IC 242 – practise problems on tensor algebra

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Problem 1. Expand and simplify:

(i) $\delta_{ij}\delta_{ji}$

(ii) $\delta_{ij}\delta_{jk}\delta_{ki}$

(iii) $\epsilon_{jkq}\epsilon_{jkq}$

Problem 2.

(i) Demonstrate that the tensor $B_{ij} = \epsilon_{ipj}a_p$ is anti-symmetric.

(ii) If v_i is the *dual vector* of the above tensor B, show that $B_{ij} = \frac{1}{2}\epsilon_{kij}v_k$. (Note: dual vector of a tensor is defined as $v_i = \epsilon_{ijk}B_{jk}$)

Problem 3. Show that $(A_{ijk} + A_{jki} + A_{jik})\, x_i x_j x_k = 3A_{ijk}x_i x_j x_k$.

Problem 4.

(i) If A_{ij} is symmetric and B_{ij} is anti-symmetric, then show that $A_{ij}B_{ij} = 0$.

(ii) Show that $\mathbf{a} \times \mathbf{b} \cdot \mathbf{a} = \mathbf{0}$.

Problem 5. Show that the triple cross product $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ simplifies to $(\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$ (*hint*: use indicial notation).