Calculus 0320 Quiz I

Name : ID :

2017/10/13 by Y.C.J.

Do not use L'Hospital's Rule , but you can use the known property: $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=1$

1. (6%)Find the **largest** δ such that if $0<|x-1|<\delta$ then $|f(x)-1|<0.1$, where

$$f(x)=\begin{cases}\sqrt{x}, & x\geq 1\\ \sqrt[3]{x}, & x<1\end{cases}$$

2. (5%)Simplify the function $\cos(\tan^{-1}(x))$ in x.
3. (6%×4)Evaluate:

$$\lim_{x\to\infty}\frac{\sin(x)}{x} \qquad \lim_{x\to 1}\frac{[\![x]\!]}{x}$$

$$\lim_{x\to\frac{\pi}{2}^{+}} e^{\tan(x)}$$

$$\lim_{x\to\frac{2\pi}{3}}\csc^{-1}\left(\frac{3}{2\pi}x+\frac{\sqrt{3}}{2}\cot(x)\right)$$

4. (16%)Find all the vertical/horizontal asymptotes of $\frac{(\sqrt{x^2+9}-3)(x-2)}{x(x-1)}$.
5. (6%)Determine the continuity of $g(x)$ on its domain **with your proof**:

$$g(x)=\begin{cases}x, & x\in \mathrm{Q}\\ 0, & x\in \mathrm{R}-\mathrm{Q}\end{cases}$$

BONUS

6. (8%)Prove that $\lim_{x\to\frac{1}{2}}\frac{1}{x}=2$ by $\varepsilon-\delta$ definition.
7. (6%)Choose **one** to evaluate:

$$\lim_{x\to 0} x\left[\!\!\left[\frac{1}{x}\right]\!\!\right] \qquad \lim_{x\to 0}\frac{\sqrt{x}}{\tan^{-1}(x)} \qquad \lim_{x\to\infty} x\left[\!\!\left[\frac{1}{x}\right]\!\!\right]$$

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Calculus I-0320 Quiz II

2017/10/27 by Y. C. J.

Name：　　　　　　　　　ID：

1. (6%)Find the values of a and b to make f differentiable everywhere.

$$f(x)=\begin{cases} b\sin x - a\cos x, & x\geq 0 \\ x^2-2ax+b-3, & x<0 \end{cases}$$

2. (6%)Determine its differentiability at $x=0$, **Show your proof:**

$$f(x)=\begin{cases} x^2\cos\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$$

3. (5%)Evaluate:

$$\lim_{x\to 0}\frac{\cos^{-1}(x)-\frac{\pi}{2}}{x}$$

4. (25%)Find $y'(x)$ of the following functions:

$y=\sqrt{x^3-\sqrt{x^2+\sqrt{x}}}$　　　$y=\csc^{-1}(x)$　　　$y=\cot^{-1}\left(\frac{e^x}{x}\right)$

