National Chiao Tung University
College of Management
1590 Operations Research I
Final Examination

Examination Date: 07 Jan 2020 Examination Time: 10:10 – 12:00

Notes

The question paper has FOUR questions and SIX pages (including cover page). The full mark is 100.

All solutions **MUST** be written on the answer booklet.

ONLY answer booklet is collected after the examination with name written on it.

This paper can be answered by blue pens, black pens, or pencils.

Mobile phones and calculators with graph plotting functions are **NOT** allowed.

No printed or electronic materials are allowed.

Each question should be answered in a **NEW** page.

Question	Q1	Q2	Q3	Q4	Total
Full mark	25	25	25	25	100
Score					

Student name: ______________________________

Student number:_____________________________

1. YEN Motor Corp. has three plants in Taipei, Taichung, and Kaohsiung, and two major distribution centers in Hsinchu and Tainan. The quarterly capacities of the three plants are 1000, 1500, and 1200 cars and the demands at the two distribution centers for the same period are 2000 and 1500 cars. The trucking company in charge of transporting the cars charges 10 NTD per kilometer. The distance matrix (in kilometer) between the plants and the distribution centers is given in Table 1:

	Hsinchu	Tainan
Taipei	100	210
Taichung	150	160
Kaohsiung	200	80

Table 1.1: Distance matrix (in km) between the plants and the distribution centers

(a) [2%] Formulate this problem as a transportation problem by constructing the corresponding parameter table.

(b) [3%] Use the Northwest corner rule to obtain an initial BF solution.

(c) [10%] Apply Vogel's approximation method for **TWO** iterations. Indicate the basic variables and their values determined in these two iterations.

(d) [10%] Given a basic feasible solution **x** = (x_{11}, x_{22}, x_{23}, x_{31}, x_{32}) = (1000, 1300, 200, 1000, 200), solve it to obtain the optimal solution by transportation simplex method.

※You are required to provide the following information *per iteration* :

(i) Justify, with reason, whether the current solution is an optimal solution.

(ii) Indicate the entering variable and the leaving variable.

(iii) Mark the chain reaction, if it is needed.

(iv) Specify the new basic feasible variables and their values.

2. An advertisement company handles a project that involves 8 jobs with the activity list shown in Figure 2.1. When using the CPM method of time-cost trade-offs, the company has obtained the data in Table 2.1.

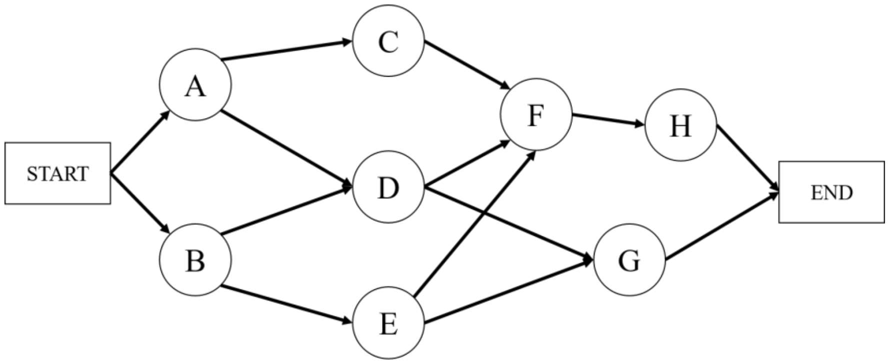


Figure 2.1: Activity list of the project

Job	Normal Time	Crash Time	Normal Cost	Crash Cost
A	5 weeks	3 weeks	$24 million	$36 million
B	3 weeks	2 weeks	$13 million	$25 million
C	4 weeks	2 weeks	$21 million	$29 million
D	6 weeks	3 weeks	$30 million	$51 million
E	5 weeks	4 weeks	$26 million	$36 million
F	7 weeks	4 weeks	$35 million	$56 million
G	9 weeks	5 weeks	$30 million	$52 million
H	8 weeks	6 weeks	$35 million	$48 million

Table 2.1: Normal and crash times and costs of all jobs

(a) [4%] By firstly listing **ALL** seven possible paths, determine the critical path and the corresponding project durations.

