Corso di Elaborazione di Segnali Biomedici

Ripasso dei Filtri Numerici

Esercitazione 4 Aprile 2011

I^st^ PART

1. What is filtering? What is the meaning of filtering a signal?

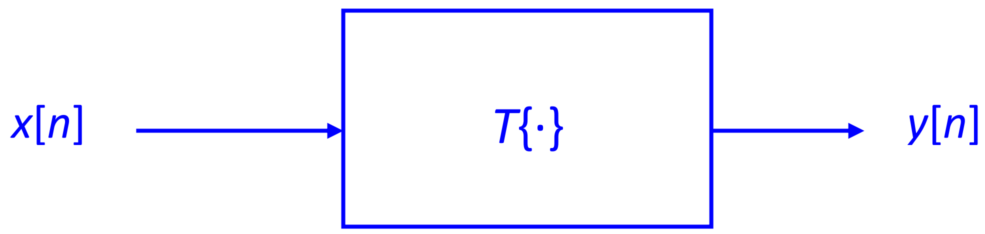
Digital filtering is just changing the frequency-domain characteristics of a given discrete-time signal.

The objectives can be:

- ✓ noise suppression
- ✓ enhancement of selected frequency ranges or edges in images
- ✓ bandwidth limiting (e.g., to prevent aliasing of digital signals or to reduce interference of neighbouring channels in wireless communications)
- ✓ removal or attenuation of specific frequencies
- ✓ special operations like integration, differentiation, etc.

2. Linear Time Invariant Filter (LTI)

A system is defined by a function $T\{\bullet\}$ which transforms the input sequence x[n] in the output sequence y[n].



A system is **linear** if it obeys the principle of superposition.

Principle of superposition. If the input of a system contains the sum of multiple signals, then the output of this system is the sum of the system responses to each separate signal.

In other words a system is linear if and only if:

$$\begin{aligned} y[n] &= T\{x[n]\} = T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\} \\ &= ay_1[n] + by_2[n] \end{aligned} \qquad (1)$$

<u>Example</u>: Let $y[n] = T\{x[n]\} = x^2[n]$, $T\{\cdot\} = (\cdot)^2$

Then $y[n] = T\{x[n]\} = T\{x_1[n] + x_2[n]\} = x_1^2[n] + x_2^2[n] + 2x_1[n] \cdot x_2[n] \neq x_1^2[n] + x_2^2[n]$

Hence, this system is nonlinear!

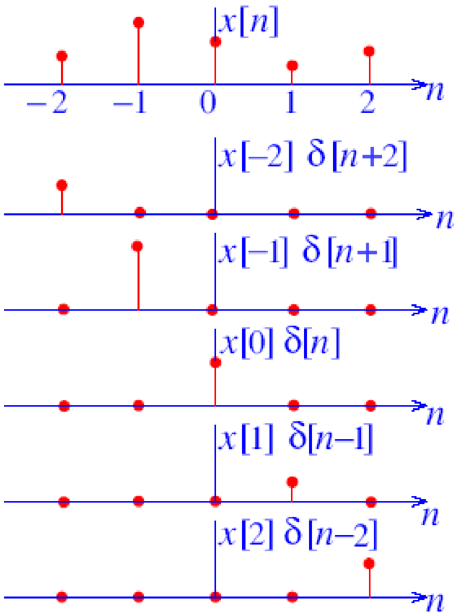
A **time-invariant system** has properties unvarying with time:

if $y[n] = \mathrm{T}\{x[n]\} \Rightarrow y[n-k] = \mathrm{T}\{x[n-k]\}$ (2)

A **linear time-invariant (LTI) system** is both linear and time-invariant; sometimes referred to as a Linear Shift-Invariant (LSI) system.

It is possible to express a sequence as a sum of weighted and translated impulses: given a sequence x[n], it can be expressed as:

$$x_n = \sum_{k=-\infty}^{\infty} x_k \delta_{n-k} \qquad (3)$$



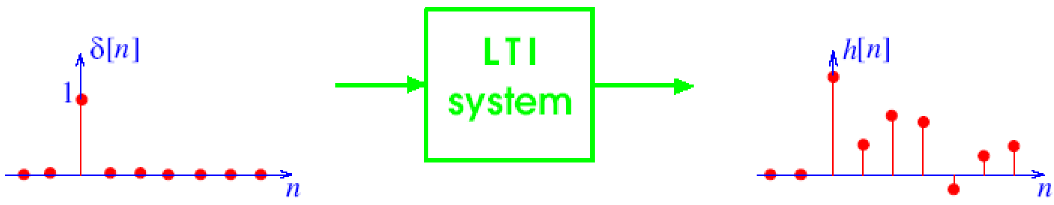
If we consider now, a linear time-invariant (LTI) system described by the function $T\{\bullet\}$, we can derive a new relation for the output of the system from equation (3):

$$y[n] = T\{x[n]\} = T\left\{\sum_{k=-\infty}^{+\infty} x[k]\delta[k-k]\right\} = \sum_{k=-\infty}^{+\infty} x[k]T\{\delta[n-k]\} = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \,, \qquad (4)$$

where $\mathrm{h}[\cdot]$ is defined as **the response to the impulse** $\delta[\mathrm{n}]$ of the LTI system with function $T\{\cdot\}$. The relation (4) comes from two important properties: the distributive property and the time-invariance property, for which the response to $\delta[\mathrm{n-k}]$ is simply $\mathrm{h[n-k]}$.

The equation (4) represents the **convolution sum**:

$$y[n] = T\{x[n]\} = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n] \tag{5}$$



We can conclude that a linear Time-Invariant (LTI) system can be simply described by its impulse response h[n]. The output of the system is univocally determined by the convolution between the input sequence and h[n].

Properties of the convolution:

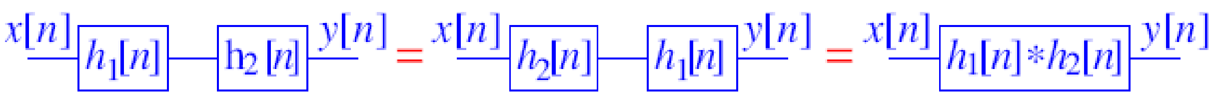
1. If two discrete signals x[n] and h[n] are N and M samples long respectively, then the convolution will be N*M-1 sample long.
2. The *meaning* of the convolution $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$ is the following: one of the two signals is twisted in respect to the time axis and translated by *k* samples, then the sum of the products is made.
3. **Commutative**:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k] = h[n] * x[n] \tag{6}$$

the order in which two sequences are convolved is unimportant!

