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Mathematical analysis of histogram equalization techniques for medical image enhancement: a tutorial from the perspective of data loss

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Abstract
This tutorial demonstrates a novel mathematical analysis of histogram equalization techniques and its application in medical image enhancement. In this paper, conventional Global Histogram Equalization (GHE), Contrast Limited Adaptive Histogram Equalization (CLAHE), Histogram Specification (HS) and Brightness Preserving Dynamic Histogram Equalization (BPDHE) are re-investigated by a novel mathematical analysis. All these HE methods are widely employed by researchers in image processing and medical image diagnosis domain, however, this has been observed that these HE methods have significant limitation of data loss. In this paper, a mathematical proof is given that any kind of Histogram Equalization method is inevitable of data loss, because any HE method is a non-linear method. All these Histogram Equalization methods are implemented on two different datasets, they are, brain tumor MRI image dataset and colorectal cancer H and E-stained histopathology image dataset. Pearson Correlation Coefficient (PCC) and Structural Similarity Index Matrix (SSIM) both are found in the range of 0.6-0.95 for overall all HE methods. Moreover, those results are compared with Reinhard method which is a linear contrast enhancement method. The experimental results suggest that Reinhard method outperformed any HE methods for medical image enhancement. Furthermore, a popular CNN model VGG-16 is implemented, on the MRI dataset in order to prove that there is a direct correlation between less accuracy and data loss.

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1 Introduction

Histogram equalization [16] is a well-known global image enhancement method which is widely used in the field of image processing, machine learning and automatic medical image diagnosis as a pre-processing step. The meaning of global method is that HE transformation is applied for all pixels in the image altogether. HE method generally stretches the Probability Density Function (PDF) [33], in order to enhance the dynamic range of the image. To the best of our knowledge, the advantage of applying any histogram equalization technique in medical images, prior to any machine learning (or deep learning) algorithm, is that (I) It provides contrast enhanced image in which edges will be more prominent to machines and consequently it can extract more prominent features [20] for classification task. Thus, it will enable the classifier to diagnose medical images faster, with higher accuracy. (II). Some HE techniques (for e.g. Histogram Specification) help to do color normalization [44] for all medical (color) images in a dataset which can reduce the computational complexity of a classifier significantly. Contrast Limited Adaptive Histogram Equalization (CLAHE) [37, 68] is a local HE technique which performs GHE in every local window. Moreover, in case of CLAHE, the contrast enhancement is limited by a clipping value. The performance of CLAHE is very much dependent on the window size and on the contrast clipping value which are chosen manually, for example CLAHE 1.0, 1.5, 2.0 etc. Thus, CLAHE method is not purely automatic. This is to clarify that, CLAHE 0.5 means it specifies a scale of contrast clipping 0.5 (this is not a version). For example, CLAHE 0.5 will do less contrast enhancement than CLAHE 1.0. Whereas, bydefault CLAHE by MATLAB software is CLAHE 2.0.

One of the reasons why those HE techniques consume huge data loss is that the mean brightness of the original or source images are not exactly preserved in the processed image [19]. Thus, some HE techniques, for example, Dynamic Histogram Equalization (DHE) [1], Brightness Preserving Bi-Histogram Equalization (BPBHE) [21], Minimum Mean Brightness Error Bi-histogram Equalization (MMBEBHE) [9], Median-Mean Sub-Image based Clipped Histogram Equalization (MMSICHE) [55], Dualistic Sub-image HE (DSIHE) [7], Brightness Preserving Dynamic Histogram Equalization (BPDHE) [19], Brightness Preserving Dynamic Fuzzy Histogram Equalization (BPDFHE) [50], Recursive Sub-Image Histogram Equalization (RSIHE) [53], Recursive Mean Separation Histogram Equalization (RMSHE) [8], Recursively Separated and Weighted Histogram Equalization (RSWHE) [22], Brightness Preserving Weight Clustering Histogram Equalization (BPWCHE) [49], Mean and Variance based Sub-Image Histogram Equalization (MVSIHE) [69], Multipeak Histogram Equalization with Brightness Preserving (MPHEBP) [64] etc. are employed in order to save the mean brightness of the source images. All of these methods overall, work based on the following steps: (I) First, they divide the image histogram into sub-histograms, thereafter, the HE stretching operation is done independently in each sub-histograms. (II) Subsequently, the mean brightness of the source image is normalized in the processed image in order to preserve the mean brightness of the source image.ł

A brief summary of table is further presented in Table 1 in Section 3.5 in order to explain all these HE techniques in brief. This has been observed that in brightness-preserving HE techniques, the data loss is comparatively lesser than that of other HE methods. In this paper,

only BPDHE among all such methods is re-investigated and this has been found that BPDHE is not also inevitable of data loss. Instead, this has been found that any HE method itself is a non-linear method, thus, causing significant data loss, which is further explained in depth in Section 3. Some of the researchers have employed the Histogram Equalization based on entropy or total information inside the image. To the best of our knowledge, any Entropy-based HE technique may not work efficiently for medical image enhancement, because, there will be huge data loss by Entropy based HE techniques. Here contrast enhancement of an image is done based on the total information, thus, often it may come up with over-contrast enhancement or over-luminance enhancement. Consequently, it may produce some artefacts inside the processed image. Brightness Preserving HE with Maximum Entropy (BPHEME) [61], Entropy based Dynamic Sub-Histogram Equalization (EDSHE) [34], Entropy based Adaptive Sub-Histogram Equalization (EASHE) [70], Iterated Adaptive Entropy Clip- Limit HE (IAECHE) [25] etc algorithms are recently proposed as entropy based HE techniques. C.H. Ooi et al. [30] have come up with a novel idea of combining both of the notions i.e. brightness preservation by dividing the histogram into sub-histograms and Clipping contrast enhancement (done by CLAHE). Their method is called as Bi-Histogram Equalization with a Plateau Limit (BHEPL) [30]. Although this method seems like an effective HE technique, but their method is unable to preserve mean brightness in their resulted image. Many researchers [5, 39] have tried to modify this method. Bi-Histogram Equalization with Median Plateau Limit (BHEPL-D) has been proposed by C.H. Ooi et al [29] in which the plateau limit for each sub-histograms are chosen based on the median value instead of mean value. Pabla B. Aquino-Morínigo et al. [5] have proposed Bi-Histogram Equalization Two Plateau Limits (BHE2PL) in which they incorporated two Plateau Limits. All these methods couldn't preserve all the information of source images in their processed images and sometimes, produce artefacts for medical images which is undesirable. Thus, these HE methods consume huge data loss.

Histogram Specification is another widely employed HE technique by several scientists [6, 11] in medical image processing. In HS method, first a reference image is chosen whose contrast is very good, thereafter, HE stretching operation is done until processed image's histogram is similar to reference image histogram. Many more HE techniques [3, 20, 26, 27, 38, 65] are recently proposed by numerous researchers. Among the aforementioned HE techniques, CLAHE is the most frequently employed in medical image diagnosis as a pre-processing technique by several researchers [2, 4, 14, 24, 47, 48, 52, 67]. Previously, S. Patel et al. [35] presented a study paper for Histogram Equalization techniques for medical image enhancement. However, our paper presents a tutorial of several histogram equalization techniques for medical image enhancement, from a different perspective. That is, whether they consume significant data loss or not during pre-processing, which is further explained in depth in Section 3.

All these HE techniques are fraught with the problem of huge data loss, since correlation coefficient value [59] between input image and processed image are found very much deviating from 1. We believe that those HE methods are not suitable for tumor detection or cancer detection task. Because such lost data may contain significant features of tumor and if those data (or significant features) are washed out during pre-processing, that will further degrade the accuracy of classifier. However, these HE techniques (especially Entropy based HE) may be suitable in other problems where we need to extract significant hidden information from the images. For example, HE method may be suitable for multi-spectral satellite images [12, 15], dark images [56], infrared images [23], ultra-sound images and fogg images [66] where the image clarity is considerably less or the image has very poor contrast. In that case, extracting more significant information from the images will be more important. Thus, in those cases, entropy [10] will be more significant quality metric than

correlation co-efficient for performance evaluation of pre-processing method, to the best of our visualization. Therefore, the applicability of these HE techniques are indeed limited on the specific problems and they are also dependent on the statistical properties [17] of those images. In this paper, all those HE techniques are analyzed in the application of medical image diagnosis, with the perspective of data loss.

The contributions of this paper are as follows.

1. First, this has been proved that by any HE technique, correlation co-efficient between processed image and input image is far deviating from the value 1, by employing a novel mathematical analysis. This indicates that any HE method consumes data loss.
2. Secondly, this has been mathematically proved that data loss in CLAHE method is not dependent on the limiting contrast enhancement parameter. Nevertheless, limiting contrast does not help the image to prevent from noise. We have identified that the actual cause of data loss is HE method itself, because any HE method is a nonlinear method.
3. Two Lemmas are proved in the appendix of this paper. Some of the frequently used HE techniques Histogram Specification (HS), Brightness Preserving Dynamic Histogram Equalization (BPDHE) have also been re-investigated, based on those two Lemmas.
4. A brief survey of several HE techniques is presented in Table 1, in Section 3.5.
5. Moreover, a mathematical analysis is done for Reinhard method in order to prove that a linear method (like Reinhard method), preserves most of the information during pre-processing. Furthermore, Reinhard method is preferred for tumor detection or cancer detection problem over any HE technique.
6. In order to support these mathematical analysis, all popular HE techniques are implemented on two datasets: MRI and Colorectal Cancer Histopathology Images. Furthermore, a CNN model VGG-16 is employed on MRI dataset, in order to prove that there is a high correlation between data loss and less accuracy.

