NATIONAL CHIAO TUNG UNIVERSITY
COLLEGE OF MANAGEMENT

FINAL EXAMINATION FOR

Operations Research (II)

(June 18th, 2019)
10:10am~noon
Time Allowed: 110 minutes

INSTRUCTIONS TO CANDIDATES:

1. 本次測驗計有八題,共四頁.
   This question paper contains **EIGHT (8)** questions and comprises **FOUR (4)** printed pages.

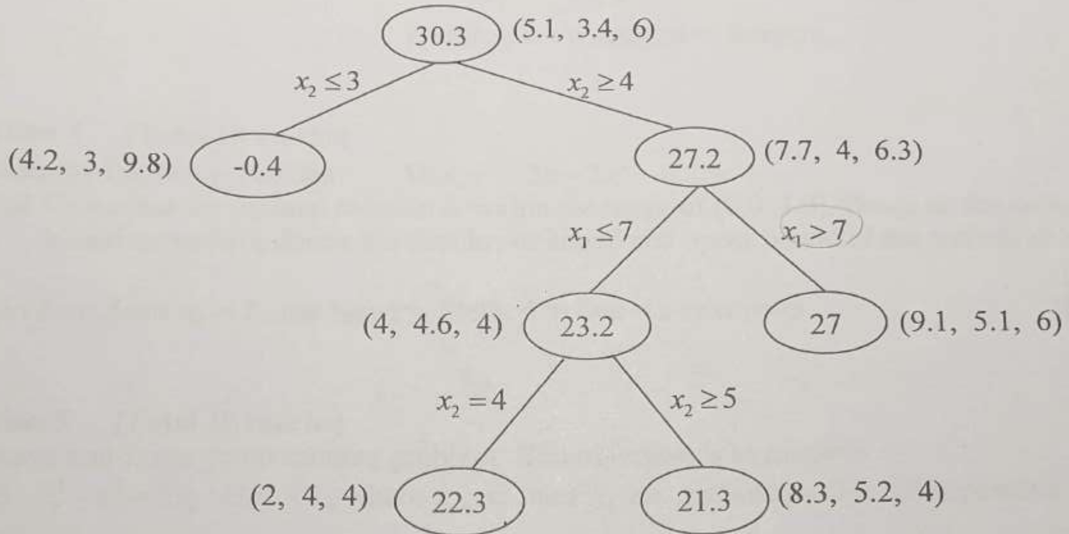
2. 回答所有問題,總分為一百分.
   Answer **ALL** questions. The total mark for this question paper is **100**.

3. 清楚標明題號,並依序在答題本上作答.
   Sequentially write down your answers in the **answer book** and clearly mark the question number.

4. 作答於試卷紙上部分,不予計分.
   Any answer written on this question paper will **NOT** be graded.

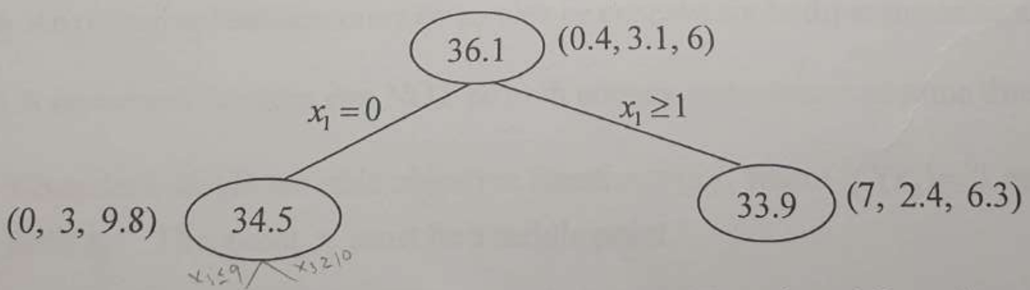
5. 本測驗採部份給分,請以指定的方法作答,並列出完整的計算過程.
   Follow the instruction and show **ALL** your work. Partial credits may be given.

Question 1 **[Total 12 marks]**
Consider the following partial Branch and Bound tree for a Maximization Integer Programming Problem where all variables are nonnegative integers. Note that the number in the circle is the optimal value of the Linear Programming Relaxation and the solution attached to the circle is the optimal solution of the Linear Programming Relaxation.



The above Branch and Bound tree seems to be **incorrect**. Identify **ALL** mistakes with this tree. For each mistake, use no more than two sentences to explain why.

Question 2 **[Total 9 marks]**
Consider the following partial Branch and Bound tree for a Maximization Integer Programming Problem where all variables are nonnegative integers. Note that the number in the circle is the optimal value of the Linear Programming Relaxation and the solution attached to the circle is the optimal solution of the Linear Programming Relaxation.