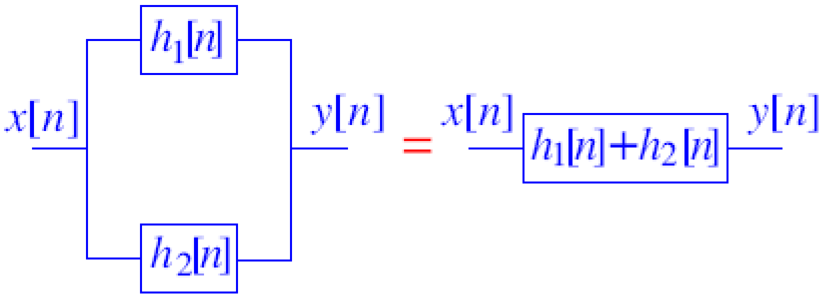
4. **Associative**

$$x[n] * \{h_1[n] * h_2[n]\} = \{x[n] * h_1[n]\} * h_2[n] \tag{7}$$



5. **Distributive**

$$x[n] * \{h_1[n] + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n]$$



Stability

A system is define **stable** if the response to an input sequence, limited in amplitude, is itself limited in amplitude.

A sufficient and necessary condition for the stability is that a linear time-invariant system have a impulse response so that $\sum_{k=-\infty}^{+\infty} |h[n]| < \infty$.

Proof: Let the input x[n] be upper bounded so that $x[n] < L_x < \infty, \forall n \in [-\infty, \infty]$. Then

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} x[n-k]h[k] \right| \leq \sum_{k=-\infty}^{\infty} |x[n-k]||h[k]| \leq L_x \sum_{k=-\infty}^{\infty} |h[k]| \quad \Rightarrow \quad |y[n]| < \infty \Leftrightarrow \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

Causality

A **causal** system is one for which the output y[n] depends on the inputs {…,x[n−2],x[n−1],x[n]} only. In other words, the outputs of the system depend on the past of inputs only.

An LTI system is causal if and only if its impulse response $h[n]=0$ for $n<0$.

Proof:

$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k]$ if $h[n]=0$ for $n<0$

This equation clearly satisfies the definition given above.

Now it remains to be proven that if $h[n] \neq 0$ for $n<0$, then the system can be noncausal. Let $h[n]=0$ for $n<-1$, but $h[-1]=-1$. Then:

$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k] + h[-1]x[n+1] \Rightarrow y[n]$ depends on $x[n+1]$.

The system is noncausal.

EXERCISE

Verify the stability and the causality of the system, whose impulse response is:

$$h[n] = a^n u[n] = \begin{cases} a^n \ , \ n \geq 0 \\ 0 \ \ , \ n < 0 \end{cases}.$$

Solution:

1. Since h[n]= 0 for n<0, the system is causal.
2. To decide on stability, we must compute the sum:

$$S = \sum_{k=-\infty}^{+\infty} |h[n]| = \sum_{k=-\infty}^{+\infty} \left|a^k\right| = \begin{cases} \dfrac{1}{1-|a|}, |a|<1 \\ \infty \ \ , \quad |a|>1 \end{cases}$$

Then the system is stable only if $|a|<1$.

The **Z-transform** of x(n) is equivalent to the DTFT X(ω). If x(n) is a sequence of N samples, its Fourier transform is: $X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-jn\omega}$

By assuming that $z = e^{j\omega}$ $\quad X(\omega) = \sum_{n=0}^{N-1} x(n) z^{-n} = Z(x)$.

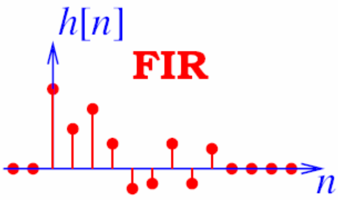
Property of unitary delay: $Z(y(k-1)) = z^{-1}\, Y(z)$

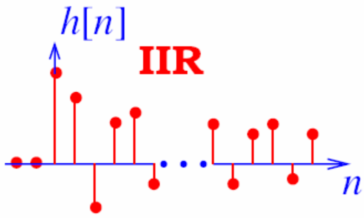
$Z(y(k-1)) = ...+z^{-1}y(0)+ z^{-2}y(1)+... = z^{-1}\, (...+y(0)+ z^{-1}y(1)+....)$

3. Finite and Infinite Impulse Responses (FIR and IIR filter)

If h[n] is an infinite duration sequence, the corresponding filter is called an infinite impulse response(**IIR**) filter.

In turn, if h[n] is a finite duration sequence, the corresponding filter is called a finite impulse response(**FIR**) filter.





A very general form of digital filter can be obtained from the familiar equation: ARMA (AutoRegressive Moving Averaging) model

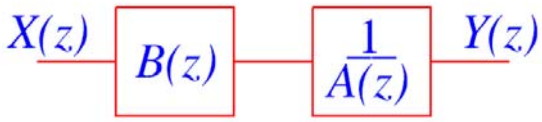
$$y(n) = \sum_{k=1}^{N} a[k] y[n-k] + \sum_{k=0}^{M} b[k] x[n-k] \qquad (1)$$

where $x[n]$ is the filter input signal and $y[n]$ is the filter output signal.

As already seen, the transfer function corresponding to this equation is the following rational function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b[k] z^{-k}}{1 - \sum_{k=1}^{N} a[k] z^{-k}} = \frac{B(z)}{A(z)} \qquad (2)$$

If N= 0, the system is an FIR (nonrecursive) filter. If N>0, the system is an IIR (recursive) filter.



Attention!

The ARMA system equation allows us to design an IIR filter using a finite number of filter coefficients, in order to simplify the computation of the system output.

An IIR filter implemented via the ARMA system equation has M+N+2 filter coefficients, compared to M+1 coefficients for an FIR. Thus, an IIR filter of order max(M,N) has N+1 more degrees of freedom for fitting a desired frequency response than an FIR filter of order M. Consequently, an IIR filter of a particular order can have a sharper frequency response than an FIR filter of the same order.

Basic filter types:

- **lowpass (LP)** filters: to pass low frequencies from zero to a certain cut-off frequency ω_C and to block higher frequencies
- **highpass (HP)** filters: to pass high frequencies from a certain cut-off frequency ω_C to π and to block lower frequencies
- **bandpass (BP)** filters: to pass a certain frequency range $[\omega_{min}, \omega_{max}]$, which does not include zero, and to block other frequencies
- **bandstop (BS)** filters: to block a certain frequency range $[\omega_{min}, \omega_{max}]$, which does not include zero, and to pass other frequencies.

