# Lecture notes: Iterative Learning for Gravity Compensation

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# Outline

This lecture note is based on

- De Luca, Alessandro, and Stefano Panzieri. ”An iterative scheme for learning gravity compensation in flexible robot arms.” Automatica 30.6 (1994): 993-1002.

# Set point tracking

PD + gravity compensation
?

- Goal: regulation of arbitrary equilibium configurations in the presence of gravity.
- Missing information: Do not have explicit knowledge of robot dynamic coefficients (nor of the structure of the gravity term).

Introduce iterative learning control:

- based on an iterative control scheme that uses: PD control+ **constant** feedforward term.
- **iterative update of the feedforward term** at successive steady-state conditions.

We will derive sufficient conditions for the global convergence.

# Revision

robot dynamic model:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) = \tau$$

Assume the bound on the gradient of the gravity term

$$\|\frac{\partial N(q)}{\partial q}\| \leq \alpha$$

set point tracking control without gravity compensation:

$$u = -K_p(q - q_d) - K_D(\dot{q} - \underbrace{\dot{q}_d})$$

set point : $q_d$ const

$\dot{q}_d = 0$

at steady state, there is a non-zero residual error

$$M\underbrace{\ddot{q}}_{0} + C\underbrace{\dot{q}}_{0} + N(q) = -K_p(q - q_d) - \underbrace{K_D\dot{q}}_{0}$$

steady state $\bar{q}$

$$N(\bar{q}) = -K_p(\bar{q} - q_d)$$

$$\bar{q} - q_d = -K_p^{-1} N(\bar{q})$$

Non zero steady state error.

# Iterative control scheme

Let

$$u = \frac{1}{\beta}(-K_P(q - q_d) - K_D\dot{q}) + u_{i-1}, \quad \beta > 0$$

where $u_{i-1}$ is a constant **compensation term.**

Iterarive update of $u_{i-1}$:

- Start with $u_0 = 0$, $q_0$ is the initial configuration.
- At the steady state, $q = q_i$, $\dot{q} = 0$, one have

Update law:

$$u_i = \frac{1}{\beta}(-K_P(q - q_d) + u_{i-1}$$

**Theorem**: if $\left\| \frac{\partial N(q)}{\partial q} \right\| \leq \alpha$

- $\lambda_{min}(K_P) \geq \alpha$.
- $0 \leq \beta \leq \frac{1}{2}$,

guarantee that the sequence $q_0, q_1, \ldots,$ converges to $q_d$ from any initial value $q_0$ (and $\dot{q}_0$), i.e., globally asymptotically stable in set point tracking.

proof

Introduce tracking error at iteration $i$

$$e_i = q_i - q_d$$

$q_i$ ↑ joint conf. at the iter. $i$.

$$u = \frac{1}{\beta}\left(-K_P(q_i - q_d) - K_D \dot{q}\right) + u_{i-1}$$

at the steady state.

$$M\underbrace{\ddot{q}_i}_{\downarrow\, 0} + C\underbrace{\dot{q}_i}_{\downarrow\, 0} + N(q_i) = \frac{1}{\beta}(-K_p(q_i - q_d) - K_D \underbrace{\dot{q}}_{\downarrow\, 0}) + u_{i-1}$$

$$N(q_i) = \underbrace{\frac{1}{\beta}(-K_p(q_i - q_d)) + u_{i-1}}_{\parallel \atop u_i}$$

$$u_i = N(q_i) \quad \cdots \; q_i \text{ steady state at the } i\text{th iter.}$$

$$u_{i-1} = N(q_{i-1})$$

$$\|u_i - u_{i-1}\| = \|N(q_i) - N(q_{i-1})\| \leq \underbrace{\alpha \|q_i - q_{i-1}\|}_{RHS} \qquad (1)$$

$$\text{Given } \left\|\frac{\partial N}{\partial q}\right\| \leq \alpha$$

$$q_i - q_d = e_i$$

$$q_i = e_i + q_d$$

RHS of (1) $\quad \alpha \| q_i - q_{i-1} \| = \alpha \| e_i + q_d - e_{i-1} - q_d \|$

$$= \alpha \| e_i - e_{i-1} \|$$

$$\| u_i - u_{i-1} \| \leq \underbrace{\alpha \| e_i - e_{i-1} \|}_{\leq \alpha ( \| e_i \| + \| e_{i-1} \| )} \quad \cdots\cdots \quad (2)$$

The update law:

$$u_i = \underbrace{\frac{1}{\beta} ( - K_p ( q_i - q_d ))}_{} + u_{i-1}$$

$$u_i - u_{i-1} = ( \quad )$$

$$\Rightarrow \| u_i - u_{i-1} \| = \| \frac{1}{\beta} ( - K_p ( q_i - q_d )) \|$$

$$= \frac{1}{\beta} \| K_p \| \underbrace{\| q_i - q_d \|}_{}$$

$$= \frac{1}{\beta} \| K_p \| \| e_i \| \quad \cdots \quad (3)$$

$$\frac{1}{\beta}\underbrace{\|K_p\|}\|e_i\| \leq \alpha(\|e_i\| + \|e_{i-1}\|)$$

For any matrix $A$, $\lambda_{min}(A) \leq \|A\| \leq \lambda_{max}(A)$

$$\frac{1}{\beta}\lambda_{min}(K_p)\|e_i\| \leq \frac{1}{\beta}\|K_p\|\|e_i\|$$

$$\lambda_{min}(K_p) \geq \alpha$$

$$\frac{1}{\beta}\cdot\cancel{\alpha}\|e_i\| \leq \cancel{\alpha}(\|e_i\| + \|e_{i-1}\|)$$

$$\boxed{\frac{1}{\beta}\|e_i\| - \|e_i\| - \|e_{i-1}\| \leq 0}$$

To shown $\|e_i\| < \|e_{i-1}\| \Rightarrow \boxed{\frac{\|e_i\|}{\|e_{i-1}\|} < 1}$

$$\left(\frac{1}{\beta} - 1\right)\|e_i\| \leq \|e_{i-1}\|$$

$$\frac{\|e_i\|}{\|e_{i-1}\|} \leq \underbrace{\frac{1}{\frac{1}{\beta}-1}} < 1$$

$$\frac{1}{\beta} - 1 \overset{<}{>} 1 \Rightarrow \frac{1}{\beta} > 2 \Rightarrow \beta < \frac{1}{2}$$

$$\| e_i \| < \frac{\beta}{1-\beta} \| e_{i-1} \|$$

a contraction for $\beta < \frac{1}{2}$

$$\lim_{i \to \infty} \| e_i \| = 0 \Rightarrow \| e_i \| \to 0$$

$$\| q_i - q_d \| \to 0$$

$$q_i \to q_d$$

# Concluding remarks

- Given the bound $\alpha$ for the derivative of gravity term, we will determine the gain $K_P$ such that

$$\alpha < \lambda_{min}(K_P)$$

- in diagonal matrix of $K_P$, this just means all diagonal terms will be greater than $\alpha$.
- again, it is a sufficient condition.
- the scheme can be interpreted as using an integral term which is updated only in correspondence of a discrete sequence of time instants and has guaranteed convergence and global asymptotic stability.