A

Mid-Term Examination

*Computer Algorithms*

**Department of Information Management & Finance**

**National Chiao Tung University**

November 7, 2017

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ACABD

CAABDB

ACCDBA

1. (5 pts) Describe the key differences between dynamic programming and greedy method.

DP: overlapping 的 subproblem，前面解完的子問題會影響後面的部分

2. (15 pts) (a) Prove that for the Activity Selection problem, there exists an optimal solution that selects the job with the earliest finish time. (b) Each activity $j$ is now assigned a value $v_j$, and we want to select from among the activities to maximize the total value earned. Design your *greedy* algorithm. The algorithm could be correct or incorrect. Prove it or disprove it by a counter-example. (b) max{選它 or 不選它}

w/h

3. (10 pts) Given three sequences $X = (x_1, \ldots, x_l), Y = (y_1, \ldots, y_m)$, and $Z = (z_1, \ldots, z_n)$. Design a dynamic programming algorithm for finding the longest common subsequence of $X, Y$, and $Z$. Analyse the run time of your algorithm. 起始，過程，目標

4. (6 pts) Let $T(n)$ be the number of moves required for solving the Tower of Hanoi with $n$ discs. Give a recurrence formula of $T(n)$ and solve it.

5. (9 pts) Use the master method to give tight asymptotic bounds for the following recurrences. (a) $T(n) = 2T(n/2) + n$; (b) $T(n) = 2T(n/4) + \sqrt{n}$; ~~(b)~~ $T(n) = 2T(n/4) + n \log n$.

$\frac{n \log n}{n^{\frac{1}{2}}}$

$\sqrt{n} \log n$

6. (9 pts) For each of functions $f_1(n) = 2n^4 - 32n^3 + 125n + 6$, $f_2(n) = n^3$, and $f_3(n) = n \log n + 7n - 9$, find from the following asymptotic notations that match the function:
(a) $O(n \log n)$ (b) $O(n^2)$, (c) $(n^2 \log n)$, (d) $O(n^3)$, (e) $O(n^4)$,
(f) $\Theta(n \log n)$, (g) $\Theta(n^2)$, (h) $\Theta(n^2 \log n)$, (i) $\Theta(n^3)$, (j) $\Theta(n^4)$,
(k) $\Omega(n \log n)$, (l) $\Omega(n^2)$, (m) $\Omega(n^2 \log n)$, (n) $\Omega(n^3)$, (o) $\Omega(n^4)$.

polynomial

V log V

7. (10 pts) Prove that Kruskal's algorithm correctly finds a minimum spanning tree of a weighted graph. 已知解和Kruskal做出來的解差別在哪?，用Kruskal做可能不會更差，更好

8. (6 pts) Analyze the minimum run time of Prim's algorithm. The run time will depend on the data structures you'd use. V + E log E

9. (3+2+5 pts) Give the worst-case run time required by each of the following operations (a) Construct a min-heap out from an array of $n$ integers; (b) Delete the minimum integer from

O(n) O(log n)

$O(n\log n)$

the min-heap of (a) with its minimality property maintained. (c) Use the min-heap to sort the $n$ integers in ascending order.

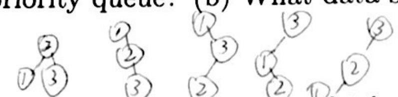
10. (3+7 pts) Prove that "If a directed weighted graph contains a negative weight cycle reachable from the source node, then Bellmand-Ford's algorithm will report FALSE". Hint: The test on each edge $(u, v)$ is $d[v] > d[u] + w(u, v)$.

→不需多餘的空間做儲存

11. (2+2 pts) (a) Explain the "in-place" property of sorting algorithms. (b) Which of HEAP SORT, QUICK SORT, MERGE SORT, and INSERTION SORT are in-place?

→ ① 所有data都有priority ② queue原本沒有排好但要output時要能照順序output

12. (2+3 pts) (a) What is a priority queue? (b) What data structures will you choose for implementing a priority queue.



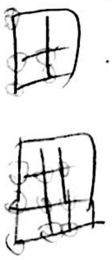
node 的右大於 node
node 的左小於 node

13. (3+7 pts) (a) Construct ALL binary search trees of 3 keys $\{1, 2, 3\}$. (b) Given $n$ keys $a_1 < a_2 < \cdots < a_n$ and their associated success probabilities $p_1, \ldots, p_n$ and failure probabilities $q_0, q_1, \ldots, q_n$, formulate a recurrence formula to find the optimal binary search tree.

14. (10 pts) On a two-dimensional grid with integer coordinates, let $T(n)$ denote the number of ways to move from node $(0, 0)$ to node $(n, n)$, $n > 0$, subject to the constraints that only upward and rightward moves are allowed and that no moves to any node $(x, y)$ with $x < y$ are allowed. Find $T(n)$ and its relation with the number of parenthesizations of Matrix Chain Multiplications.

