Lecture notes: Robot Interaction with the Environment

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a robot (end-effector) may interact with the environment

- lifting objects, moving it — modifying the state of the environment (e.g., pick-and-place operations)
- exchanging forces (e.g., assembly or surface finishing tasks).

Video of robot finishing: `https://youtu.be/DbCDVHBuOwI`

Tasks with environment interaction

- mechanical machining: deburring, surface finishing, polishing, assembly,...
- tele-manipulation: force feedback improves the performance of human operators in master-slave systems.
- cooperation of multi-manipulator systems:
  - dexterous robot hands
  - power grasp and fine in-hand manipulation require force/motion cooperation and coordinated control of the multiple fingers
- physical exploration for shape identification: force and velocity/vision sensor fusion allow 2D/3D geometric identification of unknown objects and their contour following, with applied contact forces kept under explicit control

for physical interaction tasks, the desired motion generation and execution should be integrated to achieve the desired force - hybrid planning and control objectives

- **active stiffness control:**
  contact forces are handled through position (or velocity) control of the robot end-effector and robot reacts as a compressed spring (or damper) in selected/all directions.
- **impedance control:**
  a desired dynamic behavior is imposed to the robot-environment interaction, e.g., a "model" with forces acting on a mass-spring-damper.
- **hybrid force-motion control:**
  decomposes the task space in complementary sets of directions where either force or motion is controlled.

Interaction tasks of interest

interaction tasks with the environment that require

- **accurate following/reproduction by the robot end-effector of desired trajectories** (even at high speed) defined on the **surface of contact.**
- **control of forces/torques applied at the contact** with environments having low (soft) or high (rigid) stiffness.

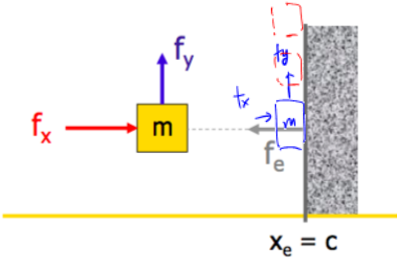
`https://youtu.be/aFuA50H9uek`

Impedance control

- environment = mechanical system undergoing small but finite deformations.
- contact forces arise as the result of a balance of two coupled dynamic systems (robot+environment).
- Control to achieve desired dynamic characteristics are assigned to the force/motion interaction.

example: robot openning the door.

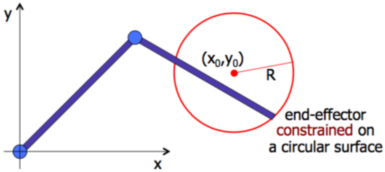
Simple constrained contact situation



- No contact: $m\ddot{x} = f_x$
  $m\ddot{y} = f_y$
- ideal contact = robot (sketched here as a Cartesian mass) + environment are both infinitely STIFF (and without contact friction): $m\ddot{x} = f_x - f_e = 0$
  $m\ddot{y} = f_y$

Example: Constrained dynamics

Problem statement: How can we describe the dynamics of a robot manipulator when its end-effector is constrained to move on an environment surface with nonlinear geometry?



$$M(q)\ddot{q} + C(\,)\dot{q} + N(q) = \tau$$

$$L = K - P$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau$$

Constrained robot dynamic model

Augmented Lagrangian method:
Recall the Lagrangian of the robot dynamics is

$$L = K - P$$

K: kinetic energy.
P: potential energy.
suppose that the task variables are subject to $M < N$ (bilateral) geometric constraints in the general form

$$k(X) = 0$$

where $X \in \mathbb{R}^6$ represents the end effector pose using any minimal representation of $SO(3)$.
and define, using $X = F(q)$.

$$h(q) = K(F(q)) = 0$$

$L = K - P$

Augmented Lagrange: $\boxed{L_a = L(q, \dot{q}) + \lambda^T h(q)}$

λ: Lagrangian multiplier

To make L_a bounded. only if $h(q) = 0$.

$$\frac{d}{dt}\frac{\partial L_a}{\partial \dot{q}} - \frac{\partial L_a}{\partial q} = \tau$$

$$L_a = L + \lambda^T h(q)$$

$$\underbrace{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}} + \underbrace{\frac{d}{dt}\cancel{\frac{\partial}{\partial \dot{q}}(\lambda^T h(q))}^{\to 0} - \frac{\partial(\lambda^T h(q))}{\partial q}} = \tau$$

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q) - \left(\frac{\partial h}{\partial q}\right)^T \lambda = \tau$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) = \tau + \underbrace{\left(\frac{\partial h}{\partial q}\right)^T \lambda}_{\text{Reaction force}}$$

? what is λ

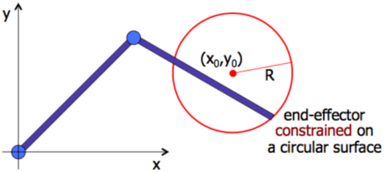
$$h(q) = 0 \quad \Rightarrow \quad \frac{d\, h(q)}{dt} = 0 \quad \Rightarrow \quad \frac{d^2}{dt^2} h = 0$$

$$\frac{d\, h(q)}{dt} = \frac{\partial h}{\partial q} \cdot \frac{dq}{dt} = \frac{\partial h}{\partial q} \dot{q}$$

let $\frac{\partial h}{\partial q} = A(q)$

$$\ddot{h} = 0 \quad \Rightarrow \quad A(q)\ddot{q} + \dot{A}(q)\dot{q} = 0$$

Example cont.



A compliant contact environment