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Comparative controller design of an aerial robot

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ABSTRACT

Flying capability opens novel opportunities in robotic applications, such as search and rescue, surveillance navigations and mapping operations. In this article, considering an aerial robot, i.e. an unmanned aerial vehicle (UAV), a few comparable controllers are designed to manage the system performance during various maneuvers. After introducing a nonlinear dynamics model of the system, an adaptive controller is proposed based on feedback linearization approach and using Lyapunov design method. Next, an optimal controller is designed to compare its performance with the designed adaptive controller. Stability analysis for the designed adaptation law is also studied and discussed. To evaluate the performance of designed controllers for a given aerial robot, a comprehensive simulation program is developed. It is shown that tracking errors for the state variables exponentially converge to zero, even in the presence of parameters uncertainty. In particular, it is shown that the proposed adaptive controller, based on its feedback linearization approach and Lyapunov stabilized characteristics, is able to perform perfect path tracking maneuvers, compared to the optimal controller that contains minor errors due to its feed-forward nature.

1. Introduction

To improve on-orbit servicing capabilities, space free-flying robots (SFFR) in which one or more manipulators are mounted on a thruster-equipped base, have been proposed [2]. It is expected that robotic systems play an important role in future space applications, including servicing, construction, and maintenance of space structures on orbit. Therefore, dynamics modeling and motion control of SFFR have been extensively studied [21,3,13,12,11]. Flying capability opens new opportunities in terrestrial applications as well, in performing field services and tasks like search and rescue, observation and mapping operations [6,14,10]. An aerial robot or unmanned aerial vehicle (UAV) may be defined as an aerial vehicle (mostly without on-board manipulators) that uses aerodynamic forces to support its flight in a desired manner, so that a modern UAV is a fully autonomous flying system. Recent technological advancements in navigation and guidance systems, airframe types, payload varieties, and propulsion systems promise more complex goals to be achievable and yet remain cost-effective. The interaction of the air flows generated by propeller contribute to complex aerodynamic forces that affects the vehicle's motion, and in turn makes the motion control of a UAV a challenging task. The system dynamics is not only coupled and nonlinear, but also difficult to be characterized due to the complexity of the system aerodynamic properties [22].

Despite an ordinary controller, an adaptive controller exploits a mechanism for online adjustment of the controller parameters based on measured variables. There are two main approaches for constructing adaptive controllers. One is the so-called model reference adaptive control method, and the other is the so-called self-tuning method. Various nonlinear control methods, fuzzy, and adaptive laws have been applied to UAVs in case of specific longitudinal and lateral maneuvers [4,17,8,7]. Besides using linear controllers for UAVs [15,23], adaptive controllers with a single hidden layer adaptive element have been successfully used on a number of aircraft [19,18,16].

In this article, various nonlinear adaptive and optimal controllers are proposed for an aerial robot. First, nonlinear dynamics model of longitudinal motion is extracted, which will be used to develop the controller. Stability condition for the designed adaptation law is investigated using Lyapunov method to guarantee the stability of controller. Then, based on feed-forward approach, an optimal controller is designed to compare the performance of these controllers, in terms of system input rate and the state variables errors. To evaluate performance of the designed controllers, a comprehensive simulation program has been prepared. Exploiting this simulation routine, the system is simulated under the proposed control laws, and comparison between state variables errors and trajectory tracking characteristics will be discussed.

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Nomenclature

δ_e	Elevator angle
δ_T	Thrust input
γ_i	Positive constant
λ_i	Positive constant
θ	Pitch angel
$\mathbf{\Gamma}$	Adaptive gain matrix
$\mathbf{\Psi}$	Filter matrix
$\mathbf{C}_d$	Transformation matrix
g	Gravity acceleration
$\mathbf{k}$	Control law constant
p	Roll rate
q	Pitch rate
r	Yaw rate
u	Velocity in x direction of body
u_0	Velocity along x direction of body coordinates
v	Velocity in y direction of body
w	Velocity in z direction of body
m	Main body mass
$\mathbf{P}$	Parameter
$\mathbf{x}$	State variable
X	Force in x direction
Y	Force in y direction
Z	Force in z direction
$X_u = \partial X/\partial u$	Stability derivatives that defined for each X parameter relative to u

2. Dynamics modeling

Considering the base of aerial robot as a rigid body, its equations of motion are supposed to be ODEs with constant coefficients. Coefficients in these ODEs are representations of aerodynamic stability derivatives of mass and inertia properties of the plane. These equations could be stated as first order ODEs. For instance, using equation of motion for a rigid body and considering Euler angles and gravity and lifting forces, dynamics equation along longitudinal axis of the plane can be written as:

$$X - mg\,\mathrm{Sin}(\theta) = m(\dot{u} + qw - rv) \tag{1}$$

Each variable in this equation is substituted with its initial value added with a perturbed value as:

$$\begin{aligned} &u = u_1 + \Delta u, \quad v = v_1 + \Delta v, \quad w = w_1 + \Delta w \\ &X = X_1 + \Delta X, \quad q = q_1 + \Delta q, \quad r = r_1 + \Delta r \end{aligned} \tag{2}$$

