Dem. că $(x+1)^2 > x$, $(\forall) x \in \mathbb{R}$

V1

$(x+1)^2 > x \iff x^2 + 2x + 1 > x \iff x^2 + 2x + 1 - x > 0 \iff$

$x^2 + x + 1 > 0 \iff x^2 + 2 \cdot x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \frac{3}{4} > 0 \iff \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0$ (A)

deorece: $\left.\begin{array}{c} \left(x + \frac{1}{2}\right)^2 \geq 0 \\[4pt] \frac{3}{4} > 0 \end{array}\right\} \implies \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0$

$A = \left\{ x \in \mathbb{Z} \mid \dfrac{3x+4}{2x-3} \in \mathbb{Z} \right\} \implies A = ?$

$\dfrac{3x+4}{2x-3} \in \mathbb{Z} \iff (2x-3) \mid (3x+4) \implies (2x-3) \mid (3x+4) \cdot 2 \implies$

$\qquad (2x-3) \mid (2x-3) \implies (2x-3) \mid (2x-3) \cdot 3 \implies$

$\left.\begin{array}{c}(2x-3) \mid (6x+8) \\[4pt] (2x-3) \mid (6x-9) \end{array}\right\} \implies (2x-3) \mid \left[(6x+8) - (6x-9)\right] \implies (2x-3) \mid (6x+8-6x+9)$

$\implies (2x-3) \mid 17 \implies 2x-3 \in D_{17} \implies 2x-3 \in \{1, -1, 17, -17\} \implies$

$\qquad 2x \in \{4, 2, 20, -14\} \implies$

$\qquad x \in \{2, 1, 10, -7\} \implies A = \{2, 1, 10, -7\}$

$B = \left\{ x \in \mathbb{N} \mid \dfrac{7x-3}{5x+1} \in \mathbb{Z} \right\}$

$\dfrac{7x-3}{5x+1} \in \mathbb{Z} \iff (5x+1) \mid (7x-3) \implies (5x+1) \mid (7x-3) \cdot 5 \implies (5x+1) \mid (35x-15)$

$\qquad (5x+1) \mid (5x+1) \qquad (5x+1) \mid (5x+1) \cdot 7 \implies (5x+1) \mid (35x+7)$

$\implies (5x+1) \mid (35x+7-35x+15) \implies (5x+1) \mid 22 \implies 5x+1 \in D_{22}$

$5x+1 \in \{1, -1, 2, -2, 11, -11, -22, 22\} \implies 5x \in \{0, -2, 1, -3, 10, -12, -23, 21\}$

$x \in \left\{ \dfrac{0}{5}, \dfrac{-2}{5}, \dfrac{1}{5}, -\dfrac{3}{5}, \dfrac{10}{5}, \dfrac{-12}{5}, -\dfrac{23}{5}, \dfrac{21}{5} \right\} \implies B = \{0; 2\}$

$$D = \left\{ x \in \mathbb{Z} \;\middle|\; \frac{3x-1}{3x+2} \in \mathbb{N} \right\}$$

$$\frac{3x-1}{3x+2} = \frac{\boxed{3x+2} - 3}{\boxed{3x+2}} = \frac{3x+2}{3x+2} - \frac{3}{3x+2} = 1 - \frac{3}{3x+2} \in \mathbb{N} \quad \Rightarrow$$

$$(3x+2) \mid 3 \Rightarrow 3x+2 \in \{1, -1, 3, -3\} \Rightarrow 3x \in \{-1, -3, 1, -5\}$$

$$\Rightarrow x \in \left\{ -\frac{1}{3}, -1, \frac{1}{3}, -\frac{5}{3} \right\} \Rightarrow x \in \{-1\}$$
$$x \in \mathbb{Z}$$

$$x = -1 \Rightarrow \frac{3 \cdot (-1) - 1}{3 \cdot (-1) + 2} = \frac{-4}{-1} = 4$$

$$D = \{-1\}$$

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