

Inequazioni in \mathbb{R}

$$(1) \sqrt{x^2 - 6x + 9} + \sqrt{y^2 + 10y + 25} \leq 0 \Leftrightarrow$$

GRUPPI TRIPLICE:

$$\underbrace{5 = \sqrt{25} = \sqrt{16+9}}_{\text{coefficient}} \neq \underbrace{\sqrt{16} + \sqrt{9} = 4+3=7}_{\text{coefficient}}$$

$$\sqrt{7} = \sqrt{16-9} \neq \sqrt{16} - \sqrt{9} = 4-3=1$$

$$\sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b} \quad \left\{ \begin{array}{l} \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \\ \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, b \neq 0 \end{array} \right.$$

$$a^2 \pm 2ab + b^2 = (a \pm b)^2$$

$$x^2 - 6x + 9 = x^2 - 2 \cdot 3 \cdot x + 3^2 = (x-3)^2$$

$$y^2 + 10y + 25 = y^2 + 2 \cdot 5 \cdot y + 5^2 = (y+5)^2$$

$$\boxed{\sqrt{x^2} = |x|}$$

$$3 = \sqrt{9} = \sqrt{(-3)^2} = |-3| = 3$$

$$(1) \Leftrightarrow \sqrt{(x-3)^2} + \sqrt{(y+5)^2} \leq 0 \Leftrightarrow$$

$$|x-3| + |y+5| \leq 0$$

$$\left\{ \begin{array}{l} |-7| = 7 > 0 \quad |0| = 0 \quad |5| = 5 > 0 \\ (*) x \in \mathbb{R} \Rightarrow |x| \geq 0 \end{array} \right.$$

$$\left. \begin{array}{l} |x-3| \geq 0, (\forall) x \in \mathbb{R} \\ |y+5| \geq 0, (\forall) y \in \mathbb{R} \\ |x-3| + |y+5| \leq 0 \end{array} \right\} \begin{array}{l} \text{POSIBIL DOAR DACĂ} \\ |x-3| = 0 \text{ și } |y+5| = 0 \\ x-3 = 0 \quad y+5 = 0 \\ x = 3 \quad y = -5 \end{array}$$

$$S = \{(3; -5)\} \quad x=3 \text{ și } y=-5$$

$$\sqrt{x^2 - 4x + 4} + \sqrt{y^2} \leq 0 \Leftrightarrow$$

$$\sqrt{x^2 - 2 \cdot 2x + 2^2} + |y| \leq 0 \Leftrightarrow$$

$$\left. \begin{array}{l} \sqrt{(x-2)^2} + |y| \leq 0 \Leftrightarrow |x-2| + |y| \leq 0 \\ |x-2| \geq 0 \\ |y| \geq 0 \end{array} \right\}$$

$$\Leftrightarrow \left. \begin{array}{l} |x-2| = 0 \quad \text{și} \quad |y| = 0 \\ x-2 = 0 \quad y = 0 \\ x = 2 \end{array} \right\} \Rightarrow \begin{cases} x = 2 \\ y = 0 \end{cases}$$

$$S = \{(2, 0)\}$$

$$\frac{3(x-1) - 2 \cdot (x+3)}{-6} < 5 \Leftrightarrow$$

$$\frac{3x-3-2x-6}{-6} < 5 \Leftrightarrow$$

$$\frac{x-9}{-6} < 5 \quad | \cdot (-6) \Leftrightarrow \frac{x-9}{\cancel{-6}} \cdot (\cancel{-6}) > 5 \cdot (-6)$$

$$x-9 > -30 \Leftrightarrow x \geq -30+9 \Leftrightarrow x \geq -21$$

$$S = [-21, +\infty)$$

$$\left. \begin{array}{l} 2x^3 - x^2 \geq 0 \Leftrightarrow x^2 \cdot (2x-1) \geq 0 \\ x^2 \geq 0, (\forall) x \in \mathbb{R} \quad \underbrace{\begin{array}{cc} + & ? \\ + & + \end{array}}_{+} \end{array} \right\} \Rightarrow$$

$$\rightarrow 2x-1 \geq 0 \Leftrightarrow 2x \geq 1 \Leftrightarrow x \geq \frac{1}{2} \Rightarrow$$

$$S = [\frac{1}{2}, +\infty)$$

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