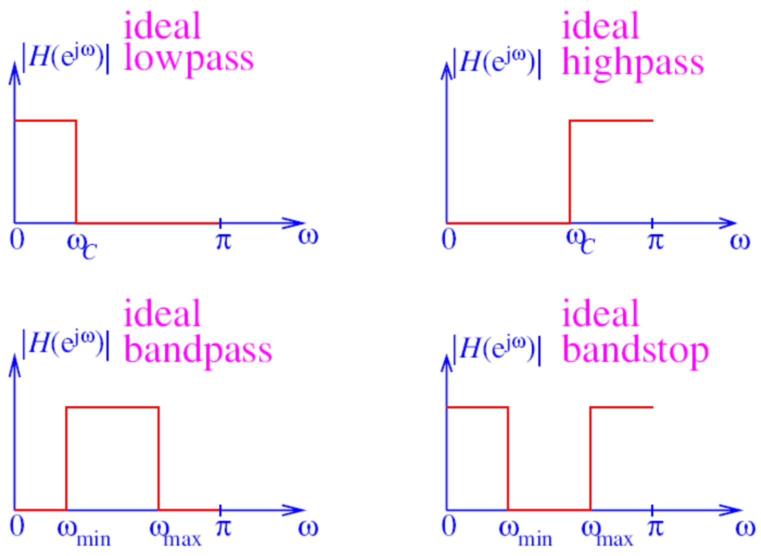


Figure 1: Ideal filters

Frequency responses of practical filters are not shaped in straight lines, i.e., they vary continuously as a function of frequency: they are neither exactly 1 in the passbands, nor exactly 0 in the stopbands.

Example: Lowpass filter specifications:

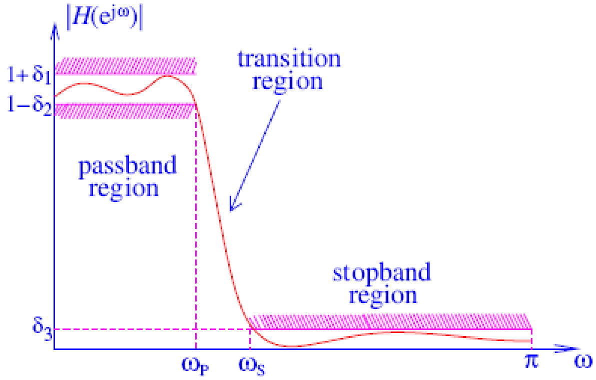


Figure 2: Real Lowpass filters

$$1-\delta_2 \le \left|H(e^{j\omega})\right| \le 1+\delta_1 \qquad \omega \in [0,\omega_P] \tag{3}$$

$$0 \le \left|H(e^{j\omega})\right| \le \delta_3 \qquad \omega \in [\omega_S,\pi] \tag{4}$$

The quantity *max*{δ_1, δ_2} is called **passband (PB) ripple**, and the quantity δ_3 is called **stopband (SB) attenuation**. These filter parameters are usually specified in decibels (dB):

Example: $\delta_1 = \delta_2 = \delta_3 = 0.1$

$$\begin{aligned} A_P &= \max\{20\log_{10}(1+\delta_1), -20\log_{10}(1-\delta_2)\} = \max\{0.828, 0.915\}\text{dB} = 0.915\text{dB} \\ A_S &= -20\log_{10}\delta_3 = 20\text{dB} \end{aligned} \tag{5}$$

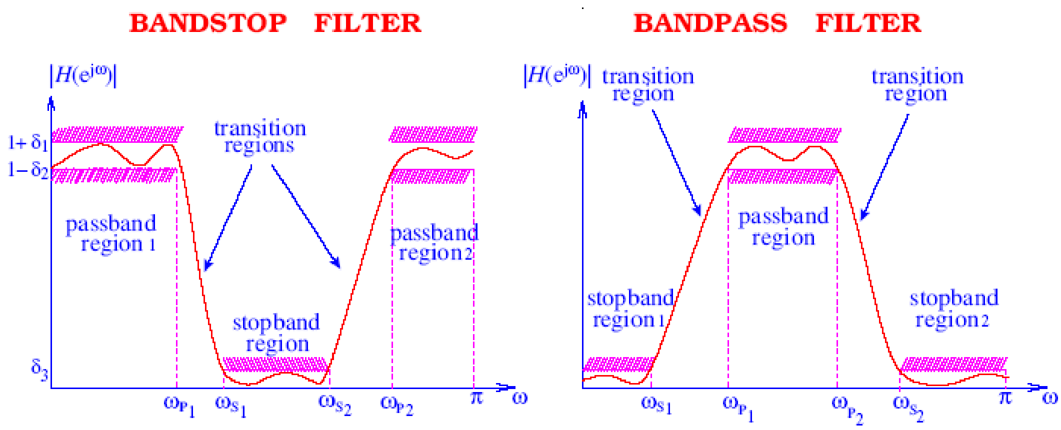


Figure 2: Real BandStop and BandPass filters with their specifications.

4. The Phase Response and Distortionless Transmission

In most filter applications, the magnitude response $|H(e^{j\omega})|$ is of primary concern. However, the phase response may also be important:

$$H(e^{j\omega}) = \left|H(e^{j\omega})\right| e^{j\angle H(e^{j\omega})} \tag{6}$$

where $\angle H(e^{j\omega})$ is the phase response.

If a signal is transmitted through a system (filter) then, this system is said to provide a **distortionless transmission** if the *signal form* remains unaffected, i.e., *if the output signal is a delayed and scaled replica of the input signal*.

Two are the conditions of distortionless transmission:

- ✓ the system must amplify (or attenuate) each frequency component uniformly, i.e., the magnitude response must be uniform within the signal frequency band;
- ✓ the system must delay each frequency component by the same discrete-time value (i.e., number of samples)→linear-phase system

