$C^7_4 (2x)^4 \cdot (-5y)^3 = 35 \cdot 16 \cdot x^4 \cdot (-125)\, y^3$

Mid-Term Examination of Discrete Mathematics, Fall Semester of 2011

Dept. of Information & Finance Management, National Chiao Tung University

1. Find the coefficient of the term x^4y^3 in $(2x - 5y)^7$. (5%)

2. How many integer solutions to the equations $x_1 + x_2 + x_3 + \cdots + x_7 = 42$ for $x_1, x_2, x_3 > 0$ and $x_4, x_5, x_6, x_7 \geq 0$? (10%)

3. Prove that if 201 integers are selected from the set $A = 1, 2, 3, \ldots, 400$, then there are two integers such that one divides the other. (5%)

4. In an integer grid, from point (x, y) we are allowed to move to only point $(x+1, y)$ or point $(x, y+1)$. Find the number of paths from point $(0, 0)$ to point $(6, 6)$ subject to the constraint that no path is allowed to rise above the line $x = y$. (10%)

5. (a) A chemist who has five assistants is engaged in a research project that calls for nine compounds that must be synthesized. In how many ways can the chemist assign these syntheses to the five assistants so that each is working on at least one synthesis? (5%)
   (b) Let $S(m, n)$ be the Stirling number of the second kind. Given $S(7, 3) = 301$ and $S(7, 4) = 350$, find $S(8, 4)$. (5%)

6. (a) Describe the properties required for a relation to be an equivalence relation. (3%) (b) Same as in (a) but for a relation to be a partial order. (3%)

7. Let $A = \{x, a, b, c, d\}$. (a) How many closed binary operations f on A satisfy $f(a, b) = c$? (5%) (b) How many of the binary operations f in part (a) have x as an identity? (5%)

8. (a) What kind of trees have exactly two pendant vertices? (3%) (b) If a tree has four vertices of degree 2, one vertex of degree 3, two of degree 4, and one of degree 5, how many pendant vertices does it have? (3%) (c) Given the subset of integers $A = \{2, 3, 4, 6, 8, 12, 16, 18, 28\}$, define a partial-order relation on A and draw the corresponding Hasse Diagram. (10%)

9. What is the necessary and sufficient conditions for the existence of Eulerian circuits in a undirected connected graph? (5%)

10. Assume that symbols $a, b, \ldots, i, j$ occur with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3. Construct an optimal prefix code of these symbols. (5%)

$S(m+1, n) = S(m, n-1) + nS(m, n)$

$H^3_2 = C^4_3 = 4$

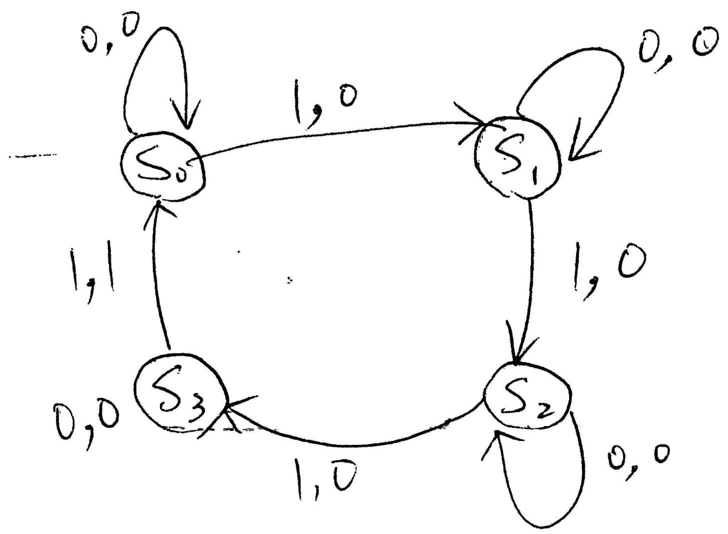
$H^3_3 = C^4_2 = 6$

11. (a) Let $G = (V, E)$ be a loop-free connected planar graph with $|V| = v, |E| = e > 2$, and r regions. Prove that $3r \leq 2e$ and $e \leq 3v - 6$ must hold. (5%) (b) Prove that K_5 and $K_{3,3}$ are non-planar. (3%+5%)

12. Describe the roles of K_5 and $K_{3,3}$ in Kuratowski's theorem for determining non-planar graphs. (5%)

13. Prove that Kruskal's algorithm produces an optimal spanning tree, if exists. (10%)

14. Describe what the finite state machine (shown below) does? (5%)



v= 4
e= 6
r=3.