

An Effect of Background Population Sample Size on the Performance of a Likelihood Ratio-based Forensic Text Comparison System: A Monte Carlo Simulation with Gaussian Mixture Model

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Abstract

This is a Monte Carlo simulation-based study that explores the effect of the sample size of the background database on a likelihood ratio (LR)-based forensic text comparison (FTC) system built on multivariate authorship attribution features. The text messages written by 240 authors who were randomly selected from an archive of chatlog messages were used in this study. The strength of evidence (= LR) was estimated using the multivariate kernel density likelihood ratio (MVKD) formula with a logistic-regression calibration. The results are reported along two points: the system performance (= accuracy) and the stability of performance based on the standard metric for LR-based systems; namely the log-likelihood-ratio cost (C_{llr}). It was found in this study that the system performance and its stability improve as a function of the sample size (= author count) in the background database in a non-linear manner, and that the more features used for modelling, the more background data the system generally requires for optimal results. The implications of the findings to the real casework are also discussed.

1 Introduction

1.1 Forensic text comparison and the likelihood-ratio framework

The conceptual framework of likelihood ratio (LR) has received or has started receiving wide support from various areas of forensic comparative sciences as the logically and legally correct framework for assessing forensic evidence, and presenting the strength of the evidence (Balding, 2005; Evett et al., 1998; Marquis et al., 2011; Morrison, 2009; Neumann et al., 2007). Although forensic text

comparison (FTC) is lagging behind other areas of forensic comparative sciences, studies in which the LR framework was applied to authorship attribution have started emerging (Ishihara, 2012, 2014b).

As expressed in equation (1), the LR, the quantified strength of evidence, is a ratio of two conditional probabilities: one is the probability (p) of observed evidence (E) assuming that the prosecution hypothesis is true (H_p) and the other is the probability of the same observed evidence assuming that the defence hypothesis (H_d) is true (Robertson & Vignaux, 1995).

$$LR = \frac{p(E|H_p)}{p(E|H_d)} \quad (1)$$

For FTC, for instance, it will be the probability of observing the difference (referred to as the evidence, E) between the offender's and the suspect's text messages if they had been produced by the same author (H_p) relative to the probability of observing the same evidence (E) if they had come from different authors (H_d).

In practice, an LR is estimated as a ratio of two terms: similarity and typicality, which correspond to the numerator and denominator of equation (1). Similarity means the similarity (or difference) between the offender and the suspect samples (e.g. text messages). Typicality means, in general terms, the typicality (or atypicality) of the offender sample against the relevant population. If the offender and the suspect samples are more similar or more atypical, the LRs will be greater than when the same samples are more different or more typical.

It is important to emphasise that for example, $LR = 100$ does not mean that it is 100 times more likely that the offender and the suspect are the same person, but it means that the evidence is 100 times more likely to arise if the offender and the s-

uspect samples had been produced by the same individual, than by different individuals.

As can be well understood from the concept of typicality, besides the offender and the suspect samples, it is an essential part of the LR framework to have samples from a relevant population for typicality. It goes without saying that an appropriate amount of data is required as relevant population data (= background data) to build an accurate model for typicality. Yet, how much do we need?

1.2 Research question

Having briefly outlined the key concepts of the LR framework, the present study investigates how the sample size of the background data influences the performance of the LR-based FTC system.

For this, a series of experiments was repeatedly carried out with the synthetic background data generated by the Monte Carlo technique, which are different in sample size (= different numbers of authors). The performance of the FTC system was assessed by the log likelihood ratio cost (C_{llr}) (Brümmer & du Preez, 2006). Three different lengths: 500, 1000 and 1500 words and four feature vectors: two, four, six and eight features were used in the experiments to see how these factors also contribute to the performance.

1.3 Previous studies

It can be considered that the greater the amount of representative data, the more accurate the model of the reference population, leading to a more accurate estimate of strength of evidence. A small number of studies have investigated the effect of sample size in the background database on the system performance, in particular, in the field of forensic voice comparison (Hughes et al., 2013; Ishihara & Kinoshita, 2008), and reported a similar outcome that the performance of a system becomes stable with greater than 20 reference individuals. However, those studies are voice/speech as evidence and did not consider the number of features in vectors.

