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INDUSTRIAL APPLICATION

Weld optimization with stress constraints and thermal load

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Abstract We consider weld optimization with stress constraints and thermal load. Our interest for weld optimization is motivated by the development of exhaust aftertreatment systems, because most designs of these systems involve components that are assembled with welds and the designs are exposed to heat from the exhaust. We find that it is possible to optimize the welds with commercial software. An example is given in the end of the paper.

Keywords Weld · Topology optimization · Stress constraint · Thermal strain

1 Introduction

The purpose of this paper is to present a methodology for weld optimization of components where thermal strains are present. By weld optimization we refer to a calculation methodology to determine where the weld material should be located for a welded design. We consider the problem with topology optimization techniques. That is, we consider the maximum weld layout as a design domain where the density of weld material is allowed to vary between an upper and a lower bound.

Our experience from vehicle design is that a trial and error technique is time consuming. In particular, a lot of design work in the vehicle design involve several different load cases, which is difficult to manage manually. A systematic and automatic calculation methodology reduces the effort in this process. Thus, an aim with the weld optimization methodology is that it should save work for the engineer, and at the same time save cost and weld material.

One topic in this paper is how we include stress constraints in the weld optimization process. We suggest an engineering approach, where the weld is split in two domains, where one domain is used for monitoring the stress and the other domain is used as a design domain. The reason is that it is difficult to monitor the stress in the design domain in general (Bendsoe and Sigmund 2003; Duysinx and Bendsoe 1998). An example is given in the end, to illustrate the methodology. Commercial software is used for this purpose.

Our interest for weld optimization is motivated by the development of exhaust aftertreatment systems for vehicles. The exhaust aftertreatment systems are usually welded designs. They are exposed to heat from the exhaust and mechanical load from the operation of the vehicle.

In the next section we review some earlier work, and then we formulate the weld optimization problem with thermal strains more carefully. The last section presents an example to show the methodology.

Part of the results and methodology of this paper was presented on the Nordic HyperWorks Users Meeting 2011 by the author, see Rietz (2011).

2 Background

Topology optimization formulated as a problem of material distribution was considered in for example Bendsoe (1989) and Rossow and Taylor (1973), and is now a well established methodology. In this paper we find that this approach is also useful for optimization of welds. We note however, that weld optimization can be done late in the

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design process, while topology optimization is usually most profitable as one of the first steps in the design work.

We can consider two different types of weld optimization problems, depending on the equipment for welding. Either you can let the weld thickness vary between a higher and a lower value continuously, or otherwise you have to choose from welding or no welding. In the first case we have a continuous optimization problem, and in the second case we have a so called discrete optimization problem. A common method to solve the discrete material distribution problems is to study a continuous optimization problem and use some kind of penalization of intermediate densities. This is studied in Bendsoe and Sigmund (2003), Bendsoe (1989), Rietz (2000) and Stolpe and Svanberg (2001a) among others. We use the SIMP power law method for this purpose (Solid Isotropic Material with Penalization).

Material distribution problems are also well known to require some kind of regularization to give well defined solutions, see for example Borrvall (2001) or Rietz (2007). Otherwise the optimization problem is likely to end up with designs that require some kind of composite materials (Bendsoe 1989). To avoid this problem in weld optimization, we can for example require a minimum length of the welds. A seam weld length shorter than 1 cm is usually not preferred by the welding engineer in vehicle production.

Several challenges are involved with the introduction of stress constraints in the design domain. The problem is difficult to solve and to formulate (Bendsoe and Sigmund 2003; Rozvany and Birker 1994; Duysinx and Bendsoe 1998). For our application we suggest an engineering approach, which avoids some difficulties by considering the stresses in the neighboring structure of the design domain.

