

Activity 11

OBJECTIVE

To interpret geometrically the meaning of $i = \sqrt{-1}$ and its integral powers.

MATERIAL REQUIRED

Cardboard, chart paper, sketch pen, ruler, compasses, adhesive, nails, thread.

METHOD OF CONSTRUCTION

1. Paste a chart paper on the cardboard of a convenient size.
2. Draw two mutually perpendicular lines $X'X$ and $Y'Y$ intersecting at the point O (see Fig. 11).
3. Take a thread of a unit length representing the number 1 along OX . Fix one end of the thread to the nail at O and the other end at A as shown in the figure.
4. Set free the other end of the thread at A and rotate the thread through angles of 90° , 180° , 270° and 360° and mark the free end of the thread in different cases as A_1 , A_2 , A_3 and A_4 , respectively, as shown in the figure.

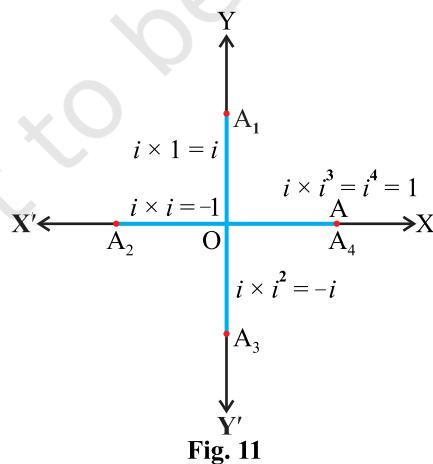


Fig. 11

DEMONSTRATION

1. In the argand plane, OA , OA_1 , OA_2 , OA_3 , OA_4 represent, respectively, 1 , i , -1 , $-i$, 1 .
2. $OA_1 = i = 1 \times i$, $OA_2 = -1 = i \times i = i^2$, $OA_3 = -i = i \times i \times i = i^3$ and so on. Each time, rotation of OA by 90° is equivalent to multiplication by i . Thus, i is referred to as the multiplying factor for a rotation of 90° .

OBSERVATION

1. On rotating OA through 90° , $OA_1 = 1 \times i = \underline{\hspace{2cm}}$.
2. On rotating OA through an angle of 180° , $OA_2 = 1 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$.
3. On rotation of OA through 270° (3 right angles), $OA_3 = 1 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$.
4. On rotating OA through 360° (4 right angles),
 $OA_4 = 1 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$.
5. On rotating OA through n -right angles
 $OA_n = 1 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \dots n \text{ times} = \underline{\hspace{2cm}}$.

APPLICATION

This activity may be used to evaluate any integral power of i .

Activity 12

OBJECTIVE

To obtain a quadratic function with the help of linear functions graphically.

MATERIAL REQUIRED

Plywood sheet, pieces of wires.

METHOD OF CONSTRUCTION

1. Take two wires of equal length.
2. Fix them at O in a plane (on the plywood sheet) at right angle to each other to represent x -axis and y -axis (see Fig.12)
3. Take a piece of wire and fix it in such a way that it meets the x -axis at a distance of a units from O in the positive direction and meets y -axis at a distance of a units below O as shown in the figure. Mark these points as B and A, respectively.

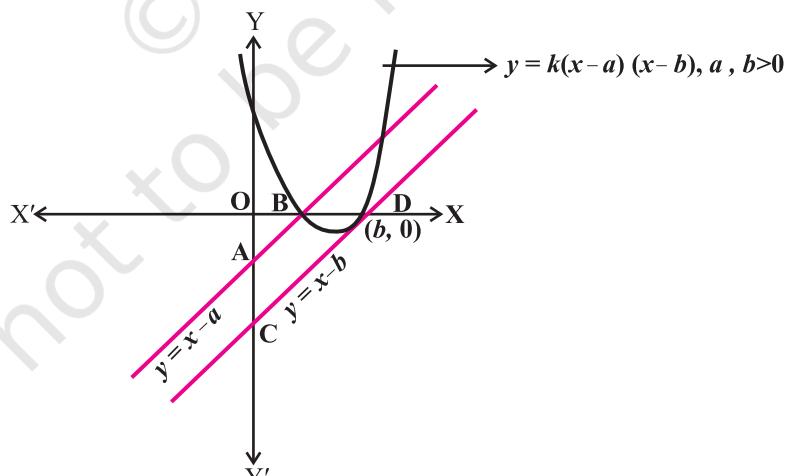


Fig. 12

- Similarly, take another wire and fix it in such a way that it meets the x -axis at a distance of b units from O in the positive direction and meets y -axis at a distance of b units below O as shown in the Fig.12. Mark these points as D and C, respectively.
- Take one more wire and fix it in such a way that it passes through the points where straight wires meet the x -axis and the wire takes the shape of a curve (parabola) as shown in the Fig.12.

