9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization
4-6 September 2002, Atlanta, Georgia

AIAA 2002-5504

MULTI-FIDELITY MULTIDISCIPLINARY DESIGN OPTIMIZATION BASED ON COLLABORATIVE OPTIMIZATION FRAMEWORK

Parviz M. Zadeh and Vassili V. Toropov
P.M.Zadeh@bradford.ac.uk and V.V.Toropov@bradford.ac.uk

School of Engineering, Design and Technology, University of Bradford
Bradford, West Yorkshire, BD7 1DP, UK.

Abstract

This paper is concerned with the development of a multi-fidelity modelling methodology to be used within the collaborative multidisciplinary design optimization (MDO) framework. The main objective is to reduce the computational costs associated with high fidelity models in MDO by utilizing approximations based on tuned low-fidelity models in the disciplinary optimization runs. The methodology is based on simplified numerical models treated as low-fidelity models, and the global response surface modelling techniques. Numerical results for a benchmark test problem indicate that simplified numerical models can provide a basis for sufficiently high quality approximations resulting in fewer high-fidelity analysis calls in the optimization process.

Key Words: Multidisciplinary design optimization, collaborative optimization, multi-fidelity modelling, approximation concepts, simplified numerical model, response surface modelling

Introduction

Important challenges associated with MDO are often characterized by interdisciplinary couplings, high computational cost of an analysis in individual disciplines and a large number of design variables and constraints. These issues result in a very high overall computational cost and organizational complexity, limiting real-life applications of MDO based on high-fidelity models (e.g. detailed computational fluid dynamics or finite element analysis simulation models, etc.). The development of new strategies and algorithms to address these challenges has become the focus of many research programmes within academia and industry. The concurrent subspace optimization (CSSO) approach[1], the collaborative optimization (CO) framework[2] and the bi-level integrated synthesis (BLISS)[3] represent some of the main developments in this field. This study aims at the development of an efficient multi-fidelity methodology for MDO problems. The key objective is to reduce the cost of using high-fidelity models in MDO by utilizing tuned low-fidelity models.

Computational issues of using high-fidelity models in engineering design optimization have been addressed over the past three decades. A survey on the use of approximations in structural optimization has been carried out by Barthelemy and Haftka[4]. Recently, Toropov[5] reviewed some of the modelling and approximation strategies in design optimization. The use of response surface methodology (RSM) in design optimization in general and in MDO in particular has become popular for reducing the computational cost of high-fidelity models in the optimization process. The main benefits of using RSM in MDO include[6] that: *(i)* it uses low order polynomials in place of high fidelity simulation models for reducing the number of expensive high-fidelity analyses during the optimization and smoothing out numerical noise; *(ii)* it enables separation of the analysis code from the optimization routines and eases the integration of codes from various disciplines; *(iii)* it makes possible utilization of parallel computer architecture.

While RSM has now become an integral part of optimizatioin process allowing a significant reduction of computational costs, it has some limitations. The cost of providing high-fidelity data for fitting the global approximations increases rapidly with the number of design variables. In addition, in some cases it is difficult to construct a good global approximation with low order polynomials. In order to build a higher quality approximation model, Toropov and Markine[7] suggested the use of low-fidelity models based on simplified numerical models as the basis for approximation building. They considered three types of the structure of such approximations. These are reviewed in section 'Choice of Approximation Models' below.

In recent years, there have been several approaches that aim at combining low-fidelity models with correction response surfaces. In Mason et al.[8] a coarse 2-D finite element model is used as a low-fidelity model to predict failure stresses and corrections are calculated using a full 3-D finite element model. Venkataraman et al.[9] demonstrated the effectiveness of correcting inexpensive solutions based on simplified models by results from more expensive and accurate models in design of shell structures for buckling. Vitali et al.[10] used a coarse finite element model as a low-fidelity model to predict the stress intensity factor, and corrected it with a high-fidelity model results based on detailed finite element model for optimizing a blade stiffened composite panel with cracks. In the field of MDO the variable-complexity modelling proposed by Giunta[11] simultaneously utilized low and high fidelity models. The low-fidelity predictions were corrected using a scaling factor obtained from the high-fidelity models, during the optimization process. The trust-region approximation management framework proposed by Alexandrov et al.[12] manages the solution of MDO process by alternating the use of high and low fidelity models during the optimization process.

In this study, a multi-fidelity modelling approach introduced within the collaborative optimization framework aims at significant reduction of computational cost as well as improving the computational efficiency of multidisciplinary design optimization with high-fidelity models. The approximations are based on a simplified numerical model, which is treated as a low-fidelity model. The main advantage of using a simplified numerical model over the conventional empirical approximation models is that it improves the quality of approximation. The effectiveness of the proposed technique is studied on a test problem.

Approximation Concept

A general design optimization problem can be stated as:

$$F_0(\boldsymbol{x}) \rightarrow \min \quad (1)$$

subject to: $F_j(\boldsymbol{x}) \le 1 \quad (j = 1,..,M) \quad (2)$

and $A_i \le x_i \le B_i \quad (3)$

where $F_0(\boldsymbol{x})$ and $F_j(\boldsymbol{x})$ are objective and constraint functions, respectively, $\boldsymbol{x} = (x_1,...,x_N)^{\mathrm{T}}$ is a vector of design variables, A_i and B_i are lower and upper bounds on the design variables, respectively .

