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Design Sensitivity Analysis of Structure-Induced Noise and Vibration

A continuum design sensitivity analysis (DSA) method for dynamic frequency responses of structural-acoustic systems is developed using the adjoint variable and direct differentiation methods. A variational approach with a non-self-adjoint operator for complex variables is used to retain the continuum elasticity formulation throughout derivation of design sensitivity results. It is shown that the adjoint variable method is applicable to the variational equation with the non-self-adjoint operator. Sizing design variables such as the thickness and cross-sectional area of structural components are considered for the design sensitivity analysis. A numerical implementation method of continuum DSA results is developed by postprocessing analysis results from established finite element analysis (FEA) codes to obtain the design sensitivity of noise and vibration performance measures of the structural-acoustic systems. The numerical DSA method presented in this paper is limited to FEA and boundary element analysis (BEA) is not considered. A numerical method is developed to compute design sensitivity of direct and modal frequency FEA results. For the modal frequency FEA method, the numerical DSA method provides design sensitivity very efficiently without requiring design sensitivities of eigenvectors. The numerical method has been tested using passenger vehicle problems. Accurate design sensitivity results are obtained for analysis results obtained from established FEA codes.

1 Introduction

Interior noise and structural vibration of motorized vehicles, such as automobiles, aircraft and marine vehicles, are of increasing significance due to the lightweight design of these structures (Dowell, 1980, and Flanigan and Borders, 1984). Vibration of a structural component can be undesirable either because of excessive vibration levels or because the vibration produces sound waves in adjacent fluid regions. For instance, noise in an automobile interior occurs because forces transmitted from the suspension and powertrain excite the vehicle compartment boundary panels. The variational formulation (Gladwell and Zimmermann, 1966) of the structural-acoustic system and recent developments in FEA (Nefske et al., 1982) provide reliable solutions, thus encouraging the study of DSA and optimization.

There are several published works on DSA and optimization of vibrating structures. Mroz (1970) used a variational principle to derive necessary and sufficient conditions for optimal design. Lekszycki and Olhoff (1981) derived a general set of necessary conditions for optimal design of one-dimensional, viscoelastic structures acted on by harmonic loads. A non-self-adjoint operator was used by means of variational analysis and the concept of complex stiffness modulus was adopted. Yoshimura performed (1983) DSA of the structural frequency response of machine structures and presented a numerical example of design sensitivity using a simplified structural model of a lathe. Lekszycki and Mroz (1983) extended their previous work to find necessary conditions for optimal support reactions to minimize stress and displacement amplitudes. A variational approach with a non-self-adjoint operator was used to consider a one-dimensional viscoelastic structure subject to harmonic loads. Choi and Lee (1992) developed a continuum DSA method of dynamic frequency responses of structural systems using the adjoint variable and direct differentiation methods. A variational approach with a non-self-adjoint operator for complex variables was used

to retain the continuum elasticity formulation throughout derivation of design sensitivity expressions.

Discrete methods of DSA of the structural-acoustic system based on the finite element formulation were presented recently. Brama (1990) applied the semi-analytic approach and presented implementations with FEA. Hagiwara et al. (1991) developed a DSA based on the modal frequency analysis of the structural-acoustic system.

In this paper, a continuum DSA method for dynamic frequency responses of structural-acoustic systems is developed using the non-self-adjoint operator for complex variables to define the complex adjoint system. The continuum DSA results can be numerically implemented outside established FEA codes (Choi and Lee, 1992; Haug et al., 1986; Choi et al., 1987) using postprocessing data, since it does not require derivatives of the stiffness, damping, and mass matrices.

2 Variational Formulation of a Structural-Acoustic System

A structural-acoustic system with a fully enclosed volume is shown in Fig. 1. All members of the structure are assumed to be plates and/or beams in three-dimensional space. The structure encloses a three-dimensional fluid region whose dynamic response is coupled to that of the structure.

The coupled dynamic motion of the structure and acoustic medium can be described using the following system of differential equations (Dowell et al., 1977):

Structure:

$$m(x, u)z_{tt}(x, u, t) + C_u z_t(x, u, t) + A_u z(x, u, t)$$
$$= f(x, u, t) + f_p(x, t), \quad x \in \Omega^s, \quad t \geq 0 \quad (1)$$

with the boundary condition

$$Gz = 0, \quad x \in \Gamma^s \quad (2)$$

and the initial condition

$$z(x, u, 0) = z_t(x, u, 0) = 0, \quad x \in \Omega^s \quad (3)$$

Fig. 1 Structural-acoustic system

$$f(x, u, t) = \text{Re}\{\mathbf{f}(x, u)e^{i\omega t}\}$$

$$z(x, u, t) = \text{Re}\{\mathbf{z}(x, u)e^{i\omega t}\}$$

$$p(x, u, t) = \text{Re}\{\mathbf{p}(x, u)e^{i\omega t}\} \tag{9}$$

where $\mathbf{f}$, $\mathbf{z}$, and $\mathbf{p}$ are complex phasors that are independent of time. Then, Eqs. (1)–(8) can be reduced to the following time-independent system of equations:

Structure:

$$D_u \mathbf{z} \equiv -\omega^2 m(x, u)\mathbf{z} + i\omega C_u \mathbf{z} + A_u \mathbf{z}$$

$$= \mathbf{f}(x, u) + \mathbf{f}_p(x), \quad x \in \Omega^s \tag{10}$$

with the boundary condition

$$G\mathbf{z} = 0, \quad x \in \Gamma^s \tag{11}$$

Acoustic Medium:

$$B\mathbf{p} \equiv -\frac{\omega^2}{\beta}\mathbf{p} - \frac{1}{\rho_0}\nabla^2\mathbf{p} = 0, \quad x \in \Omega^a \tag{12}$$

with the boundary condition

$$\nabla \mathbf{p}^T n = 0, \quad x \in \Gamma^{ar} \tag{13}$$

Interface Conditions:

$$\mathbf{f}_p = \mathbf{p}n, \quad x \in \Gamma^{as} \equiv \Omega^s \tag{14}$$

and

$$\nabla \mathbf{p}^T n = \omega^2 \rho_o \mathbf{z}^T n, \quad x \in \Gamma^{as} \equiv \Omega^s \tag{15}$$

The non-self-adjoint differential operator D_u in Eq. (10) depends on the design u explicitly, while the symmetric differential operator B in Eq. (12) does not because the shape of the acoustic medium is assumed to be fixed.

