BABEŞ-BOLYAI UNIVERSITY FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Mate-Info Contest – March 2022 Written test in Computer Science

IMPORTANT NOTE:

In the absence of further specification, assume that all arithmetic operations are performed on unlimited data types (no overflow/underflow).

Also, index numbering of all arrays starts at 1.

1. Let us consider the algorithm magic(x), where x is a natural number $(1 \le x \le 32000)$.

```
Algorithm magic(x):
st ← 1
dr ← x
While st ≤ dr execute
mj ← (st + dr) DIV 2
If mj * mj = x then
return True
EndIf
If mj * mj < x then
st ← mj + 1
else
dr ← mj - 1
EndIf
EndWhile
return False
EndAlgorithm
```

Which of the following statements are true?

- A. For any input value x strictly less than 10 the algorithm returns False.
- B. The algorithm decomposes the number x into its prime factors.
- C. The algorithm returns True if the number x is a perfect square.
- D. The algorithm does not return True for any valid value of input parameter x.
- **2.** Let us consider the algorithm calculeaza(a,b), where a and b are natural numbers $(1 \le a, b \le 10000)$.

```
Algorithm calculeaza(a, b):
    x ← 1
    For i ← 1, b execute
        x ← (x MOD 10) * a
    EndFor
    return x
EndAlgorithm
```

- A. If a = 107 and b = 101, the algorithm returns the value 107.
- B. If a = 1001 and b = 101, the algorithm returns the value 1001.
- C. For all algorithm calls with $1 \le a \le 10000$ and b = 101, the returned value is equal to a.
- D. For all algorithm calls with a = 1001 and $1 \le b \le 10000$, the returned value is equal to 1001.

3. Let us consider the algorithm afis(n), where n is a natural number $(0 \le n \le 10000)$.

```
Algorithm afis(n):
    Write n, " "
    If n > 0 then
        afis(n DIV 2)
        Write n, ", "
    EndIf
EndAlgorithm
```

Which of the following statements are true for the call afis(n)?

- A. The algorithm prints an array of numbers in which the first element is equal to the last element, the second element is equal to the second to last element, etc (except for the middle element).
- B. The algorithm prints an array of even numbers.
- C. The algorithm prints an array of numbers in ascending order followed by numbers in descending order.
- D. The algorithm prints an array of numbers in descending order followed by numbers in ascending order.
- **4**. Let us consider the algorithm cauta(n, b), where n and b are natural numbers ($0 \le n \le 10^6$, $2 \le b < 10$).

```
Algorithm cauta(n, b):
    v ← 0
    If n = 0 then
        return 1
    else
        m ← n
        While m > 0 execute
        If m MOD b = 0 then
        v ← v + 1
        EndIf
        m ← m DIV b
        EndWhile
        return v
    EndIf
EndAlgorithm
```

- A. The algorithm computes and returns the number of digits of n.
- B. The algorithm returns 1 if the number n is a power of b and 0 otherwise.
- C. The algorithm computes and returns the number of digits equal to 0 from the representation in base b of number n.
- D. The algorithm returns 1 if number n ends in digit b and 0 otherwise.
- 5. Let us consider the algorithm abc(a, n, p), where n is a natural number $(1 \le n \le 10000)$, p is an integer number $(-10000 \le p \le 10000)$, and a is an array of n non-zero natural numbers (a[1], a[2], ..., a[n]).

```
Algorithm abc(a, n, p):

If n < 1 then
return -1
else
If (1 ≤ p) AND (p ≤ n) then
return a[p]
else
return 0
EndIf
EndIf
EndAlgorithm
```