So, it is obtained:

$$\begin{aligned} &X_1 + \Delta X - mg\,\mathrm{Sin}(\theta_1 + \Delta\theta) \\ &\quad = m\bigg(\frac{d}{dt}(u_1 + \Delta u) + (q_1 + \Delta q).(w_1 + \Delta w) \\ &\qquad - (r_1 + \Delta r).(v_1 + \Delta v)\bigg) \end{aligned} \tag{3}$$

The force ΔX indicates a change in thrust and aerodynamic forces along x direction, which can be presented as a Taylor series in terms of perturbed variables. Assuming ΔX as a function of u, w, δ_e, δ_T parameters then ΔX can be written as:

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\delta_T \tag{4}$$

where $\partial X/\partial u$, $\partial X/\partial w$, $\partial X/\partial\delta_e$, $\partial X/\partial\delta_T$ are known as stability derivatives and their values are defined in the reference flight condition. The variables δ_e and δ_T define the elevator angle and the fuel gate attitude. Eq. (1) for the initial flight condition is written as:

$$X_1 - mg\,\mathrm{Sin}(\theta_1) = m(\dot{u}_1 + q_1 w_1 - r_1 v_1) \tag{5}$$

Assuming symmetric flight conditions yields:

$$v_1, w_1, q_1, r_1 \approx 0 \tag{6}$$

By subtracting Eq. (3) from previous equation and substituting ΔX while reformatting the result, the nonlinear equation of rigid body motion along x direction is obtained as:

$$\begin{aligned} &\left(\frac{d}{dt} - X_u\right)\Delta u - X_w \Delta w + g.S_\theta + \Delta q.\Delta w - \Delta r.\Delta v \\ &\quad = X_{\delta_e}\Delta\delta_e + X_{\delta_T}\Delta\delta_T \end{aligned} \tag{7}$$

where $X_w = \partial X/\partial w/m$ and $X_u = \partial X/\partial u/m$. For the two remained equations of longitudinal motion a similar approach is performed and the following equations will be obtained:

$$\begin{aligned} &-Z_u\Delta u + \left((1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right)\Delta w \\ &\quad - \left((u_0 + Z_q)\frac{d}{dt} - g\sin\theta_0\right)\Delta\theta \\ &\quad + \Delta p.\Delta v - \Delta q.\Delta u = Z_{\delta_e}\Delta\delta_e + Z_{\delta_T}\Delta\delta_T \end{aligned} \tag{8}$$

$$\begin{aligned} &-M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w \\ &\quad + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta + \left(\frac{I_x - I_z}{I_y}\right)\Delta r.\Delta p \\ &\quad + \left(\frac{I_{xz}}{I_y}\right)\Delta p^2 - \left(\frac{I_{xz}}{I_y}\right)\Delta r^2 = M_{\delta_e}\Delta\delta_e + M_{\delta_T}\Delta\delta_T \end{aligned} \tag{9}$$

These nonlinear equations are used to design the controller. Thus, according to two inputs of the system, feedback linearization controller will be designed:

$$\begin{aligned} \Delta\dot{u} &= X_u.\Delta u + X_w.\Delta w - g.\Delta\theta - \Delta q.\Delta w \\ &\quad + X_{\delta_e}.\Delta\delta_e + X_{\delta_T}.\Delta\delta_T \end{aligned} \tag{10}$$

$$\begin{aligned} \Delta\dot{w} &= Z_u.\Delta u + Z_w.\Delta w + u_0.\Delta q + \Delta q.\Delta u + Z_{\delta_e}.\Delta\delta_e \\ &\quad + Z_{\delta_T}.\Delta\delta_T \end{aligned} \tag{11}$$

3. Nonlinear controller design

Various approaches have been proposed for nonlinear controller design, which include feedback linearization, robust control, adaptive control and gain scheduling, and each of these are most suitable for a specific kind of control problem. Feedback linearization has attracted a great deal of research interests in recent years. The idea of simplifying the form of a system's dynamics by choosing a different state representation is not entirely unfamiliar. In mechanics, for instance, it is well known that the form and complexity of a system model depends considerably on the choice of reference frames or coordinate systems. Feedback linearization techniques can be viewed as way of transforming original system models into equivalent models of a simpler form. Thus, they can also be used in the development of robust or adaptive nonlinear controllers.

3.1. Canonical form of feedback linearization

In this form of controller, feedback linearization yields cancellation of nonlinearities in so that the closed-loop dynamics matches a linear form. The idea of feedback linearization, i.e. canceling the nonlinearities and imposing a desired linear dynamics, can be simply applied to a class of nonlinear systems described by the so-called companion or controllability canonical form [1]. In this research, we focus on displacement state variables ($\dot{\mathbf{x}}$) among velocity variables ($\mathbf{V}$) and we deal with the control problem based on the obtained equations of motion. So:

$$\dot{\mathbf{x}} = \mathbf{V} \quad \rightarrow \quad \ddot{\mathbf{x}} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{B}\mathbf{u} \tag{12}$$

where $\mathbf{u}$ is input of the system, $\mathbf{B}$ is input matrix and $\mathbf{A}$ is the state matrix which contains nonlinear terms. Now assuming a control law as:

$$\boldsymbol{\upsilon} = \ddot{\mathbf{x}}_d + 2\mathbf{k}_p\dot{\tilde{\mathbf{x}}} + \mathbf{k}_p^2\tilde{\mathbf{x}} \tag{13}$$

