Introductory Statistics II

First Midterm Exam

姓名: ____________

學號: ____________

1. 交卷時請將試卷連同答案本一起交回（未繳回試卷者視同未繳答案本）;
2. 請使用試卷後方做為計算紙使用（嚴禁使用其他計算紙）;
3. 請依照座位表入座，並準備學生證供查驗;
4. 所有計算題必須列出計算過程，且與最後答案相符合才給分。計算題僅有最後答案不予任何分數。;
5. 請寫出計算過程以便給部分分數;
6. 答卷時間：10：10 – 12：00 ;
7. 總分：100

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$s_p = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1)+(n_2-1)}$$

1. (Total: 15%) Suppose you are conducting a statistical test for $H_0: \mu = 255$ against $H_a: \mu > 255$ when the population variance $\underbrace{\sigma^2 = 63}$ (known) is known. The decision rule is "reject H_0 if the sample mean of a random sample of 81 items is more than 270".
   (a) Find the probability of making type I error. (5%)
   (b) Find the probability of making type II error when $\mu = 265$. (5%) (non-reject H_0 | H_a)
   (c) Find the decision power when $\mu = 270$. (5%) (reject H_0 | H_a)

2. (Total: 5%) A random sample of 10 (n) observations is taken from a normal population having the variance $\sigma^2 = 42$ (known). Find the approximate probability of obtaining a sample standard deviation between 3.94 and 8.886.

3. (Total: 30%) Assume that the time in days required for maturation of seeds of a species of guardiola (a flowering plant found in Mexico) is $N(\mu, \sigma^2)$ (unknown). A random sample of $n = 13$ seeds, yielded the sample mean is $\bar{x} = 18.97$ days and $s^2 = 10.7$ squared days.
   10.3 μ
   10.7 σ
   (a) Given $\alpha = 0.05$, test hypotheses $H_0: \mu = 20$ against $H_a: \mu < 20$. (10%)
   (b) Construct a 95% upper confidence bound for μ. (5%)
   (c) Given $\alpha = 0.05$, test hypotheses $H_0: \sigma^2 = 9$ against $H_a: \sigma^2 \neq 9$. (10%)
   (d) Construct a 95% confidence interval for σ^2. (5%)

4. (Total: 15%) An advertisement for a supermarket claims A that it's price is significantly lower than other full-service supermarkets. As part of a survey, the average weekly total, based on the prices of approximately 95 items, is given for two different supermarket chains A and B, recorded during 4 consecutive weeks in a particular month. 10.5

Week	A	B
1	254.26	256.03
2	240.62	255.65
3	231.90	255.12
4	234.13	261.18

pair

$\frac{\bar{d}-0}{\frac{S_d}{\sqrt{n}}}$

△(a) Is there a significant difference in the average prices for these two different supermarket chains? Please perform a hypothesis testing. (10%)

(b) What is the approximate p-value? (5%)

5. (Total: 30%) As part of an industrial training program, some trainees are instructed by Method A, which is straight computer-based instruction, and some are instructed by Method B, which also involves the personal attention of an instructor. Assume that the population sampled can be approximated closely with normal distributions. If random sample of size 10 are taken from large groups of trainees instructed by each of these two methods, and the scores which they obtained in an appropriate achievement test are:

Method A:	72	77	84	69	78	75	65	73	77	70
Method B:	71	75	65	69	73	66	88	71	74	68

σ大

(a) Given $\alpha = 0.05$, test the claim that Method B is more variable. (10%)

(b) Given $\alpha = 0.05$, test the claim that Method A is more effective. Assume that the populations have the same variance. (10%) μ大 Sp

(c) Construct a 95% confidence interval for the ratio of two population variances, σ_A^2 / σ_B^2. (10%)

1 6. (Total: 5%) Prove that the sample variance S^2 is an unbiased estimator to the population variance σ^2.

Appendix:

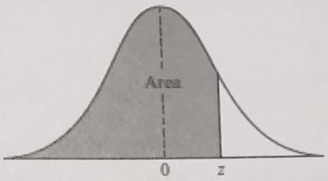

TABLE 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

- t **table:**

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20

- χ^2 **table:**

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953

$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$	df
13.3616	15.5073	17.5346	20.0902	21.9550	8
14.6837	16.9190	19.0228	21.6660	23.5893	9
15.9871	18.3070	20.4831	23.2093	25.1882	10
17.2750	19.6751	21.9200	24.7250	26.7569	11
18.5494	21.0261	23.3367	26.2170	28.2995	12
19.8119	22.3621	24.7356	27.6883	29.8194	13
21.0642	23.6848	26.1190	29.1413	31.3193	14

$F_{8,8,0.05} = 3.438,\ F_{9,9,0.05} = 3.179,\ F_{10,10,0.05} = 2.978,\ F_{11,11,0.05} = 2.818$

$F_{8,8,0.025} = 4.433,\ F_{9,9,0.025} = 4.026,\ F_{10,10,0.025} = 3.717,\ F_{10,10,0.025} = 3.474$