The well-established approximation concept leads to the iterative approximation of the implicit functions $F_j(\boldsymbol{x})$ by simpler functions $\tilde{F}_j^k(\boldsymbol{x})$ ($j = 1,...,M$). The original optimization problem (1) - (3) is replaced by a succession of approximated sub- problems:

$$\tilde{F}_o^k(\boldsymbol{x}) \rightarrow \min \quad (4)$$

subject to: $\tilde{F}_j^k(\boldsymbol{x}) \le 1 \quad (j = 1...,M) \quad (5)$

$$A_i^k \le x_i \le B_i^k,\ A_i^k \ge A_i, B_i^k \le B_i \quad (i = 1,...N) \quad (6)$$

where k is the iteration number. The function $\tilde{F}_j^k(\boldsymbol{x})$ is the approximation of the original function $F_j(\boldsymbol{x})$ which can be valid in the whole range of the design variables (global approximation) or in a domain defined by the move limits (mid-range approximations). In the later case, the solution of an individual sub-problem becomes a starting point for the next step. The move limits A_i^k and B_i^k are changed and the optimization is repeated iteratively until the optimum is reached. The approximation functions $\tilde{F}_j^k(\boldsymbol{x})$ must be inexpensive as compared to the original response functions, $F_j(\boldsymbol{x})$, and they should not possess any significant level of numerical noise.

Multi-fidelity Modelling Methodology for MDO

The multi-fidelity modelling methodology for multidisciplinary design optimisation, adopted in this study, is based on the use of high-fidelity, low-fidelity and approximation models in the optimization process. This methodology has been implemented in collaborative design optimization framework. For a detailed formulation of CO see Braun and Kroo[13]. The organization of the optimization process and the main components of the methodology are shown in Figure 1.

In this approach, a design optimization problem is decomposed into two levels, namely, system level and discipline level. The system level optimizer is used to minimize the system level objective while satisfying consistency requirements among the various disciplines by enforcing equality constraints, which coordinate the interdisciplinary couplings. The discipline level consists of computationally efficient low-fidelity models and much more expensive high-fidelity simulation models. The interaction of these is controlled by the model building module (design of experiments and approximation update). These two levels of fidelity may involve, as an example, a coarse finite element model as a low-fidelity model and a fine-meshed detailed finite element model as a high-fidelity model. The correction (or tuning) of a low-fidelity model is done using the information from a relatively small number of calls for the high-fidelity model. The tuned low-fidelity model, used as an approximation, approaches the same level of accuracy as a high-fidelity model but at the same time remains inexpensive to be used extensively during the optimization process. The interaction of these models in approximation building process is shown in Figure 2. In this process, only the corrected low-fidelity models are used within the disciplinary optimization.

Choice of Approximation Model

The quality of approximations has a profound effect on the computational cost and convergence characteristics of an approximation based optimization. It was shown by Box and Draper[14] that a mechanistic model (i.e. the one that is built upon some knowledge about the system under consideration) can provide higher quality approximation than a purely empirical model. An

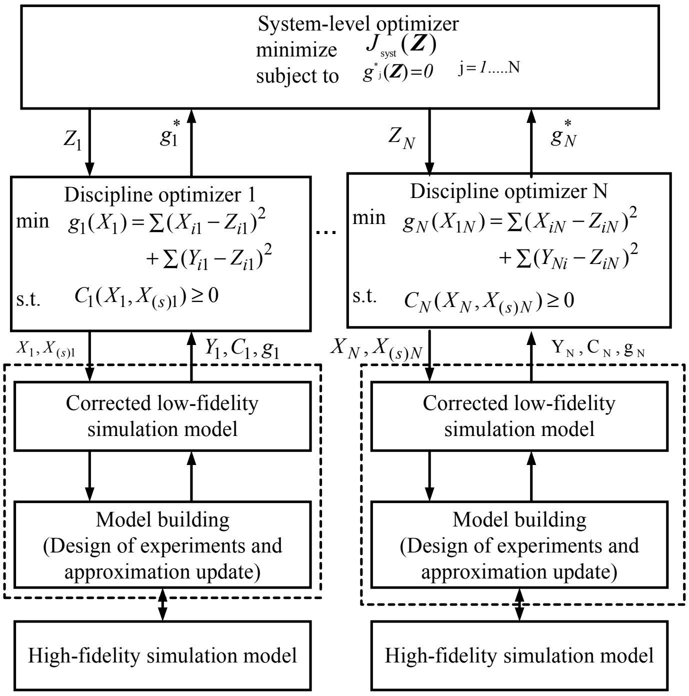

Figure 1. Collaborative multi-fidelity multidisciplinary design optimization framework

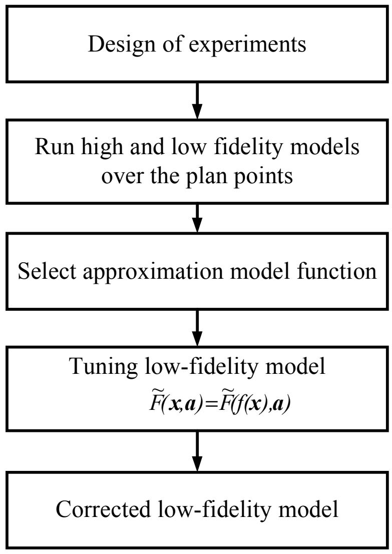

Figure 2. Approximation model building

example of application of such a model to a problem of material parameter identification can be seen in Toropov and van der Giessen[15]. The main disadvantage of using such models, however, is that they depend on the specific features of the problem.

The approximation, on the other hand, need not necessarily be an explicit function. It could be an implicit one if some numerical procedure is involved in its formulation. The requirements to such a model can be stated as (Toropov et al.[16]): *(i)* the model must depend on the same design variables as the original model; *(ii)* it should contain some tuning parameters to be defined using the least squares method; *(iii)* it must be simple enough to be used in numerous repeated calculations; *(iv)* it should not contain any considerable level of noise in order not to cause convergence problems in the optimization process. In this study, the approximation model is based on a simplified numerical model, which satisfies the above requirements. This is described in the following section.

Simplified Numerical Models

A general way of constructing high quality approximations adopted here is based on the use of a simplified numerical model. This can be achieved either by simplifying the analysis model (e.g. coarser finite element mesh) or simplifying the modelling concept (e.g. geometry and boundary conditions, use of 2D instead of 3D model, etc.). Such a model should reflect the most prominent physical features of the system under the consideration and at the same time remain computationally inexpensive. The simplified numerical model can then be used to build the approximation function as follows:

$$\widetilde{F}(\boldsymbol{x},\boldsymbol{a}) \equiv \widetilde{F}(f(\boldsymbol{x}),\boldsymbol{a}) \tag{7}$$

where $f(\boldsymbol{x})$ is the function presenting response using the simplified numerical model (i.e. low-fidelity model). The tuning parameters $\boldsymbol{a}$ are used for minimizing the discrepancy between the high-fidelity and the low-fidelity models.

