The Relational Data Model and Relational Database Constraints

Relational Model Concepts

- The relational model represents the database as a collection of relations. Informally, each relation resembles a table of values.
- When a relation is thought of as a **table** of values, each row in the table represents a collection of related data values. A row represents a fact that typically corresponds to a real-world entity or relationship. The table name and column names are used to help to interpret the meaning of the values in each row.
- In the formal relational model terminology, a row is called a tuple, a column header is called an attribute, and the table is called a relation. The data type describing the types of values that can appear in each column is represented by a domain of possible values.

3.1 Domains, Attributes, Tuples, and Relations

- A **domain** D is a set of atomic values. A**tomic** means that each value in the domain is indivisible as far as the formal relational model is concerned.
  Example: **Social_security_numbers:** The nine-digit Social Security numbers.
  Names: The set of character strings that represent names of persons.

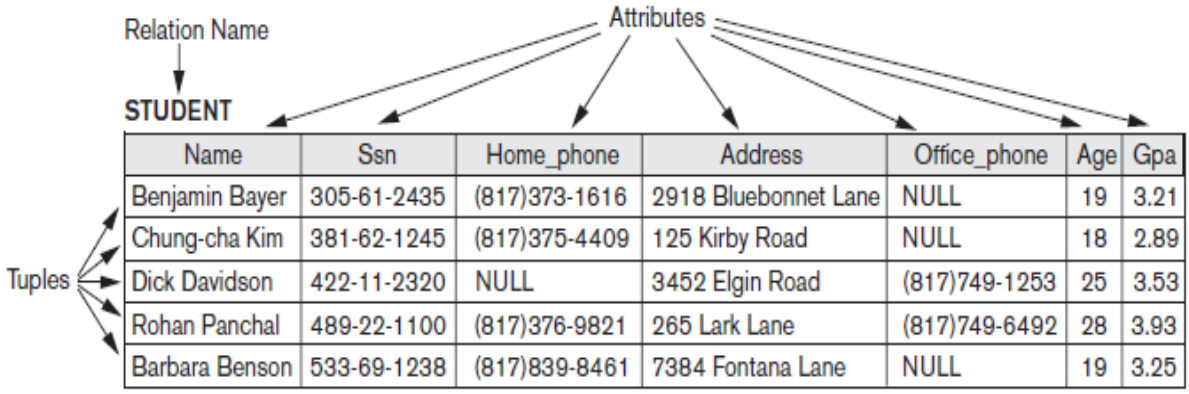
- A **data type** or **format** is also specified for each domain.
  Example: The data type for Employee_ages is an integer number between 15 and 80.

- A **relation schema R**, denoted by **$R(A_1, A_2, ..., A_n)$** is made up of a relation name R and a list of attributes **$A_1, A_2, ..., A_n$**. A relation schema is used to describe a relation; R is called the **name** of this relation.

- The **degree** (or **arity**) of a relation is the number of attributes n of its relation schema. A relation of degree seven, would contain seven attributes describing each student. STUDENT(Name, Ssn, Home_phone, Address, Office_phone, Age, Gpa)

- A **relation** (or **relation state**) r of the relation schema R(A1, A2, ..., An), also denoted by r(R), is a set of n-tuples r = {t1, t2, ..., tm}. Each **n-tuple** t is an ordered list of n values t =<v1, v2, ..., vn>, The terms **relation intension** for the schema R and **relation extension** for a relation state r(R) are also commonly used.

- Figure 3.1 shows an example of a STUDENT relation, which corresponds to the STUDENT schema. Each tuple in the relation represents a particular student . NULL values represent attributes whose values are unknown or do not exist for some individual STUDENT tuple.



Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Benjamin Bayer	305-61-2435	(817)373-1616	2918 Bluebonnet Lane	NULL	19	3.21
Chung-cha Kim	381-62-1245	(817)375-4409	125 Kirby Road	NULL	18	2.89
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	(817)749-1253	25	3.53
Rohan Panchal	489-22-1100	(817)376-9821	265 Lark Lane	(817)749-6492	28	3.93
Barbara Benson	533-69-1238	(817)839-8461	7384 Fontana Lane	NULL	19	3.25

Figure 3.1
The attributes and tuples of a relation STUDENT.

3.1.2 Characteristics of Relations

1. Ordering of Tuples in a Relation.

- A relation is defined as a set of tuples. Elements of a set have no order among them; hence, tuples in a relation do not have any particular order.
- The definition of a relation does not specify any order: There is no preference for one ordering over another. Hence, the relation displayed in Figure 3.2 is considered *identical* to the one shown in Figure 3.1.

Figure 3.2
The relation STUDENT from Figure 3.1 with a different order of tuples.

STUDENT

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	(817)749-1253	25	3.53
Barbara Benson	533-69-1238	(817)839-8461	7384 Fontana Lane	NULL	19	3.25
Rohan Panchal	489-22-1100	(817)376-9821	265 Lark Lane	(817)749-6492	28	3.93
Chung-cha Kim	381-62-1245	(817)375-4409	125 Kirby Road	NULL	18	2.89
Benjamin Bayer	305-61-2435	(817)373-1616	2918 Bluebonnet Lane	NULL	19	3.21

2. Ordering of Values within a Tuple and an Alternative Definition of a Relation.

- The order of attributes and their values is *not* that important as long as the correspondence between attributes and values is maintained.
- An **alternative definition** of a relation can be given, making the ordering of values in a tuple unnecessary. In this definition, a relation schema R = {A1, A2, ..., An} is a set of attributes and a relation state r(R) is a finite set of mappings r = {t1, t2, ..., tm}, where each tuple t_i is a **mapping** from R to D, and D is the **union** (denoted by ∪) of the attribute

domains; that is, D = dom(A1) ∪ dom(A2) ∪... dom(An). In this definition, t[Ai] must be in dom(Ai) for $1 \leq i \leq n$ for each mapping t in r. Each mapping t_i is called a tuple.

- According to this definition of tuple as a mapping, a **tuple** can be considered as a **set** of (<attribute>, <value>) pairs. The ordering of attributes is not important, because the attribute name appears with its value. By this definition, the two tuples shown in Figure 3.3 are identical.

Figure 3.3
Two identical tuples when the order of attributes and values is not part of relation definition.

$t = <$ (Name, Dick Davidson),(Ssn, 422-11-2320),(Home_phone, NULL),(Address, 3452 Elgin Road), (Office_phone, (817)749-1253),(Age, 25),(Gpa, 3.53)>

$t = <$ (Address, 3452 Elgin Road),(Name, Dick Davidson),(Ssn, 422-11-2320),(Age, 25), (Office_phone, (817)749-1253),(Gpa, 3.53),(Home_phone, NULL)>

3. Values and NULLs in the Tuples.

- Each value in a tuple is an **atomic** value; that is, it is not divisible. Hence, composite and multivalued attributes are not allowed.
- The multivalued attributes must be represented by separate relations, and composite attributes are represented only by their simple component attributes in the basic relational model.
- An important concept is that of NULL values, which are used to represent the values of attributes that may be unknown or may not apply to a tuple.
  Example: STUDENT tuples have NULL for their office phones because they do not have an office . Another student has a NULL for home phone, presumably because either he does not have a home phone or he has one but we do not know it (value is unknown).
- In general, we can have several meanings for NULL values, such as **value unknown**, **value** exists but is **not available**, or **attribute does not apply** to this tuple. An example of the last type of NULL will occur if we add an attribute Visa_status to the STUDENT relation that applies only to tuples representing foreign students.
- During database design, it is best to avoid NULL values as much as possible

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4. Interpretation (Meaning) of a Relation.

- Each tuple in the relation can be interpreted as a **fact** or a particular instance of the assertion. For example, a STUDENT relation whose Name is Benjamin Bayer, Ssn is 305-61-2435, Age is 19, and so on.
- Relations may represent facts about entities, whereas other relations may represent facts about relationships. For example, a relation schema MAJORS (Student_ssn, Department_code) asserts that students major in academic disciplines. A tuple in this

relation relates a student to his or her major discipline. Hence, the relational model represents facts about both entities and relationships uniformly as relations.

3.2 Relational Model Constraints

- There are generally many restrictions or **constraints** on the actual values in a database state. These constraints are derived from the rules in the miniworld that the database represents. Constraints on databases can generally be divided into **three** main categories:

1. **Inherent model-based constraints** or **implicit constraints**.
   - Are the constraints that are inherent in the data model. The characteristics of relations**(Please refer 3.1.2)** are the inherent constraints of the relational model.
   - **For example**, the constraint that a relation cannot have duplicate tuples is an inherent constraint

2. **Application-based** or **semantic constraints** or **business rules**.
   - Are the constraints that cannot be directly expressed in the schemas of the data model, and hence must be expressed and enforced by the application programs.
   - **Examples** of such constraints are the salary of an employee should not exceed the salary of the employee's supervisor and the maximum number of hours an employee can work on all projects per week is 56.
   - Such constraints can be specified and enforced within the application programs that update the database, or by using a general-purpose **constraint specification language**. Mechanisms called **triggers** and **assertions** can be used. In SQL, CREATE ASSERTION and CREATE TRIGGER statements can be used for this purpose.