A multi-integral form is obtained:

$$\ddot{\mathbf{x}} = \boldsymbol{\upsilon} \tag{14}$$

So control input of the system is obtained:

$$\boldsymbol{\upsilon} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{B}\mathbf{u} \quad \Rightarrow \quad \mathbf{u} = \mathbf{B}^{-1}(\boldsymbol{\upsilon} - \mathbf{A}\dot{\mathbf{x}}) \tag{15}$$

Assuming control laws:

$$\begin{cases} \upsilon = \ddot{x}_d - 2.\lambda_1.(\dot{x} - \dot{x}_d) - \lambda_1^2.(x - x_d) \\ \mu = \ddot{z}_d - 2.\lambda_2.(\dot{z} - \dot{z}_d) - \lambda_2^2.(z - z_d) \end{cases} \tag{16}$$

Below multi-integral form is achieved:

$$\begin{cases} \Delta\ddot{x} = \Delta\dot{u} = \upsilon \\ \Delta\ddot{z} = \Delta\dot{w} = \mu \end{cases} \tag{17}$$

which satisfies an exponential convergence criterion:

$$\begin{cases} \Delta\ddot{x} - \upsilon = 0 \\ \Delta\ddot{z} - \mu = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} \ddot{\tilde{x}} + 2.\lambda_1.\tilde{x} + \lambda_1^2.\tilde{x} = 0 \\ \ddot{\tilde{z}} + 2.\lambda_2.\dot{\tilde{z}} + \lambda_2^2.\tilde{z} = 0 \end{cases} \tag{18}$$

By substituting these variables in nonlinear dynamic equations for calculating control input we have:

$$\begin{cases} X_{\delta_e}.\Delta\delta_e + X_{\delta_T}.\Delta\delta_T \\ \quad = \upsilon - X_u.\Delta u - X_w.\Delta w + g.\Delta\theta + \Delta q.\Delta w = \Upsilon \\ Z_{\delta_e}.\Delta\delta_e + Z_{\delta_T}.\Delta\delta_T \\ \quad = \mu - Z_u.\Delta u - Z_w.\Delta w - u_0.\Delta q - \Delta q.\Delta u = \Lambda \end{cases} \tag{19}$$

Solving these two equations results in:

$$\begin{cases} \Delta\delta_e = \frac{Z_{\delta_T}.\Upsilon - X_{\delta_T}.\Lambda}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}} \\ \quad = \frac{Z_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Upsilon - \frac{X_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Lambda \\ \Delta\delta_T = \frac{X_{\delta_e}.\Lambda - Z_{\delta_e}.\Upsilon}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}} \\ \quad = \frac{X_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Lambda - \frac{Z_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}\Upsilon \end{cases} \tag{20}$$

and by defining the following parameters which are combinations of input matrix elements:

$$P_1 = \frac{Z_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}, \qquad P_2 = -\frac{X_{\delta_T}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}$$
$$P_3 = -\frac{Z_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}}, \qquad P_4 = \frac{X_{\delta_e}}{X_{\delta_e}.Z_{\delta_T} - Z_{\delta_e}X_{\delta_T}} \tag{21}$$

a simplified model for input control is obtained:

$$\begin{Bmatrix} \Delta\delta_e \\ \Delta\delta_T \end{Bmatrix} = \mathbf{K}.\mathbf{P} \tag{22}$$

where $\mathbf{P} = [P_1 \ P_2 \ P_3 \ P_4]^T$ and $\mathbf{K}$ matrix is defined as:

$$\mathbf{K} = \begin{bmatrix} \Upsilon & \Lambda & 0 & 0 \\ 0 & 0 & \Upsilon & \Lambda \end{bmatrix} \tag{23}$$

3.2. Adaptive control

Adaptive control is an approach to dealing with uncertain systems or time-varying systems [20]. Although the term adaptive can have broad meanings, current adaptive control designs apply mainly to systems with the known dynamic structure, but unknown constants or slowly-varying parameters. Adaptive controllers, whether developed for linear systems or for nonlinear systems, are inherently nonlinear.

3.3. Using Lyapunov design

The controller design procedure is stated by using obtained input control equations using feedback linearization method in the previous section and four unknown parameters Z_{δ_T}, X_{δ_T}, Z_{δ_e}, X_{δ_e}.

Using four unknown parameters, four new unknown parameters are defined as $\hat{P}_1$, $\hat{P}_2$, $\hat{P}_3$, $\hat{P}_4$ and as a result control input will become:

$$\begin{cases} \Delta\delta_e = \hat{P}_1.\Upsilon + \hat{P}_2.\Lambda \\ \Delta\delta_T = \hat{P}_3.\Upsilon + \hat{P}_4.\Lambda \end{cases} \tag{24}$$

The control law which completes first stage of design procedure is similar to feedback linearization method. It is assumed that $\hat{\mathbf{P}}$ is the indicator of unknown parameter and $\tilde{\mathbf{P}}$ is the indicator of estimated parameter error and the relationship between these two parameters is defined as:

$$\mathbf{P} - \hat{\mathbf{P}} = \tilde{\mathbf{P}} \quad \Rightarrow \quad \mathbf{P} = \hat{\mathbf{P}} + \tilde{\mathbf{P}} \tag{25}$$

