

Design of Engine Exhaust-Washed Structures for an Efficient Supersonic Air Vehicle MDO Application

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Structural components located aft of embedded engines on low observable aircraft, known as engine exhaust-washed structures (EEWS), are exposed to an extreme, configuration specific, combined thermal/structural loading environment. As a result of this operating environment, and the unique design challenges it poses, the effectiveness of EEWS designs is a key factor in the overall performance of an embedded engine aircraft. This means that design requirements and effects of overall configuration variability on exhaust structures must be addressed in early stages of a new embedded engine aircraft design to ensure that the final system can continue to meet performance requirements as development progresses. This paper highlights the design considerations for engine exhaust-washed structures design, steps taken to incorporate EEWS design requirements into a configuration-level design process of an efficient supersonic air vehicle (ESAV), as well as multidisciplinary design optimization methods developed that focus directly on structural design in a thermal/structural environment.

I. Introduction

To meet the growing demands for improved mission capability, combat survivability, and versatility of aerospace systems, future military aircraft will continue to rely on low observable technology. As demonstrated on modern aircraft, including the B-2 Spirit, the Predator-C Avenger, and future air vehicle concepts, embedded engines are a critical technology for reducing the observability of an aircraft. Burying an aircraft's engines inside the airframe allows for a smooth outer mold line (OML), reduced level of exhaust noise, and cooler exhaust gases. This reduces the vehicle's observability by decreasing radar, acoustic, and infrared detectability.¹ While such a configuration affords tremendous tactical capabilities, it also comes with increased structural design complexity resulting primarily from the extreme environment created by hot exhaust gases as they are passed to the rear of the aircraft. The structures located in this environment, aft of the embedded engines, that make up the exhaust path are known as *engine exhaust-washed structures* (EEWS).² EEWS commonly include a ducted exhaust nozzle and supporting substructure like the conceptual reference EEWS shown in Figure 1. These structures require design considerations that are not found in legacy aircraft, where exhaust gases from engines situated either under the wings or in the aft fuselage can be expelled directly into the airstream. In addition, their unique structural design challenges are not typically encountered in conventional thermal structures or hot structures design.

Since the advent of supersonic flight in the 1950s, the effect of elevated temperatures on structural components has been a recurring challenge faced in the design of new aircraft and space platforms. These challenges have been continually addressed by the thermal structures community, which has merged the disciplines of thermal and heat transfer analysis with structural analysis to solve thermoelastic design issues with each new generation of aircraft or space vehicles.³ When designing a component that must operate in a thermal environment, there have historically been two basic design challenges, which are still present in exhaust-washed structures design, and several proven design solutions that unfortunately are not effective for EEWS components because of operating conditions and aircraft configuration level design constraints.

The first of these challenges stems from the effect of the thermal environment on the mechanical properties of a structural system. Most aerospace materials, including both metallic and composite systems, tend to lose stiffness and strength properties and degrade at a higher rate as operating temperatures increases. To design against this, insulation or a thermal protection system (TPS) is often utilized to shield load bearing structures from the high temperature environment. Such a solution, which is sometimes referred to as "cool structure," is only effective for situations with transient, short duration exposure to high temperatures, such as that experienced by the space shuttle during atmospheric reentry. EEWS components must survive long periods of high temperature operation during a

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several long mission, which prevents application of a cool structure design solution without active cooling because given enough time, elevated temperatures will eventually saturate any insulation system. In addition, insulating material is completely parasitic to minimum weight or maximum range design objectives, which are the standard in aircraft design processes.⁴

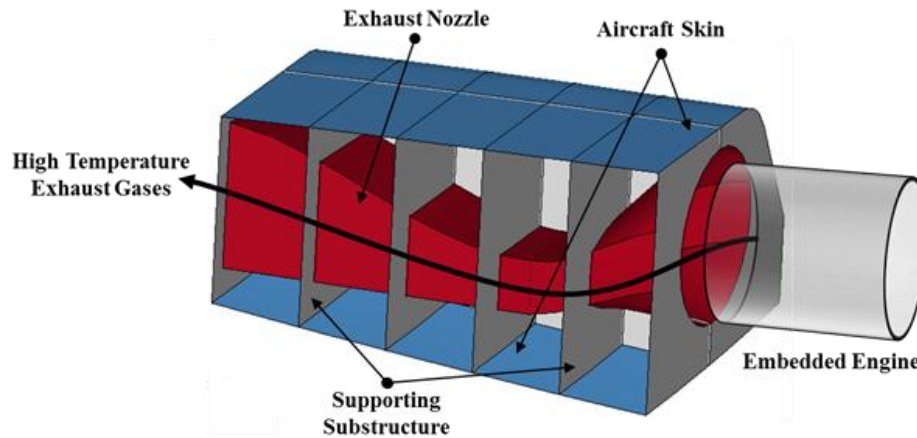


Figure 1. Conceptual reference EEWS configuration located aft of embedded engines and its primary components and loading.

