

**Calculation (70%)**

Please answer and calculate the following questions in detail. If you use some notations please define them clearly.

1. Will Wisp will live for exactly two periods. His utility function is  $U(c_1, c_2) = c_1 c_2$ , where  $c_1$  is consumption in period 1 and  $c_2$  is consumption in period 2. He will have no income in period 2. His income in period 1 is \$80,000. If the interest rate rises from 10 to 12%, what are the effects on his savings and  $c_2$ ? Calculate in detail. (5%)

0.02

$$\frac{1}{2} \times \frac{2}{1.1} \times 80000$$

$$\frac{1.12}{1.1} \times 80000$$

2. Rollo would love to have a Mercedes. His preferences for consumption in the next year are represented by a utility function  $U(x, y)$ , where  $x = 0$  if he has no Mercedes and  $x = 1$  if he has a Mercedes for the year and where  $y$  is the amount of income he has left to spend on other stuff. If  $U(0, y) = \text{the square root of } y$  and  $U(1, y) = (10/9)(y^{0.5})$  and if Rollo's income is \$50,000 a year, how much would he be willing to pay per year to have a Mercedes? Calculate in detail. (5%)

3. In a certain kingdom, the demand function for rye bread was  $q = 381 - 3p$  and the supply function was  $q = 5 + 7p$ , where  $p$  is the price in zlotys and  $q$  is loaves of bread. The king made it illegal to sell rye bread for a price above 32 zlotys per loaf. To avoid shortages, he agreed to pay bakers enough of a subsidy for each loaf of bread so as to make supply equal demand. How much would the subsidy per loaf have to be? Calculate in detail. (5%)

$$\frac{32}{1.1} \times 7$$

4. Suppose that Ms. Lynch can make up her portfolio using a risk-free asset that offers a surefire rate of return of 5% and a risky asset with an expected rate of return of 10%, with standard deviation 5%. If she chooses a portfolio with an expected rate of return of 6.25%, what is the standard deviation of her return on this portfolio? Calculate in detail. (5%)

5. You have discovered a wine. Wine drinkers are willing to pay 45 dollars to drink it right now. The amount that wine drinkers are willing to pay will rise by 15 dollars each year that the wine ages. The interest rate is 10%. In how many years will you sell it? Calculate in detail. (5%)

6. Billy Pigskin from your workbook has a von Neumann-Morgenstern utility function  $U(c) = c^{1/2}$ . If Billy is not injured this season, he will receive an income of \$25 million. If he is injured, his income will be only \$10,000. The probability that he will be injured is 0.1 and the probability that he will not be injured is 0.9. What is his expected utility? Calculate in detail. (5%)

7. Albin has quasilinear preferences and he loves pretzels. His inverse demand function for pretzels is  $p(x) = 49 - 6x$ , where  $x$  is the number of pretzels that he consumes. He is currently consuming 8 pretzels at a price of \$1 per pretzel. If the price of pretzels rises to \$7 per pretzel, what is the change in Albin's consumer surplus? Calculate in detail. (5%)



8. Shivers's annual fuel bill for home heating is 800 dollars per year. He considers three alternative plans for insulating his house. Plan A would reduce his annual fuel bill by 15%, plan B would reduce it by 20%, and plan C would eliminate his need for heating fuel altogether. The plan A insulation job would cost Shivers 800 dollars, plan B would cost him 1,100 dollars, and plan C would cost him 8,800 dollars. If the interest rate is 10% and his house and the insulation job last forever, which plan is the best for Shivers? Calculate in detail. (5%)

9. Fenner Smith is contemplating dividing his portfolio between two assets, a risky asset that has an expected return of 30% and a standard deviation of 10%, and a safe asset that has an expected return of 10% and a standard deviation of 0%.

(a) What is the budget line for Mr. Smith? Calculate in detail. (5%)

(b) If Mr. Smith's utility function is  $U(r_x, \sigma_x) = \min\{r_x, 30 - 2\sigma_x\}$ , what are Smith's optimal choices of  $r_x$  and  $\sigma_x$ ? Calculate in detail. (5%)

10. Suppose that a consumer has a utility function  $U(x_1, x_2) = \sqrt{x_1 x_2}$ . He originally faces prices (1, 1) and has income 100. Then the price of good 1 increases to 2. What are the compensating and equivalent variations? Calculate in detail. (Given that  $\sqrt{2} = 1.4$ ) (10%)

11. Tom Cruiser's car is worth \$100,000. But Tom is careless and leaves the top down and the keys in the ignition. Consequently his car will be stolen with probability 0.5. If it is stolen, he will never get it back. Tom has \$100,000 in other wealth and his von Neumann-Morgenstern utility function for wealth is  $U(w) = \ln(w)$ . Suppose that Tom can buy \$K worth of insurance at a price of \$0.6K. How much insurance will Tom buy? Calculate in detail. (10%)

### Essay (30%)

Please answer and **explain** the following essay questions **in detail**. If you use some **notations** and/or **figures**, please **define** them **clearly**.

1. True or False. Explain in detail.

(a) If current and future consumption are both normal goods, an increase in the interest rate will necessarily cause borrowers to borrow less and cause savers to save more. (Hint: Use Slutsky equation.) (5%)

(b) If there are only two goods, a person with a quasilinear utility function will have an income elasticity of demand equal to one for some goods. (5%)

(c) If government put a quantity tax on a good, it would always generate some deadweight loss. (5%)

2. Please show that a price increase raises sellers' revenue only when the demand curve slopes negative and its price elasticity is inelastic. Explain in detail by using calculus. (5%)

3. A consumer, who is initially a lender, remains a lender even after a decline in interest rates. Is this consumer better off or worse off after the change in interest rates? If the consumer becomes a borrower after the change is he better off or worse off? Please draw diagrams to supplement your explanation. (10%)