In this paper we model the welds with shell elements, and we study the stress in these finite elements to determine the mechanical integrity of the weld. However, we recommend that additional calculation should be done after the optimization to justify the mechanical integrity with more elaborate methods such as the effective notch stress methodology, fracture mechanics or the hot spot stress methods for critical welds (see Lassen and Rechom 2006). Another topic in this paper is to consider thermal strains. For more background on thermal strains we refer to books such as Boley and Weiner (1960).

3 Problem formulation

In this section we introduce some notation to formulate the assumptions and the problem. We consider a body that defines a domain Ω in $\mathbb{R}^3$ with boundary Γ. Let us denote the displacement at each position in Ω by $\mathbf{u}$, the temperature at each position by T and the density of material at each point by ρ. The temperature T is calculated with the Fourier equation, that is, the temperature in Ω fulfills

$$\frac{\partial T}{\partial t} - \lambda \nabla^2 T = 0 \tag{1}$$

with appropriate boundary conditions and where λ is the thermal diffusivity.

The (linearized) total strain tensor is denoted by $\boldsymbol{\epsilon}$, and its components are calculated from the displacements as

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

where $i, j \in \{1, 2, 3\}$ and x_i is the i:th coordinate. Let $\Delta T = T - T_0$ for some reference temperature T_0. The thermal strain tensor ϵ_{ij}^T is then defined as

$$\epsilon_{ij}^T = \alpha_{ij} \Delta T.$$

where α_{ij} is the thermal expansion coefficient tensor. The mechanical strain tensor ϵ_{ij}^M is calculated as

$$\epsilon_{ij}^M = \epsilon_{ij} - \epsilon_{ij}^T \tag{3}$$

The mechanical strain tensor is related to the stresses in the body. We denote the stress tensor by $\boldsymbol{\sigma}$. The relation between the mechanical strain and the stress tensor can be written

$$\boldsymbol{\sigma} = \mathbb{E}\boldsymbol{\epsilon}^M$$

for a linear elastic body, where $\mathbb{E}$ is the elasticity tensor. The elasticity tensor $\mathbb{E}$ depends in general on the density of material $\mathbb{E} = \mathbb{E}(\rho)$.

For our application, we assume that the stresses fulfill the static equilibrium, which can be written

$$\nabla\boldsymbol{\sigma} + \mathbf{F} = 0$$

for some volume force $\mathbf{F}$. The boundary conditions associated with this equation can for example be prescribed displacements or prescribed force $\mathbf{f}$ on part of the boundary Γ

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{f}$$

where $\mathbf{n}$ a unit normal directed out from Ω.

The temperature field gives thermal strains that interact with the mechanical strains according to (3). This can give rise to stresses that affect the fatigue life of the component. An example is when a thick plate and a thin plate are welded and exposed to a transient ambient temperature. Then the thin plate responds faster to the temperature change. The result is that the thermal expansion and contraction of the thin plate is prevented by the slower expansion and contraction of the thick plate. Thus stresses appear in the weld that attaches the plates.

In our case, the body Ω is a component with welds, and we would like to find an optimal distribution of the weld material in some part of Ω so that the stresses in the welds are below some acceptance value C. The optimization problem that we consider can be formulated as

$$\begin{cases} \min_\rho & \int_\Omega \mathbf{F}\cdot\mathbf{u}d\Omega + \int_\Gamma \mathbf{f}\cdot\mathbf{u}d\Gamma \\ \text{such that} & \max_{\mathbf{f},\mathbf{F},T} ||\boldsymbol{\sigma}|| \le C \\ & \int_\Omega \rho d\Omega \le V_{\max} \\ & \rho_{\min} \le \rho \le \rho_{\max} \end{cases} \tag{4}$$

for some positive constants $V_{\max}$, $\rho_{\min}$, $\rho_{\max}$ and C. The objective function

$$\int_\Omega \mathbf{F}\cdot\mathbf{u}d\Omega + \int_\Gamma \mathbf{f}\cdot\mathbf{u}d\Gamma$$

is called the compliance of the design with respect to some loads $\mathbf{F}$ and $\mathbf{f}$, and $\mathbf{u}$ is the corresponding displacement. The compliance is a general measure of the stiffness of the design. It corresponds well with the aim of the weld optimization, since the most common reason for the weld is to attach the parts in the design as thoroughly as possible. By $\max_{\mathbf{f},\mathbf{F},T}$ we mean the maximum over a certain number of interesting load cases and temperature fields, and we put a limit on the stresses in these load cases. For our application, we also note that ρ is interpreted as a thickness of the weld, rather than a material density, since we model welds with shell elements.