The second, and often most important, consideration is the potential for damaging thermal stresses that result from thermal expansion or contraction of structures. The best way to design against these stresses is to simply allow for some amount of thermal expansion or contraction to occur, as found in engine liners, tailpipes, the corrugated wing sections of the SR-71,⁵ and expansion joints in concrete structures. A complication in the design of engine exhaust-washed structures is the dominance of low observability design criteria over conventional thermal structures concerns, including accommodating thermal expansion. In a typical configuration, such as that shown in Figure 1, the internal geometric shape of the exhaust flow-path is fixed by propulsion system efficiency and low observability design criteria. In addition, all components must be smoothly integrated within the outer mold line of the aircraft. This creates a design space with strict shape and fixivity constraints on structural components. Since all of these components are subjected to spatially and often temporally varying elevated temperatures, their thermoelastic response will undoubtedly include restrained expansion. Thus, for EEWS, the question exists how to design this exhaust structure, the geometric shape parameters of which are fixed, and the surrounding substructure layout that must support it. Designers must somehow accommodate thermal expansion to prevent excessive thermal stresses, which can lead to buckling and fatigue failures, while simultaneously preventing excessive thermal loads from being transferred to surrounding structures. This task is even more challenging because the thermal effects that drive the basic response of the structural system are design dependent. This creates a complex, multidisciplinary, and often non-intuitive structural design space.

The following sections highlight the structural response of a conceptual engine exhaust-washed structure, the basic design trade-space that is encountered, and a method for integrating the effects of EEWS weight into the conceptual/preliminary design framework of an Efficient Supersonic Air Vehicle (ESAV) concept.

II. EEWS Structural Responses

In this section, finite element analysis is used to explore the basic structural responses and challenges related to the design of engine exhaust-washed structures. The first model is a built-up EEWS model used to demonstrate the thermal and structural response. The second model is a reduced shell model that is used to further characterize the EEWS structural response and investigate the effects of design variability.

A. Basic Thermal and Structural Responses

A 2D finite element model, similar to the conceptual reference EEWS design previously shown in Figure 1, is given in Figure 2. The finite element mesh shown is utilized for both heat transfer and structural analysis. It is composed of two primary material systems. The exhaust nozzle surface, indicated by red elements in Figure 2, is a ceramic matrix composite (CMC) material that can withstand direct exposure to the high temperature exhaust gases

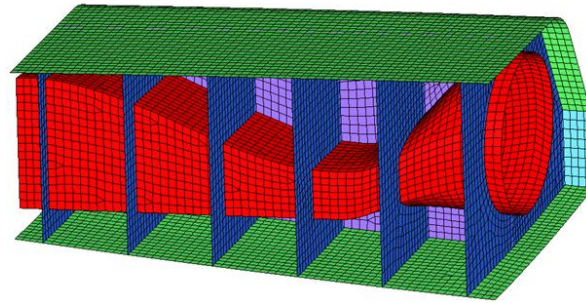


Figure 2. Finite element model of a notional reference EEWS system utilized for both heat transfer and structural analysis.

that are in excess of 1000°F. The rest of the model is composed of composite sandwich panels that contain graphite bismaleimide (Gr/BMI) facesheets with a titanium honeycomb core. This composite material system is designed for moderate temperatures of about 400-500°F. The heat transfer model contains all modes of heat transfer. This includes conduction throughout the structure originating from a fixed temperature boundary condition on the nozzle surface of 1500°F. Convection boundary conditions are placed on both the outside skin (denoted with green elements in Figure 1) as well as on the surfaces of internal cavities throughout the model to represent the thermal effects of fluid flow on the outer surface of the aircraft and ventilation in internal bays. Radiation is also included between faces inside internal cavities and from the outer skins (green elements) to ambient conditions. The structural model contains thermal loading (nodal temperature distribution from heat transfer analysis), pressure loads exerted normal to the exhaust surface to represent the exhaust pressure, and elastic boundary conditions that accurately represent the fixivity of the EEWS system inside the airframe. MD Nastran 2010 is used to solve both the steady-state heat transfer analysis and nonlinear structural analysis.⁶ Note that even though a simple steady-state solution is utilized in this demonstration case, assuming a quasi-steady relationship between heat transfer and structural solutions and using transient heat transfer analysis could also be utilized to capture time varying loading or changing operating conditions. A typical temperature distribution obtained from heat transfer analysis of the reference EEWS is shown in Figure 3.