This paper is organized as follows. In Section 2, we introduced, why measuring correlation co-efficient is equivalent to measuring data preservation. A unique mathematical analysis of various Histogram Equalization (HE) methods is explored in the Section 3. Furthermore, in Section 4, quantitative and qualitative results of various HE methods are compared. In Section 5, we present our concluding remarks.

2 Correlation Co-efficient

The correlation co-efficient measures the degree of linear correlation [59] between two random variables. In this case, this is considered images (which is nothing but array of numbers) as random variables. Suppose P and Q are two random variables. If they are exactly linearly correlated, this means

$$P = cQ \tag{1}$$

where 'c' is real constant. Now from the probability theory [33], we know, if two random variables are related by an equation and the PDF of one random variable is known, then PDF of other random variable can be easily found by the following formula.

$$f(P) = \sum_{j=1}^{r} f(Q_j) \left| \frac{dQ_j}{dP} \right| \tag{2}$$

where 'r' is the total number of roots of the (1) which is equal to 1. $f(P)$ is the Probability Density Function (PDF) [33] of random variable P, $f(Q)$ is the Probability Density function (PDF) of random variable Q. Thus, from (1), substituting the value in (2), we get,

$$f(P) = \frac{1}{c} f(Q) \tag{3}$$

(3) reveals that if two images are exactly linearly correlated, then their PDF or normalized histogram will also be linearly correlated. This also reveals that the shape of histogram of one image will be exactly same with the histogram of other image, because one histogram is scaled by a real constant, from (3). Hence, this can be concluded, if correlation co-efficient between input and processed image are exactly equal to 1, then the shape of the histogram of processed image will be exactly preserved. Thus, no significant data should be lost from the input image. Hence, we can claim that correlation co-efficient can be utilized as a measurement of data preservation between two images. More depth explanation can be found in [45]. Many more researchers [59, 62] also concluded the same regarding correlation co-efficient.

Another interesting conclusion can be done from the aforementioned analysis, that is, if any non-linear method is employed as image transformation or as image processing technique, this may cause little bit data loss. Although, many non-linear transformations we use to study in image processing book [16], according to our analysis, there will be always little bit data loss happening if we employ a non-linear transformation. Human eyes generally cannot detect such lower order statistics [32], thus, we never visualize whether there is any data loss happens or not in terms of lower order statistics, due to employing a non-linear method. Furthermore, a proof is given in the appendix that any linear transformation can provide correlation co-efficient exactly equals to 1, between source and processed image. This reveals that there should not be any data loss if the transformation is chosen a linear method.

3 Mathematical analysis of several histogram equalization techniques

In this paper, a unique mathematical analysis is employed, which is as follows. Previously, similar kind of mathematical analysis was done by S. Roy et al. [41, 45] for color normalization of H and E-stained histopathology images. However, this kind of analysis has not been employed before for evaluating HE techniques, to the best of our knowledge.

This is to clarify that this kind of mathematical analysis is employed in this paper due to some unique statistical or texture property of medical images which was already invented in [45], for histopathology images. The texture property of medical images is further explained in depth in the following points:

1. This is observed that in medical images, the autocorrelation coefficient [17] between pixels of the medical image is greater than that of natural images. In other words, the image intensity variation per single window is more in medical images than that of natural images.
2. To be more specific, the medical images does not contain large homogeneous regions like lands, sky, trees etc which are generally present in conventional natural images. In case of histopathology images, there is no background at all, whereas, in case of MRI images there is background which is totally black which is outside the brain part. Hence, more or less in both of the datasets, auto-correlation co-efficients are comparatively higher than that of natural images. Therefore, the overall statistics of these images exactly reflects

the same as local statistics of these images. This enables us to take directly the global mean or global standard deviation of those images. Because local statistics will be kind of same as global statistics of these medical images.

3. Another important texture property of the histopathology images observed by S. Roy et al. [45] and [28] is that the overall texture of the images is kind of repetitive throughout the images. This is called periodic texture [28]. This gives us more solid reason to incorporate global statistical methods as mathematical analysis.
4. Moreover, these statistical analysis are supported by the experimental results, done in Section 4.

More depth information can be found about image statistics or image texture properties in [17] and [18].

3.1 Mathematical analysis of Global Histogram Equalization (GHE)

The transformation function T of histogram equalization is given by (4).

$$s = T(r) \tag{4}$$

where s is the processed image intensity, r is the original image intensity, such that

$$p_s(s) \approx 1 \tag{5}$$

$p_s(s)$ is the PDF of the processed image. Now from the statistical formula [33] we know that

$$p_s(s) = p_r(r)|\frac{dr}{ds}| \tag{6}$$

where $p_r(r)$ is the PDF of original image, here this is considered that the number of roots of transformation function T for Histogram Equalization is 1. Because HE transformation function is monotonically increasing function and it can not be higher degree of polynomial function, according to R.C. Gonzalez et al. [16]. The (6) is employed from [16].

Putting $p_s(s) = 1$, in the (6)we get,

$$\int p_r(r)dr = \int ds \tag{7}$$

$$or,\ s = \int p_r(r)dr + k \tag{8}$$

where k is the integration constant. (8) also reveals that transformation function of GHE is an integrator [31], thus, this GHE technique is not purely a linear method.

Now from the theory of statistics [33], we can get the following formula, given in (9).

$$\sigma^2(pY + q) = p^2\sigma^2(Y) \tag{9}$$

Where p, q are real constant and Y is a random variable.

Now by taking global variance [45] to both side in (8), according to the formula (9), we get,

$$\sigma^2_{gl}(s) = \sigma^2_{gl}(\int p_r(r)dr) \tag{10}$$

$$or,\ \sigma_{gl}(s) = \sigma_{gl}(\int p_r(r)dr) \tag{11}$$

Here, in (10), σ_{gl}^2 means global variance. Now by taking global mean [5] to both side of (8), we get,

$$or,\ \mu_{gl}(s) = \mu_{gl}(\int p_r(r)dr) + k \tag{12}$$

Here, in (12), μ_{gl} means global mean.

Preservation of source information can be measured by estimating the value of correlation coefficient between source image and processed image. Covariance between processed image (s) and source image (r) is given by the (13).

$$\sigma_{sr} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(s_i - \mu_{gl}(s_i)) * (r_j - \mu_{gl}(r_j)) \tag{13}$$

By substituting the value from (8) and (12), into (13), we get,

$$\sigma_{sr} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(\int p_r(r_i)dr_i + k - \mu_{gl}(\int p_r(r_i)dr_i) - k) * (r_j - \mu_{gl}(r_j)) \tag{14}$$

For simplicity of computation, let us assume that

$$\int p_r(r)dr = q(r) \tag{15}$$

$q(r)$ is nothing but the Cumulative Distribution Function (CDF) [33] of the original image 'r'. And $\mu_{gl}(k) = k$, as k is just a real constant.

By replacing this value into (14) we get,

$$\sigma_{sr} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(q(r_i) - \mu_{gl}q(r_i)) * (r_j - \mu_{gl}(r_j)) \tag{16}$$

Now Correlation coefficient [59] between source image and processed image is given by

$$Corr_{sr} = \frac{\sigma_{sr}}{\sigma_{gl}(s) * \sigma_{gl}(r)} \tag{17}$$

By substituting the value from (8), (11) and (16), into (17), we get,

$$Corr_{sr} = \frac{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(q(r_i) - \mu_{gl}q(r_i)) * (r_j - \mu_{gl}(r_j))}{\sigma_{gl}(q(r)) * \sigma_{gl}(r)} \tag{18}$$

This can be clearly observed from the (18), $Corr_{sr} \approx 1$, if and only if Cumulative Distribution Function (CDF) $q(r)$ is equal to 'r'.

$$i.e.\ q(r) = \int p_r(r)dr = r \tag{19}$$

However, (19) can not be true. Ideally it is possible when $p_r(r) = 1$, then only $q(r) = r$. Moreover, $p_r(r)$ is the Probability Density Function (PDF) of original image, thus, it is likely to be very much deviating from 1. Because any original image's PDF is nothing but normalized histogram and this histogram contains all the statistics (or intensity variations) of image. Therefore, it is likely that this PDF will never be a flat histogram,

$$Hence,\ p_r(r) \neq 1 \tag{20}$$

Hence, for Global Histogram Equalization (GHE), this is proved that correlation co-efficient between processed image and source image is far deviating from the value 1. Because, practically $p_r(r)$ will never be equal to 1 or $q(r)$ will never be equal to 'r'. There is another way to prove this statement. This has been observed that whether correlation coefficient between original image and processed image will be closer to 1 or not, that actually depends on their PDF's ratio.

Lemma1: For any contrast enhancement method, (with having transformation function which has number of roots 1),

$$If,\ \frac{p_r(r)}{p_s(s)} \approx 1,\ then\ Corr_{sr} \approx 1. \tag{21}$$

whereas $p_r(r)$ is the PDF of source image, $p_s(s)$ is the PDF of the processed image, $Corr_{sr}$ is the correlation co-efficient between processed image and source image.

The aforementioned lemma also interprets that more difference between $p_r(r)$ and $p_s(s)$, that will further push the value of the ratio, deviating from 1. And consequently, the correlation co-efficient between source image and processed image will be deviating more from the value 1. Here, we limit our mathematical analysis within those transformation functions which has number of roots 1, because HE transformation function will always have number of roots 1.

