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A note on stress-constrained truss topology optimization

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Abstract The purpose of this brief note is to demonstrate that general-purpose optimization methods and codes should not be discarded when dealing with stress-constrained truss topology optimization. By using a disaggregated formulation of the considered problem, such methods may find also "singular optima", without using perturbation techniques like the ε-relaxed approach.

Key words truss topology optimization, stress constraints, local buckling constraints

1 Introduction

We consider the nonconvex problem of minimizing the weight of a truss structure subject to stress and local buckling constraints, using the cross-section areas as design variables. The feasible set of these problems may contain degenerate parts with zero measure. Typically the global optimal solution is located in one of these degenerate parts, so called singular optima. To remedy this difficulty, it has been suggested that the stress constraints should be perturbed so that the degenerate parts are expanded. In Cheng and Guo (1997), it is proposed that the stress constraints and the lower bounds on the variables should be perturbed by a small positive parameter. This is commonly called the ε-relaxed approach, and it is extended to include also local buckling constraints in Guo, Cheng and Yamazaki (2001).

However, the ε-relaxed approach and similar perturbation methods will in general not converge to a global minimizer since the perturbed problems are intrinsically nonconvex with several local minima, see e.g. Stolpe and Svanberg (2001).

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The purpose of this paper is to indicate that good solutions to truss topology optimization problems can be obtained by modern state-of-the-art optimization codes without using any perturbations, if a different but equivalent formulation is used. The intrinsic difficulty of finding global optima however remains.

The work in this note was initiated by the recent article Guo, Cheng and Yamazaki (2001), in which it was (erroneously) claimed that a certain truss topology optimization problem was solved to global optimality.

2 Two formulations of the considered problem

If there is only one load case, the problem under consideration is defined as follows, where the subscript i refers to the i-th bar in the structure.

$$\begin{aligned}
&\underset{\mathbf{a}}{\text{minimize}} && \sum_{i=1}^{n} \ell_i \rho_i a_i \\
&\text{subject to} && \sigma_i^{\min} \le \sigma_i(\mathbf{a}) \le \sigma_i^{\max}, \quad \text{if } a_i > 0\,, \\
& && \sigma_i(\mathbf{a}) \ge \sigma_i^{cr}(a_i), \quad \text{if } a_i > 0\,, \\
& && \mathbf{a} \ge 0\,,
\end{aligned} \tag{1}$$

where (for the i-th bar) a_i is the cross-section area, ℓ_i is the length, ρ_i is the density, $\sigma_i^{\max} > 0$ is the stress limit in tension, $\sigma_i^{\min} < 0$ is the stress limit in compression, $\sigma_i(\mathbf{a})$ is the stress given by $\sigma_i(\mathbf{a}) = (E_i/\ell_i)\mathbf{r}_i^T\mathbf{u}$, where E_i is the Young's modulus, $\mathbf{u}$ is the displacement vector, and $\mathbf{r}_i$ is a vector of direction cosines such that $\mathbf{r}_i^T\mathbf{u}$ is the elongation of the i-th bar. The displacement vector $\mathbf{u}$ satisfies the equilibrium equations $\mathbf{K}(\mathbf{a})\mathbf{u} = \mathbf{p}$, where $\mathbf{p}$ is the vector of external loads and $\mathbf{K}(\mathbf{a}) = \sum_{i=1}^{n} a_i(E_i/\ell_i)\mathbf{r}_i\mathbf{r}_i^T$ is the stiffness matrix of the structure. $\sigma_i^{cr}(a_i)$ is the Euler buckling stress, which under the assumption that the cross-section is circular is given by $\sigma_i^{cr}(a_i) = -a_i\pi E_i/(4\ell_i^2)$. Finally, n is the number of bars in the ground structure. The following alternative, but equivalent, formulation of the above problem is obtained by considering not only the cross-section areas as variables, but also the member forces and the nodal displacements, see e.g. Achtziger (1999).