Features	Two	Four	Six	Eight
Unusual word ratio	√	√	√	√
Punctuation character ratio	√	√	√	√
Type-token ratio (TTR)		√	√	√
Average word count per message line		√	√	√
Honoré's R			√	√
Digit character ratio			√	√
Average character count per message line				√
Special character ratio (, . ? ! ; : ' ")				√

Table 1: List of eight features and four different feature vectors. √ = feature used.

2 Research Design

2.1 Database

An archive of chatlog messages¹, which is a collection of real pieces of chatlog evidence used to prosecute paedophiles, was employed in this study. From the archive, 240 authors were randomly selected. Two non-overlapping fragments (in other words, two message groups) of 500 words were extracted from each author's messages so that one fragment can represent the offender and the other the suspect. The same was repeated for 1000 and 1500 words. As a result, there are altogether 480 message groups (= 240 authors × 2 message groups). The chatlog messages were tokenised into word tokens using `WhitespaceTokenizer()` stored in the *Natural Language Tool Kit (NLTK)* (version 2.0)².

The 240 authors were further divided into mutually-exclusive test (50 authors), background (140 authors) and development (50 authors) databases. The test database is used to assess the performance of the FTC system by comparing the message groups with the derived LRs. A more detailed explanation for testing is given in §2.4. The background database is used as the reference database (in terms of typicality) for calculating LRs. The development database is to calculate weights for calibrating the derived LRs from the test database. §2.5 explains calibrations in detail.

2.2 Features

In this study, the four different feature vectors given in Table 1 were used for modelling each message group. These four vectors consist of either two, four, six or eight features. These were select-

