Final Exam

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Contents

Notations: For a given variable, x, dx is its time derivative, ddx is

2nd-order derivative.

```
clc
clear all;
close all;
% the following parameters for the arm
global l1 l2
I1=10; I2=10; m1=5; r1=.5; m2=5; r2=.5; l1=1; l2=1; g=9.8;
```

specify your initial and final condition.

```
qi = [1.0;1.0];
X = ForwardKinematics(qi(1),qi(2));
x0= [qi(1),qi(2),0,0,X(1),X(2)];
tf=5;
global torque Force
torque = [];
```

Implement the Iterative Learning control for set point tracking.

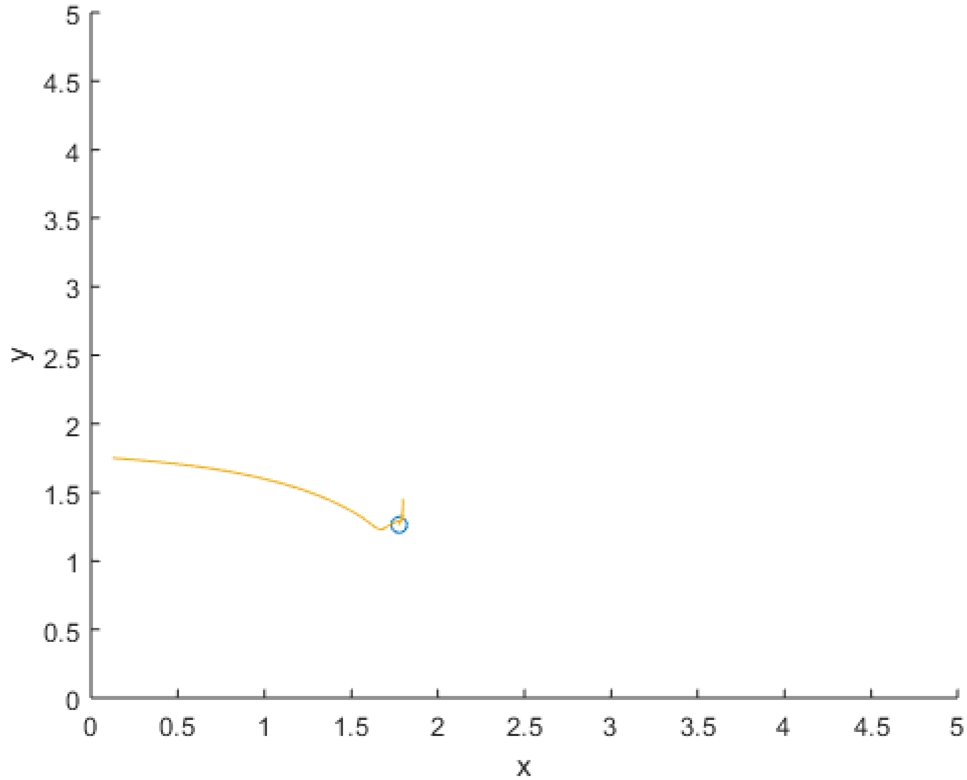
```
options = odeset('RelTol',1e-4,'AbsTol',[1e-4, 1e-4, 1e-4, 1e-4, 1e-4, 1e-4]);
[T,X] = ode45(@(t,x) ComplianceControl(t,x),[0 tf],x0, options);

figure('Name','End effector position Compliance Control');
comet(X(:,5), X(:,6));
xlabel('x')
ylabel('y')
axis([0 5 0 5])

figure('Name','End effector position Compliance Control');
plot(T, X(:,5));
xlabel('time')
ylabel('x')

figure('Name','Reaction Force: Compliance Control');
plot(T, Force(1,1:size(T,1)),'-');
hold on
plot(T, Force(2,1:size(T,1)),'r--');
xlabel('time')
ylabel('Force at the end effector')

figure('Name','Theta_1 under Compliance Control');
```