Define $\bar{\mathbf{z}}$ and $\bar{\mathbf{p}}$ as the kinematically admissible virtual states of the displacement $\mathbf{z}$ and pressure $\mathbf{p}$. The variational equation of Eqs. (10) and (12) can be obtained by multiplying both sides of Eqs. (10) and (12) by the transpose of complex conjugates $\bar{\mathbf{z}}^*$ and $\bar{\mathbf{p}}^*$ of $\bar{\mathbf{z}} \in Z$ and $\bar{\mathbf{p}} \in P$, respectively, integrating by parts over each physical domain, adding them, and using the boundary and interface conditions,

$$b_u(\mathbf{z}, \bar{\mathbf{z}}) - \iint_{\Gamma^{as}} \mathbf{p}\bar{\mathbf{z}}^{*T}nd\Gamma + d(\mathbf{p}, \bar{\mathbf{p}})$$

$$- \omega^2 \iint_{\Gamma^{as}} \bar{\mathbf{p}}^*\mathbf{z}^Tnd\Gamma = l_u(\bar{\mathbf{z}}) \tag{16}$$

which must hold for all kinematically admissible virtual states $\{\bar{\mathbf{z}}^*, \bar{\mathbf{p}}^*\} \in Q$ where Q is a complex vector space,

$$Q = \{(\mathbf{z}, \mathbf{p}) \in Z \otimes P | \mathbf{f}_p = \mathbf{p}n \quad \text{and}$$

$$\nabla \mathbf{p}^T n = \omega^2 \rho_o \mathbf{z}^T n, \quad x \in \Gamma^{as} \equiv \Omega^s\} \tag{17}$$

and

$$\left.\begin{array}{l} Z = \{\mathbf{z} \in [\mathbf{H}^2(\Omega^s)]^3 | G\mathbf{z} = 0, \quad x \in \Gamma^s\} \\ P = \{\mathbf{p} \in \mathbf{H}^1(\Omega^a) | \nabla \mathbf{p}^T n = 0, \quad x \in \Gamma^{ar}\} \end{array}\right\} \tag{18}$$

and $\mathbf{H}^1$ and $\mathbf{H}^2$ are complex Sobolev spaces of orders one and two, respectively (Adams, 1975). In Eq. (16), the sesquilinear forms $b_u(\bullet, \bullet)$ and $d(\bullet, \bullet)$, and semilinear form $l_u(\bullet)$ (Horvath, 1966) are defined, using complex L_2-inner product $(\bullet, \bullet)$ on a complex function space, as
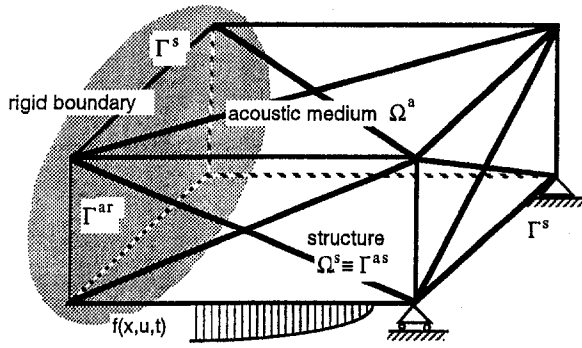

Acoustic Medium:

$$\frac{1}{\beta}p_{tt}(x, u, t) - \frac{1}{\rho_o}\nabla^2 p(x, u, t) = 0, \quad x \in \Omega^a, \quad t \geq 0 \tag{4}$$

with the boundary condition

$$\nabla p^T n = 0, \quad x \in \Gamma^{ar} \tag{5}$$

and the initial condition

$$p(x, u, 0) = p_t(x, u, 0) = 0, \quad x \in \Omega^a \tag{6}$$

Interface Conditions:

$$f_p(x, t) = p(x, t)n, \quad x \in \Gamma^{as} \equiv \Omega^s \tag{7}$$

and

$$\nabla p^T n = -\rho_0 z_{tt}^T n, \quad x \in \Gamma^{as} \equiv \Omega^s \tag{8}$$

Equation (1) describes structural vibration where Ω^s is the domain of the structure; $m(x, u)$ is the mass of the structure; C_u is the linear differential operator that corresponds to the damping of the structure; A_u is the fourth-order symmetric partial differential operator for the structure; $f(x, t, u)$ is the time dependent applied load; f_p is the acoustic pressure applied to the structure at the structure-acoustic medium interface; and n is the outward unit normal vector at the boundary of the acoustic medium. The design variable $u(x)$ is time-independent and the dynamic response $z(x, u, t) = [z_1, z_2, z_3]^T$ is the displacement field of the structure. The boundary condition of Eq. (2) is imposed on the structural boundary Γ^s using the trace operator G (Haug et al., 1986).

Equation (4) describes propagation of linear acoustic waves in the acoustic medium Ω^a where $\beta = \rho_o c_o^2$ is the adiabatic bulk modulus, ρ_o is the equilibrium density of the medium, and c_o is the acoustic velocity. The acoustic wave equation is modified to Eq. (4) to make an analogy to structural mechanics (MacNeal et al., 1980 and Flanigan and Borders, 1984). The dynamic response $p(x, u, t)$ is the acoustic or excess pressure. The normal gradient of the pressure vanishes at the rigid wall Γ^{ar} as shown in Eq. (5).