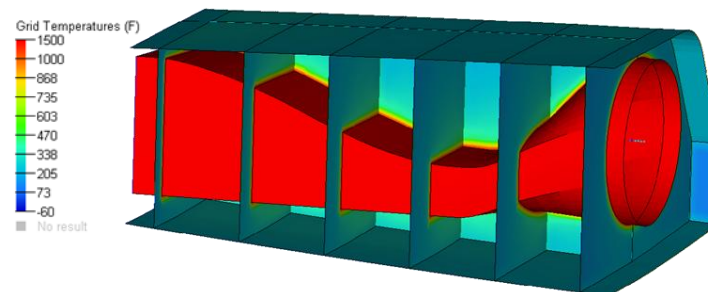


Figure 3. Typical temperature response for the reference EEWS system with high temperature nozzle structure and lower temperature adjoining support structures.

We note in the figure, a typical temperature distribution throughout the structure is characterized by a very hot inner exhaust surface that is directly exposed to the hot exhaust gases. It can be assumed that during prolonged exposure, without active cooling, that the nozzle surface structure will be approximately the same temperature as the exhaust gases. The adjoining substructure is maintained at a cooler temperature by ventilation (which is represented as film convection in the model) and conduction through the substructure to the outer aircraft skin.

When the temperature distribution is mapped onto the structural model and relevant mechanical loading is applied, structural deformations result as shown in Figure 4. Note that the magnitude of deformation is exaggerated in the figure and the elements that make up the substructure are hidden to place emphasis on the deformation of the exhaust nozzle surface. Deformations in the structure result primarily from the spatial temperature distribution, which leads to different amounts of thermal expansion within the EEWS assembly. In the case demonstrated here, the significantly higher temperature exhaust nozzle undergoes a great deal of expansion, which is restrained by the lower temperature adjoining substructure. We see in Figure 4, as a result of this restrained expansion, the panel-like

members of the exhaust nozzle surface deform out-of-plane. Often, these out-of-plane deformations are significant enough to generate damaging tensile stresses along the edges of the panels where they are affixed to the substructure. Out-of-plane deformation and resulting excessive thermal stresses similar to those seen here were determined to be the primary cause of failure on an exhaust-washed structure on the B-2 bomber called the aft-deck.² Further exploration of the thermoelastic response of the exhaust surface panels is discussed in the following section.

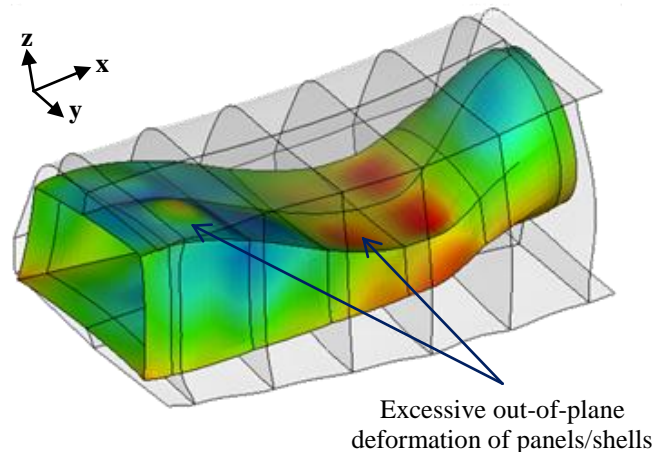


Figure 4. Vertical (z) component of displacement of the exhaust nozzle surface for the reference EEWS model. Note out-of-plane deformation of exhaust surface panels.

