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Nonlinear Response and Sonic Fatigue of National Aerospace Space Plane Surface Panels

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Introduction

IT is becoming increasingly apparent that the development of supersonic/hypersonic vehicles such as the NASP cannot become a reality until the fatigue problem under severe aerodynamic, acoustic, and thermal environment is solved. Furthermore, with an increasing use of light-weight materials such as fiber-reinforced composites, metal matrix composites, and various intermetallic compounds, there is an urgent need for improved analytical methods for nonlinear response and sonic fatigue predictions.

The proposed NASP concept involves operations at subsonic, supersonic, and hypersonic speeds. Thus, aerodynamic, acoustic, and thermal loadings should be considered for various phases of flight.

In the early stages of the takeoff, the aft surface thermal protection systems will be in the near field of the engine exhaust noise. As the speed of the vehicle increases, the effect of engine exhaust noise (except in the near vicinity of exhaust nozzles) will decrease, and at Mach 1 and higher speeds the acoustic loads are expected to become negligible. However, at supersonic and hypersonic speeds, the fluctuating surface pressures due to convecting turbulent boundary layer will become significant. In addition, local impinging shocks on the structural surface could induce severe dynamic loads.

Supersonic/hypersonic vehicles will be subjected to severe surface temperatures exceeding 3000°F.[1–4] Structural surface components and control devices can be expected to behave in a highly nonlinear fashion with respect to both geometry and materials.[4–8] No reliable analytical procedures have yet been developed which could handle these complexities and give meaningful results that are useful for the design of these vehicles.

The thermal surface protection systems of aircraft structures are usually constructed from discretely stiffened panels and stiffened shells. A typical discretely stiffened panel is shown in Fig. 1. Due to anticipated high-surface temperatures of advanced supersonic/hypersonic vehicles, multi-wall and/or multilayer constructions might need to be utilized for the design of thermal surfaces. High-cycle fatigue failures have occurred in discretely stiffened surface panels with the majority of cracks appearing in the near vicinity of the stiffening element.[5–13] Thus, proper dynamic interaction between the panel and various stiffening elements should be taken into account when calculating the nonlinear response of the surface panels. A single bay panel with clamped or simple support boundary conditions might provide reasonable estimates on deformations,[14–23] but the predictions of principal stresses of a discretely stiffened panel by a single panel model could be unrealistic. Since fatigue damage is controlled by local stress conditions, accurate stress predictions are essential for fatigue life estimates.

A time domain Monte Carlo-type approach has been successfully applied to a variety of problems of a linear and nonlinear nature with complex random inputs.[24–33] Utilizing the nonlinear time domain stress response solutions and fatigue information from constant amplitude coupon tests, preliminary fatigue damage models have been constructed. The adverse thermal conditions could result in degradation of strength, stiffness, and fatigue life. In addition, thermal in-plane loads could induce buckling and "snap through" type vibrations of surface panels. The time domain procedures could account for these effects in the general formulation of the total response problem.

General Features of Time-Domain Method

For structural dynamic problems of nonlinear/probabilistic nature where close form or effective approximate solutions are not possible, the time domain Monte Carlo method could provide a feasible approach to construct practical solutions. The recent advent of high-speed digital computers has made this procedure a useful and effective method. To illustrate the basic features of the time domain approach, consider a generalized form of a second-order nonlinear equation

$$\ddot{X}_{ij} + 2\zeta_{ij}\omega_{ij}\dot{X}_{ij} + \omega_{ij}^2 X_{ij} + N(X_{ij}, \dot{X}_{ij}) = F_{ij}(t) \qquad (1)$$

where X_{ij} are the components of generalized coordinates, ω_{ij} are the natural frequencies of a linear system, ζ_{ij} are the various modal damping coefficients, N is the nonlinear system operator, and $F_{ij}(t)$ are the generalized random forces. Continuous nonlinear systems governed by partial differential equations can be discretized using modal solutions and represented as a system of nonlinear ordinary differential equations in a similar fashion as Eq. (1).

The time domain Monte Carlo method consists of three basic steps: 1) realizations of random inputs, $F_{ij}(t)$, are generated utilizing simulation procedures of random processes; 2) the equations of motion [Eq. (1)] are solved numerically

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for each realization of $F_{ij}(t)$; and 3) statistical moments, distributions, crossing rates and other needed quantities of the response processes $X_{ij}(t)$ are computed from ensemble averages. For the cases where the ergodicity condition is applicable, the ensemble statistics can be replaced with temporal averages thereby saving computation time. The Monte Carlo procedure can be further extended to problems with random system coefficients (material properties, geometric conditions, etc.).

Mission Requirements and Local Aerodynamic/Acoustic Surface Conditions

The first objective is to identify the various flight environments that the aerospace plane would experience through its useful design life and construct local surface pressure models that are consistent with the present time domain approach. Then, composite space-time surface pressure realizations that are acting on the structural thermal protection systems are developed.