Another case, which can be visualized is that,

Lemma2: For any contrast enhancement method,

$$If,\ p_r(r) = \frac{1}{c} p_s(s),\ then\ Corr_{sr} \approx 1. \tag{22}$$

whereas, c is a real constant, $p_r(r)$ is the PDF of source image, $p_s(s)$ is the PDF of processed image, $Corr_{sr}$ is the correlation co-efficient between processed image and source image.

Both of the aforementioned Lemmas are mathematically proved in the appendix of this paper.

From (2) and (3), this can be noticed that $p_r(r)$ and $p_s(s)$ are linearly correlated to each other, if the transformation function is linear. Therefore, (22) also reveals that if the transformation function of any contrast enhancement method is linear, then correlation co-efficient between processed image and source image will be equal to 1, according to Lemma2. This also reveals that any linear transformation does not prone to data loss.

This is already proved in (8) that GHE method is not purely a linear method, indeed it's an integrator. Thus, it is likely that $p_r(r) \neq \frac{1}{c} p_s(s)$ for GHE method, hence, *Lemma 2* will not be satisfied by GHE method. In this way, this can also be proved that GHE method doesn't preserve all the information during processing and hence, it is prone to huge data loss. Indeed, many researchers [1, 19] claimed that by this method, the processed image might be highly affected by unwanted artefacts or noise.

3.2 Adaptive Histogram Equalization (AHE) and Contrast Limited Adaptive Histogram Equalization (CLAHE)

In Adaptive Histogram Equalization (AHE) [36, 37] method, HE is employed to enhance contrast for each local regions of an image. By conventional GHE, contrast of the whole image is enhanced unless the whole image histogram is stretched and become a uniform PDF. This may cause over-enhancement in some regions of the image. Moreover, by GHE, in some local regions the intensity might be so bright or so dark that it can lose its local information [37]. Therefore, AHE should work better than GHE in terms of local information preservation. However, in both AHE and GHE, there is no limit of contrast enhancement inside a local

region, thus, it may cause over-enhancement in some local regions and consequently some artefacts may appear in the image by the AHE method. Contrast Limited Adaptive Histogram Equalization (CLAHE) [36] is a modified version of AHE, where the contrast enhancement inside a local region of image is limited by a maximum clipping value, thus, CLAHE should avoid noise and artifacts slightly better than AHE. However, we have mathematically proved that the data loss in CLAHE is not dependent on this limit of contrast enhancement. Moreover, choosing this maximum value of contrast enhancement in CLAHE method (along with the choosing the size of local window) is entirely manual and must be provided by the user. Thus, CLAHE method is not a purely automatic method. A mathematical analysis of CLAHE is presented in the following:

Let's assume, Probability Density Function (PDF) inside a local region of the processed image by CLAHE is $p_{sL}(s)$, where L indicates a local region. This is clear that by the method of CLAHE,

$$p_{sL}(s) \neq 1 \tag{23}$$

Let us assume that this parameter of limiting contrast by CLAHE is 'q', whose value lies between 0 to 1. If $q = 1$, that means local histogram stretching is maximum, that means it will be exactly same as AHE. But in CLAHE, this parameter 'q' must be less than 1, because here contrast stretching is limited (not maximum).

$$Thus,\ let\ assume,\ p_{sL}(s) = q \tag{24}$$

Now from the statistical formula in (6) we get,

$$q = p_{rL}(r)|\frac{dr}{ds}|_L \tag{25}$$

where $p_{rL}(r)$ is the PDF of a local region 'L' in original image, and $\frac{dr}{ds}_L$ is the ratio of dr and ds in that local region. By taking integration both side in (25), we get,

$$q * \int_L ds = \int_L p_{rL}(r)dr \tag{26}$$

Hence, the transformation function for CLAHE in a local region will be given by (27).

$$s_L = \frac{1}{q} * \int_L p_{rL}(r)dr + k \tag{27}$$

(27) reveals that transformation function for CLAHE is an integrator [31], thus, CLAHE method is non-linear method as well, here 'k' is the integration constant in (27).

By employing similar mathematical analysis which was employed in the Section 3.1, we can compute the covariance between processed image and source image inside a local region (L) i.e. $\sigma_{sr}(L)$, by the following (28).

$$\sigma_{sr}(L) = (\frac{1}{q} * \int_L p_{rL}(r)dr + \frac{1}{q} * \mu_L \int_L p_{rL}(r)dr) * (r_L - \mu_L(r)) \tag{28}$$

where μ_L is the mean of a local region and for simplicity let assume that $\int_L p_{rL}(r)dr = q_L(r)$ where $q_L(r)$ is Cumulative Distribution Function (CDF) of a local region 'L' in original image 'r'. Therefore, final correlation coefficient between original image and processed image inside a local area can be computed from (17).

$$Corr_L = \frac{\frac{1}{q} * (q_L(r) - \mu_L q_L(r)) * (r_L - \mu_L(r))}{\frac{1}{q} * \sigma_L(q_L(r)) * \sigma_L(r)} \tag{29}$$

$$or,\ Corr_L = \frac{(q_L(r) - \mu_L q_L(r)) * (r_L - \mu_L(r))}{\sigma_L(q_L(r)) * \sigma_L(r)} \tag{30}$$

Now, again if and only if, Cumulative Distribution Function (CDF) inside a local region of image is equal to intensity of that image, that is $q_L(r) = r_L$, then only local correlation co-efficient $Corr_L$ in (30), will become equal to 1. This will only happen if $p_{rL}(r) = 1$. Now again Probability Density Function (PDF) inside a local region of an image is less likely to be equal to 1, because PDF of any source image is less likely to be flat, which was already discussed in Section 3.1. Hence, the local correlation co-efficient between processed image and original image will not be equal to 1, in case of CLAHE. This reveals that by CLAHE method there might be data loss in the image as well. (Hence Proved)

Another interesting observation is that the correlation co-efficient in (30) is not dependent on the value 'q', which is nothing but limiting co-efficient of contrast enhancement given by the user. Because $1/q$ factor has been cancelled in the previous step, which can be noticed from (29). This reveals that no matter how much you choose the limit of contrast enhancement, there will be always some data loss by CLAHE method. The physical interpretation behind this theory is that CLAHE is employing local Histogram Equalization method which itself is a non-linear method, shown in (27). Therefore, it is not capable of preserving all the significant information inside the image, even if you control the contrast enhancement for CLAHE. This theory is further supported by a series of experiments, which is explained in the Section 4.

3.3 Histogram Specification (HS)

Histogram Specification (HS) or Histogram Matching [6, 11] is a well-recognized histogram equalization technique which is widely employed by researchers. In histogram specification, first, user has to choose a reference image or target image whose contrast or edge information is very prominent and thereafter global histogram stretching operation is done of a poor contrast (source) image until the histogram of the processed image will be similar to that of reference image. In both of histogram specification and CLAHE method, the histogram stretching or contrast enhancement task is limited. However, in CLAHE method user has to put some value as contrast enhancement factor and in histogram specification user has to choose a good quality image.

According to our observation, user generally do not know how much contrast of image should be enhanced, because human eyes can not detect all the lower order statistics of images. Moreover, human visualization system is more biased to the higher order statistics [32], thus, in CLAHE method mostly we end up with enhancing contrast little bit more than it should be. Therefore, Histogram Specification (HS) is little bit better method than CLAHE. Because the parameters do not have to be chosen manually in HS method, rather users have to choose a good quality image as a target image which is comparatively easier task. However, Histogram Specification is also employing the Global Histogram Equalization (GHE) method which is not a purely linear method. Hence, HS method is not also inevitable of data loss, to the best of our knowledge. This statement is further mathematically proved below and supported by some experiments, in Section 4.

Suppose the PDF of target image is $p_t(t)$. According to HS method, PDF of the processed image will be similar to the PDF of target image. For simplicity of computation, let's assume that PDF of processed image will be exactly equal to the PDF of target image for HS method, i.e.

$$p_s(s) = p_t(t) \tag{31}$$

Now, substituting the value from (31), into (6), we get,

$$\int p_r(r)dr = \int p_t(t)ds \tag{32}$$

Now $p_t(t)$ is not dependent on 's' in (32). Hence,

$$s * p_t(t) = \int p_r(r)dr + k \tag{33}$$

$$or,\ s = \frac{\int p_r(r)dr}{p_t(t)} + \frac{k}{p_t(t)} \tag{34}$$

From, (34), this can be observed that transformation function 's' for HS method is an equation of an integrator [31]. Thus, HS method is a non-linear method as well. Now from the Lemma 1, in (21), for any contrast enhancement method, data preservation between processed image and original image actually depends on the ratio of their PDF's. For HS method, from (31) we get,

$$\frac{p_r(r)}{p_s(s)} \approx \frac{p_r(r)}{p_t(t)} \neq 1 \tag{35}$$

Here, $p_r(r)/p_t(t) \neq 1$, in (35), because target image and source image are completely different image, in terms of their contextual information. Thus, for HS method, correlation coefficient between processed image and source image will be deviating from the value 1, according to *Lemma1*.

However, according to our visualization, HS method might be slightly better than GHE and CLAHE method. Because unlike GHE or CLAHE, $p_s(s)$ does not reduce to a real constant value. Rather $p_s(s) = p_t(t)$, where $p_t(t)$ is the PDF of target image. Therefore, there is possibility that the ratio of $p_r(r)/p_s(s)$ for HS method can have value slightly closer to 1, comparative to other HE methods. Consequently, correlation co-efficient between processed image and source image by HS method may be slightly improved, if proper target image is chosen. However, the performance of HS method is very much dependent on the statistics of target image. If the target image has very dis-similar statistics than the statistics of source image, then the ratio of $p_r(r)/p_s(s)$ may deviate a lot from the value 1. Consequently, HS method may consume huge data loss.

