National Chiao Tung University, College of Management

Linear Algebra (線性代數)

Exam 2: Ch4 – Ch5

8:00-9:50 (5/11/2018)

1. Close book exam; 2. Do not use pencils to write your answers(不能用鉛筆作答).

1. Determine whether the set of vectors in P_2 is linearly independent or linearly dependent. **(5%)**

$$S = \{7 - 4x + 4x^2, 6 + 2x - 3x^2, 20 - 6x + 5x^2\}$$

2. Find (a) a basis for the column space **(5%)** and (b) the rank of the matrix. **(5%)**

$$\begin{bmatrix} 4 & 20 & 31 \\ 6 & -5 & -6 \\ 2 & -11 & -16 \end{bmatrix}$$

3. Find the nullspace of the matrix. **(5%)**

$$A = \begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ -2 & -8 & -4 & -2 \end{bmatrix} \quad Ax = 0$$

4. Determine whether the nonhomogeneous system $A\mathbf{x} = b$ is consistent. If it is, write the solution in the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$, where $\mathbf{x}_p$ is a particular solution of $A\mathbf{x} = b$ and $\mathbf{x}_h$ is a solution of $A\mathbf{x} = 0$. **(5%)**

$$x + 2y - 4z = -1$$
$$-3x - 6y + 12z = 3$$

5. Find (a) the transition matrix from B to B' **(5%)**
   (b) the transition matrix from B' to B **(5%)**
   (c) the coordinate matrix $[\mathbf{x}]_B$, given the coordinate matrix $[\mathbf{x}]_{B'}$ **(5%)**

$$B = \{(1,1,1), (1,-1,1), (0,0,1)\},$$

$$B' = \{(2,2,0), (0,1,1), (1,0,1)\},$$

$$[\mathbf{x}]_{B'} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

6. Find (a) $\langle A, B\rangle$ **(4%)**, (b) $\|A\|$ **(3%)**, and (c) $d(A, B)$ **(3%)** for the matrices in $M_{2,2}$ using the inner product $\langle A, B\rangle = 2a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + 2a_{22}b_{22}$.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

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7. Find (a) $\text{proj}_v u$ **(5%)** and (b) $\text{proj}_u v$ **(5%)** using the Euclidean inner product.

$$u = (-1, 4, -2, 3),\ v = (2, -1, 2, -1)$$

8. Find the coordinate matrix of w relative to the orthonormal basis B in R^n. **(5%)**

$$w = (2, -1, 4, 3)$$

$$B = \{\left(\frac{5}{13}, 0, \frac{12}{13}, 0\right), (0, 1, 0, 0), \left(-\frac{12}{13}, 0, \frac{5}{13}, 0\right), (0, 0, 0, 1)\}$$

9. Apply the alternative form of the Gram-Schmidt orthonormalization process to find an orthonormal basis for the solution space of the homogeneous linear system. **(10%)**

$$-x_1 + x_2 - x_3 + x_4 - x_5 = 0$$
$$2x_1 - x_2 + 2x_3 - x_4 + 2x_5 = 0$$

10. Let $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$ be vectors in P_2 with $<p, q> = a_0b_0 + a_1b_1 + a_2b_2$. Determine whether the polynomials form an orthonormal set, and if not, apply the Gram-Schmidt orthonormalization process to form an orthonormal set. **(10%)**

$$\{x^2, 2x + x^2, 1 + 2x + x^2\}$$

11. Find the projection of the vector v onto the subspace S. **(5%)**

△

$$S = \left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}\right\}, v = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

12. Find the least squares solution of the system $A\mathbf{x}=b$. **(10%)**

$$A = \begin{bmatrix}1 & -1 & 1\\1 & 1 & 1\\0 & 1 & 1\\1 & 0 & 1\end{bmatrix} \quad b = \begin{bmatrix}2\\1\\0\\2\end{bmatrix}$$

$$\begin{bmatrix}\frac{53}{30}\\ \frac{-3}{10}\\ \frac{-1}{10}\end{bmatrix}$$

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