$$
\begin{aligned}
&\underset{\mathbf{a},\mathbf{f},\mathbf{u}}{\text{minimize}} && \sum_{i=1}^{n} \ell_i \rho_i a_i \\
&\text{subject to} && \sum_{i=1}^{n} \mathbf{r}_i f_i = \mathbf{p}\,, \\
& && f_i = a_i (E_i/\ell_i) \mathbf{r}_i^T \mathbf{u}\,, \qquad i = 1, \ldots, n\,, \\
& && a_i \sigma_i^{\min} \le f_i \le a_i \sigma_i^{\max}\,, \quad i = 1, \ldots, n\,, \\
& && f_i \ge a_i \sigma_i^{cr}(a_i)\,, \qquad i = 1, \ldots, n\,, \\
& && \mathbf{a} \ge 0\,,
\end{aligned}
\tag{2}
$$

where f_i is the normal force in the i-th bar. The constraints $\sum_{i=1}^{n} \mathbf{r}_i f_i = \mathbf{p}$ correspond to equilibrium of forces, while the constraints $f_i = a_i(E_i/\ell_i)\mathbf{r}_i^T \mathbf{u}$ correspond to geometric compatibility together with Hooke's law. The constraints $a_i \sigma_i^{\min} \le f_i \le a_i \sigma_i^{\max}$ imply that $\sigma_i^{\min} \le \sigma_i(a) \le \sigma_i^{\max}$ when $a_i > 0$, while the constraints $f_i \ge a_i \sigma_i^{cr}(a_i)$ imply that $\sigma_i(\mathbf{a}) \ge \sigma_i^{cr}(a_i)$ when $a_i > 0$.

3
Two different solutions of the ten-bar truss structure

One of the truss structures considered in Guo, Cheng and Yamazaki (2001) is the ten-bar truss shown in Fig. 1. The height of the structure is $h = 360$ and the external load is given by $P = 100$. The stress limits and the material data are given by

$$\sigma_i^{\min} = -20,\ \sigma_i^{\max} = 20,\ E_i = 10^4,\ \rho_i = 0.1,\ \text{for all } i.$$

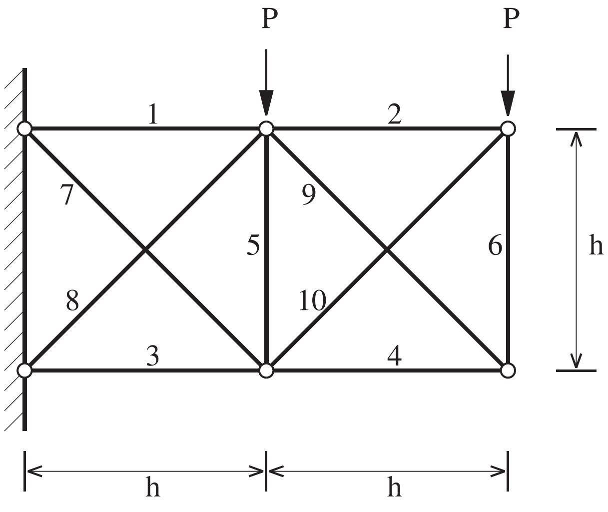

Fig. 1 Ten-bar truss structure

In Sect. 4.4 of that paper a solution which is claimed to be a global optimum is presented. The solution is shown in Fig. 2 below. However, there exists a better solution, namely the one shown in Fig. 3. It satisfies all the constraints and it has a strictly lower weight.

Now follows a brief description of how this solution was obtained. Using formulation (2), the ten-bar truss problem was solved with the sequential quadratic programming package SNOPT, see e.g. Gill, Murray and Saunders (2002). The starting point $(\mathbf{a}^0, \mathbf{f}^0, \mathbf{u}^0)$ supplied to SNOPT was calculated as follows. First, a vector $\mathbf{a}^0 \in \mathbb{R}^n$ was chosen (see Table 1 below). Then the initial displacement vector $\mathbf{u}^0$ was calculated as the unique solution to $\mathbf{K}(\mathbf{a}^0 + \delta\mathbf{e})\mathbf{u}^0 = \mathbf{p}$, where $\mathbf{e} = (1, \ldots, 1)^T$ and $\delta = 10^{-10}$. Then the initial member forces f_i^0 were calculated by $f_i^0 = a_i^0 (E_i/\ell_i)\mathbf{r}_i^T \mathbf{u}^0$, for $i = 1, \ldots, n$.

