Statistics I 2007/11/05

Midterm Exam

1. (a) Find $E\{\frac{1}{1+X}\}$, where X is a Poisson random variable. (5%)

   (b) If a random variable X has a Poisson distribution such that $P(X = 1) = P(X = 2)$, find $P(X = 4)$. (5%)

2. Let X be an exponential distribution with parameter λ.

   (a) Find the cumulative distribution function of X. (5%)

   (b) Find the median of X. (5%)

   (c) The time to failure of a certain type of electrical component is assumed to follow an exponential distribution with a mean of 4 years. The manufacturer replaces free all components that fail while guarantee. If the manufacturer wants to replace a maximum of 3% of the components, for how long should the manufacturer's stated guarantee on the component be? (5%)

3. If $X \sim N(\mu, \sigma^2)$, what are the distributions of $Y = \frac{X-\mu}{\sigma}$ and $W = \{\frac{X-\mu}{\sigma}\}^2$? (10%)

4. Given the joint probability mass function of (X, Y) with $P(-1, -1) = P(-1, 1) = P(1, -1) = P(1, 1) = 1/6$ and $P(0, -1) = 1/3$.

   (a) Find the covariance $Cov\{X, Y\}$. (5%)

   (b) Are X and Y independent? Explain. (5%)

5. If X and Y are two random variables, and a and b are constants.

   (a) Show $E\{aX + b\} = aE\{X\} + b$. (5%)

   (b) Show $Var\{aX + b\} = a^2Var\{X\}$. (5%)

   (c) Find a and b such that $E\{aX + b\} = 0$ and $Var\{aX + b\} = 1$. (5%)

   (d) Express $Cov\{X + Y, X - Y\}$ in terms of $Var(X)$, $Var(Y)$ and $Cov\{X, Y\}$. (5%)

7. The joint density of X and Y is given by

$$f(x, y) = \begin{cases} xe^{-(x+y)}, & x > 0, y > 0, \\ 0, & \text{otherwises.} \end{cases}$$

   (a) Compute the density of X. (5%)

   (b) Compute the density of Y. (5%)

   (c) Are X and Y independent? (5%)

8. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you have to wait longer than 15 minutes? (5%)

   (b) What is the average waiting time? (5%)

9. Let X equal the birth weight (in grams) of babies in United States. Assuming that the distribution of X is normal distribution with $\mu = 3315$ and $\sigma = 575$.

   (a) Compute $P(2584.75 \leq X \leq 4390.25)$. (5%)

   (b) Let Y equal the number of babies that weigh less than 2719 grams at birth among 25 of these babies selected independently. Compute $P(Y \leq 4)$. (5%)

Total: 100 points