$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) = \tau$$

$$Y(q,\dot{q},\ddot{q})\underset{\Delta}{\theta} = \tau$$

Linearity:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} \qquad q = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$$

$$M(q)\ddot{q} = \begin{bmatrix} a\ddot{\theta}_1 + 2b\ddot{\theta}_1\cos\theta_2 + d\ddot{\theta}_2 + b\ddot{\theta}_2\cos\theta_2 \\ d\ddot{\theta}_1 + b\ddot{\theta}_1\cos\theta_2 + d\ddot{\theta}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \ddot{\theta}_1 & 2\ddot{\theta}_1\cos\theta_2 + \ddot{\theta}_2\cos\theta_2 & \ddot{\theta}_2 \\ 0 & \ddot{\theta}_1\cos\theta_2 & \ddot{\theta}_1 + \ddot{\theta}_2 \end{bmatrix}\begin{bmatrix}a\\b\\d\end{bmatrix} = \underset{\sim}{Mat}^1 \cdot \text{param\_vector}$$

$$C(q,\dot{q})\dot{q} = \begin{bmatrix} -b\dot{\theta}_1 S(\theta_2)\dot{\theta}_1 - b\dot{\theta}_2(\sin\theta_2)(\dot{\theta}_1+\dot{\theta}_2) \\ b\dot{\theta}_1 \sin\theta_2 \dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -(\dot{\theta}_1\dot{\theta}_1 S_2 + \dot{\theta}_2 S_2(\dot{\theta}_1+\dot{\theta}_2)) & 0 \\ 0 & \dot{\theta}_1 S_2 \dot{\theta}_1\dot{\theta}_2 & 0 \end{bmatrix}\begin{bmatrix}a\\b\\d\end{bmatrix} = Mat^2 \cdot \text{para\_vector}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} \underset{+N(q)}{\overset{\wedge}{=}} \left[Mat^1 + Mat^2\right]\begin{bmatrix}a\\b\\d\end{bmatrix} = \underbrace{Y(q,\dot{q},\ddot{q})}_{\checkmark}\underset{\Delta\checkmark}{\theta} = \underset{\delta}{\tau}$$

Adaptive Control:

θ: actual sys. param.

$\bar{\theta}$: model param.

$$Y(q,\dot{q},\ddot{q})\theta = \tau$$

$$Y(q,\dot{q},a_q)\bar{\theta} = \tau$$

$$Y(q,\dot{q},\ddot{q})\theta = Y(q,\dot{q},a_q)\bar{\theta}$$

$$Y(q,\dot{q},\ddot{q})\theta - Y(q,\dot{q},\ddot{q})\bar{\theta} = \underline{Y(q,\dot{q},a_q)\bar{\theta}} - Y(q,\dot{q},\ddot{q})\bar{\theta}$$

$$Y(q,\dot{q},\ddot{q})(\theta-\bar{\theta}) = \bar{M}(q)a_q + \cancel{\bar{C}(q,\dot{q})\dot{q}} + \cancel{\bar{N}(q)} - (\bar{M}(q)\ddot{q} + \cancel{\bar{C}(\cdot)\dot{q}} + \cancel{\bar{N}(q)})$$

$$Y(q,\dot{q},\ddot{q})\tilde{\theta} = \bar{M}(a_q - \ddot{q})$$

introduce $\tilde{\theta} = \theta - \bar{\theta}$

$$a_q = \bar{M}^{-1}Y(q,\dot{q},\ddot{q})\tilde{\theta} + \ddot{q}$$

$$a_q = -K_p e - K_D\dot{e} + \ddot{q}_d \qquad e = q - q_d$$

$$-\ddot{e} - K_p e - K_D\dot{e} = \bar{M}^{-1}Y(q,\dot{q},\ddot{q})\tilde{\theta} \quad \cdots\cdots (\star)$$

introduce $x = \begin{bmatrix}e\\ \dot{e}\end{bmatrix}$

$$\dot{x} = Ax + B\phi\tilde{\theta}$$

$$\star: \quad \ddot{e} = -K_P \underbrace{e}_{x_1} - K_D \underbrace{\dot{e}}_{x_2} - \bar{M}^{-1} Y(q,\dot{q},\ddot{q})\tilde{\theta}.$$

$$A = \begin{bmatrix} 0 & I \\ -K_P & -K_D \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \phi = -\bar{M}^{-1} Y(q,\dot{q},\ddot{q})$$

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \xrightarrow[\text{additional state } \tilde{\theta}]{\text{Augment } x \text{ with}} \hat{x} = \begin{bmatrix} e \\ \dot{e} \\ \tilde{\theta} \end{bmatrix} \qquad \boxed{\tilde{\theta} = \theta - \bar{\theta}}$$

$\dot{\tilde{\theta}}$ is undetermined $\rightarrow$ select $\dot{\tilde{\theta}}$ such that $\lim_{t\to\infty} \hat{x}(t) \to 0.$

$$V = x^T P x + \tilde{\theta}^T H \tilde{\theta} \qquad P, H \text{ pos. def.}$$

$$\dot{V} < 0$$

$$\dot{V} = \underbrace{-x^T Q x}_{<0} + \underbrace{2x^T PB\phi\tilde{\theta} + 2\tilde{\theta}^T H \dot{\tilde{\theta}}}_{=0} < 0$$

$$\dot{\bar{\theta}} \neq \boxed{\dot{\tilde{\theta}} = -H^{-1}\phi^T B^T P x} \rightarrow \tilde{\theta} = \theta - \bar{\theta}$$

$\dot{V} = 0$ only if $x = 0 \rightarrow e = 0$ & $\dot{e} = 0$

$$\bar{\theta}_i \qquad \dot{\tilde{\theta}} = \dot{\bar{\theta}}$$