3. **Schema-based constraints** or **explicit constraints**
   - Are the constraints that can be directly expressed in schemas of the data model, typically by specifying them in the DDL (data definition language). The schema-based constraints include **domain constraints**, **key constraints**, **constraints on NULLs, entity integrity constraints, and referential integrity constraints.**

3.2.1 Domain Constraints

- Domain constraints specify that within each tuple, the value of each attribute A must be an atomic value from the domain dom(A). The data types associated with domains typically include standard numeric data types for integers, Characters, Booleans, fixed-length strings, variable-length strings, date, time and timestamp.

3.2.2 Key Constraints and Constraints on NULL Values.

- In the formal relational model, a relation is defined as a set of tuples. By definition, all elements of a set are distinct. This means that no two tuples can have the same combination of values for all their attributes.
- The **subsets of attributes** of a relation schema *R* with the property that no two tuples in any relation state *r* of *R* should have the same combination of values for these attributes i.e for any two distinct tuples t1 and t2 in a relation state r of R, we have the constraint that: **t1[SK] ≠ t2[SK]** any such set of attributes SK is called a **superkey** of the relation schema R.
- A superkey can have redundant attributes, however, so a more useful concept is that of a key, which has no redundancy.
- A **key** K of a relation schema R is a superkey of R with the additional property that removing any attribute A from K leaves a set of attributes K that is not a superkey of R any more. Hence, a key satisfies **two properties**:
  1. Two distinct tuples in any state of the relation cannot have identical values for (all) the attributes in the key. This first property also applies to a superkey.
  2. It is a minimal superkey—that is, a superkey from which we cannot remove any attributes and still have the uniqueness constraint in condition 1 hold. This property is not required by a superkey.
- The first property applies to both keys and superkeys, the second property is required only for keys. Hence, a key is also a superkey but not vice versa.
- Consider the STUDENT relation. The attribute set {Usn} is a **key** of STUDENT because no two student tuples can have the same value for Usn.
- Any set of attributes that includes Usn for example, {Ssn, Name, Age}—is a **superkey**. However, the superkey {Ssn, Name, Age} is not a key of STUDENT because removing Name or Age or both from the set still leaves us with a superkey.

CAR

License_number	Engine_serial_number	Make	Model	Year
Texas ABC-739	A69352	Ford	Mustang	02
Florida TVP-347	B43696	Oldsmobile	Cutlass	05
New York MPO-22	X83554	Oldsmobile	Delta	01
California 432-TFY	C43742	Mercedes	190-D	99
California RSK-629	Y82935	Toyota	Camry	04
Texas RSK-629	U028365	Jaguar	XJS	04

Figure 3.4
The CAR relation, with two candidate keys: License_number and Engine_serial_number.

- **In general,** a relation schema may have more than one key. In this case, each of the keys is called a **candidate key**. For example, the CAR relation in Figure 3.4 has two candidate keys: License_number and Engine_serial_number. It is common to designate one of the candidate keys as the **primary key** of the relation. This is the candidate key whose values are used to identify tuples in the relation. The other candidate keys are designated as **unique keys**, and are not underlined.

Example 2: Consider the following relation : Book (BookId, BookName, Author)

- Super Keys : A Super Key is a set of one or more attributes that are taken collectively and can identify all other attributes uniquely.
  For Example,
  (BookId)
  (BookId,BookName)
  (BookId, BookName, Author)
  (BookId, Author)
  (BookName, Author)

- Candidate Keys Candidate keys are a super key which are not having any redundant attributes. In other words candidate keys are minimal super keys.
  For Example,
  (BookId)
  (BookName,Author)

- These two keys can be candidate keys, as remaining keys are having redundant attributes. Means in super key (BookId, BookName) record can be uniquely identify by just bookid and therefore Bookname is redundant attribute
- Primary Key: A key which is used to uniquely identify each record is known as primary key. From above Candidate keys any one can be the primary key.

3.2.3 Relational Databases and Relational Database Schemas.

- A **relational database schema** S is a set of relation schemas S = {R1, R2, ..., R_m} and a set of **integrity constraints** IC.
- A **relational database state** DB of S is a set of relation states DB = {r1, r2, ..., r_m} such that each ri is a state of Ri and such that the ri relation states satisfy the integrity constraints specified in IC.
- Figure 3.5 shows a relational database schema that we call COMPANY = {EMPLOYEE, DEPARTMENT, DEPT_LOCATIONS, PROJECT, WORKS_ON, DEPENDENT}. The underlined attributes represent primary keys. A database state that does not obey all the integrity constraints iscalled an **invalid state**, and a state that satisfies all the constraints in the defined set of integrity constraints IC is called a **valid state.**

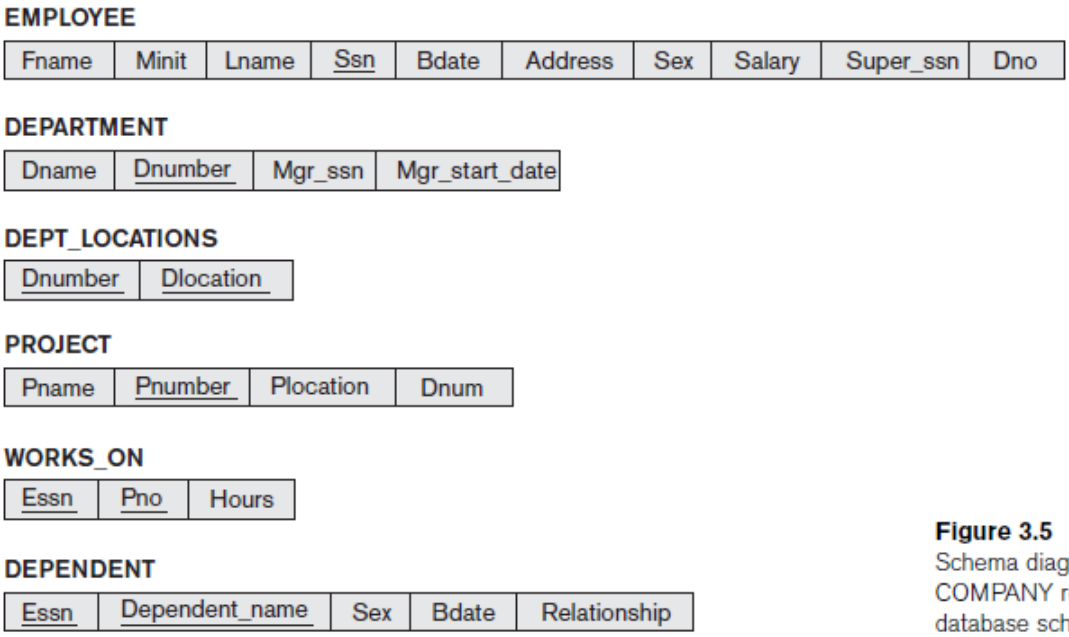


Figure 3.5
Schema diagram for the COMPANY relational database schema.

3.2.4 Integrity, Referential Integrity, and Foreign Keys

- The **entity integrity constraint** states that no primary key value can be NULL. This is because the primary key value is used to identify individual tuples in a relation. Having NULL values for the primary key implies that we cannot identify some tuples.
  For example, if two or more tuples had NULL for their primary keys, we may not be able to distinguish them if we try to reference them from other relations. Key constraints and entity integrity constraints are specified on individual relations.
- The **referential integrity constraint** is specified between two relations and is used to maintain the consistency among tuples in the two relations. Informally, **the referential integrity constraint states that a tuple in one relation that refers to another relation must refer to an existing tuple in that relation.**
  For example, The attribute Dno of EMPLOYEE gives the department number for which each employee works; hence, its value in every EMPLOYEE tuple must match the Dnumber value of some tuple in the DEPARTMENT relation.
- The formal definition of referential integrity is, Consider a set of attributes FK in relation schema R1 is a **foreign key** of R1 that **references** relation R2 if it satisfies the following rules:
  1. The attributes in FK have the same domain(s) as the primary key attributes PK of R2; the attributes FK are said to **reference** or **refer to** the relation R2.
  2. A value of FK in a tuple t1 of the current state r1(R1) either occurs as a value of PK for some tuple t2 in the current state r2(R2) or is NULL. According to previous case, t1[FK] = t2[PK]. The tuple t1 **references** or **refers to** the tuple t2.
- In this definition, R1 is called the **referencing relation** and R2 is the **referenced relation**. If these two conditions hold, a **referential integrity constraint** from R1 to R2 is said to hold. In a database of many relations, there are usually many referential integrity constraints.