$y=2^{\cot(x)}$　　　(e) $xy^2-3x^2y=\sec(xy)$

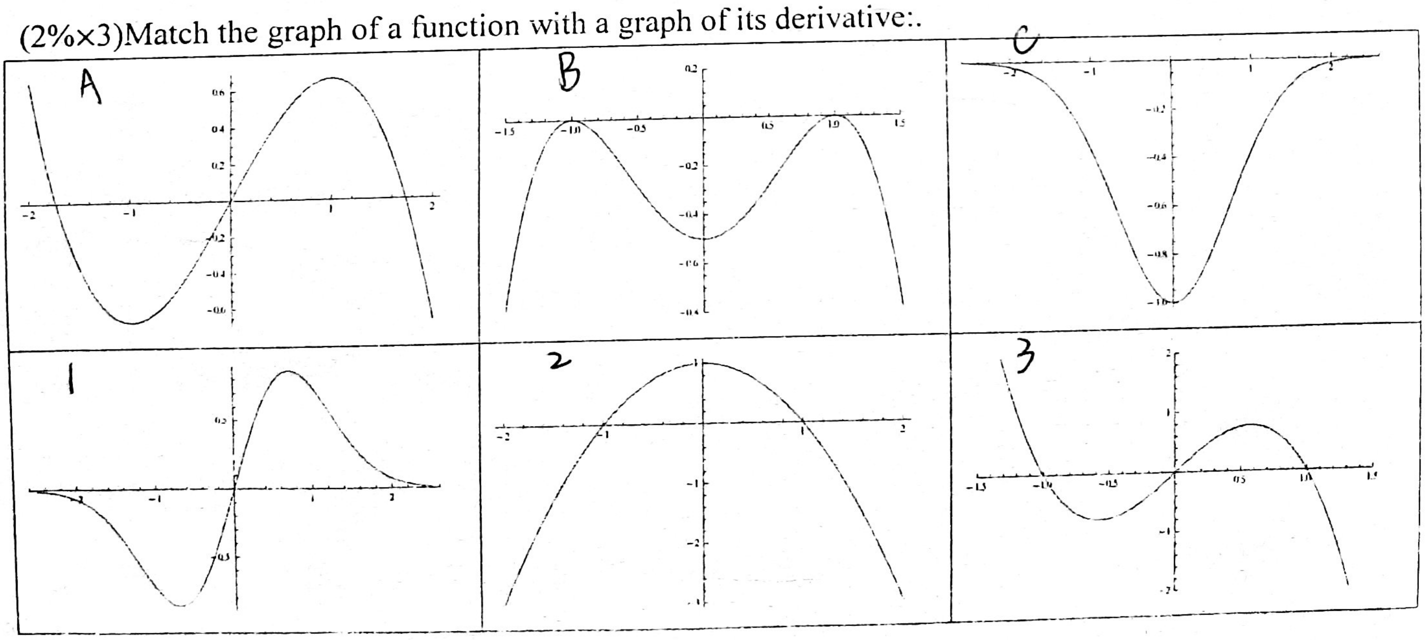
5. (5%)Find the second derivative $y''(x)$ for Question4(e)

6. (12%)Let $r(x)=f(g(h(x)))$, $w(x)=\frac{h(x)}{f(x)}$, find $r'(1)$, $w'(1)$, $(g^{-1})'(3)$.

x	1	2	3
$h(x)$	2	3	1
$g(x)$	1	3	2
$f(x)$	9	4	-5

x	1	2	3
$h'(x)$	8	0.5	-2
$g'(x)$	6	3	5
$f'(x)$	7	-9	4

7. (2%×3)Match the graph of a function with a graph of its derivative:.



BONUS

8. (6%)In the Question 2, how about the continuity of $f'(x)$ at $x=0$?

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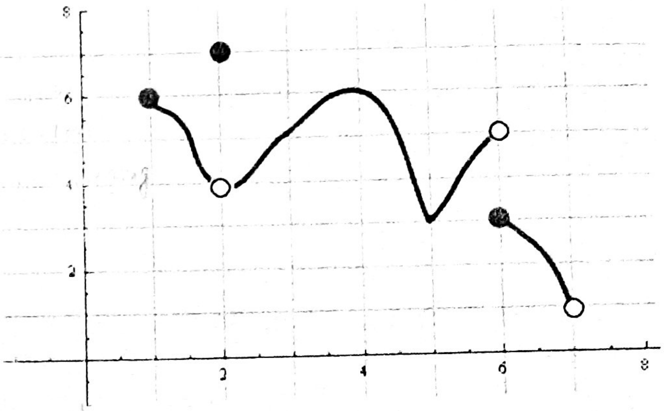
Calculus I 0320 Quiz 3

Name :   student ID :   2017/11/24 by Y. C. J.

