Bioimmagini – Esercitazione 2 – 2 aprile 2012

2D FOURIER ANALYSIS

MATLAB FUNCTIONS

- COMPUTATION OF THE DISCRETE 2D FOURIER TRANSFORM

  - ***fft***: function computing DFT coefficients of a signal with algorithms minimizing the number of required operations.
  - 2D DFT coefficients of an image A can be computed by:
    - Applying ***fft*** on rows and then on columns

      B=fft(A); apply 1D DFT on A columns
      C=fft(B'); apply 1D DFT on B rows
      D=C'; transpose the image to obtain initial conventions
    - Or applying ***fft2*** function. To make the computation faster (if the size of A is not power of 2) it is possible to perform a zero-padding before computing the DFT by using ***fft2*****(**A,Nrows,Ncolumns**)**.
  - NOTE: ***fft2*** supports inputs of data types `double` and `single` and only intensity-type images.

  EXAMPLE
  1. Load and convert an image
     load mri
     im=double(ind2gray(D(:,:,:,10),map));
  2. Compute the DFT2 transform
     IM=fft2(im);
  3. Visualize the absolute value of the DFT2
     figure,imshow(abs(IM),[]),colorbar continuous component in (1,1)
     figure,imshow(fftshift(abs(IM)),[]),colorbar continuous component into the centre
     figure,mesh(fftshift(abs(IM)))
     figure,imshow(log(1+fftshift(abs(IM))),[]),colorbar
  4. Visualize the phase
     figure,imshow(angle(IM),[]),colorbar

- COMPUTATION OF THE INVERSE 2D DISCRETE FOURIER TRANSFORM

  - ***ifft2***: function computing inverse 2D discrete Fourier transform (IDFT 2D)

  EXAMPLE
  im2=ifft2(IM);

  To understand the meaning of module and phase of the DFT2, try to compute the inverse discrete Fourier transform of the two quantities, separately.

- ***immcos.m*** FUNCTION TO BUILD 2D COSINUSOIDAL PATTERNS

```
% N = size of A
% THETA =  tilt of the cosinusoidal pattern with respect to x axis in rad
% FREQ = spatial frequency in cycles/sample (1/FREQ=samples per cycle)
% FI = phase of the cosinusoidal pattern (e.g. 0=cos; -pi/2=sin)

function A=immcos(amp,N,THETA,FREQ,FI);

    WX=2*pi*cos(THETA)*FREQ; % pulsation along x axis
    WY=2*pi*sin(THETA)*FREQ; % pulsation along y axis
    for IX=1:N,
        for IY=1:N,
            ICOL=IX; % column index representing x
            IRIGA=IY; % row index representing y
            A(IRIGA,ICOL)=amp*cos(WX*IX+WY*IY+FI);
        end
    end
```

A=immcos(amp,N,THETA,FREQ,FI) creates a 2D cosinusoidal pattern of NxN elements, of amplitude amp, rotated by THETA (expressed in rad), of spatial frequency FREQ and initial phase FI.

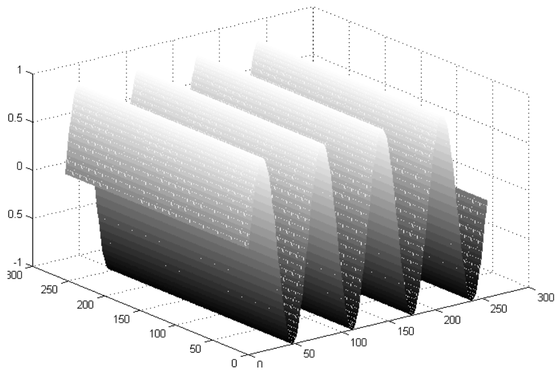
EXAMPLE 1: basis functions $\omega_x \neq 0$, $\omega_y = 0$.

a=immcos(1,256,0,1/64,-pi/2);

64 samples per cycle (4 cycles in 256 samples); phase =-pi/2 (sine);

*f(x,y)= 1*cos(2*PI*x/64-PI/2)*

Create the matrix and visualize it with imshow and mesh. Compute the 2D Fourier transform and visualize module and phase by positioning the continuous component in the frequency plane centre.

