Prepared for submission to JHEP

# Notes on Gravity as a Quantum Theory

**Aditya Vijaykumar**

*International Centre for Theoretical Sciences, Bengaluru, India.*

*E-mail:* aditya.vijaykumar@icts.res.in

## Contents

1 Classical Fields 1

## Contents

## 1 Classical Fields

$\phi(\vec{\mathbf{x}}, t)$ gives the value of the classical field at every point in spacetime. The simplest classical field is the *real scalar field*, which is characterized only by real numbers. The Klein-Gordon equation governs a free massive scalar field.

$$\frac{\partial^2 \phi}{\partial t^2} - \sum_{x_j} \frac{\partial^2 \phi}{\partial x_j^2} + m^2 \phi = 0$$

An interesting part about the free scalar field is that one can describe it as an infinite set of decoupled harmonic oscillators. Put this field into a box of length $L$ and volume $V = L^3$, and having periodic boundary conditions. One can Fourier decompose this as,

$$\phi(\vec{\mathbf{x}}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{\mathbf{k}}} \phi_{\mathbf{k}}(t) \exp\Big(i\vec{\mathbf{k}} \cdot \vec{\mathbf{x}}\Big) \text{ where } k_x = \frac{2\pi n_x}{L}, \ldots$$

Substituting this into the first equation, we find that the harmonic oscillators get nicely decoupled into an infinite set of ODEs of the form,

$$\ddot{\phi}_{\mathbf{k}} + (k^2 + m^2)\phi_{\mathbf{k}} = 0$$

which is basically the harmonic oscillator equation with frequency $\omega_k = \sqrt{k^2 + m^2}$.The energy of oscillators in simply equal to the sum of individual energies of the oscillators,

$$E = \sum_{\mathbf{k}} \left[\frac{1}{2}\dot{\phi}_{\mathbf{k}}^{\,2} + \frac{1}{2}\omega_k^2 \phi_{\mathbf{k}}^2\right]$$