TOPOLOGY OPTIMIZATION FOR ACOUSTIC-STRUCTURE INTERACTION PROBLEMS

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Abstract: We propose a gradient based topology optimization algorithm for acoustic-structure (vibro-acoustic) interaction problems without an explicit interfacing boundary representation. In acoustic-structure interaction problems, the pressure field and the displacement field are governed by the Helmholtz equation and the linear elasticity equation, respectively, and it is necessary that the governing equations should be properly evolved with respect to the design variables in the design domain. Moreover, all the boundary conditions obtained by computing surface coupling integrals should be properly imposed to subdomain interfaces evolving during the optimization process. In this paper, we propose to use a mixed finite element formulation with displacements and pressure as primary variables (u/p formulation) which eliminates the need for explicit boundary representation. In order to describe the Helmholtz equation and the linear elasticity equation, the mass density as well as the shear and bulk moduli are interpolated with the design variables. In this formulation, the coupled interface boundary conditions are automatically satisfied without having to compute surface coupling integrals. Two-dimensional acoustic-structure interaction problems are optimized to show the validity of the proposed method.

Keywords: Mixed formulation, acoustic-structure interaction, dynamics, harmonic loading, coupled problems.

1. INTRODUCTION

Topology optimization has been applied to a variety of engineering problems and extensions to multiphysics systems seems to be a promising future direction [1]. In this paper, the computational framework for topology optimization of acoustic-structure interaction problems is proposed.

First, however, we are required to address the following issues. During the optimization process, two distinct governing equations – Helmholtz equation and the linear elasticity equation – should be modeled without explicit bound-

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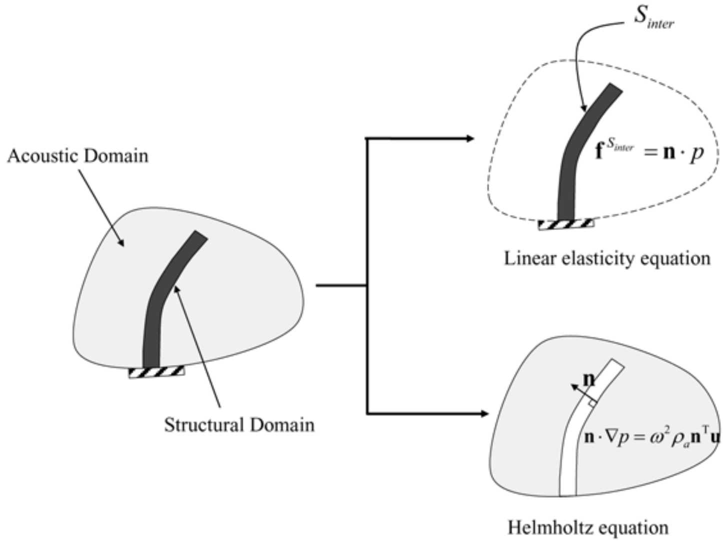


Figure 1. The interaction boundary conditions between acoustic and structure (where **n**: the normal vector from the fluid to the solid, p: the pressure, **u**: the displacement, ω: the angular speed, ρ_a: the structural density, and S_{inter}: the interfacing boundary).

ary representation to allow for free topological variations. Because the pressure and the displacements are the primal variables for the acoustic and the linear elasticity equation, respectively, alternating these two equations during optimization is difficult. Moreover, at the evolving interfacing boundaries, some conditions, illustrated in Figure 1, coming from the conservation of mass and the equation of motion should be properly imposed. This implies that positions and parameters of boundary conditions depend on a given topology. This explicit boundary issue is also observed in hydrostatics and electromagnetic structures [2]. To make it possible to compute these design dependent boundary conditions, an explicit boundary representation as known from shape optimization must be used. However, in topology optimization relying on the localized density, this is not possible.

2. u/p MIXED FORMULATION FOR THE ACOUSTIC-STRUCTURE INTERACTION PROBLEM

In this paper, instead of separately solving the Helmholtz equation and the linear elasticity equation, we propose to adopt the mixed displacement/pressure (u/p) finite element formulation, in which the displacements as well as the pressure are the primal variables for both physical regions [3, 4]. Moreover, the material interpolation scheme based on the SIMP (Solid Isotropic Material with Penalization) method [1] must account for the interpolation between the material properties of air (fluid) and solid.

