NATIONAL CHIAO TUNG UNIVERSITY
COLLEGE OF MANAGEMENT

MIDTERM EXAMINATION (I) FOR

Operations Research (II)

(March 26th, 2019)
10:10am~noon
Time Allowed: 110 minutes

INSTRUCTIONS TO CANDIDATES:

1. 本次測驗計有六題，共四頁．最後頁頁尾標有“END OF PAPER”記號．如有缺頁，請立刻提出更換試卷．
   This question paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages. The end of last page is marked with “END OF PAPER”. If there is any page missing, please ask for replacement as soon as possible.

2. 回答所有問題，總分為一百分．
   Answer **ALL** questions. The total mark for this question paper is **100**.

3. 清楚標明題號，並依序在答題本上作答．
   Sequentially write down your answers in the **answer book** and clearly mark the question number.

4. 作答於試卷紙上部分，不予計分．
   Any answer written on this question paper will **NOT** be graded.

5. 本測驗採部份給分，請以指定的方法作答，並列出完整的計算過程．
   Follow the instruction and show **ALL** your work. Partial credits may be given.

6. 關書測驗，不允許使用計算機、手機、字典、電子字典．
   Closed book Exam. Calculator, mobile phone, smart phone, dictionary, and electronic dictionary are **NOT** allowed.

L=λW

Question 1 **[Total 17 marks]**

Given the following 6-state Markov chain with state space, $S = \{0, 1, 2, 3, 4, 5\}$, and one step transition probability matrix

$$\mathbf{P} = \begin{array}{c} \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

a) Draw the transition diagram for this Markov chain. **[2 marks]**
b) Identify the recurrent, transient, and absorbing states. **[3 marks]**
c) How many classes does this Markov chain contain? **[2 marks]**
d) Determine the period of each class. **[4 marks]**
e) Is this Markov chain irreducible? (Yes/No) Justify your answer in 2 sentences. **[3 marks]**
f) Is this Markov chain ergodic? (Yes/No) Justify your answer in 2 sentences. **[3 marks]**

Question 2 **[Total 8 marks]** **HW 29.7-2**

A video cassette recorder manufacturer is so certain of its quality control that it is offering a complete replacement warranty if a recorder fails within 2 years. Based upon compiled data, the company has noted that only 1 percent of its recorders fail during the first year, whereas 5 percent of the recorders that survive that first year will fail during the second year. The warranty does not cover the replacement recorders.

a) Formulate the evolution the status of a recorder as a Markov chain whose states include two absorption states that involve needing to honor the warranty or having the recorder survive the warranty period. Then, construct the one-step transition matrix. **[4 marks]**

b) Find the probability that the manufacturer will have to honor the warranty. **[4 marks]**

Question 3 **[Total 19 marks]**

Dave plays a game repeatedly and the result of each play is that he either wins or loses. For the next play, Dave is going to win depends upon whether he won past play and this play. If Dave has lost past play and won this play, he will win next play with probability 0.6. If the Dave has won past play and lost this play, he will win next play with probability 0.5. The game ends when Dave either wins two consecutive plays or loses two consecutive plays.

a) Definite all states. **[4 marks]**
b) Continuing from (a), does the Markovian property hold in your system? (Yes/No) Explain why. **[3 marks]**
c) Find the transition probability matrix. **[2 marks]**
d) Draw the transition diagram. Remember to label all transition probabilities. **[2 marks]**
e) Suppose Dave just lost past play and won this play. What is **the expected number of plays** that Dave can have before winning two consecutive plays? **[3 marks]**
f) Suppose Dave just lost past play and won this play. What is **the probability** that the game ends with Dave winning two consecutive plays? **[5 marks]**

Question 4 [Total 30 marks]

reason

Label each of the following statements with **TRUE** or **FALSE**. (justify your answer in no more than 2 sentences.)

a) Stationary transition probabilities (stationary property) means that $P\{X_{t+1}=j \mid X_t=i, X_{t-1}=k_{t-1}, X_{t-2}=k_{t-2},\cdots, X_0=k_0\}=P\{X_{t+1}=j \mid X_t=i\}$ for $t=0, 1,\cdots$ and every sequence $i, j, k_0, k_1,\ldots, k_{t-1}$.

b) Markovian property means that any future event is independent of the past events and depends only upon the present state. Therefore, a Markov chain with Markovian property implies that its n-step transition probabilities for $n \geq 2$ do NOT exist.

c) In a Markov chain, a state with Markovian property and stationary transition probabilities (stationary property) is said to be ergodic.

d) A Markov chain with Markovian property and stationary transition probabilities (stationary property) is said to be ergodic.

e) If a Markov chain has its steady-state probabilities existed, then this Markov chain must be irreducible and ergodic.

f) An irreducible Markov chain can have some transient state.

g) In a Markov chain, any state can only be in one of the three conditions: transient, recurrent, or absorbing.

For (h), (i), and (j), consider the following Markov chain, where $\mathbf{P}=\begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \end{array}$. Note that there exists a chance to stay in State 1 forever (never leave State 1 by leaving and reentering State 1 as infinite loops).

$0 \rightleftarrows 1$

h) State 0 is transient.

i) State 1 is absorbing.

j) This Markov chain is reducible.

$\frac{3+2+4}{4}$

Question 5 **[Total 20 marks]** **Textbook example**

Dave's photography store has the following inventory problem. The store stocks a particular type of camera that can be ordered weekly. Let D_t represents the demand for the type of cameras in week t and D_t are independent an identically distributed random variables talking on the values, 0, 1, 2, each with probability of 1/3. There are 2 cameras on hand at the beginning of week 1. ***At the end of each week, Dave will order 2 cameras from the manufacturer only if the camera is sold out.*** Otherwise, no ordering will take place. And the ordered 2 cameras will be delivered be delivered by the Monday morning of next week. Dave worries about the ordering cost and the penalty cost for unsatisfied demand. For the ordering cost, Dave pays $10 fixed shopping charge for each order and $25 for each camera. For unsatisfied demand, Dave pays $50 for each camera order that cannot be satisfied.

a) Suppose Dave just checked the inventory at the end of week and there are 2 cameras on hand. Find **the expected number of weeks** that Dave can have the camera sold out at the end of week.

b) Find Dave's long-run expected average ordering and penalty cost per week.

[hint: *Remember to show all your works, which includes the transition matrix and steady state probabilities.*]

Question 6 **[Total 6 Marks]** HW 29.5-5

Consider the following blood inventory problem facing a hospital. There is need for a rare blood type, namely, type AB, Rh negative blood. The demand D (in pints) over any 3-day period is given by $P\{D=0\}=0.25$, $P\{D=1\}=0.25$, $P\{D=2\}=0.25$, $P\{D=3\}=0.25$. Note that Suppose that there are 3 days between deliveries. The hospital proposes a policy of receiving 1 pint at each delivery and using the oldest blood first. If more blood is required than is on hand, an expensive emergency delivery is made. Blood is discarded if it is still on the shelf after 9 days. Denote the state of the system as the number of pints on hand just after a delivery. Thus, because of the discarding policy, the largest possible state is 3.

a) Define states for this Markov chain. **[2 marks]**

b) Construct the (one-step) transition matrix for this Markov chain. **[4 marks]**