Mathematical Statistics, Exam 3. December 31, 2019

1. (10%) Find percentiles of the following distributions.
   (a) (5%) If X follow a χ^2_{10} distribution. Find t_0 such that $P(X < t_0) = 0.10$.
   (b) (5%) If X follow an $F_{3,10}$ distribution. Find t_0 such that $P(X < t_0) = 0.95$.
2. (10%) Distributions derived from normal distributions.
   (a) (5%) Show that if $X \sim F_{n,m}$, then $X^{-1} \sim F_{m,n}$.
   (b) (5%) Show that if $T \sim t_n$, then $T^2 \sim F_{1,n}$.
3. (10%) Suppose that an i.i.d. sample of size 13 from a normal distribution gives $\bar{X} = 20$ and $s^2 = 16$.
   (a) (5%) Find the 90% confidence interval for μ.
   (b) (5%) Find the 90% confidence interval for σ^2.
4. (20%) Let X be a continuous random variable with density function $f(x) = 2x$, $0 \le x \le 1$.
   (a) (10%) Find the moment-generating function of X, $M(t)$.
   (b) (10%) Verify that $E(X^2) = M''(0)$.
5. (10%) Suppose that you bet \$5 on each of a sequence of 50 independent fair games. Let X denote the random variable for the amount you win or lose each time in the game. Then,
   $$P(X = -5) = P(X = 5) = 0.5.$$
   Use the central limit theorem to approximate the probability that you will lose more than \$75.
6. (10%) Suppose that you throw a coin 100 times and 75 heads show up.
   (a) Suppose the coin is fair and you throw the coin 100 times, estimate the probability of observing 75 heads or more?
   (b) Based on (a), conclude if this coin is fair or not.
7. (30%) Let $X_1, \dots, X_n$ be i.i.d. random variable with density function
   $$f(x|\theta) = (\theta + 1)x^\theta, \quad 0 \le x \le 1.$$
   (a) (5%) Find the method of moment estimate of θ.
   (b) (5%) Find the mle of θ.
   (c) (5%) Find the asymptotic variance of the mle.
   (d) (5%) Find a sufficient statistic for θ.
   (e) (10%) With the sufficient statistic you derive in (d), denoted by $T = T(X1, \dots, X_n)$, show that $f(X_1, \dots, X_n|T)$ does not depend on θ.