Structure-acoustic medium interaction can be seen in Eqs. (7) and (8). In Eq. (7), the structural load f_p is imposed by the acoustic pressure. Equation (8) is the interface condition that the normal gradient of the pressure is proportional to the normal component of the structural acceleration. As can be seen in Fig. 1, the structure-acoustic medium interface Γ^{as} is the domain Ω^s of the structure.

When the harmonic force $f(x, u, t)$ with a frequency ω is applied to the structure of the coupled system, the corresponding dynamic responses $z(x, u, t)$ and $p(x, u, t)$ are also harmonic functions with the same frequency ω. These can be represented using complex harmonic functions as

$$b_u(\mathbf{z}, \overline{\mathbf{z}}) \equiv (D_u \mathbf{z}, \overline{\mathbf{z}})$$

$$= -\iint_{\Omega^s} \omega^2 m \overline{\mathbf{z}}^{*T} \mathbf{z} d\Omega + i\omega c_u(\mathbf{z}, \overline{\mathbf{z}}) + a_u(\mathbf{z}, \overline{\mathbf{z}}) \quad (19)$$

where

$$c_u(\mathbf{z}, \overline{\mathbf{z}}) \equiv \iint_{\Omega^s} \overline{\mathbf{z}}^{*T} C_u \mathbf{z} d\Omega \quad \text{and}$$

$$a_u(\mathbf{z}, \overline{\mathbf{z}}) \equiv \iint_{\Omega^s} \overline{\mathbf{z}}^{*T} A_u \mathbf{z} d\Omega \quad (20)$$

$$d(\mathbf{p}, \overline{\mathbf{p}}) \equiv (B\mathbf{p}, \overline{\mathbf{p}})$$

$$= \iiint_{\Omega^a} \left(-\frac{\omega^2}{\beta} \mathbf{p}\overline{\mathbf{p}}^* + \frac{1}{\rho_0} \nabla \mathbf{p}^T \nabla \overline{\mathbf{p}}^* \right) d\Omega \quad (21)$$

and

$$l_u(\overline{\mathbf{z}}) \equiv \iint_{\Omega^s} \mathbf{f}^T \overline{\mathbf{z}}^* d\Omega \quad (22)$$

If there is no acoustic medium, then the variational Eq. (16) can be simplified by dropping all terms corresponding to the acoustic medium, including interface conditions, and the result will be the same as the variational equation obtained by Choi and Lee (1992).

3 Finite Element Analysis and Solution Methods

Structural-acoustic systems can be solved using FEA or BEA. In this paper, FEA is utilized (MSC/NASTRAN, 1991 and ABAQUS, 1989) for analysis. The variational equation of harmonic motion of the continuum model, Eq. (16), can be reduced to a set of linear algebraic equations by discretizing the model into finite elements and introducing shape functions and nodal variables for each element. The acoustic pressure $\mathbf{p}(x)$ and the structural displacement $\mathbf{z}(x)$ are approximated, using shape functions and nodal variables for each element of the discretized model, as

$$\left. \begin{array}{l} \mathbf{z}(x) = N(x)\mathbf{z}^e \\ \mathbf{p}(x) = L(x)\mathbf{p}^e \end{array} \right\} \quad (23)$$

where $N(x)$ and $L(x)$ are matrices of shape functions, and $\mathbf{z}^e$ and $\mathbf{p}^e$ are the element nodal variable vectors. Substituting Eq. (23) into Eq. (16) and carrying out integration will yield a matrix equation

$$\begin{bmatrix} [-\omega^2 M_{ss} + i\omega C_{ss} + K_{ss}] & [K_{sf}] \\ [-\omega^2 M_{fs}] & [-\omega^2 M_{ff} + K_{ff}] \end{bmatrix} \begin{bmatrix} \{\mathbf{z}\} \\ \{\mathbf{p}\} \end{bmatrix}$$

$$= \begin{bmatrix} \{\mathbf{f}\} \\ \{0\} \end{bmatrix} \quad (24)$$

where M_{ss}, C_{ss}, and K_{ss} are the mass, damping, and stiffness matrices of the structure, respectively, and $\{\mathbf{f}\}$ is the loading vector that can be obtained from Eq. (22). Similarly, M_{ff} and K_{ff} are, respectively, the equivalent mass and stiffness matrices of the acoustic medium. The coupling terms between the structure and acoustic medium are off-diagonal submatrices M_{fs} and K_{sf} in Eq. (24). These off-diagonal submatrices correspond to the coupling terms in Eq. (16). The global matrix in Eq. (24) is not symmetric because of the off-diagonal coupling submatrices.

In solving Eq. (24), efficiency is an important factor that cannot be overlooked in practical applications. Direct and modal frequency FEA methods can be used to solve the coupled equation. In the direct frequency FEA method, Eq. (24) is directly

solved as a linear algebraic equation with complex variables (ABAQUS, 1989). Even though the method is straightforward in application and gives very accurate solutions, it requires a large amount of computational time for repeated analyses of a large system at several frequencies and with several different loading conditions. The modal frequency FEA method is efficient and practical solution method for large size coupled system (Flanigan and Borders, 1984). In this method, a finite number of modes of the structure and acoustic medium are obtained independently, and a set of selected modes are used to diagonalize the mass and stiffness submatrices, even though the off-diagonal submatrices in Eq. (24) cannot be diagonalized in this process since the modes are not orthogonal with respect to the off-diagonal submatrices.