From *Lemma2*, in (22), for any contrast enhancement method if the ratio $p_r(r)/p_s(s)$ is real constant, (that means if the transformation function is linear) then correlation co-efficient will be equal to or closer to 1. However, from (34), it was already proved that HS method is a non-linear method. Hence, this can be concluded that the correlation co-efficient between processed image and source image for HS method will not be equal to 1. Therefore, HS method does not preserve all the information during processing. This statement is further supported by some experiments in Section 4.

3.4 Brightness Preserving Dynamic Histogram Equalization (BPDHE)

Brightness Preserving Dynamic Histogram Equalization (BPDHE) [19] is another well-known HE technique, which has been widely employed in the field of image processing. The main purpose of this HE technique is to preserve the mean brightness of the source images, since it has been observed that many HE techniques don't preserve the mean brightness of the source images. The methodology of this HE technique is explained below by four steps:

1. First, the histogram of the source image is smooth out by Gaussian blur filter, so that the local maximus in its histogram can be easily detected.
2. Partition the histogram into sub-histograms based on those local maxima values.
3. Assign new dynamic range for each sub-histograms and do the histogram equalization operation in each sub-histogram separately.
4. Compute mean brightness of original image and processed image. Thereafter, do the luminance (or, brightness) normalization to the processed image.

Although this HE technique can enhance the contrast of the image with preserving its mean brightness, but it does not guarantee that it will preserve all the contextual information of the source image. Indeed, this has been found that BPDHE is not inevitable of data loss. A mathematical analysis of BPDHE is given below:

After employing Gaussian filter on original image 'r', we got,

$$G(r;\sigma) = e^{-\frac{r^2}{2\sigma^2}} \tag{36}$$

where σ is the standard deviation of Gaussian filter and its mean value is 0. Now let us partition $G(r)$ into 'K' sub-histograms. Let us assume that expression of sub-histogram is $p_{sub}(G(r_i))$. After employing histogram equalization operation, assume that processed sub-histogram's expression is $p_{sub}(s_i)$.

On top of that a normalization factor must be multiplied which is the ratio of their mean luminance. Now, from the notion of HE technique,

$$p_{sub}(s_i) \approx 1 \tag{37}$$

Final processed sub-histogram after luminance normalization will be

$$p_{sub}(s_i^{norm}) = p_{sub}(s_i) * \frac{\mu_{gl}(r_i)}{\mu_{gl}(s_i)} \tag{38}$$

However, in this case, $p_{sub}(s_i)$ value may not be very closer 1, since the length of each sub-histogram during BPDHE is not uniform. Hence, there is a possibility that one sub-histogram will not be stretched extremely. Furthermore, $p_{sub}(s_i)$ is multiplied by a normalization factor, shown in (38).

Hence, for BPDHE, the ratio $\frac{p_{sub}(G(s_i))}{p_{sub}(s_i^{norm})}$ may not be deviating very much from the value 1.

Thus, according to *Lemma1*, BPDHE may have slightly better results (of correlation coefficient) than that of other previously discussed HE techniques. Now for the sake of simplicity of computation, let us assume that $p_{sub}(s_i) = 1$, which is not always true. The final transformation function for BPDHE can be estimated from (6) and (38),

$$\frac{\mu_{gl}(r_i)}{\mu_{gl}(s_i)} * \int ds = \int p_{sub}(G(r_i))dr \tag{39}$$

$$or,\ s = \frac{\mu_{gl}(s_i)}{\mu_{gl}(r_i)} * \int p_{sub}(e^{-\frac{r^2}{2\sigma^2}})dr + k \tag{40}$$

Transformation function 's' of BPDHE is represented in (40), which is a non-linear function, 'k' is an integration constant. Here, $\mu_{gl}(s_i)$ is not dependent on 's', because $\mu_{gl}(s_i)$ is a constant value which was introduced at the time of luminance normalization. Hence, from (40), this is proved that BPDHE also employs a non-linear transformation function. Thus, according to *Lemma2*, correlation coefficient between processed image and original image

will be deviating from 1, hence, this can be concluded that BPDHE consumes data loss. This mathematical proof is further supported by number of experiments in results and analysis Section.

3.5 A brief survey of he techniques

A brief survey of several Histogram Equalization techniques are also presented in the Table 1. This is to clarify that the methods which are already explained in Sections 3.1, 3.2, 3.3 and 3.4, are not repeated in this table. Moreover, in this survey, limitations of these Histogram Equalization techniques are identified based on the perspective of whether it consumes data loss or not. More specifically we reviewed all these techniques as an application of medical image enhancement.

3.6 Reinhard method

Reinhard method [40] was first time introduced just to transfer color statistics from target image to original image. Although Reinhard method has been widely used only for color images so far, this has been that this particular method can be employed for grey-scale medical images as well, just for the task of pre-processing or contrast enhancement. Especially for tumor or cancer diagnosis this method is preferable over other recognized HE contrast enhancement methods because

1. It preserves 100 % data, which has been mathematically proved in this subsection as well as we have found the same in experimentation, in Section 4.
2. This is a purely automatic method. First, user need to choose a target image or reference image whose contrast or luminance statistics is very good or appropriate. Thereafter, color as well as contrast statistics can be transferred from target image to original image. Unlike CLAHE method, we don't have to choose any parameter (like contrast clipping) manually.

The mathematical analysis for the Reinhard method is presented in the following. The transformation function for the Reinhard method is given by (41),

$$s = \mu_{gl}(t) + (r - \mu_{gl}(r)) * \frac{\sigma_{gl}(t)}{\sigma_{gl}(r)} \tag{41}$$

's' is the intensity of the processed image, 'r' is the intensity of the original image, 't' is the intensity of a target image or reference image, μ_{gl} is the global mean value, σ_{gl} is the global standard deviation value. Another important observation can be done from (41) is that transformation function for Reinhard method is a linear function. Thus, according to Lemma 2, it is likely that it should not lose any significant information, during pre-processing.

By taking global variance [45] both side of (41), and by applying (9), we get

$$\sigma^2_{gl}(s) = \sigma^2_{gl}(r) * \frac{\sigma^2_{gl}(t)}{\sigma^2_{gl}(r)} \tag{42}$$

$$or,\ \sigma^2_{gl}(s) = \sigma^2_{gl}(t) \tag{43}$$

$$or,\ \sigma_{gl}(s) = \sigma_{gl}(t) \tag{44}$$

Table 1 A brief survey of various HE techniques

Sl No	Name	Methods	Limitations
1.	Local HE (LHE) [16]	Histogram Stretching is done in local window.	1. Computational cost is very higher 2. Huge data loss 2.
2.	Dynamic Histogram Specification (DHS) [58]	It generates a specified histogram from the source image itself.	It does contrast enhancement very little.
3.	Bi-Histogram Equalization (BBHE) [21]	It divides the image into two sub-histograms by mean value, thereafter, stretching the sub-histograms separately.	1. Computational cost is higher. 2. Unable to preserve all the local information inside image.
4.	Dualistic Sub-Image HE (DSIHE) [7]	It divides the image into two sub-histograms by median value.	1. Computational cost is higher. 2. It creates artefacts.
5.	Minimum Mean Brightness Error Bi-HE (MMBEBHE) [9]	It performs the division of two histograms based on optimal threshold value such that difference between input and output brightness will be minimum.	1. Computational cost is higher than BBHE. 2. This method is not entirely inevitable of data loss.
6.	Recursive Mean-Separate HE (RMSHE) [8]	It divides histogram into two sub-histograms by mean value. Then, it iteratively divides histogram further into 2^n number of histograms and equalize all sub-histograms.	1. Computational cost increased significantly, since this is an iterative method. 2. If n is higher, then processed image will be same as source image.
7.	Recursive Sub-Image HE (RSIHE) [53]	It follows the exact same method as RMSHE except it divides the histograms based on median value.	It has the same limitation of RMSHE, i.e., for larger n there is no contrast enhancement at all.
8.	Dynamic Histogram Equalization (DHE) [1]	It divides the global image histogram into number of sub-histograms based on local minima of histogram.Thereafter it equalizes each sub-histograms independently.	1. It couldn't remap the peaks efficiently which leads to poor brightness pre-servation. 2. Computational cost is higher.
9.	Multipeak HE with Brightness Preserving (MHEBP) [64]	Histogram is first smoothed by a 1D smoothing filter. Then, histograms are divided by local maxima and equalized.	1. Computational cost is higher. 2. Local maximas sometimes don't work.
10.	Iterated Adaptive Entropy Clip-Limit HE (IAECHE) [25]	First it divides image into sub-images. Then, in each sub image, histogram is stretched based on Entropy clipping and Rayleigh transform.	1. Computational cost is significantly higher. 2. Sometimes produce artefacts.
11.	Mean and Variance based Sub-Image HE (MVSIHE) [69]	It divides the image into 4 segments by mean and variance of luminance component. Then subhisto-grams are equalized by HE.	1. Computational cost is higher. 2. It produces artefacts in the resulted image.