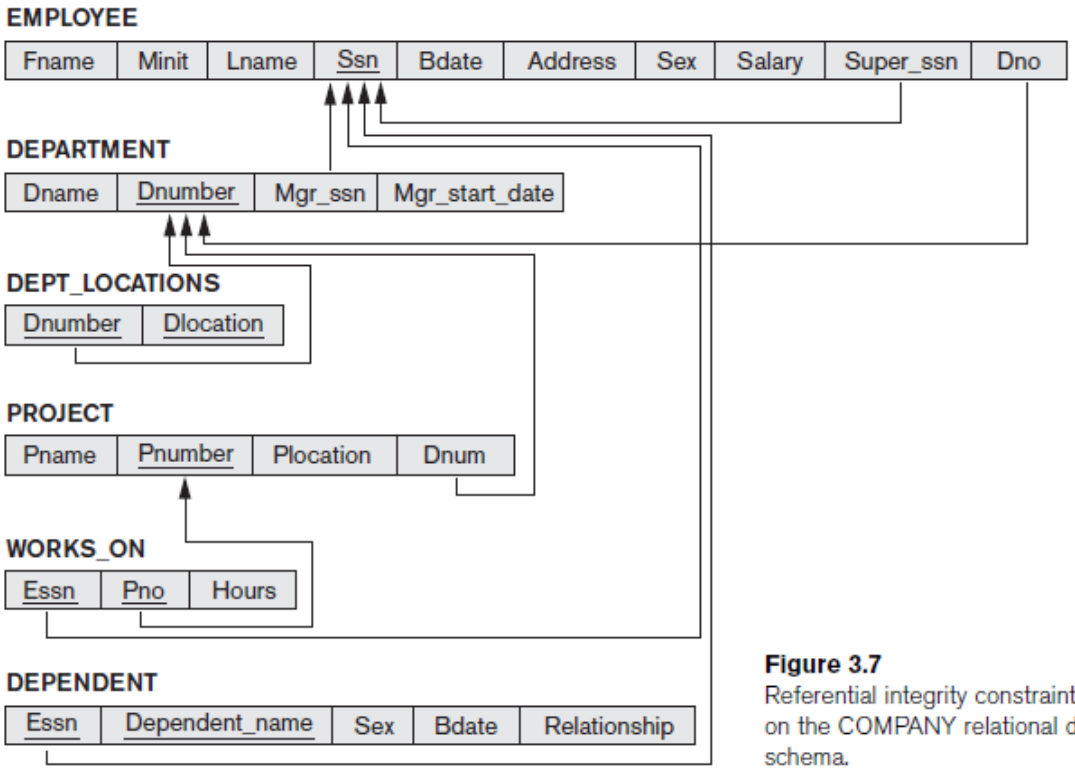


Figure 3.7
Referential integrity constraints displayed on the COMPANY relational database schema.

- For example, In the EMPLOYEE relation, the attribute Dno refers to the department for which an employee works; hence, we designate Dno to be a foreign key of EMPLOYEE referencing the DEPARTMENT relation. This means that a value of Dno in any tuple t1 of the EMPLOYEE relation must match a value of the primary key of DEPARTMENT.

3.3 Update Operations, Transactions & Dealing with Constraint Violations

There are three basic operations that can change the states of relations in the database: Insert, Delete, and Update. **Insert** is used to insert one or more new tuples in a relation, **Delete** is used to delete tuples, and **Update** is used to change the values of some attributes in existing tuples

3.3.1 The Insert Operation

- The **Insert** operation provides a list of attribute values for a new tuple t that is to be inserted into a relation R.
- Insert can violate any of the four types of constraints.
  1. **Domain constraints** can be violated if an attribute value is given that does not appear in the corresponding domain or is not of the appropriate data type.
  2. **Key constraints** can be violated if a key value in the new tuple t already exists in another tuple in the relation r(R).
  3. **Entity integrity** can be violated if any part of the primary key of the new tuple t is NULL.
  4. **Referential integrity** can be violated if the value of any foreign key in t refers to a tuple that does not exist in the referenced relation.

- **Operation:**
  Insert <‘Cecilia’, ‘F’, ‘Kolonsky’, NULL, ‘1960-04-05’, ‘6357 Windy Lane, Katy, TX’, F, 28000, NULL, 4> into EMPLOYEE.
  Result: This insertion violates the entity integrity constraint (NULL for the primary key Ssn), so it is rejected.
- **Operation:**
  Insert <‘Alicia’, ‘J’, ‘Zelaya’, ‘999887777’, ‘1960-04-05’, ‘6357 Windy Lane, Katy,TX’, F, 28000, ‘987654321’, 4> into EMPLOYEE.
  Result: This insertion violates the key constraint because another tuple withthe same Ssn value already exists in the EMPLOYEE relation, and so it is rejected.
- **Operation:**
  Insert <‘Cecilia’, ‘F’, ‘Kolonsky’, ‘677678989’, ‘1960-04-05’, ‘6357 Windswept, Katy, TX’, F, 28000, ‘987654321’, 7> into EMPLOYEE.
  Result: This insertion violates the referential integrity constraint specified on Dno in EMPLOYEE because no corresponding referenced tuple exists in DEPARTMENT with Dnumber = 7.
- **Operation:**
  Insert <‘Cecilia’, ‘F’, ‘Kolonsky’, ‘677678989’, ‘1960-04-05’, ‘6357 Windy Lane, Katy, TX’, F, 28000, NULL, 4> into EMPLOYEE.
  Result: This insertion satisfies all constraints, so it is acceptable.

3.3.2 The Delete Operation

- The **Delete** operation can violate only referential integrity. This occurs if the tuple being deleted is referenced by foreign keys from other tuples in the database. To specify deletion, a condition on the attributes of the relation selects the tuple (or tuples) to be deleted. Here are some examples.
- Operation:
  Delete the WORKS_ON tuple with Essn = ‘999887777’ and Pno =10.
  Result: This deletion is acceptable and deletes exactly one tuple.
- Operation:
  Delete the EMPLOYEE tuple with Ssn = ‘333445555’.
  Result: This deletion will result in even worse referential integrity violations, because the tuple involved is referenced by tuples from the EMPLOYEE, DEPARTMENT,WORKS_ON, and DEPENDENT relations.
- Several options are available if a deletion operation causes a violation. The first option, called **restrict,** is to reject the deletion. The second option, called **cascade,** is to attempt to cascade (or propagate) the deletion by deleting tuples that reference the tuple that is being deleted. A third option, called **set null** or **set default,** is to modify the referencing attribute values that cause the violation; each such value is either set to NULL or changed to reference another default valid tuple.

3.3.3 The Update Operation

The **Update** (or **Modify**) operation is used to change the values of one or more attributes in a tuple (or tuples) of some relation R. It is necessary to specify a condition on the attributes of the relation to select the tuple (or tuples) to be modified. Here are some examples.

- Operation:
  Update the salary of the EMPLOYEE tuple with Ssn = '999887777' to 28000.
  Result: Acceptable.
- Operation: Update the Dno of the EMPLOYEE tuple with Ssn = '999887777' to 1.
  Result: Acceptable.
- Operation:
  Update the Dno of the EMPLOYEE tuple with Ssn = '999887777' to 7.
  Result: Unacceptable, because it violates referential integrity.
- Operation:
  Update the Ssn of the EMPLOYEE tuple with Ssn = '999887777' to '987654321'.
  Result: Unacceptable, because it violates primary key constraint by repeating a value that already exists as a primary key in another tuple; it violates referential integrity constraints because there are other relations that refer to the existing value of Ssn.

3.3.4 The Transaction Concept

- A **transaction** is an executing program that includes some database operations, such as reading from the database, or applying insertions, deletions, or updates to the database. At the end of the transaction, it must leave the database in a valid or consistent state that satisfies all the constraints specified on the database schema.
- For example, a transaction to apply a bank withdrawal will typically read the user account record, check if there is a sufficient balance, and then update the record by the withdrawal amount. A large number of commercial applications running against relational databases in **online transaction processing (OLTP**) systems are executing transactions at rates that reach several hundred per second.

The Relational Algebra

- The basic set of operations for the relational model is the **relational algebra.**
- Its a Procedural Query Language.
- The relational algebra is very important for several reasons. First, it provides a formal foundation for relational model operations. Second, , it is used as a basis for implementing and optimizing queries in the query processing and optimization modules that are integral parts of relational database management systems (RDBMSs)

6.1 Unary Relational Operations: SELECT and PROJECT.

6.1.1 The SELECT Operation.

- The SELECT operation is used to choose a subset of the tuples from a relation that satisfies a **selection condition**.
- The SELECT operation is visualized as a horizontal partition of the relation into two sets of tuples. Those tuples that satisfy the condition are selected, and those tuples that do not satisfy the condition are discarded.
- In general, the SELECT operation is denoted by σ <selection condition>(R) where the symbol σ (sigma) is used to denote the SELECT operator and the selection condition is a Boolean expression (condition) specified on the attributes of relation R.
- For example, to select the EMPLOYEE tuples whose department is 4, or those whose salary is greater than $30,000, we can individually specify each of these two conditions with a SELECT operation as follows:

$$\sigma_{Dno=4}(EMPLOYEE)$$

$$\sigma_{Salary>3000}(EMPLOYEE)$$

- The Boolean expression specified in <selection condition> is made up of a number of **clauses** of the form

<attribute name> <comparison op> <constant value>

or

<attribute name> <comparison op> <attribute name>

- For example, to select the tuples for all employees who either work in department 4 and make over $25,000 per year, or work in department 5 and make over $30,000, we can specify the following SELECT operation:

$$\sigma_{(Dno=4\ AND\ Salary>25000)\ OR\ (Dno=5\ AND\ Salary>30000)}(EMPLOYEE)$$

- Figure 6.1 shows the result of SELECT operation.

