Computational Methods for Fluid Mechanics – lecture 4 Fluid-structure interaction

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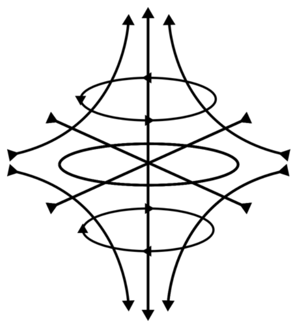
Today

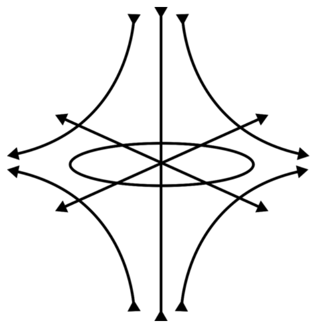
- Structure of turbulent flow
- Boundary layers, flow separation, and wall modelling.
- The structure of turbulent flow separation, analytical models, and the d’Alembert’s paradox.
- Deforming domains and fluid-structure interaction

Some structures of incompressible flow

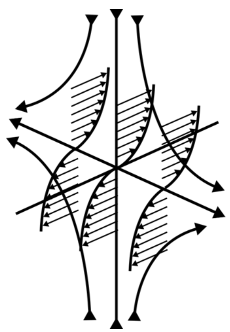
Vortex tube



Pancake



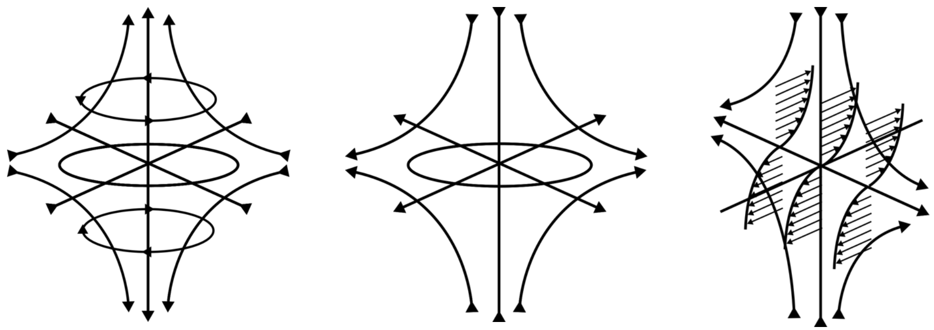
Shear layer



$$\nabla u \qquad \begin{bmatrix} \lambda_R & 0 & 0 \\ 0 & \alpha & -\beta \\ 0 & \beta & \alpha \end{bmatrix} \quad \begin{bmatrix} 2\lambda_R & 0 & 0 \\ 0 & -\lambda_R & 0 \\ 0 & 0 & -\lambda_R \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & \zeta \\ 0 & \lambda_R & 0 \\ 0 & 0 & -\lambda_R \end{bmatrix}$$

Some structures of incompressible flow

Vortex tube　　　Pancake　　　Shear layer



$$\nabla u \qquad V\begin{bmatrix} \lambda_R & 0 & 0 \\ 0 & \lambda_C & 0 \\ 0 & 0 & \overline{\lambda_C} \end{bmatrix} V^* \qquad \begin{bmatrix} 2\lambda_R & 0 & 0 \\ 0 & -\lambda_R & 0 \\ 0 & 0 & -\lambda_R \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & \zeta \\ 0 & \lambda_R & 0 \\ 0 & 0 & -\lambda_R \end{bmatrix}$$

$$\lambda_C, \overline{\lambda_C} = \alpha \pm i\beta$$

Analysis of flow structures

- Vorticity $\omega = \nabla \times u$
- Double decomposition of velocity gradient tensor (VGT) into a strain rate tensor $S(u)$ and a spin tensor $\Omega(u)$ (used for Q-criterion, etc.)

$$\nabla u = \frac{1}{2}(\nabla u + \nabla u^T) + \frac{1}{2}(\nabla u - \nabla u^T) = S(u) + \Omega(u)$$

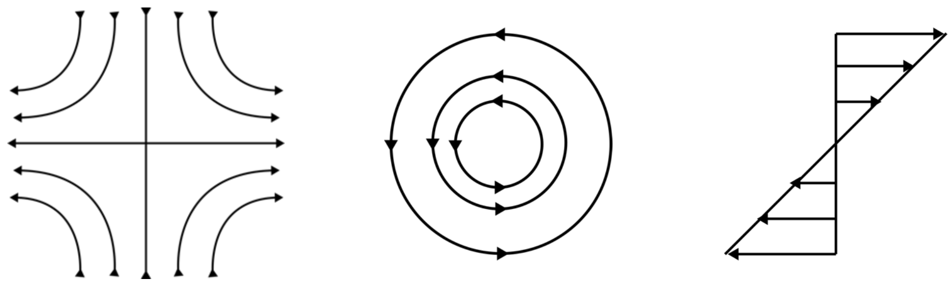
- These methods have a weakness: they do not distinguish shear flow from rotational flow and straining flow.
- Pointed out by V. Kolár [2007] in the context of vortex identification. Suggested a triple decomposition of VGT.
- Followed up by others [e.g. Liu et al. 2018, Keylock 2019, Nagata et al. 2020].