- A. The algorithm returns 0 if and only if p is a negative number or is greater than n.
- B. The algorithm returns the element on position p if p is strictly greater than 0 and less than or equal to the length of the array.
- C. The algorithm never returns 0 for parameter values that meet the given preconditions.
- D. The algorithm returns the element at position p if p is greater than or equal to 0 and strictly less than the length of the array.
- **6.** In order to generate all the numbers with n digits composed only of digits 0, 6, 7, there is an algorithm which, for n = 2, generates in ascending order the numbers 60, 66, 67, 70, 76, 77.
- If n = 4 and the same algorithm is used, what is the number generated immediately after the number 6767?
 - A. 7667
 - B. 6760
 - C. 6776
 - D. None of the other variants are correct.
- 7. For a natural number nr ($1000 \le nr \le 1000000$), we define the decrement operation as follows: if the last digit of nr is not 0, we subtract 1 from nr, otherwise, we divide nr by 10 and we keep the integer part of the quotient. Which of the following algorithms, for the call decrementare(nr, k), return the number obtained by applying the decrement operation k times ($0 \le k \le 100$) on the number nr? For example, for nr = 15243 and nr = 10, the result will be 151.

```
Algorithm decrementare(nr, k):
        If k = 0 then
            return nr
        else
            If nr MOD 10 ≠ 0 then
                 return decrementare(nr - 1, k - 1)
                 return decrementare(nr DIV 10, k - 1)
        EndIf
    EndAlgorithm
B.
    Algorithm decrementare(nr, k):
        While k > 0 execute
            If nr MOD 10 = 0 then
                 nr ← nr DIV 10
            else
                 nr \leftarrow nr - 1
            EndIf
             k \leftarrow k - 1
        EndWhile
        return nr
    EndAlgorithm
C.
    Algorithm decrementare(nr, k):
        For i \leftarrow 1, k execute
            If nr MOD 10 > 0 then
                nr ← nr DIV 10
                nr \leftarrow nr - 1
            EndIf
        EndFor
        return nr
    EndAlgorithm
```

8. Let us consider the algorithm fn with the following parameter specification: an array v with n natural numbers (v[1], v[2], ..., v[n]) and the integer number n $(1 \le n \le 10000)$.

```
Algorithm fn(v, n):
    a ← 0
    For i \leftarrow 1, n execute
         ok ← True
         b \leftarrow v[i]
         While (b \neq 0) AND (ok = True) execute
              If b \text{ MOD } 2 = 1 \text{ then}
                   ok ← False
              EndIf
              b ← b DIV 10
         EndWhile
         If ok = True then
              a ← a + 1
         EndIf
    EndFor
    return a
EndAlgorithm
```

Which of the following statements are true?

- A. The algorithm returns the number of odd elements from vector v.
- B. The algorithm returns the number of elements from vector \mathbf{v} that are powers of 2.
- C. The algorithm returns the number of elements from vector \mathbf{v} that contain only even digits.
- D. The algorithm returns the number of elements from vector \mathbf{v} that contain only odd digits.
- 9. The algorithm magic(s, n) has as input parameters an array s of n characters (s[1], s[2], ..., s[n]) and the integer number n ($1 \le n \le 10000$).

```
Algorithm magic(s, n):
    i ← n
    While 1 ≤ i execute
        If s[i] ≠ s[n - i + 1] then
            return 0
        EndIf
        i ← i - 1
        EndWhile
    return 1
EndAlgorithm
```

- A. The algorithm returns 1 if s has an even number of characters.
- B. The algorithm returns 1 if \mathbf{s} is a palindrome.
- C. The algorithm contains an error since the expression n i + 1 can have negative values during execution.
- D. The algorithm returns 1 if s contains only alphanumeric characters.
- 10. Let us consider the following sequence of pseudocode statements:

```
Read a

For i ← 1, a - 1 execute

For j ← i + 2, a execute

If i + j > a - 1 then

Write a, " ", i, " ", j

Write new line

EndIf

EndFor

EndFor
```

How many pairs of solutions will be printed after executing the pseudocode sequence for a = 9?

- A. 13
- B. 15
- C. 19
- D. None of the other variants are correct.
- 11. The algorithm ceFace(n) has as input parameter a natural number n ($0 \le n \le 10000$).

```
Algorithm ceFace(n):

s ← 0

While n > 0 execute

c ← n MOD 10

If c MOD 2 = 0 then

s ← s + c

EndIf

n ← n DIV 10

EndWhile

return s

EndAlgorithm
```

What is the value returned by the call ceFace(9876)?