4 Design Sensitivity Analysis of Dynamic Frequency Response

4.1 Direct Differentiation Method. To develop the direct differentiation method of DSA, take the first variation of Eq. (16) with respect to design u and rearrange to obtain (Choi and Lee, 1992)

$$b_u(\mathbf{z}', \overline{\mathbf{z}}) - \iint_{\Gamma^{as}} \mathbf{p}' \overline{\mathbf{z}}^{*T} n d\Gamma + d(\mathbf{p}', \overline{\mathbf{p}})$$

$$- \omega^2 \iint_{\Gamma^{as}} \overline{\mathbf{p}}^* \mathbf{z}'^T n d\Gamma = l'_{\delta u}(\overline{\mathbf{z}}) - b'_{\delta u}(\mathbf{z}, \overline{\mathbf{z}}) \quad (25)$$

which must hold for all kinematically admissible virtual states $\{\overline{\mathbf{z}}^*, \overline{\mathbf{p}}^*\} \in Q$. In Eq. (25),

$$\mathbf{z}' \equiv \frac{d}{d\tau} \mathbf{z}(x, u + \tau\delta u)|_{\tau=0} \quad (26)$$

$$\mathbf{p}' \equiv \frac{d}{d\tau} \mathbf{p}(x, u + \tau\delta u)|_{\tau=0} \quad (27)$$

are the first variations of $\mathbf{z}$ and $\mathbf{p}$ with respect to design u in the direction δu of design change (Haug et al., 1986). Also, the first variations of the sesquilinear form b_u and semilinear form l_u with respect to explicit dependence on design u are

$$b'_{\delta u}(\mathbf{z}, \overline{\mathbf{z}}) \equiv \frac{d}{d\tau} b_{u+\tau\delta u}(\tilde{\mathbf{z}}, \overline{\mathbf{z}})|_{\tau=0} \quad (28)$$

$$l'_{\delta u}(\overline{\mathbf{z}}) \equiv \frac{d}{d\tau} l_{u+\tau\delta u}(\overline{\mathbf{z}})|_{\tau=0} \quad (29)$$

where $\tilde{\mathbf{z}}$ denotes the state $\mathbf{z}$ with dependence on τ (design variable) suppressed. Equation (25) is a variational equation in which the design sensitivities $\mathbf{z}'$ and $\mathbf{p}'$ are unknowns.

If the solution $\mathbf{z}$ of Eq. (16) is obtained using the FEA Eq. (24), the fictitious load that is the right side of Eq. (25) can be computed, using the shape functions of the finite element to evaluate integrands at Gauss points (Cowper, 1973) and integrate numerically. The same FEA Eq. (24) can be used with the fictitious load to solve for $\mathbf{z}'$ and $\mathbf{p}'$. This yields the direct differentiation method of DSA. The method is applicable to both the direct and modal frequency FEA methods. Moreover, for the modal frequency FEA method, the numerical DSA method provides design sensitivity without requiring design sensitivities of eigenvectors. That is, the modal superposition method and shape functions of the finite element can be used to compute the fictitious load in Eq. (25) by evaluating integrands at Gauss points and integrating numerically.

4.2 Adjoint Variable Method. Harmonic performance measures of the structural-acoustic system can be expressed in terms of complex phasors of the structural displacement and the acoustic pressure. For the adjoint variable method, first con-

sider the pressure at a point $\hat{x}$ in the acoustic medium enclosed by the structure under harmonic excitation

$$\psi_p = \iiint_{\Omega^a} \delta(x - \hat{x})\mathbf{p}\,d\Omega \qquad (30)$$

The first variation of the performance measure is

$$\psi_p' = \iiint_{\Omega^a} \delta(x - \hat{x})\mathbf{p}'\,d\Omega \qquad (31)$$

To use the adjoint variable method, define a conjugate operator D_u^a of the non-self-adjoint operator D_u in Eq. (10), as

$$(D_u\mathbf{z}, \lambda) \equiv (\mathbf{z}, D_u^a\lambda) \qquad (32)$$

which must hold for all $\mathbf{z}$, $\lambda \in Z$. That is,

$$D_u^a\lambda \equiv -\omega^2 m(x, u)\lambda - i\omega C_u\lambda + A_u\lambda \qquad (33)$$

Then, using the definitions of $b_u(\bullet, \bullet)$ in Eq. (19) and $d(\bullet, \bullet)$ in Eq. (21), we obtain

$$b_u(\bar{\lambda}, \lambda) = (\bar{\lambda}, D_u^a\lambda) = -\iint_{\Omega^s} \omega^2 m\bar{\lambda}^T\lambda^*\,d\Omega$$

$$+ i\omega c_u(\bar{\lambda}, \lambda) + a_u(\bar{\lambda}, \lambda) \qquad (34)$$

and

$$d(\bar{\eta}, \eta) = (\bar{\eta}, B\eta)$$

$$= \iiint_{\Omega^a} \left[-\frac{\omega^2}{\beta} \bar{\eta}\eta^* + \frac{1}{\rho_0} \nabla\bar{\eta}^T \nabla\eta^* \right] d\Omega \qquad (35)$$

To obtain the design sensitivity in Eq. (31) explicitly in terms of perturbations of the design variable, define an adjoint equation for the performance measure of Eq. (30) by replacing $\mathbf{p}'$ in Eq. (31) by a virtual pressure $\bar{\eta}$ and equating the term to the sesquilinear forms as

$$b_u(\bar{\lambda}, \lambda) - \iint_{\Gamma^{as}} \bar{\eta}\lambda^{*T}n\,d\Gamma + d(\bar{\eta}, \eta) - \omega^2 \iint_{\Gamma^{as}} \eta^*\bar{\lambda}^T n\,d\Gamma$$

$$= \iiint_{\Omega^a} \delta(x - \hat{x})\bar{\eta}\,d\Omega \qquad (36)$$

which must hold for all kinematically admissible virtual states $\{\bar{\lambda}, \bar{\eta}\} \in Q$. It is very important to note that the solution of Eq. (36) is the complex conjugate $\{\lambda^*, \eta^*\}$ of the adjoint response $\{\lambda, \eta\}$. To take advantage of the adjoint equation, we may evaluate Eq. (36) at $\bar{\lambda} = \mathbf{z}'$ and $\bar{\eta} = \mathbf{p}'$, to obtain

$$b_u(\mathbf{z}', \lambda) - \iint_{\Gamma^{as}} \mathbf{p}'\lambda^{*T}n\,d\Gamma + d(\mathbf{p}', \eta)$$

