RBE 502

HW #1

Solution

P1] a.) $\dot{x}_1 = x_2 \qquad \dot{x}_2 = u \qquad \dot{x} = Ax + Bu \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u$$

Reachability

$W_r = [B, AB, \ldots, A^{n-1}B] = [B, AB]$

$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \det[B\ AB] \neq 0$

$W_r = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ full Rank so reachable

b.) chose K

$u = -Kx$

$s^2 + 2\zeta_0 \omega_0 s + \omega_0^2 \qquad$ w/ $\omega_0 = 1$ & $\zeta_0 = 0.7$

$\dot{x} = Ax - BKx = (A - BK)x$

$A - BK = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K_1 & -K_2 \end{bmatrix}$

$\det(sI - (A - BK)) = \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -K_1 & -K_2 \end{bmatrix}\right) = \det\begin{bmatrix} s & -1 \\ K_1 & s + K_2 \end{bmatrix}$

$= s(s + K_2) - (-1)(K_1) = s^2 + K_2 s + K_1$

$K_1 = \omega_0^2 = 1^2 = 1 \qquad K_2 = 2\zeta_0\omega_0 = 2(0.7)(1) = 1.4$

$K = \begin{bmatrix} 1 & 1.4 \end{bmatrix}$

c.) $x_f = (1,1)$ $x_0 = (-5,0)$ $X_d(0) = x_0$ $X_d(5) = x_f$

$\dot{x}_d = A x_d + B u_d$ $X_{1d}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$X_{2d}(t) = \dot{X}_{1d}(t) = a_1 + 2a_2 t + 3a_3 t^2$

find: a_0, a_1, a_2, a_3

at $t=0$:

$X_{1d}(0) = -5 = a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 = a_0$

$X_{2d}(0) = 0 = a_1 + 2a_2(0) + 3a_3(0)^2 = a_1$

at $t = 5$:

$X_{1d}(5) = 1 = -5 + (0)(5) + a_2(5)^2 + a_3(5)^3$

$X_{2d}(5) = 1 = 0 + (2)a_2(5) + (3)a_3(5)^2$

c.) continued

$$a_2 = \frac{6}{25} - 5a_3$$

$$10\left(\frac{6}{25} - 5a_3\right) + 75a_3 = 1$$

$$25a_3 = \frac{25 - 60}{25}$$

$$a_3 = \frac{-7}{125}$$

$$a_2 = \frac{6}{25} - 5\left(\frac{-7}{125}\right) = \frac{6+7}{25} = \frac{13}{25}$$

$a_0 = -5$

$a_1 = 0$

$a_2 = 13/25$

$a_3 = \frac{-7}{125}$

$$x_{1d}(t) = -5 + \frac{13}{25}t^2 - \frac{7}{125}t^3$$

$$x_{2d}(t) = \frac{26}{25}t - \frac{21}{125}t^2$$

$$\dot{x}_{2d}(t) = u_d(t) = \frac{26}{25} - \frac{42}{125}t$$

d.) $e(t) = X(t) - X_d(t)$

derive dynamic model

$$\dot{x}_d = Ax_d + Bu_d$$

$$\dot{e}(t) = \dot{x}(t) - \dot{x}_d(t) = Ax + Bu - Ax_d - Bu_d = A(x - x_d) + B(u - u_d)$$

d.) continued

$$\dot{e}(t) = A e + B(u - u_d)$$

$$u = -ke + u_d$$

$$\boxed{\dot{e}(t) = Ae - Bke = (A - Bk)e}$$

e.) Design trajectory controller

$$u(t) = -K e(t) + u_d(t)$$

$$K = [k_1 \quad k_2] = [1 \quad 1.4]$$

$$u(t) = -[1 \quad 1.4] \begin{bmatrix} x_1(t) - (-5 + \frac{13}{25}t^2 - \frac{7}{125}t^3) \\ x_2(t) - (\frac{26}{25}t - \frac{21}{125}t^2) \end{bmatrix} + \frac{26}{25} - \frac{42}{125}t$$

$$u(t) = -x_1(t) - 1.4x_2(t) - \frac{99}{25} - \frac{7}{125}t^3 + \frac{178}{625}t^2 + \frac{140}{125}t$$