DEMONSTRATION

- The wire through the points A and B represents the straight line given by $y = x - a$ intersecting the x and y -axis at $(a, 0)$ and $(0, -a)$, respectively.
- The wire through the points C and D represents the straight line given by $y = x - b$ intersecting x and y axis at $(b, 0)$ and $(0, -b)$, respectively.
- The wire through B and D represents a curve given by the function $y = k(x - a)(x - b) = k[x^2 - (a + b)x + ab]$, where k is an arbitrary constant.

OBSERVATION

- The line given by the linear function $y = x - a$ intersects the x -axis at the point _____ whose coordinates are _____.
- The line given by the linear function $y = x - b$ intersects the x -axis at the point _____ whose coordinates are _____.
- The curve passing through B and D is given by the function $y = \text{_____}$, which is a _____ function.

APPLICATION

This activity is useful in understanding the zeroes and the shape of graph of a quadratic polynomial.

Activity 13

OBJECTIVE

To verify that the graph of a given inequality, say $5x + 4y - 40 < 0$, of the form $ax + by + c < 0$, $a, b > 0$, $c < 0$ represents only one of the two half planes.

MATERIAL REQUIRED

Cardboard, thick white paper, sketch pen, ruler, adhesive.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines $X'OX$ and $Y'OY$ to represent x -axis and y -axis, respectively.
3. Draw the graph of the linear equation corresponding to the given linear inequality.
4. Mark the two half planes as I and II as shown in the Fig. 13.

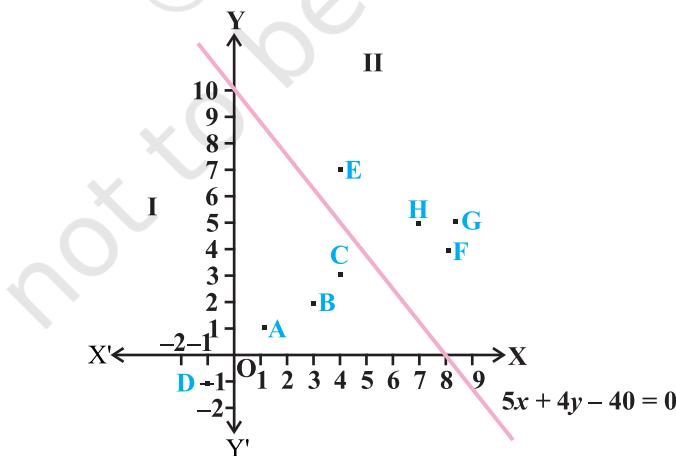


Fig. 13

DEMONSTRATION

1. Mark some points O(0, 0), A(1, 1), B(3, 2), C(4, 3), D(-1, -1) in half plane I and points E(4, 7), F(8, 4), G(9, 5), H(7, 5) in half plane II.
2. (i) Put the coordinates of O (0,0) in the left hand side of the inequality.

$$\text{Value of LHS} = 5(0) + 4(0) - 40 = -40 < 0$$

So, the coordinates of O which lies in half plane I, satisfy the inequality.

- (ii) Put the coordinates of the point E (4, 7) in the left hand side of the inequality.

Value of LHS = $5(4) + 4(7) - 40 = 8 \not< 0$ and hence the coordinates of the point E which lie in the half plane II does not satisfy the given inequality.

- (iii) Put the coordinates of the point F(8, 4) in the left hand side of the inequality. Value of LHS = $5(8) + 4(4) - 40 = 16 \not< 0$

So, the coordinates of the point F which lies in the half plane II do not satisfy the inequality.

- (iv) Put the coordinates of the point C(4, 3) in the left hand side of the inequality.

$$\text{Value of LHS} = 5(4) + 4(3) - 40 = -8 < 0$$

So, the coordinates of C which lies in the half plane I, satisfy the inequality.

- (v) Put the coordinates of the point D(-1, -1) in the left hand side of the inequality.

$$\text{Value of LHS} = 5(-1) + 4(-1) - 40 = -49 < 0$$

So, the coordinates of D which lies in the half plane I, satisfy the inequality.

- (iv) Similarly points A (1, 1), lies in a half plane I satisfy the inequality. The points G (9, 5) and H (7, 5) lies in half plane II do not satisfy the inequality.

Thus, all points O, A, B, C, satisfying the linear inequality $5x + 4y - 40 <$ lie only in the half plane I and all the points E, F, G, H which do not satisfy the linear inequality lie in the half plane II.

