Calculus I 0353 Quiz I

Name : ID : 2019/03/14 by Y. C. J.

Do not use L'Hospital's Rule , but you can use the known property: $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1$

1. (6%)Find the **largest** δ such that if $0 < |x-8| < \delta$ then $\left|\sqrt[3]{x} - 2\right| < 0.1$.
2. (6%)Simplify the function $\csc(\cos^{-1}(x))$ in x.

sin cos
tan cot
sec csc

3. (5%×5)Evaluate:

$\lim_{x\to(-3)}(⟦-x⟧ - ⟦x⟧)$ $\lim_{x\to 0} x\sin\left(\frac{1}{\sqrt{x}}\right)$ $\lim_{x\to 0}\frac{\sin(x)}{\sqrt{x}}$

$\lim_{x\to\left(\frac{2\pi}{3}\right)}\csc^{-1}\left(\frac{-9}{4\pi}x - \frac{\sqrt{3}}{2}\cot(x)\right)$ $\lim_{x\to(0^-)} e^{\cot(x)}$

4. (8%)Find all the horizontal/vertical asymptotes of $f(x) = e^{\frac{1}{x}}\cdot\tan^{-1}(e^x)$.
5. (5%) Suppose that $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x)$ exists, prove that $\lim_{x\to 0}\left[\frac{f}{g}\right](x) = \infty$.

BONUS

6. (5%)Choose **one** to evaluate:

$\lim_{x\to 0}\frac{⟦x⟧}{x}$ $\lim_{x\to 0}\frac{\tan^{-1}(x^2)}{x}$ $\lim_{x\to(-\infty)}\frac{⟦x⟧}{x}$

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$\lim_{x\to 0}\frac{\sin\left(\frac{1}{\sqrt{x}}\right)}{\frac{1}{\sqrt{x}}}\cdot\sqrt{x} = \lim_{x\to 0}\frac{\sin}{}$

cos
cot
csc

4?

Calculus I-0353 Quiz II

Name : ID : 2019/03/28 by Y. C. J.

1. (6%)Determine its differentiability at $x = 0$, **Show your proof**:

$$f(x) = \begin{cases} x^2 \cdot \cos\left(\frac{1}{x}\right), & x > 0 \\ \sqrt[3]{x^4}, & x \le 0 \end{cases}$$

$f'(0)$ exist $x^{-1} = -1x^{-2}$

$= x^{\frac{4}{3}}$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(h)-f(0)}{h}$

2. (20%)Find $y'(x)$ of the following functions:

$y = \sqrt{x^3 - 3\sqrt{x^2 - 2\sqrt{x}}}$ $y = 10^{\tan^{-1}(x)}$ $y = \cos^{-1}(\ln(x))$

(d) $x^2 \ln(y) - 3e^x y^2 = \csc(x + \sqrt{y})$ $\frac{d}{dx} b^x = b^x \ln b$

3. (5%)Find the second derivative $y''(x)$ for Question 4-(d) $\left(y' + \frac{1}{2} y^{-\frac{1}{2}} y'\right)$

4. (12%)Let $r(x) = f(g(h(x)))$, $w(x) = \frac{f(x)}{g(x)}$, find $r'(2)$, $w'(2)$, $(g^{-1})'(3)$.

x	1	2	3
$h(x)$	2	1	3
$g(x)$	3	2	1
$f(x)$	9	4	-5

x	1	2	3
$h'(x)$	8	0.5	-2
$g'(x)$	6	3	5
$f'(x)$	7	-9	4

5. (6%)Estimate $\ln(1.1)$ by linear approximation.

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

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Calculus I 0353 Quiz 3

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姓名：　　　　學號：

2019/04/18 by Y. C. J.

1. (6%)Find $f'(x)$ for $f(x) = (\sec^{-1}(x))^{\frac{1}{\sqrt{x}}}$.

$\frac{1}{\sqrt{x}\cdot x\cdot(\sqrt{x^2-1})} = \frac{1}{x\sqrt{x}(\sqrt{x^2-1})}$

2. (6%×3)Choose **3** to calculate:

$\lim_{x\to1}\left(\frac{x}{1-x}+\frac{1}{\ln x}\right)$ $\quad\infty\quad\infty$

$\lim_{x\to0^+}(\sin x)^{\tan x}$

$(\tan x)^{-1} = -1(\tan x)^{-2} = -\frac{1}{\tan^2 x}$

$\lim_{x\to0^-} xe^{\frac{1}{x^2}}$

$\lim_{n\to\infty}\left(1-\frac{2}{n}+\frac{3}{n^2}\right)^n$

3. (15%)Find all of the critical points, local/absolute maxi/minimum points, inflection points, Asymptotes and intervals with concavities. Then sketch its graph for $g(x) = -xe^{\frac{1}{x}}$ $\boxed{x\neq0}$ ($e\approx2.72$)

$-\frac{1}{e}$ +　　$-1e$　　$-e$

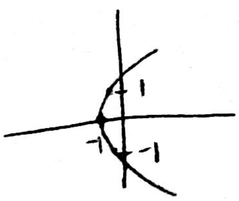
4. (6%)If $f'(x) = 0,\ \forall x\in(0,1)$ and $f(x)$ is continuous on $[0,1]$, then show that $f(x) = \cancel{f(a)}$ $f(0)$, $\forall x\in[0,1]$

constant　MVT

$f'(c) = \frac{f(1)-f(0)}{1} = 0$

5. (6%)Find the point on the parabola $y^2 = 2x+2$ which is closest to the point $(0,0)$.