To simplify the optimization in our example, we assume that the temperature T does not depend on the distribution of material ρ in the design domain. Our experience from real applications is that we should keep a 10–20% margin in the stress constraints associated with the thermal strains if we use this approximation. For simple example problems where the heat transfer of the welds are of importance to the temperature distribution we find that the stresses from thermal strains can differ 30–40% between the approximated temperature field and the temperature field corresponding to the updated weld design. However, it is conservative to compare the approximated temperature with the results from a temperature calculation where the heat transfer is completely omitted at positions without weld, because we note that there is some heat transfer between steel plates at close distance by radiation and convection by air.

Clearly, the approximate temperature field is a restriction in the methodology to cases where the heat transfer in the welds is low. The main reason we make the approximation is however that we are able to use our existing commercial software, and the gain in accuracy with an updated temperature field has not yet motivated us to develop software or search for better commercial software. Our strategy is thus to calculate a temperature field with maximum weld length, and then use that temperature field for the optimization, as demonstrated in the example in the next section.

We are most interested in the discretized weld optimization problem, where we have to choose maximum weld thickness $\rho = \rho_{\max}$ or minimum weld thickness $\rho_{\min}$. The penalty method SIMP (Solid Isotropic Material with Penalization) solves the discrete problem by considering a continuous optimization problem where the intermediate densities are made less favorable. This is done by considering the elasticity tensor

$$\mathbb{E}_P(\rho) = \mathbb{E}\left(\rho_{\min} + (\rho_{\max} - \rho_{\min})\left(\frac{\rho - \rho_{\min}}{\rho_{\max} - \rho_{\min}}\right)^P\right) \tag{5}$$

where P is a penalty exponent. We usually chose $1 \le P \le 5$, where $P = 1$ gives the continuous optimization problem. The SIMP method is a common methodology, and considered more detail in for example Bendsoe (1989), Rietz (2000) and Stolpe and Svanberg (2001a).

It is generally difficult to solve topology optimization problems with local stress constraints, and one of the main reasons is that the stress constraint for the discrete optimization problem is difficult to translate to a constraint that is easy to use in the optimization for the continuous problem with elasticity tensor (5). The formulation of the stress constraint for the continuous optimization problem is discussed in for example Duysinx and Bendsoe (1998). One of the simplest formulations with accurate asymptotic behavior as $\rho_{\min}$ decreases to zero and which does not artificially favor intermediate thicknesses is given by

$$\max_{f,F,T}\left\|\boldsymbol{\sigma}\left(\frac{\rho - \rho_{\min}}{\rho_{\max} - \rho_{\min}}\right)^{-P}\right\| \le C \tag{6}$$

for $\rho > \rho_{\min}$. It is important that (6) is well behaved for intermediate thicknesses, so that the solution avoid these thicknesses in the search of a solution to the discrete problem.