Thus, the graph of the given inequality represents only one of the two corresponding half planes.

OBSERVATION

Coordinates of the point A _____ the given inequality (satisfy/does not satisfy).

Coordinates of G _____ the given inequality.

Coordinates of H _____ the given inequality.

Coordinates of E are _____ the given inequality.

Coordinates of F _____ the given inequality and is in the half plane _____.
The graph of the given inequality is only half plane _____.

APPLICATION

This activity may be used to identify the half plane which provides the solutions of a given inequality.

NOTE

The activity can also be performed for the inequality of the type $ax + by + c > 0$.

Activity 14

OBJECTIVE

To find the number of ways in which three cards can be selected from given five cards.

MATERIAL REQUIRED

Cardboard sheet, white paper sheets, sketch pen, cutter.

METHOD OF CONSTRUCTION

1. Take a cardboard sheet and paste white paper on it.
2. Cut out 5 identical cards of convenient size from the cardboard.
3. Mark these cards as C_1, C_2, C_3, C_4 and C_5 .

DEMONSTRATION

1. Select one card from the given five cards.
2. Let the first selected card be C_1 . Then other two cards from the remaining four cards can be : $C_2C_3, C_2C_4, C_2C_5, C_3C_4, C_3C_5$ and C_4C_5 . Thus, the possible selections are : $C_1C_2C_3, C_1C_2C_4, C_1C_2C_5, C_1C_3C_4, C_1C_3C_5, C_1C_4C_5$. Record these on a paper sheet.
3. Let the first selected card be C_2 . Then the other two cards from the remaining 4 cards can be : $C_1C_3, C_1C_4, C_1C_5, C_3C_4, C_3C_5, C_4C_5$. Thus, the possible selections are: $C_2C_1C_3, C_2C_1C_4, C_2C_1C_5, C_2C_3C_4, C_2C_3C_5, C_2C_4C_5$. Record these on the same paper sheet.
4. Let the first selected card be C_3 . Then the other two cards can be : $C_1C_2, C_1C_4, C_1C_5, C_2C_4, C_2C_5, C_4C_5$. Thus, the possible selections are : $C_3C_1C_2, C_3C_1C_4, C_3C_1C_5, C_3C_2C_4, C_3C_2C_5, C_3C_4C_5$. Record them on the same paper sheet.
5. Let the first selected card be C_4 . Then the other two cards can be : $C_1C_2, C_1C_3, C_2C_3, C_1C_5, C_2C_5, C_3C_5$. Thus, the possible selections are: $C_4C_1C_2, C_4C_1C_3, C_4C_2C_3, C_4C_1C_5, C_4C_2C_5, C_4C_3C_5$. Record these on the same paper sheet.

6. Let the first selected card be C_5 . Then the other two cards can be: C_1C_2 , C_1C_3 , C_1C_4 , C_2C_3 , C_2C_4 , C_3C_4 . Thus, the possible selections are: $C_5C_1C_2$, $C_5C_1C_3$, $C_5C_1C_4$, $C_5C_2C_3$, $C_5C_2C_4$, $C_5C_3C_4$. Record these on the same paper sheet.
7. Now look at the paper sheet on which the possible selections are listed. Here, there are in all 30 possible selections and each of the selection is repeated thrice. Therefore, the number of distinct selection $= 30 \div 3 = 10$ which is same as $5C_3$.

OBSERVATION

1. $C_1C_2C_3$, $C_2C_1C_3$ and $C_3C_1C_2$ represent the _____ selection.
2. $C_1C_2C_4$, _____, _____ represent the same selection.
3. Among $C_2C_1C_5$, $C_1C_2C_5$, $C_1C_2C_3$, _____ and _____ represent the same selection.
4. $C_2C_1C_5$, $C_1C_2C_3$, represent _____ selections.
5. Among $C_3C_1C_5$, $C_1C_4C_3$, $C_5C_3C_4$, $C_4C_2C_5$, $C_2C_4C_3$, $C_1C_3C_5$, $C_3C_1C_5$, _____ represent the same selections.
6. $C_3C_1C_5$, $C_1C_4C_5$, _____, _____, represent different selections.

APPLICATION

Activities of this type can be used in understanding the general formula for finding the number of possible selections when r objects are selected from given n distinct objects, i.e., $nC_r = \frac{n!}{r!(n-r)!}$.

Activity 15

OBJECTIVE

To construct a Pascal's Triangle and to write binomial expansion for a given positive integral exponent.

MATERIAL REQUIRED

Drawing board, white paper, matchsticks, adhesive.