Figure 6.1
Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4\ \textbf{AND}\ Salary>25000)\ OR\ (Dno=5\ \textbf{AND}\ Salary>30000)}$ (EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

- In SQL, the SELECT condition is typically specified in the WHERE clause of a query.
- For example, the following operation: σDno=4 **AND** Salary>25000 (EMPLOYEE) would correspond to the following SQL query:

```
SELECT *
FROM EMPLOYEE
WHERE Dno=4 AND Salary>25000;
```

6.1.2 The PROJECT Operation

- The SELECT operation chooses some of the rows from the table while discarding other rows. The **PROJECT** operation, on the other hand, selects certain columns from the table and discards the other columns.
- For example, to list each employee's first and last name and salary, we can use the PROJECT operation as follows:

$$\pi_{\textbf{Lname, Fname, Salary}}\textbf{(EMPLOYEE)}$$

- The resulting relation is shown in Figure 6.1(b).

Figure 6.1
Results of SELECT and PROJECT operations.
(b) $\pi_{Lname, Fname, Salary}$(EMPLOYEE). (c) $\pi_{Sex, Salary}$(EMPLOYEE).

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

- The general form of the PROJECT operation is **π<attribute list>(R)** where π (pi) is the symbol used to represent the PROJECT operation, and <attribute list> is the desired sub list of attributes from the attributes of relation R

- The PROJECT operation removes any duplicate tuples, so the result of the PROJECT operation is a set of distinct tuples, and hence a valid relation. This is known as **duplicate elimination**.
- For Ex: consider the following PROJECT operation: $\pi_{Sex, Salary}(EMPLOYEE)$. In SQL, the PROJECT attribute list is specified in the SELECT clause of a query.

  SELECT DISTINCT Sex, Salary
  FROM EMPLOYEE

6.1.3 Sequences of Operations and the RENAME Operation

- For example, to retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a SELECT and a PROJECT operation.

$$\pi_{Fname, Lname, Salary}(\sigma_{Dno=5}(EMPLOYEE))$$

- Alternatively, we can explicitly show the sequence of operations, giving a name to each intermediate relation, as follows:

$$DEP5\_EMPS \leftarrow \sigma_{Dno=5}(EMPLOYEE)$$

$$RESULT \leftarrow \pi_{Fname, Lname, Salary}(DEP5\_EMPS)$$

- To rename the attributes in a relation, list the new attribute names in parentheses, as in the following example:

$$TEMP \leftarrow \sigma Dno=5(EMPLOYEE)$$

$$R(First\_name, Last\_name, Salary) \leftarrow \pi_{Fname, Lname, Salary}(TEMP)$$

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2
Results of a sequence of operations. (a) $\pi_{Fname, Lname, Salary}(\sigma_{Dno=5}(EMPLOYEE))$. (b) Using intermediate relations and renaming of attributes.

- The general RENAME operation when applied to a relation R of degree n is denoted by any of the following three forms:

ρS(B1, B2, ..., Bn)(R) or ρS(R) or ρ(B1, B2, ..., Bn)(R)
where the symbol ρ (rho) is used to denote the RENAME operator, S is the new relation name, and B1, B2, ..., Bn are the new attribute names.

SELECT E.Fname **AS** First_name, E.Lname **AS** Last_name, E.Salary **AS** Salary
FROM EMPLOYEE **AS** E
WHERE E.Dno=5.

6.2 Relational Algebra Operations from Set Theory

6.2.1 The UNION, INTERSECTION, and MINUS Operations

- For example, To retrieve the Social Security numbers of all employees who either work in department 5 or directly supervise an employee who works in department 5.

DEP5_EMPS ← σDno=5(EMPLOYEE)
RESULT1 ← πSsn(DEP5_EMPS)
RESULT2(Ssn) ← πSuper_ssn(DEP5_EMPS)
RESULT ← RESULT1 ∪ RESULT2

- The relation RESULT1 has the Ssn of all employees who work in department 5, whereas RESULT2 has the Ssn of all employees who directly supervise an employee who works in department 5. The UNION operation produces the tuples that are in either RESULT1 or RESULT2 or both (see Figure 6.3), while eliminating any duplicates. Thus, the Ssn value '333445555' appears only once in the result.

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Figure 6.3
Result of the UNION operation RESULT ← RESULT1 ∪ RESULT2.

We can define the three operations UNION, INTERSECTION, and SET DIFFERENCE on two union-compatible relations R and S as follows:

- UNION: The result of this operation, denoted by R∪S, is a relation that includes all tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated.
- INTERSECTION: The result of this operation, denoted by R ∩S, is a relation that includes all tuples that are in both R and S.
- SET DIFFERENCE (or MINUS): The result of this operation, denoted by R – S, is a relation that includes all tuples that are in R but not in S.

- The relations STUDENT and INSTRUCTOR in Figure 6.4(a) are union compatible and their tuples represent the names of students and the names of instructors, respectively.
- The result of the UNION operation in Figure 6.4(b) shows the names of all students and instructors. Note that duplicate tuples appear only once in the result.
- The result of the INTERSECTION operation, Figure 6.4(c) includes only those who are both students and instructors.
- Figure 6.4(d) shows the names of students who are not instructors.
- Figure 6.4(e) shows the names of instructors who are not students.

Figure 6.4
The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT – INSTRUCTOR. (e) INSTRUCTOR – STUDENT.

(a) **STUDENT**

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

- UNION and INTERSECTION are commutative operations; that is,

$$R \cup S = S \cup R \text{ and } R \cap S = S \cap R$$

- UNION and INTERSECTION can be treated as n-ary operations applicable to any number of relations because both are also associative operations; that is,

$$R \cup (S \cup T) = (R \cup S) \cup T \text{ and } (R \cap S) \cap T = R \cap (S \cap T)$$

- The MINUS operation is not commutative; that is, in general, $R - S \neq S - R$
- INTERSECTION can be expressed in terms of union and set difference as follows: $R \cap S = ((R \cup S) - (R - S)) - (S - R)$.

- In SQL, there are three operations **UNION, INTERSECT, and EXCEPT** that correspond to the set operations.

6.2.2 The CARTESIAN PRODUCT (CROSS PRODUCT) Operation.

- The **CARTESIAN PRODUCT** operation—also known as **CROSS PRODUCT** or **CROSS JOIN**—which is denoted by **X.**
- Cartesian Product produces a new element by combining every member (tuple) of one relation (set) with every member (tuple) from the other relation (set).
- The result of R(A1, A2, ..., A_n) X S(B1, B2, ..., B_m) is a relation Q with degree n + m attributes Q(A1, A2, ..., An, B1, B2, ..., Bm).
- The CARTESIAN PRODUCT creates tuples with the combined attributes of two relations. We can SELECT related tuples from the two relations by specifying an appropriate selection condition after the Cartesian product,
- For example, suppose that we want to retrieve a list of names of each female employee's dependents.

FEMALE_EMPS ← σSex='F'(EMPLOYEE)
EMPNAMES ← πFname, Lname, Ssn(FEMALE_EMPS)
EMP_DEPENDENTS ← EMPNAMES × DEPENDENT
ACTUAL_DEPENDENTS ← σSsn=Essn(EMP_DEPENDENTS)
RESULT ← πFname, Lname, Dependent_name(ACTUAL_DEPENDENTS)

- The resulting relations from this sequence of operations are shown in Figure 6.5.

Figure 6.5
The Cartesian Product (Cross Product) operation.

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1986-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

6.3 Binary Relational Operations: JOIN and DIVISION

6.3.1 The JOIN Operation

- The JOIN operation, denoted by ⋈ , is used to combine related tuples from two relations into single "longer" tuples.

- The general form of a JOIN operation on two relations $R(A_1, A_2, ..., A_n)$ and $S(B_1, B_2, ..., B_m)$ is $\mathbf{R} \bowtie_{\text{<join condition>}} \mathbf{S}$.
- The result of the JOIN is a relation Q with n + m attributes $Q(A_1, A_2, ..., A_n, B_1, B_2, ..., B_m)$ in that order; Q has one tuple for each combination of tuples—one from R and one from S—whenever the combination satisfies the join condition. This is the main difference between CARTESIAN PRODUCT and JOIN. In JOIN, only combinations of tuples satisfying the join condition appear in the result, whereas in the CARTESIAN PRODUCT all combinations of tuples are included in the result.

- To illustrate JOIN, suppose that we want to retrieve the name of the manager of each department. To get the manager's name, we need to combine each department tuple with the employee tuple whose Ssn value matches the Mgr_ssn value in the department tuple.

$$\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE}$$
$$\text{RESULT} \leftarrow \pi_{\text{Dname, Lname, Fname}}(\text{DEPT\_MGR})$$

DEPT_MGR

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

Figure 6.6
Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\bowtie_{\text{Mgr\_ssn=Ssn}}$ EMPLOYEE.