Lyapunov design method for the control law satisfies two needs of selecting an adaptation rule for regulating parameters, and analyzing convergence characteristics of the controller. For these purposes and considering uncertainty in parameters, the calculated control input will be substituted in nonlinear dynamic equations of the system. Using above definitions, dynamic equations are simplified as:

$$\begin{cases} P_1.(\Delta\ddot{x} - M_1) + P_2.(\Delta\ddot{z} - M_2) = \hat{P}_1.\Upsilon + \hat{P}_2.\Lambda \\ P_3.(\Delta\ddot{x} - M_1) + P_4.(\Delta\ddot{z} - M_2) = \hat{P}_3.\Upsilon + \hat{P}_4.\Lambda \end{cases} \tag{26}$$

where:

$$\begin{cases} M_1 = X_u.\Delta u + X_w.\Delta w - g.\Delta\theta - \Delta q.\Delta w \\ M_2 = Z_u.\Delta u + Z_w.\Delta w + u_0.\Delta q + \Delta q.\Delta u \end{cases} \tag{27}$$

and using the definition:

$$\begin{cases} (\hat{P}_1 + \tilde{P}_1).(\Delta\ddot{x} - M_1) + (\hat{P}_2 + \tilde{P}_2).(\Delta\ddot{z} - M_2) \\ \quad = \hat{P}_1.\Upsilon + \hat{P}_2.\Lambda \\ (\hat{P}_3 + \tilde{P}_3).(\Delta\ddot{x} - M_1) + (\hat{P}_4 + \tilde{P}_4).(\Delta\ddot{z} - M_2) \\ \quad = \hat{P}_3.\Upsilon + \hat{P}_4.\Lambda \end{cases} \tag{28}$$

and after some simplifications we achieve:

$$\begin{bmatrix} \Delta\ddot{x} - \upsilon \\ \Delta\ddot{z} - \mu \end{bmatrix} = \begin{bmatrix} \hat{P}_1 & \hat{P}_2 \\ \hat{P}_3 & \hat{P}_4 \end{bmatrix}^{-1} \begin{bmatrix} -\Delta\ddot{x} & -\Delta\ddot{z} & 0 & 0 \\ 0 & 0 & -\Delta\ddot{x} & -\Delta\ddot{z} \end{bmatrix} \begin{bmatrix} \tilde{P}_1 \\ \tilde{P}_2 \\ \tilde{P}_3 \\ \tilde{P}_4 \end{bmatrix} \tag{29}$$

If the right-hand side tends to be zero, an exponential convergence condition is guaranteed. This situation is exactly equal to the situation which estimation error of parameters tend to be zero using an adaptive law. So, we can write:

$$\begin{bmatrix} \ddot{\tilde{x}} + 2.\lambda_1.\dot{\tilde{x}} + \lambda_1^2.\tilde{x} \\ \ddot{\tilde{z}} + 2.\lambda_2.\dot{\tilde{z}} + \lambda_2^2.\tilde{z} \end{bmatrix} = \hat{\mathbf{B}}.\mathbf{K}.\tilde{\mathbf{P}} \tag{30}$$

By substituting the controller input in dynamic equations of the system, we go through designing adaptation rule and stability guarantee of the system, with uncertain parameters. So:

$$\ddot{\mathbf{X}} = \mathbf{A}\dot{\mathbf{X}} + \mathbf{B}\mathbf{U} = \mathbf{A}\dot{\mathbf{X}} + \mathbf{B}.(\hat{\mathbf{B}}.\mathbf{K}.\tilde{\mathbf{P}})$$
$$\mathbf{Y} = \dot{\tilde{\mathbf{X}}} + \boldsymbol{\Psi}\tilde{\mathbf{X}} = \mathbf{C}\mathbf{X} \tag{31}$$

where $\boldsymbol{\Psi} = \mathrm{diag}[\Psi_1\ \Psi_2\ \ldots\ \Psi_r]$ describes a filter matrix, $\tilde{\mathbf{X}} = [\tilde{x}\ \tilde{z}]^T$ is tracking error and $\mathbf{Y}$ is the filtered output. Assuming that the system is stable and by means of stability theorems, it could be stated that if the defined system is stable, there exist symmetric and positive definite $\boldsymbol{\rho}$ and $\mathbf{Q}$ matrices which satisfy the following equation:

$$\mathbf{A}^T\boldsymbol{\rho} + \boldsymbol{\rho}\mathbf{A} = -\mathbf{Q} \tag{32}$$

$$\boldsymbol{\rho}\mathbf{B} = \mathbf{C} \rightarrow \mathbf{B}^T\boldsymbol{\rho}^T = \mathbf{C} \rightarrow \mathbf{B}^T\boldsymbol{\rho} = \mathbf{C} \tag{33}$$

Now, we define a positive definite Lyapunov function candidate as:

$$\mathbf{V}(\mathbf{X}, \tilde{\mathbf{P}}) = \mathbf{X}^T\boldsymbol{\rho}\boldsymbol{X} + \tilde{\mathbf{P}}^T\boldsymbol{\Gamma}^{-1}\tilde{\mathbf{P}} \tag{34}$$

where $\boldsymbol{\Gamma} = \mathrm{diag}[\gamma_1\ \gamma_2\ \ldots\ \gamma_r]$, $\gamma_i \succ 0$, is the gain matrix. Now, the derivative of Lyapunov function is obtained as:

$$\dot{\mathbf{V}} = \dot{\mathbf{X}}^T\boldsymbol{\rho}\mathbf{X} + \mathbf{X}^T\boldsymbol{\rho}\dot{\mathbf{X}} + 2\tilde{\mathbf{P}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\mathbf{P}}} \tag{35}$$