METHOD OF CONSTRUCTION

1. Take a drawing board and paste a white paper on it.
2. Take some matchsticks and arrange them as shown in Fig.15.

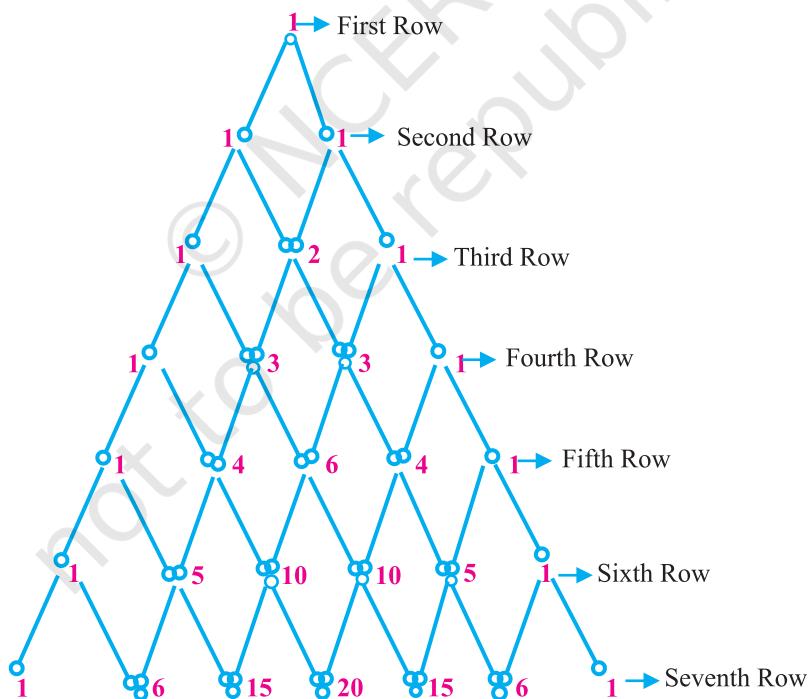


Fig. 15

3. Write the numbers as follows:

1 (first row)

1 1 (second row)

1 2 1 (third row)

1 3 3 1 (fourth row), 1 4 6 4 1 (fifth row) and so on (see Fig. 15).

4. To write binomial expansion of $(a + b)^n$, use the numbers given in the $(n + 1)^{\text{th}}$ row.

DEMONSTRATION

1. The above figure looks like a triangle and is referred to as Pascal's Triangle.
2. Numbers in the second row give the coefficients of the terms of the binomial expansion of $(a + b)^1$. Numbers in the third row give the coefficients of the terms of the binomial expansion of $(a + b)^2$, numbers in the fourth row give coefficients of the terms of binomial expansion of $(a + b)^3$. Numbers in the fifth row give coefficients of the terms of binomial expansion of $(a + b)^4$ and so on.

OBSERVATION

1. Numbers in the fifth row are _____, which are coefficients of the binomial expansion of _____.
2. Numbers in the seventh row are _____, which are coefficients of the binomial expansion of _____.
3. $(a + b)^3 = \underline{\quad} a^3 + \underline{\quad} a^2b + \underline{\quad} ab^2 + \underline{\quad} b^3$
4. $(a + b)^5 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$.
5. $(a + b)^6 = \underline{\quad} a^6 + \underline{\quad} a^5b + \underline{\quad} a^4b^2 + \underline{\quad} a^3b^3 + \underline{\quad} a^2b^4 + \underline{\quad} ab^5 + \underline{\quad} b^6$.
6. $(a + b)^8 = \underline{\quad} + \underline{\quad}$.
7. $(a + b)^{10} = \underline{\quad} + \underline{\quad}$.

APPLICATION

The activity can be used to write binomial expansion for $(a + b)^n$, where n is a positive integer.

Activity 16

OBJECTIVE

To obtain formula for the sum of squares of first n -natural numbers.

MATERIAL REQUIRED

Wooden/plastic unit cubes, coloured papers, adhesive and nails.

METHOD OF CONSTRUCTION

1. Take 1 ($= 1^2$) wooden/plastic unit cube Fig.16.1.
2. Take 4 ($= 2^2$) wooden/plastic unit cubes and form a cuboid as shown in Fig.16.2.
3. Take 9 ($= 3^2$) wooden/plastic unit cubes and form a cuboid as shown in Fig.16.3.
4. Take 16 ($= 4^2$) wooden/plastic unit cubes and form a cuboid as shown in Fig. 16.4 and so on.
5. Arrange all the cube and cuboids of Fig. 16.1 to 16.4 above so as to form an echelon type structure as shown in Fig.16.5.
6. Make six such echelon type structures, one is already shown in Fig. 16.5.
7. Arrange these five structures to form a bigger cuboidal block as shown in Fig. 16.6.