- A. 16
- B. 48
- C. 14
- D. 63
- **12.** The algorithm generare(n) processes a natural number n (0 < n < 100).

```
Algorithm generare(n):
    nr ← 0
    For i \leftarrow 1, 1801 execute
        used[i] ← False
    EndFor
    While not used[n] execute
        sum ← 0
        used[n] ← True
        While n \neq 0 execute
             digit ← n MOD 10
             n ← n DIV 10
             sum ← sum + digit * digit * digit
        EndWhile
        n ← sum
        nr \leftarrow nr + 1
    EndWhile
    return nr
EndAlgorithm
```

- A. If n = 10, the algorithm returns 2.
- B. If n = 10, the algorithm returns 1.
- C. If n = 3, the algorithm returns 4.
- D. The two calls generare(3) and generare(30) will return the same value.
- **13.** The algorithm f(a, b) has as input parameters two natural numbers a and b ($1 \le a < b \le 1000$):

```
Algorithm f(a, b):
    If a > 0 then
        return b + f(a DIV 2, b * 2)
    EndIf
    return b + f(a * 2, b DIV 2)
EndAlgorithm
```

Unfortunately, the algorithm calls itself recursively an infinite number of times. State what will be the value of b when variable a becomes 0 for the first time. The algorithm is called using the instruction: $c \leftarrow f(20, 10)$

- A. 100
- B. 160
- C. 320
- D. 640
- **14.** Which of the following expressions have the value true if and only if the natural number n is divisible by 3 and has the last digit 4 or 6:

```
 \begin{array}{l} A.\; (\text{n MOD } 3 = \emptyset) \;\; \text{OR} \;\; ((\text{n MOD } 10 = 4) \;\; \text{AND} \;\; (\text{n MOD } 10 = 6)) \\ B.\; (\text{n MOD } 6 = \emptyset) \;\; \text{AND} \;\; ((\text{n MOD } 10 = 4)) \;\; \text{OR} \;\; (\text{n MOD } 10 = 6)) \\ C.\; ((\text{n MOD } 9 = \emptyset) \;\; \text{AND} \;\; (\text{n MOD } 10 = 4)) \;\; \text{OR} \;\; ((\text{n MOD } 3 = \emptyset) \;\; \text{AND} \;\; (\text{n MOD } 10 = 6)) \\ D.\; (\text{n MOD } 3 = \emptyset) \;\; \text{AND} \;\; ((\text{n MOD } 2 = \emptyset) \;\; \text{AND} \;\; (\text{n MOD } 5 = 0)) \;\; \text{OR} \;\; ((\text{n MOD } 2 = \emptyset) \;\; \text{AND} \;\; (\text{n MOD } 5 = 1))) \\ \end{array}
```

15. Let us consider the following logical expression (X **OR** Z) **AND** (X **OR** Y). Choose the values for X, Y, Z such that the evaluation of the expression gives the result True:

```
A. X ← False; Y ← False; Z ← True;
B. X ← True; Y ← False; Z ← False;
C. X ← False; Y ← True; Z ← False;
D. X ← True; Y ← True; Z ← True;
```

- **16.** Consider all strings of length $l \in \{1, 2, 3\}$ consisting of letters in the set $\{a, b, c, d, e\}$. How many of these strings have elements ordered in strictly ascending order and also having an odd number of consonants? (b, c) and d are consonants)
 - A. 14
 - B. 13
 - C. 26
 - D. 81
- 17. In order to display a square together with its diagonals, we will use the characters * (asterisk) and . (dot) (for the space inside the square except the diagonals). The following example displays a square having a side of n = 6 asterisks. For this representation, 28 asterisks and 8 dots were required.

```
A. For n = 5, precisely 22 asterisks and 4 dots are required.
```

