

$$P(x) = \frac{(x^2+2x-2)^2 + 9(x^2+2x-2) + 18}{x^4+x^3-8x-8}$$

$$\text{Notam: } x^2+2x-2=t$$

$$(x^2+2x-2)^2 + 9(x^2+2x-2) + 18 = t^2 + 9t + 18 = t^2 + 3t + 6t + 18 = \\ = t(t+3) + 6(t+3) = (t+3)(t+6) = \\ = (x^2+2x-2+3) \cdot (x^2+2x-2+6) = (x^2+2x-1)(x^2+2x+4)$$

$$x^2+2x-1=0 \quad | \Rightarrow \Delta = b^2 - 4ac = 4 - 4 \cdot 1 \cdot (-1) = 8 \\ a=1 \quad b=2 \quad c=-1 \quad | \quad \sqrt{\Delta} = 2\sqrt{2} \quad \frac{2 \cdot (-1 - \sqrt{2})}{2} = -1 - \sqrt{2} \\ x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm 2\sqrt{2}}{2} = \frac{2 \cdot (-1 + \sqrt{2})}{2} = -1 + \sqrt{2}$$

$$x^2+2x-1 = (x - (-1 - \sqrt{2}))(x - (-1 + \sqrt{2})) = (x + 1 + \sqrt{2})(x + 1 - \sqrt{2})$$

$$x^2+2x+4=0 \quad | \Rightarrow \Delta = b^2 - 4ac = 4 - 4 \cdot 1 \cdot 4 = 4 - 16 = -12 < 0 \\ a=1 \quad b=2 \quad c=4 \quad | \quad x^2+2x+4 = \text{nn ne decompune in } \mathbb{R}$$

$$Q(x) =$$

$$-x^4 + x^3 - \underbrace{8x - 8}_{\text{VI}} = x^3(x+1) - 8(x+1) = (x+1)(x^3 - 8)$$

$$\left. \begin{array}{l} a^3 + b^3 = (a+b)(a^2 - ab + b^2) \\ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \end{array} \right\} \Rightarrow x^3 - 8 = (x-2)(x^2 + 2x + 4) \quad \text{VII}$$

$$P(x) = x^3 - 8 = 1 \cdot x^3 + 0 \cdot x^2 + 0 \cdot x - 8$$

$$D_8 = \{ \pm 1, \pm 2, \pm 4, \pm 8 \} \quad P(1) = 1^3 - 8 = 1 - 8 \neq 0 \\ P(-1) = (-1)^3 - 8 = -1 - 8 \neq 0$$

$$P(2) = 2^3 - 8 = 8 - 8 = 0$$

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & -8 \\ \hline 2 & 1 & 2 & 4 & 0 \end{array} \Rightarrow$$

$$P(x) = (x-2) \cdot (x^2 + 2x + 4)$$

$$Q(x) = (x+1)(x-2)(x^2 + 2x + 4)$$

f. c.  $Q(x) \neq 0 \Leftrightarrow x+1 \neq 0 \text{ und } x-2 \neq 0 \text{ und } \cancel{x^2 + 2x + 4} \neq 0$   
 $x \neq -1 \quad x \neq 2$  am dem.  
 $\rightarrow x \in \mathbb{R} \setminus \{-1; 2\}$

$$\tilde{f}(x) = \frac{(x+1+\sqrt{2})(x+1-\sqrt{2}) \cancel{(x^2 + 2x + 4)}}{(x+1)(x-2) \cancel{(x^2 + 2x + 4)}}$$