¹ <http://pjfi.org/>

² <http://www.nltk.org/>

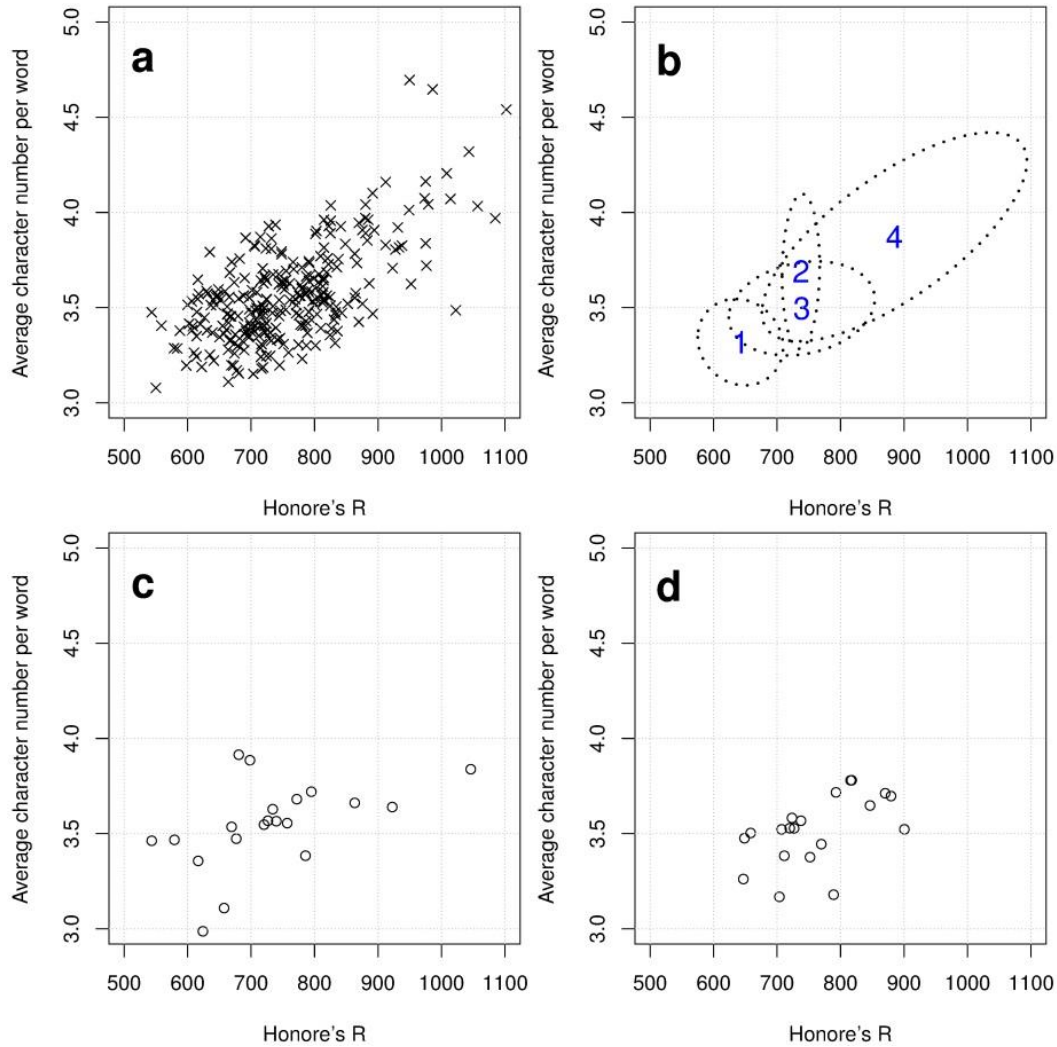


Figure 1: Panel a = the distribution of the two features: ‘Honore’s R’ and ‘Average character count per message line’; Panel b = A GMM (four components) of the distribution; Panels c and d = two sets of randomly generated feature values (20 samples) based on the GMM.

ed from 11 features, which were previously reported as carrying good authorial information (De Vel et al., 2001; Iqbal et al., 2010; Zheng et al., 2006). They are: 1) Yule’s I (the inverse of Yule’s K), 2) Type-token ratio (TTR), 3) Honore’s R, 4) Average word count per message line, 5) Unusual word ratio, 6) Average character count per message line, 7) Upper case character ratio, 8) Digit character ratio, 9) Average character count per word, 10) Punctuation character ratio and 11) Special character ratio (, . ? ! ; : ’ ”). Based on these, a series of FTC experiments was carried out with all possible combinations of two, four, six and eight features in order to identify which combinations perform best. The combinations listed in Table 1 returned the

best C_{lr} values, respectively for the sets of two, four, six and eight features.

Many of the features given in Table 1 are self-explanatory. The `unusual_words()` function³ of the *NLTK* was used to obtain “Unusual word ratio” (e.g. unusual and misspelt words). TTR and Honore’s R are so-called vocabulary richness features.

2.3 Repeated experiments using Monte Carlo techniques

If the current study had been conducted with natural data, sufficiently large amounts of text message-

³ <http://www.nltk.org/book/ch02.html#code-unusual>

es written by a substantial number of authors would have been required. However, due to a lack of such a database of extensively large natural data, the Monte Carlo simulations were employed for this study (Fishman, 1995). The Monte Carlo simulations enable us to generate synthetic values from the specified statistical properties of a distribution. It is common to use a single Gaussian component to model a distribution in the Monte Carlo simulations. However, the Gaussian mixture model (number of components = 4) was utilised in this study. This is because the distributional patterns of the features concerned in the current study do not always conform to a normal distribution as can be seen in Panel a of Figure 1, in which the sampled values of ‘Honoré’s R’ and ‘Average character count per message line’ are plotted.

The process of the Monte Carlo simulation is illustrated in Figure 1, using the feature values of ‘Honoré’s R’ and ‘Average character count per message line’, as an example. First of all, the distributional pattern of the two features are modelled using four Gaussian components as shown in Figure 1b. Figure 1c and Figure 1d are two examples of synthetic data, each of which contains randomly generated 20 values of the two features based on the model given in Figure 1b. The number of Gaussian components was set as four because the log likelihood value remains relatively stable with four components. Thus, in this study, the feature values of the X number of authors ($X = (10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140)$) were randomly generated 200 times for building the background model using the necessary statistics (the mean vectors, covariance matrices and mixture weights from all component densities) obtained from the original background database of 140 authors. A single GMM (a dimension of eight) was used in all experiments (even when features of less than eight are evaluated). The mixtools and mixAK libraries of R statistical package were used for the Monte Carlo simulations.

