

# Trefethen and Bau: Lecture #6

September 25, 2018

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## Problem 1

As  $P$  is orthogonal,  $P^* = P$  and  $P^2 = P$ . Consider  $I - 2P$ ,

$$\begin{aligned}(I - 2P)^*(I - 2P) &= I - 2P - 2P^* + 4P^*P \\ &= I - 2P - 2P + 4P \\ &= I\end{aligned}$$

Hence,  $I - 2P$  is unitary.

## Problem 2

Given,  $E = \frac{1}{2}(1 + F)$ , where  $F$  just reverses the sequence of elements of the vector.  $F$  is hence just the anti-diagonal matrix of shape  $m \times m$  with anti-diagonal elements 1. Also,  $F^* = F \implies E^* = E \implies$  *orthogonality*. If  $m$  is odd, then  $E$  has all diagonal and anti-diagonal elements equal to  $\frac{1}{2}$ , except the middle element which will be equal to 1. If  $m$  is even, then  $E$  has all diagonal and anti-diagonal elements equal to  $\frac{1}{2}$ . Two examples are given below.

## Problem 3