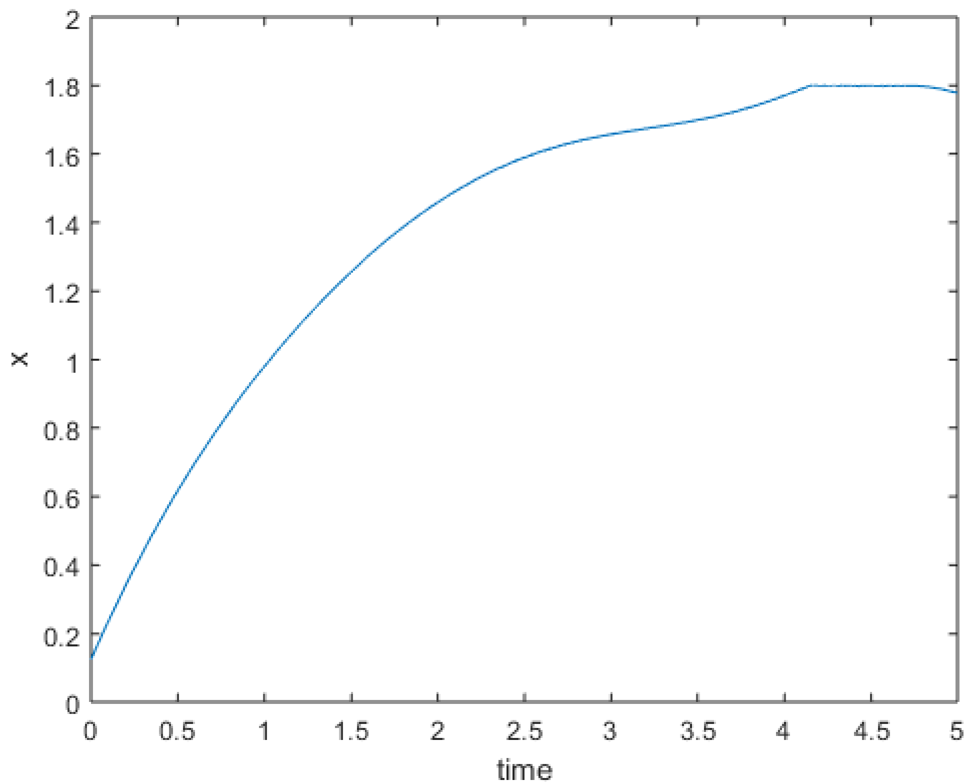
```
plot(T, X(:,1),'r-');
xlabel('time')
ylabel('Theta1')

figure('Name','Theta_2 under Compliance Control');
plot(T, X(:,2),'r--');
xlabel('time')
ylabel('Theta2')

figure('Name','Torque: Compliance Control');
plot(T, torque(1,1:size(T,1)),'-');
hold on
plot(T, torque(2,1:size(T,1)),'r--');
xlabel('time')
ylabel('torque')
hold off
torque=[];
```