Design of Experiments

The selection of points in the design variable space where the response must be evaluated is commonly referred to as design of experiments. The planning of design of experiments (i.e. the choice of the set of points in the design variable space) can have a considerable effect on the accuracy and the efficiency of the approximation building.

There are many schemes available in the literature for generating plans of experiments. In this study, the scheme suggested by Audze and Eglais[17] is adopted. It is a space-filling design of experiments that is based on a Latin hypercube design but is accompanied by an optimality criterion as follows:

$$\sum_{p}^{P} \sum_{q=p+1}^{P} \frac{1}{L^2_{pq}} \rightarrow \min \tag{8}$$

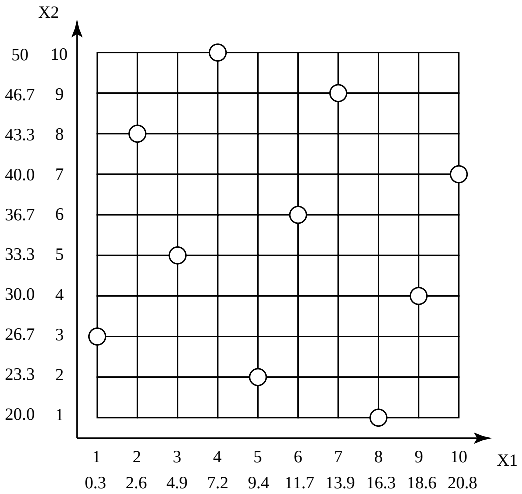

Figure 3. Audze-Eglais plan of experiments for N =2 and P =10

where L_{pq} is the distance between the points p and q ($p \neq q$), P is the total number of plan points. For the case of N =2 and P =10 the plan is shown in Figure 3.

Approximations based on Simplified Numerical Models

In building approximation models, $\widetilde{F}(\boldsymbol{x})$, it is necessary to select an appropriate structure of the approximation model (i.e. to define them as a function of design variables $\boldsymbol{x}$ and tuning parameters $\boldsymbol{a}$). The efficiency of the optimization process depends strongly on the accuracy of such a model. In this study, six types of approximation functions (linear and multiplicative of types 1 and 2, quadratic and cubic) were examined on a test problem. The selection of the best approximation function was based on the error of the corrected low-fidelity model as compared to the high-fidelity model at points of a selected plan of experiments referred to as the verification plan.

Type 1, Linear and Multiplicative Approximation with Two Tuning Parameters

The simplest form of the approximation functions is a linear or intrinsically linear function with two tuning parameters which can be represented as follows:

Type 1 linear: $\widetilde{F}(\boldsymbol{x},\boldsymbol{a}) = a_0 + a_1 f(\boldsymbol{x})$ (9)

or

Type 1 multiplicative: $\widetilde{F}(\boldsymbol{x},\boldsymbol{a}) = a_0 f(\boldsymbol{x})^{a_1}$ (10)

where $\boldsymbol{a} = [a_0 \;\; a_1]^{\mathrm{T}}$.

Type 2, Correction Functions

The type 2 approximations are based on an explicit correction function $C(\boldsymbol{x},\boldsymbol{a})$ which depends on the design variables and tuning parameters. The following correction functions can be used:

Multiplicative function:

$\widetilde{F}(\boldsymbol{x},\boldsymbol{a}) = f(\boldsymbol{x})\, C(\boldsymbol{x},\boldsymbol{a})$, where

$$C(\boldsymbol{x},\boldsymbol{a}) = a_0 \prod_{l=1}^{N} x_l^{a_l} \tag{11}$$

or, alternatively, a linear function:

$$\widetilde{F}(\boldsymbol{x},\boldsymbol{a}) = f(\boldsymbol{x}) + C(\boldsymbol{x},\boldsymbol{a}) \tag{12}$$

where $C(\boldsymbol{x},\boldsymbol{a})$ can be a low order polynomial in $\boldsymbol{x}$ with coefficients $\boldsymbol{a}$, e.g. a linear, quadratic or a cubic function.

Type 3: The low-fidelity model $f(\boldsymbol{x})$ which is used as a basis for building approximation model depends not only on the design variables $\boldsymbol{x}$ but also on other parameters including geometry and material properties, etc. Some of these parameters can be considered as tuning parameters $\boldsymbol{a}$ so that an approximation function can be represented in the following form:

$$\widetilde{F}(\boldsymbol{x},\boldsymbol{a}) = f(\boldsymbol{x},\boldsymbol{a}) \tag{13}$$

The tuning parameters $\boldsymbol{a}$ included in the approximation model are estimated by minimizing the sum of squares of the errors:

$$G(\boldsymbol{a}) = \sum_{p=1}^{P_t} \left\{ w_p \left(F_p - \widetilde{F}_p(\boldsymbol{a}) \right)^2 \right\} \rightarrow \min \tag{14}$$

where w_p is a weight coefficient that determines the relative contribution of the information at the p-th plan point.

The linear and multiplicative functions of types 1 and 2 have been successfully used for a variety of design optimization problems. The advantage of these approximation functions is that a relatively small number of tuning parameters $\boldsymbol{a}$ are to be obtained. Higher order approximation functions of type 2, such as full quadratic and cubic polynomials, can be expected to produce higher quality of approximations. However, these approximation functions require a considerably larger number of designs to be evaluated. In this study the performance of six different types of approximation functions have been examined on a test problem.