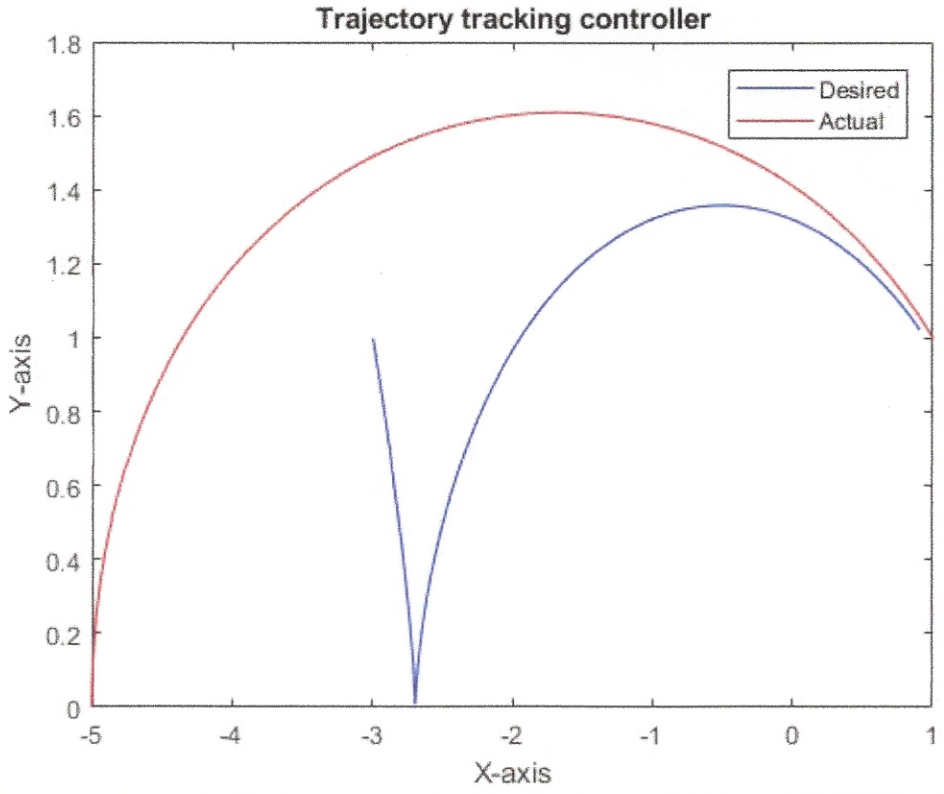
(f)

```
clc

clear all
close all
A = [ 0 1 ; 0 0 ];
B = [ 0 ; 1 ];
K = [1 1.4];
temp_x=[];
temp_x_d=[];
x_0=[-3;1];
dt = 0.01;
for t=0:0.01:5
x1_d = - (0.0560*t^3) + (0.52*t^2) - 5;
x2_d = (1.04*t) - (0.1680*t^2);
x_d  = [x1_d; x2_d];
u_d = 1.04 - (0.3360*t);
u_t=-K*(x_0-x_d)+u_d;
x_dot=A*x_0+B*u_t;
temp_x=[temp_x,x_0];
temp_x_d=[temp_x_d,x_d];
x_0=x_dot*dt+x_0;
```

```
end
figure

plot(temp_x(1,:),temp_x(2,:),'b-');
hold on
plot(temp_x_d(1,:), temp_x_d(2,:), 'r-');
legend('Desired','Actual')
title('Trajectory tracking controller');
xlabel('X-axis')
ylabel('Y-axis')
hold off
```



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