B. Curved Shell Response

In an actual application, it is very likely that the thermal stresses generated by the out-of-plane deformation of the exhaust nozzle surface panels would govern design criteria for the exhaust-washed structure assembly. A common design prescription for alleviating high structural stresses is to simply increase the thickness or cross-section of the high stress member. In the presence of thermal loading, this straightforward design solution may not be sufficient due to the design dependency of thermal loads. In this section, the challenges associated with design in this environment and the relationship between the exhaust nozzle and surrounding structure in the EEWS system is explored using a curved shell model shown in Figure 5. This simple model is a convenient idealization of the individual panels that make up the exhaust surface in the full EEWS model.

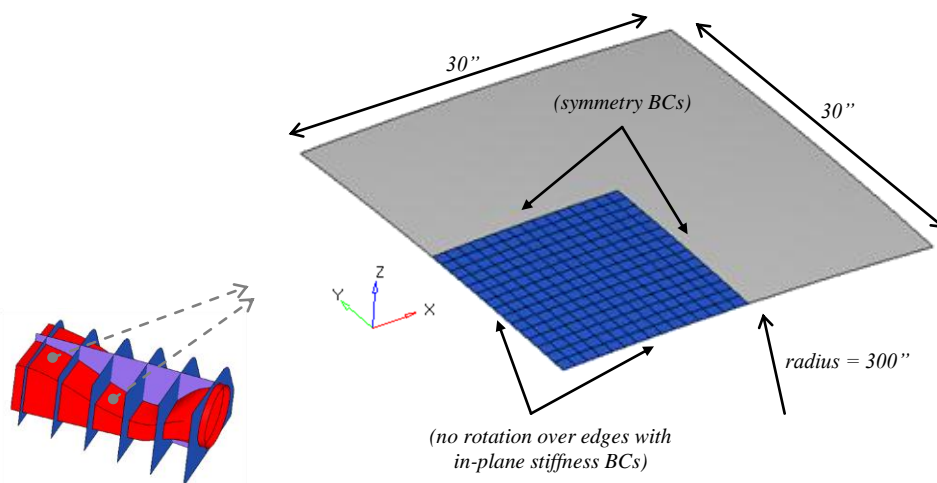


Figure 5. Curved shell model (quarter symmetry finite element model) to idealize a panel in the exhaust nozzle surface of the EEWS assembly with boundary conditions.

The shell is assumed to have spherical curvature in the z-direction of 300 inches. Note that the quarter-symmetry of the geometry is exploited in the finite element model. Boundary conditions, as shown in Figure 5, consist of no

rotation over the edge and an in-plane stiffness condition. In this direction, elastic boundary conditions are modeled using scalar spring elements to more closely represent the fixivity of the panel to a non-rigid structural member in an actual EEWS assembly. A high stiffness boundary corresponds to stiff surrounding structure and vice versa.

The total stiffness applied down each side of the panel is determined by Equation 1, where K_s is the total applied stiffness (which is distributed along an edge in the finite element model), t_o is the baseline thickness of the panel, and k is simply a stiffness scaling parameter ranging from 0.01 to 5.0 to explore the effects of different boundary stiffness.

$$K_s = 6 \times 10^7 k t_o \quad (1)$$

When subjected to a uniform temperature change of 1500°F and a relatively stiff boundary condition ($k=1$), a shell of thickness $t = 0.3$ inches deforms out-of-plane and regions of high stress develop along the boundary as demonstrated in Figure 6a and 6b. In reality, these regions of high stress would correspond to locations of fixivity to surrounding structures and consequently, also are likely failure points due to fatigue or fracture. It is acknowledged that the stress results in these locations are dependent on element sizes, but since only a comparison of stresses in these locations under different conditions is desired, mesh dependency does not impede the conclusions drawn in this analysis.

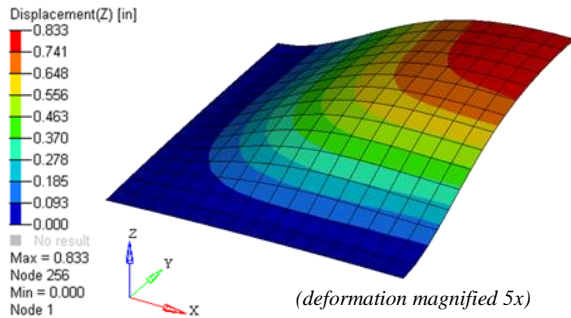


Figure 6a. Out-of-plane (z-direction) displacement of curved shell model.