(b) [9%] By firstly determining the crash cost per week saved of **ALL** jobs, determine the least expensive way of crashing some jobs *if* the project duration needs to be reduced by 3 weeks.

The advertisement company finally decides **not** to crash any jobs. Meanwhile, some of the jobs require outsourcing companies (OC) to accomplish. The weekly costs for these OCs handling each job are listed as below in Table 2.2 (the dash '-' denotes that the company cannot handle that job). Due to commercial reasons, each OC can be only assigned to at most one job, whereas the advertisement company aims to minimize the total cost.

(c) [6%] Formulate this problem as an assignment problem by constructing the appropriate cost table.

(d) [6%] Table 2.3 shows the second-last tableau based on the constructed cost table using the Hungarian algorithm, where X is a dummy job. Based on this table, compute the final tableau and determine the optimal outsourcing plan and total cost.

OC\Job	A	B	C	D	E
1	\$0.5m	\$0.7m	\$0.8m	\$0.4m	\$1m
2	\$0.55m	\$0.6m	-	\$0.3m	\$1.1m
3	-	-	\$0.7m	\$0.5m	\$1.2m
4	\$0.6m	\$0.75m	\$1m	\$0.8m	\$0.9m
5	\$0.4m	\$0.55m	-	\$0.35m	\$0.95m
6	\$0.9m	\$0.8m	\$0.85m	-	\$1.15m

Table 2.2: Weekly cost of the jobs for all outsourcing companies

OC\Job	A	B	C	D	E	X
1	0.1	0.05	0	0.2	0.1	0
2	0.75	0.15	M	0	1	0.4
3	M	M	0	1.2	1.5	0.4
4	1	0.6	1.2	3	0	0.4
5	0	0	M	0.3	0.25	0.4
6	2.1	0.35	0.2	M	0.85	0

Table 2.3: Second-last tableau based on the constructed cost table in part (c)

3. A freight driver needs to deliver a container from port to a place he has never visited before. Therefore, he is studying a map to determine the shortest route from the origin to the destination. There are five other towns (call them A, B, C, D, E) that he may pass through on the way. The map shows the mileage along each road that directly connects two towns without any intervening towns. The mileages are summarized in Table 3.1, where a dash ('-') indicates that there is no road directly connecting these two towns without going through any other towns. It is also assumed that all roads are undirected.

Town	Miles between adjacent towns					
	A	B	C	D	E	Destination(T)
Origin(O)	47	25	33	-	-	-
A		17	-	12	-	-
B			16	30	24	-
C				-	15	-
D					8	7
E						12

Table 3.1: Distance matrix (in miles) between the towns

(a) [5%] Draw the network of this problem, where nodes represent towns, links represent roads, and numbers denote the lengths of the links in miles.

(b) [15%] Determine the shortest path from origin to destination **using** the following table format. (The **first step** is given as an example). Hence, determine the total miles of and the visiting sequence*(s)* of towns on each of the shortest path*(s)*.

n	Solved nodes	Closest connected unsolved nodes	Total distance involved	n^{th} nearest node	Minimum distance	Last connection
1	O	B	0+25=25	B	25	OB

(c) [2%] If each number in the matrix represents travel time (in minutes) instead of mileage, given that the vehicle drives at a constant speed, does the shortest path between OT change? Briefly explain the answer.

(d) [3%] Using the network constructed in part (a), determine the involved roads and the total mileages of the minimum spanning tree.

4. Hsinchu Science Park has suffered from traffic congestion problems in recent years due to the park expansion. The local government aims to build more roads between downtown and the Park. Figure 4.1 shows the existing roadway network (in solid lines) and the proposed additional roads (in broken lines). The road capacity (in vehicle per hour, vph) of each link is placed on top of the link (while the capacities of the proposed links are underlined). It is assumed that all roads do not have congestion effect and the vehicle flow on each road cannot exceed the road capacity.

