Lecture notes: Robot Interaction with the Environment

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Part of the lecture notes are from Prof. Alessandro De Luca's lecture in force control. http://www.diag.uniroma1.it/~deluca/rob2_en/15_ImpedanceControl.pdf

- imposes a desired dynamic behavior, specified through a **generalized dynamic impedance**, namely a complete set of mass-spring-damper equations.
- a model describing how reaction forces are generated is **not** explicitly required.
- suited for tasks in which contact forces should be “small” and no need to modulate the force.
- the choice of a specific stiffness in the impedance model along a Cartesian direction results in a trade-off between contact forces and position accuracy in that direction.

Cartesian dynamics

When system is in contact, the environment introduces a **generalized Cartesian force** F.

$$J_a^T T_a^{-T}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) = \tau + J^T(q)F.$$

where $J(q)$ is the geometric jacobian of the robot manipulator.

The force $F = \begin{bmatrix} \gamma \\ \mu \end{bmatrix}$ performs work on $v = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(q)\dot{q}$. ①

$$\dot{x} = J_a(q)\dot{q} \qquad ⓪$$

$J^T T_a^T = J_a^T$

where $J_a(q)$ is the analytic Jacobian. $\dot{X} = T_a(\phi) v$ ②

$\dot{X} = T_a(\phi) v = T_a(\phi) J \dot{q} = J_a(q)\dot{q} \Rightarrow T_a(\phi) J = J_a$

$J^T = J_a^T T_a^{-T}$

Dynamic model in Cartesian coordinates

$$M(q)\ddot{q} + C\dot{q} + N = \tau + J_a^T \underbrace{T_a^{-T} F}_{\downarrow \text{ Rename } F_a} = \tau + J_a^T F_a$$

F_a: generalized force performing work on x

$$\dot{x} = J_a \dot{q}$$

$$\ddot{x} = \dot{J}_a \dot{q} + J_a \ddot{q} \Rightarrow \ddot{q} = J_a^{-1}(q)(\ddot{x} - \dot{J}_a \dot{q})$$

$$= J_a^{-1}(q)(\ddot{x} - \dot{J}_a J_a^{-1} \dot{x})$$

$$M_x(q)\ddot{x} + C_x(q,\dot{q})\dot{x} + N_x(q) = J_a^{-T}\tau + F_a$$

- $M_x(q) = J_a^{-T}(q)\, M(q)\, J_a^{-1}(q)$
- $C_x = J_a^{-T}(q)\, C\, J_a^{-1} - M_x(q)\, \dot{J}_a J_a^{-1}$
- $N_x = J_a^{-T}(q)\, N(q)$

Properties of the dynamic model in Cartesian coordinates

- $M_x(q) > 0$
- $\dot{M}_x(q) - 2C_x(q,\dot{q})$ is skew symmetric.
- The Cartesian dynamic model of the robot is linearly parameterized in terms of a set of dynamic coefficients.

Impedence control design

$$M_x \ddot{x} + C_x \dot{x} + N_x = J_a^{-T} \tau + F_a$$

introduce a_x

① Feedback linearization: $\ddot{x} = \underset{\sim}{a_x}$

$$\ddot{x} = M_x^{-1}(-C_x \dot{x} - N_x) + M_x^{-1}(J_a^{-T} \tau + F_a) = a_x$$

$$\tau = J_a^{T}(q)\left[M_x a_x + C_x \dot{x} + N_x - F_a \right]$$

② Impose a desired dynamic impedance model:

M_d, K_d, B_d $\in$ 6x6 matrices. $x \in R^6$

inertia, stiffness, damping.

$x - x_d$

$$M_d(\ddot{x} - \ddot{x}_d) + B_d(\dot{x} - \dot{x}_d) + K_d(x - x_d) = F_a$$

$\ddot{X} = a_x$

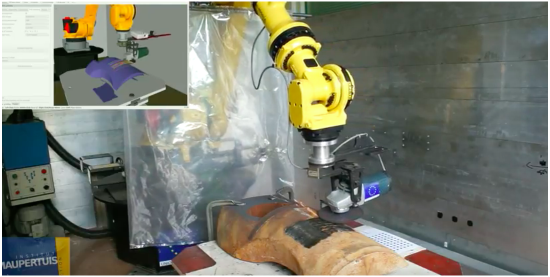
$$\ddot{X} = M_d^{-1}\left[F_a - B_d(\dot{X} - \dot{X}_d) - K_d(X - X_d)\right] + \ddot{X}_d = a_x$$

To satisfy the desired impedance:

$$\tau = J_a^T(q)\left[M_x \underbrace{\left(M_d^{-1}\left[F_a - B_d(\dot{X} - \dot{X}_d) - K_d(X - X_d)\right] + \ddot{X}_d\right)}_{a_x} + C_x\dot{X} + N_x - F_a\right]$$

Examples

x_d is often **slightly inside the environment** to maintain contact.



x_d is often the rest position in a human-robot interaction task.