The present day approach to predict structural response and fatigue life is based on linearized models and a single surface-pressure profile for which a partial damage is calculated. Then, linear superposition procedures are utilized to obtain the total fatigue damage. However, due to anticipated nonlinear response of surface panels, prediction of structural reliability and fatigue life of high-performance supersonic/hypersonic aircraft requires that a composite random surface pressure spectra be constructed. These aircraft are designed to perform a variety of missions for different aerodynamic flight regimes. Therefore, response calculations and fatigue life estimates of the surface structures should reflect the different mission profiles since drastic changes in local flow and acoustic conditions could be induced. To account for these conditions, typical composite surface pressure spectra which combines the different mission profiles and operational constraints should be developed.

The local surface pressure distribution is needed for the entire vehicle (or regions of most severe aerodynamic, acoustical, and thermal loading) so that response calculations and fatigue estimates for a particular structural component of the thermal surface protection system can be accomplished. Typical categories of random surface pressure that a structural component might be exposed are power plant noise, turbulent boundary-layer flow, oscillating, and impinging shocks. To simulate the required realizations of generalized random forces $F_{ij}(t)$ corresponding to these surface pressures the cross-spectral densities of the local pressures are needed.[34] Empirical and experimental information is readily available to characterize the turbulent flow and power plant noise.[35–44]

The various segments of surface pressure acting on a critical structural component could include stationary, nonstationary, Gaussian, and nonGaussian characteristics. The statistical quantities such as the expected mean value, root mean square (rms), probability distribution, crossing rates, peak exceedances, spectral energy associated with the simulated individual surface pressure segments, and the entire composite spectra (single mission and multimissions) will need to be determined. In addition, there are regions on the flight vehicle where the local surface pressure is produced by a combination and interaction of various sources such as the turbulent flow, impinging shocks, and acoustic noise from engine exhaust. These random surface pressures need to be combined probabilistically to produce a generic simulated surface pressure model.

Simulation of Stationary-Homogeneous Multidimensional Random Processes

The random sample functions of pressure $p(x, y, t)$ with a zero mean can be generated utilizing simulation procedures of stationary Gaussian random processes and fast Fourier transforms (FFT) techniques.[24,26,34] Then, $p(x, y, t)$ can be simulated by the series

$$p(x, y, t) = \mathrm{Re}\left(\sum_{i=0}^{M_1-1}\sum_{j=0}^{M_2-1}\sum_{r=0}^{M_3-1} A_{ijr}\right. \left. e^{i\phi_{ijr}} e^{i(k_{1i}x + k_{2j}y + \omega_r t)}\right) \tag{2}$$

where

$$A_{ijr} = [2S_p(k_{1i}, k_{2j}, \omega_r)\Delta k_1 \Delta k_2 \Delta\omega]^{1/2} \tag{3}$$

ϕ_{ijr} are the realized values of independent random phase angles uniformly distributed between 0 and 2π, Re indicates the real part, and the values of spectral density S_p are selected at wave numbers k_{1i}, k_{2j}, and frequencies ω_r. The Δk_1, Δk_2, and $\Delta\omega$ are the wave number and frequency intervals, respectively.

The numerical details and the limitations of simulating a multidimensional random process can be found in Refs. 24 and 34. It is expected that for a number of flight segments and specific local flow conditions, surface random pressures will be simulated in the form of Eq. (2). As the need arises, Eq. (2) can be simplified to two- or one-dimensional cases to meet the physical/computational constraints.

Simulation of Nonstationary and Nonhomogeneous Random Pressure

When a surface pressure is idealized as a stationary and Gaussian vector process, the mean value vector and the cross-spectral density function matrix completely characterizes the random process. However, for rapid spatial or time changes in local aerodynamic and acoustic conditions, a stationary-homogeneous idealization could lead to significant errors when estimating structural response and accumulated damage.

Various forms of nonstationary random process models have been proposed.[45–62] These nonstationary models can be classified into two categories. One category consists of a class of time domain models such as filtered Poisson processes[52,53] and data-based nonstationary processes.[51] The other consists of frequency domain models based on the generalized spectrum,[55,56] the instantaneous spectrum,[57,58] and the evaluationary spectrum.[46–50,60,61] The general difficulties associated with these frequency domain models are that they characterize the nonstationary processes in terms of modified forms of the mean square spectral density which is not particularly amenable to nonstationary conditions. Furthermore, such a frequency domain representation limits the analysis to linear response and systems with deterministic and time-invariant material properties.

A nonstationary model called "data based" has been constructed in the time domain on the basis of observed records.[51] The model is written in the form of the inversion of the Fourier transform of the original record (the first kind) or of its periodic-symmetric extension (the second kind) with randomly shifted phase angles. This model lends itself to a tractable implementation of Monte Carlo analyses for structural dynamic applications. The data based nonstationary model has been mainly applied to simple one-dimensional uses, and the accuracy of simulation is strongly dependent on an adequate data base.

