

Mid-Term Examination

Computer Algorithms

Department of Information Management & Finance

National Chiao Tung University

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ACABD

CAABDB

ACCDDBA

✓ 1. (5 pts) Describe the key differences between dynamic programming and greedy method.

DP: overlapping subproblem, 前面解完的子問題會影響後面自給部分

✓ 2. (15 pts) (a) Prove that for the Activity Selection problem, there exists an optimal solution that selects the job with the earliest finish time. (b) Each activity j is now assigned a value v_j , and we want to select from among the activities to maximize the total value earned. Design your greedy algorithm. The algorithm could be correct or incorrect. Prove it or disprove it by a counter-example. (b) max {選它 or 不選它}

✓ 3. (10 pts) Given three sequences $X = (x_1, \dots, x_l)$, $Y = (y_1, \dots, y_m)$, and $Z = (z_1, \dots, z_n)$. Design a dynamic programming algorithm for finding the longest common subsequence of X , Y , and Z . Analyse the run time of your algorithm. 遞迴, 動態, 目標

✓ 4. (6 pts) Let $T(n)$ be the number of moves required for solving the Tower of Hanoi with n discs. Give a recurrence formula of $T(n)$ and solve it.

✓ 5. (9 pts) Use the master method to give tight asymptotic bounds for the following recurrences. (a) $T(n) = 2T(n/2) + n$; (b) $T(n) = 2T(n/4) + \sqrt{n}$; (c) $T(n) = 2T(n/4) + n \log n$.

$n \log n$
 n^2

✓ 6. (9 pts) For each of functions $f_1(n) = 2n^4 - 32n^3 + 125n + 6$, $f_2(n) = n^3$, and $f_3(n) = n \log n + 7n - 9$, find from the following asymptotic notations that match the function:

(a) $O(n \log n)$, (b) $O(n^2)$, (c) $(n^2 \log n)$, (d) $O(n^3)$, (e) $O(n^4)$,
(f) $\Theta(n \log n)$, (g) $\Theta(n^2)$, (h) $\Theta(n^2 \log n)$, (i) $\Theta(n^3)$, (j) $\Theta(n^4)$,
(k) $\Omega(n \log n)$, (l) $\Omega(n^2)$, (m) $\Omega(n^2 \log n)$, (n) $\Omega(n^3)$, (o) $\Omega(n^4)$.

polynomial

✓ 7. (10 pts) Prove that Kruskal's algorithm correctly finds a minimum spanning tree of a weighted graph. 已知解和 Kruskal 做出來的解差別在哪? 用 Kruskal 做可能不會更差, 更文字

✓ 8. (6 pts) Analyze the run time of Prim's algorithm. The run time will depend on the data structures you'd use.

$V \in \log E$

✓ 9. (3+2+5 pts) Give the worst-case run time required by each of the following operations (a) Construct a min-heap out from an array of n integers; (b) Delete the minimum integer from

$O(n)$

$O(\log n)$

$O(n \log n)$

the min-heap of (a) with its minimality property maintained. (c) Use the min-heap to sort the n integers in ascending order.

- ✓ 10. (3+7 pts) Prove that "If a directed weighted graph contains a negative weight cycle reachable from the source node, then Bellman-Ford's algorithm will report FALSE". Hint: The test on each edge (u, v) is $d[v] > d[u] + w(u, v)$.
 → 不需要多餘的空間去做儲存
- ✓ 11. (2+2 pts) (a) Explain the "in-place" property of sorting algorithms. (b) Which of HEAP SORT, QUICK SORT, MERGE SORT, and INSERTION SORT are in-place?
 → 所有 data 都有 priority ② queue 原本沒有排好但要 output 時要能依照順序 output
- ✓ 12. (2+3 pts) (a) What is a priority queue? (b) What data structures will you choose for implementing a priority queue.
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1. (8+8+6+8 pts) Given a directed, weighted graph $G = (V, E)$, we want to find all-pairs shortest paths. (a) Define $f_{ij}^{(k)}$ as the minimum weight of any path from vertex i to vertex j that contains at most k edges. Write a recursive formulation to find $f_{ij}^{(k)}$. (b) Define Let $g_{ij}^{(k)}$ be the minimum weight of any path from vertex i to vertex j for which all intermediate vertices belong to the set $\{1, 2, \dots, k\}$. Write a recursive formulation to find $g_{ij}^{(k)}$. (c) Give the lowest possible run times of the two dynamic programs of (a) and (b). (d) Describe the causes of the difference between the two run times.
- 20 ✓ 2. (8+12 pts) (a) In a binary search tree, how to find the immediate successor of node z ? (b) Describe the purpose of the instructions from Step 6 to Step 9.

