

## Part I: Multiple Choice (20%, 5% for each question)

Choose the **single** best alternative that completes the statement or answers the question.

1. In a Rubinstein's strategic bargaining game where players A and B are deciding how to divide the pie in turn at each period. Which of the following statements is **correct**?

- a. If player A is more patient than player B, then in equilibrium player B will get more than player A.
- b. If player B becomes more impatient, then in equilibrium player A will get more of the pie.
- c. If player B is indifferent between getting the pie now and getting it later, while player A is not, then player A will get the entire pie.
- d. All the statements above are incorrect.

2. In a  $2 \times 2$  game, which of the following statements is **incorrect**?

- a. In the battle of the sexes and assurance games, there are two pure strategy Nash equilibriums.
- b. In the game of competition, there is only one mixed strategy Nash equilibriums.
- c. In the games of assurance, battle of the sexes, and chicken, assigning one player to move first can lead these coordination games to be coordinated.
- d. If the dominant strategy exists, there is no mixed-strategy Nash equilibrium.

3. In a sequential game, which of the following statements is **incorrect**?

- a. The first player's action is both irreversible and observable by the second player.
- b. The first player knows that his action is seen by the second player.
- c. The dominant strategy of the both players does not matter in a sequential game.
- d. The backward induction is necessary to derive the subgame perfect Nash equilibrium.

4. Professor Binmore has a monopoly in the market for undergraduate game theory textbooks. The time-discounted value of Professor Binmore future earnings is \$2,000. Professor Ditt is considering writing a book to compete with Professor Binmore book. With two books amicably splitting the market, the time-discounted value of each professor future earnings would be \$200. If there is full information (each professor knows the profits of the other), under what conditions could Professor Binmore deter the entry of Professor Ditt into his market?

- a. Professor Binmore threatens to cut his price and attack the credibility of Professor Ditt book so that Professor Ditt would lose \$8. In so doing, Professor Binmore would still make \$190 over time.
- b. Professor Binmore threatens to cut his price and attack the credibility of Professor Ditt book so that Professor Ditt would only make \$8. In so doing, Professor Binmore would still make \$100 over time.
- c. Professor Binmore threatens to cut his price and attack the credibility of Professor Ditt book so that Professor Ditt would lose \$2. In so doing, Professor Binmore would still make \$210 over time.
- d. Professor Binmore threatens to cut his price and attack the credibility of Professor Ditt book so that Professor Ditt would only make \$2. In so doing, Professor Binmore would still make \$100 over time.



## Part II: Essay (80%)

Please calculate and explain the following essay questions in detail. If you use some notations and/or figures, please define them clearly.

1. First Fiddler's Bank has foreclosed on a home mortgage and is selling the house at auction. There are three bidders for the house, Ernie, Minnie, and Betsy. First Fiddler's does not know the willingness to pay of these three bidders for the house, but on the basis of its previous experience, the bank believes that each of these bidders has a probability of  $1/3$  of valuing it at \$700,000, a probability of  $1/3$  of valuing it at \$400,000, and a probability of  $1/3$  of valuing it at \$300,000. First Fiddler's believes that these probabilities are independent among buyers. If First Fiddler's sells the house by means of a second-bidder, sealed-bid auction (Vickrey auction), what will be the bank's expected revenue from the sale?

(8%)

$$\begin{array}{r}
 108 \\
 140 \\
 135 \\
 \hline
 383 \\
 120 \\
 \hline
 263 \\
 48 \\
 \hline
 130
 \end{array}
 \quad
 \begin{array}{r}
 490 \\
 620 \\
 210 \\
 \hline
 1320
 \end{array}
 \quad
 \begin{array}{r}
 111 \rightarrow 1 \\
 113 \rightarrow 3 \\
 114 \rightarrow 3 \\
 \hline
 333 \rightarrow 1 \\
 113 \rightarrow 3 \\
 433 \rightarrow 3 \\
 \hline
 490000 + 520000 + 210000 \\
 \hline
 1220
 \end{array}$$

2. A dealer decides to sell an oil painting by means of an English auction with a reservation price just slightly below \$45,000. If he fails to get his reservation price for the painting, he will burn it. There are two bidders. The dealer believes that there are only three possible values, \$90,000, \$45,000, and \$18,000, that each bidder's willingness to pay might take. Each bidder has a probability of  $1/3$  of having each of these willingnesses to pay, and the probabilities for each of the two bidders are independent of the other's valuation. Assuming that the two bidders bid rationally and do not collude, what is the dealer's expected revenue from selling the painting? (6%)

$$\begin{array}{r}
 90 \ 90 \rightarrow 1 \\
 90 \ 45 \rightarrow 2 \\
 90 \ 18 \rightarrow 2 \checkmark \\
 45 \ 45 \rightarrow 1 \checkmark \\
 45 \ 18 \rightarrow 2 \checkmark \\
 18 \ 18 \rightarrow 1 \checkmark
 \end{array}$$