2.1 The Mixed Finite Element Formulation for the Acoustic-Structure Interaction Problem

Rather than using only the displacements of nodes as the primal variables, in the mixed finite element procedure, pressure is added as an additional primal variable and the constitutive equation involving pressure and displacements is implemented in a finite element context.

2.1.1 Basic Principle of the Mixed Finite Element Formulation. In the mixed finite element formulation, the (2D) governing equation, without consideration of body force, and the constitutive equation are formulated as follows:

$$\text{Frequency domain equilibrium equation: } \nabla\boldsymbol{\sigma} = -\omega^2\rho\mathbf{u} \quad \text{on } \Omega, \tag{1}$$

$$\text{Stress and strain relationship: } \boldsymbol{\sigma} = K\varepsilon_v\boldsymbol{\delta} + 2G\mathbf{e}, \tag{2}$$

$$\text{Pressure and volumetric strain relationship: } p = -K\varepsilon_v \tag{3}$$

$$\mathbf{e} = \boldsymbol{\varepsilon} - \frac{\varepsilon_v}{2}\boldsymbol{\delta}, \quad \varepsilon_v = \frac{\Delta V}{V} = \varepsilon_{kk}, \tag{4}$$

where K, G and ρ are the bulk modulus, the shear modulus, and the density in the analysis domain Ω, respectively. The deviatoric strain components and the volumetric strain are denoted by $\mathbf{e}$ and ε_v, respectively, and δ is Kronecker's delta.

The basic approach of displacement/pressure finite element formulations is to interpolate the displacements and the pressure, simultaneously. This requires that we express the principle of virtual work in terms of the independent variables $\mathbf{u}$ and p, which gives

$$\int_V \delta\mathbf{e}^T\mathbf{C}\boldsymbol{\sigma}\,\mathrm{d}V - \int_V p\delta\varepsilon_v\,\mathrm{d}V = \int_\Omega -\omega^2\rho\delta\mathbf{u}^T\mathbf{u}\,\mathrm{d}\Omega + \int_{S^f}\delta\mathbf{u}^T\mathbf{f}^{S^f}\,\mathrm{d}S^f, \tag{5}$$

$$\int_V (p/K + \varepsilon_V)\delta p\,\mathrm{d}V = 0, \tag{6}$$

$$(\boldsymbol{\sigma} + p\delta) = \mathbf{C}'\left(\boldsymbol{\varepsilon} - \frac{1}{2}\varepsilon_v\delta\right). \tag{7}$$

The virtual displacement and the corresponding strains are denoted by $\delta\mathbf{u}$ and $\delta\boldsymbol{\varepsilon}$, respectively, and $\mathbf{C}'$ is the stress-strain matrix for the deviatoric stress and strain component.

In this mixed displacement/pressure finite element formulation, three involved material properties, the bulk modulus (K), the shear modulus (G), and the density (ρ) in Equations (1)–(7), are alternated with respect to the acoustic domain and the structural domain. For instance, if the analysis domain Ω is assumed to be divided into a structural domain Ω_s and an acoustic domain Ω_a, these three material properties are varying as follows:

$$\Omega = \Omega_s \cup \Omega_a, \quad \Omega_s \cap \Omega_a = \mathbf{0}, \tag{8}$$

$$\text{For structural domain: } K \equiv K_s, \quad G \equiv G_s, \quad \rho \equiv \rho_s \text{ on } \Omega_s, \tag{9}$$

$$\text{For acoustic domain: } K \equiv K_a, \quad G \equiv G_a = 0, \quad \rho \equiv \rho_a \text{ on } \Omega_a, \tag{10}$$

where the subscripts 's' or 'a' on the bulk, the shear and the density denote whether the corresponding material properties are belonging to the structural domain or the acoustic domain, respectively.

2.1.2 The Derivation of the Wave Equation from the Mixed Displacement/Pressure Formulation. The analysis procedure for response of linear solid media by the mixed displacement/pressure finite element procedure is well understood. However, in case of the acoustic domain, it requires some algebra to derive the Helmholtz equation from the mixed displacement/pressure formulation by assigning the appropriate material properties.