Numerical Example

The test problem for the multidisciplinary design optimization developed in this study deals with the weight minimization of a cantilevered composite beam subjected to a parabolic distributed load, $q = q_0(1 - \frac{x^2}{L^2})$, where $x=0$ at the clamped end. The design data for this benchmark problem are outlined in Table 1. The maximum stress and deflection of the beam can be calculated analytically as follows:

$$\sigma_{\max} = \frac{M_{\max} h}{2I} = \frac{q_0 L^2 h}{8I} \qquad (15)$$

$$\delta_{\max} = \frac{19 q_0 L^4}{360 EI} \qquad (16)$$

Based on the rule of mixtures for a continuous fiber-reinforced composite material with a fiber volume fraction v_f and a matrix volume fraction v_m the following relationship must be satisfied for the longitudinal (fiber direction) Young's modulus E_l and the composite weight density ρ :

$$E_l = E_f\, v_f + E_m\,(1 - v_f), \qquad (17)$$

$$\rho = \rho_f\, v_f + \rho_m\,(1 - v_f), \qquad (18)$$

$$v_f + v_m = 1. \qquad (19)$$

where E_f and E_m are the elastic moduli for graphite and epoxy resin, respectively, and ρ_f and ρ_m are the weight density of the graphite fiber and epoxy resin, respectively. The fiber volume fraction v_f can vary from zero (i.e. no fiber is used) to the maximum value defined by the maximum amount of fiber packed in the composite, $V_f^{max} = 0.9069$. In this test problem v_f = 0.4 was taken as a lower limit.

The test problem was solved using a finite element (FE) beam model. Two levels of model fidelity were considered, a low-fidelity FE model consisting of 2 elements, and a high-fidelity FE model consisting of 100 elements. Numerical results have been compared to the analytical results.

Formulation

The design variables x_1, x_2 and x_3 are the second moment of area *I,* the depth of the beam *h*, and the fiber volume fraction v_f, respectively.

The objective function, $F_o(\boldsymbol{x}) = AL\rho$ is the weight of the beam, $A = \frac{12I}{h^2}$ (mm^2), $\rho = (12 + 5.2\, v_f)10^{-6}$ (N/mm^3).

The optimization problem is then formulated as follows:

$$F_o(\boldsymbol{x}) = \frac{x_1}{x_2^2}(1440 + 624\, x_3) \rightarrow \quad \min \qquad (20)$$

subject to:

$$F_1(\boldsymbol{x}) = \frac{\sigma_{max}}{[\sigma]} \le 1, \quad [\sigma] = 166.66667 \ \left(N/mm^2\right) \qquad (21)$$

$$F_2(\boldsymbol{x}) = \frac{\delta_{\max}}{[\delta]} \le 1, \quad [\delta] = 12.93223 \ (mm) \qquad (22)$$

$$F_3(\boldsymbol{x}) = \frac{x_2^4}{1.2E6\; x_1} \le 1, \qquad (23)$$

$$A_i \le x_i \le B_i\ . \qquad (24)$$

The inequalities (21) to (23) represent, respectively, the constraints on the maximum stress, maximum deflection at the free end of the beam, and the geometric requirement that the depth of the beam must be equal or smaller than ten times the width of the composite beam. The side constraints on the design variables A_i and B_i are shown in Table 1.

Optimization was first carried out using the conventional all-at-once strategy and MARS[5] (Multipoint Approximation method based on the Response Surfaces) optimization routine. The models used in this optimization run include high and low fidelity FE models as well as an analytical model of the cantilevered composite beam. Results for the minimum value of the objective function, corresponding design variables and constraints are shown in Table 2.

Design parameter			Design variable				
Description (notation)	Unit	Value	Description (notation)	Unit	Baseline design	Range	
						Min.	Max.
Parabolic distributed load q_0	*N/mm*	1	2nd moment of area *I*	mm^4	2.25E4	3.3E3	20.833E5
Length of the beam *L*	*mm*	1000	Depth of the beam *h*	*mm*	30	20	50
Elastic modulus graphite fiber E_f	N/mm^2	2.3E5	Fiber vol. fraction v_f		0.785	0.4	0.9069
Elastic modulus epoxy resin E_m	N/mm^2	3.45E3	-	-	-	-	-
Weight density graphite fiber ρ_f	N/mm^3	1.72E-5	-	-	-	-	-
Weight density epoxy resin ρ_m	N/mm^3	1.2E-5	-	-	-	-	-

Table 1. Data for cantilevered composite beam example

Design variables	Solution (All-At-Once Method)		
	Analytical model	High-fidelity FE model	Low-fidelity FE model
x_1 $(10^4)[\mathrm{mm}^4]$	3.361	3.361	2.993
x_2 [mm]	44.814	44.814	43.533
x_3	0.521	0.521	0.528
$F_o(\boldsymbol{x})$	2.954	2.954	2.795
$F_1(\boldsymbol{x})$	1.0	1.0	1.0
$F_2(\boldsymbol{x})$	1.0	1.0	1.0
$F_3(\boldsymbol{x})$	1.0	1.0	1.0

Table 2. Results of all-at-once optimization for high and low-fidelity and analytical models

Multidisciplinary Design Optimization

The above formulated benchmark problem was then used to test the proposed multi-fidelity MDO based on collaborative optimization framework. The problem is decomposed into two disciplines and system level. The two disciplines involved in this problem are the stress constrained problem and deflection constrained problem. The objective at the system level is to find target values that satisfy consistency requirements for both disciplines and at the same time minimize the system level objective function. This objective function is the total weight of the composite beam:

$$F(\boldsymbol{x_s}) = \frac{x_{s1}}{x_{s2}^2}(1440 + 624\, x_{s3}) \rightarrow \quad \min \tag{25}$$

where x_{s1}, x_{s2} and x_{s3} represent the second moment of area, depth of the beam and the fiber volume fraction, respectively. These are treated as system level target values corresponding to discipline level design variables x_1, x_2 and x_3, respectively. The constraints at the system level are:

$$g_1^*(x_{s1}, x_{s2}) = 0\,,\, g_2^*(x_{s1}, x_{s3}) = 0 \tag{26}$$

$$\text{and } A_i \le x_i \le B_i\,,\ i = 1, 2, 3 \tag{27}$$

where g_1^* and g_2^* are the discrepancies between the actual and target values returned from the discipline 1 and discipline 2, respectively. The system level must satisfy these consistency requirements among the disciplines 1 and 2 by enforcing the equality constraints (26). These functions treated as constraints at the system level are the objective functions of the disciplines 1 and 2, respectively.