sin $\frac{1}{\sqrt{1-x^2}}$

tan $\frac{1}{x^2+1}$

sec $\frac{1}{x\sqrt{x^2-1}}$



1. (6%)Find f' for $f(x) = (\ln(x))^{\frac{1}{\sqrt{x}}}$.
2. (6%)On [1,7], state where the absolute/local maxi/minimum are and their values.
3. (6%)Estimate $\cot^{-1}(1.1)$ by linear approximation.在1做
4. (6%×3)Choose **3** to calculate:

   $\lim_{x\to 1}\left(\frac{1}{\ln x}+\frac{x}{1-x}\right)$ $\quad\quad$ $\lim_{x\to 0^+} x^{\sqrt{x}}$ $\quad\quad$ $\lim_{x\to 0^-} xe^{\frac{1}{x^2}}$ $\quad\quad$ $\lim_{n\to\infty}\left(1+\frac{2}{n}+\frac{5}{n^2}\right)^n$

5. (6%)For $f(x) = \frac{\sqrt{e^x+x^2}}{1-e^x}$, find its all Asymptotes(漸進線)(horizontal/vertical/slant).
6. (16%)For $g(x) = \frac{x^3}{1-x^2}$, find all of the critical, local/absolute maxi/minimum and inflection points with Asymptotes(3 types) and intervals' concavities. Then sketch its graph ($\sqrt{3} \approx 1.732$)
7. (6%)Show that $|\ln(a) - \ln(b)| < |a - b|,\ \forall a, b > 1$.

   Hint: Mean Value Theorem

BONUS

8. (6%)Extra one in Problem 4.

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Calculus I 0320 Quiz 4

Name : ID : $\frac{1}{\sqrt{u}} = u^{-\frac{1}{2}}$ 2017/12/15 by Y. C. J

1. (44%)Choose 7 to evaluate:

(6%)$\int_{-2}^{3} |x^2 - 1| \, dx$ (6%)$\int_{2}^{3} \frac{x}{\sqrt{x-1}} \, dx$ (6%)$\int_{-1}^{1} x^{11} \cos(x) \, dx$

(6%)$\int \frac{1}{\sqrt{-2x-x^2}} \, dx$ (6%)$\int \sqrt{1 - \sin(x)} \, dx$ (8%)$\int \frac{x^2}{(x^2+1)^2} \, dx$

(6%)$\int \csc^{-1}(x) \, dx$ (8%)$\int \frac{\sin^2 x}{\cos^3 x} \, dx$ (8%)$\int \sqrt{\frac{1+x}{2-x}} \, dx$

2. (6%)Evaluate the limit of sum: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n\sqrt{i^2+n^2}}$? Hint: Riemann sum

3. (5%+6%)Let $f(x) = \int_{x}^{\sqrt{x}} \sin(t^2) \, dt, x > 0$, find $f'(x)$ and $(f^{-1})'(0)$.

4. (6%)Find the average value of $f(t) = \ln(x)$ on the interval [2,3].

$\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \, f(x_i)$

$= \int f(x) \, dx$

BONUS

5. (5%)Continued from Problem 3, find $\lim_{x\to 0^+} \frac{f(x)}{\sqrt{x^3}}$.

6. Choose extra **1** in the Problem 1.

$u = x^2+1$ $\sqrt{u-1} = x$ $\frac{1}{2\sqrt{u-1}} du = dx$

$f(x) = \sin x$ $f(t^2) = \sin(t^2)$ $x = t^2$

$f'(t^2) = \sin(t^2)$

$\int \sqrt{\frac{1+x}{1-x}}$

$\int \frac{1}{(x^2+1)^2} dx$

$= \int \frac{1}{u \, 2\sqrt{u-1}} du$

$= \int \frac{1}{2u\sqrt{u-1}} du$

$\int_{t}^{t^2} \sin(t^2) dt$

$f'(x^2)$

$u = \frac{\sqrt{1+x}}{\sqrt{2-x}}$

$u^2 = \frac{1+x}{2-x}$ $u^2(2-x)-1 = x$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}, \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$\int \tan x \, dx = \ln|\sec x| + c$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