By substituting from dynamic equations and reformatting and simplifications we achieve:

$$\dot{\mathbf{V}} = -\mathbf{X}^T\mathbf{Q}\mathbf{X} + 2\tilde{\mathbf{P}}^T\left[\boldsymbol{\Gamma}^{-1}\dot{\tilde{\mathbf{P}}} + \mathbf{K}^T\hat{\mathbf{B}}^T\mathbf{Y}\right] \tag{36}$$

As the first term in right-hand side is always negative, if the second term become equal to zero, derivative of Lyapunov function will be negative definite, i.e. $\dot{\mathbf{V}} < 0$, and thus the system will be stable. So:

$$\dot{\tilde{\mathbf{P}}} = -\boldsymbol{\Gamma}\mathbf{K}^T\hat{\mathbf{B}}^T\mathbf{Y} \tag{37}$$

where according to the definition of estimated parameters we have $\dot{\tilde{\mathbf{P}}} = -\dot{\hat{\mathbf{P}}}$ and then the adaptation law obtained from Lyapunov method for unknown parameters of the system is:

$$\dot{\hat{\mathbf{P}}} = \boldsymbol{\Gamma}\mathbf{K}^T\hat{\mathbf{B}}^T\mathbf{Y} \tag{38}$$

4. Optimal controller design

An optimal control system is developed in this section as an autopilot. Other kinds of autopilot systems are available that they reduce human operation and help them to land in undesired conditions. It is assumed that all state variables are available, so that all state variables are measurable by airplane sensors. If one or some variables are not measurable, a state estimator can be used for estimating state variables which are not measured, the so-called observer. State observer estimates state variables based on measurement of output and control variables. It should be noted that one should not take derivatives from a state variable to achieve another variable. Differentiation from a signal often decreases signal precession due to measurement noise.

The optimal controller design method is based on considering a predefined performance index as below:

Table 1
Mass and geometrical specifications of the aerial robot.

$S = 17.1\ \mathrm{m}^2$	$b = 10.18$ m
$m = 1247$ kg	$\bar{c} = 1.74$ m
$I_y = 4067\ \mathrm{kg\,m}^2$	$I_x = 1470\ \mathrm{kg\,m}^2$
$I_{xz} = 0$	$I_z = 4786\ \mathrm{kg\,m}^2$
$Q\,S = 301.3\mathrm{e}6$ N	$u_0 = 53.65$ m/sec
$Q\,S\bar{c} = 524.3\mathrm{e}6$ N m	$Q = \frac{1}{2}\rho u_0^2 = 17.62\mathrm{e}6\ \mathrm{N/m}^2$

$$\mathbf{J} = \int_0^\infty \mathbf{L}(\mathbf{x}, \mathbf{u})\,dt = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}\right)dt \tag{39}$$

In which $\mathbf{Q}$ and $\mathbf{R}$ are assumed as below, then we will get the desired result by solving Riccati equation [9]:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{40}$$

We define the control law as below:

$$\mathbf{u} = -\mathbf{k}\mathbf{e} - \mathbf{k}_r\mathbf{x}_r \tag{41}$$

where $\mathbf{x}_r$ is the reference input and $\mathbf{k}_r$ is the corresponding gain. In feed-forward control, the goal is designing a controller for tracking a reference input. Usually, for such a system, the number of output errors that can be reduced to zero must be equal to the number of inputs [5]. Therefore, for the system with equations in sate space form, and also desirable reference input as below:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \qquad \dot{\mathbf{x}}_r = \mathbf{A}\mathbf{x}_r \tag{42}$$

The reference input gain is computed as [5]:

$$\mathbf{k}_r = \left(\mathbf{C}_d(\mathbf{A} - \mathbf{B}\mathbf{K})^{-1}\mathbf{B}\right)^{-1}\mathbf{C}_d(\mathbf{A} - \mathbf{B}\mathbf{K})^{-1}(\mathbf{A} - \mathbf{A}_r) \tag{43}$$

where $\mathbf{C}_d$ is a transformation matrix that describes the desired outputs. Here, we assume a transformation matrix $\mathbf{C}_d$ as below:

$$\mathbf{C}_d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{44}$$

5. Simulation results and discussions

To study the performance of the proposed adaptive and optimal controllers for an aerial robot, a comprehensive simulation routine has been conducted. This program after taking the time of simulation calculates the reference input value $\mathbf{x}_r$, the ideal assumed angle, the time and distance for performing defined maneuver, and then according to the entered coefficients by the operator, illustrates the results. First, this program calculates stability derivatives for the given UAV based on geometrical characteristics and stability coefficients. It is noticeable that designing controller using feedback linearization method and followed discussions to complete design and simulation of adaptive controller are based on two horizontal and vertical velocity state variables. The system specification is described in Table 1, for more details see [24,25].