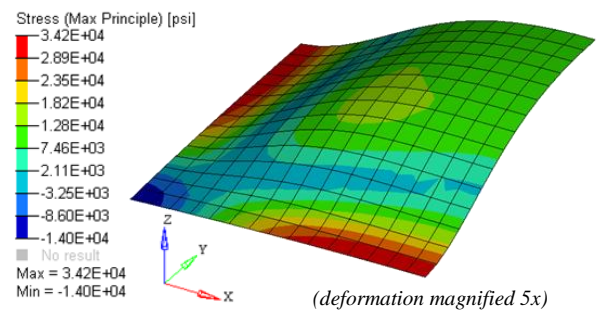


Figure 6b. Maximum principle stress (psi) in the curved shell model.

In mechanical and aerospace design, it is common practice to use thickness design variables to reduce excessive deformation and high stress areas in structural members. Figure 7 shows the effects of varying the thickness of the shell on the edge stress levels for various values of boundary stiffness. The nominal thickness in the Figure is $t_o = 0.3$ inches. One would expect that increasing the thickness of a structural member would lead to a reduction in the stresses in that component. However, as we note from Figure 7, in the presence of thermal loading, the thickness increase may not have the intended effect. For high values of boundary stiffness, which would correspond to a stiff adjoining structure in an actual application, a thickness increase in the panel may actually result in an increase in the stress along the edges of the panels. This occurs due to the design dependency of the thermal loading. The added material from the thickness increase also undergoes thermal expansion, which with a sufficiently stiff boundary, can lead to greater out-of-plane deformation and higher stresses. Figure 8 shows the effect of the same thickness increase on the in-plane reaction load along the edges of the panel. We see that increasing the panel thickness, regardless of the boundary stiffness, results in an increase in boundary reaction force. In a real application, this translates to a potentially significant increase in load transferred to surrounding structures. To validate these results, we note that similar trends in stress response were identified by Haney et. al. using a very simple beam strip model subjected to thermal loads.⁷

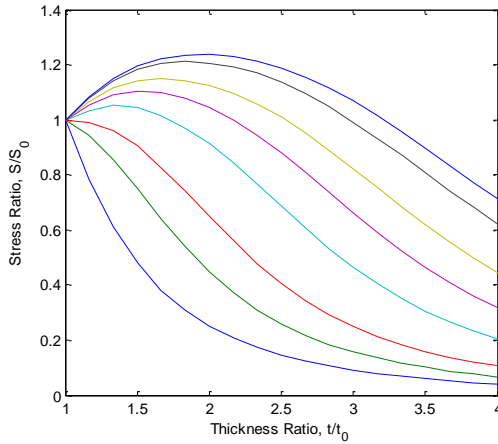


Figure 7. Stress ratio versus thickness ratio for various values of boundary stiffness.

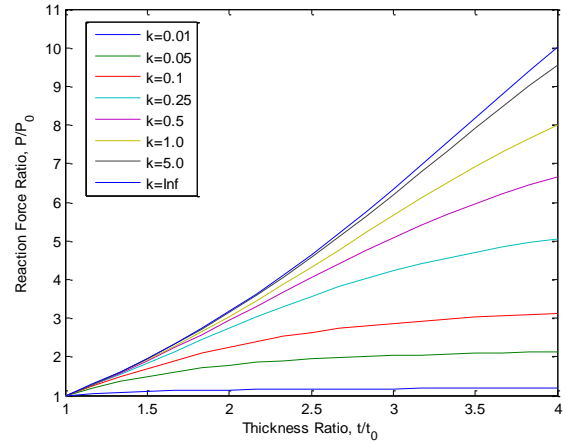


Figure 8. Boundary reaction load ratio versus thickness ratio for various values of boundary stiffness.

From this study it appears that a better design alternative for reducing stress levels in the shell and limiting reaction load to adjoining structure is to decrease the stiffness of the boundaries, which will reduce the restraints on the thermal expansion of the panels. Such a design prescription is only possible by utilizing structural design optimization methods at the EEWS system level with a design model that includes design freedom in both the exhaust nozzle surface and the adjoining substructure.

As a result of the challenging design space briefly demonstrated in the previous studies, the dependency of EEWS loading on operating conditions, and the impact exhaust-washed structure failure can have on vehicle performance, it is important to consider EEWS effects on the design of any new embedded aircraft configuration. The following section describes steps that have been taken to include these effects, at a first-order level, of exhaust-washed structure design into a multidisciplinary design framework of an embedded engine ESAV aircraft.

