Effect of food for two cats
Randomized complete block design

Presented to the Department of Mathematics and Statistics
California State University, Long Beach

In Partial Fulfillment of the Requirements for the Degree
Master of Science in Mathematics
Option in Statistics

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DEC 2015

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1 Introduction

Pets are like human, they have their own preference to food. In this paper, Randomized Complete Block Design (RCBD) is conducted to check which food the cats like most. Three kinds of food are being tested. Food 1 and Food 2 are the same brand . Food 3 is different brand and is designed for adult cats. Because only two cats are being tested, Latin square and Graeco Latin square design are not satisfied. After the experiment the best food will be chose to feed the cats in the future.

2 Method

Randomized Complete Block Design is conducted with two cats as blocks. It is assuming that if the cats like the food then they will eat more. If they do not like the food, then they will hesitate to eat and eat less. We know that two cats eat totally different by experience. The fat (white) cat eats more, and the slim (black) cat eats less. So it is preferred to be block effect when comparing the effect of day. We can unify the effect of day so that its effect will not be too large to affect the error of analysis.

We separate the cats and supervise them when they are eating and put them in the same room, feeding at the same time. By doing this can unify the effect of days so that they can eat under the same temperature, same time and so forth.

We will test three meals, breakfast, lunch and dinner. We will look into the data and find out that the black cat may eat only 5 gram one meal. The error of measurement is 0.1 gram by estimation of the last digit. Measure twice for test then the error will be 0.2, so 0.2/5=0.04. 4 percent of error can be large enough to affect the analysis. If measuring three meals then 0.2/15=0.013, which is pretty smaller than 0.04.

Feeding at the midnight before go to bed, one more extra meal is offered with three kinds of food. This guarantee at least one food they like to eat and will be at the same status of being full or hungry for the test. By doing so can also prevent the interference of two food tested in two continuous days. This should be offered one day before the experiment begins.

3.1 Model

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}, \quad i = 1, \ldots, a, \quad j = 1, \ldots, b.$$

y_{ij} is the observation of ith food jth cat.

μ is overall mean.

τ_i is effect of ith food.

β_j is the effect of jth cat.

ε_{ij} is a random error.

a=3 and b=2.

3.2 Assumption

$\sum_{i=1}^{a} \tau_i = 0$, $\sum_{j=1}^{b} \beta_j = 0$ and ε_{ij}'s are i.i.d. $N(0, \sigma^2)$

3.3 Hypothesis

$H_0 : \tau_1 = \tau_2 = \cdots = \tau_a = 0$

$H_1 : \tau_i \neq 0$ for some i.

We also need to test the block effect of cat.

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3.4 Formula

$$y_{i.} = \sum_{j=1}^{b} y_{ij}, \quad \bar{y}_{i.} = \frac{y_{i.}}{b}, \quad i = 1, \ldots, a,$$

$$y_{.j} = \sum_{i=1}^{a} y_{ij}, \quad \bar{y}_{.j} = \frac{y_{.j}}{a}, \quad j = 1, \ldots, b,$$

$$y_{..} = \sum_{i=1}^{a} \sum_{j=1}^{b} y_{ij} = \sum_{i=1}^{a} y_{i.} = \sum_{j=1}^{b} y_{.j}, \quad \bar{y}_{..} = \frac{y_{..}}{N}, \quad N = ab.$$

Table 1
Formula for ANOVA table

Source of Variation	SS	DF	MS	F
Treatments	$\sum \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{ab}$	$a-1$	$\frac{SS_{tr}}{a-1}$	$\frac{MS_{tr}}{MS_E}$
Blocks	$\sum \frac{y_{.j}^2}{a} - \frac{y_{..}^2}{ab}$	$b-1$	$\frac{SS_{bl}}{b-1}$	
Error	$SS_T - SS_{tr} - SS_{bl}$	$(a-1)(b-1)$	$\frac{SS_E}{(a-1)(b-1)}$	
Total	$\sum \sum y_{ij}^2 - \frac{y_{..}^2}{ab}$	$ab-1$		

4 Data

The order of food assigned to cats are random. One cell present the total weight of three meal one day. The experiment pertains 9 days. The order of cells do not present the order of food assigned to cats. By looking at the raw data, we can find that both cats like food 2 and dislike food 3. But conclusion is not given before running the statistic analysis.