$$- \omega^2 \iint_{\Gamma^{as}} \eta^*\mathbf{z}'^T n\,d\Gamma = \iiint_{\Omega^a} \delta(x - \hat{x})\mathbf{p}'\,d\Omega \qquad (37)$$

which is the term on the right of Eq. (31) that we would like to write explicitly in terms of δu. Similarly, evaluate Eq. (25) at $\bar{\mathbf{z}}^* = \lambda^*$ and $\bar{\mathbf{p}}^* = \eta^*$ to obtain

$$b_u(\mathbf{z}', \lambda) - \iint_{\Gamma^{as}} \mathbf{p}'\lambda^{*T}n\,d\Gamma + d(\mathbf{p}', \eta)$$

$$- \omega^2 \iint_{\Gamma^{as}} \eta^*\mathbf{z}'^T n\,d\Gamma = l_{\delta u}'(\lambda) - b_{\delta u}'(\mathbf{z}, \lambda) \qquad (38)$$

Since the left sides of Eqs. (37) and (38) are equal, the desired explicit design sensitivity expression can be obtained from Eqs. (31), (37), and (38),

$$\psi_p' = l_{\delta u}'(\lambda) - b_{\delta u}'(\mathbf{z}, \lambda)$$

$$= \iint_{\Omega^s} \mathbf{f}_u^T\lambda^*\delta u\,d\Omega + \iint_{\Omega^s} \omega^2 m_u\lambda^{*T}\mathbf{z}\delta u\,d\Omega$$

$$- i\omega c_{\delta u}'(\mathbf{z}, \lambda) - a_{\delta u}'(\mathbf{z}, \lambda) \qquad (39)$$

To evaluate the design sensitivity of Eq. (39), the solution λ^* of Eq. (36), which is the complex conjugate of λ, must be used. For this, the same FEA Eq. (24) can be used with the adjoint load to solve for $\{\lambda^*, \eta^*\}$. Like the direct differentiation method, this method is applicable to both the direct and modal frequency FEA methods and, for the modal frequency FEA method, the numerical DSA method provides design sensitivity without requiring design sensitivities of eigenvectors.

Another performance measure of the structural-acoustic system is the structural displacement at a point $\hat{x}$. For instance, the performance measure could be the vibration amplitude at a seat of the passenger vehicle, aircraft, or ship. The performance measure can be written as

$$\psi_{z_i} = \iint_{\Omega^s} \delta(x - \hat{x})\mathbf{z}_i\,d\Omega, \quad i = 1, 2, 3 \qquad (40)$$

The adjoint equation for this performance measure is defined as

$$b_u(\bar{\lambda}, \lambda) - \iint_{\Gamma^{as}} \bar{\eta}\lambda^{*T}n\,d\Gamma + d(\bar{\eta}, \eta) - \omega^2 \iint_{\Gamma^{as}} \eta^*\bar{\lambda}^T n\,d\Gamma$$

$$= \iint_{\Omega^s} \delta(x - \hat{x})\bar{\lambda}_i\,d\Omega \qquad (41)$$

which must hold for all kinematically admissible virtual states $\{\bar{\lambda}, \bar{\eta}\} \in Q$. Once the complex conjugate λ^* of the adjoint response is obtained from Eq. (41), the same design sensitivity expression of Eq. (39) can be used to obtain design sensitivity information. Also, since sizing design u is defined only on the structural part, Eq. (39) requires only the structural response λ^* of the adjoint Eqs. (36) or (41). The design sensitivity result of Eq. (39) is general since it is valid for structural systems without the acoustic medium.

5 Design Components

To use the continuum DSA method, the first variations of the sesquilinear and semilinear forms must be derived for each structural design component so that these can be used to evaluate the design sensitivity of Eq. (39). In this paper, structures with structural damping are considered.

5.1 Beam Design Component. The sesquilinear form of the beam design component of length L and structural damping coefficient φ is (Choi and Lee, 1992)

$$b_u(\mathbf{z}, \bar{\mathbf{z}}) = -\int_0^L \omega^2 (\rho h \sum_{i=1}^{3} \mathbf{z}_i\bar{\mathbf{z}}_i^* + J\mathbf{z}_4\bar{\mathbf{z}}_4^*)\,dx_1$$

$$+ (1 + i\varphi) \int_0^L (Eh\mathbf{z}_{1,1}\bar{\mathbf{z}}_{1,1}^* + EI_3\mathbf{z}_{2,11}\bar{\mathbf{z}}_{2,11}^*$$

$$+ EI_2\mathbf{z}_{3,11}\bar{\mathbf{z}}_{3,11}^* + GJ\mathbf{z}_{4,11}\bar{\mathbf{z}}_{4,1}^*)\,dx_1 \qquad (42)$$

where $\mathbf{z}_1$, $\mathbf{z}_2$, $\mathbf{z}_3$, and $\mathbf{z}_4$, are the axial displacement, two orthogonal lateral displacements, and the angle of twist, respectively, and $\mathbf{z} = [\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4]^T$. In Eq. (42), ρ is the mass density, E is Young's modulus, G is shear modulus, and h, I_2, I_3, and J are the cross-sectional area, two moments of inertia and the torsional moment of inertia, respectively. The semilinear form of external loads is

$$l_u(\bar{\mathbf{z}}) = \int_0^L (\sum_{i=1}^{3} \mathbf{f}_i\bar{\mathbf{z}}_i^* + T_1\bar{\mathbf{z}}_4^* + M_2\bar{\mathbf{z}}_{3,1}^* + M_3\bar{\mathbf{z}}_{2,1}^*)\,dx_1 \qquad (43)$$

where $\mathbf{f}_1$, $\mathbf{f}_2$, and $\mathbf{f}_3$ are the axial and two orthogonal lateral

harmonic loads, respectively. Also, T_1 is the harmonic torque and M_2 and M_3 are two harmonic moments.