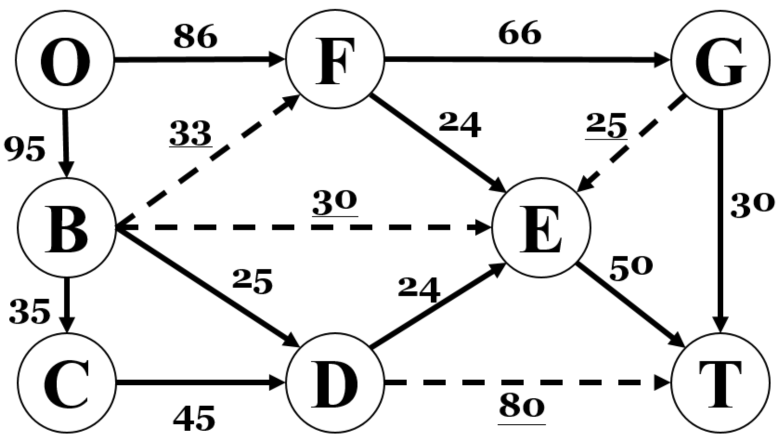


Figure 4.1: Sketch map of the existing and proposed roads

(Note: if using max-flow min-cut theorem, the links intersect with the min-cut <u>must</u> be provided; if using augmented path algorithm, all augmenting paths <u>must</u> be listed)

(a) [10%] *Without* any newly added roads, it is known that no vehicle flow on link BC in the resultant network.

(i) Determine the maximum flow (in vph) of the existing network, and

(ii) Identify the links that the resultant vehicle flow is less than half of its capacity.

(b) [9%] *With* **ALL** the newly added roads, determine the maximum flow of the new network.

(c) [6%] After all roads are added, the government plans to widen **one** of the roads <u>*directly*</u> *connected with T* by ***doubling*** the road capacity to further increase the maximum vehicle flow of the network. Determine, with explanations, the best road for road widening such that the flow of the network can be maximized.

END OF PAPER

Solutions

Question 1

(a) (2%) 有錯即沒分

Plant	Cost (NTD)			Supply
	Hsinchu	Tainan	Dummy	
Taipei	1000	2100	0	1000
Taichung	1500	1600	0	1500
Kaohsiung	2000	800	0	1200
Demand	2000	1500	200	

(b)

Northwest rule：(2%) 圖對 2 分 (1%) 標示出 initial basic solution

	Hsinchu	Tainan	Dummy	Supply
Taipei	1000 1000	2100	0	1000
Taichung	1500 1000	1600 500	0	1500
Kaohsiung	2000	800 1000	0 200	1200
Demand	2000	1500	200	

∴ The initial BF solution = (x_{11}, x_{21}, x_{22}, x_{32}, x_{33}) = (1000,1000,500,1000,200) (1%)

(b) (4%)圖全對(包含 row, column diff)才給，錯一個扣 1% (1%)給 basic variable

Vogel's approximation method：

(iteration 1) (4%)

	Hsinchu	Tainan	Dummy	Supply	Row diff
Taipei	1000	2100	0	1000	1000
Taichung	1500	1600	0 200	~~1500~~ 1300	1500 (Max)
Kaohsiung	2000	800	0	1200	800
Demand	2000	1500	~~200~~		

			0		
Col diff	500	800	0		

∴ basic variable = x_{23} = 200 (1%)

(iteration 2) (4%)

	Hsinchu	Tainan	Dummy	Supply	Row diff
Taipei	1000	2100	0	1000	1100
Taichung	1500	1600	0 200	~~1500~~ 1300	100
Kaohsiung	2000	800 1200	0	~~1200~~ 0	1200 (Max)
Demand	2000	~~1500~~ 300	~~200~~ 0		
Col diff	500	800	0		

∴ basic variable = x_{32} = 1200 (1%)

（以下只是求出解而已，不算分）

(iteration 3)

	Hsinchu	Tainan	Dummy	Supply	Row diff
Taipei	1000 1000	2100	0	~~1000~~ 0	1100 (Max)
Taichung	1500	1600	0 200	~~1500~~ 1300	100
Kaohsiung	2000	800 1200	0	~~1200~~ 0	1200
Demand	~~2000~~ 1000	~~1500~~ 300	~~200~~ 0		
Col diff	500	500	0		