$= 2(x+1)$



1

$y = \sqrt{2x+2}$

$y' = \frac{1}{2}(2x+2)^{-\frac{1}{2}}\cdot2 = (2x+2)^{-\frac{1}{2}}$　　$\frac{1}{\sqrt{-2}}$

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$x = -1$

$y'' = -\frac{1}{2}(2x+2)^{-\frac{3}{2}}\cdot2 = -(2x+2)^{-\frac{3}{2}}$

x	$f(x)$	$f'(x)$	$f''(x)$
-1	0	0	0
		+	−

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Calculus I 0353 Quiz 4

Name： ID： 2019/5/09 by Y.C.J

1. (42%)Choose 7 to evaluate:

(6%)$\int_0^2 [\![x^2]\!]dx$ (6%)$\int_2^3 x\sqrt{x-1}\,dx$ (6%)$\int_{-1}^{1} \frac{\tan^{-1}(x)}{1+x^2+x^4+x^6}dx$

(6%)$\int \frac{1}{\sqrt{2x+x^2}}dx$ (6%)$\int \frac{x}{\sqrt{1-x^4}}dx$ (8%)$\int \frac{x^2-1}{(x^2+1)^2}dx$

(6%)$\int \tan^{-1}(x)\,dx$ (8%)$\int \frac{\sin^2 x}{\cos^3 x}dx$ (8%)$\int \sqrt{\frac{2-x}{1+x}}dx$ (6%)$\int \ln(\sqrt{x})\,dx$

$u\ dv$

2. (6%)Evaluate the limit of sum: $\lim_{n\to\infty}\left(\frac{1}{2-3n}+\frac{1}{4-3n}+\cdots+\frac{1}{2n-3n}\right)$

3. (5%+6%)Let $f(x)=\int_{\sqrt{x}}^{x^3}\frac{\sin t}{t^3}\,dt$, $x>0$, find $f'(x)$ and $(f^{-1})'(0)$.

BONUS

4. (6%)Prove that $\frac{1}{2}\left(e^{\frac{-1}{4}}+e^{-1}\right)\le\int_0^1 e^{-x^2}dx\le\frac{1}{2}\left(1+e^{\frac{-1}{4}}\right)$. Hint : $\int_0^1=\int_0^{\frac{1}{2}}+\int_{\frac{1}{2}}^1$

e^0

$\sin^2\theta=\frac{1-\cos 2\theta}{2}$, $\cos^2\theta=\frac{1+\cos 2\theta}{2}$ $\sin 2\theta = 2\sin\theta\cos\theta$

[\![ ]\!]: Gauss function

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$x\ln(\sqrt{x})-\cancel{2x}$

$=\ln\sqrt{x}+x\frac{1}{\sqrt{x}}\frac{1}{2}x^{-\frac{1}{2}}-x$

$=\ln\sqrt{x}+\frac{x}{2}\frac{1}{\sqrt{x}}\cdot\frac{1}{\sqrt{x}}-x$

$=\ln\sqrt{x}+\frac{1}{2x}-x$

$x\ln(\sqrt{x})-\frac{1}{2}$

$=\ln\sqrt{x}+\cancel{x}\frac{1}{\sqrt{x}}\cdot\frac{1}{2}\frac{1}{\sqrt{x}}-\frac{1}{2}$

$=\ln\sqrt{x}+\frac{1}{2}-\frac{1}{2}$

$\int \ln x\,dx$

$= x\ln x - x$

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Calculus I 0353 Quiz5

Name： ID：

2019/05/23 by Y.C.J.

$y=\sqrt{x}$

x	1	4	$\frac{1}{4}$	1
y	1	2	$\frac{1}{2}$	-1

1. One region is bounded by $y = \sqrt{x}$, $y = x$.
   i. (3%)Find the area of this region.
   ii. (4+4%)Use the disk (washer) method to calculate the solid volume generated by rotating that region about $y = 1$.
   $\int (r_1^2 - r_2^2)\pi \, dx$
   iii. (4+4%)Use the shell method to calculate the solid volume generated by rotating that region about the $x = -1$.
   $\int 2\pi x f(x) \, dx$
2. (6%)Find the average value of $f(x) = e^x$ on the interval $[0,1]$.
3. (3+3%+3+3%)For a curve: $y = \dfrac{x^2}{4} - \dfrac{\ln(x)}{2}$, $1 \le x \le 3$. Find its arc length and surface area obtained by rotating it about y-axis.

$L = \int_a^b \sqrt{1+[f'(x)]^2} \, dx$

$S = \int_a^b 2\pi x \, ds$

$= \int_a^b 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2} \, dy$

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