For the time domain analysis of nonlinear problems for NASP applications, an amplitude modulated nonstationary and nonhomogeneous random process can be introduced

$$X(x, y, t) = G(x, y, t)\cdot p(x, y, t) \tag{4}$$

where $G(x, y, t)$ is a modulating function of space-time and $p(x, y, t)$ is a stationary and Gaussian random pressure simulated by Eq. (2). The $G(x, y, t)$ is usually a deterministic function equal to zero outside of the space $(0, X_0)$, $(0, Y_0)$, and time $(0, T_0)$ intervals in which X_0, Y_0, and T_0 are the bounds of the simulated processes. An amplitude modulated process has been used for a number of linear and nonlinear applications.[45,62] To model the nonstationary characteristics of a random surface pressure, a suitable and useful form of the modulating function $G(x, y, t)$ is

$$G(x, y, t) = \begin{cases} F(x, y)[\exp(-c_1 t) - \exp(-c_2 t)] & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases} \tag{5}$$

where $F(x, y)$ is selected to represent gradual changes of surface pressure distribution. By a suitable choice of constants c_1 and c_2, $G(x, y, t)$ can be made to resemble an envelope of short time duration burst of surface pressure. It should be noted that the variation of $G(x, y, t)$ is much slower than that of the sample function $p(x, y, t)$. To account for possible random variations in the modulating function $G(x, y, t)$, a procedure can be developed to assign distributions functions to the shape parameters of $G(x, y, t)$ and simulate pressure realizations $X(x, y, t)$ compatible with the Monte Carlo analysis method.

Simulation of NonGaussian Random Pressure

The nonGaussian random process can be constructed by performing a nonlinear transformation on a simulated Gaussian process as given by Eqs. (2) or (4). For many engineering applications, this is a highly economical procedure to include the nonGaussian characteristics observed in reality. For example, a symmetric nonlinear transformation could be of the form[34]

$$Y(x, y, t) = |p(x, y, t)|^{\gamma} \operatorname{sgn}(x, y, t) \tag{6}$$

where $\gamma > 0$ and $\operatorname{sgn}(x, y, t)$ is used to adjust the sign of the simulated process $p(x, y, t)$. Other forms similar to Eq. (6) can be used to perform a nonlinear transformation on the simulated random process in order to match the nonGaussian characteristics of the localized surface pressure that is acting on the flight vehicle.

Aerodynamic and Mechanical Loads Acting on Surface Panels

In addition to random surface pressures produced by boundary-layer turbulent flow and engine exhaust noise, there are various other dynamic input sources that are acting on the thermal surface panels. Depending on local flow and geometric conditions, these inputs could have a significant effect on structural response and alter the fatigue life of a structural component. These include nonsteady aerodynamic pressure (supersonic and hypersonic flow), cavity pressure, impinging localized shock waves, parametric in-plane excitations, pressurization, and aerodynamic/engine exhaust heating.

Nonsteady Aerodynamic Pressure

It has been demonstrated by numerous investigations that convecting high-speed flow (supersonic/hypersonic) could induce large nonlinear vibrations (flutter) of surface panels.[26-28,63,64] Exact-type formulations of the nonsteady aerodynamic pressure lead to nonlinear integro-differential equations for structural vibrations, thereby complicating the numerical solution procedure.[65] However, for high-speed supersonic (Mach number larger than 1.6) and hypersonic flow, a piston theory aerodynamic approximation can be used to represent the nonsteady aerodynamic pressure.[63,64] For most applications of panel flutter, a linearized form of the piston theory has been used to represent the nonsteady surface flow pressure. However, for the cases where vibrations of the structural surface are nonlinear, the interaction between the convecting flow and the vibrating structure might need to be modeled by a nonlinear aerodynamic/structural system.

Cavity Pressure

Cavity (or backup pressure) effects develop on a structural panel if the acoustic enclosure underneath the panel has finite dimensions. The general solution for cavity pressure involves solving a three-dimensional acoustic wave equation with a vibrating flexible boundary (surface panel).[65] This type of formulation leads to a set of nonlinear integro-differential equations. Useful approximations can be obtained for deep and shallow cavities.[27,65] A deep cavity has no significant effect on structural response, while a shallow cavity could have a large influence on response of surface panels.[68,69]

Pressurization

The main effect of pressurization is to increase stiffness of a structural surface. Furthermore, pressurization induces out-of-plane bending, resulting in a significant static mean stress value and earlier initiation of buckling.

Aerodynamic/Engine Exhaust Heating

The high-speed surface flow friction drag induces large thermal effects on surface structural components. In addition, hot gases from engine exhaust could produce intense thermal conditions in the aft portion of the vehicle. Surface heating produces large thermal stresses and large in-plane loads which could lead to thermal buckling and snap through type response of surface panels.[32,33,66,67]

The result is a significantly shorter fatigue life of surface thermal protection panels. New composite materials are now being developed to withstand these intense thermal conditions. In addition, active cooling might need to be implemented at some locations to protect the interior components of the vehicle.