Setting the shear modulus to zero makes it possible to derive the Helmholtz equation on the acoustic domain from the mixed displacement/pressure formulation.

$$K \equiv K_a, \quad G = G_a = 0, \quad \rho \equiv \rho_a. \tag{11}$$

The governing equation (1) and the constitutive equation (3) may then be simplified as follows:

$$\nabla p - \omega^2 \rho_a \mathbf{u} = 0, \tag{12}$$

$$\nabla \cdot \mathbf{u} + \frac{p}{K_a} = 0. \tag{13}$$

Note that Equations (12) and (13) can be regarded as the *linearized Euler's equation* and *the linear continuity equation*, respectively, which together constitute the basis of the linear wave equation [5]. Substituting the displacement in (12) into Equation (13), the Helmholtz equation (the frequency dependent wave equation for the pressure variable) can be derived:

$$\nabla \cdot \left(\frac{1}{\rho_a} \nabla p \right) + \omega^2 \frac{1}{K_a} p = 0. \tag{14}$$

This shows that we can imitate the wave equation (or Helmholtz equation) using the mixed displacement/pressure finite element problems with the proper bulk, shear modulus, and the fluid density corresponding to air or fluid domain as well as the proper boundary conditions for the pressure and displacements. (This can be conceptually understood because the acoustic pressure is in fact generated by the harmonic movement of the fluid element.)

2.2 Parameterization Method for Topology Optimization

For the mixed finite element governing equation to alternate between the Helmholtz equation and the linear elasticity equation, the involved material properties, i.e., the bulk modulus, the shear modulus, and the mass density, should be properly interpolated with respect to the design variables follwing Equations (8)–(10) [2]. In this paper, the nodal design variable method based on the SIMP (Solid Isotropic Material with Penalization) is used:

$$K(\gamma) = K_s\gamma^{n_1} + (1 - \gamma^{n_1})K_a, \tag{15a}$$

$$G(\gamma) = G_s\gamma^{n_1} + G_a(1 - \gamma^{n_1}) = G_s\gamma^{n_1}, \tag{15b}$$

$$\rho(\gamma) = \rho_s\gamma^{n_2} + (1 - \gamma^{n_2})\rho_a, \tag{15c}$$

$$0 \leq \gamma \leq 1, \tag{15d}$$

where γ is the nodal design variable. The penalty factors for the bulk modulus, the shear modulus, and the density are denoted by n_1 and n_2, respectively. In these interpolation functions, the solid media can be represented when is one and the acoustic media when γ is zero. Positive values between 1 and 3 are used for n_1 and $n_2 = 1$.

3. TOPOLOGY OPTIMIZATION OF ACOUSTIC-STRUCTURE INTERACTION PROBLEMS

In this section, an analysis example as well as topology optimization problems for the acoustic-structure interaction structures will be solved with the developed mixed displacement/pressure ($\mathbf{u}/p$) finite element procedures and the method of moving asymptotes [6].

Case 1: The pressure calculation by the mixed finite element formulation
In order to verify the analysis code we first solve the simple wave propagation problem shown in Figure 2(a) by the Helmholtz equation as well as the mixed finite element formulation with the same discretization. Figure 2(b) shows the pressure distributions along cross-section AA' obtained by the mixed finite element formulation and the Helmholtz equation, respectively. It is hereby shown