∴ basic variable = x_{11} = 1000

(iteration 4)

	Hsinchu	Tainan	Dummy	Supply	Row diff
Taipei	1000 1000	2100	0	~~1000~~ 0	1100
Taichung	1500 1000	1600 300	0 200	~~1500~~ 1300	100
Kaohsiung	2000	800 1200	0	~~1200~~ 0	1200
Demand	~~2000~~ 1000	~~1500~~ 300	~~200~~ 0		
Col diff	500	500	0		

The rest basic variables we can get easily are (x_{21}, x_{22}) = (1000, 300).
Above all, by Vogel's approximation method we can get the initial BF solution = (x_{11}, x_{21}, x_{22}, x_{23}, x_{32}) = (1000,1000,300,200,1200).

(d)

(iteration 1) (3%)- 基本圖 2% + 連鎖反應 1%

	Hsinchu	Tainan	Dummy	Supply	u_i
Taipei	1000 1000	2100 2300	0 1800	1000	-1800
Taichung	1500 -1300 (+)	1600 1300 (-)	0 200	1500	0
Kaohsiung	2000 1000 (-)	800 200 (+)	0 800	1200	-800
Demand	2000	1500	200		
v_j	2800	1600	0		

In the 1st iteration, the current basic feasible solution is **not optimal**(1%) since there still exist negative Δ_{ij}. Δ_{ij} are marked in red fonts. As we can see in the tableau, x_{21} has negative value, thus we **choose x_{21} as entering variable**. To maintain the balance, we need to find a chain reaction (see above). We **choose x_{31} as leaving variable**(1%), because it has the min value to reduce.

(iteration 2) (3%)

	Hsinchu	Tainan	Dummy	Supply	u_i
Taipei	1000 1000	2100 1000	0 500	1000	-500
Taichung	1500 1000	1600 300	0 200	1500	0
Kaohsiung	2000 1300	800 1200	0 800	1200	-800
Demand	2000	1500	200		
v_j	1500	1600	0		

In the 2 st iteration, **the current basic feasible solution is optimal** (1%)since there exist no negative Δ_{ij}. **The optimal BF solution = (x_{11}, x_{21}, x_{22}, x_{23}, x_{32}) = (1000, 1000, 300, 200, 1200).**(1%)

各小題分數標準：

(a) 2%：圖全對才有 **2%**，有任何錯即沒分

(b) 3%：圖全對才有 **2%**，另外寫出 initial basic solution 和其值才有 **1%**

(c) 10%：圖全對(要有 row, column diff)才給 **4%**，若有錯，基本上全扣∵；有寫 basic variable 和值才有 **1%)**

(情況一)若用 Vogel's method 解出全部解(只用一張圖解釋)，10%全扣掉

(情況二)若表中寫對，值寫錯，不扣分

(情況三)選擇 Max 有錯，代表方法有錯、觀念不對，該 iteration 5%全扣掉

(d) 10%：圖全對才有 **2%**，缺一個 nonbasic variable 係數扣 1%；連鎖反應寫對才有 **1%**；optimality 有寫才有 **1%**；第 1 iteration 的 entering 和 leaving variable 均有寫才給 **1%**；第 2 iteration optimal BF solution 和其值有寫才有 **1%**

Question 2

Comments:

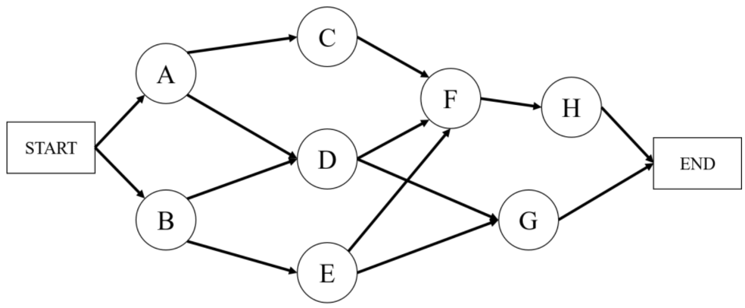
2a) Most students can identify all seven paths. Mark deductions are due to wrong calculations (due to missing numbers). Meanwhile, some students forget to give an indication of which path being the critical path. Few students does not understand the concept of critical path by pointing out the shortest path to be the critical one.