Table 1 continued

Sl No	Name	Methods	Limitations
12.	Recursively Separated Weighted HE (RSWHE) [22]	It divides the histogram by same method as RMSHE except sub-histogram probabilities are modified by a power-law distribution function.	1. Computational cost is more than RSWHE. 2. Data loss is higher by this method.
13.	Brightness Preserving Dynamic Fuzzy HE (BPDFHE) [50]	This method is a modification of BPDHE. Here fuzzy membership function is incorporated to create a fuzzy histogram.	This method is computationally less complex than BPDHE, however, this method is not inevitable of data loss.
14.	Entropy-based Dynamic Sub-Histogram Equalization (EDSHE) [34]	It divides the histogram into two sub-histograms recursively such that their energy will be equal.	1. Computational cost is significantly higher, since this is an iterative method. 2. Sometimes it produces artefacts in image.
15.	Entropy-based Adaptive Sub-Histogram Equalization (EASHE) [70]	It divides the histogram into 4 sub-histograms based on Entropy. A novel algorithm is employed to control the PDF of sub-histograms.	1. Computational cost is significantly higher. 2. Sometimes it produces artefacts in image.
16.	Bi-Histogram Equalization using Two Plateau Limits (BHE2PL) [5]	First it divides histogram into 2 sub-histograms, then it modifies those sub-histograms by two plateau limits.	1. Computational cost is higher. 2. Sometimes it produces artefacts inside image.
17.	Dynamic Clipped HE (DCLHE) [39]	It removes all zero frequency grey levels from image and preserves other non-zero frequency levels by a minimum clip value. Then it stretches the histogram by HE.	1. Computational cost is higher. 2. Can't preserve mean brightness inside the processed image.
18.	White Balance and CLAHE [52]	They used a combination White Balance noise and CLAHE 2.0 (by-default CLAHE) for Covid-19 detection from Chest X-Ray Images.	Their method does excess contrast enhancement for CXR images, thus, data loss is higher.
19.	CLAHE 0.5 and CLAHE 1.0 [43]	They used the contrast clip of CLAHE 0.5 and 1.0 (which are lesser in strength) for Chest X-Ray images.	Their method consumes lesser data loss than by-default CLAHE 2.0. However, it is not inevitable of data loss.

By taking the global mean both side in (41), we get

$$\mu_{gl}(s) = \mu_{gl}(t) \tag{45}$$

Now, contrast of the image can be defined by the ratio of standard deviation of its intensity per unit mean intensity value [45]. From the (44) and (45) we get,

$$C(s) = \frac{\sigma_{gl}(s)}{\mu_{gl}(s)} = \frac{\sigma_{gl}(t)}{\mu_{gl}(t)} = C(t) \tag{46}$$

In (46), $C(s)$ is the global contrast of the processed image and $C(t)$ is the global contrast of the target image. Global contrast of image is already introduced in [45]. Hence, this is proved that overall contrast of the processed image is exactly same as the contrast of the target image, after employing Reinhard method. The covariance between processed image and source image can be computed by substituting value from (41) and (45).

$$\sigma_{sr} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(r_i - \mu_{gl}(r_i))^2 * \frac{\sigma_{gl}(t)}{\sigma_{gl}(r)} \tag{47}$$

$$or,\ \ \sigma_{sr} = \sigma_{gl}(t) * \sigma_{gl}(r) \tag{48}$$

Now, substituting values from (48) and (44) into (16), we get,

$$Corr_{sr} = 1 \tag{49}$$

Equation (49) indicates that correlation co-efficient between processed image and source image by Reinhard method is exactly equal to 1. This reveals that Reinhard method preserves all its information during contrast enhancement or color transfer. Hence, Reinhard method consumes no data loss. This is to clarify that no color transfer is done for for experimentation of Reinhard method for both of the dataset in this paper. Rather, we have employed Reinhard method for only contrast enhancement. For grey-scale MRI dataset, we have employed the (41) for contrast enhancement. For color histopathology dataset, we have employed the same equation for l channel for contrast enhancement, but we didn't change α and β space [46] or any color information. The provided statistical analysis of Reinhard method in this subsection, is done only in one color space. Similarly, statistical analysis can also be done in all three-color channels in [41], i.e., $l\alpha\beta$ space [46], but here we avoided it because we did not change intensity in $\alpha\beta$ space.

4 Experimental results and analysis

Experimental results of several histogram equalization techniques e.g. GHE, CLAHE (with 1.5 and 2.0), HS method and BPDHE are compared with Reinhard method in Fig. 1. The hardware which has been incorporated to implement those methods is Intel CoreTM i3 PC with 2.00 GHz CPU and 8GB RAM. Moreover, MATLAB 2019 software has been employed to implement those methods. Brain tumor MRI images and Colorectal cancer H and E-stained histopathology images are taken from publicly available databases (https://github.com/MohamedAliHabib/Brain-Tumor-Detection) and [57] respectively. From MRI database, 200 number of source images are taken randomly for experimentation, 100 images are associated with tumor and other 100 images are non-tumor images. This MRI dataset contains only gray-scale images. Colorectal cancer H and E-stained histopathology images are taken from Glass challenge dataset [57], 150 images are taken randomly for experimentation, all these images are color images. Moreover, one image from each dataset is chosen as reference image for implementing HS method and Reinhard method, which is further shown in Fig. 1. References images are named as Target 1 and Target 2 for histopathology and MRI images respectively. This reference image should have good contrast. Furthermore, three quality indexes Pearson Correlation Co-efficient (PCC) [59] and Structural Similarity Index Matrix (SSIM) [62] and Absolute Mean Luminance Error (AMLE) [42] are computed for both of the datasets and their mean value are presented in Tables 2 and 3 respectively. In

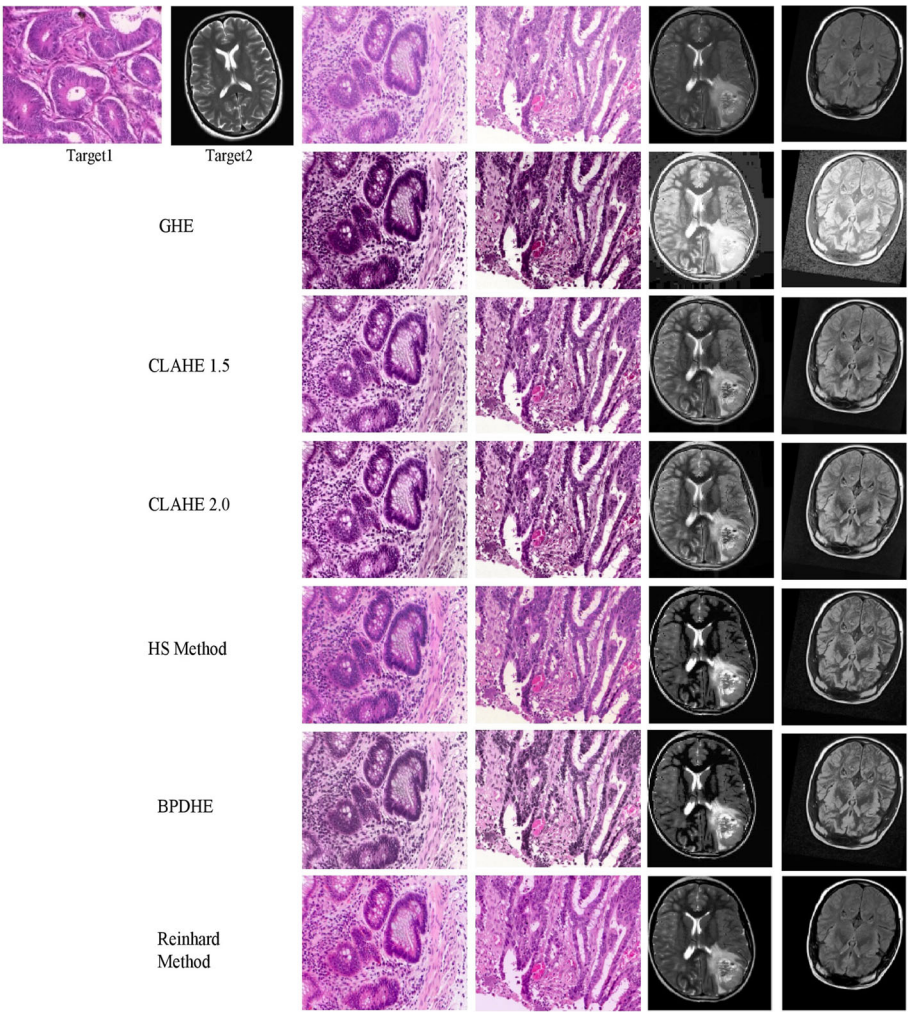

Fig. 1 Comparisons of several HE techniques for Colorectal cancer histopathology images (3rd and 4th columns) and MRI brain tumor images (last two columns). 1st row 3rd, 4th, 5th and 6th column represent source images, row 2nd to 7th represent processed images by several Histogram Equalization techniques

order to prove that there is higher correlation between data loss and degrading accuracy, one more experimentation is done in Section 4.4.

4.1 Quality metrics

PCC and SSIM are presented in Equations (50-51), which are given below:

$$PCC = \frac{1}{Z}\sum_{i=1}^{Z}\frac{\sigma_{s_i r_i}}{\sigma_{s_i}\sigma_{r_i}} \tag{50}$$

$$SSIM = \frac{1}{Z}\sum_{i=1}^{Z}\frac{(2\mu_{r_i}\mu_{s_i} + k_1) * (2\sigma_{s_i r_i} + k_2)}{(\mu_{r_i}^2 + \mu_{s_i}^2 + k_1) * (\sigma_{r_i}^2 + \sigma_{s_i}^2 + k_2)} \tag{51}$$

where, s_i and r_i are processed image and original image content at i^{th} window, Z is total number of window in (50–51), k_1 and k_2 are deployed in (51), in order to avoid infinity and zeros. PCC determines what is the correlation between two images, means contextually how much they match each other. On the other hand, SSIM not only measures correlation, but also contrast and brightness distortion are incorporated, which can be observed from (51). In other words, SSIM is more supporting human visualization [62]. This is to clarify that we computed both SSIM and PCC locally, that is, by taking a window 23 x 23 for MRI images and 17 x 17 for color histopathology images, thereafter, taking the average value (i.e. summing up all the values and dividing it by no of windows). The reason for taking different window size for different dataset is that their texture properties differ a little bit. We have observed that auto-correlation co-efficient [45] of histopathology images is relatively higher than that of MRI images. Therefore, window size for histopathology images is chosen little smaller. A more depth explanation of auto-correlation co-efficient can be found in [45]. For color histopathology images, we computed those metrics in all R, G and B space and thereafter taking their average (i.e. dividing it by 3).