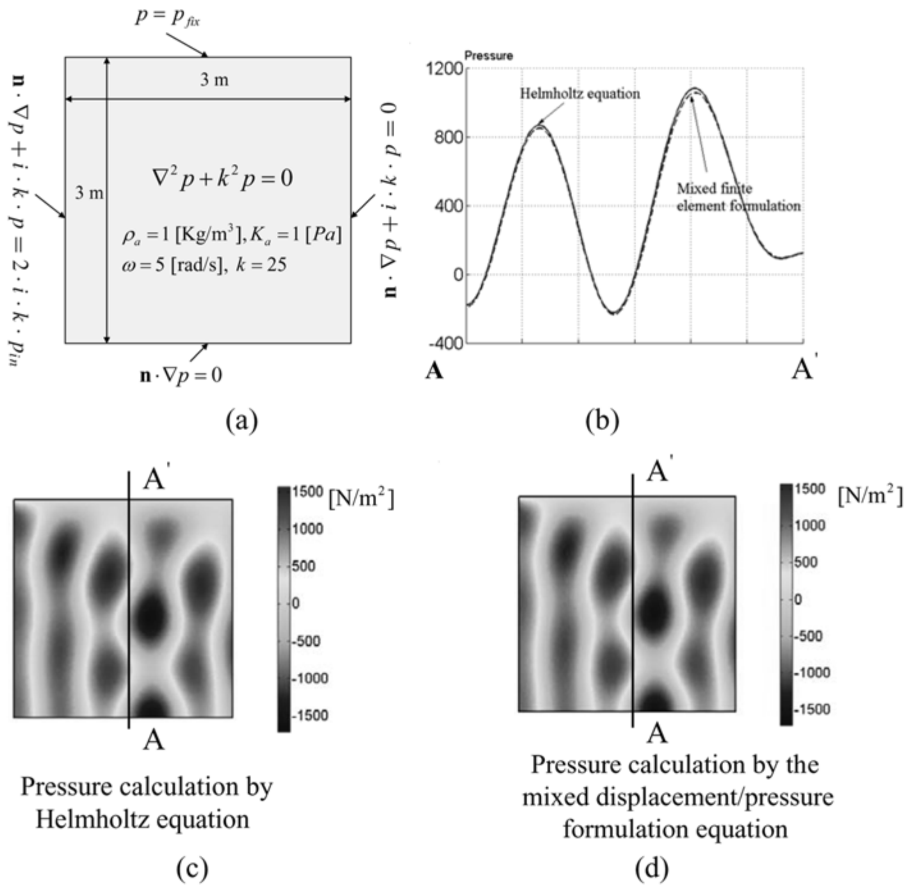


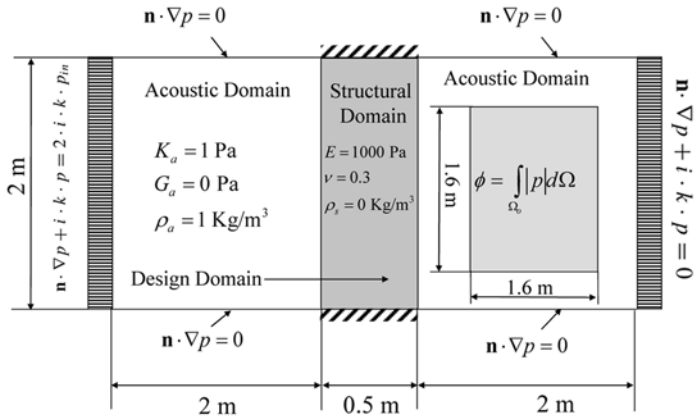
Figure 2. Analysis example 1: Acoustic domain analysis with the mixed formulation with various boundary conditions. (a) Problem definition (where $p_{\text{fix}} = 123$ Pa and $p_{\text{in}} = 1000$ Pa), (b) the pressure distribution of the cross section, (c) the pressure distribution by the Helmholtz equation, and (d) the pressure distribution by the mixed finite element procedure.

that it is possible to implement the Helmholtz equation by the studied mixed finite element formulation.

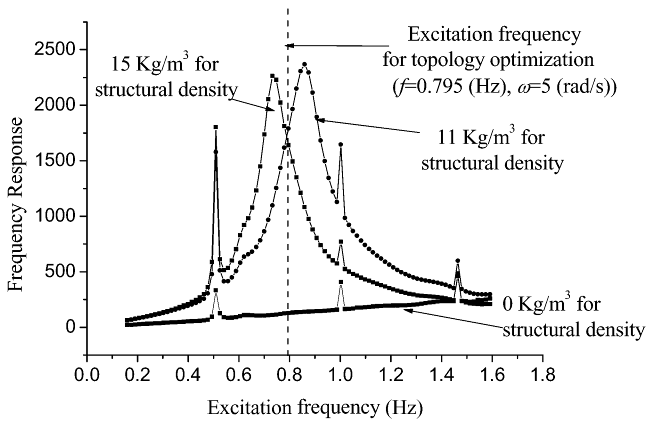
Case 2: Topology optimization for flexible partition

For an illustrative topology optimization example, a design problem shown in Figure 3 is considered. By designing a structure inside the structural (design) domain, the noise level in a defined domain should be minimized. The objective function is defined as the minimizing of the integral of the pressure.

