Chapter 14: The Schwarzschild Solution

Stationary Solutions

In some sense, stationary solution is a solution that does not contain time coordinate explicity. Note that this does not mean that the solution is not evolutionary. On the other hand, if the solution is not evolutionary, it is necessarily static.

So then, a metric is defined as stationary when, in a special coordinate system, the metric is visibly time-independent, ie.,

$$\boxed{\frac{\partial g_{ab}}{\partial x^0} = 0}$$

But this is a weird definition, it relies on the existence of a "special coordinate system". We have to make this more precise. If we define $X^a = \delta^a_0$, in the special coordinate system,

$$\mathcal{L}_X g_{ab} = \cancelto{0}{X^c g_{ab,c}} + \cancelto{0}{g_{ac} X^c{}_{,b}} + \cancelto{0}{g_{bc} X^c{}_{,a}}$$

Since $X^a = \delta^a_0$

$$= 0$$

since $\delta^a_0 g_{ab,0} = 0$

$$\Rightarrow \boxed{\mathcal{L}_X g_{ab} = 0}$$

Since $\mathcal{L}_X g_{ab}$ is a tensor, it should vanish in all coordinate systems if it does in one coordinate system.

So then, we can say a metric is stationary if we can find a timelike Killing vector field.

$\boxed{\text{Hypersurface-orthogonal vector fields}}$