Examples on Groups

Example 1. Show the set of integers …,-4,-3,-2,-1,0,1,2,3,4,… is and infinite Abelian group with respect to the operation of addition of integers.

Example 2. Show the set of non-zero rational numbers with respect to operation is a group.

Example 3. Show that C, the set of all non-zero complex numbers is multiplicative group.

Example 4
Is the set $(N,+)$ a Group ?

Example 5
Is the set $(N,-)$ a Group ?

Example 6
Is the set $(N,\cdot)$ a Group ?

Example 7
Is the set $(N,\div)$ a Group ?

Example 8
Is the set $(Z,-)$ a Group ?

Example 9
Is the set $(Z,\cdot)$ a Group ?

Example 10
Is the set $(Z,\div)$ a Group ?

Example 11
Are the sets $(Q,+)$, $(Q,-)$, $(Q,\cdot)$, $(Q,\div)$ the Groups ?

Example 12
Are the sets $(R,+)$, $(R,-)$, $(R,\cdot)$, $(R,\div)$ the Groups ?

Examples on Subgroups

1. Is the following set $(R,*)$ subgroup with respect to operation $(*)$, if

a) $a*b=a+b+3;$

b) $a*b=a(1-b)+b;$

c) $a*b=\sqrt{a\cdot b}+1;$

d) $a*b=\dfrac{1}{a\cdot b};$

e) $a*b=-a-b+ab+3;$

2. Are the following sets

$(N,+),\ (N,\cdot),\ (Z,+),(Z,\cdot),(Q,+),(Q,\cdot),(R,+),(R,\cdot)$

Subgroups?

Examples on Rings

1. Let Z is the set of whole numbers. $(+),(-),(\cdot)$ are operations of addition, difference and multiplication on Z and $x, y \in Z$.

Is the set $(Z,+_1,-_1,\cdot_1,e)$ Ring, if

a) $x+_1 y=x+y-2;\quad x\cdot_1 y=xy-2x-2y+6.$

b) $x+_1 y=x+y-1;\quad x\cdot_1 y=xy-x-y+2.$

c) $x+_1 y=x+y+3;\quad x\cdot_1 y=xy-3x-3y++6.$

Examples on Fields

1. Is the set $(Z,+_1,-_1,\cdot_1,e)$ field, if

a) $x+_1 y=x+y-\dfrac{1}{2};\quad x\cdot_1 y=xy-\dfrac{1}{2}x-\dfrac{1}{2}y+\dfrac{3}{4}.$

b) $x+_1 y=x+y-\dfrac{1}{2};\quad x\cdot_1 y=\dfrac{3}{2}xy-\dfrac{3}{4}x-\dfrac{3}{4}y+\dfrac{7}{8}.$

c) $x+_1 y=x+y+\dfrac{1}{2};\quad x\cdot_1 y=xy+\dfrac{1}{2}x+\dfrac{1}{2}y-\dfrac{1}{4}.$