① Let a_n be a convergent sequence of real numbers with limit L. Let b_n be a sequence of real numbers converging to 1 & ϵ_n a sequence converging to zero.

Show $a_n b_n + \epsilon_n$ is a sequence that also converges to L.

② Let a_n be a sequence of real numbers with limit L.

Let $M > 0$ a fixed real number

& let S be the interval $[L-M, L+M]$

Let f_n be a sequence of functions $f_n : \mathbb{R} \longrightarrow \mathbb{R}$

domain of f_n is any real number & range of f_n is a real number

Further suppose that $\lim\limits_{n\to\infty} \left(\sup\limits_{x \in S} | f_n(x) - x | \right) = 0$

a) Show that $f_n(a_n) \longrightarrow L$ as $n \to \infty$.

b) Is the "sup" necessary in the lim condition for f_n?

What if $\lim\limits_{n\to\infty} | f_n(x) - x | = 0$ for each $x \in S$?