Governing Equations of Motion and Nonlinear Response of Surface Panels

The thermal surface protection systems of NASP type vehicles will be constructed from a variety of structural components such as plates, stiffened panels, multiwall/multilayered designs, etc. The basic structural component which reflects the key dynamic characteristics is a discretely stiffened panel. Such a panel is shown in Fig. 1. For response and fatigue analysis, it is pertinent to select a structural model which contains the main structural features of the actual surface panel. Furthermore, this model should be capable of simulating the local stress field, and it is tractable mathematically and numerically. A discretely stiffened subpanel system might be selected which is taken to be clamped or simply supported along two boundaries (frame supports) and stiffened by stiffeners in the spanwise direction y (dotted line in Fig. 1). This structural model should give a reasonable estimate of local stresses at critical locations marked by an x in Fig. 1. Experiments tend to indicate the fatigue cracks usually appear at these locations.[5-9,10-13] Furthermore, the selected stiffened panel should be compatible with the experimental models to be tested at various government, industrial, and university

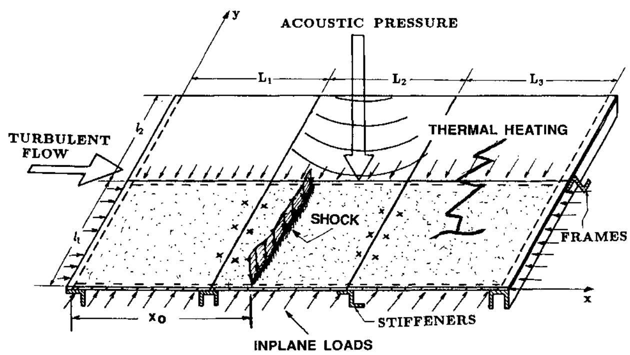

Fig. 1 Stiffened surface panel.

facilities in order to validate analytical response and fatigue life predictions.

The proposed time domain Monte Carlo method requires a formulation of the governing equations of motions in the space-time domain. Tests on aircraft panels[2,10–13] and anticipated severe aerodynamic, acoustic, and thermal loads for the supersonic-hypersonic vehicles[1–3,39–44] indicated that nonlinear large deflection theories are needed to properly model the vibrations of these panels. Analytical formulations on nonlinear vibrations are available for single isotropic/orthotropic/composite plates,[14–23] isotropic double walls,[30,31] and isotropic discretely stiffened panels.[32,33] The majority of these formulations are limited to isothermal conditions, single-mode approximations, no transverse shear and simple input pressures such as band limited Gaussian white noise. Research is now being conducted to extend these procedures to discretely stiffened composite panels which will include geometric nonlinearities, transverse shear, thermal effects, and other loading conditions described earlier. Utilizing Newton's second law, compatibility requirements of inplane strains, von Karman nonlinear strain-displacement relations, and thermoelastic conditions for large deformations, the nonlinear coupled partial differential equations can be developed.

In the development of analytical formulations for linear/nonlinear response, a damping model needs to be selected. The energy dissipating mechanisms which contribute to damping are very complex and depend on many factors.[70,71] However, damping plays a very significant role in controlling linear or nonlinear response. Most analytical formulations for linear or nonlinear analysis tend to model damping as a linear process. However, for large deformations and structures at elevated temperatures, such a linear damping representation might not be adequate.[70] Nonlinear damping models have been introduced to study the response of thin panels by time domain simulation.[72]

The solution to the governing nonlinear partial differential equations of the thermal surface panels can be developed using the finite element method or modal Galerkin-like procedure. Even though the finite element method provides a realistic method in solving geometrically complex problems, the time domain approach requires a very large memory storage capacity and extensive computation time. Since the nonlinear time domain solutions by the finite element routines are based on the marching sequence in time, i.e., nonlinear space distributed solutions are developed for each time increment, computation time and costs become prohibitive even for a small number of response realizations. The Galerkin-like procedure provides a much faster and less costly alternative, but it limits the analysis to simplified geometric configurations. The results presented in this article are based on the Galerkin-like method. After the required statistical information on nonlinear stress response is known, a fatigue damage model can be constructed.

Fatigue Life Prediction Methodology

The ultimate objective is to develop analytical models of predicting fatigue life of thermal surface protection structures subjected to a severe input environment that the NASP will experience through its service life. This involves construction of fatigue damage models that are consistent with the time domain methods developed. The determination of fatigue damage in composite and intermetallic materials is a very complex and fast-changing area of technology. In what follows, a preliminary fatigue damage model based on data from constant amplitude coupon tests and analytical predictions of the total number of stress peaks per unit time and stress peak distribution is developed. The key elements in this procedure are detailed local stress state and reliable cumulative damage rules for nonlinear and nonGaussian random stress amplitudes. The present research and design efforts for fatigue life estimation are basically proceeding along two lines: 1) stress-type cumulative damage theories, and 2) fracture mechanics (crack growth) approach.

The fracture mechanics theories are nonlinear and deal with the stress intensity at the crack tip rather than the continuum state stress used in the cumulative damage approach. Even though significant gains have been achieved in damage estimation by fracture mechanics, there are many deficiencies such as proper models for random load spectra, predictions for crack initiation, accounting for material and manufacturing imperfections, and reliable knowledge of two- or three-dimensional stress field states at the crack tip. For composite and intermetallic materials, the fracture mechanics models seem to be in early stages of development.[78] In addition, the stress response of surface panels to random pressure is dominated by relatively high frequencies in the range of 100–1000 Hz. Thus, the crack growth stage for high-cycle fatigue might be very short as compared to the crack initiation stage (before a crack can be measured and assigned an initial crack size in the crack growth model). In the present time domain approach, it is possible to incorporate the stress response time histories in the crack growth models and then estimate the damage on a cycle-by-cycle basis.

