

Aduceți la formă cea mai simplă expresiile:

$$E(x) = \frac{4x-12}{x^2-9} - \frac{x+2}{x-3} : \frac{x^2+5x+6}{x-3}$$

P.e. $x^2-9 \neq 0 \Leftrightarrow x^2-3^2 \neq 0 \Leftrightarrow (x-3)(x+3) \neq 0 \Leftrightarrow x-3 \neq 0$ și $x+3 \neq 0 \Leftrightarrow$
 $x \neq 3$ și $x \neq -3$

$$x^2+5x+6 \neq 0 \Leftrightarrow \overbrace{x^2+2x+3x+6} \neq 0 \Leftrightarrow x(x+2)+3(x+2) \neq 0 \Leftrightarrow$$

$$(x+2)(x+3) \neq 0 \Leftrightarrow \left. \begin{array}{l} x+2 \neq 0 \\ x+3 \neq 0 \end{array} \right\} \Rightarrow x \in \mathbb{R} \setminus \{-2, -3\}$$

$$E(x) = \frac{4 \cancel{(x-3)}}{\cancel{(x-3)}(x+3)} - \frac{\cancel{x+2}}{\cancel{1}} \cdot \frac{\cancel{x-3}}{\cancel{(x-2)}(x+3)} = \frac{4}{x+3} - \frac{1}{x+3} = \frac{4-1}{x+3} = \frac{3}{x+3}$$

$$F(x) = \left(\frac{x+1}{x} - \frac{x-1}{x+1} + \frac{1-x}{x^2+x} \right) : \frac{2x-4}{x^2+3x}$$

P.e. $x \neq 0$ $x+1 \neq 0 \Rightarrow x \neq -1$

$$x \neq 0, x+1 \neq 0 \Rightarrow x(x+1) \neq 0 \Rightarrow x^2+x \neq 0$$

$$\left. \begin{array}{l} 2x-4 \neq 0 \Leftrightarrow 2x \neq 4 \Leftrightarrow x \neq \frac{4}{2} \\ \Leftrightarrow x \neq 2 \\ x^2+3x \neq 0 \Leftrightarrow x(x+3) \neq 0 \\ \Leftrightarrow x \neq 0 \text{ și } x+3 \neq 0 \\ x \neq -3 \end{array} \right\}$$

Deci $x \in \mathbb{R} \setminus \{0, -1, 2, -3\} \Rightarrow x \in \mathbb{R}^* \setminus \{-1, 2, -3\}$

$$F(x) = \left(\frac{x+1}{x} - \frac{x-1}{x+1} + \frac{1-x}{x(x+1)} \right) \cdot \frac{x(x+3)}{2(x-2)} \left\{ \begin{array}{l} (x+1)(x+1) = (x+1)^2 = x^2+2x+1 \\ x \cdot (x-1) = x^2-x \end{array} \right.$$

$$F(x) = \left(\frac{x^2+2x+1}{x(x+1)} - \frac{x^2-x}{x(x+1)} + \frac{1-x}{x(x+1)} \right) \cdot \frac{x(x+3)}{2(x-2)}$$

$$F(x) = \frac{x^2+2x+1-\cancel{x^2}+\cancel{x}+1-\cancel{x}}{x(x+1)} \cdot \frac{x(x+3)}{2(x-2)} \Rightarrow F(x) = \frac{2x+2}{x(x+1)} \cdot \frac{x(x+3)}{2(x-2)}$$

$$F(x) = \frac{\cancel{x}(\cancel{x+1})}{\cancel{x+1}} \cdot \frac{x+3}{\cancel{2}(x-2)} \Rightarrow F(x) = \frac{x+3}{x-2}, x \in \mathbb{R}^* \setminus \{2, -3, -1\}$$

$$A = \{x \in \mathbb{Z} \mid F(x) \in \mathbb{Z}\} \Rightarrow A = ?$$

$$\frac{x+3}{x-2} = \frac{x-2+5}{x-2} = \frac{x-2}{x-2} + \frac{5}{x-2} = 1 + \frac{5}{x-2} \in \mathbb{Z} \quad \left| \begin{array}{l} \hookrightarrow \in \mathbb{Z} \end{array} \right. \Rightarrow \frac{5}{x-2} \in \mathbb{Z} \Rightarrow$$

$$(x-2) \mid 5 \Rightarrow x-2 \in D_5 \Leftrightarrow x-2 \in \{1, -1, 5, -5\}$$

$$\Rightarrow x \in \{3, 1, 7, -3\} \quad \left| \begin{array}{l} x \in \mathbb{N}^* \setminus \{2, -3, -1\} \end{array} \right. \Rightarrow A = \{3, 1, 7\}$$

$$\left\{ \begin{array}{l} x-2 = -1 \\ x = -1+2 \\ x = 1 \end{array} \right.$$

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