Figs. 1 to 7 illustrate take off maneuver for the considered system, applying the proposed adaptive controller, in the presence of uncertainty in parameters. It is observed that using this controller tracking errors of two selected state variables go to zero with an exponential rate (Figs. 4 and 5) which can be realized by comparing each variable with its reference value (Fig. 6). Only fourth state variable is not able to reach its expected value and shows an offset. Considering the fact that designed adaptive controller using Lyapunov method is based on feedback linearization, this offset value could be explained by nonlinear terms in pitch angle rate and its

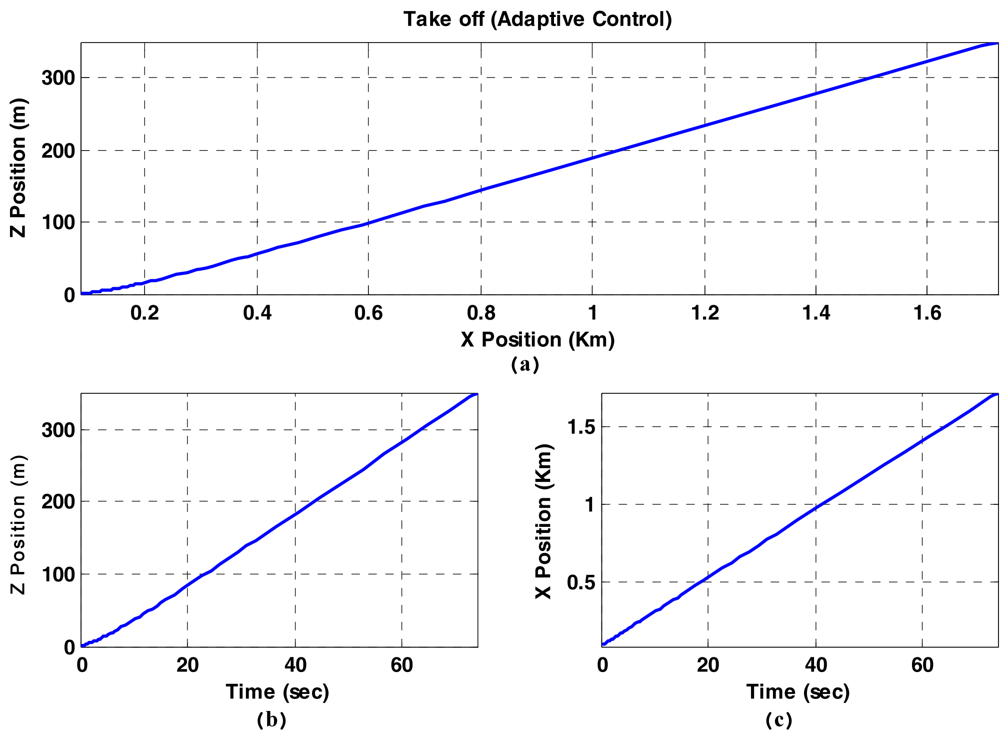

Fig. 1. Flight path in take off maneuver for adaptive controller, (a) path in vertical plane, (b) vertical displacement versus time, (c) horizontal displacement versus time.

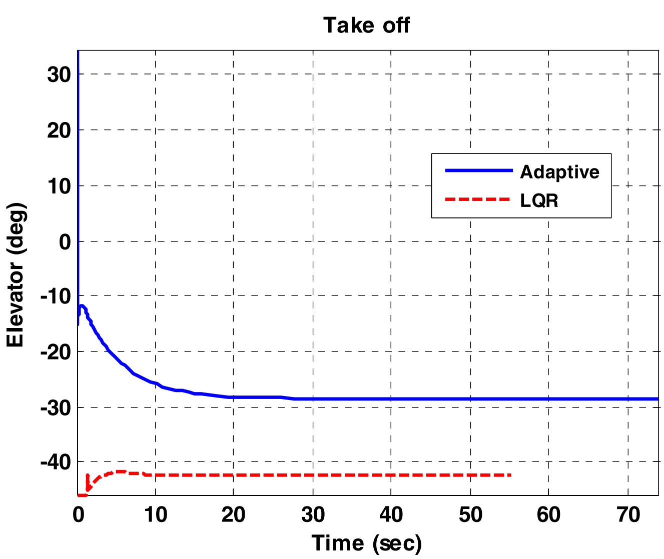

Fig. 2. Elevator input in take off maneuver using adaptive and optimal controllers.

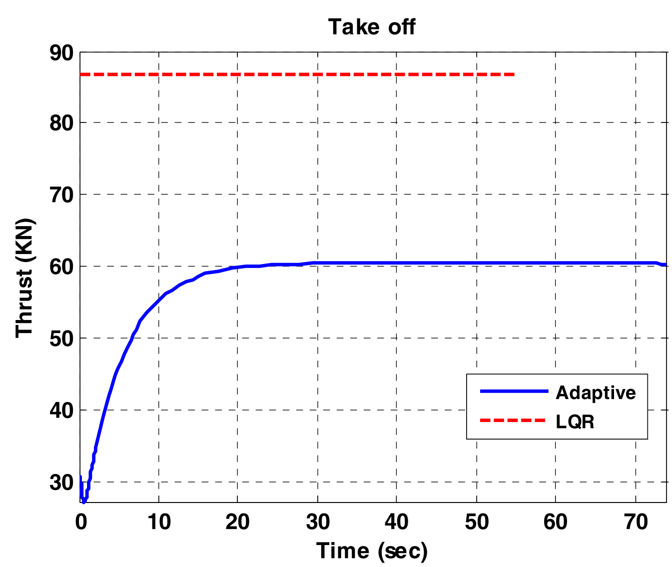

Fig. 3. Thrust input in take off maneuver for adaptive and optimal controllers.

effects on pitch angle. Also, considering a two input UAV system in terms of longitudinal equations (Figs. 2 and 3) and controller design which was used for perfect tracking of first and second state variables in case of feedback linearization, this error occurs in fourth state variables.

