Assignment 1

— Aishwary Jagetia

Considering the double integrator

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

A] State vector $x = [x_1, x_2]^T$ and input u
writing in the form $\dot{x} = Ax + Bu$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad — (1)$$

$$\therefore \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad — (2)$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad — (3)$$

For the system to be reachable rank $([B \;\; AB])$ = Full rank.

$$[B \;\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \text{rank}([B \;\; AB]) = 2 = \text{no. of rows}$$

$\therefore$ det $([B \;\; AB]) \neq 0 \;\therefore$ System is reachable.

B] Feedback $u = -kx$

$$\dot{x} = Ax + Bu$$
$$\dot{x} = (A - Bk)x$$

characteristic polynomial $\det[(A-Bk) - \lambda I] = 0$
considering A, B from eq^n (2) & (3)

$$\det[(A-Bk)-\lambda I] = \det\left[\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix}\right) - \lambda I\right]$$

$$= \det\left[\begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right] = \det\begin{bmatrix} -\lambda & 1 \\ -k_1 & -k_2-\lambda \end{bmatrix} = 0$$

$$\det \begin{bmatrix} -\lambda & 1 \\ -k_1 & -k_2-\lambda \end{bmatrix} = 0$$

$$-\lambda(-k_2-\lambda) - (-k_1) = 0$$
$$\lambda k_2 + \lambda^2 + k_1 = 0$$
$$\lambda^2 + \lambda k_2 + k_1 = 0 \quad - (4)$$

Given characteristic polynomial

$s^2 + 2\zeta_0 \omega_0 s + \omega_0^2$ has $\omega_0 = 1$, $\zeta_0 = 0.7$

i.e $s^2 + 1.4s + 1 = 0 \quad - (5)$

Comparing 4 & 5 :-

$$k_1 = 1$$
$$k_2 = 1.4$$

c] Given initial state $x_0 = (-5, 0) \rightarrow x_{1d}(0) = -5$, $x_{2d}(0) = 0$

$x_f = (1, 1) \rightarrow x_{1d}(f) = 1$, $x_{2d}(f) = 1$

Desired trajectory $\dot{x}_d = Ax_d + Bu_d$

$$x_d(0) = x_0$$
$$x_d(5) = x_f$$

$$x_{1d}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$
$$x_{2d}(t) = \dot{x}_{1d}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

For $t=0$, $x_{1d}(0) = a_0 = -5$

$x_{2d}(0) = a_1 = 0$

For $t=5$, $x_{1d}(5) = -5 + 25a_2 + 125a_3 = 1 \quad (a_1 = 0)$

$x_{2d}(5) = 10a_2 + 75a_3 = 1$

$$25 a_2 + 125 a_3 = 6$$
$$10 a_2 + 75 a_3 = 1$$

Solving for a_2 & a_3,

$$a_2 = 0.52$$
$$a_3 = -0.056$$

$$\therefore \; x_{1d}(t) = -0.056\, t^3 + 0.52\, t^2 - 5$$
$$x_{2d}(t) = -0.168\, t^2 + 1.04\, t$$

we know $\dot{x}_2^d = u^d$

$$\underline{u^d(t) = -0.336\, t + 1.04} \quad -⑧$$

D] Given state $\dot{x}_d = A x_d + B u_d$

$$e(t) = x(t) - x^d(t)$$

$$\dot{e}(t) = \dot{x}(t) - \dot{x}^d(t)$$
$$= Ax + Bu - Ax^d - Bu^d$$
$$= A(x - x^d) + B(u - u^d) \quad -⑥$$

$$Bv = Bke = B(u - u^d) \quad -⑦$$

Substituting ⑦ in eq^n ⑥

$$\dot{e}(t) = Ae + Bke$$
$$\dot{e}(t) = \underline{(A + Bk)e} \quad \{ e \text{ as the new state} \}$$

E]

To design the control input u such that the system eventually converges to a desired trajectory $x^d(t)$

From eq^n (7) $\quad Ke = u - u^d$

or $u = ke + u^d$

$$u = -[k_1 \;\; k_2]\begin{bmatrix} x_1 - (-0.056t^3 + 0.52t^2 - 5) \\ x_2 - (-0.168t^2 + 1.04t) \end{bmatrix} + (-0.336t + 1.04)$$

We know $k_1 = 1$, $k_2 = 1.4$

$$u = -((x_1 + 0.056t^3 - 0.52t^2 + 5) + 1.4(x_2 + 0.168t^2 - 1.04t) + (-0.336t + 1.04))$$

$$u = -(0.056t^3 - 0.2848t^2 - 1.792t + 6.04 + x_1^{(t)} + 1.4x_2^{(t)})$$

```
clc;
clear;

A = [0, 1; 0, 0];
B = [0; 1];
K = [1, 1.4];
dt = 0.1;

x_t = [-3;1];
x_d = [-5;0];
desired_posi = [x_d];
new_posi = [x_t];

for t=0:dt:5
    x1_d = -(0.056*(t^3))+ (0.52*(t^2))-5;
    x2_d = -(0.168*(t^2))+ (1.04*t);
    x_d = [x1_d;x2_d];
    u_d = -(0.336*t)+1.04;
    u_t = (-K*(x_t - x_d))+ u_d;
    x_t_dot = (A*x_t) + (B*u_t);
    desired_posi = [desired_posi,x_d];
    new_posi = [new_posi,x_t];
    x_t = (x_t_dot*dt) + x_t;
end

figure
hold on
plot(desired_posi(1,:),desired_posi(2,:))
plot(new_posi(1,:),new_posi(2,:))
hold off
```