III. MDO Incorporation

As with most aircraft subsystems, for example a landing gear or control surface, design criteria including loading and operating conditions result from decisions made at the early configuration level concept design. In addition, these subsystems can be adequately represented in early stages of design using historical databases, empirical relations, or low fidelity models that capture first-order effects on overall vehicle performance. While certainly the design of exhaust-washed structures depends on conceptual level design parameters, including propulsion system variables, structural configuration, and specified mission conditions, there currently exists no direct method for representing the impact of these components in a new vehicle design process. This is due in large part to the lack of legacy design references and the need for multiphysics modeling that is typically not performed at early design stages. As a result, a candidate aircraft configuration with embedded engines that results from a conceptual design study may need significant modification once design criteria for exhaust structures are imposed. It is very likely that unanticipated performance penalties, resulting from the added structural weight required to design an effective EEWS system, on the overall vehicle occur. To prevent this, a method is presented here in which a surrogate EEWS weight model is populated based on reference optimum EEWS designs. This surrogate can then be utilized at the conceptual level in MDO to account for the effects of configuration variability on the EEWS's contribution to an overall minimum weight design objective.

A. ESAV MDO Framework

Figure 9 shows the data flow of an Efficient Supersonic Air Vehicle (ESAV) multidisciplinary design optimization framework. The colored boxes down the diagonal indicate an individual discipline specific analysis or sub-optimization that provides a response(s) to the overall framework. A set of system level design variables, constraints, and objective function control the overall vehicle configuration, while each discipline may contain local variables, constraints, and design parameters utilized only in a localized sub-system or component analysis or optimization. In this framework, the objective function is minimum take-off gross weight (TOGW) and configuration level constraints consist primarily of mission parameters including cruise speed and altitude, range, and payload. In the figure, the connections running from the right of a block and down to the top of another block

represent a forward flow of design information while the connections running from the left of a block to the bottom of another block represent a backwards flow. This information includes loading or boundary conditions for one discipline that must be obtained as the result of another. A complete pass down the diagonal represents one design iteration of the overall aircraft configuration. More information regarding this framework can be found in Reference 8. Relevant to this work is how engine exhaust-washed structures, which represent a localized subsystem themselves, can be efficiently integrated into this configuration-level design framework.

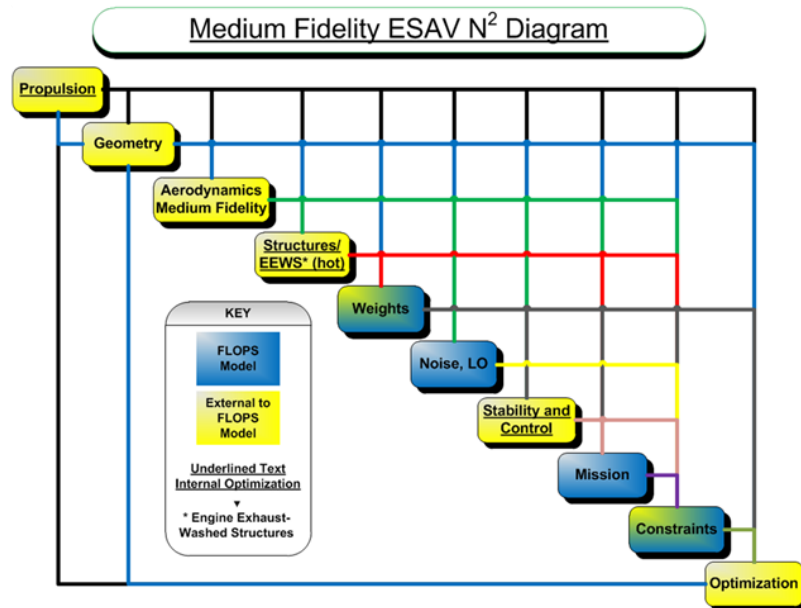


Figure 9. Data flow relationships for a multidisciplinary optimization framework for an Efficient Supersonic Air Vehicle (ESAV).