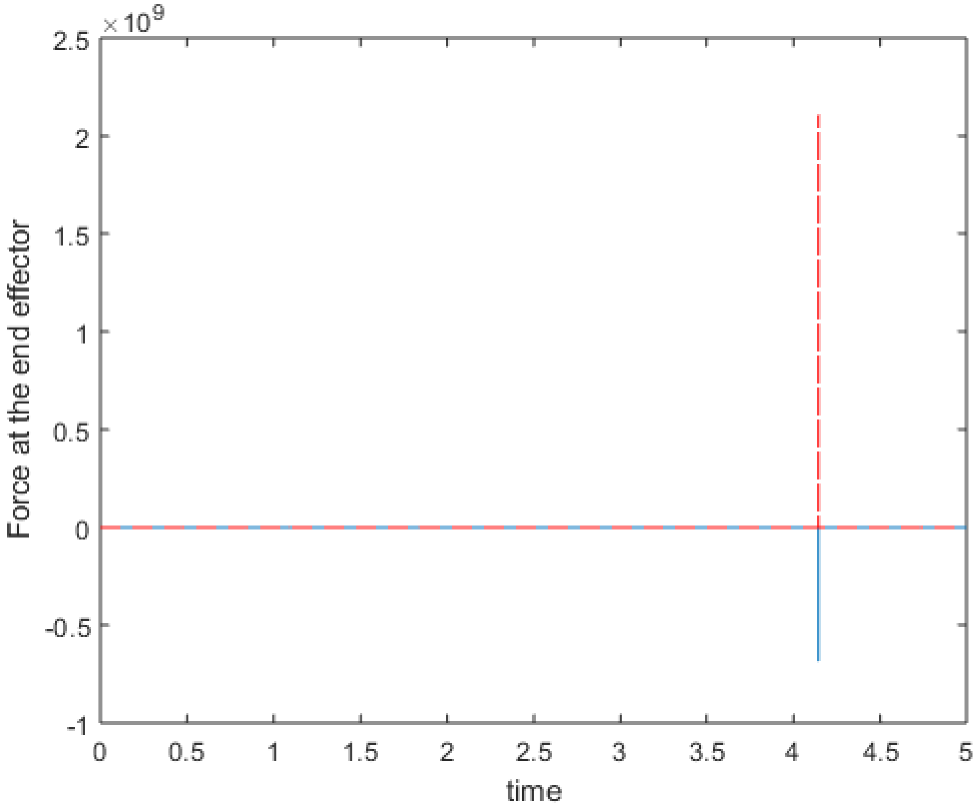
Compliance Control

```
function dx = ComplianceControl(t,x)
    Xd = [2.0;0.5]; % Desired end effector position
    Xe = x(5:6,1); % Current end effector position

    theta=x(1:2,1); % [x1;x2]=[x(1);x(2)]
    dtheta=x(3:4,1); % [x1_dot;x2_dot]=[x(3);x(4)]

    global l1 l2
    I1=10;  I2 = 10; m1=5; r1=.5; m2=5; r2=.5; l1=1; l2=1; g=9.8;

    % we compute the parameters in the dynamic model
    a = I1+I2+m1*r1^2+ m2*(l1^2+ r2^2);
    b = m2*l1*r2;
    d = I2+ m2*r2^2;

    global M C J he
    M = [a+2*b*cos(x(2)), d+b*cos(x(2));
        d+b*cos(x(2)), d];
    C = [-b*sin(x(2))*x(4), -b*sin(x(2))*(x(3)+x(4)); b*sin(x(2))*x(3),0];

    invM = inv(M);
    invMC= inv(M)*C;

    J = getJacobian(theta(1),theta(2)); % Current Jacobian

    tau = Controler(Xd,Xe,theta,dtheta);

    global torque
    torque = [torque, tau];

    dx = zeros(6,1);
    dx(1)= x(3); %dtheta1
    dx(2)= x(4); %dtheta2
    dx(3:4) = -invMC* x(3:4) + invM*tau - invM*(transpose(J)*he);
    dx(5:6)= J*dx(3:4);
end

function tau = Controler(Xd,Xe,theta,dtheta)

   global he Force
   Xr = [1.8; 0]; % Wall location
```

