微積分(乙)期末考　　2018/06/08

(1) 10% Kimberly Austin deposits \$5000 in an IRA at 6% interest compounded continuously for her retirement in 10 years. She intends to make continuous deposits at the rate of \$3000 a year until she retires. How much will she have accumulated at that time?

(2) 10% After use of an experimental insecticide, the rate of decline of an insect population is $\frac{dy}{dt}=\frac{-10}{1+5t}$, where t is the number of hours after the insecticide is applied. Assume that there are 50 insects initially.

(a) How many are left after 24 hours?

(b) How long will it take for the entire population to die?

(3) 10% Megan Donnelly wants to buy a car that she estimates will cost \$24000 in a 5 years. How much money must she deposit at the end of each quarter at 4% interest compounded quarterly in order to have enough in 5 years to pay for her car?

(4) 10% Kristin Walters buys a new car costing \$22000. She agrees to make payments at the end of each month for 5 years. If she pays 6% interest, compounded monthly, what is the amount of each payment? Find the total amount of interest Kristina will pay.

(5) 10% Find the Taylor series for the following functions and give the interval of convergence (a) $f(x)=x^5e^x$ and (b) $f(x)=\ln\left(1+2x^4\right)$.

(6) 10% Find (a) $\lim_{x\to\infty}\frac{\ln\left(e^x+1\right)}{5x}$, and (b) $\lim_{x\to 0^+} x\ln(e^x-1)$.

(7) 10% Use Newton's method to approximate $\sqrt[4]{102.6}$ to the nearest thousandth.

(8) 30% Find the general solution for (a) $3\frac{dy}{dx}+6xy+x=0$ (b) $\frac{dy}{dx}=\frac{y(1-y)e^{\sqrt{x}}}{\sqrt{x}}$ and

(c) $\frac{dy}{dx}=\frac{\cos xe^{x}}{y^{3}\ln y}$.

(9) 5% Find the sum of $1-\frac{1}{e}+\frac{1}{2!e^{2}}-\frac{1}{3!e^{3}}+\frac{1}{4!e^{4}}-\cdots$

(10) 5% If $f(x)=5x^{4}e^{x^{2}}$, find $f^{(10)}(0)$.

$e^{0.1}=1.105171, e^{0.2}=1.221403, e^{0.3}=1.349859, e^{0.4}=1.491825, e^{0.5}=1.648721$
$e^{0.6}=1.822119, e^{0.7}=2.013753, e^{0.8}=2.225541, e^{0.9}=2.459603, e^{1.0}=2.718282$

$e^{2}=7.3891, e^{5}=148.41, e^{10}=22026.5, e^{15}=3269017$
$e^{20}=485165195, e^{25}=7.2005\cdot10^{10}$

$1.005^{10}=1.05114, 1.005^{20}=1.10490, 1.005^{30}=1.16140,$
$1.005^{40}=1.22079, 1.005^{50}=1.28323, 1.005^{60}=1.34885$

$1.01^{10}=1.10462, 1.01^{20}=1.22019, 1.01^{30}=1.34785,$
$1.01^{40}=1.48886, 1.01^{50}=1.64463, 1.01^{60}=1.81670$

$\ln(9)=2.1972, \ln(10)=2.3026, \ln(11)=2.3979, \ln(81)=4.3944$
$\ln(120)=4.7875, \ln(121)=4.7958, \ln(125)=4.8283$

$\frac{dy}{dx}+P(x)y=Q(x)$

Step 1 find $I(x)=e^{\int P(x)dx}$.

Step2 $y=\frac{\int Q(x)I(x)dx}{I(x)}$.