It is illustrated clearly in Fig. 7 that the designed adaptation law using Lyapunov method had been able to estimate unknown parameters according to initial values of parameters and gain of adaptation law which are defined by operator, such that the UAV would be able to perform a satisfactory tracking and guarantee stability of the system, in addition to guaranteeing convergence of parameters. Variations in adaptation rule gains change the convergence speed and convergence rate while these parameters are convergent all the time. Observing the fact that changes in coefficients of error polynomial and adaptation law gains which are defined by operator can affect the error convergence rate of state variables, it should be noted that although there is an offset for tracking forth variable, the UAV has been successful in performing a take off maneuver with uncertainty in parameters.

Comparing the results for system inputs rate in adaptive controller with respect to optimal controller shows that they have lower quantities (Figs. 2 and 3). This is due to the fact that adaptive controller estimates unknown parameters in input matrix of the system. However, we must not neglect that adaptive controller is an online control system and requires more calculations. Also, first state variable or horizontal component of velocity that is weighted in feed-forward control contains higher errors in comparison with adaptive controller (Fig. 4). We note that nonlinear terms are considered for adaptive controller in this condition. Similarly, the results are shown for second state variable or vertical component of velocity in adaptive controller compared to those of optimal controller (Fig. 5).

6. Conclusions

In this article, different controllers were designed for an aerial robot or unmanned aerial vehicle (UAV) and the system performance were compared during various maneuvers. First, a canonical feedback linearization controller was designed. According to nonlinear effects in the first and second state variables for longitudinal equations, namely vertical and horizontal components, this controller was designed. Then, an adaptation law was designed to encounter uncertainty in the system based on feedback linearization controller using Lyapunov design method and the obtained controller was implemented on a UAV system. Then, based on feed-forward approach, an optimal controller was designed, and the system input rate and corresponding errors of the state variables were compared those of the designed adaptive controller. A simulation program was used for studying a takeoff maneuver.

Applying the proposed adaptive controller, tracking errors for the two state variables with uncertainty in parameters tends to zero with an exponential rate. Also, the proposed adaptation law

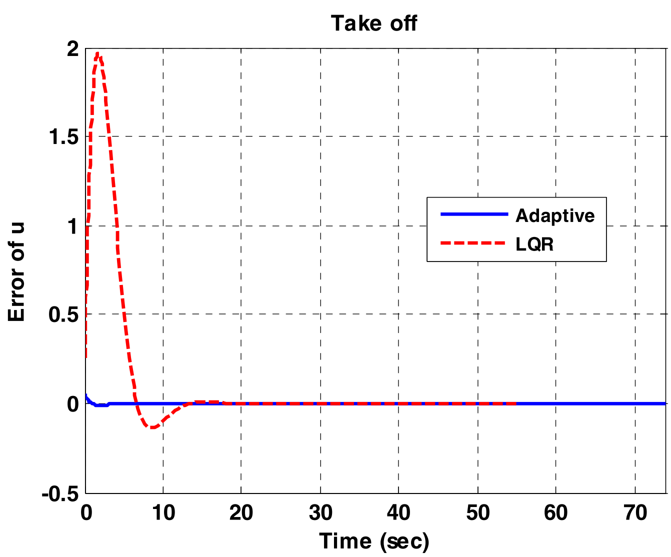

Fig. 4. Error for horizontal component of velocity in take off maneuver for adaptive controller compared to optimal controller.

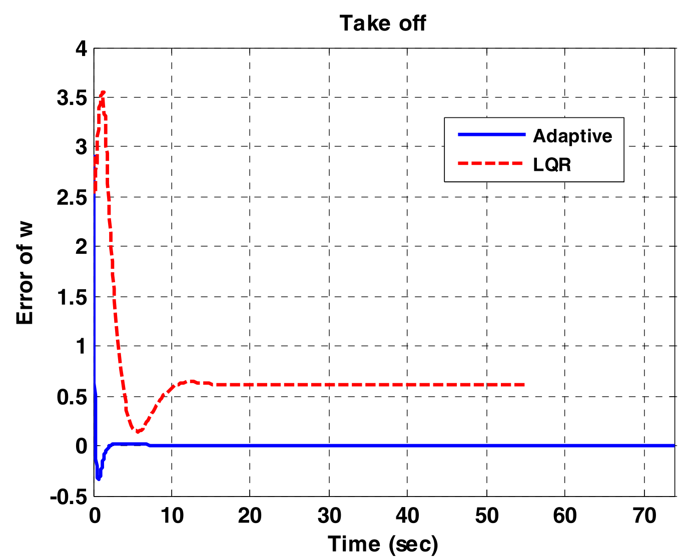

Fig. 5. Error for vertical component of velocity in take off maneuver for adaptive controller compared to optimal controller.

