Fluid Mechanics: Test #2

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Aditya Vijaykumar

Problem 1

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Problem 2

We assume homogenous, isotropic turbulence. These assumptions tell us that the second-order structure function S_2 calculated between two spatial points should depend *only* on the distance between the two points. More mathematically,

$$\left\langle \left(\vec{\mathbf{u}}(\vec{\mathbf{r}}+\vec{\mathbf{l}}) - \vec{\mathbf{u}}(\vec{\mathbf{r}})\right)^2 \right\rangle = S_2\left(\left|\vec{\mathbf{l}}\right|\right) = S_2(l)$$

Further, we define the *energy spectrum* $E(k)$ such that $E(k)dk$ gives the mean kinetic energy contained within k and $k + dk$. It follows from the definition that,

$$\int_0^\infty E(k)dk = \frac{1}{2}\left\langle u^2 \right\rangle \tag{1}$$

The *Wiener-Khinchin* theorem tells us that energy spectrum is the Fourier Transform of the spatial auto-correlation function,

$$E(k) \sim \int_0^\infty e^{ikl} S_2(l) \implies S_2(l) \sim \int_0^\infty e^{ikl} E(k)dk \tag{2}$$

Now we try to guess the scaling form of $E(k)$ as $E(k) \sim k^{-n}$. Substituting this ansatz into (1),

$$\begin{aligned}\int_0^\infty E(k)dk &\sim \int_0^\infty k^{-n}dk \\ &\sim \left.\frac{k^{1-n}}{1-n}\right|_\infty\end{aligned}$$

As the RHS of (1) is finite, it follows from above that $n > 1$.

Problem 3