A.3.2 A discrete valued time series; The polio dataset

Model description Zeger (1988) analyzed a time series of monthly numbers of poliomyelitis cases during the period 1970–1983 in the US. We make comparison to the performance of the Monte Carlo Newton-Raphson method as reported in Kuk & Cheng (1999). We adopt their model formulation.

Let y_i denote the number of polio cases in the *i*th period (i = 1, ..., 168). It is assumed that the distribution of y_i is governed by a latent stationary AR(1) process $\{u_i\}$ satisfying

$$u_i = \rho u_{i-1} + \varepsilon_i$$

where the $\varepsilon_i \sim N(0, \sigma^2)$ variables. To account for trend and seasonality the following covariate vector is introduced

$$\mathbf{x}_i = \left(1, \frac{i}{1000}, \cos\left(\frac{2\pi}{12}i\right), \sin\left(\frac{2\pi}{12}i\right), \cos\left(\frac{2\pi}{6}i\right), \sin\left(\frac{2\pi}{6}i\right)\right).$$

Conditionally on the latent process $\{u_i\}$, the counts y_i are independently Poisson distributed with intensity

$$\lambda_i = \exp(\mathbf{x}_i'\beta + u_i).$$

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Results Estimates of hyper-parameters are shown in the following table.

	β_1	β_2	β_3	β_4	β_5	β_6	ρ	σ
ADMB-RE	0.242	-3.81	0.162	-0.482	0.413	-0.0109	0.627	0.538
Std. dev.	0.270	2.76	0.150	0.160	0.130	0.1300	0.190	0.150
Kuk & Cheng (1999)	0.244	-3.82	0.162	-0.478	0.413	-0.0109	0.665	0.519

We note that not the standard deviation is large for several regression parameters. The ADMB-RE estimates (which are based on the Laplace approximation) very are very similar to the exact maximum likelihood estimates as obtained with the method of Kuk & Cheng (1999).

Files http://otter-rsch.com/admbre/examples/polio/polio.html

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