Another metric Absolute Mean Luminance Error (AMLE) is recently proposed by S. Roy et al. [42], to find the mean luminance difference between processed image and source image. The mathematical expression of AMLE is given in (52). In other words, this metric is important for measuring whether luminance of the source image has been preserved in the processed image or not.

$$AMLE = \frac{1}{Z}\sum_{i=1}^{Z}\mu(l_{ip}) - \frac{1}{Z}\sum_{i=1}^{Z}\mu(l_{is}) \tag{52}$$

where l_{ip} is the local content at the i^{th} window of processed image in l space, l_{is} is the local content at the i^{th} window of source image in l space, μ indicates local mean value and Z indicates total number of windows. In this paper, we consider this metric AMLE, because some of the histogram equalization methods like BPDHE works based on notion that it preserves the brightness of the source image.

According to our perspective, AMLE is better metric than Absolute Mean Brightness Error (AMBE) for measuring brightnes preservation in the image. Because AMBE computes mean value in R, G, B space. Whereas, AMLE computes mean value in l space which is more accurate for measuring mean brightness. Moreover, unlike AMBE, AMLE metric is computed locally, thus, it is able to provide more accurate value. For local computation, a local window 23 x 23 is taken for MRI images and 17 x 17 local winodw is taken empirically for color histopathology images and thereafter mean value is computed of such windows, mentioned in the (52).

This is to clarify that in this paper, we did not employ Entropy [10], Peak Signal to Noise Ratio (PSNR) [60], Visual Information Fidelity (VIF) [51] etc. as quality metric. Because, in this paper, our direction is to find whether there is any information lost or not, during pre-processing from medical images. Therefore, we focus more on PCC and SSIM as quality metrics. Moreover, some of the Histogram equalization methods (e.g. BPDHE) are based on preserving source luminance in the processed images. Thus, additionally we brought AMLE [42] in order to evaluate the performance of HE method. Computational complexity or time taken by each HE method is also given in Tables 2 and 3. This computational complexity is not only dependent on the HE method, but also it is dependent on the image quality (or, the

Table 2 Mean values of quality metrics for 200 images for various he techniques (for MRI Dataset)

Metrics	GHE	CLAHE 1.5	CLAHE 2.0	HS Method	BPDHE	Reinhard Method
PCC	0.8152	0.9234	0.9037	0.9287	0.9568	**0.9852**
SSIM	0.6334	0.7568	0.6964	0.7034	0.7157	**0.9405**
AMLE	130.56	65.66	69.57	38.61	**33.39**	34.52
Time taken (in sec)	1.8	2.0	2.0	2.2	2.4	3.1

number of pixels). Since Histopathology images are color images and also their quality is comparatively better than MRI images, this is so obvious that for histopathology images all HE techniques took more time. This can be further observed from Tables 2 and 3.

4.2 Implementating HE techniques on different datasets

For implementing conventional HE techniques like GHE, CLAHE, by default MATLAB code is employed. In CLAHE technique, there is an option to be chosen by user that is contrast clipping. In this paper, we have taken both CLAHE 1.5 and CLAHE 2.0 for experimentation. In case of Reinhard method, only one transformation function, mentioned in (41), is utilized. In case of MRI image, this transformation function is directly applied on the grey scale image. In case of color image (histopathology images), the image first need to be converted to lab color space, thereafter, incorporating this transformation function (mentioned in 41) in only l channel, but not in ab channel. The reason his was already mentioned in the Section 3.6.

4.3 Observation and siscussions

This can be observed from Fig. 1 second row, that Global HE method has done excess contrast enhancement of the source images, for both of the datasets. For first two histopathology images, due to over contrast enhancement, the color of all the nuclei (blue) have been converted into black color. This is kind of undesirable, because blue color is an important feature to recognize nuclei in histopathology images and this may cause miss-classification at the final step of cancer detection. Moreover, if we zoom enough those figures, this can be noticed that some of the significant information of nuclei (e.g., intra and inter intensity

Table 3 Mean values of quality metrics for 150 images for various he techniques (for Histopathology Dataset)

Metrics	GHE	CLAHE 1.5	CLAHE 2.0	HS Method	BPDHE	Reinhard Method	tab3 tab3
PCC	0.7671	0.8898	0.8479	0.9471	0.8383	**0.9752**	tab3
SSIM	0.6345	0.7697	0.7341	0.7987	0.6975	**0.9191**	tab3
AMLE	106.04	62.39	65.61	52.49	56.23	**45.73**	tab3
Time taken (in sec)	3.1	3.4	3.4	3.6	4.1	3.9	tab3 tab3

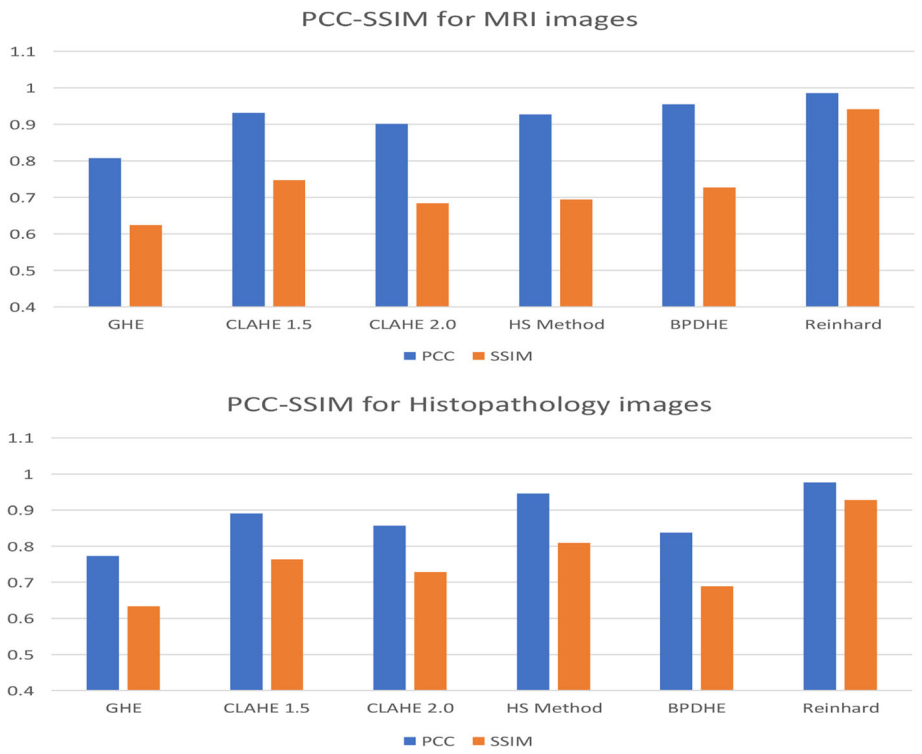

Fig. 2 Visualization of Mean PCC-SSIM values for both MRI and histopathology images

variation of nuclei, nuclei boundary etc.) has been washed out from the processed image, which are significant features for cancer detection. From the last two MRI images in second row, this can be observed that GHE provides very different images, the local areas of processed images are mostly turned into white and loosing local information. Thus, mean value of PCC and SSIM of GHE for both of the datasets deviates from 1 significantly. This can be further observed from Tables 2 and 3. This can be also visualized from Fig. 2. Furthermore, AMLE value we found is the largest for GHE method, that means this method didn't preserve the local brightness of the image as well. Hence, these experimental results support our mathematical analysis, presented in Section 3.1.

