Fluid Mechanics: Assignment #1

Due on 2nd September, 2018

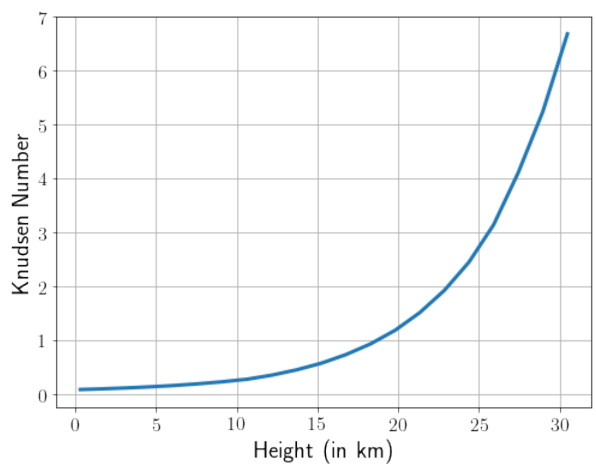
Aditya Vijaykumar

Problem 1

The *Knudsen Number* is given by,

$$\mathrm{Kn} = \frac{\lambda}{L} = \frac{k_B T}{\sqrt{2}\pi d^2 p L}$$

where L is the characteristic length scale, and T and p are the temperature and pressure respectively.
We can take some approximations for the the variations of T and p, but we note that data for the variation of T and p is also available publicly `http://www.hyvac.com/tech_support/atmosphere%20vs%20pressure%202.htm`. We import that data and use it to do our calculations. We also assume some standard values for all the other parameters.



Problem 2

$$\frac{\mathrm{d}C}{\mathrm{d}\eta} = \kappa \exp\left(\frac{-\eta^2}{4D}\right)$$

where $\eta = x/\sqrt{t}$. Performing the indefinite integral and substituting for η, one gets,

$$C(\eta) = \sqrt{D\pi}\kappa \ \mathrm{erf}\left(\frac{\eta}{2\sqrt{D}}\right) + \alpha = \sqrt{D\pi}\kappa \ \mathrm{erf}\left(\frac{x}{2\sqrt{D}\sqrt{t}}\right) + \alpha$$

where α is some integration constant. We now go ahead and impose boundary conditions

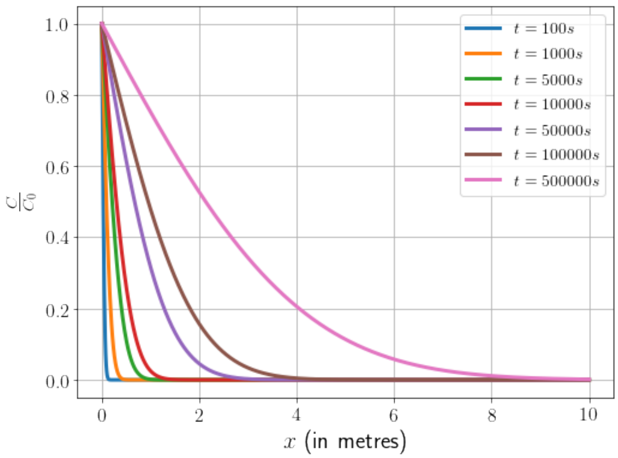
- The concentration at the flower ($x = 0$) is assumed to be a constant C_0 at all times. As $\mathrm{erf}(0) = 0$, $\alpha = C_0$

- At $t = 0$, any x would have zero concentration. As $\text{erf}(x \to \infty) = 1$, We get the condition that $\sqrt{D\pi}\kappa + C_0 = 0$ ie. $\kappa = -C_0/\sqrt{D\pi}$

Therefore, the final solution is,

$$C(x,t) = C_0 \left[1 - \text{erf}\left(\frac{x}{2\sqrt{D}\sqrt{t}}\right)\right]$$

The function is plotted below for different t using standard value of diffusivity $D \approx 10^{-5}\ m^2/s$,



Problem 3