2.4 Testing

In order to assess the performance of an FVC system, two types of comparisons, namely same-author (SA) and different-author (DA) comparisons, are necessary. In SA comparisons, the two groups of messages produced by the same individuals are compared and evaluated with the derived

LRs. Given their same origin, it is expected that the derived LR is higher than 1, to the extent that the features are valid. In DA comparisons, *mutatis mutandis*, they are expected to receive LR lower than 1.

Out of the 50 authors in the test database, in total, 50 SA comparisons and 2450 ($= {}_{50}C_2 \times 2$) DA comparisons are possible. The LR was calculated for these comparisons with the synthetic background databases which are different in the author count. Following the common practice, a logarithmic scale (base 10) was used in this study, in which case unity is $\log_{10}LR = 0$.

2.5 Likelihood ratio calculation and calibration

LRs were estimated using the Multivariate Kernel Density Likelihood Ratio (MVKD) formula, which is one of the methods that can be used in FTC (Ishihara, 2012, 2014d). A full mathematical exposition of the MVKD formula is given in Aitken & Lucy (2004). One of the advantages of this formula is that an LR can be estimated from multiple variables (e.g. the eight features given in Table 1), considering the correlation between them. The MVKD formula assumes normality for within-group (within-author) variance while it uses a kernel-density model for between-group (between-author) variance.

2.6 Logistic-regression calibration

The outputs of the MVKD formula explained in §2.5 are actually *scores* (not LR) (Rose, 2013). Scores are logLRs in that their values indicate degrees of similarity between two samples in comparison having taken into account their typicality against a background population (Morrison, 2013, p. 2). A logistic-regression calibration (Brümmer & du Preez, 2006) was applied to the outputs (scores) of the MVKD formula in order to convert them to interpretable logLRs. The conversion is carried out by linearly shifting and scaling the scores in the logged odd space, relative to a decision boundary. The FoCal toolkit⁴ was used for the logistic-regression calibration in this study (Brümmer & du Preez, 2006). The logistic-regression weight was obtained from the development database.

⁴ <https://sites.google.com/site/nikobrummer/focal>

2.7 Performance evaluation

It is common to use metrics based on the accuracy or error rate in order to assess the systems which carry identification or classification tasks. However, accuracy or error rate is binary and categorical (e.g. correct or not correct), and it is not suited for the nature of LR, which is gradient and continuous. It has been argued that a more appropriate metric for assessing LR-based systems is the log-likelihood-ratio cost (henceforth C_{llr}) (Brümmer & du Preez, 2006). C_{llr} can be calculated using (2).

$$C_{llr} = \frac{1}{2} \left(\frac{1}{N_{H_p}} \sum_{i \text{ for } H_p = \text{true}}^{N_{H_p}} \log_2 \left(1 + \frac{1}{LR_i} \right) + \frac{1}{N_{H_d}} \sum_{j \text{ for } H_d = \text{true}}^{N_{H_d}} \log_2 (1 + LR_j) \right) \quad (2)$$

N_{H_p} and N_{H_d} refer to the numbers of SA and DA comparisons. LR_i and LR_j refer to the LRs derived from these SA and DA comparisons, respectively. In this approach, LRs are given penalties in proportion to their magnitudes, and, in particular, the LRs which support the counter-factual hypotheses are more severely penalised. The C_{llr} is based on information theory, and if the C_{llr} value is higher than 1, the system is performing worse than not utilising the evidence at all. The FoCal toolkit⁴ was used for calculating C_{llr} values in this study.

3 Pre-analysis

Before presenting the results of the Monte Carlo simulations, it is useful to see how the system performs with the original raw data (test database = 50 authors; development database = 50 authors and the background database = 140 authors). As described in §2.2, four different feature vectors: two, four, six or eight features, were trialled. Furthermore, each message group was modelled using three different amounts of data: 500, 1000 and 1500 words. The results of the pre-analysis are given in Table 2 in terms of C_{llr} .

As can be well expected, the performance improves as the sample size increases; for example, $C_{llr} = 0.6765$ (500 words) \rightarrow 0.5992 (1000 words) \rightarrow 0.5448 (1500 words) for the two features. More data in the background database will naturally lead to building a better and more accurate background model for typicality; consequently the experi-

mental result improves. This result aligns with the general rule of thumb in statistics: “more is better”.

	two	four	six	eight
500	0.6765	<u>0.5774</u>	0.5812	0.7590
1000	0.5992	<u>0.4690</u>	0.4694	0.4835
1500	0.5448	0.3697	0.3817	<u>0.3619</u>

Table 2: C_{llr} values of the experiments with the original raw data, but differing in the sample size (500, 1000 or 1500 words) for modelling each message group and the number of features (two, four, six or eight). The underlined figures = the best C_{llr} values for the sample sizes.