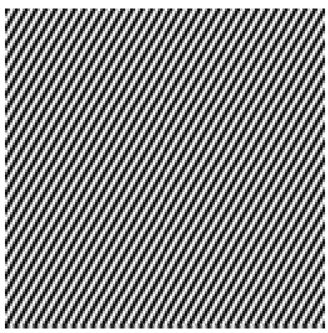


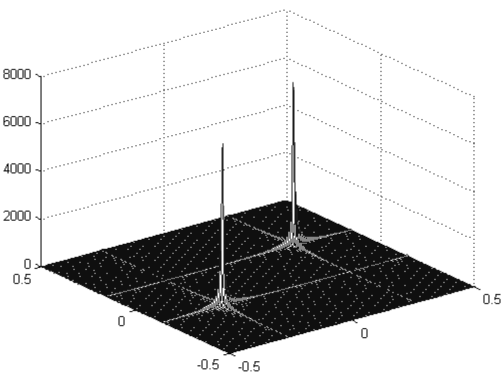
EXAMPLE 2: basis functions $\omega_x \neq 0$, $\omega_y \neq 0$.

b=immcos(1,128,pi/6,0.3,0);

$\omega_x = 2\pi * 0.3 * \cos(\pi/6) = 0.25$; $\omega_y = 2p * 0.3 * \sin(\pi/6) = 0.15$

Create the matrix b and visualize it. Compute the 2D Fourier transform and visualize the module with the continuous component in the centre and with the normalized frequencies along x and axis.

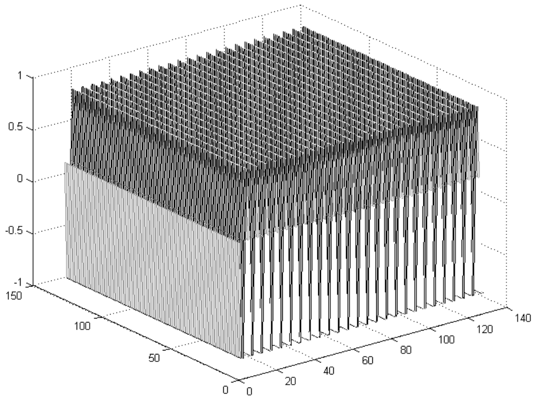


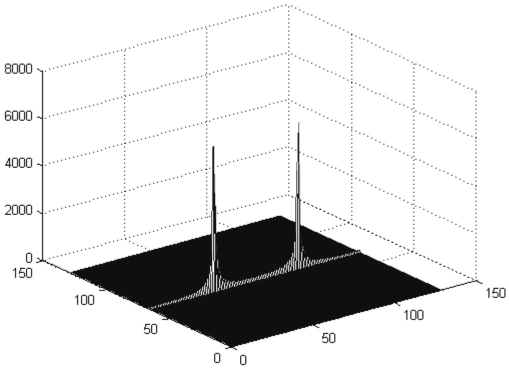


EXAMPLE 3: non-integer number of periods.

c= immcos(1,128,0,1/5,0);

Visualize the c function and the module of the 2D Fourier transform. Compare results with those obtained in example 1.

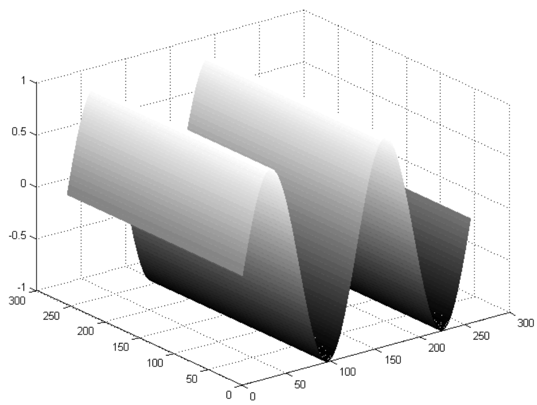


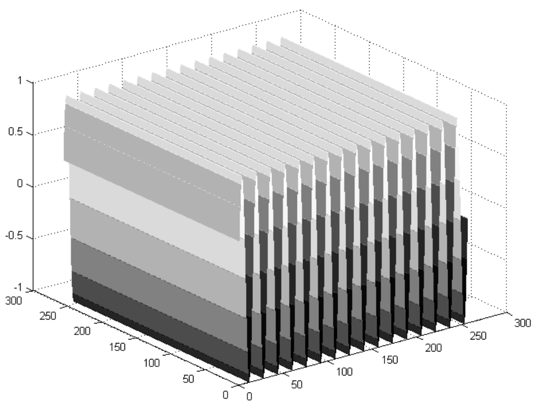


EXAMPLE 4: low spatial frequencies, high spatial frequencies.

d=immcos(1,256,0,1/128,-pi/2); *e=immcos(1,256,0,1/16,-pi/2);*

Visualize functions d and e and the module of the 2D Fourier transform. Note that spatial frequency content of an image provides information about the size of the objects into the image. Low frequencies imply large objects, while high frequencies are generated by small objects.