TREE-DELETE(T, z)

```

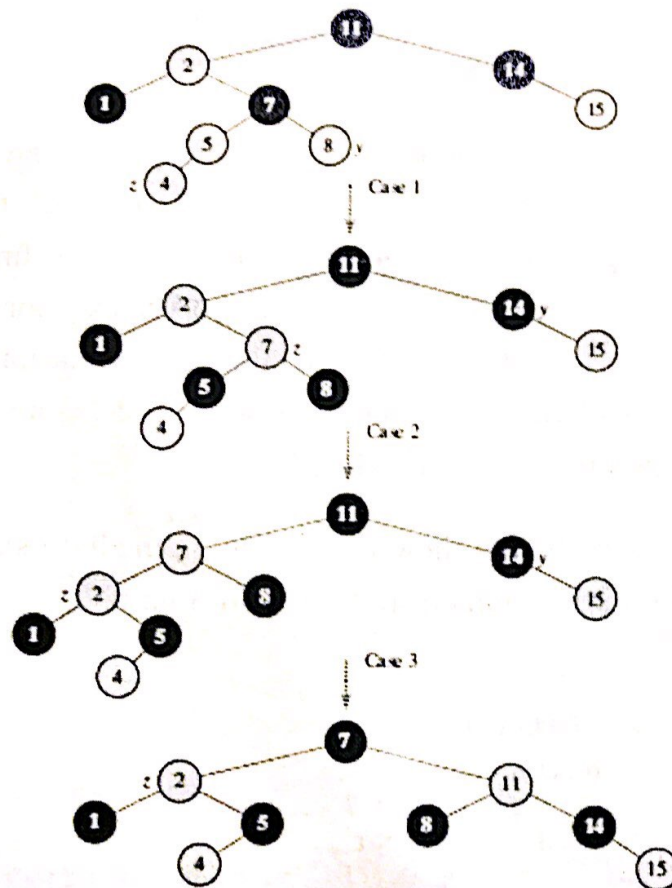
1  if  $z.left == \text{NIL}$ 
2      TRANSPLANT( $T, z, z.right$ )
3  elseif  $z.right == \text{NIL}$ 
4      TRANSPLANT( $T, z, z.left$ )
5  else  $y = \text{TREE-MINIMUM}(z.right)$ 
6      if  $y.p \neq z$ 
7          TRANSPLANT( $T, y, y.right$ )
8           $y.right = z.right$ 
9           $y.right.p = y$ 
10         TRANSPLANT( $T, z, y$ )
11          $y.left = z.left$ 
12          $y.left.p = y$ 
    
```



$$\begin{aligned}
 h &< 2 \lg n + 1 \\
 \frac{h}{2} &\lg n + 1 \\
 2^{\frac{h}{2} - 1} &< n + 1
 \end{aligned}$$

- 15 3. (7+8+5 pts) (a) Describe the red-black properties that a red-black tree must satisfy. (b) A red-black tree is guaranteed to be *approximately balanced*. What does “approximately balanced” mean? (c) Describe the advantages of red-black trees over binary search trees.
- 10 4. (10 pts) Show that a red-black tree with n internal nodes has height at most $2 \lg(n + 1)$.
- 10 5. (5+5 pts) Describe double hashing used in open addressing. What are the major advantages of double hashing.

6. (10 pts) Describe the purposes for handling Case 1 and Case 2 in a red-black tree.



*** End of Test ***