$$A = \left\{ x \mid x \in \mathbb{N}, \; \frac{x}{2} < \frac{20}{x} < \frac{x}{1.7} \right\} \Rightarrow A = ?$$

$$\overset{I}{\frac{13}{4}} \; ? \; \overset{IV}{\frac{11}{6}} \qquad 13 \cdot 6 > 4 \cdot 11 \Rightarrow \quad \frac{13}{4} > \frac{11}{6}$$
$$\underset{III}{} \qquad \underset{II}{} \qquad \underbrace{78} \qquad \underbrace{44}$$

$$-\frac{3}{4} \; ? \; -\frac{2}{3} \qquad \text{Comparăm} \quad \frac{3}{4} \; ? \; \frac{2}{3}$$

$$\frac{3 \cdot 3}{9} > \frac{4 \cdot 2}{8} \Rightarrow \frac{3}{4} > \frac{2}{3} \Rightarrow -\frac{3}{4} < -\frac{2}{3}$$

$$x \in \mathbb{N}, \; x \neq 0 \Rightarrow \frac{x}{2} < \frac{20}{x} \Leftrightarrow x \cdot x < 2 \cdot 20 \Leftrightarrow$$

$$x^2 < 40 \Leftrightarrow \sqrt{x^2} < \sqrt{40} \Leftrightarrow |x| < \sqrt{40} \Leftrightarrow x < \sqrt{40}$$
$$\text{(deoarece } x > 0 \Rightarrow |x| = x)$$

$$40 < 49 \Rightarrow \sqrt{40} < \sqrt{49} \Rightarrow \sqrt{40} < 7 \Rightarrow \boxed{x < 7}$$

$$\overset{I}{\frac{20}{x}} < \overset{IV}{\frac{x}{1.7}} \Leftrightarrow 20 \cdot 1.7 < x \cdot x \Leftrightarrow 34 < x^2 \Leftrightarrow$$
$$\underset{III}{} \qquad \underset{II}{}$$

$$\sqrt{34} < \sqrt{x^2} \Leftrightarrow \sqrt{34} < |x| \Leftrightarrow \sqrt{34} < x$$
$$(x > 0)$$

$$25 < 34 \Leftrightarrow \sqrt{25} < \sqrt{34} \Leftrightarrow 5 < \sqrt{34} < x \Rightarrow \boxed{5 < x}$$

$$A = \{ x \in \mathbb{N} \mid 5 < x < 7 \} = \{6\}$$

OBS: Dacă n, $m \in \mathbb{N}$ și $n^2 < m < (n+1)^2 \Rightarrow$
   m nu este pătrat perfect.
Între două pătrate perfecte consecutive nu mai avem
niciun pătrat perfect (în mț. nr. naturale).

$n = 4 \Rightarrow n+1 = 5 \Rightarrow 16; \underbrace{17, 18, 19, 20, 21, 22, 23, 24,}_{\text{nu sunt p.p.}} 25$
$\qquad\qquad\qquad\quad 4^2 \qquad\qquad\qquad\qquad\qquad\qquad\quad 5^2$

Dem. că $(\forall) n \in \mathbb{N} \Rightarrow n^2 < \underline{n \cdot (n+1)} < (n+1)^2$

Dem: $n^2 = n \cdot n < n \cdot \underline{(n+1)} < (n+1) \cdot \underline{(n+1)} = (n+1)^2$
$\qquad\qquad\qquad\quad 2$
$\Rightarrow n^2 < n(n+1) < (n+1)^2$

Dem. că $(\forall) n \in \mathbb{N}^* \Rightarrow \sqrt{n^2 + n} \notin \mathbb{Q}$

$n^2 + n = n(n+1) \Rightarrow n^2 < n(n+1) < (n+1)^2 \Rightarrow$

$\Rightarrow n(n+1)$ nu $\in$ p.p. $\Rightarrow n^2 + n$ nu e p.p. $\Rightarrow$

$\underline{\sqrt{n^2 + n} \notin \mathbb{Q}}$

Dem. că $(\forall) n \in \mathbb{N} \Rightarrow \sqrt{5n + 2} \notin \mathbb{Q}$

Dacă $n \in \mathbb{N} \Rightarrow u.c. (n) \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \Rightarrow$
$\Rightarrow u.c. (n^2) \in \{0, 1, 4, 9, 6, 5\} \Rightarrow$ dacă $u.c. (m) \in \{2, 3, 7, 8\} \Rightarrow$
$\Rightarrow m$ nu e p.p.

$(\forall) m \in \mathbb{N} \Rightarrow u.c.(5m) \in \{0;5\} \Rightarrow u.c.(5m+2) \in \{2;7\}$

$\Rightarrow 5m+2$ nu e p.p. $\Rightarrow \overline{\sqrt{5m+2}} \notin \mathbb{Q}$

Efectuați:

$\mathbb{N} \cap \mathbb{Z} = \mathbb{N}$

$\mathbb{N} \cup \mathbb{Q} = \mathbb{Q}$

$\mathbb{Z} \cap (\mathbb{R} \setminus \mathbb{Q}) = \overline{\Phi}$

$\mathbb{N} \setminus \mathbb{Z} = \Phi$

$\mathbb{Q} \setminus (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{Q}$

$\mathbb{R} \setminus (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{Q}$

$\mathbb{Z} - \mathbb{N} = \{\ldots, -4, -3, -2, -1\} = \mathbb{Z}_-$

Știm că $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$\mathbb{R} = \mathbb{Q} \cup \mathring{\mathbb{I}}$

$\mathbb{I} = \mathbb{R} \setminus \mathbb{Q}$

$\mathbb{R} \setminus \mathbb{Q} \subset \mathbb{R}$

$A = \{1,2,3\}$ $B = \{2\}$ $C = \{3,4\}$ $D = \{9\}$

$A \cup B = A$

$A \cap B = B$

$A - B = \{1,3\}$

$B - A = \Phi$

$A \cup C = \{1,2,3,4\}$

$A \cap C = \{3\}$

$A - C = \{1,2\}$

$C - A = \{4\}$

$A \cup D = \{1,2,3,9\}$

$A \cap D = \Phi$ → MULȚIMI

$A - D = A$ DISJUNCTE

$D - A = D$

$A \cap B = B$

DACĂ $B \subset A \Rightarrow \begin{cases} A \cup B = A \\ B - A = \Phi \end{cases}$

...