Fig 16.1



Fig 16.2

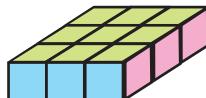


Fig 16.3

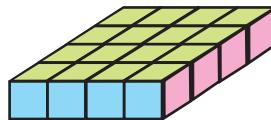


Fig 16.4

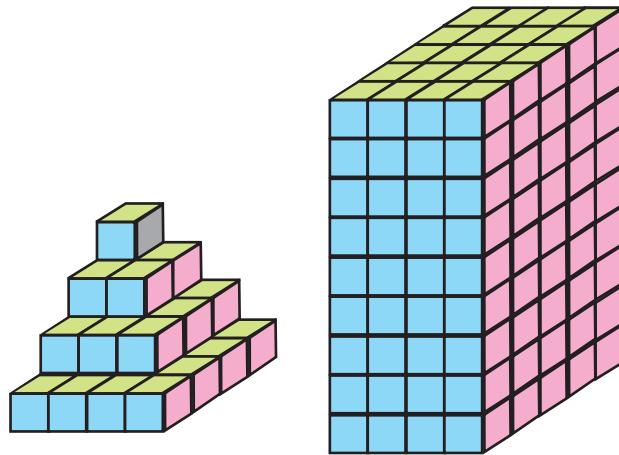


Fig. 16.5

Fig. 16.6

DEMONSTRATION

1. Volume of the structure as given in Fig. 16.5

$$= (1 + 4 + 9 + 16) \text{ cubic units} = (1^2 + 2^2 + 3^2 + 4^2) \text{ cubic units.}$$

2. Volume of 6 such structures $= 6 (1^2 + 2^2 + 3^2 + 4^2)$ cubic units.

3. Volume of the cuboidal block formed in Fig. 16.6 (which is cuboid of dimensions $= 4 \times 5 \times 9$) $= 4 \times (4+1) \times (2 \times 4 + 1)$.

4. Thus, $6 (1^2 + 2^2 + 3^2 + 4^2) = 4 \times (4+1) \times (2 \times 4 + 1)$

$$\text{i.e., } 1^2 + 2^2 + 3^2 + 4^2 = \frac{1}{6} [4 \times (4+1) \times (2 \times 4 + 1)]$$

OBSERVATION

$$1. 1^2 + 2^2 + 3^2 + 4^2 = \frac{1}{6} (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}).$$

$$2. 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6} (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}).$$

$$3. 1^2 + 2^2 + 3^2 + 4^2 + \dots + 10^2 = \frac{1}{6} (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}).$$

$$4. \ 1^2 + 2^2 + 3^2 + 4^2 \dots + 25^2 = \frac{1}{6} (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}).$$

$$5. \ 1^2 + 2^2 + 3^2 + 4^2 \dots + 100^2 = \frac{1}{6} (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}).$$

APPLICATION

This activity may be used to obtain the sum of squares of first n natural numbers

$$\text{as } 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

Activity 17

OBJECTIVE

An alternative approach to obtain formula for the sum of squares of first n natural numbers.

MATERIAL REQUIRED

Wooden/plastic unit squares, coloured pencils/sketch pens, scale.

METHOD OF CONSTRUCTION

1. Take unit squares, 1, 4, 9, 16, 25 ... as shown in Fig. 17.1 and colour all of them with (say) Black colour.

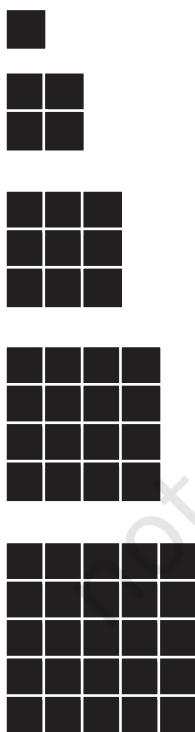


Fig. 17.1

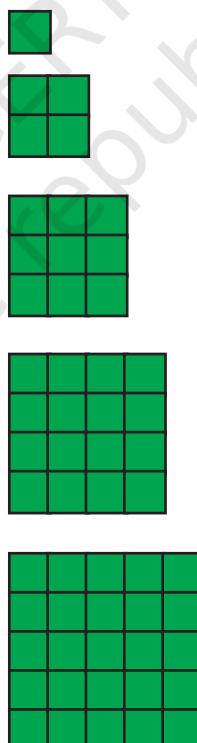


Fig. 17.2

- Take another set of unit squares 1, 4, 9, 16, 25 ... as shown in Fig. 17.2 and colour all of them with (say) green colour.
- Take a third set of unit squares 1, 4, 9, 16, 25 ... as shown in Fig. 17.3 and colour unit squares with different colours.
- Arrange these three set of unit squares as a rectangle as shown in Fig. 17.4.

