

12. reale - probleme

$a \in \mathbb{R}, b \in \mathbb{R}$

$$\min(a, b) = \begin{cases} a, & \text{dacă } a \leq b \\ b, & \text{dacă } b \leq a \end{cases}$$

$$\max(a, b) = \begin{cases} a, & \text{dacă } a \geq b \\ b, & \text{dacă } b \geq a \end{cases}$$

Ex:



$$\min(-3, -4) = -4 \quad \max(-10, -1) = -1$$

$$\min(3\sqrt{5}, 5\sqrt{3}) = ?$$

$$3\sqrt{5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{9 \cdot 5} = \sqrt{45}$$

$$5\sqrt{3} = \sqrt{25} \cdot \sqrt{3} = \sqrt{25 \cdot 3} = \sqrt{75}$$

$$45 < 75 \Rightarrow \sqrt{45} < \sqrt{75} \Rightarrow 3\sqrt{5} < 5\sqrt{3}$$

$$\min(3\sqrt{5}, 5\sqrt{3}) = 3\sqrt{5}$$

$$\max(2\sqrt{3} - 3\sqrt{2}, 4\sqrt{2} - 5\sqrt{3}) = M$$

$$2\sqrt{3} - 3\sqrt{2} < 4\sqrt{2} - 5\sqrt{3} \Leftrightarrow -3\sqrt{2} - 4\sqrt{2} < -5\sqrt{3} - 2\sqrt{3}$$

$$-7\sqrt{2} < -7\sqrt{3} \quad | : (-7) \Leftrightarrow \sqrt{2} > \sqrt{3} \quad (\text{F})$$

$$\Rightarrow 2\sqrt{3} - 3\sqrt{2} < 4\sqrt{2} - 5\sqrt{3} \quad (\text{F})$$

$$\Rightarrow 2\sqrt{3} - 3\sqrt{2} > 4\sqrt{2} - 5\sqrt{3} \quad (A)$$

$$\Rightarrow M = 2\sqrt{3} - 3\sqrt{2}$$

Înmulțirea cu -1:

$$-2 = -2 \mid \cdot (-1) \Rightarrow 2 = 2$$

$$-1 < 10 \mid \cdot (-1) \Rightarrow 1 > -10$$

$$-5 < -1 \mid \cdot (-1) \Rightarrow 5 > 1$$

(Validabil din la comparație)

$$\min\left(\frac{4}{5}; \frac{6}{7}\right) = C \Rightarrow C = ?$$

$$\begin{array}{l} \frac{4}{5} ? \frac{6}{7} \\ \hline \end{array} \quad \left. \begin{array}{l} \frac{4}{5} \\ \frac{6}{7} \end{array} \right\} \text{Înmulțim în ordinele prezente:}$$

$$\frac{4 \cdot 7}{28} < \frac{5 \cdot 6}{30} \Rightarrow \frac{4}{5} < \frac{6}{7}$$

$$C = \frac{4}{5}$$

$$\begin{array}{l} \frac{8}{7} ? \frac{10}{9} \\ \hline \end{array} \Rightarrow \frac{8 \cdot 9}{72} > \frac{7 \cdot 10}{70} \Rightarrow \frac{8}{7} > \frac{10}{9}$$

$$\begin{array}{l} -\frac{2}{3} ? \frac{4}{5} \\ \hline \end{array} \left\{ \begin{array}{l} -\frac{2}{3} < 0 \\ 0 < \frac{4}{5} \end{array} \right\} \Rightarrow -\frac{2}{3} < \frac{4}{5}$$

$$-\frac{4}{3} ? -\frac{5}{7} \left\{ \begin{array}{l} \text{Comparați: } \frac{4}{3} ? \frac{5}{7} \\ \frac{4 \cdot 7}{28} > \frac{5 \cdot 3}{15} \Rightarrow \frac{4}{3} > \frac{5}{7} \end{array} \right. \cdot (-1) \Rightarrow$$

$$\Rightarrow -\frac{4}{3} < -\frac{5}{7} \left\{ \begin{array}{l} 201 < 1997 \Rightarrow \frac{1997}{201} > 1 \\ 6 < 7 \Rightarrow \frac{6}{7} < 1 \end{array} \right\} \Rightarrow$$

$$\frac{1997}{201} ? \frac{6}{7} \Rightarrow \frac{1997}{201} > \frac{6}{7}$$

$$|x| = \begin{cases} x, & \text{dacă } x \geq 0 \\ 0, & \text{dacă } x = 0 \\ -x, & \text{dacă } x < 0 \end{cases} \quad \underline{\text{Ex:}}$$

$$|-10| = -(-10) = 10$$

$$|9| = 9$$

$$|0| = 0 \quad |x| \geq 0 \quad (\forall) x \in \mathbb{R}$$

Aflați nr. x :

$$|x| = 4 \Rightarrow x = -4 \text{ SAU } x = 4$$

$$|x| = -\sqrt{2} \Rightarrow \begin{cases} |x| \geq 0, (\forall) x \in \mathbb{R} \\ -\sqrt{2} < 0 \end{cases} \Rightarrow |x| = -\sqrt{2} \text{ (imposibil)}$$

$$\underline{\underline{x \in \emptyset}}$$

$$|x-3|=0 \Leftrightarrow x-3=0 \Leftrightarrow x=3$$

$$|x| \leq 0 \left\{ \begin{array}{l} \text{Știm că } |x| \geq 0 \\ \text{Știm că } |x| \leq 0 \end{array} \right. \Leftrightarrow x=0$$

$$|x| = |-x| \quad (\forall) \quad (\forall) x \in \mathbb{R}$$

$$|3| = |-3| = 3 \quad \hookrightarrow \text{ORICE: } (\forall)$$

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