One of the ways to overcome the limitations of GHE is to employ CLAHE which is a local Histogram Equalization technique in which amount of contrast enhancement is also limited. In this paper, we compared the performance of both CLAHE 1.5 and CLAHE 2.0 in Fig. 1, which can be observed from second and third rows of Fig. 1 respectively. Generally, most of the researchers employ by default CLAHE 2.0. From Fig. 1, this can be observed that due to employing CLAHE, the nuclei color is little bit changed to dark color which is undesirable. Furthermore, CLAHE 2.0 has done little bit over contrast enhancement for both of the datasets, according to our visualization and due to that some of the information is lost. However, it seems like for MRI dataset CLAHE 1.5 and CLAHE 2.0 both of the methods worked very efficiently. Therefore, for MRI dataset, PCC is found 0.9234 and 0.9037 for CLAHE 1.5 and CLAHE 2.0 respectively. However, their method is not entirely unavoidable

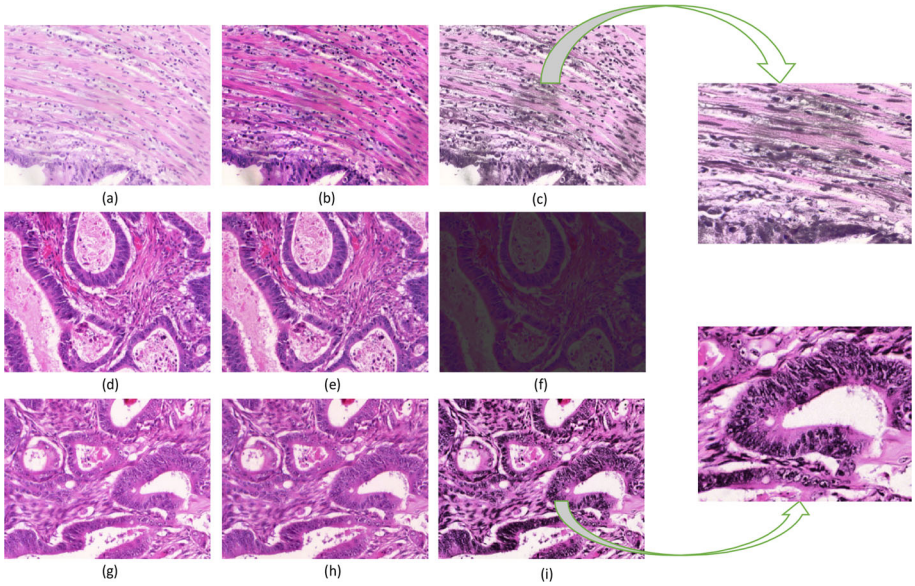

Fig. 3 Some of the histopathology images where BPDHE didn't work. In first column, Figs. 3 (a), (d), (g) represent histopathology source images, In second column, Figs. 3 (b), (e), (h) represent processed images by HS method, In third column, Figs. 3 (c), (f), (i) represent processed images by BPDHE method

of data loss. Moreover, human eyes can not detect lower order statistics [32], thus, we can not detect such little data

loss which is reflected in Table 2 and 3. Furthermore, a CNN model VGG-16 is implemented on this CLAHE pre-processed MRI images, which shows in Section 4.4 that due to data loss during pre-processing,the accuracy of the model decreases. AMLE value is quite higher for both of the CLAHE methods. This indicates that CLAHE could not preserve the local brightness in the processed images as well. Hence, we can conclude that CLAHE consumes significant data loss for histopathology dataset and little bit data loss in MRI dataset. This also supports our mathematical analysis, done in Section 3.2.

From Fig. 1, in row 5-6, this can be observed that HS method and BPDHE both are working efficiently for both of the datasets. The reason why they might work better than other HE techniques were already pre-estimated and analyzed in depth in Sections 3.3 and Section 3.4 respectively. However, we have found that BPDHE does not work well for some of the histopathology color images which is further shown in Fig. 3. This can be observed that in Fig. 3f, BPDHE produced a completely dark image, thus, correlation coefficient for that image we got 0.203 which is very less. Similarly, in Figs. 3c and 3i, the processed images' nuclei color have turned into black and there are many black artefacts, which are further highlighted in the zoomed portion in Fig. 3. For source image 3 (in Fig. 3i), BPDHE does over contrast enhancement, thus, it converted the nuclei color into black which is not desirable for further cancer classification task. Consequently, in Table 3 and Fig. 2, this is found that both PCC and SSIM for BPDHE method deviate considerably from the value 1, for histopathology images. This indicates BPDHE method consumes significant data loss for histopathology image dataset.

On the other hand, HS method seems like working efficiently for both of the datasets. In the processed images we did not notice too much changes for HS method. However, still we

noticed from Tables 2, 3and Fig. 2 that HS method is not entirely avoidable of data loss, it consumes approx 6-8 percent data loss from both of the datasets which is bearable. Overall HS method is the only HE technique we found that works well for both of the datasets. Additionally, this can also be observed that BPDHE works very efficiently for only MRI dataset. Tables 2 and Fig. 1 showed that BPDHE only consumes around 5% data loss (PCC 0.9568) which is bearable. We have got significant deviation of SSIM (0.7157) due to the fact that it sometimes does excess-contrast enhancement for MRI images. The main reason behind BPDHE works well for MRI images is that it has been able to preserve background luminance of the source images, thus, there is no big difference noticed between processed image and source images for MRI dataset. Additionally, we have computed AMLE metric for BPDHE and HS method, given in Tables 2and 3. We have found that for HS method and for BPDHE, mean value of AMLE is considerably lesser than that of other HE techniques, despite of the fact that by their method data loss is not entirely unavoidable.

The aforementioned data analysis revealed that there are considerable data loss for any HE techniques. Generally, for natural images that much of data loss may not be so relevant, however, for medical image diagnosis this much of data loss is undesirable. Especially, in case of tumor or cancer detection, this lost data may contain significant features for the final classification task. Consequently, it may lead to lesser accuracy, this is further shown in Section 4.4. Hence, these HE techniques are not suitable for medical image diagnosis as a pre-processing task. HS method has worked slightly better than other HE methods, for both of the datasets, due to the fact that it does the contrast enhancement operation according to the histogram of a reference image. Thus, contrast enhancement here is limited to a certain extent. Perhaps, the applicability of these HE techniques are also dependent on statistical properties of the images. For example, one particular HE technique (e.g. BPDHE) may work efficiently for one (MRI) dataset, but there is no guarantee that it would work for other dataset as well. We have observed that for other medical image datasets like CT images, Satellite images, Ultrasound and Chest X-Ray, the images' clarity may not be very good. In that case, HE technique may be suitable in order extract extra valuable information from the images. However, if the contrast is already higher, then further employing HE technique or CLAHE technique can further do over-contrast enhancement and consequently, it may lose some significant data from the image. Hence, actually, it is very much dependent on the statistics of the images' dataset.

We conduct Reinhard method [40] on the same datasets. Unlike any HE method, Reinhard method is a linear transformation. Thus, it is likely that it will preserve all the data during pre-processing according to the Lemma 2. In Section 3, we already proved mathematically that Reinhard method doesn't consume any data loss. Furthermore, from Fig. 1, this can be observed that, the processed images by Reinhard method have not changed too much with respect to source image and it's preserving most of its local information. The same can be noticed from Table 2, Table 3 and Fig. 2. The mean values of both PCC and SSIM for Reinhard method are found very closer to the value 1. Therefore, experimental results support our mathematical analysis. Moreover, from Fig. 1, MRI first image (last row third column), this can be noticed that it didn't do over-contrast enhancement like CLAHE, GHE or BPDHE, but it enhance the tumor information, without changing the mean brightness or any other information of the image. Furthermore, from Fig. 1 (last row first two column) this can be observed that Reinhard method did not convert the nuclei color into black, like CLAHE or BPDHE method. Indeed, Reinhard method has been able to preserve all the significant nuclei informations in histopathology images, which is desirable. Hence, for tumor detection or for cancer diagnosis, we prefer Reinhard method over any other HE methods. Because it

Table 4 Testing Accuracy and testing loss for pre-trained VGG-16 Model on MRI Dataset

Metrics	Original MRI Dataset	preprocessed with CLAHE 0.5	preprocessed with CLAHE 2.0
Accuracy	82.5%	54.8%	51.52%
loss	0.66	0.98	1.13

can improve the contrast of very poor contrast image and at the same time, it preserves all the contextual information inside that image.

This is to clarify that some of the HE techniques like CLAHE 1.5, BPDHE and HS method are little bit closer to the performance of linear method (e.g. Reinhard method), which can be observed from Table 2 and Table 3. Because it limits the contrast enhancement operation to a certain extent. However, we found still they are prone to data loss. Since, CLAHE (or, any other HE techniques) are widely used in medical image detection, we believe that this paper would give researchers a kind of insight that they should not blindly incorporate any HE technique as a pre-processing technique. Otherwise, they may lose some important data even before feeding it into a classifier. We found that for medical images, data preservation is more significant than extracting some hidden information from the images. Any small data loss may miss-classify the disease. This is further proved with an experiment in the next subsection. Hence, linear transformations like Reinhard method would be certainly a better choice in that scenario.

4.4 VGG-16 Implementation on MRI images

One may argue that, if there is huge data loss by those HE techniques (as pre-processing), why should we bother? In other words, is there any direct correlation between data loss (by HE pre-processing technique) and degrading accuracy in the final classifier? In order to prove that, we included some implementations of CNN model, in this subsection. We have implemented one of the most popular CNN models i.e. VGG-16 [54], which is pre-trained from the ImageNet dataset [13], which is readily available in Keras library. Table 4 shows that after employing CLAHE 0.5 and CLAHE 2.0, the accuracy of the classifier decreases significantly and the testing loss has increased for both CLAHE 0.5 and CLAHE 2.0. The reason behind that is CLAHE is prone to huge data loss, which we already addressed in Section 3.2. Thus, we have lost some important features (especially in case of cancer or tumor images), even before feeding these images into the classifier. As a result, the accuracy after utilizing CLAHE is getting worse. One important observation is that dataset preprocessed with CLAHE 0.5 has little bit better accuracy than that of dataset preprocessed with CLAHE 2.0. Because CLAHE 2.0 does more contrast enhancement than CLAHE 0.5, it is likely that there will be more loss. However, the data loss by CLAHE 0.5 technique (or, any HE technique) is also unavoidable, which is shown in Table 4. Therefore, this experiment also supported the conclusion we have done from mathematical analysis that "Data loss in CLAHE method does not depend on the limiting contrast enhancement (or, contrast clip) parameter". That means, there will be always data loss by any CLAHE method.

The training specifications for the VGG-16 model are given below:

1. For all the three experiments (given in Table-4) a pre-trained model VGG-16 is used. That means, transfer learning [63] is done from ImageNet dataset to MRI dataset.