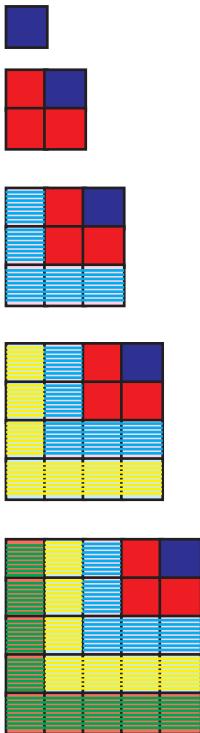


Fig. 17.3

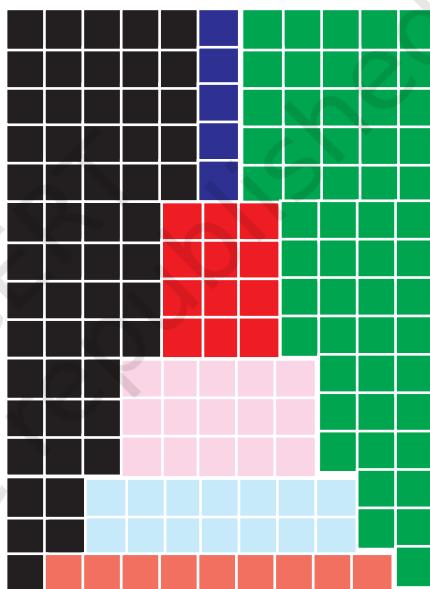


Fig. 17.4

DEMONSTRATION

- Area of one set as given in Fig. 17.1

$$= (1 + 4 + 9 + 16 + 25) \text{ sq. units}$$

$$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2) \text{ sq. units.}$$

- Area of three such sets = $3 (1^2 + 2^2 + 3^2 + 4^2 + 5^2)$

3. Area of rectangle = $11 \times 15 = [2(5) + 1] \left[\frac{5 \times 6}{2} \right]$

$\therefore 3(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{1}{2} [5 \times 6] [2(5) + 1]$

or $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6} [5 \times (5 + 1)] [2(5) + 1].$

OBSERVATION

$$3(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{1}{2} (\underline{\quad} \times \underline{\quad}) (\underline{\quad} + 1)$$

$$\Rightarrow 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6} (\underline{\quad} \times \underline{\quad}) (\underline{\quad} + 1)$$

$$\therefore 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = \frac{1}{6} (\underline{\quad} \times \underline{\quad}) (\underline{\quad} + 1)$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + 10^2 = \frac{1}{6} (\underline{\quad} \times \underline{\quad}) (\underline{\quad} + 1).$$

APPLICATION

This activity may be used to establish

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n + 1)(2n + 1).$$

Activity 18

OBJECTIVE

To demonstrate that the Arithmetic mean of two different positive numbers is always greater than the Geometric mean.

METHOD OF CONSTRUCTION

1. From chart paper, cut off four rectangular pieces of dimension $a \times b$ ($a > b$).
2. Arrange the four rectangular pieces as shown in figure. 18.

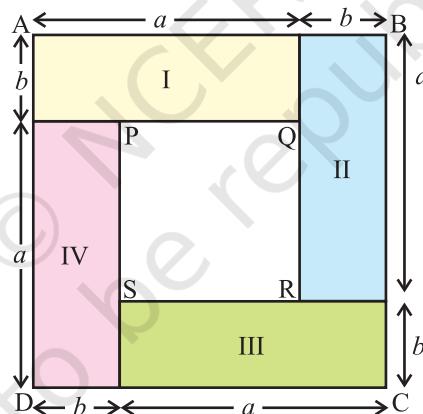


Fig. 18

DEMONSTRATION

1. ABCD is a square of side $(a + b)$ units.
2. Area ABCD = $(a + b)^2$ sq. units.
3. Area of four rectangular pieces = $4(ab) = 4ab$ sq. units.

4. PQRS is a square of side $(a - b)$ units.
5. Area ABCD = Sum of the areas of four rectangular pieces + area of square PQRS.

