Examination I, *Computer Algorithms*

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1. (3 × 5 pts) Let $T_B(n)$ and $T_Q(n)$ denote the run times required by MERGE SORT and QUICK SORT on n input elements, respectively. (a) Give the recurrence relation of $T_B(n)$. (b) Give the recurrence relation of $T_Q(n)$ for the worst case. (c) Derive the functions of $T_B(n)$ and $T_Q(n)$ (of the worst case) as described in (a) and (b).

2. (7+3 pts) (a) Use a recursion tree to solve $T(n) = 3T(\lfloor n/4 \rfloor) + \theta(n^2)$. (b) Can we apply the same technique to $T(n) = 9T(\lfloor n/10 \rfloor) + \theta(n^2)$?

3. (4 × 5 pts) Let $A = a_1, a_2, \cdots, a_m$ and $B = b_1, b_2, \cdots, b_n$ be two strings in the LCS problem. Let $L(m, n)$ denote the length of the LCS of A and B. (a) Give the recursive formula, including the boundary conditions, of $L(m, n)$ as a solution method in the form of dynamic programming. (b) Write the pseudo codes in nested loops. (c) What is the run time? Explain it. (d) Let $A = w, x, x, w, y, z$ and $B = x, y, y, x, w, z$. Find $L(6, 6)$ by filling up the table of $L_{7\times7}$.

4. (5 pts) For node i, d_i and f_i denote stamps of start and finish times in DFS. Explain the following fact: If interval $[d_j, f_j]$ is properly included in $[d_i, f_i]$, then node j is a descendant of node i in the DFS tree.

5. (10 pts) Use the master method to determine an expression of each run time $T(n)$. Give the reasoning behind your answers.

   (a) $T(n) = 3T(\frac{n}{2}) + n^2$

   (b) $T(n) = 25T(\frac{n}{5}) + n$

   (c) $T(n) = 2T(\frac{n}{2}) + n \log n$

   (d) $T(n) = 3T(\frac{n}{6}) + n^{0.5001}$

   (e) $T(n) = \frac{1}{2}T(\frac{n}{4}) + \frac{1}{n}$

6. (3 × 3 pts) Give the run time for each of the following operations on a min-heap. (a) Find the minimum. (b) Extract the minimum. (c) Insert a new element.

7. (3+7 pts) (a) Answer the value of the ?.

   BUILD-MAX-HEAP(A)

   1. *A.heap-size* = *A.length*
   2. For i = ? downto 1

3. MAX-HEAPIFY(A, i).

(b) What is the run time of BUILD-MAX-HEAP(A) given that $A.length = n$? Is it $O(n \log n)$? Describe the reasoning behind your answer. You don't have to give mathematical details.

8. (4+4 pts) In view of divide-and-conquer approach, (a) what is the run time for "divide" in MERGE SORT and QUICK SORT? (b) what is the "merge" time in MERGE SORT and QUICK SORT?

9. (5 pts) Describe the relation between dynamic programming and greedy method.

10. (5+5 pts) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?
a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21
Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

11. (3+5 pts) In the community of mathematics, there is an interesting metric called Erdös distance*. Any scientist who co-authors an article with Paul Erdös has a distance of ONE. Those who co-author articles with distance-one scientists has a distance of TWO. Other distances are defined in the way. Describe how to list all scientists whose Erdös distances are not greater than THREE.

12. (8 pts) Function $f(n) = 5n^2 + 32n + 7$ is a non-increasing function of n. Which of the followings are good to describe the membership of function f? (a) $O(n^2)$, (b) $O(n)$, (c) $O(n^{3.5})$, (d) $\Omega(n^2)$, (e) $\Omega(n)$, (f) $\Omega(n^{3.5})$, (g) $\theta(n^2)$, (h) $\theta(n)$, (i) $\theta(n^{3.5})$.

13. (10 pts) Given a set of n integers, we want to distribute them into two subsets such that the absolute difference between the sums of the two subsets is minimum. Design a dynamic programming algorithm for this problem.

*** End of Test ***

*Paul Erdös, a Hungarian mathematician. Please visit Wiki.