Discipline Levels Optimization

The objective of design optimization at the discipline level is to find optimum values of the design variables, which satisfy the discipline's own constraints and minimize the discrepancy from the target values generated by the system level optimizer. The discrepancy function to be minimized in the first discipline is:

$$g_1(x_1, x_2) = (\, x_1 - x_{s1}\,)^2 + (\, x_2 - x_{s2}\,)^2 \quad \rightarrow \quad \min$$

where x_1 (second moment of area) and x_2 (depth of the beam) are the shared and local design variables respectively, x_{s1} and x_{s2} are the associated system level target values. The constraints in discipline 1 are:

$$F_1(\boldsymbol{x}) = \frac{\sigma_{max}}{[\sigma]} \le 1\,, \tag{27}$$

$$F_2(\boldsymbol{x}) = \frac{x_2^4}{1.2E6\, x_1} \le 1\,. \tag{28}$$

$$\text{and } A_i \le x_i \le B_i\,,\ i = 1, 2 \tag{29}$$

Similarly to the discipline 1, the objective of design in the discipline 2 is to minimize the discrepancy function of system level target values and associated design variables while satisfying its constraints. The discrepancy function of this discipline can be expressed as:

$$g_1(x_1, x_3) = (\, x_1 - x_{s1}\,)^2 + (\, x_3 - x_{s3}\,)^2 \quad \rightarrow \quad \min$$

where x_i (second moment of area) and x_3 (fiber volume fraction) are the shared and local design variables respectively, x_{s1} and x_{s3} are the associated system level target values.

The second discipline has one constraint on the maximum deflection of the beam:

$$F_1(\boldsymbol{x}) = \frac{\delta_{\max}}{[\delta]} \le 1\,. \tag{30}$$

$$\text{and } A_i \le x_i \le B_i\,,\ i = 1, 3 \tag{31}$$

Implementation of CO

Optimization at the system level was carried out using the Nelder and Mead[18] method combined with the sequential unconstrained minimization and exterior penalty function techniques. Sequential quadratic programming (SQP) method was used for solving the optimization problems in the disciplines 1 and 2. It can be seen from the Table 1 that the numerical values of design variables are different by several orders of magnitude. To avoid numerical difficulties, all the design variables at the system level have been scaled between 1 and 11.

Multi-Fidelity Modeling

This section demonstrates the multidisciplinary design optimization for the test problem using multi-fidelity modelling strategy, focusing on the use of a tuned low-fidelity model in place of a high-fidelity model in the optimization process. The models used in this problem involve three levels of fidelity: a coarse FE model consisting of 2 elements as a low-fidelity model, a fine meshed FE model consisting of 100 elements as a high-fidelity model, and a corrected low-fidelity model.

The corrected low-fidelity models were constructed the constraints (27) and (30) in the disciplines 1 and 2, respectively. The data needed to build these approximation models was obtained by conducting discipline level analysis at plan points. This was carried out for both the low-fidelity and high-fidelity models. To ensure the high quality of approximation models and efficiency of the approximation model building processes a study has been carried out on the choice of the design of experiments.

Design of Experiments

In order to use only a minimum number of points needed for the construction of high quality approximation models, five separate designs of experiments involving 10 point plan (Figure 3), 5 point plan (Figure 4), 4, 3 and 2 point plans (Figure 5) were studied on the test problem. In Figure 5 the 3 point plan corresponds to the points 1, 2 and 3, and the 2 point plan corresponds to the points 1 and 4.

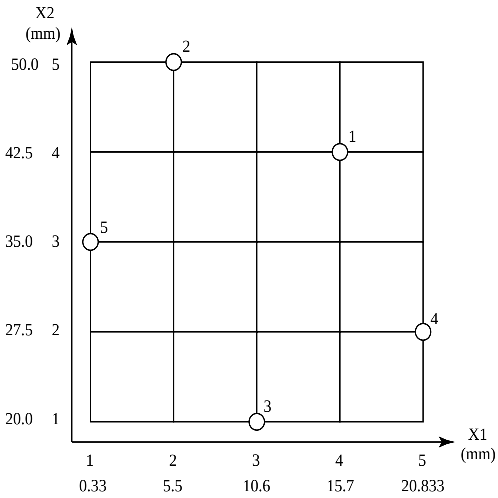

Figure 4. Five point Audze-Eglais plan

.

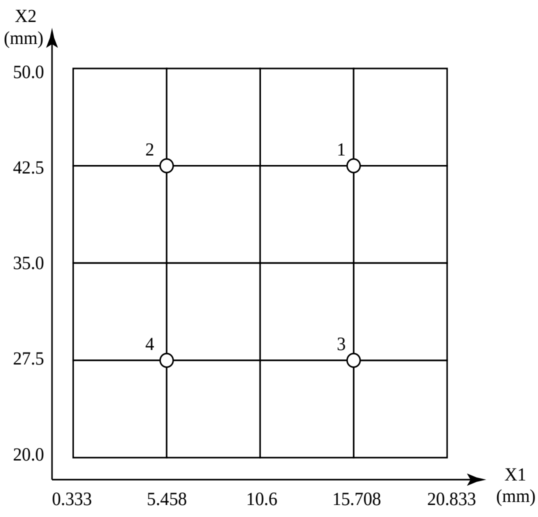

Figure 5. Four point plan

Choice of Approximation models

The performance of six types of approximation models (linear type 1 and 2, multiplicative type 1 and 2, quadratic and cubic polynomials as defined by (9)-(12)) has been studied for the constraints in both disciplines of the test problem. Each model was verified against an additional plan of experiments (a verification plan), these results are shown in Tables 3 and 4. The quality of these approximation models is measured by the root mean square (RSM) error and the maximum deviation (MD) between the high-fidelity model and the corrected low-fidelity model.

