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1.

Given: $Z \sim N(0, 1)$ (standard normal distribution)

(a) Find $P(0.2 \leq Z \leq 0.79)$

Step 1: Use the formula $P(a \leq Z \leq b) = \Phi(b) - \Phi(a)$
where Φ is the cumulative distribution function (CDF)

Step 2: Find $\Phi(0.79)$ from Z-table or calculator
$\Phi(0.79) = 0.7852$

Step 3: Find $\Phi(0.2)$ from Z-table or calculator
$\Phi(0.2) = 0.5793$

Step 4: Calculate the probability
$P(0.2 \leq Z \leq 0.79) = 0.7852 - 0.5793 = 0.2059$

ANSWER: 0.2059 or 20.59%

(b) Find $P(-1.64 \leq Z \leq 0.9)$

Step 1: Use the formula $P(a \leq Z \leq b) = \Phi(b) - \Phi(a)$

Step 2: Find $\Phi(0.9)$ from Z-table or calculator
$\Phi(0.9) = 0.8159$

Step 3: Find $\Phi(-1.64)$ from Z-table or calculator
$\Phi(-1.64) = 0.0505$

Step 4: Calculate the probability
$P(-1.64 \leq Z \leq 0.9) = 0.8159 - 0.0505 = 0.7654$

ANSWER: 0.7654 or 76.54%

2.

Given:
- Population mean: $\mu = 4.0$ g
- Population standard deviation: $\sigma = 1.5$ g
- Sample size: $n = 50$ batches
- Find: $P(3.5 \leq \bar{X} \leq 3.8)$

Step 1: Calculate the standard error (SE) of the sample mean
$SE = \sigma/\sqrt{n} = 1.5/\sqrt{50} = 1.5/7.071 = 0.2121$ g

Step 2: By Central Limit Theorem, $\bar{X} \sim N(4.0, 0.2121)$
The sampling distribution is approximately normal

Step 3: Standardize the lower bound (3.5)
$Z_1 = (3.5 - 4.0)/0.2121 = -0.5/0.2121 = -2.357$

Step 4: Standardize the upper bound (3.8)

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$Z_2 = (3.8 - 4.0)/0.2121 = -0.2/0.2121 = -0.943$

Step 5: Find the probability

$P(3.5 \leq \bar{X} \leq 3.8) = P(-2.357 \leq Z \leq -0.943)$
$= \Phi(-0.943) - \Phi(-2.357)$
$= 0.1728 - 0.0092$
$= 0.1636$

ANSWER: 0.1636 or 16.36%

Interpretation: There is approximately a 16.36% chance that the average impurity from 50 batches will be between 3.5 and 3.8 g.

3.

Given:
- Number of trials: $n = 48$
- Number of successes (ignitions): $x = 16$
- Confidence level: 95%
- Find: 95% CI for the true proportion p

Step 1: Calculate the sample proportion

$\hat{p} = x/n = 16/48 = 0.3333$

Step 2: Find the critical value for 95% confidence

For 95% CI, $\alpha = 0.05$, so $\alpha/2 = 0.025$
$z^* = 1.96$ (from Z-table)

Step 3: Calculate the standard error

$SE = \sqrt{[\hat{p}(1-\hat{p})/n]}$
$SE = \sqrt{[0.3333(1-0.3333)/48]}$
$SE = \sqrt{[0.3333 \times 0.6667/48]}$
$SE = \sqrt{[0.00463]}$
$SE = 0.0680$

Step 4: Calculate the margin of error

$ME = z^* \times SE = 1.96 \times 0.0680 = 0.1333$

Step 5: Calculate the confidence interval

Lower bound $= \hat{p} - ME = 0.3333 - 0.1333 = 0.2000$
Upper bound $= \hat{p} + ME = 0.3333 + 0.1333 = 0.4666$

ANSWER: 95% CI = (0.2000, 0.4666) or (20.00%, 46.66%)

Interpretation: We are 95% confident that the true proportion of trials that would result in ignition is between 20.00% and 46.66%.

4.

Given:
- Claimed mean: $\mu_0 = 130°F$
- Sample size: $n = 9$
- Sample mean: $\bar{x} = 131.08°F$

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- Population standard deviation: $\sigma = 1.5°F$
- Significance level: $\alpha = 0.01$
- Test: Does the data contradict the manufacturer's claim?

