

Aduceti la forma cea mai simplă expresie:

$$E(x) = \frac{4x-12}{x^2-9} - \frac{x+2}{x-3} : \frac{x^2+5x+6}{x-3}$$

P.e. $x^2-9 \neq 0 \Leftrightarrow x^2-3^2 \neq 0 \Leftrightarrow (x-3)(x+3) \neq 0 \Leftrightarrow x-3 \neq 0 \text{ și } x+3 \neq 0 \Leftrightarrow$

$x \neq 3$ și $x \neq -3$

$$x^2+5x+6 \neq 0 \Leftrightarrow \underbrace{x^2+2x+3x+6}_{\substack{\text{1} \\ \text{1}}} \neq 0 \Leftrightarrow x(x+2)+3(x+2) \neq 0 \Leftrightarrow$$

$$(x+2)(x+3) \neq 0 \Leftrightarrow \begin{cases} x+2 \neq 0 \\ x \neq -2 \end{cases} \quad \begin{cases} x+3 \neq 0 \\ x \neq -3 \end{cases} \Rightarrow x \in \mathbb{R} \setminus \{-3, -2\}$$

$$\underline{E(x) = \frac{4(x-3)}{(x-3)(x+3)} - \frac{x+2}{x-3} \cdot \frac{x+3}{(x+2)(x+3)}} = \frac{4}{x+3} - \frac{1}{x+3} = \frac{4-1}{x+3} = \frac{3}{x+3}$$

$$F(x) = \left(\frac{x+1}{x} - \frac{x-1}{x+1} + \frac{1-x}{x^2+x} \right) : \frac{2x-4}{x^2+3x}$$

P.C. $x \neq 0$ $x+1 \neq 0 \Rightarrow x \neq -1$

$$x \neq 0, x+1 \neq 0 \Rightarrow x(x+1) \neq 0 \Rightarrow x^2+x \neq 0$$

$$\left. \begin{array}{l} 2x-4 \neq 0 \Leftrightarrow 2x+4 \Leftrightarrow x \neq -2 \\ \Leftrightarrow x \neq 2 \\ x^2+3x \neq 0 \Leftrightarrow x(x+3) \neq 0 \\ \Leftrightarrow x \neq 0 \text{ și } x+3 \neq 0 \\ x \neq -3 \end{array} \right\}$$

Deci $x \in \mathbb{R} \setminus \{0, -1, 2, -3\} \Rightarrow x \in \mathbb{R}^* \setminus \{-1, 2, -3\}$

$$F(x) = \left(\frac{x+1}{x} - \frac{x-1}{x+1} + \frac{1-x}{x(x+1)} \right) \cdot \frac{x(x+3)}{2(x-2)}$$

$$F(x) = \left(\frac{x^2+2x+1}{x(x+1)} - \frac{x^2-x}{x(x+1)} + \frac{1-x}{x(x+1)} \right) \cdot \frac{x(x+3)}{2(x-2)}$$

$$F(x) = \frac{x^2+2x+1-x^2+x+1-x}{x(x+1)} \cdot \frac{x(x+3)}{2(x-2)} \Rightarrow F(x) = \frac{2x+2}{x(x+1)} \cdot \frac{x(x+3)}{2(x-2)}$$

$$F(x) = \frac{x(x+1)}{x+1} \cdot \frac{x+3}{2(x-2)} \Rightarrow F(x) = \frac{x+3}{x-2}, x \in \mathbb{R}^* \setminus \{2, -3, -1\}$$

$$A = \{x \in \mathbb{Z} \mid F(x) \in \mathbb{Z}\} \Rightarrow A = ?$$

$$\frac{x+3}{x-2} = \frac{x-2+5}{x-2} = \frac{x-2}{x-2} + \frac{5}{x-2} = 1 + \frac{5}{x-2} \in \mathbb{Z} \quad \left\{ \begin{array}{l} \frac{5}{x-2} \in \mathbb{Z} \\ x-2 \in \mathbb{Z} \end{array} \right. \Rightarrow \frac{5}{x-2} \in \mathbb{Z} \Rightarrow$$

$$(x-2) \mid 5 \Rightarrow x-2 \in \mathbb{D}_5 \Leftrightarrow x-2 \in \{1, -1, 5, -5\}$$

$$\Rightarrow x \in \{3, 1, 7, -3\} \quad \left\{ \begin{array}{l} A = \{3, 1, 7\} \\ x \in \mathbb{N}^* \setminus \{2, -3, -1\} \end{array} \right.$$

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