The five point plan was selected as an appropriate one for building approximation models for constraints in both disciplines. This was verified against the four point plan. Results of this analysis for three types of approximation models for constraints in both disciplines 1 and 2 are shown in Tables 5 and 6 respectively. On the basis of these results three types of approximations have been selected, namely, the linear and multiplicative type 1, and multiplicative type 2.

Based on results presented in Tables 5 and 6 the linear approximation model type 1 was selected as the appropriate approximation model to be used in the collaborative optimization framework.

Results of Multi-fidelity CO

The average discrepancies between the high-fidelity and low-fidelity models in the disciplines 1 and 2 are 8.33% and 9.7%, respectively. From the Tables 5 and 6 it is clear that this discrepancy between the high-fidelity and the corrected-low fidelity models became very small. The results obtained from the collaborative optimization (Table 7) show that the corrected low-fidelity model has the same performance as the high-fidelity model. The discrepancy of the optimum solutions using the high-fidelity and the corrected low-fidelity models is negligible.

Conclusions

In this paper multi-fidelity modelling methodology in conjunction with the collaborative optimization framework is presented. This methodology has been applied to a cantilevered composite beam test problem. On this test, problem performance of several approximation function obtained using the information at different number of design points has been studied. Significant computational savings over the conventional optimization, in terms of the number of calls for the high-fidelity models, have been observed. This capability suggests an effective utilization of high-fidelity models in MDO without incurring excessively high computational cost.

Table 3. Comparison of approximations based on the corrected low-fidelity FE model (discipline 1)

Number of plan points	Plan type		Root mean square (RMS) and maximum deviation (MD) for corrected low-fidelity FE model					
			Linear type 1	Mult. type 1	Linear type 2	Mult. type 2	Quad.	Cubic
10	Tuning	RMS	5.49E-5	3.68E-4	1.18E-1	2.03E-4	5.78E-2	Exact fit
		MD	6.18E-2	4.04E-2	16.25	3.97E-2	46.32	
5	Verification	RMS	1.08E-5	3.85E-5	9.57E-2	3.75E-5	8.34E-2	5.8E-2
		MD	2.84E-3	1.16E-2	22.76	1.12E-2	54.22	61.99
5	Tuning	RMS	6.25E-10	2.71E-9	1.82E-1	2.58E-10	-	-
		MD	6.78E-7	1.7E-7	126.18	1.39E-7	-	-
4	Verification	RMS	1.06E-9	8.8E-10	2.06E-1	7.72E-10	-	-
		MD	2.84E-7	2.4E-7	24.9	5.62E-8	-	-
4	Tuning	RMS	7.46E-11	9.13E-11	2.80E-3	4.06E-16	-	-
		MD	5.11E-8	3.68E-8	2.13	1.14E-13	-	-
5	Verification	RMS	1.96E-8	3.06E-8	3.02E-1	3.32E-8	-	-
		MD	1.96E-8	3.06E-8	3.02E-1	3.32E-8	-	-
3	Tuning	RMS	5.99E-11	4.41E-11	Exact fit	Exact fit		-
		MD	3.89E-8	2.72E-8			-	-
5	Verification	RMS	1.94E-8	2.84E-8	2.98E-1	3.32E-8	-	-
		MD	4.94E-7	7.25E-7	13.2	8.5E-7	-	-
2	Tuning	RMS	Exact fit	Exact fit	-	-	-	-
		MD			-	-	-	-
5	Verification	RMS	2.38E-8	4.66E-8	-	-	-	-
		MD	6.08E-7	1.19E-6	-	-	-	-

Table 4. Comparison of approximations based on the corrected low-fidelity FE model (discipline 2)

Number of plan points	Plan type		Root mean square (RMS) and maximum deviation (MD) for corrected low-fidelity FE model					
			Linear type 1	Mult. type 1	Linear type 2	Mult. type 2	Quad.	Cubic
10	Tuning	RMS	9.10E-10	1.44E-9	2.46E-1	1.47E-9	1.09E-1	Exact fit
		MD	5.054E-7	4.56E-7	163.9130	4.63E-7	85.93	
5	Verification	RMS	1.65E-9	1.65E-9	2.03E-1	1.66E-9	1.93E-1	1.34E-1
		MD	2.33E-6	2.34E-6	86.83	9.73E-8	51.834	30.89
5	Tuning	RMS	5.49E-10	2.52E-9	2.08E-1	2.64E-9	-	-
		MD	5.22E-7	4.16E-7	80.66	4.03E-7	-	-
4	Verification	RMS	1.80E-9	1.94E-9	2.28E-1	1.94E-9	-	-
		MD	5.46E-9	2.04E-7	41.53	9.67E-8	-	-
4	Tuning	RMS	1.19E-9	1.93E-9	3.73E-3	2.44E-9	-	-
		MD	5.00E-7	1.64E-7	2.65	6.09E-7	-	-
5	Verification	RMS	8.17E-9	1.30E-7	3.35E-1	9.21E-8	-	-
		MD	1.54E-6	1.91E-6	8.93	1.4E-6	-	-
3	Tuning	RMS	1.16E-9	1.61E-9	Exact fit	Exact fit	-	-
		MD	7.2E-7	9.45E-7			-	-
5	Verification	RMS	2.92E-8	2.40E-7	3.4E-1	2.53E-7	-	-
		MD	1.73E-6	2.02E-6	9.04	2.35E-6	-	-
2	Tuning	RMS	Exact fit	Exact fit	-	-	-	-
		MD			-	-	-	-
5	Verification	RMS	1.22E-8	3.32E-8	-	-	-	-
		MD	4.07E-7	8.83E-7	-	-	-	-