Currently, the tree is **completely correct** and the tree (Branch and Bound procedure) is **not finished** yet.

(a) Can (0, 4, 9) with objective value 31.2 be a **feasible** solution to the original Integer Programming problem? (Yes/No). Explain your answer with two sentences. **[3 marks]**

(b) Can (2, 4, 7.2) with objective value 33 be an **optimal** solution to the original Integer Programming problem? (Yes/No). Explain your answer with two sentences. **[3 marks]**

(c) Can (5, 3, 1) with objective value 34 be an **optimal** solution to the original Integer Programming problem? (Yes/No). Explain your answer with two sentences. **[3 marks]**

Question 3 **[Total 15 marks]**
Use the Branch and Bound Procedure to solve the following Integer Programming Problem.

$$\max \quad 3x_1 + x_2$$

subject to

$$2x_1 + 3x_2 \le 15$$
$$4x_1 \le 9$$

x_1 and x_2 are nonnegative integers.

Question 4 **[Total 10 marks]**
Consider the following problem: Max $x^3 + 2x - 2x^2 - 0.25x^4$

(a) Given that the optimal solution is within the range of [0.0, 4.0]. Based on the concept of the bisection method, derive the new lower bound and upper bound of the optimal solution. **[5 marks]**

(b) Start from $x_0 = 2$, use Newton Method to find the next point. **[5 marks]**

Question 5 **[Total 12 marks]**
Consider a non-linear programming problem. The objective is to maximize $f(x) = -x_1^2 - x_2^2 + 2x_2 - 7x_3^2 + 7$, where x_1, x_2, and x_3 are real and can be either positive or negative.

(a) Given the initial point $(x_1, x_2, x_3) = (0.5, 1, 0)$ as start. Solve this problem by the **gradient search method** with error tolerance $\varepsilon = 0.0001$. **[9 marks]**

(b) Continuing from (a), is the point where you moved to a global maximum? (Yes/No) Explain why in no more than 2 sentences. **[3 marks]**

Question 6 **[Total 14 marks]**
Label the following statements with **TRUE** or **FALSE** and explain your reasons in no more than 2 sentences.

(a) An objective function must be convex or concave (or both) at the same area.

(b) A constraint function can NOT be both convex and concave at same time.

(c) Consider a single variable objective function $f(x)$, where $f'(x_0) = 0$ and $f''(x_0) = 0$ at some point x_0. This point x_0 must be a saddle point.

(d) Consider a multiple variable objective function $f(x)$, where $\nabla f(x^0) = \mathbf{0}$ at some point x^0. This point x^0 must be a local maximum or local minimum.

(e) Newton's method is designed for single variable unconstrained non-linear programming. Thus, Newton's method can NOT be applied to multiple variable unconstrained NLP.

(f) For general NLP problem, we can use the Phase I of Two-Phase SIMPLEX method to find all KKT points.

(g) For a maximal NLP problem with concave objective function, KKT points must be global optimum.

Question 7 **[Total 20 marks]**

Consider a non-linear programming problem. The objective is to maximize $f(x) = x_1^2 - 10x_1x_2 + x_2^2$ subject to $1 \le x_1^2 + x_2^2 \le 9$, where x_1 and x_2 are real and can be either positive or negative.

(a) Is this a quadratic programming problem? (Yes/No) Explain why. **[2 marks]**

(b) Is the feasible region convex, or concave? **[1 mark]**

(c) Find the gradient of the objective function. **[2 marks]**

(d) Is the objective function convex or, concave? **[1 marks]**

(e) Write down the KKT conditions for this problem. **[6 marks]**

(f) Does $(x_1, x_2) = (0,3)$ satisfy KKT the conditions? (Yes/No and verify your answer) **[1 mark]**

(g) Find all points that satisfy the KKT conditions and their associated objective values. **[4 marks]**

(h) In this question, we claim that "any point satisfies the KKT conditions must be a global maximum". Is this claim true or false? Explain why in no more than 2 sentences. **[3 marks]**

Question 8 **[Total 8 marks]**

Consider the following quadratic programming problem.

$$\begin{aligned} \text{Maximize } & -x_1^2 - (x_1 - 2x_2)^2 + 15(x_1 + 2x_2) + 15 \\ \text{subject to: } & x_1 + 2x_2 \le 30 \\ & x_1 \ge 0, \; x_2 \ge 0 \end{aligned}$$

(a) Rewrite the objective function in matrix notation and show the Q matrix and **c** vector. **[3 marks]**

(b) Write down the KKT conditions in the convenient form (all linear equations with complementary constraints) for this problem. **[5 marks]**

- END OF PAPER -