Table2
Data

	black				white		
food1	36.7	32.2	27.2		62.2	86.8	72.8
food2	53.7	45.5	49.2		111.5	109.1	100.8
food3	21.6	17.3	15.7		49.1	48.1	59.8

5 Analysis

The P values of food and cat are less than 0.001. We are confident to conclude that the effects of food are statistically different and the effects of cats (block effect) statistically different, we should block the cats. MSE is pretty small when comparing to MS of food and cats. This means the effect of day does not affect the experiment much.

Table 3
ANOVA table

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	14898.99722	4966.33241	64.04	<.0001
Error	14	1085.76778	77.55484		
Corrected Total	17	15984.76500			

R-Square	Coeff Var	Root MSE	weight Mean
0.932075	16.20335	8.806523	54.35000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
food	2	6787.863333	3393.931667	43.76	<.0001
cat	1	8111.133889	8111.133889	104.59	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
food	2	6787.863333	3393.931667	43.76	<.0001
cat	1	8111.133889	8111.133889	104.59	<.0001

Figure 1
Normal histogram and QQ plot

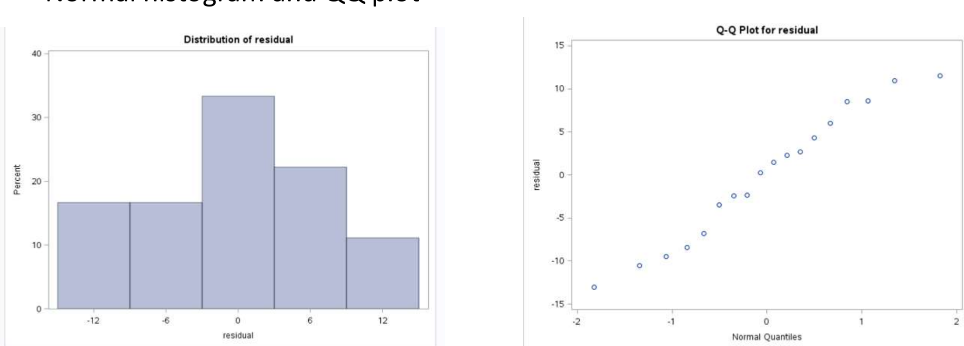

Table 4

Normal test

Tests for Location: Mu0=0				
Test	Statistic		p Value	
Student's t	t	0	Pr > |t|	1.0000
Sign	M	1	Pr >= |M|	0.8145
Signed Rank	S	1.5	Pr >= |S|	0.9661

Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.960667	Pr < W	0.6146
Kolmogorov-Smirnov	D	0.093066	Pr > D	>0.1500
Cramer-von Mises	W-Sq	0.028076	Pr > W-Sq	>0.2500
Anderson-Darling	A-Sq	0.218659	Pr > A-Sq	>0.2500

6 Model adequacy checking

Conduct normal capability test on residuals and get the normal histogram and QQ plot. The results are shown on Figure 1 and table 4. The histogram center to 0 and shows bell shape. The QQ plot seems proximate straight line. The P value of normal tests show greater than 0.1. All these evidences agree with normal distribution.

7 Estimate of effects of food

Because we are only interested in the effect of food. We need to estimate the effects of food to compare which food is the best one. It is recommended to conduct a contrast test or comparison test before making a conclusion. But here the effects are contradicted to each other visually. So it is acceptable to make a conclusion directly.

μ .hat= y../18=55.52

τ 1=y1./6- μ .hat=-2.53

τ 2=y2./6- μ .hat=22.78

τ 3=y3./6- μ .hat=-20.25

We can find out that the effect of food 2 is positive and the largest one. This agree with our first looking at the raw data.

8 Conclusion

Food 2 is the best one and should be chose to feed the cats in the future. Food 3 gets the worst result may due to the reason that it is designed for adult cat. My two cats are just approximate 7 month old. Food 3 is too early for these two baby cats. Food 1 and food 2 are the same bland but with different favor. This experiment shows food 2 favors the cats better. Furthermore, we should take the level of health into consideration. Because taste good is not equal to healthy food. For further experiment, we should take this into analysis and decide which food is healthier for cats.