Table 5. Comparisons of models for discipline 1

Plan point	x_1 (10^4) [mm^4]	x_2 [mm]	High-fidelity model $F(\mathbf{x})$	Low-fidelity model $f(\mathbf{x})$	Corrected low-fidelity model $\tilde{F}(\mathbf{x})$	Discrepancy of models (%)	
						$\frac{F(x)-f(x)}{F(x)}100$	$\frac{F(x)-\tilde{F}(x)}{F(x)}100$
1	15.7	42.5	2.030E-1	1.861E-1	2.030E-1	8.33	1.19E-7
2	5.5	50	6.818E-1	6.250E-1	6.818E-1	8.33	1.56E-7
3	10.6	20	1.415E-1	1.297E-1	1.415E-1	8.33	3.95E-7
4	20.8	27.5	9.915E-2	9.089E-2	9.915E-2	8.33	6.78E-7
5	0.3	35	8.749	8.021	8.75	8.33	8.18E-10
Linear type 1: $\tilde{F}(\mathbf{x},\mathbf{a}) = a_0 + a_1 f(\mathbf{x})$, a_0 = -7.71E-10, a_1 = 1.0909 Root mean square values: RMS_{TP}=6.25E-10; RMS_{VP}=1.06E-9 for tuning and verification plan, respectively.							

(a). Audze -Eglais 5 point tuning plan and 4 point verification plan, linear model type 1

Plan point	Corrected low-fidelity model $\tilde{F}(\mathbf{x})$	Discrepancy of models (%) $\frac{F(x)-\tilde{F}(x)}{F(x)}100$
1	2.030E-1	1.54E-7
2	6.818E-1	6.21E-8
3	1.415E-1	2.65E-7
4	9.915E-2	4.16E-7
5	8.75	6.53E-8
Multiplicative type 1, $\tilde{F}(\mathbf{x},\mathbf{a}) = a_0 f(\mathbf{x})^{a_1}$ a_0 =1.0909, a_1 =1.00 RMS_{TP}=2.7E-9; RMS_{VP} =8.8E-10 for tuning and verification plan respectively.		

(b). Multiplicative model type 1

Plan point	Corrected low-fidelity model $\tilde{F}(\mathbf{x})$	Discrepancy of models (%) $\frac{F(x)-\tilde{F}(x)}{F(x)}100$
1	2.030E-1	1.38E-7
2	6.818E-1	3.98E-8
3	1.415E-1	2.93E-7
4	9.915E-2	4.03E-7
5	8.75	6.83E-8
Multiplicative type 2, $\tilde{F}(\mathbf{x},\mathbf{a}) = f(\mathbf{x})\, a_0\, x_1^{a_1} x_2^{a_2}$ a_0 =1.0909, a_1 =-7.32E-10; a_2 =-3.26E-9 RMS_{TP}=2.58E-10; RMS_{VP}=7.7E-10 for tuning and verification plan respectively.		

(c). Multiplicative model type 2

Table 5. Comparisons of models for discipline 2

Plan point	x_1 (10^4) [mm^4]	x_3	High-fidelity model $F(\mathbf{x})$	Low-fidelity model $f(\mathbf{x})$	Corrected low-fidelity model $\tilde{F}(\mathbf{x})$	Discrepancy of models (%)	
						$\frac{F(x)-f(x)}{F(x)}100$	$\frac{F(x)-\tilde{F}(x)}{F(x)}100$
1	15.7	0.8	1.407E-1	1.271E-1	1.407E-1	9.7	1.19E-7
2	5.5	0.9	3.578E-1	3.231E-1	3.578E-1	9.7	1.55E-7
3	10.6	0.4	4.092E-1	3.696E-1	4.093E-1	9.7	3.95E-7
4	20.8	0.5	1.681E-1	1.518E-1	1.681E-1	9.7	6.78E-7
5	0.3	0.7	8.3951	7.581	8.395	9.7	8.18E-10
Linear type 1: $\tilde{F}(\mathbf{x},\mathbf{a}) = a_0 + a_1 f(\mathbf{x})$, a_0 =-4.95E-11, a_1 =1.1074 Root mean square values: RMS_{TP}=5.49E-10; RMS_{VP}=1.8E-9 for tuning and verification plan, respectively							

(b). Audze -Eglais 5 point tuning plan and 4 point verification plan using multiplicative model type 1

Plan point	Corrected low-fidelity model $\widetilde{F}(\boldsymbol{x})$	Discrepancy of models (%) $\frac{F(x)-\widetilde{F}(x)}{F(x)}100$
1	1.407E-1	2.60E-7
2	3.578E-1	8.22E-8
3	4.093E-1	1.04E-7
4	1.681E-1	1.70E-7
5	8.395	6.87E-8
Multiplicative type 1, $\widetilde{F}(\boldsymbol{x},\boldsymbol{a})=a_0 f(\boldsymbol{x})^{a_1}$ a_0=1.1074, a_1 =1.0 RMS$_{TP}$ =2.52E-9; RMS$_{VP}$=1.94E-9 for tuning and verification plan respectively.		

(b). Multiplicative model type 1

Plan point	Corrected low-fidelity model $\widetilde{F}(\boldsymbol{x})$	Discrepancy of models (%) $\frac{F(x)-\widetilde{F}(x)}{F(x)}100$
1	1.407E-1	1.39E-7
2	3.578E-1	6.72E-8
3	4.092E-1	6.52E-8
4	1.681E-1	1.39E-7
5	8.395	1.48E-9
Multiplicative type 2, $\widetilde{F}(\boldsymbol{x},\boldsymbol{a})=f(\boldsymbol{x})\,a_0 x_1^{a_1} x_2^{a_2}$ a_0=1.1074, a_1=3.52E-10; a_2=-2.77.E-10 RMS$_{TP}$ =2.52E-9; RMS$_{VP}$ =1.94E-9 for tuning and verification plan respectively.		