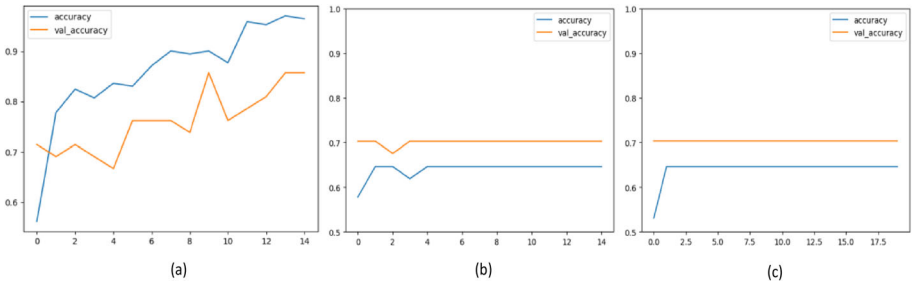

Fig. 4 Accuracy vs number of epochs graph of training and validation for VGG-16 model on (a) Original dataset, (b) Dataset preprocessed with CLAHE 0.5, (c) Dataset preprocessed with CLAHE 2.0

2. Training is done for 15 epochs. This much of epochs was enough, since we are using pre-trained model. This is to clarify that we have not monitored validation loss as early stopping, because validation loss was very unstable and after 6 epochs training was stopped.
3. Learning rate chosen for all the three experiments is $1e^{-3}$.
4. Adams-optimizer is employed as the preferred choice of optimizer.
5. A batch size of 16 is utilized for all the experiments.
6. Binary cross entropy is incorporated as loss function, since this was a binary classification task MRI yes or no. That means, either tumor is present (yes) or not (no).
7. Two dense layers are used in VGG-16 model (after the flatten layer), having number of neurons 24 and 6 respectively.
8. Sigmoid activation function was used in output layer, whereas at other layers, ReLU activation function was employed.
9. For implementing CLAHE, we take window grid size 8 x 8 and clip 0.5 and 2.0. (i.e. CLAHE 0.5 and CLAHE 2.0).

Accuracy vs number of epochs graph is shown for training (blue color) and validation (orange color) in Fig. 4. Total three experients are done here: (a) Implementing VGG-16 on Original Dataset as it is, (b) Implementing VGG-16 on Dataset preprocessed with CLAHE 0.5, (c) Implementing VGG-16 on Dataset preprocessed with CLAHE 2.0. Since the dataset is very small and VGG-16 model is quite heavy (having more than 8 million hyper-paramters to train), there is overfitting in the first experiment. However, after using CLAHE 0.5 and CLAHE 2.0, training did not happen properly, that means both of the experiments with CLAHE is prone to huge underfitting. Thus, there is a flat line of accuracy for both of the experiments; 0.7 accuracy it achieved from the very first epochs due to using pre-trained model. That means, it utilizes the weights for VGG-16 model which was already trained from large ImageNet dataset. However, after the first epoch, there was no improvement in the training, due to huge data loss in the images. Consequently, its testing accuracy has decreased considerably from 82% to 51%. These experiments are the proof that CLAHE method can significantly remove important features from Medical images, especially for tumor or cancer diagnosis. Hence, there is a direct correlation between data loss and degrading accuracy. Therefore, CLAHE method or any other HE method consuming data loss, is not suitable for pre-processing task prior to disease diagnosis.

The above research analysis provides the researchers an insight that they should not employ any pre-processing technique like CLAHE or GHE method blindly. They have to be very careful before employing any pre-processing method for reducing noise or for contrast

enhancement. The pre-processing techniques which consume data-loss, may degrade the final classification accuracy. We have observed that especially for the cancer or tumor diagnosis images, any pre-processing technique which is prone to data loss, provides worse accuracy for classification. Thus, researchers should first analyze these pre-processing techniques whether they are suitable or not for a particular dataset or for particular task. Then only they should employ these techniques prior to a classifier.

5 Conclusion

A novel mathematical analysis was presented in order to re-investigate numerous HE techniques which are frequently used as pre-processing techniques, prior to classification task. First, it was mathematically proved that any HE technique itself is a non-linear transformation, thus, for HE technique correlation co-efficient between processed image and original image was far deviating from the value 1. That means there was huge data loss, according to our mathematical analysis. Thereafter, all these HE techniques GHE, CLAHE 1.5, CLAHE 2.0, HS, BPDHE were implemented in two medical datasets, those are MRI dataset and Colorectal Cancer histopathology dataset. Moreover, by experimentation, quality metrics PCC and SSIM both were found in the range of 0.6-0.95, for all HE techniques. This indicated that there were huge data loss (5-40 percent) by those HE techniques. Thus, these experiments actually supported our employed mathematical analysis. Furthermore, a mathematical analysis was also provided for a linear method (e.g., Reinhard method), and it was proved mathematically that Reinhard method preserved most of the data during pre-processing. A Lemma had also been proved in the appendix that any linear transformation (for contrast enhancement) is actually prone to no data loss. Moreover, by experimentation it was found that both PCC and SSIM mean values for Reinhard method were very close to the value 1. Hence, we can conclude that a linear method (like Reinhard method) can indeed outperform other HE based contrast enhancement methods, for medical image enhancement task. Furthermore, a popular CNN model VGG-16 was implemented on MRI images, to prove that there was a huge correlation between data loss and degrading accuracy, especially for tumor-related diagnosis. Hence, before employing any HE technique as a pre-processing method one should analyze whether that method consumes any data loss or not. Otherwise, there may be considerable degradation of model performance. Hence, this research work provided researchers a kind of insight that they should not blindly incorporate any HE technique as a pre-processing technique.

Appendix

Lemma1: For any contrast enhancement method, (with having transformation function which has number of roots 1),

$$If,\ \frac{p_r(r)}{p_s(s)} \approx 1,\ then\ Corr_{sr} \approx 1 \tag{53}$$

where, $p_r(r)$ is the PDF of source image, $p_s(s)$ is the PDF of the processed image, $Corr_{sr}$ is the correlation co-efficient between processed image and source image.

From (6), we got (as the number of roots of the transformation function is 1),

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| \tag{54}$$

Now if $p_r\ (r) \approx\ p_s\ (s)$, according to the Lemma1, then from (54), we got, (for simplicity of calculation let's assume $p_r\ (r) = p_s\ (s)$)

$$\int dr = \int ds \tag{55}$$

$$or,\ \ s = r + k \tag{56}$$

where k is integration constant.

From (56), this is concluded that the transformation function of such contrast enhancement method will be linear.

Taking global standard deviation both of the sides in (56), we got,

$$\sigma_{gl}(s) = \sigma_{gl}(r) \tag{57}$$

Similarly, by taking global mean both side of the (56), we got,

$$\mu_{gl}(s) = \mu_{gl}(r) + k \tag{58}$$

Now, Covariance between processed image (s) and source image (r) is given by the following equation. (from 13)

$$\sigma_{sr} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(s_i - \mu_{gl}(s_i)) * (r_j - \mu_{gl}(r_j)) \tag{59}$$

Now, substituting the value from (56) and (58) into (59), we got,

$$\sigma_{sr} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(r_j - \mu_{gl}(r_j))^2 \tag{60}$$

$$or,\ \ \sigma_{sr} = \sigma_{gl}^2(r) \tag{61}$$

Now the correlation coefficient between processed image and original image is given by following equation. (from 17)

$$Corr_{sr} = \frac{\sigma_{sr}}{\sigma_{gl}(s) * \sigma_{gl}(r)} \tag{62}$$

Substituting values from (57) and (61) into (62) we got,

$$Corr_{sr} = 1\ \ (Hence\ Proved) \tag{63}$$

Lemma2: For any contrast enhancement method,

$$If,\ \ p_r(r) = \frac{1}{c}p_s(s),\ \ then\ Corr_{sr} \approx 1. \tag{64}$$

whereas, c is a real constant, $p_r(r)$ is the PDF of source image, $p_s(s)$ is the PDF of processed image, $Corr_{sr}$ is the correlation co-efficient between processed image and source image. In other words, if the transformation function of contrast enhancement method is linear, then there will be no data loss.

For simplicity of calculation let's assume $p_r(r) = 1/c * p_s(s))$ then from (54) we got,

$$\int dr = \frac{1}{c} * \int ds \tag{65}$$

$$s = \frac{1}{c} * (r + k) \tag{66}$$

where k is an integration constant.

From (66), this is concluded that the transformation function of such contrast enhancement method is linear.

Taking global standard deviation both of the sides in (66), we got,

$$\sigma_{gl}(s) = \frac{1}{c} * \sigma_{gl}(r) \tag{67}$$

Similarly, by taking global mean both side of the (66), we got,

$$\mu_{gl}(s) = \frac{1}{c} * \mu_{gl}(r) + k_1 \tag{68}$$

Now, substituting the value from (66) and (68) into (59), we got,

$$\sigma_{sr} = \frac{1}{c} * \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (r_j - \mu_{gl}(r_j))^2 \tag{69}$$

$$or,\ \sigma_{sr} = \frac{1}{c} * \sigma_{gl}^2(r) \tag{70}$$

Now, substituting values from (67) and (70), into (62) we got,

$$Corr_{sr} = 1 \quad (Hence\ Proved) \tag{71}$$

Hence, it is proved that a linear transformation doesn't prone to data loss.

Data Availability Data sharing is not applicable to this article as no dataset was generated. Only the existing datasets are experimented with HE techniques. The source of these datasets are already mentioned in Reference section and in manuscript.

Declarations

Conflicts of interest The authors declare that they have no conflict of interest for this manuscript.

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