The first variations of the sesquilinear and semilinear forms of Eqs. (42) and (43) can be obtained by taking the first variations of Eqs. (42) and (43) with respect to explicit dependency on design u as (Choi and Lee, 1992)

$$b'_{\delta u}(\mathbf{z}, \overline{\mathbf{z}}) = -\int_0^L \omega^2 (\rho h_{,u} \sum_{i=1}^3 \mathbf{z}_i \overline{\mathbf{z}}_i^* + J_{,u} \mathbf{z}_4 \overline{\mathbf{z}}_4^*) \delta u dx_1$$

$$+ (1 + i\varphi) \int_0^L (Eh_{,u} \mathbf{z}_{1,1} \overline{\mathbf{z}}_{1,1}^* + EI_{3,u} \mathbf{z}_{2,11} \overline{\mathbf{z}}_{2,11}^*$$

$$+ EI_{2,u} \mathbf{z}_{3,11} \overline{\mathbf{z}}_{3,11}^* + GJ_{,u} \mathbf{z}_{4,1} \overline{\mathbf{z}}_{4,1}^*) \delta u dx_1 \quad (44)$$

and

$$l'_{\delta u}(\overline{\mathbf{z}})$$

$$= \int_0^L (\sum_{i=1}^3 \mathbf{f}_{i,u} \overline{\mathbf{z}}_i^* + T_{1,u} \overline{\mathbf{z}}_4^* + M_{2,u} \overline{\mathbf{z}}_{3,1}^* + M_{3,u} \overline{\mathbf{z}}_{2,1}^*) \delta u dx_1 \quad (45)$$

where the subscript u denotes the derivative with respect to design u.

5.2 Plate Design Component.
The sesquilinear form of the plate design component with structural damping coefficient φ is (Choi and Lee, 1992)

$$b_u(\mathbf{z}, \overline{\mathbf{z}}) = -\int\int_\Omega \omega^2 \rho h \sum_{i=1}^3 \mathbf{z}_i \overline{\mathbf{z}}_i^* d\Omega$$

$$+ (1 + i\varphi) \int\int_\Omega \left[h \sum_{i,j=1}^2 \sigma^{ij}(\mathbf{v}) \epsilon^{ij}(\overline{\mathbf{v}}^*) \right.$$

$$\left. + \frac{h}{3} \sum_{i,j=1}^2 \sigma^{ij}(\mathbf{z}_3) \epsilon^{ij}(\overline{\mathbf{z}}_3^*) \right] d\Omega \quad (46)$$

where $\mathbf{z}_3$ is the lateral displacement due to bending and $\mathbf{v} = [\mathbf{z}_1, \mathbf{z}_2]^T$ is the in-plane displacement. For this design component, sizing design variable $u = h(x_1, x_2)$ is the thickness of the component. The semilinear form of external loads is

$$l_u(\overline{\mathbf{z}}) = \int\int_\Omega \sum_{i=1}^3 \mathbf{f}_i \overline{\mathbf{z}}_i^* d\Omega + \int_{\Gamma^2} \sum_{i=1}^2 T_i \overline{\mathbf{z}}_i^* d\Gamma \quad (47)$$

where $\mathbf{f}_1$ and $\mathbf{f}_2$ are two in-plane harmonic loads; $\mathbf{f}_3$ is the lateral harmonic load; and T_1 and T_2 are two in-plane harmonic traction loads applied at the traction boundary Γ^2.

The first variations of the sesquilinear and semilinear forms of Eqs. (46) and (47) can be obtained by taking the first variations of Eqs. (46) and (47) with respect to explicit dependency on design h as

$$b'_{\delta u}(\mathbf{z}, \overline{\mathbf{z}}) = -\int\int_\Omega \omega^2 \rho \sum_{i=1}^3 \mathbf{z}_i \overline{\mathbf{z}}_i^* \delta h d\Omega$$

$$+ (1 + i\varphi) \int\int_\Omega [\sum_{i,j=1}^2 \sigma^{ij}(\mathbf{v}) \epsilon^{ij}(\overline{\mathbf{v}}^*)$$

$$+ \frac{1}{3} \sum_{i,j=1}^2 \sigma^{ij}(\mathbf{z}_3) \epsilon^{ij}(\overline{\mathbf{z}}_3^*)] \delta h d\Omega \quad (48)$$

and

$$l'_{\delta u}(\overline{\mathbf{z}}) = \int\int_\Omega \sum_{i=1}^3 \mathbf{f}_{i,h} \overline{\mathbf{z}}_i^* \delta h d\Omega + \int_{\Gamma^2} \sum_{i=1}^2 T_{i,h} \overline{\mathbf{z}}_i^* \delta h d\Gamma \quad (49)$$

where the subscript h denotes the derivative of terms with respect to design h.
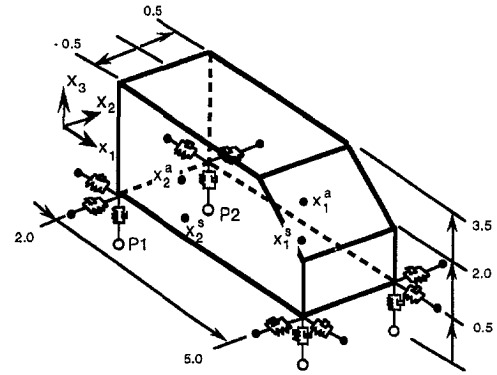

Body Structure   Acoustic Medium

Fig. 2 Simplified passenger vehicle and finite element models

6 Numerical Computations and Examples

For the adjoint variable method, the adjoint load for each performance measure needs to be computed. For the displacement performance measure, the adjoint load is a unit harmonic load applied on the structure at the node and degree of freedom for which the design sensitivity is to be computed. For the pressure performance measure, the adjoint load is the second time derivative of a unit volumetric strain at the point in the acoustic medium where the pressure performance measure is defined. The complex conjugates of the adjoint structural responses, Eqs. (36) and (41), can be solved efficiently by performing a restart of the FEA code. Using the original response and complex conjugate of adjoint structural response, the design sensitivity information can be obtained by evaluating the integrands of Eq. (39) at Gauss points (Cowper, 1973) using the shape functions of the finite element and by carrying out numerical integration. Computational procedures for continuum design sensitivity analysis can be found from Choi et al. (1987) and Haug et al. (1986). If the direct differentiation method is used, the fictitious load on the right of Eq. (25) is computed using the shape functions of the finite element and numerical integration. Equation (25) can also be solved efficiently by performing a restart of the FEA code. Two vehicle systems are used to demonstrate feasibility of the continuum DSA method.

