# Classical Mechanics - Assignment 4
# (Midterm Preparatory)
# Due date: Oct 5, 2018

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***Note:*** Submit the assignment to any one of TA's office on/before the due date. For the numerical parts of the questions take print outs of the codes along with the plots, etc. and attach it in the correct place of your solution. If you have any doubt regarding the assignment problems or the topics discussed in the class, feel free to discuss with TAs or the instructor. For numerical parts use any of your favourite programming language and plotting software. Good luck!

**Q1** $\boxed{\text{10 marks}}$

**System of Particles**

(a) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy: $\boxed{5}$

$$\frac{dT}{dt} = \vec{F} \cdot \vec{v}, \tag{1}$$

while if the mass varies with time the corresponding equation is

$$\frac{d(mT)}{dt} = \vec{F} \cdot \vec{p}, \tag{2}$$

(b) Suppose a system of two particles is known to obey the equation of motion, $M\frac{d^2\vec{R}}{dt^2} = \vec{F}^{ext}$ and $\frac{d\vec{L}}{dt} = \vec{\tau}^{ext}$. From the equations of motion of the individual particles show that the internal forces between particles satisfy both the weak law and strong laws of action and reaction. The argument may be generalized to a system with arbitrary number of particles, thus proving the converse of these relations. $\boxed{5}$

**Q2** $\boxed{\text{5 marks}}$

**Constraints**

Write down the equations of constraint for a rolling disc. Show that it's a special case of general linear differential equations of constraint of the form

$$\sum_{i=1}^{n} g_i(x_1, ..., x_n) dx_i = 0. \tag{3}$$

A constraint of this type is holonomic only if an integrating function $f(x_1, ..., x_n)$ can be found that turns it into an exact differential. Clearly the function must be such that $\frac{\partial (f g_i)}{\partial x_j} =$

$\frac{\partial(fg_j)}{\partial X_i}$ $\forall i \neq j$. Show that no such integrating factor can be found for the equations for the rolling disc.

**Q3** $\boxed{\text{15 marks}}$

**Generalized coordinates & Lagrangian**

(a) Consider a pendulum whose link is not rigid but a flexible spring. Besides that it's not constrained to move in a plane. We neglect the bending of the spring (you can include that too if you are keen to). figure out the degrees of freedom and the generalized coordinates. Write down the Lagrangian and derive the equations of motion (you can solve those too if you are interested). Now constrain the pendulum in a plane (do you think that can be achieved easily in an experiment? Try with a simple pendulum), and solve the resulting equations of motion for small stretching and angular displacements. $\boxed{10}$

(b) Consider the Lagrangian $\boxed{5}$

$$L = e^{\gamma t}\left(\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\right). \tag{4}$$

Derive the equations of motion. Are you surprised? How would you describe the system? Are there any constants of motion? Perform a transformation $s = e^{\gamma t}q$. What is the transformed Lagrangian and the equation of motion? What do these results say about the conserved quantities?

**Q4** $\boxed{\text{15 marks}}$

**Variational principle**

(a) Suppose it is known experimentally that a particle fell a given distance $y_0$ in a time $t_0 = \sqrt{2y_0/g}$, but the times of fall for distance other than $y_0$ is not known. Let's guess that the $y$ takes the form $y = at + bt^2$. Show *directly* that the integral $\int_0^{t_0} Ldt$ is an extremum for real values of $a, b$ when $a = 0$ and $b = g/2$. $\boxed{5}$

(b) Derive an Euler-Lagrange equation for $L = L(q_i, \dot{q}_i, \ddot{q}_i, t)$, given that the Hamilton's principle holds with zero variation of both $q_i$ and $\dot{q}_i$ at the end points. $i = 1, 2, ..., n$. Apply your result to the Lagrangian $L = -\frac{m}{2}q\ddot{q} - \frac{k}{2}q^2$. Do you recognize the equations of motion? $\boxed{10}$

**Q5** $\boxed{\text{15 marks}}$

**Symmetry**

A particle moves without friction in a conserved field of force produced by various mass distributions. In each instance, the force generated by a volume element of the distribution is derived from a potential that is proportional to the mass of the volume element and is a function only of the scalar distance from the volume element. For the following fixed, homogeneous mass distributions, state the conserved quantities in the motion of the particle:

(a) The mass is uniformly distributed in the plane $z = 0$. $\boxed{2}$

(b) The mass is uniformly distributed in the half-plane $z = 0, y > 0$. $\boxed{2}$

(c) The mass is uniformly distributed in a circular cylinder of *infinite* length, with axis along the $z$ axis. $\boxed{2}$

(d) The mass is uniformly distributed in a circular cylinder of *finite* length, with axis along the $z$ axis. $\boxed{2}$

(e) The mass is uniformly distributed in a right cylinder of elliptical cross section and *infinite* length, with axis along the $z$ axis. $\boxed{2}$

(f) The mass is uniformly distributed in a dumbbell whose axis is oriented along the $z$ axis. $\boxed{2}$

(g) The mass is in the form of a uniform wire wound in the geometry of an infinite helical solenoid, with axis is oriented along the $z$ axis. $\boxed{3}$

**Q6** $\boxed{\text{10 marks}}$

**Central Force** A particle moves in a potential, $V(r) = -V_0 e^{-\lambda^2 r^2}$

(a) Given angular momentum $L$, find the radius of stable circular orbit. An implicit equation is fine. $\boxed{5}$

(b) If it turns out that $L$ is too large, then no circular exists. What is largest value of $L$, for which a circular orbit does in fact exist? $\boxed{5}$

**Q7** $\boxed{\text{15 marks}}$

**Small Oscillations** A particle slides on the inside surface of frictionless cone (See figure

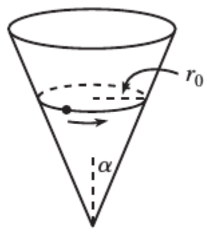


Figure 1:

above). The half angle at the tip is $\alpha$. Let $r$ be the distance from the particle to the axis and let $\theta$ be the angle around the cone.

(a) Find the equations of motion. $\boxed{2}$

(b) If the particle moves in a circle of radius $r_0$, what is the frequency, $\omega$, of this motion? $\boxed{5}$

(c) If the particle is perturbed slightly from this circular motion,What is frequency, $\Omega$, of the oscillations about the radius $r_0$? Under what conditions does $\Omega = \omega$? $\boxed{8}$

**Q8** $\boxed{\text{10 marks}}$

**Small Oscillations**

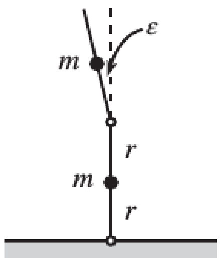


Figure 2:

Two very light sticks of length $2l$, each with a point mass $m$ fixed at its middle, are hinged at an end. One stands on top of the other (see above). The bottom end of the lower stick is hinged at the ground. They are held such that the lower stick is vertical, and the upper one is tilted at a very small angle $\epsilon$ with respect to the vertical. They are then released. At this instant, what are the angular accelerations of the two sticks?