IC 242 – Assignment 1 (due date: 23 March 2017)

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Problem 1. Simplify:

(i) $\delta_{ij}\delta_{ji}$

(ii) $\delta_{ij}\delta_{jk}\delta_{ki}$

(iii) $\epsilon_{jkq}\epsilon_{jkq}$

Problem 2.

(i) Demonstrate that the tensor $B_{ij} = \epsilon_{ipj}a_p$ is anti-symmetric.

(ii) If v_i is the *dual vector* of the above tensor B, show that $B_{ij} = \frac{1}{2}\epsilon_{kij}v_k$. (Note: dual vector of a tensor is defined as $v_i = \epsilon_{ijk}B_{jk}$)

Problem 3. Show that $(A_{ijk} + A_{jki} + A_{jik})\, x_i x_j x_k = 3A_{ijk}x_i x_j x_k$.

Problem 4.

(i) If A_{ij} is symmetric and B_{ij} is anti-symmetric, then show that $A_{ij}B_{ij} = 0$.

(ii) Using indicial notation show that $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \mathbf{0}$.

Problem 5. Show that the triple cross product $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ simplifies to $(\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$ (*hint*: use indicial notation).

Problem 6. Show that $\epsilon_{qmn} det(A) = \epsilon_{ijk}A_{iq}A_{jm}A_{kn}$.

Problem 7. For a second order tensor σ, show that σ_{ii} is invariant.

Problem 8. Find the transformation matrix A for the following transformations:

(i) $90°$ clockwise rotation about the x_1 axis,

(ii) reflection about the $x_1 - x_2$ plane.

Apply the above two transformations to the vector $\mathbf{v}$ and tensor σ, which are given as: $\mathbf{v} = (1, 2, 0)$, and $\sigma = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Problem 9. Find the principal directions and principal values of the 2nd-order tensor σ given as: $\sigma = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

Problem 10. For a scalar field $\phi(\mathbf{x})$, show that:

$$\int_{\Omega} \nabla^2 \phi \mathrm{d}\Omega = \oint_{\Gamma} \frac{\partial \phi}{\partial n} \mathrm{d}\Gamma,$$

where Γ is the surface enclosing the volume Ω, and n is the normal coordinate (to the surface).