Introductory Statistics I　　　姓名: ____________

Second Midterm Exam　　　學號: ____________

1. 交卷時請將試卷連同答案本一起交回（未繳回試卷者視同未繳答案本）；
2. 請使用試卷後方做為計算紙使用（嚴禁使用其他計算紙）；
3. 請依照座位表入座，並準備學生證供查驗;
4. 所有計算題必須列出計算過程，且與最後答案相符合才給分。計算題僅有最後答案不予任何分數。；
5. 所有計算過程均以四捨五入方式取小數點後四位數字，但最後答案可取小數兩位;
6. 請寫出計算過程以便給部分分數;
7. 答卷時間：10：10 – 12：00；
8. 總分：100

Part I. Multiple Choice (Only one answer is correct)

1. (1%) Which property is not a characteristic of the Binomial experiment?
   A) The trials are independent.
   B) The probability of success on a single trial is equal to p.
   C) The experiment consists of two identical trials.
   D) The probability of failure on a single trial is equal to 1-p.
   E) None of the above.

2. (1%) The probability of x successes when a random sample of n is drawn from a population of N (within which S units exist with the characteristic denoting success) is given by:
   A) The Binomial formula
   B) The Hypergeometric formula
   C) The Poisson formula
   D) The Exponential formula
   E) None of the above.

3. (1%) The Binomial distribution approaches a normal curve in shape when

A) n increase and p approaches 1.00.

B) n increase and p approaches 0.

C) n increase and p approaches 0.5.

D) all of the above.

E) None of the above.

4. (1%) In a Hypergeormetric distribution, when the population size N gets very large (or infinite), the Hypergeometric probability can be approximated by

A) the Binomial formula

B) the Uniform formula

C) the Poisson formula

D) None of the above

Part II: Show the work to receive partial credit. (請列出計算過程)

1. (Total: 11%) When a new machine is functioning properly, only 3% of the items produced are defective. Assume that we will random select threes items produced on the machine and we are interested in the number of defective items found.
   (a) What is the probability distribution of the number of defective items found in a sample of three items. (3%)
   (b) What is the probability that we found at least one defective products? (4%)
   (c) Compute the expected number of defective items. (2%)
   (d) Compute the standard deviation of the number of defective items. (2%)

<sol>

p = 0.03 (defective rate)

n = 3

X = # of defective

(a) $P(X = x) = \binom{3}{x}(0.03)^x(0.97)^{3-x}$, $x = 0, 1, 2, 3$

(b) $P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{3}{0}(0.03)^0(0.97)^3 = 1 - 0.9127 = 0.0873$

(c) $\mu = np = 3 \times 0.03 = 0.09$

(d) $\sigma = \sqrt{npq} = \sqrt{3 \times 0.03 \times 0.97} \cong 0.2955$

2. (Total: 11%) Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.
   (a) Compute the probability of at most one arrival in a one-minute period. (3%)
   (b) Compute the probability of no arrival in a three-minute period. (4%)
   (c) What is the mean number of arrivals in a one-minute period? (2%)
   (d) What is the standard deviation of number of arrivals in a three-minute period? (2%)

<sol>

$\lambda = 10$ 人/min

$\Delta t = 1$ min

X = # of arrival

X ~ Poisson ($\lambda = 10$)

(a) $t = 1$, $\mu = \lambda t = 10$ 人

$$\begin{aligned} P(\text{at most one arrival}) &= P(X \le 1) \\ &= P(X = 0) + P(X = 1) \\ &= \frac{10^0 e^{-10}}{0!} + \frac{10^1 e^{-10}}{1!} \\ &= 11e^{-10} \cong 0.0005 \end{aligned}$$

(b) $t = 3$, $\mu = \lambda t = 30$ 人

$$P(\text{no arrival}) = P(X = 0) = \frac{30^0 e^{-30}}{0!} \cong e^{-30} = 0$$

(c) $\mu = 10$

(d) $\sigma = \sqrt{\mu} = \sqrt{\lambda t} = \sqrt{30} = 5.4772$ or 5.48

3. (Total: 7%) Electric fuses produced by ABC Company are packaged in a box which contains 12 fuses. An inspector randomly selects three of the 12 fuses in the box for testing. Suppose the box contains exactly five defective fuses,

   (a) What is the probability that the inspector will find exactly one of defective fuses? (3%)

   (b) Find the mean and the variance for the number of defective fuses. (4%)

<sol>

N = 12, n = 3, a = 5, x = # of defective fuses in the sample of three

(a) P(X = 1) = $\frac{\binom{5}{1}\binom{7}{2}}{\binom{12}{3}} = \frac{5\times21}{220} = \frac{21}{44}$ = 0.4773 or 0.48

(b) $\mu = n \times \frac{a}{N} = 3 \times \frac{5}{12}$ = 1.25

$$\sigma^2 = n \times \frac{a}{N} \times \frac{N-a}{N} \times \frac{N-n}{N-1} = \frac{3\times5\times7\times9}{12\times12\times11} = \frac{945}{1584} = 0.59659 \text{ or } 0.60$$

4. (Total: 10%) The scores of English in a class are normally distributed. The distribution is divided into three portions, 5% of the students' scores are below 60 and 10% are above 80. Try to find the mean and the standard deviation for this course.