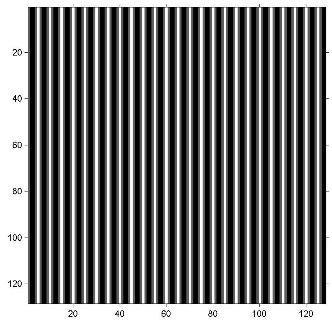


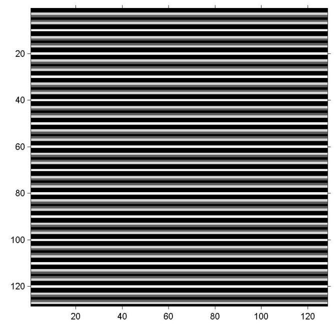


EXAMPLE 5: linearity of the Fourier Transform.

f= immcos(1,128,0,0.2,0); *g= immcos(1,128,pi/2,0.3,0);*

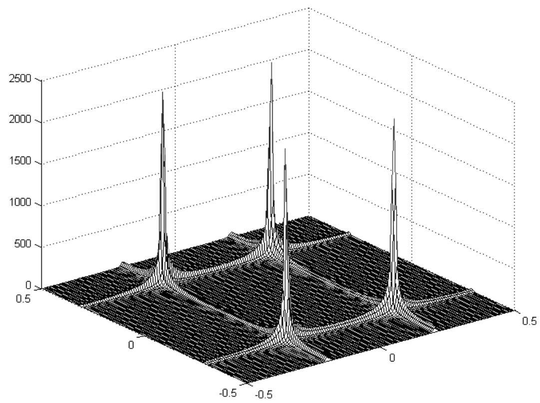
Visualize functions f and g, function f+g and the module of the corresponding Fourier transforms.





EXAMPLE 6: convolution theorem.

*Visualize function f.*g and the module of the Fourier transform.*



EXAMPLE 7: relationship between signal amplitude and spectral power.

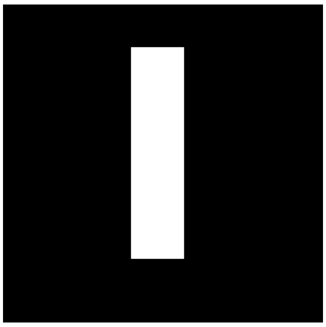
h=immcos(1,256,0,1/128,-pi/2); *i=immcos(10,256,0,1/8,-pi/2);*

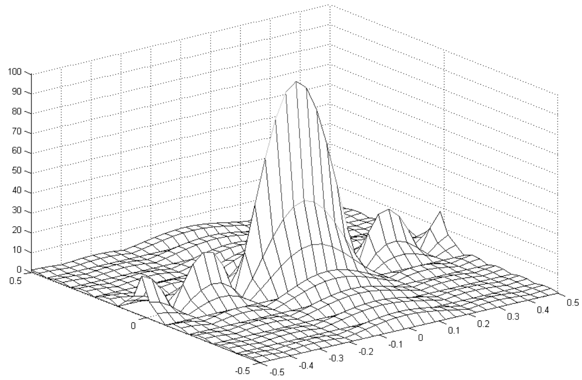
Generate the image h+i and observe the frequency content.

EXAMPLE 8: zero padding.

im1=zeros(30,30);
im1(5:24,13:17) = 1;

Visualize im1 and the Fourier transform module.



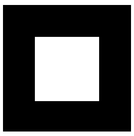


im2=imresize(im1,3,'nearest')/3;
IM2=fft2(im2); *figure,mesh(abs(fftshift(IM2))*
IM3=fft2(im1,90,90); *figure,mesh(abs(fftshift(IM3))*

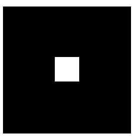
Note that the zero-padding operation doesn't increase the frequency resolution. The spectrum is simply interpolated.

EXAMPLE 9: scaling property.

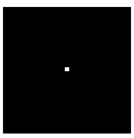
Create the following figures:



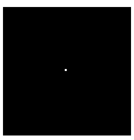
f=zeros(64,64);
f(17:48,17:48)=1;



g=zeros(64,64);
g(27:38,27:38)=1;



h=zeros(64,64);
h(32:33,32:33)=1;



i=zeros(64,64);
i(32,32)=1;

Visualize the 2D DFT module of the generated images.

EXAMPLE 10: relationship between an imaging system PSF and the image frequency content.

Compare the 2D DFT module of two available images that correspond to different imaging systems. Where is the highest power concentrated? At low or high frequencies? Does a relationship exist between the spatial resolution of the considered imaging modality and the power distribution?