\therefore Area ABCD > sum of the areas of four rectangular pieces

$$\text{i.e., } (a + b)^2 > 4 ab$$

$$\text{or } \left(\frac{a+b}{2}\right)^2 > ab$$

$$\therefore \frac{a+b}{2} > \sqrt{ab}, \text{i.e., A.M.} > \text{G.M.}$$

OBSERVATION

Take $a = 5\text{cm}$, $b = 3\text{cm}$

$$\therefore AB = a + b = \text{_____ units.}$$

$$\text{Area of ABCD} = (a + b)^2 = \text{_____ sq. units.}$$

$$\text{Area of each rectangle} = ab = \text{_____ sq. units.}$$

$$\text{Area of square PQRS} = (a - b)^2 = \text{_____ sq. units.}$$

$$\text{Area ABCD} = 4 (\text{area of rectangular piece}) + \text{Area of square PQRS}$$

$$\text{_____} = 4 (\text{_____}) + (\text{_____})$$

$$\therefore \text{_____} > 4 (\text{_____})$$

$$\text{i.e. } (a + b)^2 > 4 ab \quad \text{or } \left(\frac{a+b}{2}\right)^2 > ab$$

$$\text{or } \frac{a+b}{2} > \sqrt{ab} \quad \therefore \quad \text{AM} > \text{GM}$$

Activity 19

OBJECTIVE

To establish the formula for the sum of the cubes of the first n natural numbers.

MATERIAL REQUIRED

Thermocol sheet, thermocol balls, pins, pencil, ruler, adhesive, chart paper, cutter.

METHOD OF CONSTRUCTION

1. Take (or cut) a square sheet of thermocol of a convenient size and paste a chart paper on it.
2. Draw horizontal and vertical lines on the pasted chart paper to form 225 small squares as shown in Fig. 19.

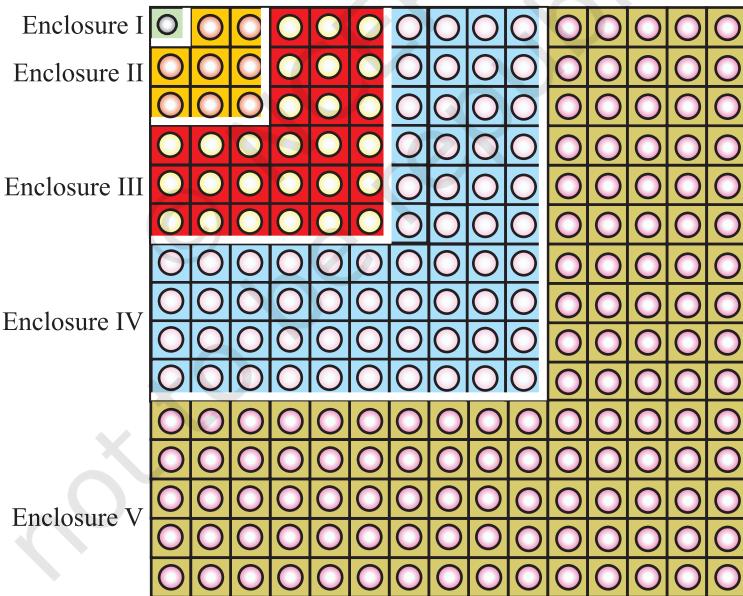


Fig. 19

3. Fix a thermocol ball with the help of a pin at the square on the upper left most corner.

- Fix 2^3 , i.e., 8, thermocol balls with the help of 8 pins on the same square sheet in 8 squares adjacent to the previous square as shown in the figure.
- Fix 3^3 , i.e., 27 thermocol balls with the help of 27 pins on the same square sheet in 27 squares adjacent to the previous 8 squares.
- Continue fixing the thermocol balls in this way till all the squares are filled (see. Fig. 19).

DEMONSTRATION

- Number of balls in Enclosure I $= 1^3 = 1 = \left(\frac{1 \times 2}{2}\right)^2$.
- Number of balls in Enclosure II $= 1^3 + 2^3 = 9 = \left(\frac{2 \times 3}{2}\right)^2$.
- Number of balls in Enclosure III $= 1^3 + 2^3 + 3^3 = 36 = \left(\frac{3 \times 4}{2}\right)^2$.
- Number of balls in Enclosure IV $= 1^3 + 2^3 + 3^3 + 4^3 = 100 = \left(\frac{4 \times 5}{2}\right)^2$.
- Total number of balls in Enclosure V $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 = \left(\frac{5 \times 6}{2}\right)^2$.

OBSERVATION

By actual counting of balls

- Number of balls in Enclosure I $= 1^3 = \text{_____} = \left(\frac{1 \times 2}{2}\right)^2$.

2. Number of balls in Enclosure II = $= 1^3 + 2^3 = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{\underline{\hspace{1cm}}} \right)^2$.

3. Number of balls in Enclosure III

$$= 1^3 + 2^3 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{2} \right)^2.$$

4. Number of balls in Enclosure IV

$$= 1^3 + 2^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{2} \right)^2.$$

5. Number of balls in Enclosure V

$$= (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{2} \right)^2.$$

APPLICATION

This result can be used in finding the sum of cubes of first n natural numbers, i.e.,

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2.$$

Activity 20

OBJECTIVE

To verify that the equation of a line passing through the point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is of the form $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$.

