Problem 1

(20pt) Consider the system

$$\dot{x} = -y - x^3$$
$$\dot{y} = x - y^3$$

- 5pt : show that the origin is an equilibrium.
- 15pt: Take $V(x,y) = \frac{x^2+y^2}{2}$, and show the system asymptotically converge to the origin.

Problem 2

(20pt)
Show that $(x(t), y(t)) = (0,0)$ is an asymptotically stable solution of

$$\dot{x} = -x^3 + 2y^3$$
$$\dot{y} = -2xy^2$$

using LaSalle's invarianc principle and function $V = \frac{1}{2}(x^2 + y^2)$.

Problem 3

(30pt) Consider a scalar system

$$\dot{x} = ax^3$$

- 10pt: Show that linearization method fails to determine stability of the origin.
- 15pt: Use the Lyapunov function
  $$V(x) = x^4$$
  to show that the system is stable for $a < 0$ and unstable for $a > 0$.
- 5pt: What can you say about the system for $a = 0$?

Problem 4

(30pt) Consider the pendulum equation

$$\dot{x}_1 = x_2 \tag{1}$$
$$\dot{x}_2 = -\frac{g}{l}\sin x_1 - \frac{k}{l}x_2. \tag{2}$$

Assume zero friction, i.e. let $k = 0$, and show that the origin is stable. (Hint. Show that the energy of the pendulum is constant along all system trajectories.)