Triple decomposition of VGT

- Triple decomposition of the velocity gradient tensor

$$\nabla u = \nabla u_{strain} + \nabla u_{rotation} + \nabla u_{shear}$$

- Structure of fluid flow: strain + rotation + shear



Algebraic derivation

- Any square matrix A has a (non-unique) Schur factorization

$$A = UTU^*$$

U is unitary, T upper triangular with eigenvalues of A on the diagonal.

- Any normal square matrix is unitary diagonalizable (spectral theorem)

$$A = UDU^*$$

U is unitary with eigenvectors as columns, D diagonal with eigenvalues.

- Any square matrix A is decomposed into sum of a normal and a non-normal part.

$$A = UTU^* = UDU^* + U(T - D)U^*, \qquad \|T - D\|_F^2 = \sum_i \sigma_i^2 - \sum_i |\lambda_i|^2$$

Algebraic derivation

- If A is a real matrix it has a real Schur form

$$A = QBQ^T$$

Q real and orthogonal, B real upper quasi-triangular with possible pairs of conjugate complex eigenvalues $(\lambda, \bar{\lambda})$ represented by real 2x2 block matrices on the diagonal with the same eigenvalues.
[Golub/Van Loan 1996]

$$\begin{bmatrix} \lambda & * \\ 0 & \bar{\lambda} \end{bmatrix} \to \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- If $a = d$ and $cb < 0$, then the real Schur form is said to be in standardized form, which can be computed by standard methods. [Bai/Demmel 1993]

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \alpha \end{bmatrix}, \qquad \beta\gamma < 0, \qquad \lambda, \bar{\lambda} = \alpha \pm \sqrt{\beta\gamma} = \alpha \pm i\sqrt{|\beta\gamma|}$$

Triple decomposition of VGT

- For the real 3x3 VGT we have a standardized real Schur form

$$\nabla u = Q\overline{\nabla u}Q^T = Q\left(\overline{\nabla u}_{diag} + \overline{\nabla u}_{skew} + \overline{\nabla u}_{nn}\right)Q^T = \nabla u_{sym} + \nabla u_{skew} + \nabla u_{nn}$$

- Assume $|\beta| > |\gamma|$, then with $\lambda_R + 2\alpha = 0$,

$$\overline{\nabla u} = \begin{bmatrix} \lambda_R & \varepsilon & \zeta \\ 0 & \alpha & \beta \\ 0 & \gamma & \alpha \end{bmatrix} = \begin{bmatrix} \lambda_R & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -\gamma \\ 0 & \gamma & 0 \end{vmatrix} + \begin{vmatrix} 0 & \varepsilon & \zeta \\ 0 & 0 & \beta + \gamma \\ 0 & 0 & 0 \end{vmatrix}$$

$$\left\|\nabla u_{sym}\right\|_F^2 = \lambda_R^2 + 2\alpha^2, \qquad \|\nabla u_{skew}\|_F^2 = 2\gamma^2, \qquad \|\nabla u_{nn}\|_F^2 = \sum_i \sigma_i^2 - \sum_i |\lambda_i|^2$$

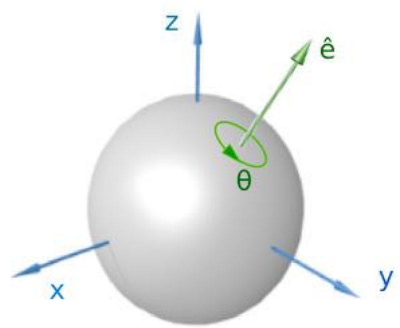
- This algebraic decomposition can be identified with a triple decomposition of VGT into strain, rotation and shear flow.
- Q represents a change of basis – how to interpret Q?

