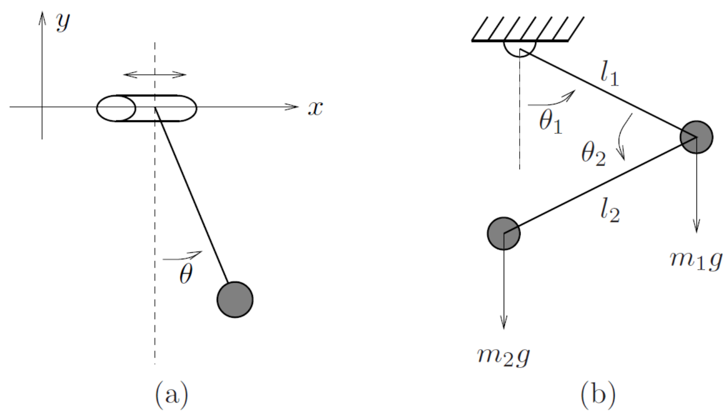
Problem 1

1(a) 20 pt and 1(b) 30pt

1. Derive the equations of motion for the systems shown below.



   (a) Pendulum on a wire: an idealized planar pendulum whose pivot is free to slide along a horizontal wire. Assume that the top of the pendulum can move freely on the wire (no friction).

   (b) Double pendulum: two masses connected together by massless links and revolute joints.

you can use the symbolic computation in matlab or mathematica for this problem. If you use the tools, please send the report with the source code for submission.

Problem 2

Revisit eigendecomposition of a matrix: Given a linear dynamical system $\dot{x} = Ax$ where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ is the state vector and matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

- (10pt) compute the eigenvalues and corresponding eigenvectors of A. Let v_1, v_2 be the computed eigenvectors for eigenvalues λ_1, λ_2.
- (10pt) show that $T = [v1, v2]$ satisfies that $T^{-1}AT = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$.
- (10pt) consider a change of basis, i.e., $z = T^{-1}x$. What is the differential equation using z as the state variable? Present your derivation steps.
- (20pt) Can you obtain the solution of the differential equation using z as the state variables? If yes, compute the solution x of the original ode using the solution of z. hint: first solve the ode with variable z, and then solve for x using the relation that $z = T^{-1}x$.