2b) Most students perform well in this question. Main problem is that some students do not calculate the crash cost per week correctly and identify wrong crashed jobs.

2c) This question is not satisfactory. Most students do not aware that the provided figures are per week cost but not actual cost. Few students forget to add dummy node.

2d) This question performs generally well. Meanwhile, due to the wrong final tableau in part (c), very few students can get full marks.

(a)



Path:

P1) S-A-C-F-H-End: 5+4+7+8 = 24; P2) S-A-D-F-H-End: 5+6+7+8 = 26

P3) S-A-D-G-End: 5+6+9 = 20; P4) S-B-D-F-H-End: 3+6+7+8 = 24

P5) S-B-D-G-End: 3+6+9 = 18; P6) S-B-E-F-H-End: 3+5+7+8 = 23

P7) S-B-E-G-End: 3+5+9 = 16

Critical path: S-A-D-F-H-End with 26 weeks

[Path 3%; each missing/wrong path -0.5%; critical path and week 1%]

2b

Crash cost per week saved:

A - \$6m; B - \$12m; C - \$4m; D - \$7m; E - \$10m; F - \$7m; G - \$5.5m; H - \$6.5m

Crash Job	Crash cost	P1	P2	P3	P4	P5	P6	P7
		24	26	20	24	18	23	16
A	\$6m	23	25	19	24	18	23	16

A	$6m	22	24	18	24	18	23	16
H	$6.5m	21	23	18	23	18	22	16

Crash Job A for two weeks and Job H for one week with total crash cost $18.5m
[Crash cost per week: 2%; Marginal cost analysis: 2% per step; Conclusion: 1% (0.5% job + 0.5% TOTAL cost]
[Each wrong crash cost per week -0.5%; excess crashed job @-1% (max 3%); wrong marginal cost in MC analysis: -1% per step]

2c
Final tableau

OC\Job	A	B	C	D	E	X
1	2.5	2.1	3.2	2.4	5	0
2	2.75	1.8	M	1.8	5.5	0
3	M	M	2.8	3	6	0
4	3	2.25	4	4.8	4.5	0
5	2	1.65	M	2.1	4.75	0
6	4.5	2.4	3.4	M	5.75	0

[Cost conversion: 2%; dummy job & 0: 1+1%; big-M: 2%]

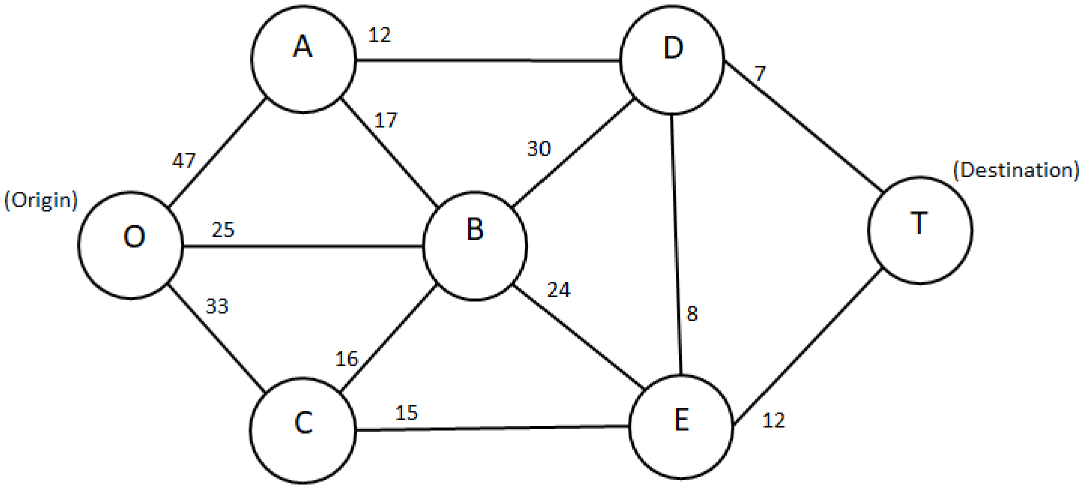
2d
Final tableau

OC\Job	A	B	C	D	E	X
1	0.05	0	0	0.15	0.05	0
2	0.75	0.15	M	0	1	0.45
3	M	M	0	1.15	1.45	0.4
4	1	0.6	1.25	3	0	0.45
5	0	0	M	0.3	0.25	0.45
6	2.05	0.3	0.2	M	0.8	0