$$\underset{\gamma}{\text{Minimize}} \quad \phi = \int_{\Omega_0} |p| \, \mathrm{d}\Omega \quad \text{(where the objective domain is defined by } \Omega_0\text{)}. \tag{16}$$



(a)



(b)

Figure 3. Topology optimization for a flexible partition. (a) The definition of optimization problem (where E, ν, and ρ_s are Young's modulus, the Poisson's ratio, and the structural density, respectively. p_{in} is set to -1.0×10^3 Pa), (b) the initial frequency response functions of the objective function with two different mass densities.

At first, the initial frequency response functions of the defined objective function for two different mass densities ($\rho = 11$ and 15 kg/m^3) are presented in Figure 3(b) with the whole design domain filled with solid. Important differences between these two frequency response functions can be observed. As expected, the eigenfrequencies for $\rho = 11$ kg/m^3 are higher than those for $\rho = 15$ kg/m^3. Setting the excitation frequency to $5/2\pi$ between two hills in Figure 3(b) and minimizing the objective function, one can imagine the different behaviors of these frequency response functions during optimization. In Figures 4 and 5, the optimized results and the frequency response func-

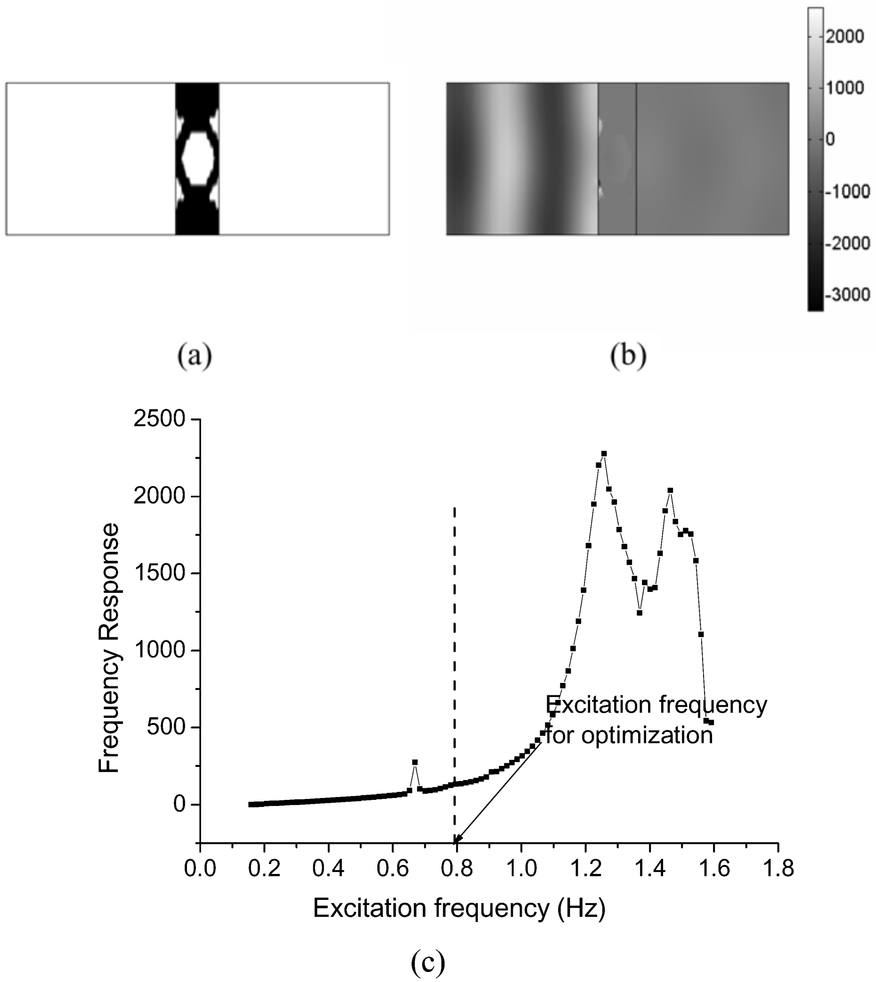


Figure 4. Optimization results with $\rho = 11$ kg/m^3. (a) An optimized result ($\phi_{\text{optimized}} = 250.93$ N), (b) the pressure distribution, and (c) the frequency response.