The constraint (6) is however not well suited to consider with gradient based optimization algorithms, since the constraint is only valid for $\rho > \rho_{\min}$ which means that $\rho = \rho_{\min}$ is a singular case. The degeneracy of the feasible design space is considered in more detail in for example Rozvany and Birker (1994), Duysinx and Bendsoe (1998) and Bendsoe and Sigmund (2003). This means that special algorithms have been developed to solve the problem, and most common is to consider a sequence of perturbed problems, see

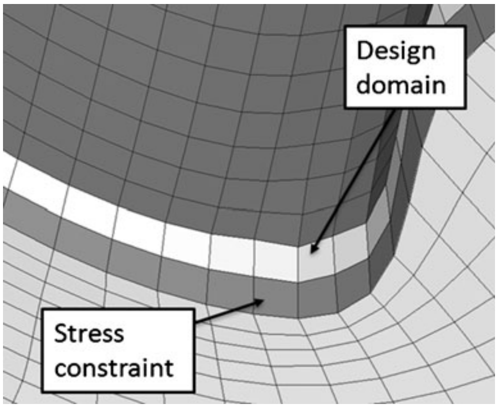


Fig. 1 Weld model. We split the welds in two domains, where one part is a design domain and another part is a domain where the stresses are monitored

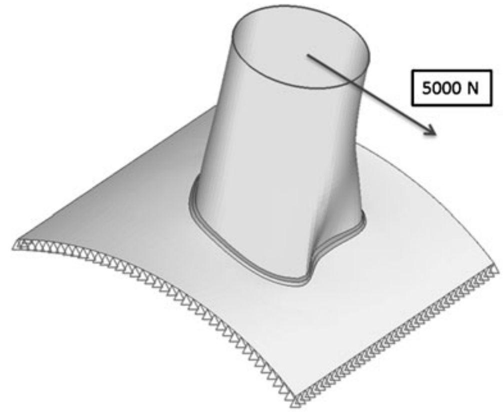


Fig. 3 The static load, a 5 kN load acts on the rim of the tube, and the plate is fixed on all edges

for example Petersson (2001). The optimal solutions to the perturbed problems converge as the perturbation decreases to a solution of the optimization problem (4) with stress constraint (6). The convergence of one such method is presented in Petersson (2001), however Stolpe and Svanberg (2001b) shows that local optima for the regularized problem makes the methodology unreliable even for some simple problems, as we are likely to end up in a local optimum.

In this paper we suggest an engineering approach for monitoring the stresses, instead of monitoring the stress in the design domain. The method is to split the welds in two domains, where one part is a design domain and the other part is a domain where the stresses are monitored. This is illustrated in Fig. 1 for the example in the next section. The method makes it possible to use optimization software that lacks functionality for constraining stress in the design domain, and in general we make the problem more well posed for the optimization algorithm which means that we increase the possibility for the algorithm to deliver a competitive design. We note also that the mechanical integrity of welds is often determined by considering the stress levels in the surrounding structure rather than in the actual weld, for example in the hot-spot stress methodology (Lassen and Rechom 2006), which in some sense justifies that we consider the stresses in the neighboring elements.

To solve the topology optimization problem with local stress constraints we have to use optimization algorithms that efficiently treat many constraints in the optimization. Usually some type of constraint screening is used, where the objective is to determine which constraints are most important to consider for each iteration. Simpler constraints makes also the constraint screening easier, so the engineering approach presented above decreases most likely also the computational effort.

4 Example problem

In this section we consider a computational problem, to illustrate methodology with the weld model presented in Fig. 1. We consider the example problem presented in Fig. 2.

