Mentos, Volcanos, and Low-Budget Physics

Gautam Mittal
AP Physics C
F Period

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1 Problem

A cart with mass m_1 is propelled up a friction-less ramp with an angle of elevation of θ using an inelastic string attached to a pulley which is being pulled downward by a counterweight with mass m_2. When the cart moves over a trip mechanism, represented by the orange dot in Figure 1, a mentos candy is released from the cart. The objective is to find the distance d between the cart's starting position and the trip's position along the ramp, such that the mentos candy will land exactly in the mouth of the volcano 1.265 meters away from the start of the ramp as shown in Figure 2.

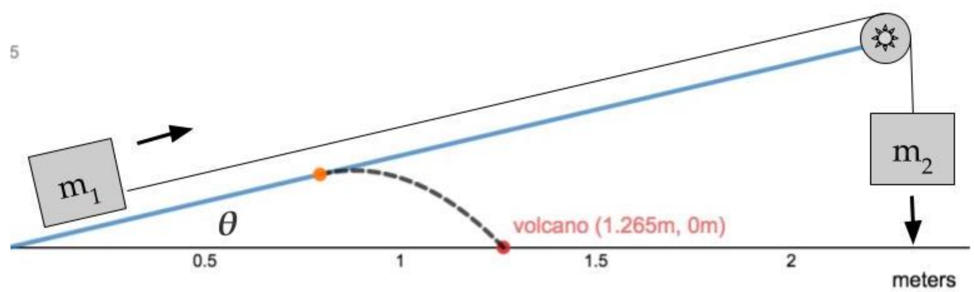


Figure 1: Problem Setup

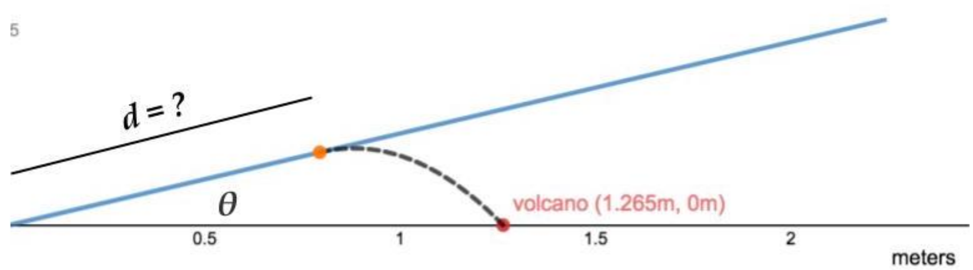


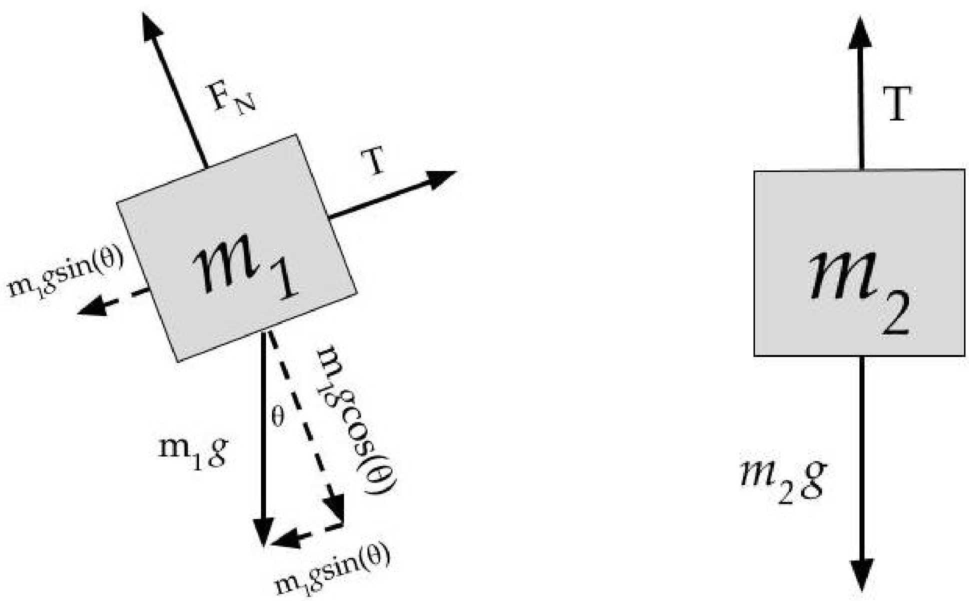
Figure 2: Objective

2 The Kinematic System

In order to optimize the mentos path such that it lands exactly in the volcano, we must first understand the respective paths and kinematic equations acting upon each object in the system. In all calculations involving gravity (g), it is assumed that gravity's magnitude is 9.8 $\frac{m}{s^2}$ down.

2.1 The Cart's Motion

The cart's motion is determined by the motion of the counterweight with mass m_2. In addition, there is a tension force exerted on the string attached to both objects. The following free-body diagrams (Figure 3) can be drawn for each mass, where T is the tension force exerted by the string and F_N is the normal force exerted by the ramp. The ramp is also friction-less and air resistance acting on each object is negligible.