tions are plotted. When the heavier mass density ($\rho = 15$ kg/m^3) is used, the second eigenfrequency is placed to the left of the excitation frequency. Thus, to minimize the objective function, the optimized result will be one having low fundamental eigenfrequency. Oppositely, with the lighter structural density $\rho = 11$ kg/m^3, the fundamental eigenfrequency is placed to the right of the excitation frequency. This leads to an optimized result having larger fundamental eigenfrequency as Figure 4 shows. Some observations can be made here. First, it can be postulated that optimization topology for frequencies between eigenfrequencies will have similar topologies. Second, minimizing

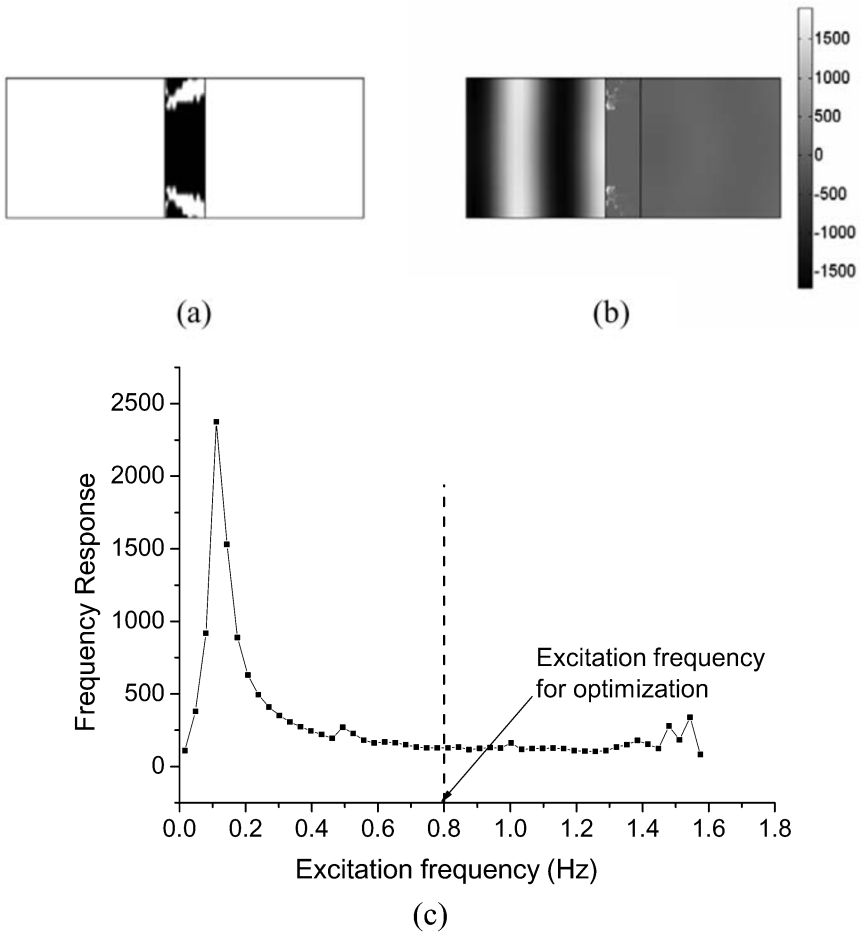


Figure 5. An optimization result with $\rho = 15$ kg/m^3. (a) An optimized result ($\phi_{\text{optimized}} = 129.27$ N), (b) the pressure distribution, and (c) the frequency response function.

the response below the fundamental frequency is very similar to maximizing the fundamental eigenfrequency as seen in Figure 4. It should also be noted that the sharp peaks seen at regular intervals in the response function correspond to acoustical eigenmodes with little influence on the structural behaviour.

4. CONCLUSION

Using a mixed displacement/pressure formulation, we can solve acoustic-structure interaction problems using a standard density based topology optimization approach. By changing the bulk modulus, the shear modulus, and the density in the mixed displacement/pressure formulation, the Helmholtz equation and the linear elasticity equation can be recovered. Topology optimization of acoustic-structure interaction structures is demonstrated and the behavior of the optimized designs was interpreted.

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