The earliest method of fatigue analysis based on constant amplitude fatigue tests is the Palmgren-Miner[74] cumulative damage model

$$D = \sum_i \frac{n_i}{N_i} \tag{7}$$

in which n_i represents the actual number of cycles at a given stress level, and N_i is the number of total cycles at which failure occurs at the same stress level. Miner has proposed that the quality $D = 1$ is at failure.

Various tests[75–79] indicate that the range for D is from about 0.3–3. To improve on fatigue life predictions, cumulative damage theories which incorporate the nonlinearity of the damage rate[80,81] and "stress interaction" effects[82–88] have been proposed. Furthermore, the large scatter of experimental data in fatigue tests has led to the general conclusion that fatigue damage is a random process. Numerous research efforts have been devoted to treating the fatigue damage as a stochastic model where the key random parameters are loading and material response to stress cycles.[89–94] More recently, a more refined cumulative damage rule for constant amplitude cycling and stochastic modeling of fatigue damage has been proposed in Ref. 95. For the constant amplitude cumulative damage, an interaction function is introduced which accounts for the effect of the frequency of high-low/low-high cycle of transitions. The stochastic model of fatigue damage is a function of random quantities such as loading and material properties. Damage is described in the form of a probability distribution function in which $F_D(x_i)$ is the probability that $D_i > 1$, for given x_i.

The nonlinear nature of the stress response leads to a nonGaussian distribution of stress amplitudes. Analytical expressions to predict fatigue damage for nonGaussian response are available only for a few limited cases.[52,96–98] A starting point in developing a damage theory that is consistent with the present time domain approach for nonlinear/nonGaussian response is to utilize fatigue data from coupon testing. Size and mean stress effects, imperfections, local stress-strain relations of the material at bonds, connections, rivets, etc. could be built into the model utilizing experimental and empirical information. The stress vs the number of cycles relationship can be written as

$$S^\lambda = B/N \tag{8}$$

where S is a fixed stress amplitude for constant amplitude loading, N is the number of stress cycles until failure at a stress level S, and λ and B are material constants which are also temperature-dependent. Due to a very large scatter of fatigue data, these constants are often modeled as random variables leading to a stochastic damage prediction model. However, at the present time, these material constants are taken as deterministic quantities.

Stress response under random surface pressure loading is a random quantity and the application of the Palmgren-Miner theory must be broadened to account for the random stress. This can be achieved by letting the number of cycles $n(S_i)$ represent the number of peaks at the stress level S_i. Following Ref. 52, it can be shown that for a stationary response the expected damage can be written as

$$E[D(\tau)] = \tau E(D) = \frac{E(M_T)\tau}{B} \int_{-\infty}^{\infty} |S|^{\lambda} p_I(S)\, \mathrm{d}S \qquad (9)$$

where $E(M_T)$ is the expected total number of peaks from $t = 0$ to $t = \tau$. The fatigue life T_f of a structural component is the time duration for the total damage to reach unity, i.e.

$$D(T_f) = \int_0^{T_f} D(t)\, \mathrm{d}t = 1 \qquad (10)$$

For the time domain approach, $E(M_T)$ and $p_I(S)$ needed in Eq. (9) are computed directly from the stress response time history. Such an approach has been used in Refs. 23, 32, and 33 to obtain preliminary guidelines of fatigue damage prediction of simple titanium panels to random loads. This procedure can be generalized to predict expected damage of more realistic surface protection structures, composite designs, and complex mission spectra inputs as previously described. In addition, the effects of thermal loading is reflected in the form of material property changes, fatigue relationship as described in Eq. (8), total expected number of peaks $E(M_T)$, and peak distribution p_I. Preliminary results presented in Refs. 32 and 33 indicate that thermal loads could have a drastic effect on the nonlinear stress response and fatigue damage. Improved analytical models to account for thermal effects not only on the global scale of surface structural components but also on a local scale (concentrated "hot spots") will be an important area of future fatigue damage research.[99–101] The expected damage calculated from Eq. (9) provides the basic statistical information on the average life of a structural component. Due to randomness of the damage prediction model, other statistical quantities might need to be computed to assess the degree of variability of the predicted fatigue damage.[52] Furthermore, as the experimental data on response and fatigue of composite and other materials under severe acoustic/thermal loading become available, the preliminary damage model given in Eq. (9) might need to be updated to reflect the realism of fatigue life prediction.

Numerical Results

To illustrate the application of the time domain approach for solution of nonlinear problems, several typical examples that are compatible with the theory developed are selected. These include a discretely stiffened panel and an orthotropic composite panel. The displacement and stress response solutions are given in Refs. 23 and 33.