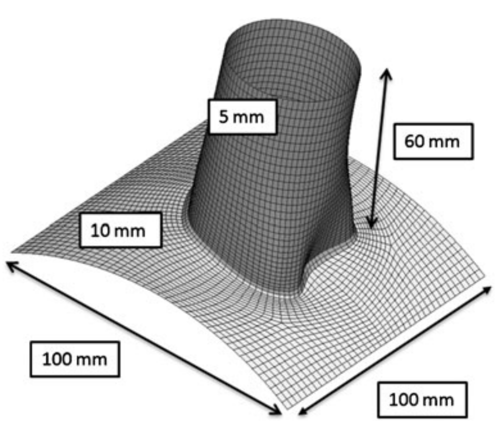


Fig. 2 Calculation model. A 5 mm thick tube is welded to a 10 mm thick plate

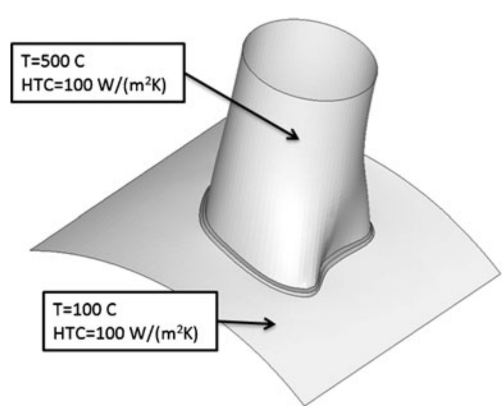


Fig. 4 Thermal boundary conditions. The ambient temperature of the tube is 500°C and the ambient temperature of the plats is 100°C

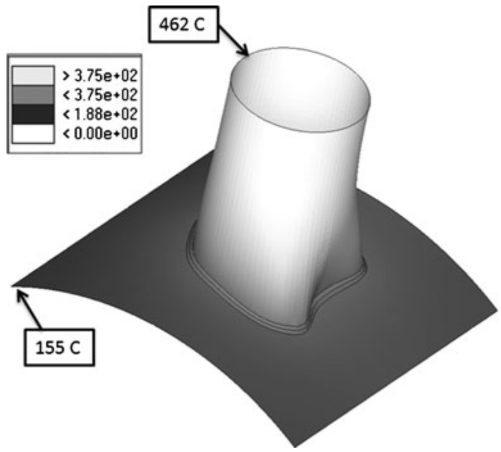


Fig. 5 Temperatures in the design after heat transfer calculation, steady state. The maximum temperature is 462°C at the rim of the tube

The example is made to present the methodology, and does not represent any existing design. We use the commercial software Optistruct v 10 for the optimization, and ABAQUS v 6.10 for the temperature calculation. The temperature data is transfered to Optistruct as text input as explained in Rietz (2011). The ABAQUS software admits several different boundary conditions for the thermal calculation, such as convection, conductivity, heat loss and radiation. In this example we consider only a convective boundary condition. Stress constraints in the design domain are possible for static loads in Optistruct v 10, however the software lacks functionality to put stress constraints in the design domain with thermal loads.

We consider a 5-mm thick tube that is welded to a 10-mm thick plate. The design is exposed to some static load as illustrated in Fig. 3. Convection from the surrounding heats the design, as presented in Figs. 4 and 5, and in this example we consider the thermal field that is the steady state temperature with these boundary conditions. The fixed boundary conditions (Fig. 3) are released in the thermal loadcase, so that the stresses from the thermal expansion is only caused by the temperature difference between the plate and tube.

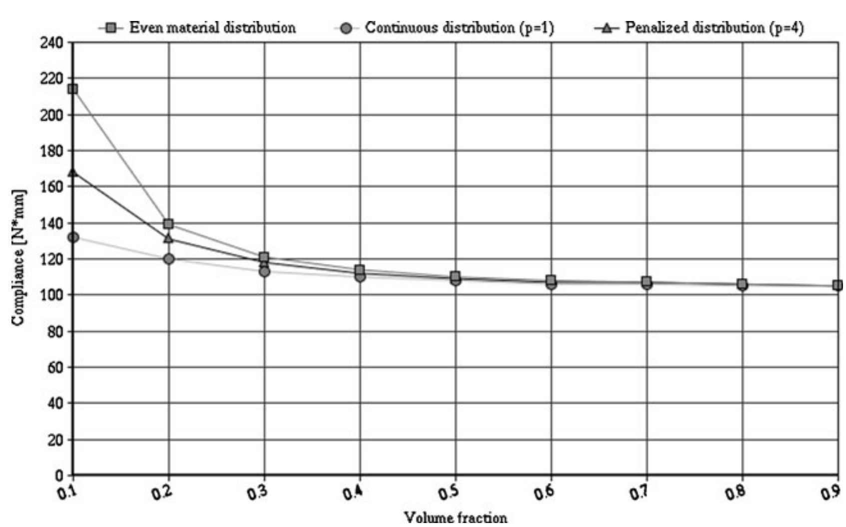
We use a steel material for the design, with Young's modulus $2.1 \cdot 10^{11}$ Pa, Poisson number 0.3, thermal expansion 10^{-5}, density 7,800 kg/m^3, specific heat 500 J/Kkg and thermal conductivity 10 W/Km.