Plan: 5-A; 1-B; 3-C; 2-D; 4-E
Total cost: 2 + 2.1 + 2.8 + 1.8 + 4.5 =13.2
[Final tableau: 2%; total cost: 1%; assigned job: 3%] [Missing final tableau -2%]

Question 3

(a) 5% Deduction (until 0): each wrong connection/missing number 1%



(b) Table 10%((Each wrong box -1%; deduct until 0) 少寫一列扣 5 分

n	Solved nodes	Closest connected unsolved nodes	Total distance involved	N^{th} nearest node	Minimum distance	Last connection
1	O	B	0+25=25	B	25	OB
2	O B	C C	0+33=33 25+16=41	C	33	OC
3	O B C	A A E	0+47=47 25+17=42 35+15=48	A	42	BA
4	A B C	D E E	42+12=54 25+24=49 33+15=48	E	48	CE
5	A B E	D D D	42+12=54 25+30=55 48+8=56	D	54	AD
6	D E	T T	54+7=61 48+13=60	T	60	DT ET

Route:

O→C→E→T (each 3%) 多寫路徑扣一分

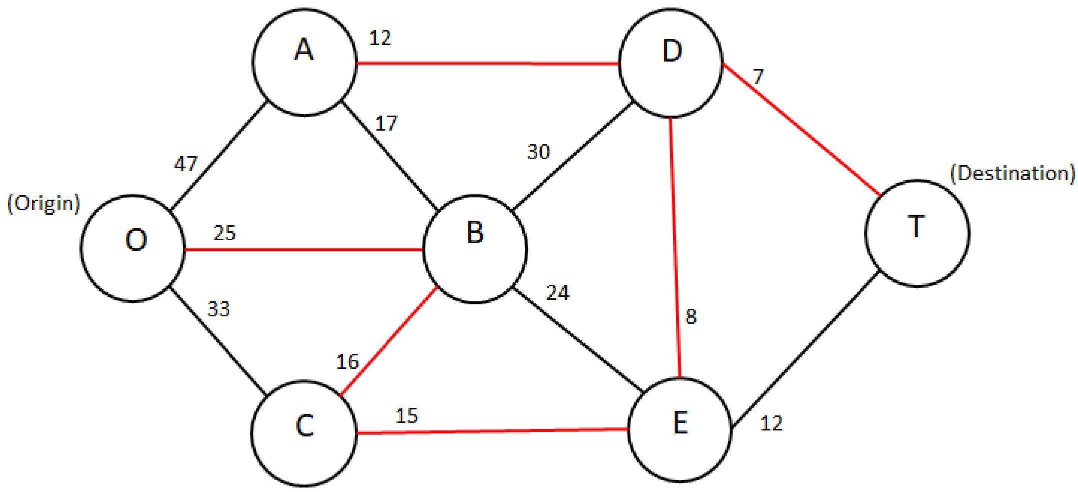
Total miles: 60(2%)

(c)

No. (1%)

As mileage = speed * travel time, when speed becomes a constant, the shortest path with respect to travel time is equivalent to the shortest path with respect to travel distance. (1%)

(d)



Involved roads: OB, BC, CE, DE, AD, DT (2%) 每條扣一分，至多扣兩分

Total mileages: 25+16+15+8+12+7=83 (1%)

學生答題狀況:

3.(a)幾乎都答對，僅有少部分同學粗心漏掉一些線和數字。

(b)很多人都在第三列那邊把 B 連到 E 而沒看到 A，或是第五列的 E 連到 T，導致計算錯誤!其他有些是沒標明最後路徑與總里程。

(c)幾乎都答對，除了寫 yes 的人被扣分外，還有寫 no 但是理由是”不會變”，則會扣分。

(d)滿多人沒連到 A，或是把 AB 連起來等等，有些則是少寫 total mileages

Question 4

Comments:

4ai) Most students can identify the three augmenting paths. Some students misunderstand the problem to consider path via BC and CD in their solutions. Some students forget to calculate the total flow but only stating the flow of 30 and 48 from the points. This does not answer the keyword of "maximum flow of the network".