The operating environment for exhaust-washed structures is defined by the temperature and pressure of the engine exhaust gases, Mach number and altitude of flight, and the heat transfer boundary conditions that exist at the outer skin of the aircraft. It is also necessary to know the desired size of the exhaust path for a given aircraft configuration. Referring to Figure 9, we note that the EEWS module receives input from the propulsion and aerodynamics disciplines. These inputs are the exhaust temperature, exhaust pressure, Mach number, and altitude. This information is utilized to determine the loading and boundary conditions on an EEWS model. Input from both propulsion and geometry provide the required size of the EEWS model. Given these governing parameters, the goal of the EEWS module in the MDO process is to provide the weight of exhaust-structures designed to operate within those conditions. This weight is then passed onto the weights and structures modules, where it is used to account for the contribution of the EEWS to the overall minimum weight objective given the current aircraft configuration parameters.

B. EEWS Structural Optimization

To determine the optimum weight EEWS design when provided with a set of governing loading and geometric parameters, a structural optimization process is required to navigate the difficult EEWS design space outlined in Section II. A reference EEWS design, which is similar to that shown previously in Figure 2, is parameterized according to governing input conditions received from other modules in the MDO framework. These input conditions are referred to as governing parameters on the EEWS optimization. These parameters represent the overall size and loading or boundary conditions for the EEWS. Structural design variables are selected as individual member thicknesses within the reference EEWS design. A basic sizing optimization problem is then defined with the objective to minimize structural weight with constraints on stresses, strains, and buckling load factor. This localized EEWS optimization is solved using a gradient-based thermal/structural optimization process. Within the optimization process, heat transfer analysis is performed to determine temperature fields, which are then utilized in a structural optimization along with structural loading and boundary conditions. Steps are taken to ensure that structural design variability is reflected in heat transfer analysis to account for the design dependency of thermal loading in the structural optimization.⁹ The overall EEWS problem is summarized for clarity as follows.

Given (for critical conditions): 1.) Exhaust Nozzle Throat Area (engine size)
 2.) Altitude
 3.) Mach number
 4.) Exhaust Pressure
 5.) Exhaust Temperature

Minimize: EEWS structural weight

Subject to: stress & strain criteria
 bifurcation buckling load factor

Design Variables: structural member thicknesses

The solution of this localized optimization problem represents the optimum weight of exhaust-washed structures (within the freedom afforded by the reference model) for the specified governing parameters regarding EEWS loading, boundary conditions, and overall geometric size.

C. EEWS Surrogate Model

The overall vehicle level optimization framework requires information from the EEWS module hundreds if not thousands of times for a complete design study of just a single configuration. It is not feasible to perform the localized EEWS optimization each time. Thus it was determined that a surrogate model should be used to provide the optimum EEWS weight given a set of configuration parameters. Surrogate models have been utilized to successfully represent high-fidelity physics and computationally intensive processes in multidisciplinary optimization in a variety of applications.¹⁰ To populate the model, a local EEWS optimization was performed at design points that span the governing parameter space as given in Table 1. Upper and lower bounds were placed on the governing parameters using constraints on the overall configuration level optimization process and engineering judgment. The levels of each parameter were determined by a design of experiments process to effectively cover the configuration design space.

Table 1. Upper and Lower Bounds of Governing Parameters for the EEWS design space.

Governing Parameter		Lower Bound (coded to $x=0$)	Upper Bound (coded to $x=1$)
x_1	Nozzle Area (in ²)	100.0	1500.0
x_2	Altitude (ft)	10000.0	70000.0
x_3	Mach Number	0.20	2.40
x_4	Exhaust Pressure (psi)	10.0	50.0
x_5	Exhaust Temperature (°F)	600.0	2800.0

After solving the EEWS optimization problem for each set of governing parameters, a second order quadratic response surface using coded variables¹¹ was fit to the EEWS weight data. Interaction terms were included to capture coupling between input parameters. The resulting beta coefficients (using coded variables between 0 and 1) of the response surface are given by Table 2.

Table 2. Beta coefficients of the second order responses surface fit of EEWS weight data.

Beta No.	Beta Coeff. Value	Response Surface Term	Beta No.	Beta Coeff. Value	Response Surface Term
β_1	204.1	constant	β_{12}	-3.031	$x_2 x_4$
β_2	1442.1	x_1	β_{13}	-25.58	$x_2 x_5$
β_3	-35.21	x_2	β_{14}	75.64	$x_3 x_4$
β_4	-281.6	x_3	β_{15}	185.8	$x_3 x_5$
β_5	38.92	x_4	β_{16}	463.7	$x_4 x_5$
β_6	73.83	x_5	β_{17}	409.5	x_1^2
β_7	-44.77	$x_1 x_2$	β_{18}	75.24	x_2^2
β_8	227.8	$x_1 x_3$	β_{19}	156.9	x_3^2
β_9	502.8	$x_1 x_4$	β_{20}	266.7	x_4^2
β_{10}	573.4	$x_1 x_5$	β_{21}	760.0	x_5^2
β_{11}	-52.69	$x_2 x_3$			

A summary of the overall process utilized to populate the EEWS weight response surface surrogate model is given in Figure 10.

