2008/11/10

Statistics I
Midterm Exam

1. (10%) For 40,000 flips of a balanced coin, the probability is at least 0.96 that the proportion of heads will fall between _____ and_____. (Detailed calculation is required)

2. (15%) In the inspection of tin plate produced by a continuous electrolytic process (Poisson process), 0.3 imperfection is spotted per minute, on average. Find the probabilities of spotting (a) one imperfection in 3 minutes; (b) at least two imperfection in 5 minutes; (c) at most one imperfection in 10 minutes.

3. (10%) Let T have an exponential distribution with mean $\beta > 0$.
Show that $P(T > t+s \mid T > t) = P(T > s)$.

4. (10%) A shipment of 120 burglar alarms contains 5 that are defective. If 3 of these alarms are randomly selected and shipped to a customer, find the probability that the customer will get one bad unit by using (a) the formula for the hypergeometric distribution; (b) the formula for the binomial distribution as an approximation.

5. (10%) The average grade for an exam is 74, and the standard deviation is 7. Suppose that the grades follow a normal distribution. (a) If 12% of the class are given A's, what is the lowest grade for A? (b) If the grade for John is 80, what is the proportion of the grades of all students that fall below his grade?

6. (20%) Suppose that the joint probability density function (pdf) of X and Y is given by:

$$f(x,y) = \begin{cases} \frac{6}{5}(x+y^2) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

(a) Verify that $f(x,y)$ is a pdf. (b) Find the marginal pdf of X, f_X.
(c) Find $P(Y \le 0.5 | X = 0.8)$. (d) Find $E(Y | X = 0.8)$.

7. (15%) Prove the following statements.
(a) If X and Y are independent random variables, then $Cov(X,Y) = 0$. Cov 定義 -3, linear 1-
(b) Suppose that X and Y are independent random variables with variances $Var(X) = 2$ and $Var(Y) = 3$. Find the variance of $Z = 3X - 2Y + 5$. 30. 式子 3分, ans -3. 證見 -3
(c) Suppose the random variables X and Y both have uniform(0,1) distributions, and are independent. Calculate $E(X-Y)$ and $Var(X-Y)$. 2. < 式 1 ans 1 3. < 式 2 ans 1

8. (10%) Several important distributions are: Bernoulli, binomial, geometric, Poisson, hypergeometric, negative binomial, and normal. What would be the most reasonable guess for the distribution in each of the following situations (give the simplest of the above that seems appropriate or specify "none of the above" if none of them fit)? (a) The number of fish caught during the first two hours of fishing. (b) The number of fish until a rainbow trout is caught. (c) Whether or not the third fish caught is a rainbow trout. (d) The number of rainbow trout among the 5 fish caught. (e) Suppose 10 fish are caught, in a randomly selected sample of 3 fish, the number of rainbow trout.