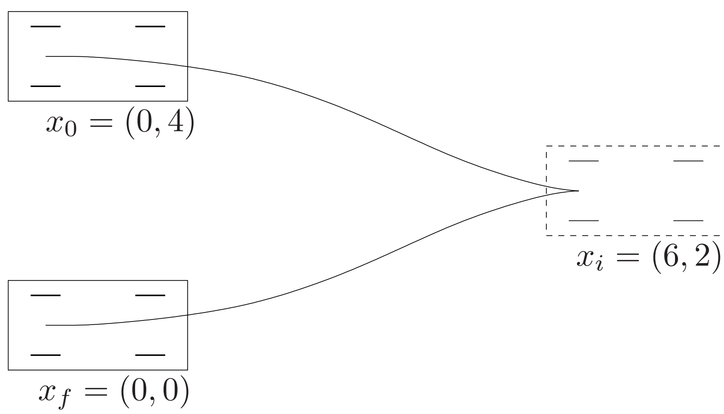
Problem 1

Consider the lateral control problem for an autonomous ground vehicle:

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v}{\ell}\tan\phi$$

- Using the fact that the system is differentially flat, find an explicit trajectory that solves the following parallel parking maneuver:



  Your solution should consist of two segments: a curve from x_0 to x_i with $v > 0$ and a curve from x_i to x_f with $v < 0$. For the trajectory that you determine, plot the trajectory in the plane (x versus y) and also the inputs v and ϕ as a function of time.

- Use the trajectory tracking code (though we have a slightly different dynamics) to generate the tracking controller for the system. Test your controller under two different initial state $x_0 = (0, 5)$, $x_0 = (-1, 4.5)$ and check how the system response given the initial error.

Problem 2

Consider the dynamics of a planar, vectored thrust flight control system as shown in Figure. This system consists of a rigid body with body fixed forces and is a simplified model for a vertical take-off and landing aircraft. Let (x, y, θ) denote the position and orientation of the center of mass of the aircraft. We assume that the forces acting on the vehicle consist of a force F_1 perpendicular to the axis of the vehicle acting at a distance r from the center of mass, and a force F_2 parallel to the axis of the vehicle. Let m be the mass of the vehicle, J the moment of inertia, and g the gravitational constant. We ignore aerodynamic forces for the purpose of this example.

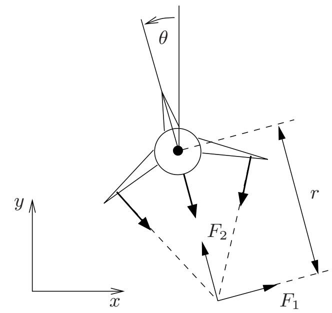


Figure 1.6: Vectored thrust aircraft (from ÅM08). The net thrust on the aircraft can be decomposed into a horizontal force F_1 and a vertical force F_2 acting at a distance r from the center of mass.

The dynamics for the system are

$$
\begin{aligned}
m\ddot{x} &= F_1 \cos\theta - F_2 \sin\theta, \\
m\ddot{y} &= F_1 \sin\theta + F_2 \cos mg, \\
J\ddot{\theta} &= rF_1
\end{aligned}
$$

show that this system is differentially flat and that one set of flat outputs is given by

$$z_1 = x - (J/mr)\sin\theta$$

$$z_2 = y + (J/mr)\cos\theta$$

Remind: To show differentially flatness of a system, you need to be able to express all the state and input variables as a function of the flat outputs and their higher-order derivatives.