2}$$

$$1(\frac{kg}{s^2})^4 = 10^{12}\frac{g}{s^2}$$

$$\det(K) = 10^{12}\det(K')$$

Problem 3

Let the identity $I = [e_1, e_2, \ldots, e_m]$, $R^{-1} = [a_1, a_2, \ldots, a_n]$. Let us write the expression $I = R^{-1}R$ as follows,

$$[e_1, e_2, \ldots, e_m] = [a_1, a_2, \ldots, a_n] \begin{bmatrix} r_{11} & r_{12} & \ldots & r_{1m} \\ r_{21} & r_{22} & \ldots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{1m} & r_{2m} & \ldots & r_{mm} \end{bmatrix}$$