матрицей Грама линейно-независимой системы столбцов $\{\mathbf{V}_{\cdot i}\}$ в метрике $\mathbf{M}$, а значит, невырожденна. Таким образом, все выражение не может быть равным нулю. Полученное противоречие завершает доказательство.

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**Dynamics of a vehicle with omniwheels with massive rollers:**

**change of the contacting roller**

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## Abstract

We study the dynamics of a vehicle with omni-wheels moving along a horizontal plane. In this work, we consider dynamics of the rollers and suggest the model for switching of contact from one roller to another using impact theory. We compare the behavior with the simplified model of the omni-wheel as a rigid disk with a non-holonomic sliding constraint (MassLess Rollers Model – MLRM).

The dynamics of a symmetrical vehicle with $N$ omni-wheels, each carrying $n$ rollers, moving along a fixed horizontal absolutely rough plane is considered under the following assumptions: the mass of each roller is nonzero, the plane and rollers are absolutely rigid, so the contact between a supporting roller and the plane occurs in one point. The slippage is allowed only at the instant just after the change of the rollers in contact (a tangent impact).

Between the impacts, the dynamics of motion is governed by the equations in pseudovelocities. Comparing to MLRM, additional terms proportional to the axial moment of inertia of the rollers and depending on the angles of wheels' rotation appear. For free motions (without control), we analytically showed the existence of the energy first integral, cyclic linear integral for the non-supporting rollers, and slow change of the

MLRM first integral. It is shown that some MLRM motions disappear. All analytical results are confirmed by simulations. A comparison of the main types of motion for symmetric three-wheeled vehicle for MLRM and the whole model is done.

For the switching between rollers, an impact theory problem is posed and solved, impact forces and energy loss being obtained in assumption of non-elastic impact and ideal constraints. Immediately before the impact instant, only holonomic constraints are imposed on the system. After the impact, a set of differential constraints is applied. The impact problem is then formulated as a system of algebraic equations. The system admits the unique solution. We consider the impact as non-elastic in the sense that it is equivalent to projection of the vector of generalized velocities onto the plane defined by constraints in the space of virtual displacements, orthogonal in the kinetic metric. Thus, the normal part of the generalized velocities vanish, and the kinetic energy of the system decreases by the value of the kinetic energy of lost generalized velocities, in accordance to Carnot's theorem. Solutions are then obtained numerically combining both smooth parts of motion and impacts.

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