(a) The cart (b) The counterweight

Figure 3: Free-body diagrams

Because the cart has no lateral movement perpendicular to the ramp, it can be inferred that the normal force F_N is equal and opposite to the vertical gravitational force component $m_1 gcos(\theta)$. As a result, the net force on each mass can be determined easily. In addition, both objects have the same acceleration (a), and using the net forces of each, the acceleration of the system can be determined given each mass.

$$a = \frac{\sum F}{m}$$

$$\sum F_{m_1} = T - m_1 gsin(\theta)$$

$$\sum F_{m_2} = m_2 g - T$$

$$a = \frac{T - m_1 gsin(\theta)}{m_1} \quad \text{and} \quad a = \frac{m_2 g - T}{m_2}$$

Given that T is an unknown in both equations, T, and subsequently a, can be solved for using substitution:

$$T = -a \cdot m_2 + m_2 g$$

$$a \cdot m_1 = -a \cdot m_2 + m_2 g - m_1 g sin(\theta)$$

$$a \cdot m_1 + a \cdot m_2 = m_2 g - m_1 g sin(\theta)$$

$$a(m_1 + m_2) = m_2 g - m_1 g sin(\theta)$$

$$a = \frac{g(m_2 - m_1 sin(\theta))}{m_1 + m_2} \tag{1}$$

Given the constant acceleration of the cart, the velocity of the cart can also be found.

$$V = \int a dt$$

$$V(t) = at \tag{2}$$

The displacement equation is given:

$$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$$

Subsequently, the displacement equation with respect to time for the cart can be found, given θ (angle of elevation of the ramp) and a (acceleration of the cart given m_1 and m_2). Because the cart starts at rest, x_0 and V_0 are zero-values.

$$C(t) =< \frac{1}{2} a t^2 \cos\theta, \frac{1}{2} a t^2 \sin\theta > \tag{3}$$

2.2 The Mentos' Motion

Solving for the mentos candy's position can be regarded as a projectile motion problem, as the mentos is released with an initial velocity, an initial position, and is in free-fall. The motion curve, however, is dependent on an external parameter: the time the mentos is released from the cart ($t_{dropped}$).

In a projectile motion problem, the parametric motion equation with respect to time is initialized with V_0, x_0, and a downward gravitational acceleration g. This equation is established with the assumption that there is no air resistance or friction in the system.

$$P(t) =< x_{0_x} + V_{0_x} t, x_{0_y} + V_{0_y} t + \frac{1}{2} g t^2 >$$

A similar equation can be established for the mentos candy's path as it is released. The initial velocity and position of the mentos candy when released can be found using the respective velocity and position of the cart at $t_{dropped}$:

$$x_{0_x} = C(t_{dropped})_x \quad \text{and} \quad x_{0_y} = C(t_{dropped})_y$$

$$V_{0_x} = V(t_{dropped}) \cos\theta \quad \text{and} \quad V_{0_y} = V(t_{dropped}) \sin\theta$$

Given these initial constants, the position of the mentos candy can be derived. The following equation has been translated by $t_{dropped}$ units so that its position is relative to the cart's position $C(t)$ with time.

$$M_x(t) = x_{0_x} + V_{0_x}(t - t_{dropped})$$
$$M_y(t) = x_{0_y} + V_{0_y}(t - t_{dropped}) + \frac{1}{2}g(t - t_{dropped})^2$$

(4)

3 Solution

The final variables needed to solve the system are as follows: $\theta = 13.19°$, $m_1 = 0.4351$kg, and $m_2 = 0.2678$kg. In addition, the problem states that the mouth of the volcano is horizontally-located 1.265 meters away from the start of the ramp. Therefore, the mentos candy's path must intersect with the point (1.265, 0). Given this information, the only value that can be calculated is the acceleration a (Eq. 1) of the cart.

$$a = 2.34952484547\frac{m}{s^2}$$

All other variables, and consequently the solution to the problem, cannot be found without first solving for the two external parameters in the following system of equations.

$$M_x(t) = 1.265 \quad \text{and} \quad M_y(t) = 0$$

$$1.265 = x_{0_x} + V_{0_x}(t - t_{dropped})$$
$$0 = x_{0_y} + V_{0_y}(t - t_{dropped}) + \frac{1}{2}g(t - t_{dropped})^2$$

The two external parameters that must be solved are t and $t_{dropped}$. The t-value solution in the parametric system is the moment in time when the mentos collides with the mouth of the volcano, and $t_{dropped}$ indicates when the mentos should be released from the cart, simultaneously allowing one to find the point on the ramp where the trip mechanism should be placed. Using a computer algebra solver written in Python, these two parameters were found to be $t = 1.08004$ seconds, and $t_{dropped} = 0.83407$ seconds.

In order to find the position of the trip mechanism, one can make the following calculation:

$$C(t_{dropped}) = (0.796, 0.186)$$

To solve for the distance d from the start of the ramp, the following equation based on the Pythagorean theorem can be used:

$$d = \sqrt{C(t_{dropped})_x^2 + C(t_{dropped})_y^2} \tag{5}$$

The final distance **d** is equal to **0.81725 meters** from the start of the ramp.