- B. For n = 7, precisely 34 asterisks and 15 dots are required.
- C. For n = 7, precisely 33 asterisks and 16 dots are required.
- D. For n = 18, precisely 100 asterisks and 224 dots are required.
- **18.** Let us consider the algorithm ceFace(T, n, e), having as a parameter an array T of n natural numbers in ascending order (T[1], T[2], ..., T[n]) and natural numbers n and n0 ($1 \le n$ 1, $1 \le n$ 2).

```
Algorithm ceFace(T, n, e):
    If e MOD 2 = 0 then
        a ← 1
        b ← n
        While a ≤ b execute
            m \leftarrow (a + b) DIV 2
            If e < T[m] then
                b ← m - 1
            else
                 If e > T[m] then
                     a ← m + 1
                 else
                     return m
                 EndIf
            EndIf
        EndWhile
        return 0
    else
        While (c \le n) AND (g = 0) execute
            If e = T[c] then
                 g = c
            EndIf
            c ← c + 1
        EndWhile
        return g
    FndTf
EndAlgorithm
```

- A. The algorithm returns 0 if number e does not belong to the array T.
- B. If the number e is odd and belongs to the array T, the algorithm returns the index of e in T using the binary search algorithm.
- C. If the number e is odd and belongs to the array T, the algorithm returns the index of e in T using the sequential search algorithm.
- D. The algorithm returns the index of e in T.

19. Let us consider the algorithm calcul(x, n), where the input parameters are the natural numbers n and x ($1 \le x \le n < 10$).

```
Algorithm calcul(x, n):
    b ← 1
    For i ← 1, n - x execute
        b ← b + i
    EndFor
    a ← b
    For i ← n - x + 1, n execute
        a ← a + i
    EndFor
    return a - b
EndAlgorithm
```

Which of the following statements are true?

```
A. If n = 5 and x = 2, the algorithm returns 20.
```

- B. If n = 3 and x = 2, the algorithm returns 5.
- C. The algorithm returns the cardinality of the set $\{\overline{c_1c_2...c_x}: c_i \neq c_j \ \forall \ 1 \leq i,j \leq x, i \neq j, 1 \leq c_i \leq n\}$
- D. The algorithm always returns a value strictly greater then 0.
- **20.** Let us consider the algorithm s(a, b, c), where a, b, c are positive natural numbers $(1 \le a, b, c \le 10000)$.

```
Algorithm s(a, b, c):
    If (a = 1) OR (b = 1) OR (c = 1) then
        return 1
    else
        If a > b then
            return a * s(a - 1, b, c)
        else
            If a < b then
                return b * s(a, b - 1, c)
        else
               return c * s(a - 1, b - 1, c - 1)
        EndIf
    EndIf
EndIf
EndIf
EndIgorithm</pre>
```

20a. Which of the following statements are true when a = b and a < c?

- A. The algorithm computes and returns c! / (c a)!
- B. The algorithm computes and returns c! / (c a + 1)!
- C. The algorithm computes and returns c! / (c a 1)!
- D. The algorithm computes and returns the number of combinations of c taken (a 1) at a time

20b. Given a = 3, b = 4, c = 7, the algorithm returns:

- A. 224
- B. 56
- C. 336
- D. 168
- 21. Let us consider the algorithm numere(a, b, c, d, e), which receives as input parameters five integer numbers a, b, c, d and e ($1 \le a$, $b \le 10000$, $2 \le c \le 16$, $1 \le d < c$).

```
Algorithm numere(a, b, c, d, e):
    If a = 0 AND b = 0 then
        If e = 0 then
            return True
        else
            return False
        EndIf
    EndIf
    If a MOD c = d then
        e = e + 1
    EndIf
    If b MOD c = d then
       e = e - 1
    EndIf
    return numere(a DIV c, b DIV c, c, d, e)
EndAlgorithm
```

Which of the following statements are true when calling numere(a, b, c, d, 0)?

- A. The algorithm returns True if the representations in base c of the numbers a and b contain the digit d occurring an equal number of times and False otherwise
- B. The algorithm returns True if digit d occurs in the base c representation of the number a and in the base c representation of the number b, False otherwise
- C. Calling numere(a, b, c, d, 0) returns the same value as when calling numere(b, a, c, d, 0)
- D. The algorithm returns True if the digit d occurs on the same positions in the base c representation of the numbers a and b and False otherwise

```
s_i = \begin{cases} x, & \text{if } i = 1 \\ x+1, & \text{if } i = 2 \\ s_{(i-1)}@s_{(i-2)} & \text{if } i > 2 \end{cases}, (i=1,2,\ldots). \text{ The operator @ concatenates the digits from the left and right}
```

operands, in this order (the digits of base 10 representation), and x is a natural number ($1 \le x \le 99$). For example, if x = 3, the array s is 3, 4, 43, 434, 43443, ... For a natural number k ($1 \le k \le 30$) state the number of digits of the item in s that precedes the item having k1 digits, where k1 is the lowest number where $k \le k1 \le 30$ and there exists an item having k1 digits.

```
A. for x = 15 and k = 8, the number of digits of the desired item is 6.
```