Discretely Stiffened Panel

The numerical results are presented for a typical discretely stiffened titanium panel. The panel is assumed to be stiffened with two stiffeners in one direction at equal distances, and simply supported at the frames and outside edges. The dimensions of the panel are $L_1 = L_2 = L_3 = 8.2$ in., $l_1 = 20$ in., and $h = 0.06$ in. (see Fig. 1). The panel and the stiffeners are made from 6A1-4V titanium material with $\rho = 0.000414$ lb$_f$-s^2/in.^{4} (material density), $E = 16.0 \times 10^6$ psi (modulus of elasticity), $G = 6.2 \times 10^6$ psi (sheer modulus), $\nu = 0.34$ (Poisson's ratio), and $\alpha = 4.6 \times 10^{-6}$ in./in./°F (thermal expansion coefficient). The geometric properties of the stiffeners are given in Ref. 33. The modal damping coefficients were taken as

$$\zeta_{mn} = \zeta_{11}(\omega_{11}/\omega_{mn}) \qquad (11)$$

where damping for the fundamental mode $\zeta_{11} = 0.02$. The modes and modal frequencies were calculated using a transfer matrix procedure.[52]

The input to the panel is taken to be uniformly distributed truncated Gaussian white noise with input pressure spectral density

$$\begin{aligned} S_p(\omega) &= (p_0^2/\Delta\omega)10^{\mathrm{SPL}/10} \qquad 0 \le \omega \le 2\pi \times 500 \\ &= 0 \qquad \text{otherwise} \end{aligned} \qquad (12)$$

where p_0 is the reference pressure, $p_0 = 2.9 \times 10^{-9}$ psi, $\Delta\omega$ is the selected bandwidth ($\Delta\omega = 2\pi$) and SPL is the sound pressure level expressed in decibels. The simulation of the random input pressure was obtained using the one dimensional form of Eq. (2) with the spectral density given in Eq. (12). The displacement and stress response time histories corresponding to this input are given in Fig. 2. These results correspond to the upper surface and the center of the panel. At 120 dB input (147 dB overall), the response of the panel reaches a transition point between linear and nonlinear solutions as illustrated in Fig. 3. For larger input levels, response becomes highly nonlinear invalidating the linear solution.

The "oil canning" or snap through phenomenon is observed when studying the dynamic response with thermal effects present. As the temperature of the panel rises above the ambient temperature, in-plane compressive stresses are induced in the panel. As the temperature increases, compressive thermal stresses produce thermal buckling of the panel. If the panel is loaded by a random pressure of small magnitude, the panel vibrates about the buckled state. If this random pressure is large enough, it may cause the panel to snap through from one position to the other. This kind of phenomenon is frequently observed in experimental tests.[66,67] The panel undergoes snap through vibrations under certain combined thermal and acoustic loads as illustrated in Figs. 4 and 5. Such a response behavior would have an adverse effect on fatigue life of those panels.

Response probability density histograms, peak distributions, total number of peaks per unit time, and threshold

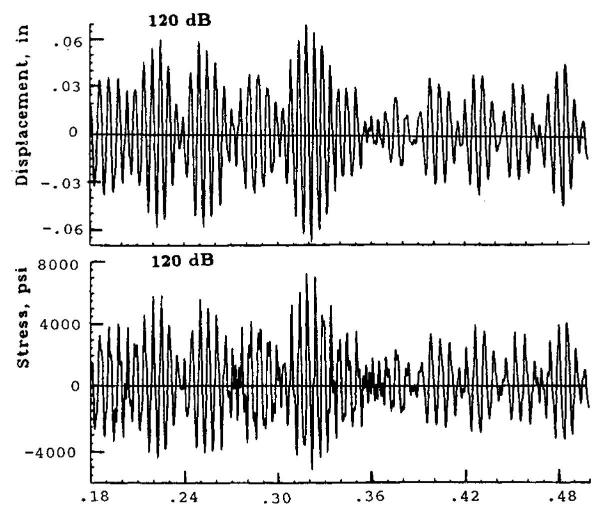

Fig. 2 Response time histories of displacement and stress.

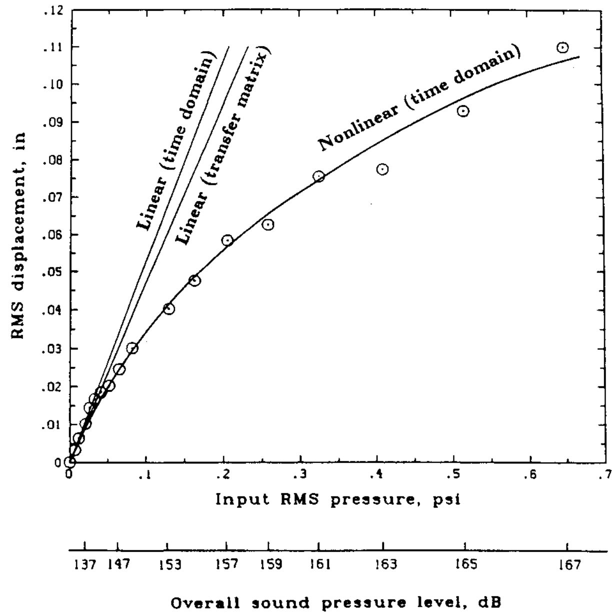

Fig. 3 Linear and nonlinear rms displacement response.