In this example, we consider 100 MPa as an upper bound for the stress in the static loadcase, and 175 MPa as an upper bound for the stress in the thermal loadcase. The reason that we allow higher stress for the thermal loadcase is that the number of load cycles is lower for thermal loads than static loads in our applications. The weld model is presented in Fig. 1, and the design domain and the part intended for monitoring and constraining the stress consist of 80 finite elements each.

Figure 6 presents the minimum compliance results from the optimization when the fraction of weld in the design domain is increased from 0.1 to 0.9. We consider both a continuous optimization problem and a penalized problem with the SIMP exponent P equal to 4. The static stress constraint is harder to fulfill for low weld fractions, and the thermal stress constraint is most difficult for large weld fractions. We note that the maximum static stress is close to 100 MPa for weld fractions up to 0.3 in this example, and we find that the optimization program is not able to produce any solution for tested weld fractions above 0.9 because the stress from thermal strains was exceeding 175 MPa. That is, the program does not use all available welding material for volume fractions above 0.9. From Fig. 6 we find also that the gain in stiffness of the design is small when more weld material is added to welds with weld fractions larger than 0.5.

Figure 7 presents an optimization result with the weld fraction 0.5. The maximum stress from thermal strains with the approximated temperature is 175 MPa and the maximum static stress is 80.5 MPa. The heat transfer in the welds

Fig. 6 Compliance results for optimized designs at different weld fractions (that is the volume fraction of material in the weld)



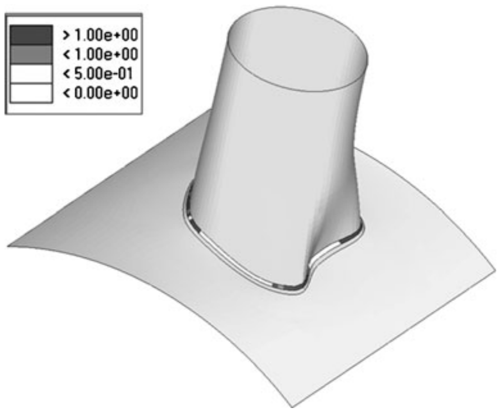


Fig. 7 Optimization results. Positions where the welds should be kept are displayed in *black*. The penalization exponent P is 4, and we consider a weld fraction of 0.5

was important in the temperature calculation in this example, so the main purpose with the example was to present the general methodology. In fact, a reference calculations with a temperature field calculated with an updated weld design gives a 36% increase in thermal stress. However, the updated temperature field assumes that the heat transfer is omitted at locations where the weld is discarded, which is a crude approximation.

5 Discussion

We find that the presented methodology with commercial software has limitations, for example the temperature field is not updated during the optimization. However, the gain with the optimization methodology compared to a trial-and-error method is considerable, both in terms of dealing with several loadcases and in terms of reduced work for the design engineer. Our experience is that the method provides us with a competitive design.

We should note that the optimization algorithm solves the problem within the premises of the optimization problem. For example it is possible that the algorithm focuses too much on problematic positions. Such position should otherwise be considered separately, and solved in some other way with a redesign, modified thermal load or additional reinforcement instead of a modified weld layout. Thus, understanding of the optimization problem and engineering monitoring of the optimization is preferable.

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