- B. for x = 2 si k = 6, the number of digits of the desired item is 6.
- C. for x = 14 si k = 27, the number of digits of the desired item is 26.
- D. for x = 5 si k = 12, the number of digits of the desired item is 8.
- 23. Let us consider the following recursive algorithm fibonacci(n), where n is a natural number $(1 \le n \le n)$ 100). Determine the number of times that the message "Aici" is displayed for a call of fibonacci(n).

```
Algorithm fibonacci(n):
    If n \le 1 then
        Write "Aici"
        return 1
        return fibonacci(n - 1) + fibonacci(n - 2)
    EndIf
EndAlgorithm
```

- A. fibonacci(n) number of times.
- B. fibonacci(n-1) number of times.
- C. fibonacci(n)-1 number of times.
- D. fibonacci(n) fibonacci(n-1) number of times.

- 24. Consider the expression: $E(x) = a_0 + a_1 * x + a_2 * x^2 + a_3 * x^3 + a_4 * x^4$, where a_0 , a_1 , a_2 , a_3 , a_4 and x are non-zero real numbers. The minimum number of multiplications needed to compute the value of the expression E(x) is:
 - A. 4
 - B. 5
 - C. 7
 - D. 3
- **25**. Let us consider the algorithm f(x, n) where x, n are natural numbers and x > 0.

```
1. Algorithm f(x, n):
        If n = 0 then
2.
3.
            return 1
4.
         EndIf
5.
        m \leftarrow n DIV 2
6.
         p \leftarrow f(x, m)
         If n MOD 2 = 0 then
7.
8.
             return p * p
9.
         EndIf
         return x * p * p
10.
11. EndAlgorithm
```

- **25a.** Which of the following statements are true?
 - A. The algorithm returns x^n running approximately n recursive calls.
 - B. The algorithm returns x^n making approximately $log_2 n$ recursive calls.
 - C. The algorithm returns x^n if and only if n is a power of 2
 - D. The algorithm returns x^n if and only if n is even.
- **25b.** Let us consider line 10 replaced with:

```
10. return x * f(x, n - 1)
```

- A. The algorithm does not return x^n anymore
- B. The algorithm returns x^{n+1}
- C. The algorithm runs approximately n^2 recursive calls.
- D. The algorithm returns x^n
- **26**. Let us consider the algorithm f2(a,b) having parameters a and b natural numbers, and the algorithm f(arr, i, n, p) having as parameters the array arr with n integers (arr[1], arr[2], ..., arr[n]), and the integers i and p.

```
Algorithm f2(a, b):
    If a > b then
         return a
    else
         return b
    EndIf
EndAlgorithm
Algorithm f(arr, i, n, p):
    If i = n then
         return 0
    EndIf
    n1 \leftarrow f(arr, i + 1, n, p)
    n2 ← 0
    If p + 1 \neq i then
        n2 \leftarrow f(arr, i + 1, n, i) + arr[i]
    return f2(n1, n2)
EndAlgorithm
```

State which is the result of calling f(arr, 1, 9, -10), if the array *arr* has the values (10, 1, 5, 4, 7, 12, 1, 12, 6).

- A. 24
- B. 37
- C. 39
- D. 56
- **27.** Let us consider the algorithm f(n), having as a parameter the nonzero natural number n which returns a natural number.

```
Algorithm f(n):
    j ← n
While j > 1 execute
    i ← 1
While i ≤ n⁴ execute
    i ← 4 * i
    Write "*"
EndWhile
If j DIV 2 > 1 then
    Write " "
EndIf
    j ← j DIV 2
EndWhile
return j
EndAlgorithm
```

27a. The time complexity of the algorithm belongs to which of the following complexity classes?