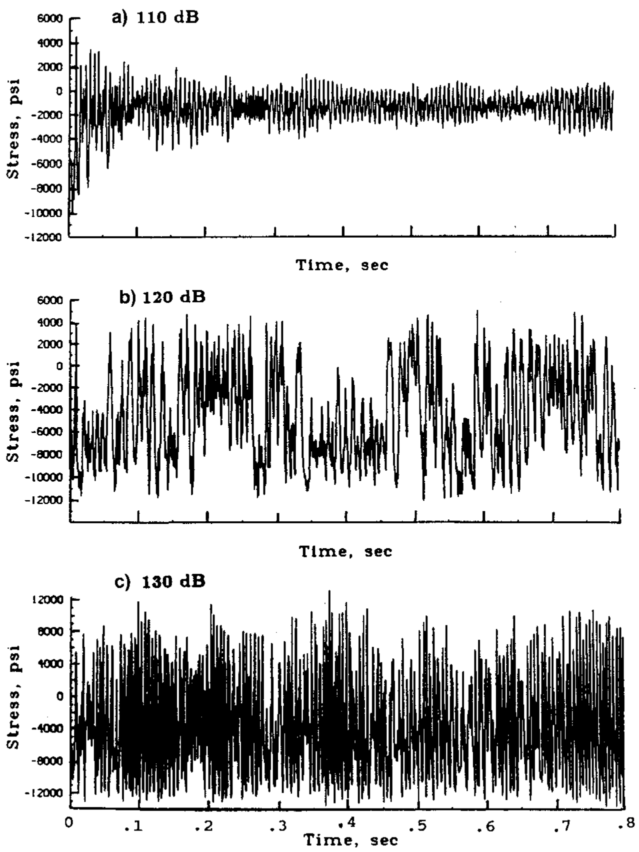

Fig. 4 Stress response time histories for average temperature rise $\Delta T = 50°F$.

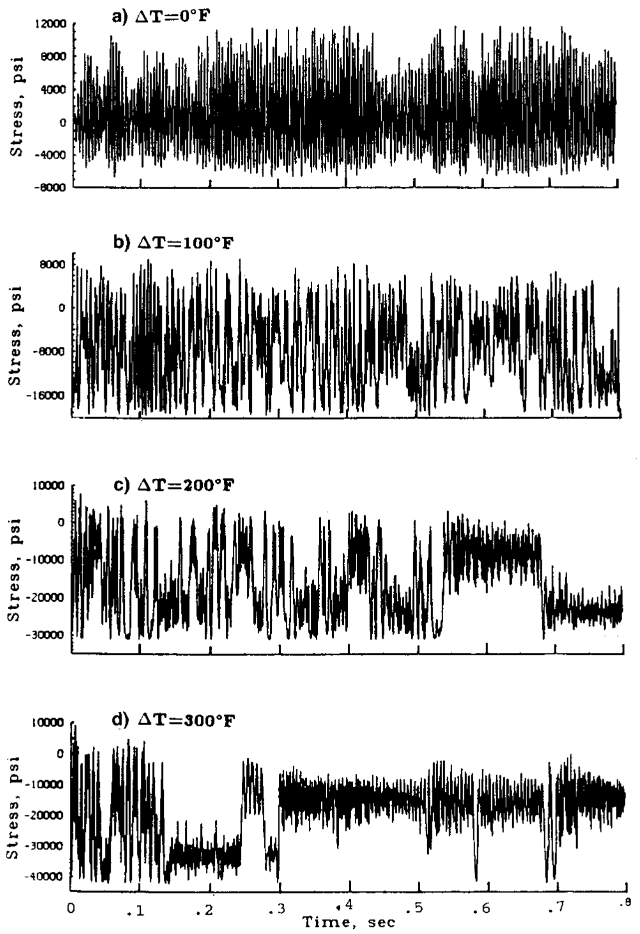

Fig. 5 Stress response time histories for a 130-dB sound pressure input.

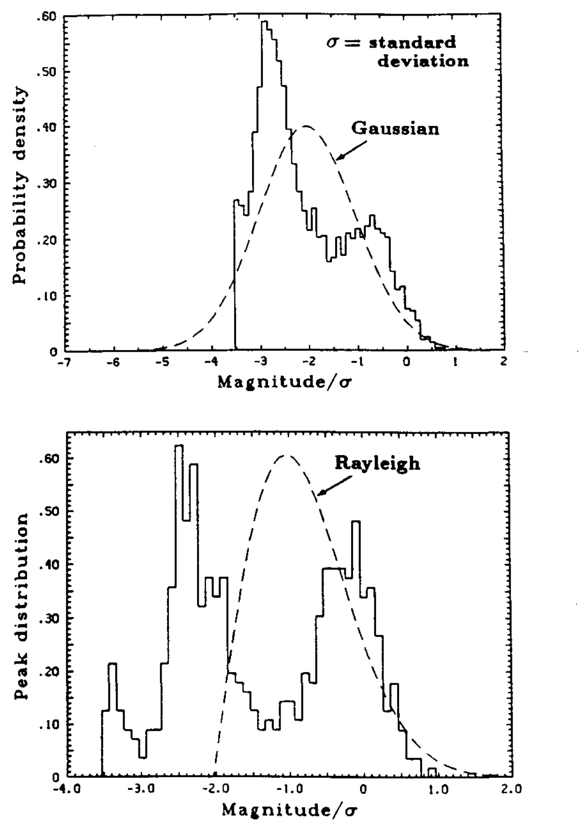

Fig. 6 Probability density and peak distribution histograms of stress at ΔT = 200°F and noise input of 130 dB.

crossing rates can be obtained from the response time histories. Figure 6 shows the probability density histograms and peak distributions for the nonlinear stress processes. For comparison, a Gaussian density function is given with each probability density histogram and a Rayleigh distribution with each histogram of peak distribution. It can be observed from these results that for high-pressure inputs, the response is no longer Gaussian and the peak distribution does not follow the Rayleigh distribution. It should be noted that the Rayleigh distribution is shifted by the amount of mean stress.

