Examination II, *Computer Algorithms*

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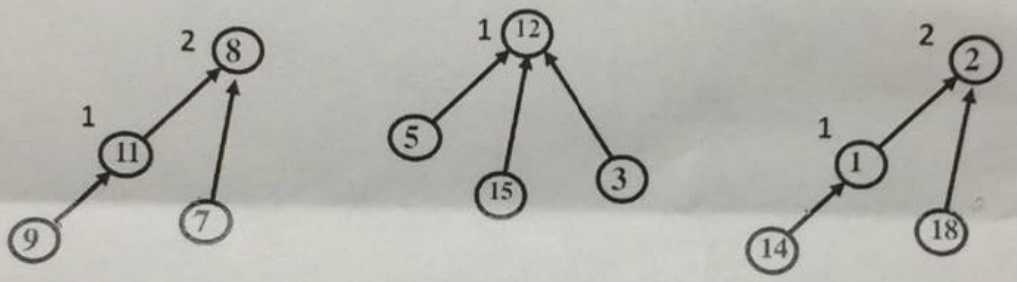
January 8, 2018

1. (6 pts) Ackermann function: For integers $k \geq 0$ and $j \geq 1$, define function $A_j(k)$

$$A_k(j) = \begin{cases} j+1, & \text{if } k = 0; \\ A_{k-1}^{(j+1)}(j), & \text{if } k \geq 1, \end{cases}$$

where $A_{k-1}^{(0)}(j) = j$ and $A_{k-1}^{(i)}(j) = A_{k-1}(A_{k-1}^{(i-1)}(j))$ for $i \geq 0$.
Find $A_0(2)$ and $A_1(2)$.

2. (4, 6 pts) (a) Describe the concept of union by rank and path compression. (b) Consider the following disjoint set forest. The number next to a node is the rank of this node. Draw the forests after UNION(7, 18) and then FIND(14).



3. (5, 5, 8 pts) In the problem of "Selecting the i-th Element", the n elements are partitioned into 5-element groups. We find the median x of the $\lceil n/5 \rceil$ medians of the groups. (a) If x is not our solution (the i-th element), then at least how many elements can be excluded for further consideration? (b) What is the run time of the prune-and-search algorithm for solving this problem? (c) What would happen if the elements are partitioned into 3-element groups or into 7-element groups?

4. (10 pts) Let $X = [1 : n]$ and $Y = [1 : n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\log n)$-time algorithm to find the median of all $2n$ elements in arrays X and Y.

5. (6, 8 pts) We consider table expansion without extraction. Define potential function of table T by $\Phi(T) = 2 \times T.num - T.size$. Recall that for each operation, "amortize cost = actual cost + potential change". Discuss the actual costs and amortized costs of the following two cases: (a) $T.num < T.size$ and insert a new element. (b) $T.num = T.size$ and insert a new element.

9. (3+7%) (a) Describe the property of prefix codes. (b) Show that in a Huffman code, the two characters having the lowest frequencies are at the bottom of the tree.

10. (2+3%) What is a priority queue? What data structures are suitable for implementing a priority queue?

11. (10%) Suppose that we have a set of activities to schedule among a large number of lecture halls. We wish to schedule all the activities using as few lecture halls as possible. Give an efficient greedy algorithm to determine which activity should use which lecture hall.