https://youtu.be/3lqVuNXHdkk

Control law in joint coordinates

$\dot{x} = J_a \dot{q}$

$$\tau = M(q)J_a^{-1}(q)\{\ddot{x}_d - \dot{J}_a(q)\dot{q} + M_d^{-1}[B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x)] + C(q,\dot{q})\dot{q} + N(q) + J_a^T[M_x(q)M_d^{-1} - I]F_a\} \quad (1)$$

while the principle of control design is based on dynamic analysis and desired (impedance) behavior as described in the **Cartesian space**, the final control implementation is always made at **robot joint level.**

When to use impedence control

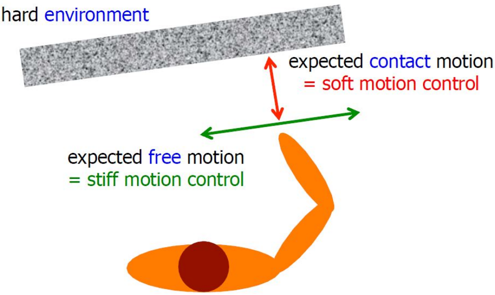
- there is uncertainty in geometric characteristics (position, orientation) of the environment
- adapt/match to the dynamic characteristics (in particular, stiffness) of the environment, in a complementary way.
- mimic the behavior of human arm. - fast and stiff in the free motion direction, and compliant and slow in "safe guard" motion.
- Require force feedback in the general form.

parameter selection:

- large $M_{d,i}$ and small $K_{d,i}$ in Cartesian directions where contact is foreseen (low contact forces).
- large $K_{d,i}$ and small $M_{d,i}$ in Cartesian directions where motion is free to keep tracking performance good.
- the damping coefficients $B_{d,i}$ are used to shape transient behaviors.

Mimic human arm

Human arm behavior



in selected directions:
the stiffer is the environment, the softer is the chosen model stiffness $K_{m,i}$

Simplication -1

When choose the apparent inertia M_d equal to the natural Cartesian inertia of the robot M_x.
note that $M_d = M_x = J_a^{-T} M J_a^{-1}$

$$\tau = M(q)J_a^{-1}(q)\{\ddot{x}_d - \dot{J}_a(q)\dot{q} + M_d^{-1}[B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x)]\}$$
$$+ C(q,\dot{q})\dot{q} + N(q) + J_a^T \underbrace{[\underbrace{M_x(q)M_d^{-1}}_{I} - I]}_{\downarrow\, 0} F_a \quad (2)$$

becomes

$$\tau = M J_a^{-1}[\ddot{x}_d - \dot{J}_a \dot{q}] + J_a^T[B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x)] + C\dot{q} + N$$

No need for force sensor.

Simplication -2

if the impedance model (now, nonlinear) is still supposed to represent a real mechanical system, then in correspondence to a desired non-constant inertia (~~Dx~~(q)) there should be Coriolis and centrifugal terms.
M_x

$M_d = M_x(q)$

$$M_d(\ddot{x} - \ddot{x}_d) + \underline{(C_x(q, \dot{q}) + B_d)}(\dot{x} - \dot{x}_d) + K_d(x - x_d) = F_a$$

the new controller becomes

$$\tau = M J_a^{-1} \ddot{x}_d + J_a^T C_x \dot{x}_d + N(q) + J_a^T [B_d (\dot{x}_d - \dot{x}) + K_d (x_d - x)]$$

for set point tracking: $x_d = \text{const.}$ $\ddot{x}_d = \dot{x}_d = 0$

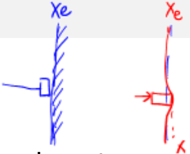
$$\tau = N(q) + J_a^T [B_d (-\dot{x}) - K_d (x_d - x)]$$

Cartesian regulation control

When x_d is constant and $F_a = 0$ (no contact) the controller is A.S. for set point tracking.

PD + Gravity compensation.

Cartesian stiffness control



Assume: $F_a = K_e (x_e - x)$

When x_d is constant and $F_a \neq 0$: Convergence to x_d is not guaranteed.

- closed-loop system behavior

$$V = \frac{1}{2}\dot{x}^T M_x \dot{x} + \frac{1}{2}(x_d - x)^T K_d (x_d - x) + \frac{1}{2}(x_e - x)^T K_e (x_e - x)$$

positive

$$\dot{V} = \dot{x}^T M_x \ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}_x \dot{x} - \dot{x}^T K_d (x_d - x) - \dot{x}^T \underbrace{K_e (x_e - x)}_{F_a}$$

$$M_x \ddot{x} + (C_x + B_d)\dot{x} + K_d (x - x_d) = F_a \qquad (\ddot{x}_d = \dot{x}_d = 0)$$

$$\dot{V} = -\dot{x}^T B_d \dot{x} \leq 0$$

$$\dot{x} = 0 = \ddot{x}$$

invariant.

$$\cancel{V = \frac{1}{2}(X_d - X)^T K_d (X_d - X)}$$

Substitute into impedance model:

$$M_x \ddot{X} + (C_x + B_d)\dot{X} + K_d(\underset{X_E}{\underset{\uparrow}{X}} - X_d) = K_e(X_e - X)$$

$$X_E = (K_d + K_e)^{-1}(K_d X_d + K_e X_e).$$

① rigid env: $K_e \gg K_d$ $\quad X_E \to X_e$

② rigid control: $K_d \gg K_e$ $\quad X_E \to X_d$