(HW2) 1)

$$\begin{cases} m\ddot{x} = F_1\cos\theta - F_2\sin\theta \\ m\ddot{y} = F_1\sin\theta + F_2\cos\theta - mg \\ J\ddot{\theta} = rF_1 \end{cases}$$

$$z_1 = x - (J/mr)\sin\theta$$
$$z_2 = y + (J/mr)\cos\theta$$

$$\begin{cases} \dot{z}_1 = \dot{x} - \left(\frac{J}{mr}\right)(\cos\theta)\dot{\theta} \\ \dot{z}_2 = \dot{y} + \left(\frac{J}{mr}\right)(-\sin\theta)\dot{\theta} \end{cases}$$

$$\begin{cases} \ddot{z}_1 = \ddot{x} - \left(\frac{J}{mr}\right)(\cos\theta)\ddot{\theta} + \left(\frac{J}{mr}\right)(\sin\theta)\dot{\theta}^2 \\ \ddot{z}_2 = \ddot{y} + \left(\frac{J}{mr}\right)(-\sin\theta)\ddot{\theta} + \left(\frac{J}{mr}\right)(-\cos\theta)\dot{\theta}^2 \end{cases}$$

$$\begin{cases} \ddot{z}_1\cos\theta = \ddot{x}\cos\theta - \left(\frac{J}{mr}\right)(\cos^2\theta)\ddot{\theta} + \left(\frac{J}{mr}\right)\cdot(\sin\theta\cos\theta)\dot{\theta}^2 \\ \ddot{z}_2\sin\theta = \ddot{y}\sin\theta - \left(\frac{J}{mr}\right)(\sin^2\theta)\ddot{\theta} + \left(\frac{J}{mr}\right)(-\sin\theta\cos\theta)\cdot\dot{\theta}^2 \end{cases}$$

from system dynamics:

$$\ddot{x} = \frac{F_1}{m}\cos\theta - \frac{F_2}{m}\sin\theta$$

$$\ddot{x}\cos\theta = \frac{F_1}{m}\cos^2\theta - \frac{F_2}{m}\sin\theta\cos\theta$$

Similarly: $\ddot{y}\sin\theta = \frac{F_1}{m}\sin^2\theta + \frac{F_2}{m}\sin\theta\cos\theta - mg\,\sin\theta$

$\sin\theta$

(HW2) 2/.

$$\ddot{x}\cos\theta + \ddot{y}\sin\theta = \frac{F_1}{m} - mg\sin\theta.$$

$$\ddot{z}_1\cos\theta + \ddot{z}_2\sin\theta = \ddot{x}\cos\theta + \ddot{y}\sin\theta - \left(\frac{J}{mr}\right)\ddot{\theta}$$

$$= \frac{F_1}{m} - mg\sin\theta - \left(\frac{J}{mr}\right)\ddot{\theta}$$

(Because $\ddot{\theta} = \frac{rF_1}{J}.$)

$$= \frac{F_1}{m} - mg\sin\theta - \frac{J}{mr}\cdot\frac{rF_1}{J}$$

$$= mg\sin\theta$$

$$\Rightarrow \ddot{z}_1\cos\theta + \ddot{z}_2\sin\theta = mg\sin\theta$$

$$\Rightarrow \ddot{z}_1\cos\theta + (\ddot{z}_2 - mg)\sin\theta = 0$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\ddot{z}_1}{\ddot{z}_2 - mg}\right)$$

except the singularity $\boxed{\ddot{z}_2 = mg}$

using $\theta = f(\ddot{z}_1, \ddot{z}_2)$

can obtain $\ddot{x}$, $x$, $\dot{x}$

$\ddot{y}$, $y$, $\dot{y}$

$F_1$, $F_2$.