Step 1: State the hypotheses

H_0: $\mu = 130°F$ (manufacturer's claim is correct)
H_1: $\mu \neq 130°F$ (two-tailed test)

Step 2: Calculate the test statistic (use Z-test since σ is known)

$Z = (\bar{x} - \mu_0)/(\sigma/\sqrt{n})$
$Z = (131.08 - 130)/(1.5/\sqrt{9})$
$Z = 1.08/(1.5/3)$
$Z = 1.08/0.5$
$Z = 2.16$

Step 3: Find the critical values for $\alpha = 0.01$ (two-tailed)

$\alpha/2 = 0.005$ in each tail
Critical values: $z^* = \pm 2.576$

Step 4: Calculate the p-value

P-value = $2 \times P(Z > 2.16)$ [multiply by 2 for two-tailed]
P-value = $2 \times (1 - 0.9846)$
P-value = 2×0.0154
P-value = 0.0308

Step 5: Make a decision

Since $|Z| = 2.16 < 2.576$ (critical value)
OR equivalently, p-value = $0.0308 > 0.01$ (α)
We FAIL TO REJECT H_0

ANSWER: No, the data does NOT contradict the manufacturer's claim at the $\alpha = 0.01$ significance level.

5.

Given:
- $X \sim N(\mu, 5)$ where 5 is the standard deviation ($\sigma = 5$)
- $P(X < 23) = 0.9192$
- Find: (i) the value of μ, (ii) $P(\mu < X < 23)$

Part (i): Find the value of μ

Step 1: Standardize the probability statement

$P(X < 23) = 0.9192$
$P[(X - \mu)/\sigma < (23 - \mu)/\sigma] = 0.9192$
$P[Z < (23 - \mu)/5] = 0.9192$

Step 2: Find the Z-value corresponding to probability 0.9192

From Z-table (inverse lookup): $\Phi(z) = 0.9192$
$z = 1.40$

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Step 3: Set up the equation

$(23 - \mu)/5 = 1.40$

Step 4: Solve for μ

$23 - \mu = 1.40 \times 5$
$23 - \mu = 7.0$
$\mu = 23 - 7.0$
$\mu = 16.0$

ANSWER (i): $\mu = 16.0$

Part (ii): Find $P(\mu < X < 23) = P(16 < X < 23)$

Step 1: Standardize both bounds

$Z_1 = (16 - 16)/5 = 0/5 = 0$
$Z_2 = (23 - 16)/5 = 7/5 = 1.40$

Step 2: Calculate the probability

$$\begin{aligned} P(16 < X < 23) &= P(0 < Z < 1.40) \\ &= \Phi(1.40) - \Phi(0) \\ &= 0.9192 - 0.5000 \\ &= 0.4192 \end{aligned}$$

ANSWER (ii): $P(\mu < X < 23) = 0.4192$ or 41.92%

Interpretation: There is a 41.92% probability that X falls between its mean (16) and the value 23.

Note: This makes sense because $P(X < 23) = 0.9192$, which includes both the left half [below the mean, probability = 0.5] and the right portion [from mean to 23, probability = 0.4192].

Problem 6:

Given:
- Scores are approximately normally distributed
- Mean: $\mu = 70$
- Standard deviation: $\sigma = 10$
- Top 10% receive A's
- Next 25% receive B's (students between 65th and 90th percentile)
- Find: The minimum scores for A's and B's

Part 1: Find the minimum score for an A

Step 1: Understand what we're looking for

Top 10% get A's means 90% of students score below the A cutoff
We need to find the score x where $P(X < x) = 0.90$

Step 2: Find the Z-score corresponding to the 90th percentile

From Z-table (inverse lookup): $P(Z < z) = 0.90$
$z = 1.28$ (approximately 1.282)

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Step 3: Convert the Z-score to the actual score
Use the formula: $X = \mu + z \times \sigma$
$X = 70 + 1.28 \times 10$
$X = 70 + 12.8$
$X = 82.8$

Step 4: Round appropriately
Minimum score for an A ≈ 83

ANSWER: Students need to score at least 83 to receive an A.