6.1 Simplified Passenger Vehicle Model.
Lightweight unibody construction and similar automobile weight-saving efforts have increased interior noise, particularly noise in the low-frequency range. This low-frequency noise occurs over a wide range of vehicle speeds, and interior measurements show it to be dominant at frequencies between 20 and 200 Hz. Previous testing has shown strong correlation between panel motion and the measured noise (Kamal and Wolf, 1982).

A simplified passenger vehicle model that can be used to identify the system characteristics prior to a practical engineering model analysis of a vehicle system is shown in Fig. 2. The body structure is made of thin aluminum plates with uniform thickness that enclose the acoustic medium (air), and the structure is mounted on a simplified suspension system consisting of springs and dampers. The air has equilibrium density $\rho_0 = 0.1205$ kg/m^3 and adiabatic bulk modulus $\beta = 139298$ N/m^2. Material properties of the structure are Poisson's ratio $\nu = 0.334$, structural damping coefficient $\zeta = 0.06$, mass density

Table 1 Sizing design sensitivity of simplified passenger vehicle model acoustic frequency responses*

Frequency	Location		$\psi(b-\delta u)$	$\psi(b+\delta u)$	$\Delta\psi$	ψ'	$\psi'/\Delta\psi(\%)$
54 Hz	x_1^a	R	0.1002E-1	0.9630E-2	-.1961E-3	-.1978E-3	100.8
		I	0.1294E-1	0.1334E-1	0.1992E-3	0.2010E-3	100.9
		D	0.1637E-1	0.1645E-1	0.4200E-4	0.4254E-4	101.3
62 Hz	x_1^a	R	-.1729E-1	-.1651E-1	0.3874E-3	0.3914E-3	101.0
		I	-.1131E-1	-.1235E-1	-.5202E-3	-.5294E-3	101.8
		D	0.2066E-1	0.2062E-1	-.1900E-4	-.1700E-4	89.4
54Hz	x_2^a	R	-.1724E-2	-.1642E-2	0.4139E-4	0.3985E-4	96.3
		I	-.9126E-2	-.9387E-2	-.1303E-3	-.1310E-3	100.6
		D	0.9288E-2	0.9529E-2	0.1208E-3	0.1218E-3	100.8
62 Hz	x_2^a	R	0.6635E-2	0.5604E-2	-.5154E-3	-.5204E-3	101.0
		I	0.1315E-1	0.1387E-1	0.3621E-3	0.3677E-3	101.6
		D	0.1473E-1	0.1496E-1	0.1170E-3	0.1200E-3	102.6

* Unit : Pascal (Pa)

$\rho = 2700 \text{ kg/m}^3$, and Young's modulus of elasticity $E = 7.1 \times 10^{10} \text{ N/m}^2$. The thickness of the body panels is chosen as a design variable and the current design value is 0.01 m.

The finite element model in Fig. 2 includes 688 hexagonal and 32 tetrahedral acoustic elements, and 928 triangular structural plate elements for the body panels. Twelve spring elements and twelve viscous dampers support the structure in three directions at each attachment point. The rear suspension supports, P1 and P2, are excited with harmonic displacements in the x_3-direction with amplitudes of 1.0×10^{-4} m, and the front supports are fixed on the ground. Direct frequency response analysis of ABAQUS 4.9 (1989) is used for analysis of the primary and adjoint problems.

The predicted design sensitivity results of the harmonic responses at 54 and 62 Hz are compared with the central finite difference results. In Tables 1 and 2, $\psi(u - \delta u)$ and $\psi(u + \delta u)$ are the frequency responses of the perturbed designs $u - \delta u$ and $u + \delta u$, respectively, where δu is the amount of variation in design. The central finite difference is denoted by $\Delta\psi = (\psi(u + \delta u) - \psi(u - \delta u))/2$, and ψ' is the predicted design sensitivity. For the design variation, the study uses a perturbation of $\pm 1.0 \times 10^{-5}$ m in the body panel thickness. Table 1 shows the design sensitivity results for the acoustic pressures in Pascals (Pa) at points $x_1^a = (4.0, 0.25, 1.0)$ and $x_2^a = (3.0,$

$-0.25, 1.0)$ in the acoustic medium. Table 2 shows design sensitivity results for structural displacements, velocities, and accelerations in the x_3-direction at points $x_1^s = (4.0, 0.25, 0.5)$ and $x_2^s = (3.0, -0.25, 0.5)$ that are located on the floor panel. The unit of displacement is meter (m). In Tables 1 and 2, the real and the imaginary parts of complex phasors are denoted by R and I, respectively, and the magnitude is denoted by D, which is the amplitude of the harmonic response. Table 2 shows the design sensitivities of the velocity and acceleration amplitudes, V and A, as well as the structural displacement. Both tables show good agreement between the continuum DSA results and the finite difference results.