MATERIAL REQUIRED

Cardboard, sketch pen, white paper, adhesive, pencil, ruler.

METHOD OF CONSTRUCTION

1. Take a cardboard of convenient size and paste a white paper on it.
2. Draw two perpendicular lines $X'OX$ and $Y'OY$ on the graph paper. Take same scale for marking points on x and y -axes.
3. Draw the graph of the given two intersecting lines and note down the point of intersection, say (h, k) (see Fig. 20.1)

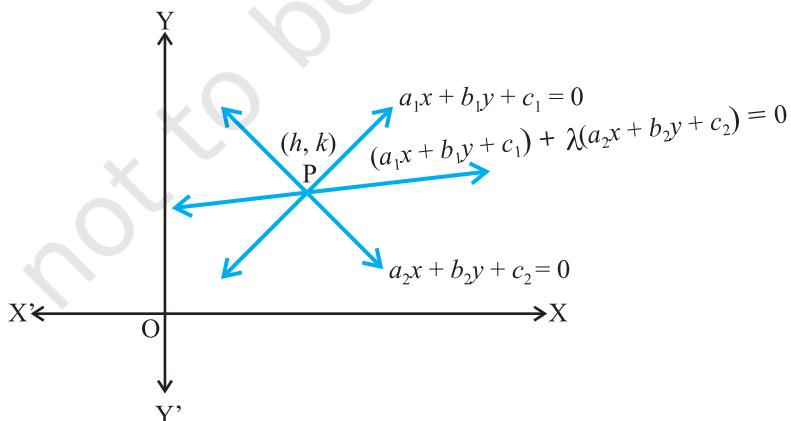


Fig. 20.1

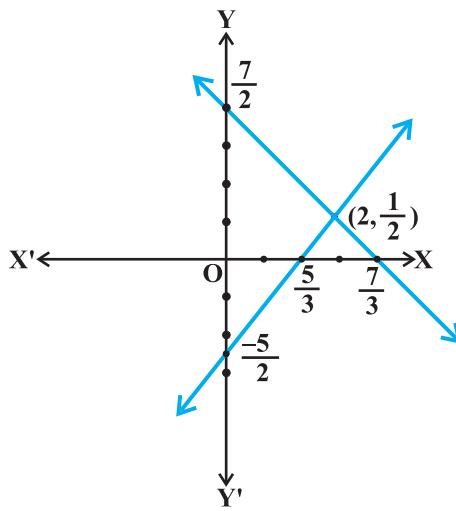


Fig. 20.2

DEMONSTRATION

1. Let the equations of the lines be $3x - 2y = 5$ and $3x + 2y = 7$.
2. The point of intersection of these lines is $\left(2, \frac{1}{2}\right)$ (See Fig. 20.2).
3. Equation of the line passing through the point of intersection $\left(2, \frac{1}{2}\right)$ of these lines is $(3x - 2y - 5) + \lambda(3x + 2y - 7) = 0$ (1)
4. Take $\lambda = 1, -1, 2, \frac{1}{2}$.
5. (i) For $\lambda = 1$, equation of line passing through the point of intersection is $(3x - 2y - 5) + 1(3x + 2y - 7)$, i.e., $6x - 12 = 0$, which is satisfied by the point of intersection $\left(2, \frac{1}{2}\right)$, i.e., $6(2) - 12 = 0$

(ii) For $\lambda = -1$, the equation of line passing through the point of intersection is

$(3x - 2y - 5) - 1(3x + 2y - 7) = 0$ is $-4y + 2 = 0$, which is also satisfied by the point of intersection $\left(2, \frac{1}{2}\right)$.

(iii) For $\lambda = 2$, the equation is $(3x - 2y - 5) + 2(3x + 2y - 7) = 0$, i.e., $9x + 2y - 19 = 0$, which is again satisfied by the point $\left(2, \frac{1}{2}\right)$.

OBSERVATION

1. For $\lambda = 3$, the equation of the line passing through intersection of the lines is _____ which is satisfied by the point $\left(2, \frac{1}{2}\right)$.
2. For $\lambda = 4$, the equation of the line passing through point of the intersection of the lines is _____ which is satisfied by the point of intersection _____ of the lines.
3. For $\lambda = 5$, the equation of the line passing through the intersection of the lines is _____ which is satisfied by the point of intersection _____ of the lines.

APPLICATION

The activity can be used in understanding the result relating to the equation of a line through the point of intersection of two given lines. It is also observed that infinitely many lines pass through a fixed point.