Final Exam of ***Computer Algorithms***
Department of Information Management & Finance
National Chiao Tung University
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1. (6%) In Order Statistics of finding the k-th smallest elements, we partition the elements into groups, each of which contains exactly 5 elements. What would happen if instead we let each group have 7 elements?

2. (5%,5%) Given a directed graph with non-negative weights on edges, we use breadth-first search equipped a priority Q for finding the shortest paths from the origin v_1 to all other vertices. Let $d[i,j]$ be the initial weight on the edge from v_i to v_j. Let δ_j denote the shortest path length from v_1 to v_j. (a) Finish the following code segment. (b) Justify "When a node is extracted from Q, its shortest path is determined and fixed."
   Step 1: Put v_1 into Q;
   Step 2: While Q is not empty do
   Step 3: Retract the first element from Q;
   Step 4: Finish the remaining part;
   ⋮

3. (5%,5%) In the chapter of MINIMUM SPANNING TREE, the concepts of cut and light edge are introduced to describe Prim's algorithm and Kruskal's algorithm. (a) Explain the use of two concepts in both algorithms. (b) Analyze the run time of Kruskal's algorithm.

4. (4%,4%,4%,4%) (a) Discuss the main considerations for adopting B-trees. (b) Given the B-tree shown in Figure 1 for $t = 3$, show the three B-tree snapshots after three operations "insert F", "insert Q", and "delete K".

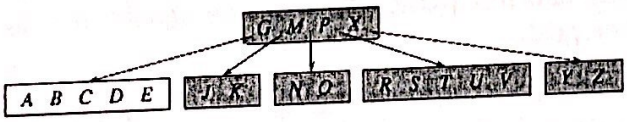

Figure 1: B-Tree

5. (8%, 6%, 3%) (a) Given the Fibonacci heap shown in Figure 2, we decrease the key value 46 as 8 and then decrease 35 as 10. Show the resulting heaps. (b) The number of root nodes in the root list increases due to some operations, like decreasing key values as in (a) or inserting new elements. By which operation(s), the number of root nodes will decrease? By which mechanism? (c) Define potential function for Fibonacci heap H by $\Phi(H) = t(H) + 2 \times m(H)$ = number of trees $+ 2 \times$ number of marked nodes. What is the potential change after uniting two heaps?

6. (5%, 4%) Bellman-Ford Algorithm shown below can detect if a given graph contains a negative-weight cycle or not. Fill up the missing part (Figure 3) for detecting the existence of negative cycles and explain why a negative-weight cycle can be successfully identified?

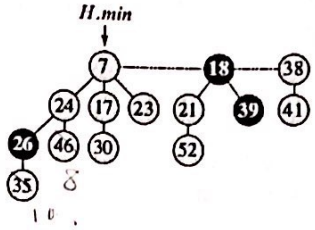

Figure 2: Fibonacci Heaps

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G, s)
    for i = 1 to |G.V| − 1
        for each edge (u, v) ∈ G.E
            RELAX(u, v, w)

    *** To be filled ***

    return TRUE
```

Figure 3: Bellman-Ford Algorithm

7. (10%) In a directed graph, $d[i, j]$ indicates the given non-negative weight from node v_i to node v_j. Let $A^{(k)}[i, j]$ indicate the shortest path distance from node v_i to node v_j subject to the condition that any intermediate node on the path has an index no greater than k. Write the recurrence relation for finding all pairs shortest paths. Your answer should include boundary conditions, recurrence formula, and the goal.

8. (4%, 8%) (a) Advantages of Red-Black trees mainly rest on "balanced heights" from the root to all leaves. Which property of Red-Black trees guarantees the balancedness? (b) Number 4 is added into as shown as node z (Figure 4). Derive the trees leading to a final balanced Red-Black tree.

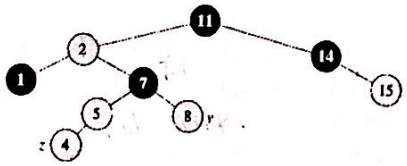

Figure 4: Red-Black Tree

9. (10%) In the binary search tree of Figure 5, the immediate successor of node 20 is node 24, and the immediate successor of node 10 is node 11. Write pseudo-codes for finding the immediate successor of node v in a binary search tree T.

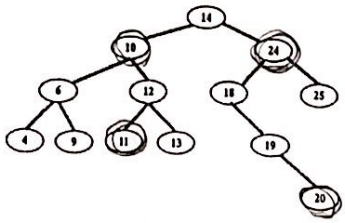

Figure 5: Binary Search Tree

10. (5%) Please provide your suggestions to the class of *Computer Algorithms*.

*** End of Test ***
*** Have a Happy Winter Break! ***