The results given in Table 2 also show that having more features does not necessarily lead to an improvement in performance. For example, the system performed best with four features for 500 and 1000 words.

4 Results and discussions

The experimental results of the Monte Carlo simulations are given in Figure 2. In the left column of Figure 2 (Panels a, b, c and d), the mean C_{llr} values (of the 200 repeated experiments) are plotted as a function of the author count in the background database (10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130 and 140 authors), but separately for the sample size (word lengths) of either 500, 1000 or 1500 words. Panels a, b, c and d of Figure 2 are for the two, four, six and eight features, respectively. The panels on the left-hand side of Figure 2 show how the performance of the system changes as a function of the author count in the background database.

In the right column of Figure 2 (Panels e, f, g and h), the standard deviation (sd) values of the pooled C_{llr} values are plotted against the number of authors in the background database, but separately for the different word counts. Panels e, f, g and h of Figure 2 are again for two, four, six and eight features, respectively. Panels e, f, g and h show how the stability of the system performance fluctuates as the author count increases in the background database.

First of all, conforming to the results of the pre-analysis given in §3, as can be seen from the left panels of Figure 2, the results of the simulated experiments also show that the performance of the system improves as the word count increases. The

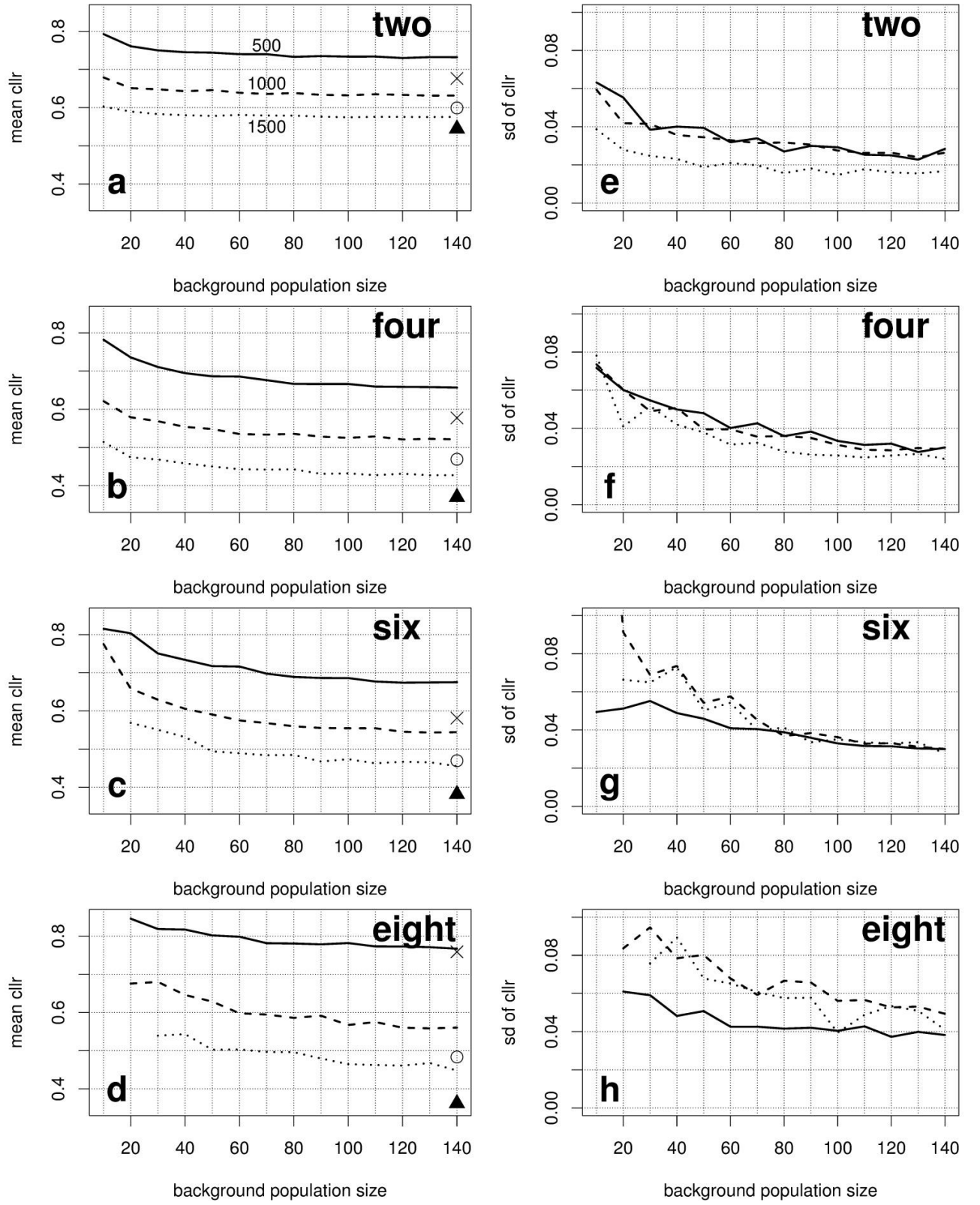


Figure 2: Mean C_{lr} (Panels a, b, c and d) and standard deviation values of the pooled C_{lr} values (Panels e, f, g and h) are plotted as the function of the author count in the background database (10 ~ 140 authors) (x-axis), separately for the word counts: 500 (solid), 1000 (dashed) and 1500 (dotted). Note that some values are missing in Panels c, d, g and h. \times , \circ , \blacktriangle = the C_{lr} values with the raw original data (background database = 140 authors).

above observation is not surprising, but it is novel to see that the three curves included in each of Panels a, b, c and d are more or less parallel to each other within the same feature number. This means that the degree of improvement which resulted from the increase in word count is there or thereabouts constant, regardless of the author count in the background database.