(c). Multiplicative model type 2

Model type	Design variables							Constraints					Obj. funct.
	System level			Discipline1		Discipline 2							
	x_1 10^4 mm^4	x_2 mm	x_3	x_1 10^4 mm^4	x_2 mm	x_1 10^4 mm^4	x_3	$F_1(\boldsymbol{x})$	$F_2(\boldsymbol{x})$	$F_3(\boldsymbol{x})$	$g_1^*(x_1,x_2)$	$g_2^*(x_1,x_3)$	$F_o(\boldsymbol{x})$
Analytical	3.361	44.807	0.520	3.361	44.807	3.360	0.521	1.0	1.0	0.999	1.3E-7	1.5E-7	2.954
High-fidelity	3.359	44.788	0.496	3.359	44.788	3.364	0.520	1.0	0.998	1.0	2.4E-7	6.7E-4	2.929
Corrected low-fidelity	3.359	44.788	0.497	3.359	44.788	3.363	0.520	1.0	1.0	0.998	2.4E-7	5.8E-4	2.930

Table 7. Results of collaborative optimization for analytical, high, low and corrected low fidelity models

References

[1] J. Sobieszczanski-Sobieski. Optimization by decomposition: A step from hierarchic to non-hierarchic systems. *NASA Conference publication 3031, Part 1, 2nd NASA / Air Force Symp. on Recent Advances in Multidisciplinary Analysis and Optimization,* Hampton, Virginia, September 28-20, 1988.

[2] I. Kroo, S. Altus, R. Braun, P. Gage and I. Sobieski. Multidisciplinary optimization methods for aircraft preliminary design", AIAA-94-4325-CP, *Proc.of the 5th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization,* pp. 697-707, Panama City, Florida, September 7-9, 1994 .

[3] J. Sobieszczanski-Sobieski , J. Agte and Jr. R. Sandusky. Bi-level integrated system synthesis (BLISS). *Proc. 7-th AIAA/USAF/NASA/ISSMO Symp. On Multidisciplinary Analysis and Optimization* (held in St. Louis, Mo, September), AIAA 98-4916, 1998.

[4] J-F. Barthelemy and R.T. Haftka. Approximation concepts for optimum structural design - a review. *Struct.l Optim., 5:129-144, 1993.*

[5] V.V. Toropov. Modelling and approximation strategies in optimization-global and mid-range approximations, response surface methods, genetic programming, and low/high fidelity models. In: Blachut, J. Eschenauer, H.A. (eds.). Emerging Methods for Multidisciplinary Optimization, CISM Courses and Lectures, No. 425, Inter. Center for Mechanical Scie., pp. 205-256, Springer-Verlag, 2001.

[6] A. A. Giunta, V. Balabanov, S. Burgee, B. Grossman, R.T. Haftka, W.H. Mason and L.T. Watson. Variable-complexity multidisciplinary design optimization using parallel computers. *Computational Mechanics'95 – Theory and Applications,* pp. 489-494, 1995. Editors: S.N. Alturi, G. Yagawa, T.A. Cruse. Springer, Berlin.

[7] V.V. Toropov and V.L. Markine. The use of simplified numerical models as mid-range approximations. *Proceedings of the 6-th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, pp. 952-958, 1996.

[8] W.H. Mason, R.T. Haftka, E. R. Johnson. Analysis and design of composite channel frames. *Proceedings of the 5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization.* AIAA Paper 94-4346-CP. Panama City, Florida, September 7-9, 1994. Vol. 2 pp. 1023-1040.

[9] S. Venkataraman, R.T. Haftka, T. F. Johnson. Design of shell structures for bucking using correction response surface approximations *Proceedings of 7th AIAA/USAF/NASA/ ISSMO Symposium on Multidisciplinary Analysis and Optimization.* St. Louis, Mo. AIAA 98-4855, September 2-4, 1988.

[10] R. Vitali, R.T. Haftka and B.V. Sankar. Correction response surface approximations for stress intensity factors of a composite stiffened plate. *Proceedings of the 39th AIAA/ASME/ASCE/AHS/ASC Structures Structural Dynamics, and Materials Conference.* Long Beach CA, AIAA-98-2047, PP. 2917-2922, April 20-23, 1998.

[11] A.A. Giunta. *Aircraft Multidisciplinary Design Optimization using Design of Experiments Theory and Response Surface Modelling Methods*. PhD thesis, Department of Aerospace and Ocean Engineering, Virginia, 1997.

[12] N. Alexandrov, J.E. Dennis, Jr., R. M. Lewis, and V. Torczon. A trust region framework for managing the use of approximation models in optimization. *Struc. Optim.* 15(1): 16-23, Feb. 1998.

[13] R.D. Braun and I.M. Kroo. Development and Application of the Collaborative Optimization Architecture in a Multidisciplinary Design Environment. *Multidisciplinary Design Optimization state-of-the-art,* N. Alexandrov and M.Y. Hassaini (Ed.), Proceedings in Applied Mathematics 80, SIAM, Philidelphia, 1995.

[14] G.E.P. Box and N.R. Draper. *Empirical Model Building and Response Surfaces: Design and Analysis.* New York, Marcel Dekker, Inc., 1987.

[15] V.V. Toropov and E. van der Giessen. Parameter identification for nonlinear constitutive models: Finite element simulation - optimization - nontrivial experiments. In: Pedersen, P. (ed.), *Optimal design with advanced materials - The Frithiof Niordson volume.* Proc. of IUTAM Symp., Elsevier Sci. Publ., 113-130, 1993.

[16] V.V. Toropov, V.L. Markine, P. Meijers, and J.P. Meijaard. Optimization of a dynamic system using multipoint approximations and simplified numerical model. In: Gutkowski, W., Mroz., Z. (eds.), *Proceedings of the Second World Congress of Structural and Multidisciplinary Optimization*, pp.613-618, 1997.

[17] P. Audze and V. Eglais. New approach for planning out of experiments. *Problems of Dynamics and Strengths* 35: 104-107, 1997 (in Russian).

[18] J. A. Nelder, R. Mead. A simplex method for minimization. *The Comp. Journal*, 7, 308-313, 1965.