Part 2: Find the minimum score for a B

Step 1: Understand what we're looking for
Top 10% get A's, next 25% get B's
So B's go to students between 65th and 90th percentile
(100% - 10% - 25% = 65% score below the B cutoff)
We need to find x where $P(X < x) = 0.65$

Step 2: Find the Z-score corresponding to the 65th percentile
From Z-table (inverse lookup): $P(Z < z) = 0.65$
$z = 0.385$ (approximately 0.39)

Step 3: Convert the Z-score to the actual score
$X = \mu + z \times \sigma$
$X = 70 + 0.385 \times 10$
$X = 70 + 3.85$
$X = 73.85$

Step 4: Round appropriately
Minimum score for a B ≈ 74

ANSWER: Students need to score at least 74 to receive a B.

SUMMARY OF GRADE CUTOFFS:
- Score ≥ 83: Grade A (top 10%)
- 74 ≤ Score < 83: Grade B (next 25%)
- Score < 74: Grade C or below (remaining 65%)

Verification:
- $P(X \geq 83) = P(Z \geq 1.28) = 1 - 0.90 = 0.10 = 10\%$ ✓
- $P(74 \leq X < 83) = P(0.385 \leq Z < 1.28) = 0.90 - 0.65 = 0.25 = 25\%$ ✓

7.

Given:
- Significance level: $\alpha = 0.05$
- Partially completed ANOVA table with values:
  * Between: SS = 28, df = C, MS = 14
  * Within: SS = A, df = 15, MS = E, F-value = 7, p-value = K
  * Total: SS = B, df = D

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- Find: A, B, C, D, E, and K

ANOVA Table Relationships:
- SS (Sum of Squares): SS_Total = SS_Between + SS_Within
- df (degrees of freedom): df_Total = df_Between + df_Within
- MS (Mean Square): MS = SS/df
- F-statistic: F = MS_Between / MS_Within

Step 1: Find C (df for Between groups)
We know: MS_Between = SS_Between / df_Between
$14 = 28 / C$
$C = 28/14$
$C = 2$

ANSWER C: df_Between = 2 (This means 3 groups, since df = k - 1)

Step 2: Find E (MS for Within groups)
We know: F = MS_Between / MS_Within
$7 = 14 / E$
$E = 14/7$
$E = 2$

ANSWER E: MS_Within = 2

Step 3: Find A (SS for Within groups)
We know: MS_Within = SS_Within / df_Within
$2 = A / 15$
$A = 2 \times 15$
$A = 30$

ANSWER A: SS_Within = 30

Step 4: Find B (Total SS)
SS_Total = SS_Between + SS_Within
$B = 28 + 30$
$B = 58$

ANSWER B: SS_Total = 58

Step 5: Find D (Total df)
df_Total = df_Between + df_Within
$D = 2 + 15$
$D = 17$

ANSWER D: df_Total = 17 (This means total sample size n = 18)

Step 6: Find K (p-value)
We need to find $P(F > 7)$ where F follows an F-distribution
with $df_1 = 2$ (numerator) and $df_2 = 15$ (denominator)

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Using F-table or calculator:
For $F(2, 15)$ distribution, the critical value at $\alpha = 0.05$ is 3.68
For $F(2, 15)$ distribution, the critical value at $\alpha = 0.01$ is 6.36

Since our F-statistic = 7 > 6.36, we know p-value < 0.01

More precisely (using F-distribution table or calculator):
$P(F_{2,15} > 7) \approx 0.0074$

ANSWER K: p-value ≈ 0.0074 (or approximately 0.007)

COMPLETED ANOVA TABLE:

Source of Variation	SS	df	MS	F-value	p-value
Between	28	2	14		
Within	30	15	2	7	0.0074
Total	58	17			

SUMMARY OF ANSWERS:
A = 30 (SS_Within)
B = 58 (SS_Total)
C = 2 (df_Between)
D = 17 (df_Total)
E = 2 (MS_Within)
K = 0.0074 (p-value)

CONCLUSION:
Since p-value = 0.0074 < α = 0.05, we REJECT the null hypothesis.
There is sufficient evidence to conclude that there are significant differences among the group means.