4aii) Diverse answers found in this question. Some students wrongly interpreted as finding links with MORE than half; some understood as whether the maximum flow reaches half of the capacity of the entering flow; and some forget to include links BC and CD in their solution. Moreover, some students overlook this question and lose the scores of this sub-section completely.

4b) Some of the students work on this question well. On the other hand, some students forget to include paths such as OBFET or OBFGET into their solutions. These come to a lower maximum flow.

4c) Most students perform poorly. Some students cannot understand the question that only DT, ET, and GT should be considered. Some students choose DT while this link is not yet full in the new network. Some students choose ET with a wrong calculation of new flow on GT. Some students pick GT without good explanations.

(a)(i):

Method (1): Augmenting path (AP) algorithm

AP1: O-F-G-T = 30; AP2: O-F-E-T = 24; AP3: O-B-D-E-T = 24;

Maximum flow = 78.

[Each path 1.5%; maximum flow 1.5%]

Method (2): Max-flow min-cut theorem

Using max-flow min-cut theorem, the maximum flow is 78 vph (with the min-cut on links DE, EF, and GT).

[Flow: 1.5%; each min-cut link: 1.5%]

4aii:

Links below half of its capacity: BC, CD, OB, FG [each 1%]

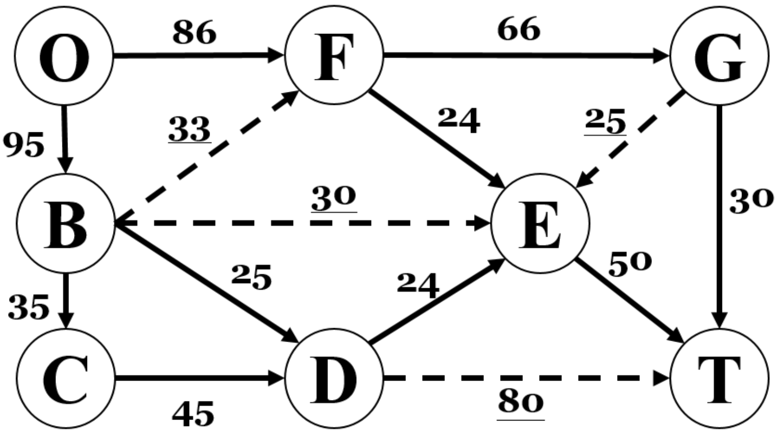
4b:

Method (1): Augmenting path algorithm

AP1: O-F-G-T = 30; AP2: O-B-C-D-T = 35; AP3: O-F-E-T = 24;

AP4: O-B-E-T = 26; AP5: O-B-D-T = 25

Max Flow: 140 vph
(Other AP combinations are acceptable, unless the sum is 140)



Method (2): Max-flow min-cut
Using max-flow min-cut theorem, the maximum flow is 140 vph (with the min-cut on links BC, BD, BE, ET, and GT).
[Flow: 1.5%; each min-cut link: 1.5%]

4c:
As the capacity of DT has not been used up, it must not be the chosen link.
By doubling the capacity of ET (i.e., 100)
AP1: O-F-G-T = 30; AP2: O-B-C-D-T = 35; AP3: O-F-E-T = 24;
AP4: O-B-E-T = **30**; AP5: O-B-D-T = 25; AP6: O-F-G-E-T = **25**;
New max-flow = 169
By doubling the capacity of GT (i.e., 60),
AP1: O-F-G-T = **60**; AP2: O-B-C-D-T = 35; AP3: O-F-E-T = 24;
AP4: O-B-E-T = 26; AP5: O-B-D-T = 25
New max-flow = 170.
Therefore, GT should be chosen.
[DT: 1%; ET calculation: 2%; GT calculation: 2%; Conclusion: 1%]