—— End of Test ——

n=2, 4 6



n=3, 9 19

$2^{n-1}\cdot n$ $2^3$ $2\times2\times2$ $2^4$ $n^2-1$

Examination II, *Computer Algorithms*
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1. (8+8+6+8 pts) Given a directed, weighted graph $G = (V, E)$, we want to find all-pairs shortest paths. (a) Define $f_{ij}^{(k)}$ as the minimum weight of any path from vertex $i$ to vertex $j$ that contains at most $k$ edges. Write a recursive formulation to find $f_{ij}^{(k)}$. (b) Define Let $g_{ij}^{(k)}$ be the minimum weight of any path from vertex $i$ to vertex $j$ for which all intermediate vertices belong to the set $\{1, 2, \ldots, k\}$. Write a recursive formulation to find $g_{ij}^{(k)}$. (c) Give the lowest possible run times of the two dynamic programs of (a) and (b). (d) Describe the causes of the difference between the two run times.

20 V 2. (8+12 pts) (a) In a binary search tree, how to find the immediate successor of node $z$? (b) Describe the purpose of the instructions from Step 6 to Step 9.

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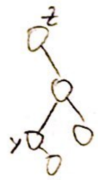
TREE-DELETE(T, z)
 1  if z.left == NIL
 2      TRANSPLANT(T, z, z.right)
 3  elseif z.right == NIL
 4      TRANSPLANT(T, z, z.left)
 5  else y = TREE-MINIMUM(z.right)
 6      if y.p ≠ z
 7          TRANSPLANT(T, y, y.right)
 8          y.right = z.right
 9          y.right.p = y
10      TRANSPLANT(T, z, y)
11      y.left = z.left
12      y.left.p = y
```




$h < 2 \lg n+1$

$\frac{h}{2}$ $\lg n+1$

$2^{\frac{h}{2}} - 1 < n+1$

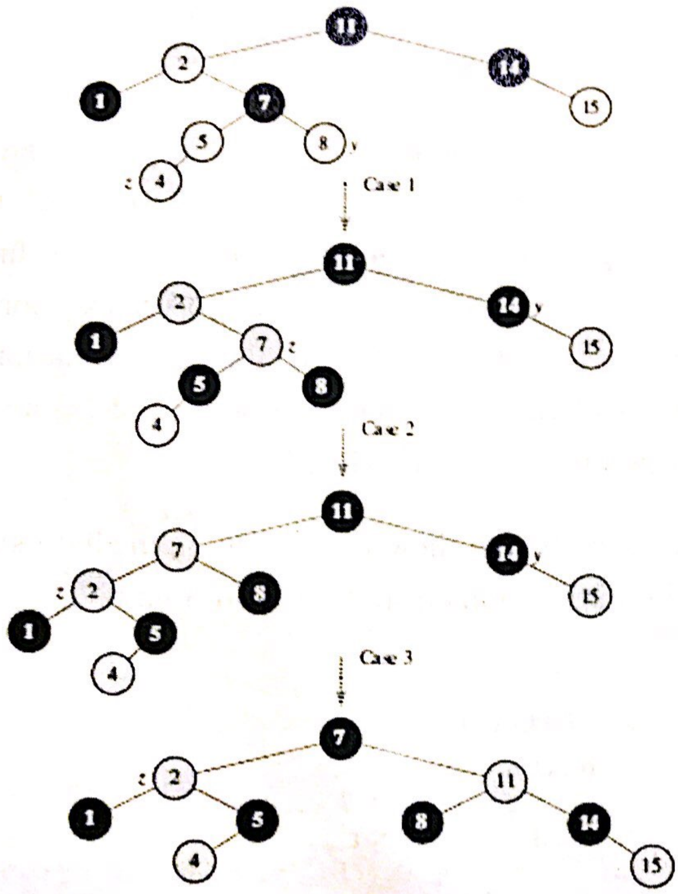
15 3. (7+8+5 pts) (a) Describe the red-black properties that a red-black tree must satisfy. (b) A red-black tree is guaranteed to be *approximately balanced*. What does "approximately balanced" mean? (c) Describe the advantages of red-black trees over binary search trees.

$h > 2^{\frac{h}{2}} - 1$

10 4. (10 pts) Show that *a red-black tree with $n$ internal nodes has height at most* $2\lg(n+1)$.

10 5. (5+5 pts) Describe double hashing used in open addressing. What are the major advantages of double hashing.

10 6. (10 pts) Describe the purposes for handling Case 1 and Case 2 in a red-black tree.




*** End of Test ***