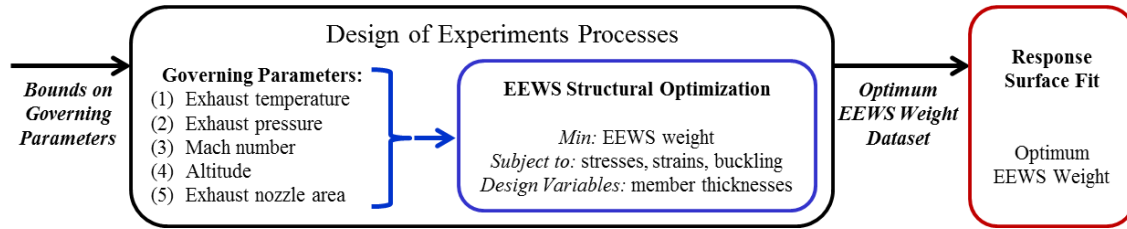


Figure 10. Summary of process utilized to generate the surrogate EEWS weight model.

The sensitivity of EEWS weight with respect to each response surface parameter is shown in Figure 11. In each plot, while the value of one parameter was varied according to the x-axis values, all other parameters were held at fixed values of $x = 0.25, 0.5$, and 0.75 , respectively. From the Figure, we observe that the nozzle area parameter has the greatest influence on EEWS weight throughout the design space, followed by the exhaust temperature and exhaust pressure. We also note that the relative sensitivity of each parameter varies with the value assigned to other parameters. This is a direct result of the coupling between governing parameters throughout the design space.

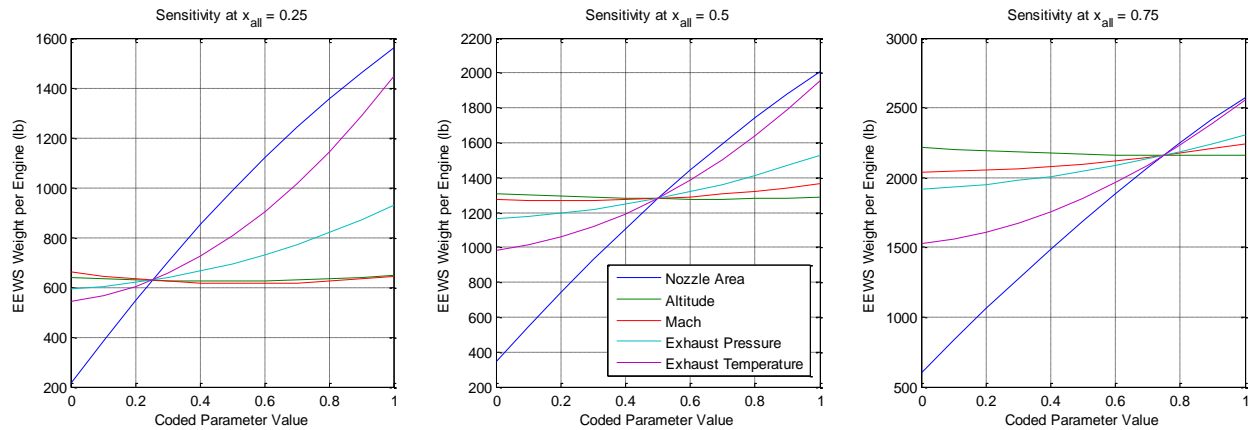


Figure 11. Sensitivity of each response surface parameter at various points throughout the design space

By simply utilizing the response surface surrogate in MDO, the optimum EEWS weight can be quickly determined for each engine. This weight is then supplied to the weights module. In the future, it is also anticipated that the weight can be used to place concentrated masses accompanying those representing engines on a lower fidelity representation of the aircraft structure. This will further allow for a better representation of the effects of EEWS in MDO without requiring additional heat transfer analysis in the structures discipline.

Acknowledgments

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