6 Instability Theorem

Theorem 6 *Let $x = 0$ be an equilibrium point of the system $\dot{x} = f(x)$. Let $V : \mathcal{D} \to \mathbb{R}$ be a continuously differentiable function such that*

$$V(0) = 0, \text{ and } V(x_0) > 0$$

for some x_0 with arbitrary small $\|x_0\|$. Define the set

$$U = \{x \in \mathcal{B}_r \mid V(x) > 0\}$$

and suppose that $\dot{V}(x) > 0$ in U. Then $x = 0$ is unstable.

$\dot{x} = ax^3$, $V(x) = x^4$, $\dot{V}(x) = 4ax^6$, when $a > 0$

$\Rightarrow V(0) = 0$, $V(x_0) > 0$

For some x_0 with arbitrary small $\|x_0\|$, $\dot{V}(x) > 0$

Then $x = 0$ is unstable.