6.2 Large-scale Vehicle System. The continuum DSA method has been applied to large-scale vehicle models in an automotive industry using the results of the modal frequency FEA. Accurate design sensitivity predictions for acoustic and structural performance measures are obtained at critical frequencies. A large-scale detailed vehicle model shown in Fig. 3 has 70,000 elements and 500,000 degrees of freedom. Dynamic responses were calculated using super-elements and the modal frequency FEA method. To reduce the rear seat sound pressure level at 70 and 81 Hz, sensitivities of the rear seat sound pressure level near two frequencies were calculated with respect

Table 2 Sizing design sensitivity of simplified passenger vehicle model structural frequency responses in x_3-direction*

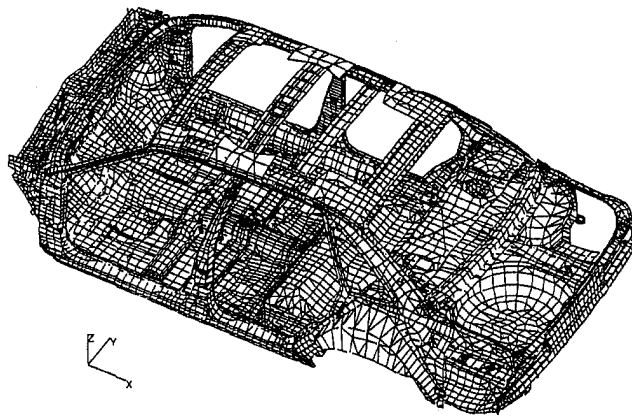| Frequency	Location		$\psi(b-\delta u)$	$\psi(b+\delta u)$	$\Delta\psi$	ψ'	$\psi'/\Delta\psi(\%)$
54 Hz	x_1^s	R	0.1070E-8	0.3797E-9	-.3454E-9	-.3465E-9	100.3
		I	-.3445E-7	-.3509E-7	-.3203E-9	-.3231E-9	100.9
		D	0.3446E-7	0.3509E-7	0.3130E-9	0.3159E-9	100.9
		V	0.1169E-4	0.1191E-4	0.1062E-6	0.1072E-6	100.9
		A	0.3967E-2	0.4039E-2	0.3603E-4	0.3637E-4	100.9
62 Hz	x_1^s	R	-.4879E-7	-.4777E-7	0.5085E-9	0.5083E-9	100.0
		I	-.3730E-7	-.3924E-7	-.9691E-9	-.9823E-9	101.4
		D	0.6141E-7	0.6182E-7	0.2035E-9	0.2118E-9	104.1
		V	0.2392E-4	0.2408E-4	0.7927E-7	0.8251E-7	104.1
		A	0.9320E-2	0.9381E-7	0.3088E-4	0.3214E-4	104.1
54 Hz	x_2^s	R	0.5631E-7	0.5635E-7	0.2412E-10	0.2134E-10	88.5
		I	0.1242E-6	0.1266E-6	0.1195E-8	0.1210E-8	101.3
		D	0.1364E-6	0.1386E-6	0.1100E-8	0.1113E-8	101.2
		V	0.4626E-4	0.4701E-4	0.3732E-6	0.3776E-6	101.2
		A	0.1570E-1	0.1595E-1	0.1266E-3	0.1281E-3	101.2
62 Hz	x_2^s	R	0.2030E-7	0.1734E-7	-.1481E-8	-.1499E-8	101.2
		I	0.9691E-7	0.9793E-7	0.5076E-9	0.5203E-9	102.5
		D	0.9902E-7	0.9945E-7	0.2175E-9	0.2264E-9	104.1
		V	0.3857E-4	0.3874E-4	0.8473E-7	0.8818E-7	104.1
		A	0.1503E-1	0.1509E-1	0.3301E-4	0.3435E-4	104.1

* Unit : Meter (m)

Fig. 3 Finite element of large-scale passenger vehicle body

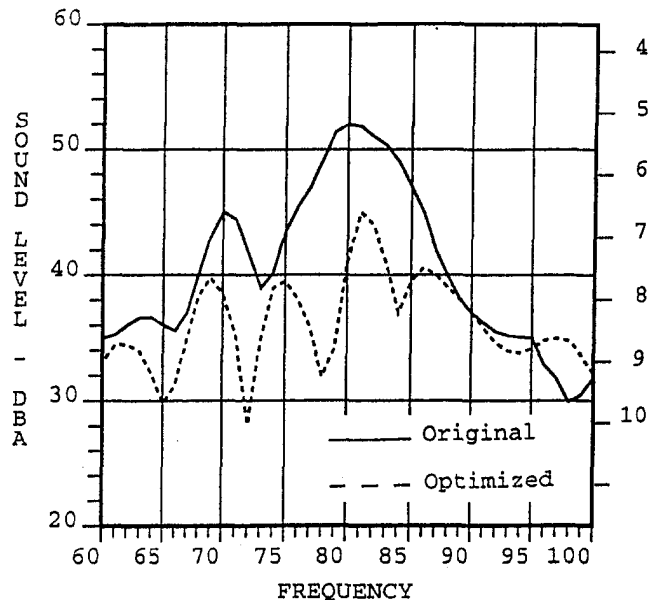
Fig. 4 Effect of optimized body design on rear seat sound pressure level

the structural-acoustic system, interface conditions are identified and sesquilinear and semilinear forms are defined. Both the direct differentiation and adjoint variable methods are developed in which continuum formulations for the structure and acoustic medium are retained throughout derivation of design sensitivity expressions. The numerical method has been implemented using direct and modal frequency FEA results from MSC/NASTRAN and ABAQUS FEA codes. Two vehicle systems are studied and good sensitivity results are obtained.

Acknowledgments

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to the thickness of 36,000 mostly warped plate elements. The thickness distribution among various body panels was optimized to reduce weight and noise using sensitivity coefficients. Weight is the objective and constraints are rear seat sound pressure levels at 69–71 Hz and 80–82 Hz. Sequential linear programming is used as an optimizer. The responses of original and optimized body models are compared in Fig. 4. It is shown in Fig. 4 that the noise is significantly reduced.

7 Conclusions

A continuum sizing DSA method is developed for the dynamic frequency responses of structural-acoustic systems using a variational approach with non-self-adjoint operators for complex variables. To derive a variational governing equation for