National Chiao Tung University, College of Management

Linear Algebra (線性代數)

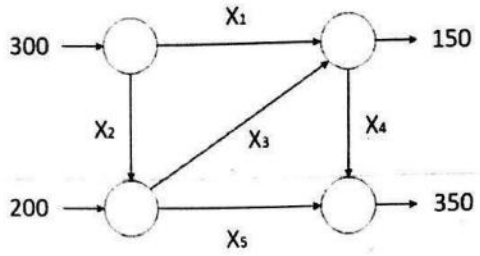
Exam 1: Ch1 – Ch3

8:10-9:50 (3/30/2018)

(Close book exam; Do not use pencils to write your answers! 不可使用鉛筆作答)

(Write the answer of each question in order! 請依照題號順序作答於答案本，否則不予計分)

1. The figure shows the flow of traffic (in vehicles per hour) through a network of streets.



1) Solve this system for x_i, i = 1,2,...,5. **(6%)**
2) Find the traffic flow when $x_2 = 200$ and $x_3 = 50$. **(2%)**
3) Find the traffic flow when $x_2 = 150$ and $x_3 = 0$. **(2%)**

2. Find all values of λ for which the homogeneous system of linear equations has nontrivial solutions. **(10%)**

$$\begin{aligned}(\lambda+2)x_1 - \quad & 2x_2 + 3x_3 = 0\\ -2x_1 + (\lambda-1)x_2 + \quad & 6x_3 = 0\\ x_1 + \quad & 2x_2 + \lambda x_3 = 0\end{aligned}$$

3. Write the column matrix b as a linear combination of the columns of A. **(10%)**

$$A = \begin{bmatrix}1 & 1 & -5\\ 1 & 0 & -1\\ 2 & -1 & -1\end{bmatrix}, \text{b}=\begin{bmatrix}3\\ 1\\ 0\end{bmatrix}$$

4. Find the inverse of the matrix (if it exists). **(10%)**

$$\begin{bmatrix}10 & 5 & -7\\ -5 & 1 & 4\\ 3 & 2 & -2\end{bmatrix}$$

5. Use an inverse matrix to solve the system of linear equations. **(10%)**

$$\begin{aligned}x_1 + \ x_2 - 2x_3 &= -1\\ x_1 - 2x_2 + \ x_3 &= \ 2\\ x_1 - \ x_2 - \ x_3 &= \ 0\end{aligned}$$

(There are still questions on the flip side. 背面尚有試題)

6. Find a sequence of elementary matrices whose product is the given nonsingular matrix. **(10%)**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$$

7. Two movie theatres that show several different movies each night compete for the same audience. Of the people who attend Theatre A one night, 10% will attend again the next night and 5% will attend Theatre B the next night. Of the people who attend Theatre B one night, 8% will attend again the next night and 6% will attend Theatre A the next night. Of the people who attend neither theatre one night, 3% will attend Theatre A the next night and 4% will attend Theatre B the next night. Find the steady state matrix for this situation. **(10%)**

8. Use an LU-factorization of the coefficient matrix to solve the linear system. **(10%)**

$$\begin{aligned} x \quad &+ z = 3 \\ 2x + \ y &+ 2z = 7 \\ 3x + 2y &+ 6z = 8 \end{aligned}$$

9. Use elementary row or column operations to find the determinant. **(10%)**

$$\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 6 & 1 & 6 \end{vmatrix}$$

10. Find an equation of the plane passing through the points. **(5%)**

$$(1, 2, 7), (4, 4, 2), (3, 3, 4)$$

11. Use Cramer's Rule to solve (if possible) the system of linear equations. **(5%)**

$$\begin{aligned} 4x - 2y + 3z &= -2 \\ 2x + 2y + 5z &= 16 \\ 8x - 5y - 2z &= \ 4 \end{aligned}$$