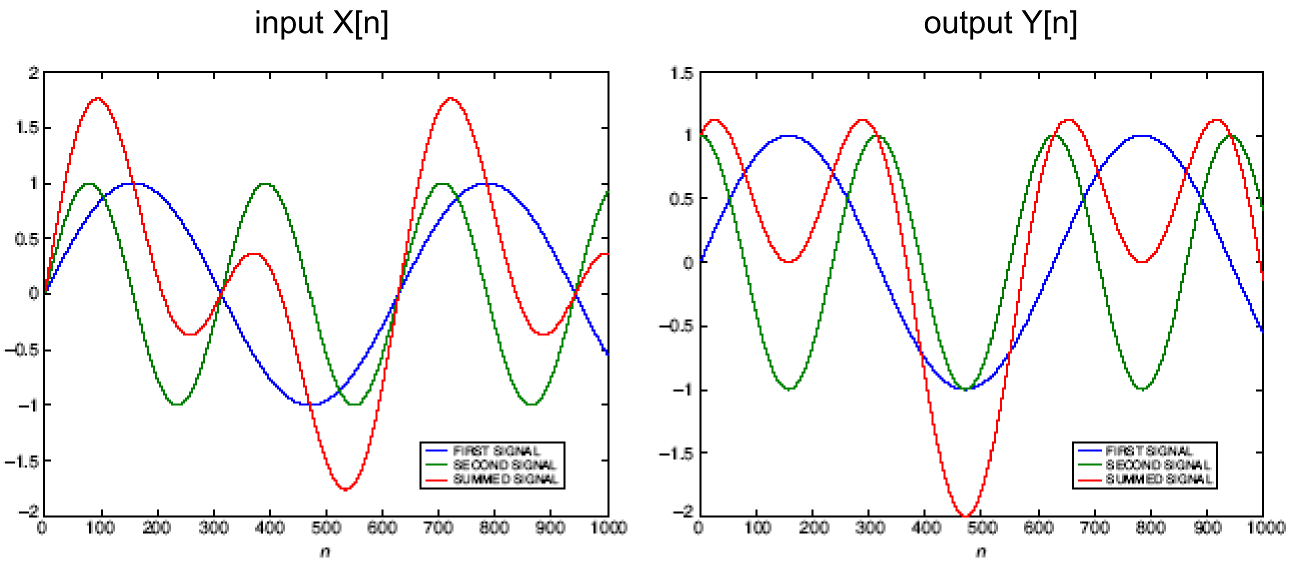


Figure 3: the original signal (red line) is constituted by two harmonics f1 and f2 (green and blue lines). The output signal is distorted because one of its harmonic (green line) was simply delayed.

Definition 1: The phase delay $\Theta(\omega)$ of a filter is the relative delay imposed on a particular frequency component of an input signal:

$$\Theta(\omega) = -\frac{\angle H(e^{j\omega})}{\omega} \tag{7}$$

measured in samples.

To satisfy the distortionless response phase condition, the phase delay must be frequency-independent, i.e., uniform for each frequency: $\Theta(\omega)=\alpha=const$. That is, all frequency components will be delayed by α samples. A filter having this property is called a **linear-phase filter** because its phase varies linearly with the frequency ω.

Property 1: Filters with symmetric impulse responses have linear phase. (However, filters with nonsymmetric impulse responses may also have linear phase!!!! sufficient but not necessary!!!).

Property 2: A filter with an impulse responses that is symmetric around the time origin n=0 is a special case of linear-phase filters, i.e., α=0, referred to as a **zero-phase filter**.

Example. Ideal zero-phase lowpass filter:

$$\widetilde{H}(e^{j\omega}) = \left|H(e^{j\omega})\right| e^{j\omega 0} = \left|H(e^{j\omega})\right|$$

$$\widetilde{h}[n] = \frac{\sin(\omega_C n)}{\pi n}$$

In practice, zero-phase filters are only perfectly zero-phase in their passband. In the stopband, it is possible to have a different value[1].

Definition 2: A system is referred to as a generalized linear-phase system if its frequency response can be expressed in the form:

$$H(e^{j\omega}) = A(e^{j\omega}) e^{j\angle(\beta-\alpha\omega)} \tag{8}$$

with a real function $A(e^{j\omega})$ and β=const, α=const. Note that the phase delay is now:

$$\Theta(\omega) = -\frac{\angle H(e^{j\omega})}{\omega} = \alpha - \frac{\beta}{\omega} \neq const \tag{9}$$

[1] Smith, J.O. Introduction to Digital Filters, http://www-ccrma.stanford.edu/~jos/filters/.

and consequently, the generalized linear-phase systems produce *phase distortions*. Generalized linear-phase systems are useful in that they can produce near-distortionless transmission of the envelope of band-limited signals.

Remarks

- ✓ Causal FIR filters can be linear-phase or generalized linear-phase. In filter design, the **symmetry** or **antisymmetry property** is typically utilized to ensure linear-phase or generalized linear-phase, respectively.
- ✓ **Causal FIR filters cannot be zero-phase**, except for the trivial case of $h[n] = c\delta[n]$, where c is a constant.

5. FIR Filter Design

Transfer function: $H(z)=\sum_{n=0}^{M} b[n]z^{-n} = \sum_{n=0}^{M} h[n]z^{-n}$

Frequency response: $H(e^{j\omega})=\sum_{n=0}^{M} b[n]e^{-j\omega n} = \sum_{n=0}^{M} h[n]e^{-j\omega n}$

Advantages of FIR Filters:

- ✓ FIR filters are inherently stable
- ✓ they can be designed to have linear phase or generalized linear phase
- ✓ they are convenient to implement

How to construct a FIR filter?

Impulse response truncation method:

1 Choose the desired amplitude response according to the *filter class* (LP, HP, BP, BS)

2 Choose the *filter's phase characteristics*: integer or fractional group delay, initial phase

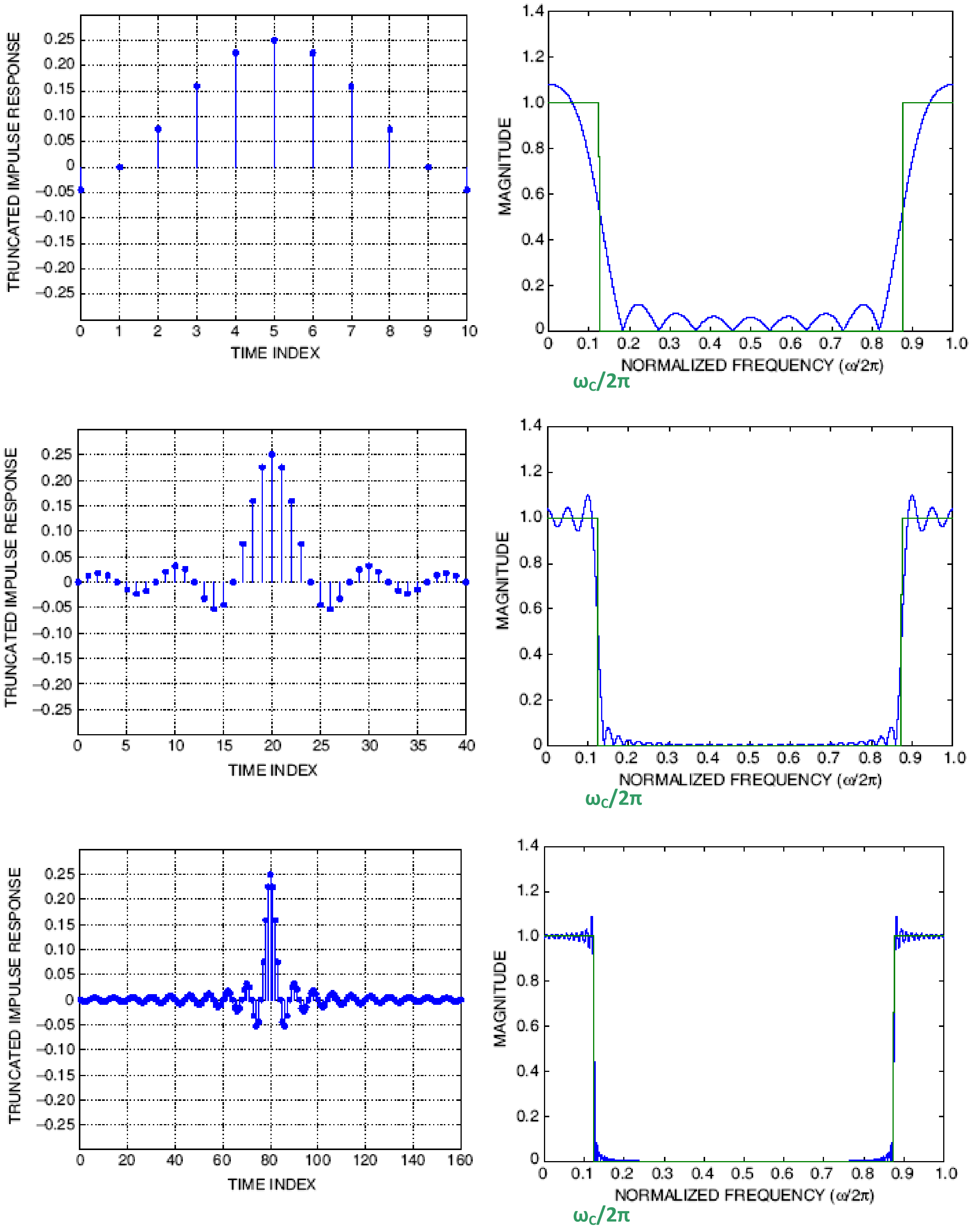
3 Write the ideal frequency response as $H_{ideal}(\omega)$ and compute the ideal impulse response $h_{ideal}[n]$ using the inverse DTFT.

4 Truncate the impulse response by applying a rectangular window of length M+1, that means to consider:

$$h[n]=\begin{cases} h_{ideal}[n] & 0 \le n \le M \\ 0 & n > M \end{cases}$$

Let's consider the ideal lowpass filter, for example let's consider a rectangular signal of amplitude 1 and duration *f* between $-\omega_C$ and ω_C (a singular period $\omega \in [-\pi, \pi]$). Its inverse DTFT is simply $h[n]=2\omega_C \cdot \mathrm{sinc}(2\omega_C n)$

Let's consider now the truncated impulse response (M=10, 40, 160) and corresponding approximation of the ideal lowpass frequency response:



Oscillations at the edges of the pass-band and stop-band cannot be reduced by the increase of *M*. For any *M*, these oscillations correspond to about 0.09 passband ripple and stopband attenuation parameters! This is also called **Gibbs phenomenon**.

So we can state that the impulse response truncation method is a simple but not very good method of filter design.

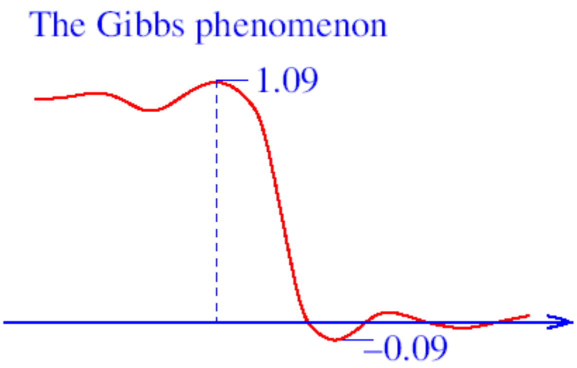
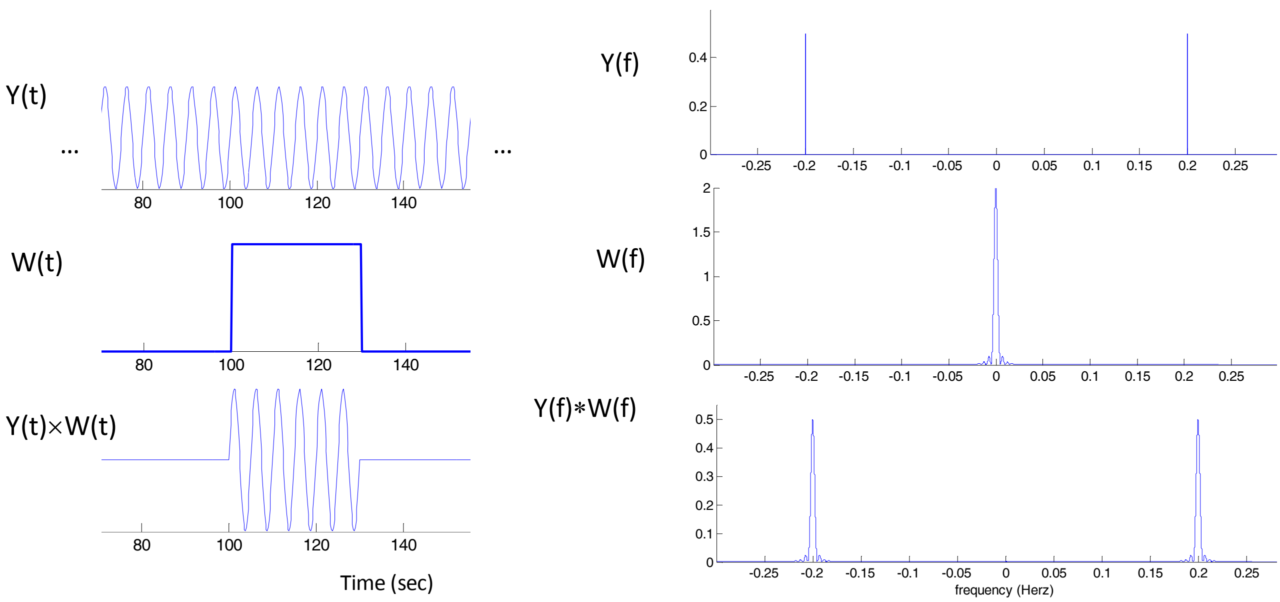