- Consider the earlier example illustrating CARTESIAN PRODUCT, which included the following sequence of operations:

$$\text{EMP\_DEPENDENTS} \leftarrow \text{EMPNAMES} \times \text{DEPENDENT}$$
$$\text{ACTUAL\_DEPENDENTS} \leftarrow \sigma_{\text{Ssn=Essn}}(\text{EMP\_DEPENDENTS})$$

- These two operations can be replaced with a single JOIN operation as follows: **ACTUAL_DEPENDENTS ← EMPNAMES** $\bowtie_{\text{Ssn=Essn}}$ **DEPENDENT**
- A general join condition is of the form :
<condition> **AND** <condition> **AND**...**AND** <condition>

where each <condition> is of the form Ai θ Bj, Ai is an attribute of R, Bj is an attribute of S, A JOIN operation with such a general join condition is called a **THETA JOIN.**

6.3.2 Variations of JOIN: The EQUIJOIN and NATURAL JOIN

- The JOIN operation where the only comparison operator used is = in join condition is called an **EQUIJOIN**.
- EQUIJOIN always have one or more pairs of attributes that have identical values in every tuple.
- For example, in Figure 6.6, the values of the attributes Mgr_ssn and Ssn are identical in every tuple of DEPT_MGR (the EQUIJOIN result) because the equality join condition specified on these two attributes requires the values to be identical in every tuple in the result.

DEPT_MGR

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

Figure 6.6
Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\bowtie_{Mgr\_ssn=Ssn}$ EMPLOYEE.

- Because one of each pair of attributes with identical values is superfluous, a new operation called **NATURAL JOIN** denoted by * was created to get rid of the second (**superfluous**) attribute in an EQUIJOIN condition.
- The standard definition of NATURAL JOIN requires that the two join attributes have the same name in both relations. If this is not the case, a renaming operation is applied first.
- Suppose we want to combine each PROJECT tuple with the DEPARTMENT tuple that controls the project. In the following example, first we rename the Dnumber attribute of DEPARTMENT to Dnum so that it has the same name as the Dnum attribute in PROJECT and then we apply NATURAL JOIN:

$$\mathbf{PROJ\_DEPT \leftarrow PROJECT * \rho_{(Dname,\ Dnum,\ Mgr\_ssn,\ Mgr\_start\_date)}\ (DEPARTMENT)}$$

- The same query can be done in two steps by creating an intermediate table DEPT as follows:

$$\mathbf{DEPT \leftarrow \rho_{(Dname,\ Dnum,\ Mgr\_ssn,\ Mgr\_start\_date)}(DEPARTMENT)}$$
$$\mathbf{PROJ\_DEPT \leftarrow PROJECT * DEPT}$$

- The attribute Dnum is called the **join attribute** for the NATURAL JOIN operation, because it is the only attribute with the same name in both relations. The resulting relation is illustrated in Figure 6.7(a).

- If the attributes on which the natural join is specified already have the same names in both relations, renaming is unnecessary. For example, to apply a natural join on the Dnumber

attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write. The resulting relation is shown in Figure 6.7(b),

DEPT_LOCS ←DEPARTMENT * DEPT_LOCATIONS

(a)

PROJ_DEPT

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Figure 6.7
Results of two NATURAL JOIN operations. (a) PROJ_DEPT ← PROJECT * DEPT.
(b) DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS.

- A single JOIN operation is used to combine data from two relations so that related information can be presented in a single table. These operations are also known as **inner joins.** An inner join is a type of match and combine operation defined formally as a combination of CARTESIAN PRODUCT and SELECTION.

6.3.3 The DIVISION Operation

- The division operator is an interesting operator that is useful in answering queries that involve "for all" statements.
- The DIVISION operation, denoted by ÷, is useful for a special kind of query that sometimes occurs in database applications.
- An example is Retrieve the names of employees who works for all the projects that 'John Smith' works on. To express this query using the DIVISION operation, proceed as follows. First, retrieve the list of project numbers that 'John Smith' works on in the intermediate relation SMITH_PNOS:

$$\text{SMITH} \leftarrow \sigma_{\text{Fname='John' AND Lname='Smith'}}(\text{EMPLOYEE})$$
$$\text{SMITH\_PNOS} \leftarrow \pi_{\text{Pno}}(\text{WORKS\_ON} \bowtie_{\text{Essn=Ssn}} \text{SMITH})$$
$$\text{SSN\_PNOS} \leftarrow \pi_{\text{Essn, Pno}}(\text{WORKS\_ON})$$

SSNS(Ssn) ← SSN_PNOS ÷ SMITH_PNOS

RESULT ← $\pi_{Fname, Lname}$(SSNS * EMPLOYEE)

- The preceding operations are shown in Figure 6.8(a).

Figure 6.8

The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

- In the formulation of the DIVISION operation, the tuples in the denominator relation S restrict the numerator relation R by selecting those tuples in the result that match all values present in the denominator.
- As illustrated in the SMITH_PNOS relation in the above example. Figure 6.8(b) illustrates a DIVISION operation where X = {A}, Y = {B}, and Z = {A, B}. Notice that the tuples (values) b1 and b4 appear in R in combination with all three tuples in S; that is why they appear in the resulting relation T. All other values of B in R do not appear with all the tuples in S and are not selected: b2 does not appear with a2, and b3 does not appear with a1.

List of the various basic relational algebra operations:

Table 6.1 Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{\text{(<join attributes 1>), (<join attributes 2>)}} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{\text{(<join attributes 1>), (<join attributes 2>)}} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

$$\pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(((\sigma_{\text{Plocation='Stafford'}}(\text{PROJECT})) \bowtie_{\text{Dnum=Dnumber}}(\text{DEPARTMENT})) \bowtie_{\text{Mgr\_ssn=Ssn}}(\text{EMPLOYEE}))$$

6.4 Additional Relational Operations

6.4.1 Aggregate Functions and Grouping

- Aggregate functions are used in simple statistical queries that summarize information from the database tuples. Some of the aggregate functions are **SUM, AVERAGE, MAXIMUM, and MINIMUM**. The **COUNT** function is used for counting tuples or values.
- AGGREGATE FUNCTION operation can be defined using the symbol □ (pronounced script F).

<grouping attributes> ℑ <function list> (R)

where <**grouping attributes**> is a list of attributes of the relation specified in R, <**function list**> is a list of **(<function> <attribute>)** pairs. In each such pair, <**function**> is one of the functions such as SUM, AVERAGE, MAXIMUM, MINIMUM, COUNT.

- For example, to retrieve each department number, the number of employees in the department, and their average salary,

$\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}$ (Dno □ COUNT Ssn, AVERAGE Salary (EMPLOYEE))

- The result of this operation on the EMPLOYEE is shown in Figure 6.10(a).

Figure 6.10
The aggregate function operation.
a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(\text{Dno } \Im \text{ COUNT Ssn, AVERAGE Salary}(\text{EMPLOYEE}))$.
b. $\text{Dno } \Im \text{ COUNT Ssn, AVERAGE Salary}(\text{EMPLOYEE})$.
c. $\Im \text{ COUNT Ssn, AVERAGE Salary}(\text{EMPLOYEE})$.

R

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

- If no **renaming** operation is applied, then the resulting relation will be in the form **<function_attribute>**. For example, Figure 6.10(b) shows the result of the following operation:

Dno ℑ COUNT Ssn, AVERAGE Salary(EMPLOYEE)

- If no grouping attributes are specified, the functions are applied to all the tuples in the relation, so the resulting relation has a single tuple only. For example, Figure 6.10(c) shows the result of the following operation:

$$\mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$$

6.4.2 Recursive Closure Operations

- **Recursive closure** operation cannot be specified in the relational algebra**.** This operation is applied to a **recursive relationship** between tuples of the same type, such as the relationship between an employee and a supervisor.
- This relationship is described by the foreign key Super_ssn of the EMPLOYEE relation.
- An example of a recursive operation is to retrieve all SUPERVISEES of an EMPLOYEE e at all levels—that is, all EMPLOYEE e' directly supervised by e; all employees e'' directly supervised by each employee e'; all employees e''' directly supervised by each employee e''; and so on

- For example, To specify the Ssns of all employees directly supervised by James Borg' at **level one.**

$$\text{BORG\_SSN} \leftarrow \pi_{\text{Ssn}}(\sigma_{\text{Fname='James' AND Lname='Borg'}}(\text{EMPLOYEE}))$$
$$\text{SUPERVISION(Ssn1, Ssn2)} \leftarrow \pi_{\text{Ssn,Super\_ssn}}(\text{EMPLOYEE})$$
$$\text{RESULT1(Ssn)} \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{BORG\_SSN})$$

- To retrieve all employees supervised by Borg at level 2 is as follows:

$$\text{RESULT2(Ssn)} \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{RESULT1})$$

- To get both sets of employees supervised at levels 1 and 2 by 'James Borg', we can apply the UNION operation to the two results, as follows:

$$\text{RESULT} \leftarrow \text{RESULT2} \cup \text{RESULT1}$$

- The results of these queries are illustrated in Figure 6.11.