Further relating to the left panels of Figure 2, although there are some minor ups and downs, the system performance improves, regardless of the number of words and features, as the author count increases in the background database. More precisely speaking, the improvement is in a decelerating manner; there is a large improvement at the beginning, after which the performance starts converging or continues to improve to a (far) less degree. In the case of the feature number of two (Panel b), for example, there is a minor improvement from the author count of 10 to that of 20-30, after which the C_{lr} values almost remain unchanged. Whereas for the feature number of four (Panel b), there is a large drop in C_{lr} value between the author counts of 10 and 50-60, but with 60 authors or more, the degree of improvement is small and linear. That is to say, the more features used for modelling, the more data is required in the background database for the system performance to start converging. However, if the discriminating potential of each feature differs significantly, the above point may not be valid. Thus, the variance ratio (between-speaker sd^2 /mean within-speaker sd^2) (Rose et al., 2006); the greater the ratio is, the higher the discriminating potential of the feature, was calculated for each feature, and given in Table 3.

Features	Ratios
Unusual word ratio	7.01
Punctuation character ratio	63.06
Type-token ratio (TTR)	13.38
Average word count per message line	11.40
Honoré's R	9.51
Digit character ratio	4.00
Average character count per message line	11.87
Special character ratio (, . ? ! ; : ' ' ')	1.71

Table 3: Variance ratios.

As can be seen from Table 3, the features of "Digit character ratio" and "Special character ratio" are relatively low in variance ratio in compari-

son to the other features. These poor-performing features (variance ratio: 4.00 and 1.71, respectively) may have functioned as noise features in the six and eight features, and the inclusion of them may not have contributed to the improvement of the system performance; thus consequently the system may have required more samples to continue to improve in the six and eight features. This entails further study.

Some values are missing in Panels c (six features) and d (eight features) – and consequently in Panels g and h – with the author counts of 10 and 20. This is because all of the relevant 200 repeated experiments returned one or more $\log_{10}LR = \inf$ or $-\inf$, which is an ill-condition for the calculation of C_{lr} . It is well known that for the higher the dimension of the feature vector, the more data is required to appropriately model the multi-dimensional density (Silverman, 1986, pp. 93-94). The occurrence of $\log_{10}LR = \inf$ or $-\inf$ indicates that having only 10-20 authors in the background database is not large enough to accurately model the multi-dimensional density of the background population with the feature numbers of six and eight.