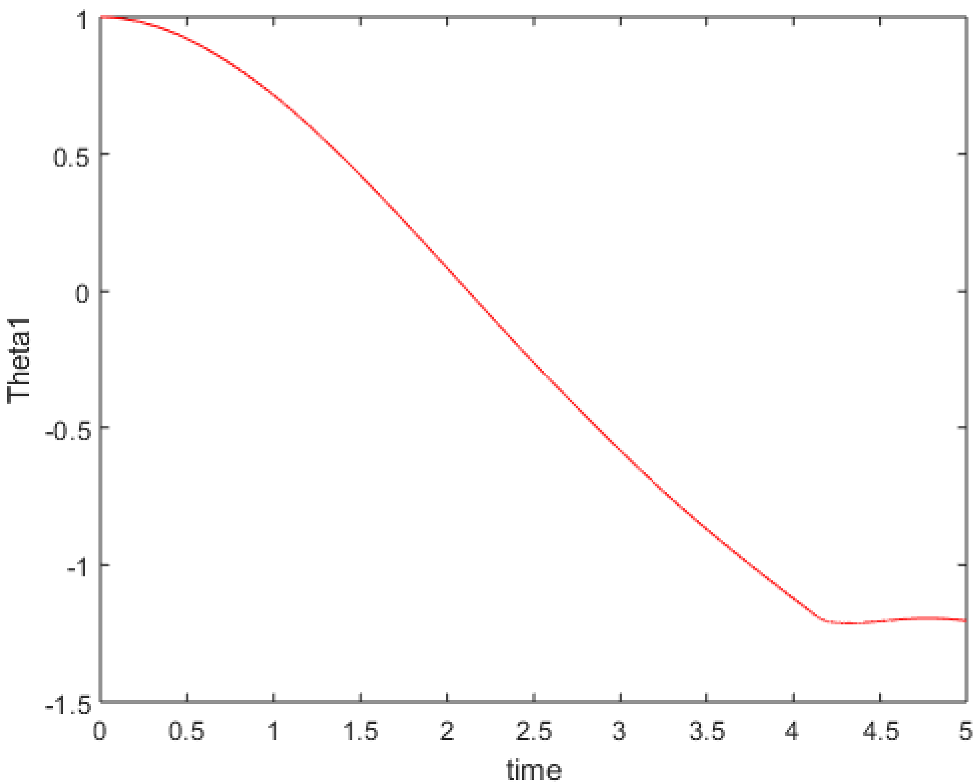
```
    Kd = [6 0; 0 4];
    % Kd makes sure that the tracking performance is optimum and it shows
    % how jerky the system is. Here we have kept x higher than y to control
    % the system in x direction. Therefore Kd is small where contact forces
    % are low.
    Bd = [1 0; 0 1];
    % Bd is the damping coef which takes care of transient behaviour which
    % smooths the system and also controls the rate aslo
    K = [1000 0; 0 1000];
    if (Xe(1) >= Xr(1))
        he = K*(Xr - Xe);
    else
        he = [0; 0];
    end
    Force = [Force, he];

    P_e = Xd - Xe; % Position error

    global J
    tau = J.'*(Kd*P_e - Bd*J*dtheta) + he;
end

function [qd] = InverseKinematics(x, y)
    global l1 l2
    q2 = acos((x*x + y*y - l1*l1 - l2*l2)/(2*l1*l2));
    q1 = atan2(y, x) - atan2((l2*sin(q2)), (l1 + l2*cos(q2)));
    qd = [q1;q2;];
end

function [X] = ForwardKinematics(q1, q2)
    global l1 l2
    x = l1*cos(q1) + l2*cos(q1 + q2);
    y = l1*sin(q1) + l2*sin(q1 + q2);
    X = [x;y;];
end

function [J] = getJacobian(q1, q2)
    global l1 l2
    j11 = -l1*sin(q1) - l2*sin(q1+q2);
    j12 = -l2*sin(q1+q2);
    j21 = l1*cos(q1) + l2*cos(q1+q2);
    j22 = l2*cos(q1+q2);
    J = [j11, j12; j21, j22];
end
```