SUPERVISION

(Borg's Ssn is 888665555)

(Ssn)	(Super_ssn)
Ssn1	Ssn2
123456789	333445555
333445555	888665555
999887777	987654321
987654321	888665555
666884444	333445555
453453453	333445555
987987987	987654321
888665555	null

RESULT1

Ssn
333445555
987654321

(Supervised by Borg)

RESULT2

Ssn
123456789
999887777
666884444
453453453
987987987

(Supervised by Borg's subordinates)

RESULT

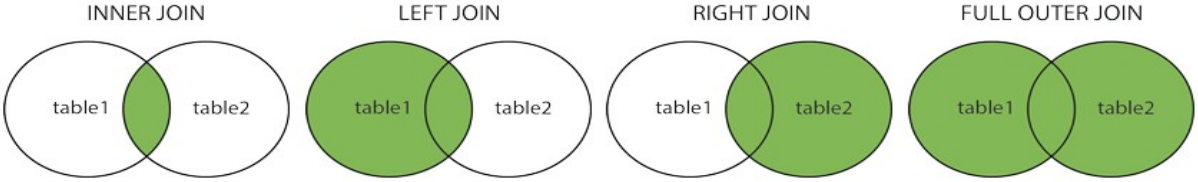
Ssn
123456789
999887777
666884444
453453453
987987987
333445555
987654321

(RESULT1 ∪ RESULT2)

Figure 6.11
A two-level recursive query.

6.4.3 OUTER JOIN Operations.

- A NATURAL JOIN operation R * S, only tuples from R that have matching tuples in S and vice versa appear in the result. Hence, tuples without a matching tuple are eliminated from the JOIN result. Tuples with NULL values in the join attributes are also eliminated. This type of join, where tuples with no match are eliminated, is known as an **inner Join.**
- A set of operations, called **outer joins**, is the case where the user wants to keep all the tuples in R, or all those in S, or all those in both relations in the result of the JOIN,
- There are three types of outer join operations**: left outer join, right outer join, and full outer join.**



Left Outer Join Operation

- The left outer join is written as R ⟕ S where R and S are relations. The result of the left outer join is the set of all combinations of tuples in R and S that are equal on their common attribute names, in addition to tuples in R that have no matching tuples in S. If no matching tuple is found in *S*, then the attributes of *S* in the join result are filled or padded with NULL values

- For an example consider the tables Employee and Dept and their left outer join:

Employee

Name	EmpId	DeptName
Harry	3415	Finance
Sally	2241	Sales
George	3401	Finance
Harriet	2202	Sales
Tim	1123	Executive

Dept

DeptName	Manager
Sales	Harriet
Production	Charles

Employee ⟕ Dept

Name	EmpId	DeptName	Manager
Harry	3415	Finance	NULL
Sally	2241	Sales	Harriet
George	3401	Finance	NULL
Harriet	2202	Sales	Harriet
Tim	1123	Executive	NULL

Right Outer Join Operation

- The right outer join of relations *R* and *S* is written as *R* ⟖ *S*. The result of the right outer join is the set of all combinations of tuples in *R* and *S* that are equal on their common attribute names, in addition to tuples in *S* that have no matching tuples in *R*.
- For example, consider the tables Employee and Dept and their right outer join:

Employee		
Name	**EmpId**	**Dept Name**
Harry	3415	Finance
Sally	2241	Sales
George	3401	Finance
Harriet	2202	Sales
Tim	1123	Executive

Dept	
DeptName	**Manager**
Sales	Harriet
Production	Charles

Employee ⋉ *Dept*			
Name	**Emp Id**	**DeptName**	**Manager**
Sally	2241	Sales	Harriet
Harriet	2202	Sales	Harriet
NULL	**NULL**	Production	Charles

- In the resulting relation, tuples in R which have no common values in common attribute names with tuples in S take a null value.

Outer join Operation.

- The **outer join** or **full outer join** in effect combines the results of the left and right outer joins.

- The full outer join is written as $R \ltimes S$ where *R* and *S* are relations. The result of the full outer join is the set of all combinations of tuples in *R* and *S* that are equal on their common attribute names, in addition to tuples in *S* that have no matching tuples in *R* and tuples in *R* that have no matching tuples in *S* in their common attribute names.

- For an example consider the tables *Employee* and *Dept* and their full outer join:

Employee

Name	EmpId	DeptName
Harry	3415	Finance
Sally	2241	Sales
George	3401	Finance
Harriet	2202	Sales
Tim	1123	Executive

Dept

DeptName	Manager
Sales	Harriet
Production	Charles

Employee ⟕ Dept

Name	EmpId	DeptName	Manager
Harry	3415	Finance	NULL
Sally	2241	Sales	Harriet
George	3401	Finance	NULL
Harriet	2202	Sales	Harriet
Tim	1123	Executive	NULL
NULL	NULL	Production	Charles

- In the resulting relation, tuples in *R* which have no common values in common attribute names with tuples in *S* take a *null* value. Tuples in *S* which have no common values in common attribute names with tuples in *R* also take a *null* value

6.4.4 The OUTER UNION Operation

- The **OUTER UNION** operation was developed to take the union of tuples from two relations that have some common attributes, but are not union (type) compatible.
- This operation will take the UNION of tuples in two relations R(X, Y) and S(X, Z) that are **partially compatible**, meaning that only some of their attributes, say X, are union compatible.
- Two tuples $t1$ in R and $t2$ in S are said to **match** if $t1[X]=t2[X]$. These will be combined (unioned) into a single tuple in t. Tuples in either relation that have no matching tuple in the other relation are padded with NULL values.
- For example, an OUTER UNION can be applied to two relations whose schemas are STUDENT(Name, Ssn, Department, Advisor) and INSTRUCTOR(Name, Ssn, Department, Rank). Tuples from the two relations are matched based on having the same

combination of values of the shared attributes—Name, Ssn, Department. The resulting relation, STUDENT_OR_INSTRUCTOR, will have the following attributes:

STUDENT_OR_INSTRUCTOR(Name, Ssn, Department, Advisor, Rank)

- All the tuples from both relations are included in the result, but tuples with the same (Name, Ssn, Department) combination will appear only once in the result. Tuples appearing only in STUDENT will have a NULL for the Rank attribute, whereas tuples appearing only in INSTRUCTOR will have a NULL for the Advisor attribute. A tuple that exists in both relations, which represent a student who is also an instructor, will have values for all its attributes.

6.5 Examples of Queries in Relational Algebra

Query 1. Retrieve the name and address of all employees who work for the 'Research' department.

$$\text{RESEARCH\_DEPT} \leftarrow \sigma_{\text{Dname='Research'}}(\text{DEPARTMENT})$$
$$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{\text{Dnumber=Dno}} \text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Address}}(\text{RESEARCH\_EMPS})$$

As a single in-line expression, this query becomes:

$$\pi_{\text{Fname, Lname, Address}}(\sigma_{\text{Dname='Research'}}(\text{DEPARTMENT} \bowtie_{\text{Dnumber=Dno}}(\text{EMPLOYEE}))$$

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$$\text{STAFFORD\_PROJS} \leftarrow \sigma_{\text{Plocation='Stafford'}}(\text{PROJECT})$$
$$\text{CONTR\_DEPTS} \leftarrow (\text{STAFFORD\_PROJS} \bowtie_{\text{Dnum=Dnumber}} \text{DEPARTMENT})$$
$$\text{PROJ\_DEPT\_MGRS} \leftarrow (\text{CONTR\_DEPTS} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(\text{PROJ\_DEPT\_MGRS})$$

Query 3. Find the names of employees who work on *all* the projects controlled by department number 5.

$$\text{DEPT5\_PROJS} \leftarrow \rho_{(\text{Pno})}(\pi_{\text{Pnumber}}(\sigma_{\text{Dnum=5}}(\text{PROJECT})))$$
$$\text{EMP\_PROJ} \leftarrow \rho_{(\text{Ssn, Pno})}(\pi_{\text{Essn, Pno}}(\text{WORKS\_ON}))$$
$$\text{RESULT\_EMP\_SSNS} \leftarrow \text{EMP\_PROJ} \div \text{DEPT5\_PROJS}$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{RESULT\_EMP\_SSNS} * \text{EMPLOYEE})$$

Query 4. Make a list of project numbers for projects that involve an employee whose last name is 'Smith', either as a worker or as a manager of the department that controls the project.

$$SMITHS(Essn) \leftarrow \pi_{Ssn}(\sigma_{Lname='Smith'}(EMPLOYEE))$$
$$SMITH\_WORKER\_PROJS \leftarrow \pi_{Pno}(WORKS\_ON * SMITHS)$$
$$MGRS \leftarrow \pi_{Lname,\ Dnumber}(EMPLOYEE \bowtie_{Ssn=Mgr\_ssn} DEPARTMENT)$$
$$SMITH\_MANAGED\_DEPTS(Dnum) \leftarrow \pi_{Dnumber}(\sigma_{Lname='Smith'}(MGRS))$$
$$SMITH\_MGR\_PROJS(Pno) \leftarrow \pi_{Pnumber}(SMITH\_MANAGED\_DEPTS * PROJECT)$$
$$RESULT \leftarrow (SMITH\_WORKER\_PROJS \cup SMITH\_MGR\_PROJS)$$

Query 5. List the names of all employees with two or more dependents.

$$T1(Ssn,\ No\_of\_dependents) \leftarrow {}_{Essn}\Im_{COUNT\ Dependent\_name}(DEPENDENT)$$
$$T2 \leftarrow \sigma_{No\_of\_dependents>2}(T1)$$
$$RESULT \leftarrow \pi_{Lname,\ Fname}(T2 * EMPLOYEE)$$