As for the stability of the system, an unexpected observation can be made from the right-hand side panels of Figure 2 in that the system does not necessarily become more stable in performance (= smaller sd values) with more words in each message group. This somehow disagrees with the earlier observation regarding the system performance and the word count in each message group. For example, the three curves included in Panel f overlap with each other to a reasonable extent, which means that the system shares a similar degree of stability in performance across the three different word counts, whereas in Panel h, the system with 500 words exhibits smaller sd values (better stability) on the whole than the systems with 1000 and 1500 words.

These results are counter-intuitive as one would ideally expect that the performance will be more stable with more samples. However, Morrison (2011) notes that in practice this is not often the case. There seems to be some degree of trade-off between the performance in accuracy (which can be represented by C_{lr}) and the stability of the system.

In light of the background population size, it is evident from the downward-trend of the curves included in the right-hand panels of Figure 2 that

the system performance becomes more stable as the sample size in the background database increases; a large improvement in stability at the beginning, but the degree of improvement in stability becomes less and less with more authors included in the background database. Additionally, similar to the system performance, it appears in many cases that for the stability to start converging, the system needs more authors in the background database with more features. This point can be seen in Panels e, f and g (1000 and 1500 words), in which the degree of falling in sd values becomes sharper as the feature number increases.

Although the usefulness of the GMM-based Monte Carlo simulation was discussed for the purpose of the current study, there is always the possibility that the GMM model did not accurately approximate the true nature of the original raw data. In particular, it needs to be pointed out that the system with the synthesised data, on average, underperformed the system with the original raw data (refer to Figure 2). However, it is not clear at this stage to what extent and how this possible inaccuracy of the GMM model influenced the results.

5 Conclusion

By generating synthetic data for the background database by means of the Monte Carlo technique, this study looked into how the performance of the system and its stability are subject to the sample size (= the number of authors) in the background database. The effect of the background sample size on the system performance and stability may differ with the dimensions of the feature vector and the number of words used for modelling. Thus, four different vectors consisting of two, four, six and eight features were tested in this study. Furthermore, the number of words used for modelling each message group was also altered as 500, 1000 and 1500 words.

Regardless of the number of features (two, four, six and eight) and words (500, 1000 and 1500), the performance of the system improved in a decelerating manner as the sample size (the number of authors) increases in the background database. This result conforms to previous studies on other types of evidence (Hughes et al., 2013; Ishihara & Kinoshita, 2008). Moreover, in general terms, it was found that the more features included in the vector, the more authors the system needs in the

background database for the performance to start converging. However, other potential factors which may have contributed to the outcomes of the current study have also been discussed.

Although there are a large number of potential features that can be used in casework – according to Abbasi and Chen (2008), the total number of features tested in previous studies exceeds 1000, the results of the current study indicate that more features may only deteriorate the performance of the system unless an appropriate amount of background data is available for the dimension of the feature vector, and that the number of features should be determined according to the size of the available background data. These two points are important, in particular, as data scarcity is a common issue in FTC casework. Some drawbacks arising from the use of the GMM-based approximation were also discussed.

It was also pointed out that the model is likely to be inaccurately built only with 10 or 20 authors when the feature number is six or more, resulting in the system returning erroneous LR values. Together with other observations, it can be judged that a system with 20 or less authors in the background database is not admissible in court in terms of performance.

In terms of the stability of the system performance, it is interesting to know that having more words in each message group does not necessarily lead to an improvement in stability. This point was in fact reported in previous studies (Frost, 2013; Ishihara, 2014a; Morrison, 2011). On the other hand, like the case of system performance, regardless of the number of features and words, it was shown that the system becomes more stable along with the number of authors in the background database.

The MVKD formula was used in this study. However, there are other methods for estimating LRs (e.g. word or character *N*-grams) (cf. Ishihara, 2014a; Ishihara, 2014c). This warrants further studies on the same topic as the current study with other methods for LR estimations.

This study focused on the performance (= accuracy) and stability of the system in order to investigate the effect of the background sample size. However, it is equally important to investigate to what extent the LR value fluctuates depending on the sample size in the background database. This also entails a future study.

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