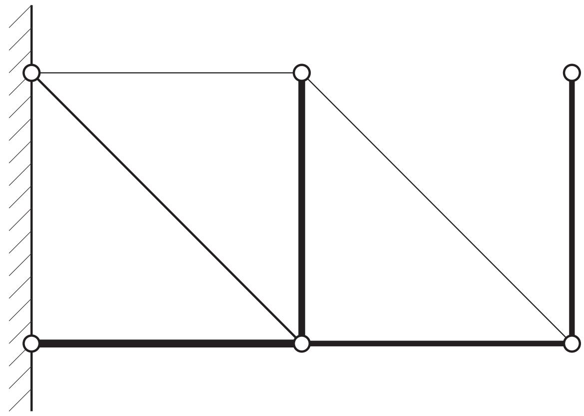

Fig. 2 Solution obtained in Guo, Cheng and Yamazaki (2001). Weight = 8785.79

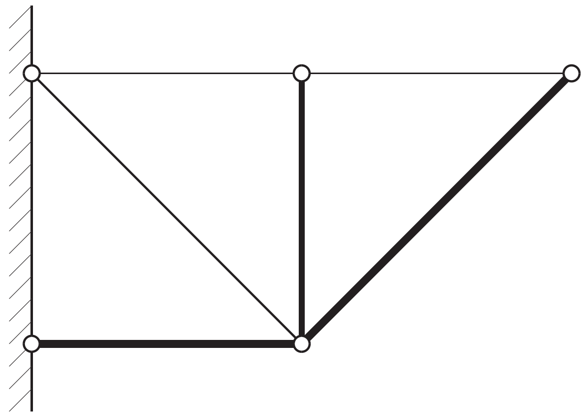

Fig. 3 Solution obtained in this note. Weight = 8553.44

Table 1 Initial and obtained areas for the ten-bar truss

Bar	$\mathbf{a}^0$	$\bar{\mathbf{a}}$	$\mathbf{a}^0$	$\hat{\mathbf{a}}$
a_1	50	5.00000	50	5.00000
a_2	0	0	50	5.00000
a_3	50	70.35876	50	70.35876
a_4	50	40.62165	50	0
a_5	50	57.44769	50	40.62165
a_6	50	40.62165	50	0
a_7	50	14.14214	50	14.14214
a_8	0	0	0	0
a_9	50	7.07107	50	0
a_{10}	0	0	50	68.31720
Weight		8785.79		8553.44

As expected, the solution found by SNOPT was dependent on the choice of starting point. If $a_2^0 = a_8^0 = a_{10}^0 = 0$ and $a_i^0 = 50$ for the other 7 bars, then SNOPT found exactly the same solution as the one found in Guo, Cheng and Yamazaki (2001). This solution is denoted $(\bar{\mathbf{a}}, \bar{\mathbf{f}}, \bar{\mathbf{u}})$. The cross-section areas $\bar{\mathbf{a}}$ are listed in Table 1 and depicted in Fig. 2. If instead $a_8^0 = 0$ and $a_i^0 = 50$ for the other 9 bars, then SNOPT found a strictly better solution! This solution is denoted $(\hat{\mathbf{a}}, \hat{\mathbf{f}}, \hat{\mathbf{u}})$. The cross-section areas $\hat{\mathbf{a}}$ are listed in Table 1 and depicted in Fig. 3.

In the solution $(\bar{\mathbf{a}}, \bar{\mathbf{f}}, \bar{\mathbf{u}})$, bars 3, 4, 5, and 6 are in compression with active buckling constraints, while bars 1, 7, and 9 are in tension with active stress constraints. In the solution $(\hat{\mathbf{a}}, \hat{\mathbf{f}}, \hat{\mathbf{u}})$, bars 3, 5, and 10 are in compression with active buckling constraints, while bars 1, 2, and 7 are in tension with active stress constraints. Hence, both designs are *fully stressed*.

Moreover, for each of the two obtained solutions there are in fact Lagrange multipliers such that the *Karush-Kuhn-Tucker conditions* for problem (2) are satisfied.

Finally, it should be emphasized that we do not claim that the obtained solution $\hat{\mathbf{a}}$ is a global optimum.

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