Figure 4: Gibbs phenomenon

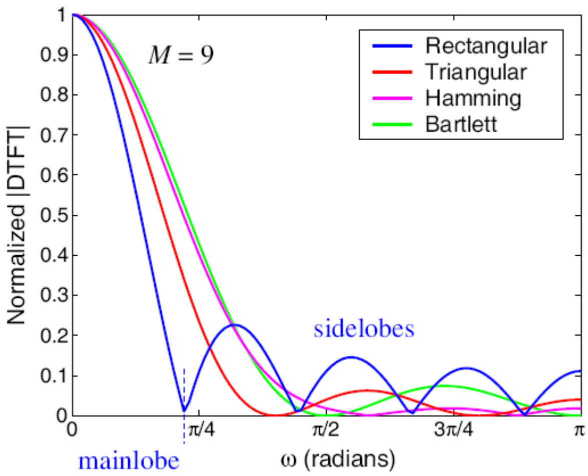
The effects of windowing on the actual frequency response. It was shown that truncation of an ideal impulse response by application of a *rectangular window* produces non-zero passband ripple and stopband attenuation parameters. However, the actual frequency response oscillates around the ideal frequency response, so we could consider a smoothing (i.e., averaging) operation in the frequency domain to reduce these oscillations. If we convolve the frequency response with a smoothing function in the frequency domain, this is equivalent to windowing (multiplying) in the time domain. However, we must be careful with our choice of windows if we want the resulting impulse response to remain (generalized) linear-phase.

Recall



Which is the more suitable window?

If we view the truncation procedure as multiplication by a rectangular window in the time domain (convolution in the frequency domain of the ideal frequency response with the DTFT of the rectangular window), we can remember that the rectangular window has the narrowest **mainlobe** of the common window types. This narrow mainlobe produces a fairly sharp transition region in the achieved frequency response. However, the rectangular window has the largest **sidelobe** magnitudes (these sidelobes are the cause of the non-zero passband ripple and stopband attenuation in the truncation method). Consequently, we may wish to use a window with *smaller magnitude sidelobes* to reduce the passband ripple and stopband attenuation, at the expense of have a *less sharp transition region*.



However the **window properties** which must hold are:

- ✓ windows are always real and symmetric, i.e., they must satisfy: $w[n] = w[M-n] \quad n = 0,...,M$ for either even or odd *M*. If the ideal FIR filter has linear phase, symmetric windows do not affect this property.
- ✓ windows must be positive to avoid any changes of sequence polarities
- ✓ a “good” window must satisfy the trade-off between the width of the mainlobe and the magnitudes of the sidelobes in the frequency domain
- ✓ a “good” window must have a smooth transition at the edges in order to reduce the **Gibbs phenomenon**

FIR filter design using windows:

1 Define the ideal frequency response as in the impulse response truncation method

2 Obtain the impulse response $h_{ideal}[n]$ of this ideal filter as in the impulse response truncation method (inverse DTFT).

3 Compute the coefficients of the filter by:

$$h[n] = \begin{cases} h_{ideal}[n]w[n] & 0 \le n \le M \\ 0 & n > M \end{cases}$$

where w[n] is some chosen window function.

Classical windows type.

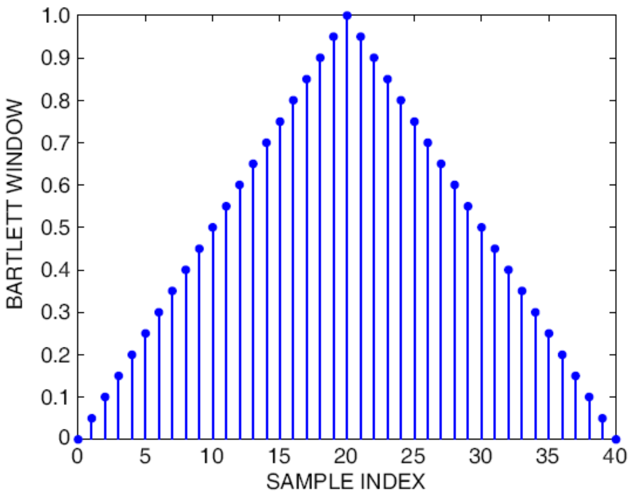
1) Rectangular window:(corresponds to natural truncation like in the impulse response truncation method!)

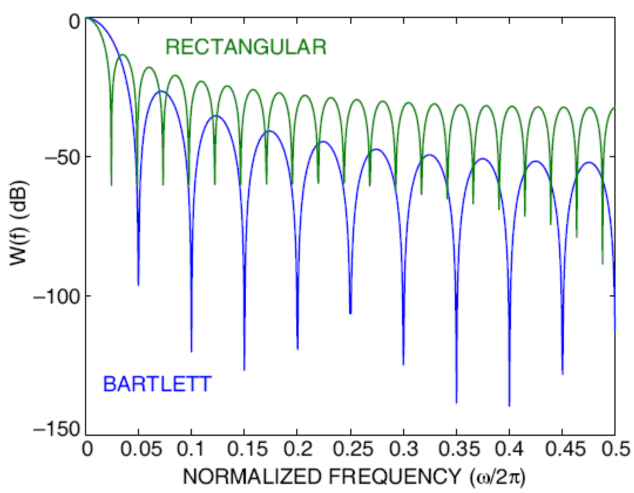
$$w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & n > M \end{cases}$$

2) Triangular (or Bartlett) window:

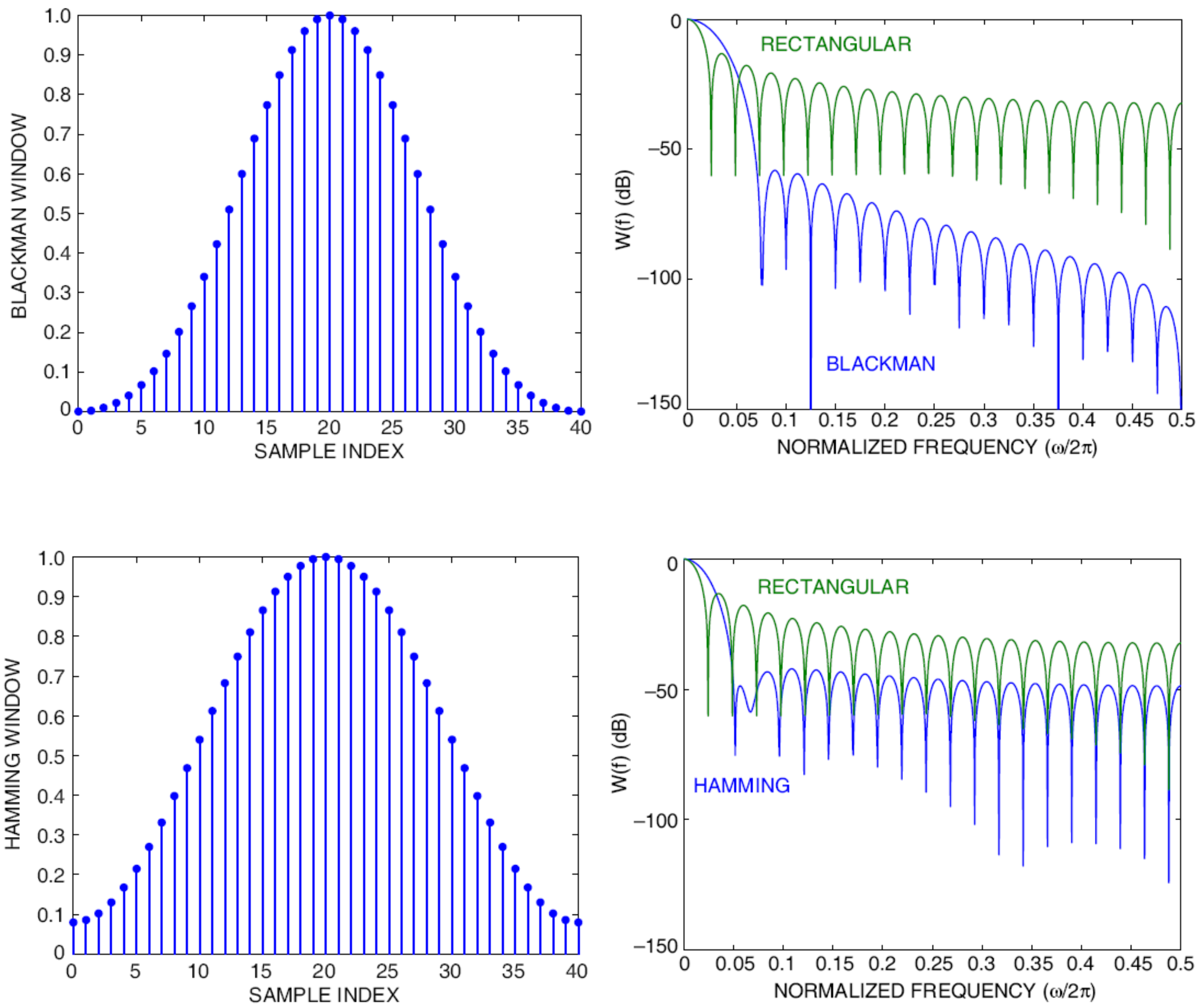
$$w[n] = \begin{cases} 2n/M & 0 \le n \le M/2 \\ 2-2n/M & M/2 < n \le M \\ 0 & n > M \end{cases}$$

Triangola window compared to the rectangular window in the frequency domain (M= 40):



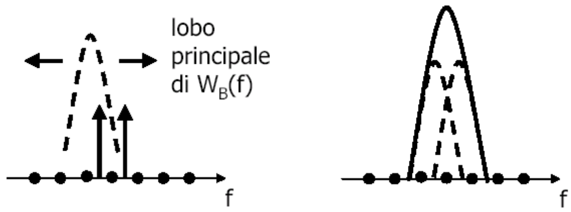


Blackman and Hamming window compared to the rectangular window in the frequency domain (M= 40):

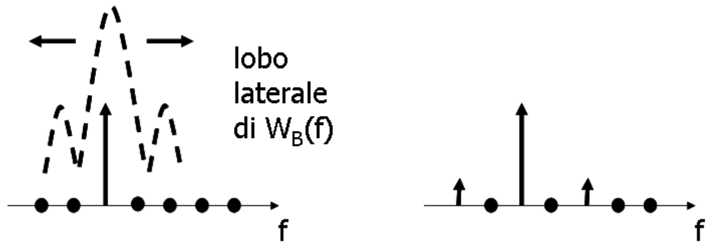


Which are the problems that may rise from a wrong choice of the filter?

1. **spectral leakage**: fusion of two spectral components into one peak. This is caused by a too large mainlobe.



2. False spectral components caused by the presence of sidelobes in the chosen window W_B.



By choosing an adequate window, it is possible to reduce these undesired effects It must taken into account the spectral features of the signal and of the chosen window. The output signal is the result of the sum of convolution.

An ideal window should have a sharp mainlobe, to avoid the local dispersion fo the spectrum, and limited amplitudes of sidelobes, to avoid the appearance of fictitious spectral components.

These two requirements cannot be satisfied at the same time!!!

Bibliografia:

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(si può trovare in biblioteca del dipartimento di Bioingegneria)

C. Prati, “Teoria dei segnali” 2. ed. - Milano : CUSL.

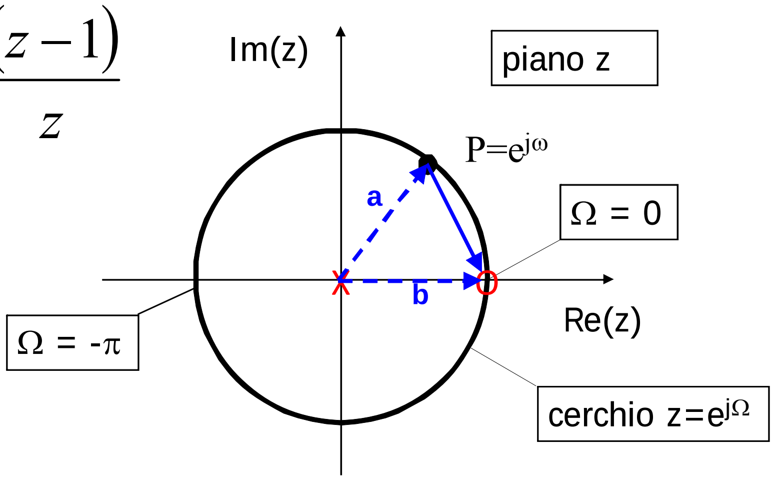
2nd PART

Geometric evaluation of frequency response of a filter

I can roughly estimate the modulus of H(ω) by moving around the unitary circle and computing the distances between poles/zeros and the point on the circle ($z=e^{j\omega}=e^{j2\pi f}$)

- ✓ to cancel a certain frequency, place a zero on the circle in correspondence of the desired frequency
- ✓ poles and zeros in the origin do not contribute to |H(w)|

$$H_1(z) = \frac{(z-1)}{z}$$



1 polo in z=0 e 1 zero in z=1

$$H(e^{j\omega}) = \frac{e^{j\omega}-1}{e^{j\omega}} = \frac{\cos\omega + jsen\omega - 1}{\cos\omega + jsen\omega}$$

$$\left|H(e^{j\omega})\right| = \sqrt{(\cos\omega - 1)^2 + sen^2\omega}$$

$$\left|H(e^{j\omega})\right| = \sqrt{(\mathrm{Re}(P)-1)^2 + \mathrm{Im}(P)^2}$$

The **amplitude of a filter** is assessed in the following way: we take as a reference a point P on the unit circle $z_P=e^{j\omega}=e^{j2\pi f}$, we compute the distances between that point and the zeros, and the distances between that point and the poles of the filter. The amplitude is the ratio between the distances of zeros and the distances of the poles. In the computation it must be taken onto account of the multiplicity of the zeros and poles.