- A. $O(\log_2 n)$
- B. $O(\log_2^2 n)$
- C. $O(\log_4^2 n)$
- D. $O(\log_2 \log_4 n)$

- A. If n = 10, the algorithm displays groups of 7 asterisks, groups being separated by a space.
- B. If n = 20, the algorithm displays 4 groups of asterisks and 4 space characters.
- C. If n = 25, the algorithm displays 48 asterisks, and after each group displays one space.
- D. If n = 100, the algorithm displays 84 asterisks and 5 space characters.

BABEŞ-BOLYAI UNIVERSITY

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

*Mate-Info UBB Contest March 25th 2022*Written Exam for Computer Science

GRADING AND SOLUTIONS 23 – 03 - 2022

DEFAULT: 10 points

1	С	3 points
2	ABD	3 points
3	AD	3 points
4	С	3 points
5	В	3 points
6	D	3 points
7	ABD	3 points
8	С	3 points
9	В	3 points
10	С	3 points
11	С	3 points
12	ACD	3 points
13	С	3 points
14	В	3 points
15	BD	3 points
16	В	3 points
17	CD	3 points
18	AC	3 points
19	BD	3 points
21	AC	3 points
22	AD	3 points
23	Α	3 points
24	Α	3 points
26	С	3 points
20a	В	3 points
20b	D	3 points
25a	В	3 points
25b	D	3 points
27a	ВС	3 points
27b	AD	3 points

3. Se consideră expresia următoare, în care x este un număr natural pozitiv.

```
(x MOD 2) + ((x + 1) MOD 2)
```

Care din afirmațiile de mai jos sunt adevărate?

- A. Expresia are valoarea 1 pentru orice număr natural pozitiv x.
- B. Expresia are valoarea 1 dacă și numai dacă x este un număr par.
- C. Expresia are valoarea 1 dacă și numai dacă x este un număr impar.
- D. Există număr natural x pentru care expresia are o valoare strict mai mare decât 1.
- **4.** Fie subalgoritmul prelucrare(x, n) definit mai jos, care primește ca și parametru un șir x cu n numere reale nenule (x[1], x[2], ..., x[n]) și numărul întreg n ($1 \le n \le 10000$). Operatorul / reprezintă împărțirea reală (ex. 3/2=1,5).

```
Subalgoritm prelucrare(x, n):
    p ← 1
    Pentru k ← 1, n - 1 execută
        p ← p + 1
    Pentru i ← 1, n - 1 execută
        Dacă x[i] > x[i + 1] atunci
              x[i] ← x[i] * x[i + 1]
              x[i] ← x[i] / x[i + 1]
              x[i] ← x[i] / x[i + 1]
              x[i] ← x[i] / x[i + 1]
              xFDacă
    SfPentru
    SfPentru
    n ← p
SfSubalgoritm
```

Care dintre următoarele afirmații descriu modificarea aplicată șirului \mathbf{x} în urma apelului subalgoritmului prelucrare(x, n)?

- A. Elementele șirului x vor rămâne nemodificate
- B. Elementele șirului x vor fi în ordine descrescătoare
- C. Elementele sirului x vor fi în ordine crescătoare
- D. Numărul **n** este decrementat cu o unitate
- **5.** Se consideră subalgoritmul calcul(a, n), care primește ca parametru un șir **a** cu **n** numere naturale (a[1], a[2], ..., a[n]) și numărul întreg **n** $(1 \le n \le 10000)$.

```
Subalgoritm calcul(a, n):
    Dacă n = 0 atunci
        returnează 0
    altfel
        returnează a[n] * (a[n] MOD 2) + calcul(a, n - 1)
    SfDacă
SfSubalgoritm
```

Pentru ce valori a numărului **n** și a șirului **a** funcția calcul(a,n) va returna valoarea 10?

```
A. n = 4, a = (2, 4, 7, 5)
B. n = 6, a = (3, 1, 2, 5, 8, 1)
C. n = 6, a = (2, 4, 5, 3, 8, 5)
D. n = 7, a = (1, 1, 2, 1, 1, 1, 3)
```