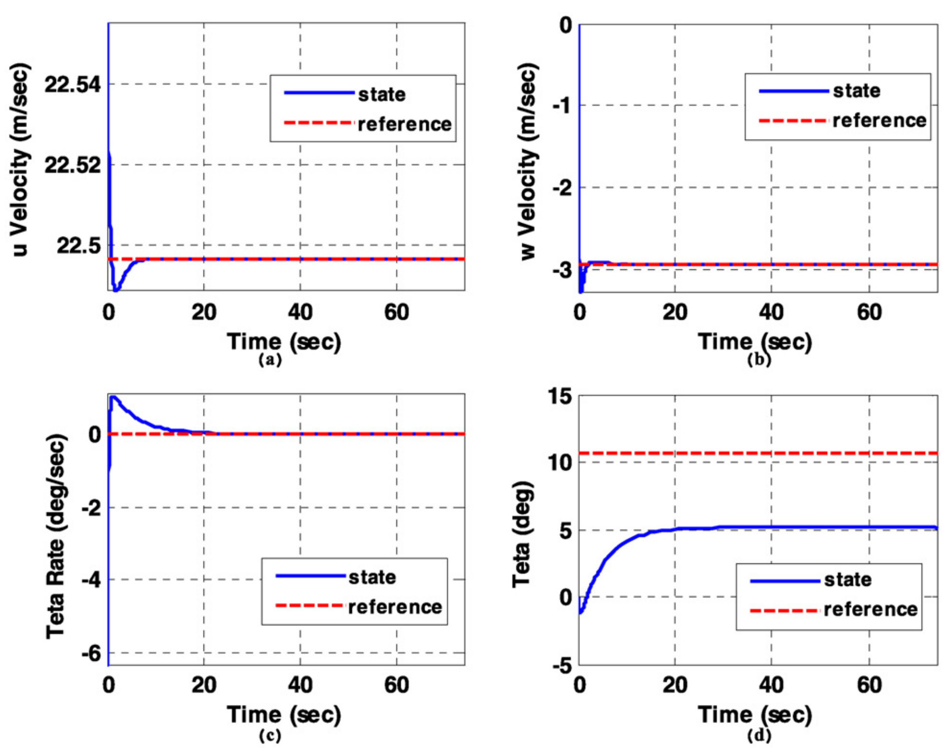

Fig. 6. State variables in take off maneuver for adaptive controller, (a) horizontal component of velocity, (b) vertical component of velocity, (c) pitch angle rate, (d) pitch angle.

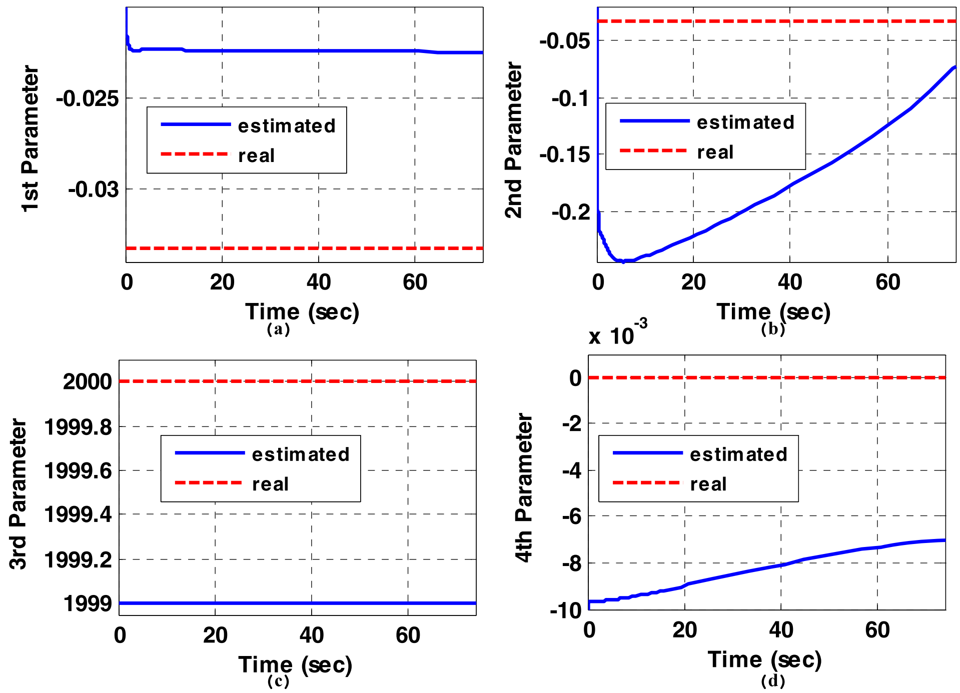

Fig. 7. Parameter estimation values in take off maneuver using adaptive controller, (a) first unknown parameter, (b) second unknown parameter, (c) third unknown parameter, (d) fourth unknown parameter.

is able to estimate the unknown parameters such that reasonable tracking performance is obtained. In addition to guaranteeing convergence of parameters, the proposed adaptive controller based on Lyapunov method guarantees stability of the system. The proposed adaptive controller reveals perfect path tracking characteristics, compared to the optimal controller that contains minor errors due to its feed-forward characteristics.

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