Query 6. Retrieve the names of employees who have no dependents.

$$ALL\_EMPS \leftarrow \pi_{Ssn}(EMPLOYEE)$$
$$EMPS\_WITH\_DEPS(Ssn) \leftarrow \pi_{Essn}(DEPENDENT)$$
$$EMPS\_WITHOUT\_DEPS \leftarrow (ALL\_EMPS - EMPS\_WITH\_DEPS)$$
$$RESULT \leftarrow \pi_{Lname,\ Fname}(EMPS\_WITHOUT\_DEPS * EMPLOYEE)$$

Query 7. List the names of managers who have at least one dependent.

$$MGRS(Ssn) \leftarrow \pi_{Mgr\_ssn}(DEPARTMENT)$$
$$EMPS\_WITH\_DEPS(Ssn) \leftarrow \pi_{Essn}(DEPENDENT)$$
$$MGRS\_WITH\_DEPS \leftarrow (MGRS \cap EMPS\_WITH\_DEPS)$$
$$RESULT \leftarrow \pi_{Lname,\ Fname}(MGRS\_WITH\_DEPS * EMPLOYEE)$$

Illustrate queries using the following new instances S3 of sailors, R2 of Reserves and B1 of boats.

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Figure 4.15 An Instance $S3$ of Sailors

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Figure 4.16 An Instance $R2$ of Reserves

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Figure 4.17 An Instance $B1$ of Boats

(Q1) Find the names of sailors who have reserved boat 103

$$\pi_{sname}(\sigma_{bid=103}(\text{Reserves} * \text{Sailors}))$$

Q2) Find the names of sailors who have reserved a red boat.

$$\text{Sailres} \leftarrow \text{Sailors} * \text{Reserves}$$

$$\pi_{sname}((\sigma_{color='red'}\text{Boats}) * \text{Sailres})$$

(Q3) Find the colors of boats reserved by Lubber.

$$\text{Resboat} \leftarrow \text{Reserves} * \text{Boats}$$

$$\pi_{color}(\sigma_{sname='Lubber'}(\text{Sailors} * \text{Resboat}))$$

(Q4) Find the names of Sailors who have reserved at least one boat

$$\pi_{sname}(\text{Sailors} * \text{Reserves})$$

(Q5) Find the names of sailors who have reserved a red or a green boat.

$$\text{Tempboats} \leftarrow (\sigma_{color='red'}\text{Boats}) \cup (\sigma_{color='green'}\text{Boats})$$

$$\pi_{sname}(\text{Tempboats} * \text{Reserves} * \text{Sailors})$$

Q6) Find the names of Sailors who have reserved a red and a green boat.

$$\text{Tempred} \leftarrow \pi_{sid}\ (\sigma_{color='red'}(\text{Boats} * \text{Reserves}))$$
$$\text{Tempgreen} \leftarrow \pi_{sid}(\sigma_{color='green'}(\text{Boats} * \text{Reserves}))$$
$$\pi_{sname}\ ((\text{Tempred} \cap \text{Tempgreen}) * \text{Sailors})$$

Q7) Find the sids of sailors with age over 20 who have not reserved

$$\text{sailortwenty} \leftarrow \pi_{sid}(\sigma_{age>20}\text{Sailors})$$
$$\text{sailredboat} \leftarrow \pi_{sid}((\sigma_{color='red'}\text{Boats}) * \text{Reserves} * \text{Sailors})$$
$$\text{Result} \leftarrow \text{sailortwenty} - \text{sailredboat}$$

(Q8) Find the names of sailors who have reserved all boats.

$$\text{Tempsids} \leftarrow (\pi_{sid,bid}\text{Reserves}) \div (\pi_{bid}\text{Boats})$$
$$\text{Result} \leftarrow \pi_{sname}(\text{Tempsids} * \text{Sailors})$$

(Q9) Find the names of sailors who have reserved all boats called Interlake.

$$\text{Tempsids} \leftarrow (\pi_{sid,bid}\text{Reserves}) \div (\pi_{bid}(\sigma_{bname='Interlake'}\text{Boats}))$$
$$\text{Result} \leftarrow \pi_{sname}(\text{Tempsids} * \text{Sailors})$$

LOGICAL DATABASE DESIGN: ER TO RELATIONAL

The ER model is convenient for representing an initial, high level database design. The following steps show how to translate an ER diagram into a collection of tables with associated constraints.

1. Entity Sets to Tables.

- An entity set is mapped to a relation in a straightforward way: Each attribute of the entity set becomes an attribute of the table.
- **Steps to convert Entity Sets to Tables.**

i. Create a table for the entity set.
ii. Make each attribute of the entity set a field of the table, with an appropriate data type.
iii. Declare the field or fields comprising the primary key

- Consider the Employees entity set with attributes ssn, name, and lot.

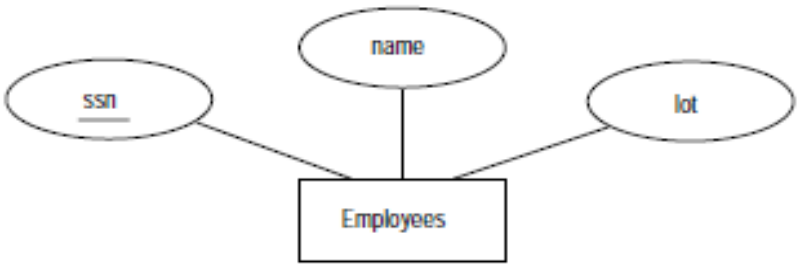


Figure 3.8 The Employees Entity Set

```
CREATE TABLE Employees ( ssn     CHAR(11),
                         name    CHAR(30),
                         lot     INTEGER,
                         PRIMARY KEY (ssn) )
```

2. Relationship Sets (without Constraints) to Tables.

- A relationship set, like an entity set, is mapped to a relation in the relational model.
- To represent a relationship, we must be able to identify each participating entity and give values to the attributes of the relationship.
- **Steps to convert Relationship sets to tables**

i. Create a table for the relationship set.
ii. Add all primary keys of the participating entity sets as fields of the table.
iii. Add a field for each attribute of the relationship.
iv. Declare a primary key using all key fields from the entity sets.
v. Declare foreign key constraints for all these fields from the entity sets.

- Consider the Works_In2 relationship set shown in Figure 3.10. Each department has offices in several locations and we want to record the locations at which each employee works

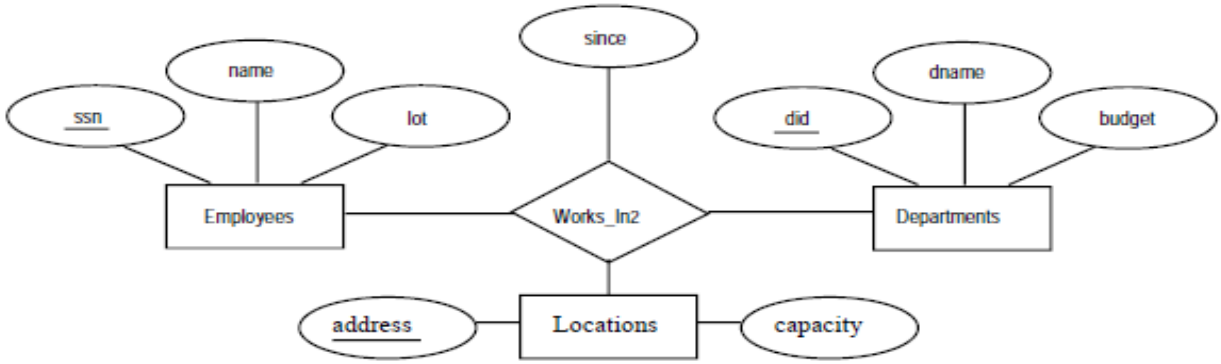


Figure 3.10 A Ternary Relationship Set

- All the available information about the Works-ln2 table is captured by the following SQL definition:

```
CREATE TABLE Works_In2 ( ssn      CHAR(11),
                         did      INTEGER,
                         address  CHAR(20),
                         since    DATE,
                         PRIMARY KEY (ssn, did, address),
                         FOREIGN KEY (ssn) REFERENCES Employees,
                         FOREIGN KEY (address) REFERENCES Locations,
                         FOREIGN KEY (did) REFERENCES Departments )
```

- The address, did. and ssn fields cannot take on null values. Because these fields are part of the primary key for Works_In2, a NOT NULL constraint is implicit for each of these fields. This constraint ensures that these fields uniquely identify a department, an employee, and a location in each tuple of Works_In.

3. Translating Relationship Sets with Key Constraints.

- If a relationship set involves n entity sets and some m of them are linked via arrows in the ER diagram, the key of these m entity sets constitutes a key for the relation to which the relationship set is mapped. Hence we have m candidate keys, and one of these should be designated as the primary key.
- Consider the relationship set Manages. The table corresponding to Manages has the attributes ssn, did, since. However, because each department has at most one manager, no two tuples can have the same did value but differ on the ssn value. A consequence of this observation is that did is itself a key for Manages; indeed, the set did, ssn is not a key (because it is not minimal).