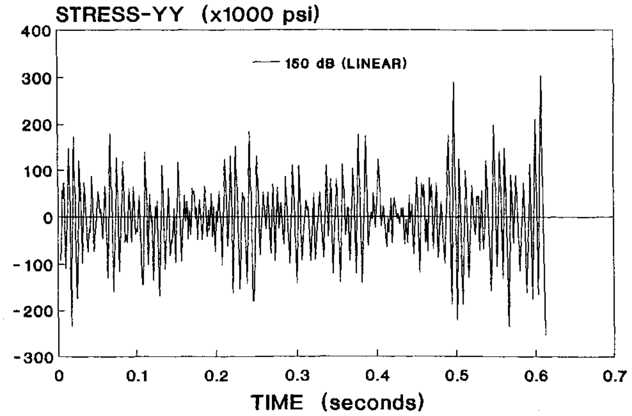

Fig. 7 Linear stress response time history of a composite panel at 150-dB input.

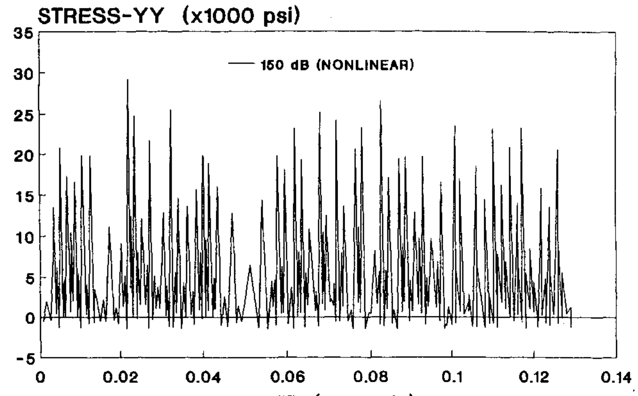

Fig. 8 Nonlinear stress response time history of a composite panel at 150-dB input.

Composite/Orthotropic Panel

Nonlinear response calculations were obtained for A-S/3501 Graphite/Epoxy composite panel.[23] The single panel is taken to be simply supported on all four edges. The overall dimensions are $L_1 = 8.2$ in., $l_1 = 20$ in. and $h = 0.0416$ in. The basic lamina constitutive properties are obtained from 0-deg lamina elastic properties with $E_l = 18.6 \times 10^6$ psi, $E_t = 2 \times 10^6$ psi, $G_{lt} = 0.8 \times 10^6$ psi, $v_{lt} = 0.31$ and $\rho = 0.0001302$ lb_f-s^2/in.[4] The stress response time histories, calculated at the middle of the panel and $z = h/2$, are shown in Figs. 7 and 8 for linear and nonlinear cases. The input is 150 dB (177 dB overall) uniformly distributed random pressure. These results indicate that the linear prediction provides unrealistic values. Note also that the mean value for nonlinear stress is not zero. Thus, the use of linear strain-displacement relationship is not applicable for high levels of acoustic loading. The probability density histograms, peak distributions, and crossing rates for nonlinear response show highly nonGaussian characteristics.[23] Additional results on nonlinear response of composite panels with thermal effects included can be found in Ref. 102.

Fatigue

Preliminary fatigue damage estimates are obtained for a stiffened titanium panel.[32,33] For linear and narrow band stress response, expected fatigue damage can be calculated in closed form from where the distribution of stress peaks $p_l(S)$ is approximated by a Rayleigh distribution. However, for the nonlinear stress response, only the histograms of peak distribution are available and Eq. (9) is evaluated numerically. The expected total number of peaks $E(M_t)$ are 303, 422, and 616 peaks/s for 120-, 130-, and 140-dB sound pressure inputs, respectively. These sound pressure levels correspond to a constant spectral level for frequency range of 0–500 Hz. The expected total number of peaks $E(M_T)$ for linear response is 232 peaks/s. The time to failure at which $E[D(\tau)] = 1$ is 23,268 h (linear response at 120 dB spectral level input), 144,700 h (120 dB), 1845 h (130 dB), 45.4 h (140 dB) for the nonlinear response analysis. These results indicate that fatigue life could be underestimated by a large amount if linear response analysis is used.

Conclusions

A time domain method can be used to solve a variety of problems of a nonlinear nature. It is anticipated that response of thermal surface protection systems of high-speed aircraft such as the NASP will be nonlinear under severe aerodynamic, acoustic, and thermal loading. The various simplified linear theories and Gaussian inputs commonly used to predict dynamic response and fatigue life of structural components would not be meaningful in the design of the thermal surface protection systems for a supersonic/hypersonic vehicle. The nonlinear response in nonGaussian and peaks do not follow a Rayleigh-type distribution. New fatigue damage models need to be developed to accommodate the nonGaussian response characteristics.

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