$$\left|H(z)\right| = b_o \frac{\Pi_{m=1,M}\left|(e^{j\omega}-z_m)\right|}{\Pi_{k=1,N}\left|(e^{j\omega}-p_k)\right|}$$

3rd PART

Progettazione dei Filtri

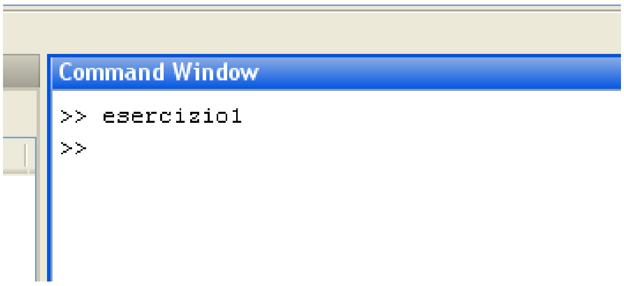
Gli esercizi proposti sono sviluppati in ambiente Matlab® (Signal Processing Toolkit)

1. *Effetto del rumore sul segnale ECG*

Funzione esercizio1.m (scaricarsi gli allegati anche il file ecg1.mat).

a. segnale ECG originale, b. segnale ECG corrotto da random noise,

c. registrazione ECG corrotta da un'interferenza di rete a 50Hz



2. *Progettazione Filtro FIR*

Prendiamo il segnale ECG campionato a 250Hz con il rumore a 50Hz (interferenza di rete).

Costruiamo dapprima un filtro FIR posizionando gli zeri sul cerchio unitario.

Dove li posizioniamo?

Filtro FIR di *ordine 2.*

2 poli nell'origine e 2 zeri complessi coniugati

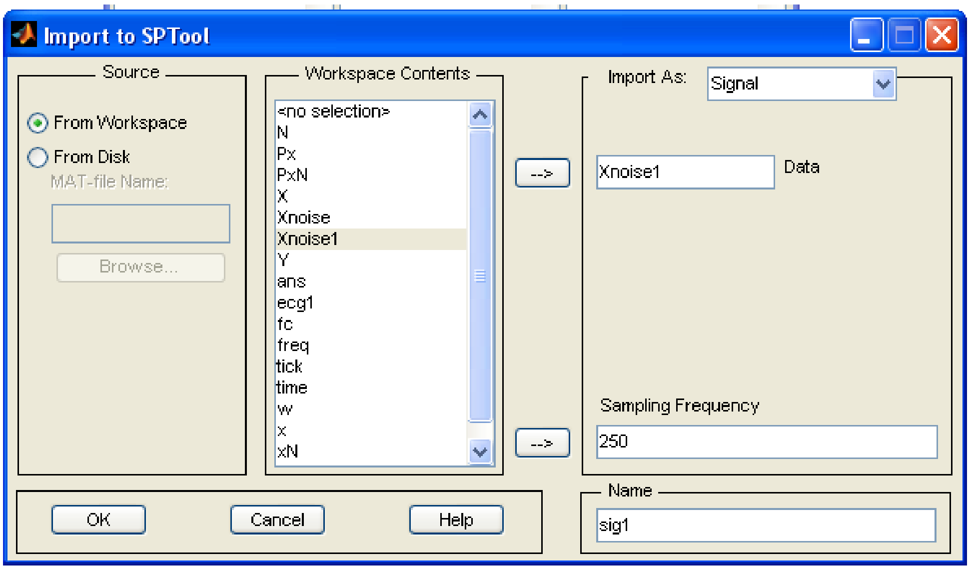
Fnoise/fNyquist=50/125=0.4

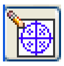
Zeri: r=1 $\omega = \pm 0.4\pi = \pm 1.2566$ radianti

Funzioni Matlab

>>sptool

File →**Import** →selezionare Xnoise1 e inserire freq di campionamento pari a 250 Hz



Filters→New → 

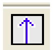
Posiziona poli e zeri sul cerchio, metti FilterGain=1

Analysis→Sampling Frequency=250Hz

Filters→Apply

Per vedere gli effetti sullo spettro selezionare il segnale

Spectra→Create e poi digitare Apply

Puoi vedere anche la risposta all'impulso!  .

E' un filtro FIR quindi il numero di coefficienti è finito

3. Progettazione Filtro IIR

Esiste una relazione precisa tra i poli e i picchi dello spettro. Ciascun picco è legato ad una coppia di poli complessi coniugati (o a un polo reale quando il picco è in DC o alla frequenza di Nyquist) e la sua forma è influenzata dalla posizione nel cerchio. Più il polo è vicino al cerchio unitario, più stretto sarà il picco, con potenza più concentrata in alcune frequenze, indicando così la presenza di un ritmo di oscillazione quasi-sinusoidale nel segnale. Al contrario, più vicino è il polo all'origine del piano z, più largo sarà il picco che modella la presenza di oscillazioni a larga banda nel segnale considerato.

In questo caso annulliamo sempre la componente a 50Hz.

Fnoise/fNyquist=50/125=0.4

2 Zeri: r=1 $\omega = \pm\, 0.4\pi = \pm\, 1.2566$ radianti

ma spostiamo i due poli dall'origine per rendere più selettivo il filtro

2 Poli: r=0.9 $\omega = \pm\, 0.4\pi = \pm\, 1.2566$ radianti

Funzioni Matlab

Filters→selezionare il filtro precedente→ Edit
Spostiamo i due poli dall'origine
Filters→Apply

Per vedere gli effetti sullo spettro selezionare il segnale
Spectra→Create e poi digitare Apply

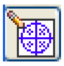
4. Cosa succede se ci spostiamo dal raggio unitario?

Proviamo a spostare i poli lungo il raggio unitario

5. *Eliminare linea di base, filtro passa alto*

Che cosa succede alla linea di base? e alle alte frequenze?

File →Import →X inserire freq di campionamento 250

Filters→New → 

2 Poli nell'origine e 2 zeri complessi coniugati vicino a f=0 (ad es. con θ= 0.0126)

FilterGain=1

Analysis→Sampling Frequency=250Hz

Filters→Apply

Per vedere gli effetti sullo spettro selezionare il segnale

Spectra→Create e poi digitare Apply

Contact

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