y
x
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

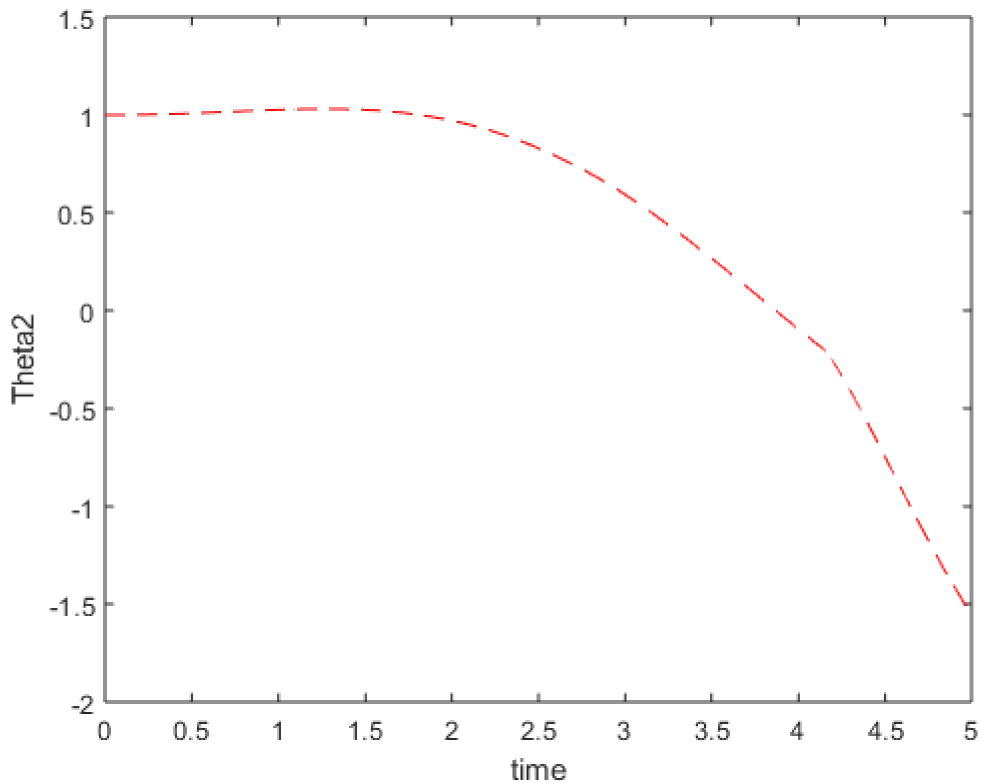

x
time
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.5
1.5
2.5
3
3.5
4
4.5
5

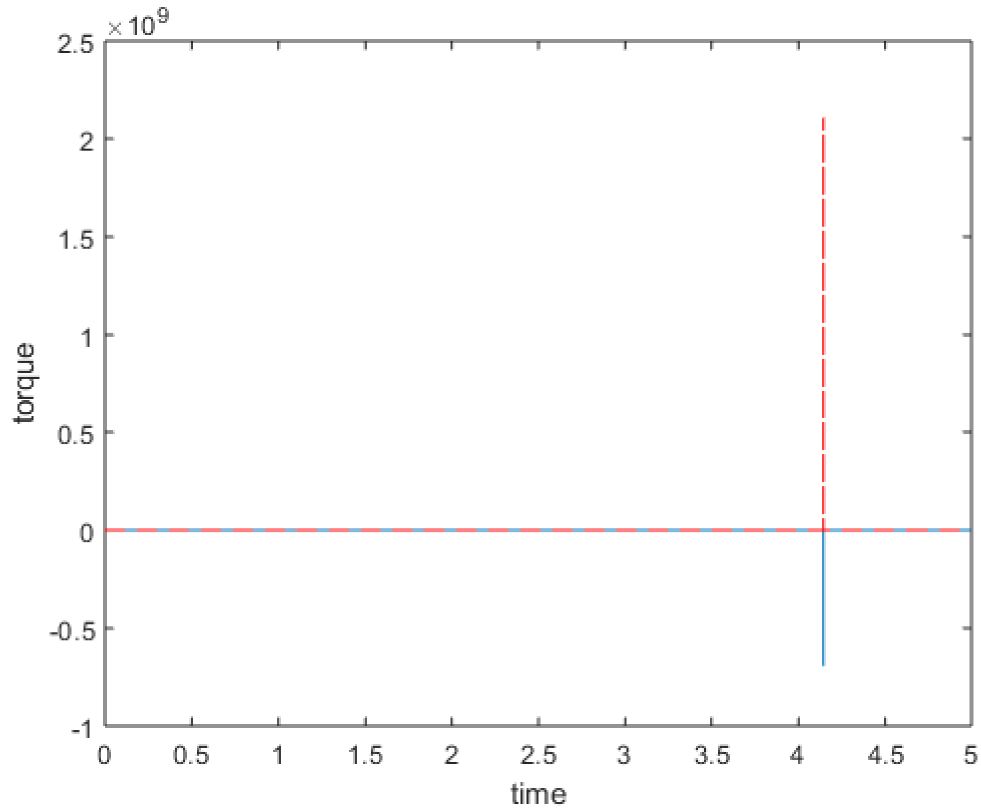


×10^9
2.5
2
1.5
1
0.5
0
-0.5
-1
Force at the end effector
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
time



1
0.5
0
-0.5
-1
-1.5
Theta1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
time





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1\] Consider force F acting at the end effector : to be $F_e = [-2, -1]^T$