- The Manages relation can be defined using the following SQL statement:

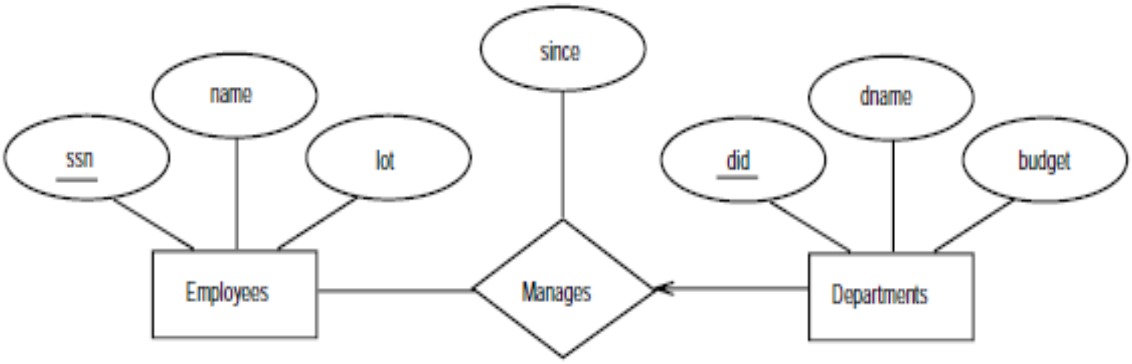


Figure 3.12 Key Constraint on Manages

```
CREATE TABLE Manages ( ssn       CHAR(11),
                       did       INTEGER,
                       since     DATE,
                       PRIMARY KEY (did),
                       FOREIGN KEY (ssn) REFERENCES Employees,
                       FOREIGN KEY (did) REFERENCES Departments )
```

- **Steps to translate Translating Relationship Sets with Key Constraints to tables.**

i. Create a table for the relationship set.
ii. Add all primary keys of the participating entity sets as fields of the table.
iii. Add a field for each attribute of the relationship.
iv. Declare a primary key using the key fields from the source entity set only.
v. Declare foreign key constraints for all the fields from the source and target entity sets.

- The following SQL statement, defining a Dept_Mgr relation that captures the information in both Departments and Manages, illustrates the second approach to translating relationship sets with key constraints. The only drawback to this approach is that space could be wasted if several departments have no managers. In this case the added fields would have to be filled with *null* values

```
CREATE TABLE Dept_Mgr ( did       INTEGER,
                        dname     CHAR(20),
                        budget    REAL,
                        ssn       CHAR(11),
                        since     DATE,
                        PRIMARY KEY (did),
                        FOREIGN KEY (ssn) REFERENCES Employees )
```

4. Translating Relationship Sets with Participation Constraints.

- Consider the ER diagram in Figure 3.13, which shows two relationship sets, Manages and "Works_In. Every department is required to have a manager, due to the participation constraint, and at most one manager, due to the key constraint.

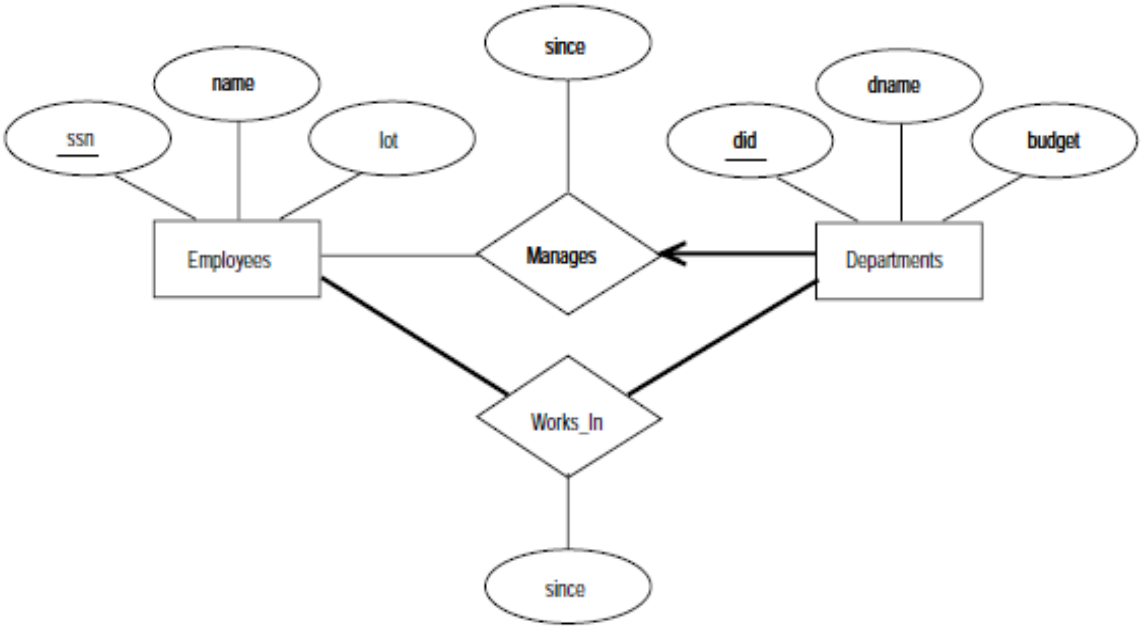


Figure 3.13 Manages and Works_In

- **Steps to translate relationship sets with participation constraints**

i. Create a table for the source and target entity sets as usual.
ii. Add every primary key field of the target as a field in the source.
iii. Declare these fields as not null.
iv. Declare these fields as foreign keys.

```
CREATE TABLE Dept_Mgr ( did      INTEGER,
                        dname    CHAR(20),
                        budget   REAL,
                        ssn      CHAR(11) NOT NULL,
                        since    DATE,
                        PRIMARY KEY (did),
                        FOREIGN KEY (ssn) REFERENCES Employees
                                ON DELETE NO ACTION )
```

- According to participation constraint every department must have a manager: because ssn cannot take on null values, each tuple of Dept_Mgr identifies a tuple in Employees (who is the manager).
- The **NO ACTION** specification, which is the default and need not be explicitly specified, ensures that an Employees tuple cannot be deleted while it is pointed to by a Dept_Mgr

tuple. If we wish to delete such an Employees tuple, we must first change the Dept_Mgr tuple to have a new employee as manager.

5 Translating Weak Entity Sets

- A weak entity set always participates in a one-to-many binary relationship and has a key constraint and total participation.
- The weak entity has only a partial key, when an owner entity is deleted, all the weak entities should be deleted.
- Consider the Dependents weak entity set shown in Figure 3.14, with partial key pname. A Dependents entity can be identified uniquely by the key of the owning Employees entity and the pname of the Dependents entity, and the Dependents entity must be deleted if the owning Employees entity is deleted.

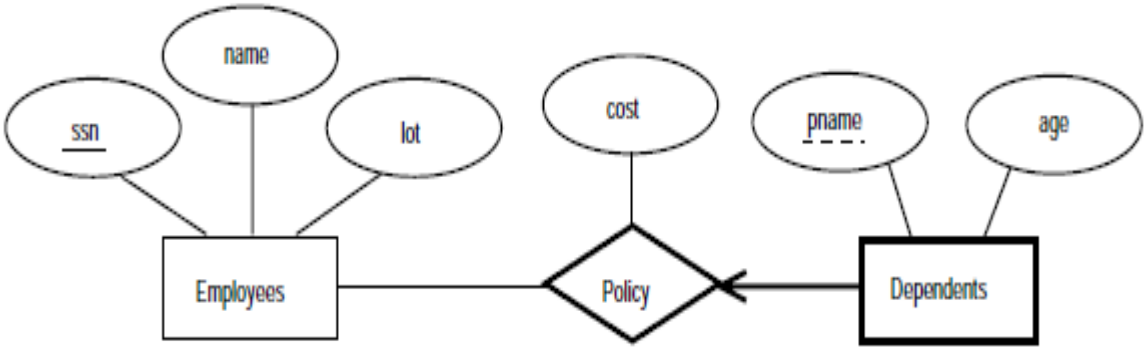


Figure 3.14 The Dependents Weak Entity Set

```
CREATE TABLE Dep_Policy ( pname   CHAR(20),
                          age     INTEGER,
                          cost    REAL,
                          ssn     CHAR(11),

                        PRIMARY KEY (pname, ssn),
                        FOREIGN KEY (ssn) REFERENCES Employees
                                ON DELETE CASCADE )
```

- **Steps to translate weak entity set to tables.**

i. Create a table for the weak entity set.
ii. Make each attribute of the weak entity set a field of the table.
iii. Add fields for the primary key attributes of the identifying owner.
iv. Declare a foreign key constraint on these identifying owner fields.
v. Instruct the system to automatically delete any tuples in the table for which there are no owners.

- The primary key is (pname, ssn), since Dependents is a weak entity. The value of ‘ssn’ cannot be null because the Dependents entity is associated with an Employees entity (the owner), as per the total participation constraint on Dependents.
- The ON DELETE CASCADE option ensures that information about an employee's policy and dependents is deleted if the corresponding Employees tuple is deleted.