<sol>

P($z \geq \frac{80-\mu}{\sigma}$)=0.1 → $\frac{80-\mu}{\sigma}$ = 1.28 → 1.28 σ+ μ= 80

P($z \leq \frac{60-\mu}{\sigma}$)=0.05 → $\frac{60-\mu}{\sigma}$ = -1.645 → -1.645 σ+ μ= 60

2.925σ= 20, σ= 6.84

1.28*6.84+ μ= 80 → μ= 71.2448→71.25

5. (Total: 23%) The time required to repair an automatic loading machine in a complex food-packaging operation of a production process is X minutes. Studies have shown that the normal distribution approximation X with the mean of 120 and the variance of 16 is quite good. If the process is down for more than 125 minutes, all equipment must be cleaned, with the loss of all product in process. The total cost of product loss and cleaning associated with the long down-time is $10,000.

(a). Given a breakdown of the packaging machine, find the expected cost. (10%)

(b). Suppose the management can reduce the mean of the service time distribution to 115 minutes by adding more maintenance personnel. Under what cost condition the management is willing to add the maintenance crew? (13%)

<sol>

(a) $P(z > \frac{125-120}{4}) = P(z > 1.25) = 0.1056$

$10000*0.1056= $1056

(b) $P(z > \frac{125-115}{4}) = P(z > 2.5) = 0.0062$

$10000*.0062= $62

$1056-$62= $994

Add the maintenance crew when the cost is lower than $994.

6. (Total: 10%) Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on the average, 10% of their prospective guests will not claim their reservation. If the hotel accepts 215 reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?

<sol>

Define x to be the number of guests claiming a reservation at the motel. Then $p = P[\text{guest claims reservation}] = 1 - .1 = .9$ and $n = 215$. The motel has only 200 rooms. Hence, if $x > 200$, a guest will not receive a room. The probability of interest is then $P(x \le 200)$. Using the normal approximation, calculate

$$\mu = np = 215(.9) = 193.5 \text{ and } \sigma = \sqrt{215(.9)(.1)} = \sqrt{19.35} = 4.399$$

The probability $P(x \le 200)$ is approximated by the area under the appropriate normal curve to the left of 200.5. The z-value corresponding to $x = 200.5$ is $z = \frac{200.5 - 193.5}{\sqrt{19.35}} = 1.59$ and

$$P(x \le 200) \approx P(z < 1.59) = .9441$$

7. (Total: 24%) Suppose that college faculty with the rank of assistant professor earn an average of $71,802 per year with a standard deviation of $4000 in the United States. A random sample of 60 assistant professors was selected from a database for all universities.

(a). Please define the population of interest in this question. (2%)

(b). Explain what is a sampling distribution of a mean? (4%)

(c). Explain what is Central Limit Theorem? (4%)

(d). Describe the sampling distribution of the sample mean $\bar{x}$, specifically, the mean and standard deviation (4%)

(e). Within what limits would you expect the sample average to lie, with probability .95? (3%)

(f). Calculate the probability that the sample mean $\bar{x}$ is greater than $73,000? (3%)

(g). If your random sample actually produced a sample mean of $73,000, would you consider this unusual? What conclusion might you draw? (4%)

<sol>

(a)All assistant professors in the US.

(b)The sampling distribution of a statistic is the probability distribution for the possible values of the statistic that results when random samples of size n are repeatedly drawn from the population.

(c)The Central Limit Theorem states that, under rather general conditions, sums and means of random samples of measurements drawn from a population tend to have an approximately normal distribution.

(d)-(g)

a Since the sample size is large, the sampling distribution of $\bar{x}$ will be approximately normal with mean $\mu = 71,802$ and standard deviation $\sigma/\sqrt{n} = 4000/\sqrt{60} = 516.3978$.

b From the Empirical Rule (and the general properties of the normal distribution), approximately 95% of the measurements will lie within 2 standard deviations of the mean:

$$\mu \pm 2SE \Rightarrow 71,802 \pm 2(516.3978)$$
$$71,802 \pm 1032.80 \text{ or } 70,769.20 \text{ to } 72,834.80$$

c Use the mean and standard deviation for the distribution of $\bar{x}$ given in part **a**.

$$P(\bar{x} > 73,000) = P\left(z > \frac{73,000 - 71,802}{516.3978}\right)$$
$$= P(z > 2.32) = 1 - .9898 = .0102$$

d Refer to part **c**. You have observed a somewhat unlikely occurrence (one that happens only about 1 time in 100), assuming that $\mu = 71,802$. Perhaps your sample was not a random sample, or perhaps the average salary of $71,802 is no longer correct.

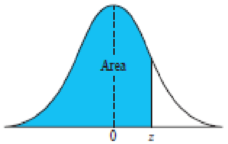

TABLE 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 3 *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5 871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998