3. A seller knows that there are two bidders for the object she is selling. She believes that with probability  $1/2$ , one has a buyer value of \$5 and the other has a buyer value of \$10 and, with probability  $1/2$ , one has a buyer value of \$8 and the other has a buyer value of \$15. She knows that bidders will want to buy the object so long as they can get it for their buyer value or less. She sells it in an English auction with a reserve price which she must set before the auction starts. To maximize her expected profits, what should she set the reserve price at? (6%)

$$\begin{array}{r}
 \frac{1}{2} \quad 5 \quad 10 \\
 \frac{1}{2} \quad 8 \quad 15
 \end{array}$$

4. Herb's Auction House in Purloined Hubcap, Oregon, holds sealed-bid used-car auctions every Wednesday. Each car is sold to the highest bidder at the second-highest bidder's bid. On average, two-thirds of the cars that are auctioned are lemons and one-third are good used cars. A good car is worth \$1,500 to any buyer. A lemon is worth only \$150 to a buyer. Normal buyers can do no better than random at picking good cars from the lot. There is only one exception, Al Crankcase. Al can sometimes but not always detect lemons by means of a subtle test. A good car will never fail Al's test, but approximately half of the lemons fail his test. Al attends every auction, tests every car, and always bids his expected value. Normal bidders bid less than the expected value for a randomly selected car but more than the value of a lemon.

(a) What price will Al bid for cars that pass his test? (6%) What price will Al bid for cars that fail his test?

(2%) Calculate and explain.

(b) Will normal bidders get good used cars? Why? (6%)

No



...savings and Social Security. If the older generation "saves" and the younger generation also "supports" them, the old folks end up with a utility level of 2 and the young folks end up with -1. If the older generation "squanders" and the younger generation "supports" them, the elders end up with a utility of 3 and the young folks end up with -1. If the younger generation "refrains" from providing support to their elders and the older generation "saves", the old folks get 1 and the young folks get 0. Finally, if the old folks "squander" and the young folks "refrain" from providing support to their elders, each ends up with utility of -2, the old folks from starving and the young folks from having to watch.

- (a) Show the payoff matrix for this two-player-two-action game. (5%)  
 (b) If this is a simultaneous game, find out all Nash equilibrium(s) in pure strategy? Explain. (5%)  
 (c) If the old folks decide first, what is the subgame perfect Nash equilibrium? Plotting a sequential game and explaining by backward induction. (5%)

6. This is an example of the game of "Chicken." Two teenagers in souped-up cars drive toward each other at great speed. The first one to swerve out of the road is "chicken." The best thing that can happen to you is that the other guy swerves and you don't. Then you are the hero and the other guy is the chicken. If you both swerve, you are both chickens. If neither swerves, you both end up in the hospital. A payoff matrix for a chicken-type game is the following.

		Leroy	
		Swerve $\pi^L$	Don't Swerve $1-\pi^L$
Joe Bob	Swerve $\pi^J$	1, 1	1, 2
	Don't Swerve $1-\pi^J$	2, 1	0, 0

- (a) Does this game have a dominant strategy? Find out all Nash equilibrium(s) in pure strategy? (4%)  
 (b) Find Nash equilibrium in mixed strategies for this game. Explain by plotting two best response curves in one figure. (6%)

7. In the little town of Gas Pump, South Dakota, there are two gas stations across the street from each other, the East station and the West station. A local ordinance requires that they are permitted to change the price of gasoline only once a week at precisely 8:00 a.m. on Monday and in 5 cent increments. For some time, the price of gasoline has been stable at \$1.50 per gallon, but midweek, the price each station pays for gasoline changed to \$1.49.

		West		
		1.45 $\pi^W$	1.50 $\pi^W$	1.55 $1-\pi^W$
East	1.45 $\pi^E$	-20, -20	-40, 0	-40, 0
	1.50 $\pi^E$	0, -40	5, 5	10, 0
	1.55 $\pi^E$	0, -40	0, 10	30, 30

- (a) Given the payoff matrix above, find out all Nash equilibrium(s) in pure strategy? Explain. (5%)  
 (b) What is the mixed strategy Nash equilibrium? Explain by plotting two best response curves in one figure. (6%)

8. Consider the goalie's anxiety at the penalty kick. Let the kicker's payoffs below represent the kicker's probability of success and the goalie's payoffs the probability of failure.

		$\pi^g$ Goalie	
		Defend left	Defend right
Kicker	Kick left $\pi^k$	90, -90	10, -10
	Kick right	50, -50	70, -70

- (a) Find the pure strategy Nash equilibrium(s) in this game. (2%)
- (b) Find the mixed strategy Nash equilibrium in this game. Explain by plotting two best response curves in one figure. (8%)

$$8 \cdot \pi^g + 10$$

$$70 - 20\pi^g$$

$$40\pi^k - 50$$

$$60\pi^k - 70$$