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$\sin^2 x + \cos^2 x = 1$

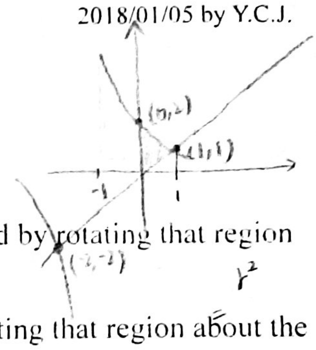
$\tan^2 + 1 = \sec^2 x$

Calculus I 0320 Quiz5

Name : ID : 2018/01/05 by Y.C.J.

$1+x=\frac{2}{x}$ x^2 $1+x=\frac{2}{x}$ 第一象限

1. One region is bounded by $y=\frac{2}{1+x}$, $y=x$ and $x=0$ in the first quadrant.



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$x=\frac{2}{1+x}$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x(1+x)-2=0$ $x=-2,1$ $(-2,-2)(1,1)$

i. (3%)Find the area of this region.

ii. (2+4%)Use the disk (washer) method to calculate the solid volume generated by rotating that region about y-Axis.

$\int_a^b r^2\pi\,dy$ r^2

iii. (3+4%)Use the shell method to calculate the solid volume generated by rotating that region about the $x=1$.

$(1,\frac{9}{16})$ $(2,\frac{9}{8})$ shell $\frac{1}{2}x^{-1}$

2. (2+3%+2+3%)For a curve: $y=\frac{x^4}{16}+\frac{1}{2x^2}$, $1\le x\le 2$. Find its arc length and surface area obtained by rotating it about y-axis.

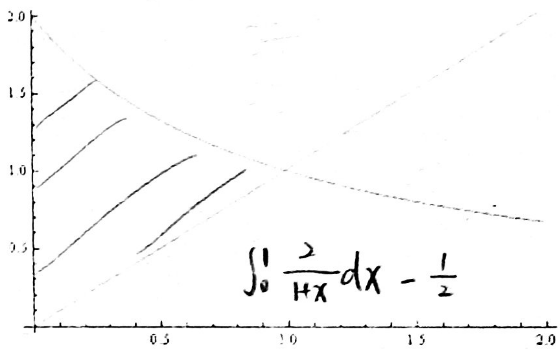
$\frac{1}{2}x^{-2}=-1x^{-3}$ $\frac{1}{16}+\frac{1}{2}=\frac{1+8}{16}$ $1+\frac{1}{8}$ $\int_a^b$ x

3. Do the following for the curve: $x(t)=\ln(t+1)-t,\ y(t)=2t^3-3t^2$

x^5

i. (5%)Its $\frac{d^2y}{dx^2}$.

ii. (4%×3)Find all its horizontal and vertical tangent points.



$\int_0^1 \frac{2}{1+x}dx-\frac{1}{2}$

$=\ln|1+x|-\frac{1}{2}$

4. (6%×3)Evaluate:

$$\int_2^{\infty}\frac{1}{x\sqrt{\ln x}}dx \qquad \int_{-2}^{2}\frac{e^{x^2}}{\sqrt[5]{x}}dx \qquad \int_1^{\infty}\frac{\ln x}{x^2}dx$$

5. (6%)Choose **one** to determine its convergence. Give your proof.

積不動 用比較

$$\int_1^{\infty}\frac{x^{1.5}}{1+x^2}dx \qquad \int_0^1\frac{-\ln x}{1+x^2}dx \qquad \int_0^1\frac{(\ln x)^4}{\sqrt{x}}dx$$

$u=\ln x$. $du=\frac{1}{x}dx$ $(\ln x)^4$

$dv=\frac{1}{1+x^2}dx$

BONUS

6. (10%)To determine the range of p in order to converge: $\int_1^{\infty}\frac{1}{x[\ln(1+x^3)]^p}dx$

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