$\therefore$ the complete dynamic model is :

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) = \tau + J^T(q) F$$

we know the relationship between geometric jacobian and analytical jacobian is

$$T_A J = J_A \Rightarrow J = T_A^{-1} J_A$$

$$J^T = (T_A^{-1} J_A)^T = J_A^T T_A^{-T}$$

Substituting back into the dynamic model

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) = \tau + J_A^T T_A^{-T} F$$

Let $T_A^{-T} F = F_A$

and we do not consider gravity matrix $N(q)$

$$\therefore M\ddot{q} + C\dot{q} = \tau + J_A^T F_A \quad \text{—— (A)}$$

The relationship between joint space and task space is given by :

$$\dot{x}_A = J_A \dot{q} \Rightarrow \dot{q} = J_A^{-1}(\dot{x}_A) \quad \text{—— (1)}$$

$$\ddot{x}_A = J_A \ddot{q} + \dot{J}_A \dot{q} \Rightarrow \ddot{q} = J_A^{-1}(\ddot{x}_A - \dot{J}_A \dot{q})$$

$$\ddot{q} = J_A^{-1}(\ddot{x}_A - \dot{J}_A J_A^{-1} \dot{x}_A) \quad \text{—— (2)}$$

Substituting (1) and (2) equations into (A) equation, we get

$$M(J_A^{-1}(\ddot{x}_A - \dot{J}_A J_A^{-1}\dot{x}_A)) + C J_A^{-1}(\dot{x}_A) = \tau + J_A^T F_A$$

$$\Rightarrow J_A^{-T} M J_A^{-1}(\ddot{x}_A) + J_A^{-T} C J_A^{-1}(\dot{x}_A) - J_A^{-T} M J_A^{-1} \dot{J}_A J_A^{-1}(\dot{x}_A) = J_A^{-T}\tau + F_A \quad \text{—— (B)}$$

Simplifying:

Let $M_x = J_A^{-T} M J_A^{-1}$ — (C)

$C_x = J_A^{-T} C J_A^{-1} - M_x \dot{J}_A J_A^{-1}$ — (D)

Therefore, the modified dynamics model is

$$M_x (\ddot{x}_A) + C_x (\dot{x}_A) = J_A^{-T} \tau + F_A$$

Jacobian for two link arm, limiting only to x and y linear velocities as the arm is planar.

$$J = J_A = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1+q_2) & -l_2 \sin(q_1+q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1+q_2) & l_2 \cos(q_1+q_2) \end{bmatrix}$$

Therefore, Analytical and Geometric jacobians are equal.
From midterm question we know,

$a = I_1 + I_2 + m_1 r_1^2 + m_2 (l_1^2 + r_2^2)$

$b = m_2 l_1 r_2$

$c = I_2 + m_2 r_2^2$

$$\therefore M = \begin{bmatrix} a + 2b\cos(q_2) & d + b\cos(q_2) \\ d + b\cos(q_2) & d \end{bmatrix}$$

$$C = \begin{bmatrix} -b\sin(q_2)\dot{q}_2 & -b\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ b\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}$$

Calculating $\dot{J}_A$ using Matlab:

$$\dot{J}_A = \begin{bmatrix} -l_1 \dot{q}_1 \cos(q_1) - l_2 \cos(q_1+q_2)(\dot{q}_1+\dot{q}_2) & -l_2 \cos(q_1+q_2)(\dot{q}_1+\dot{q}_2) \\ -l_1 \dot{q}_1 \sin(q_1) - l_2 \sin(q_1+q_2)(\dot{q}_1+\dot{q}_2) & -l_2 \sin(q_1+q_2)(\dot{q}_1+\dot{q}_2) \end{bmatrix}$$

Using equation (C) and (D)

$$M_x = J_A^{-T} M J_A^{-1}$$

$$C_x = J_A^{-T} C J_A^{-1} - M_x \dot{J}_A J_A^{-1}$$

Thus, $F_e = F_A$ as $T_A^{-1} = I$

Hence $M_x(\ddot{x}_A) + C_x(\dot{x}_A) = J_A^{-T} \tau + F_e$