Euler’s rotation theorem

- “When a sphere is moved around its center it is always possible to find a diameter whose direction in the displaced position is the same as in the initial position” (Euler, 1776)

- Rigid body rotational transformation represents the eigenvalue problem:

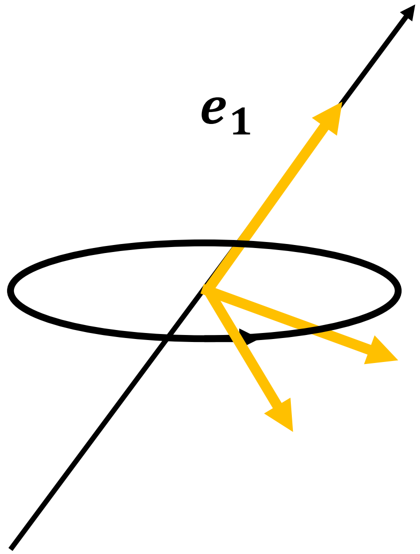
  $Rv = \lambda v$ (with $\lambda = 1$)

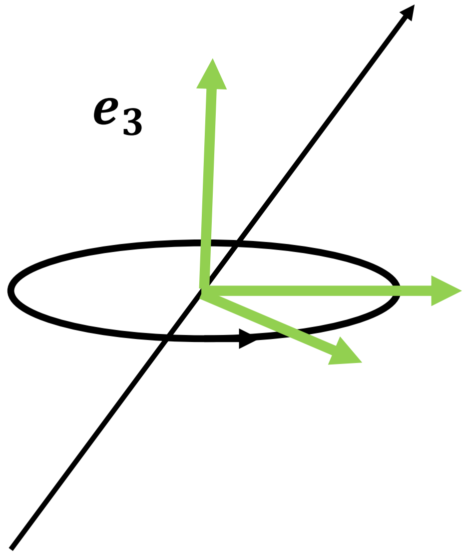


[Wikipedia]

- Here it is natural to align Q with v.
- What about a general linear transformation?

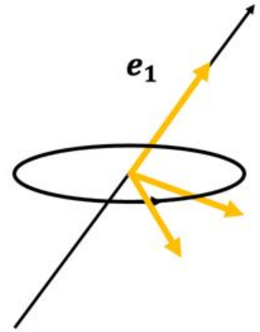
How to choose Q with shear?



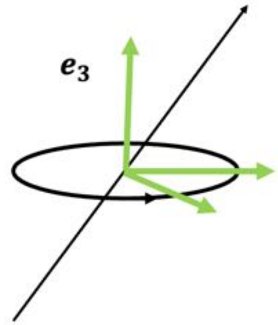


How to choose Q with shear?

$$\nabla u = Q\overline{\nabla u}Q^T, \qquad \overline{\nabla u} = \begin{bmatrix} \lambda_R & \varepsilon & \zeta \\ 0 & \alpha & \beta \\ 0 & \gamma & \alpha \end{bmatrix}$$



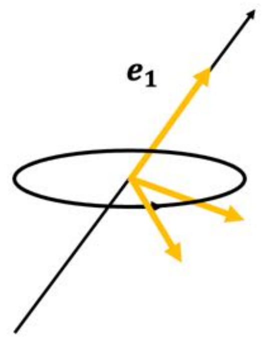
$$\nabla u = \hat{Q}\widehat{\nabla u}\hat{Q}^T, \qquad \widehat{\nabla u} = \begin{bmatrix} \alpha & \hat{\beta} & \hat{\zeta} \\ \hat{\gamma} & \alpha & \hat{\varepsilon} \\ 0 & 0 & \lambda_R \end{bmatrix}$$



How to choose Q with shear?

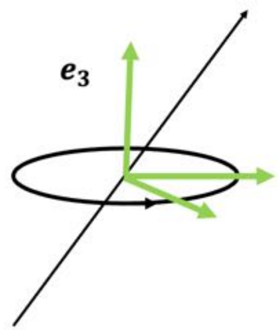
$$v_R = Qe_1, \qquad Q^T v_R = e_1$$

$$\nabla u v_R = Q\overline{\nabla u}Q^T v_R = Q\overline{\nabla u}e_1 = Q\lambda_R e_1 = \lambda_R v_R$$



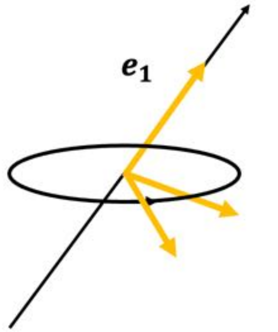
$$\hat{v}_R = \hat{Q}e_3, \qquad \hat{Q}^T \hat{v}_R = e_3$$

$$\nabla u \hat{v}_R = \hat{Q}\widehat{\nabla u}\hat{Q}^T \hat{v}_R = \hat{Q}\widehat{\nabla u}e_3 = \hat{Q}\begin{bmatrix}\hat{\zeta}\\ \hat{\varepsilon}\\ \lambda_R\end{bmatrix} = \hat{Q}\begin{bmatrix}\hat{\zeta}/\lambda_R\\ \hat{\varepsilon}/\lambda_R\\ 1\end{bmatrix}$$

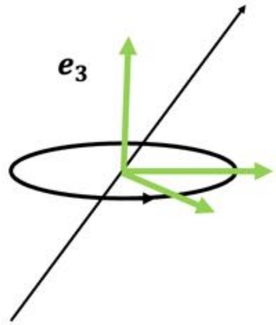


How to choose Q with shear?

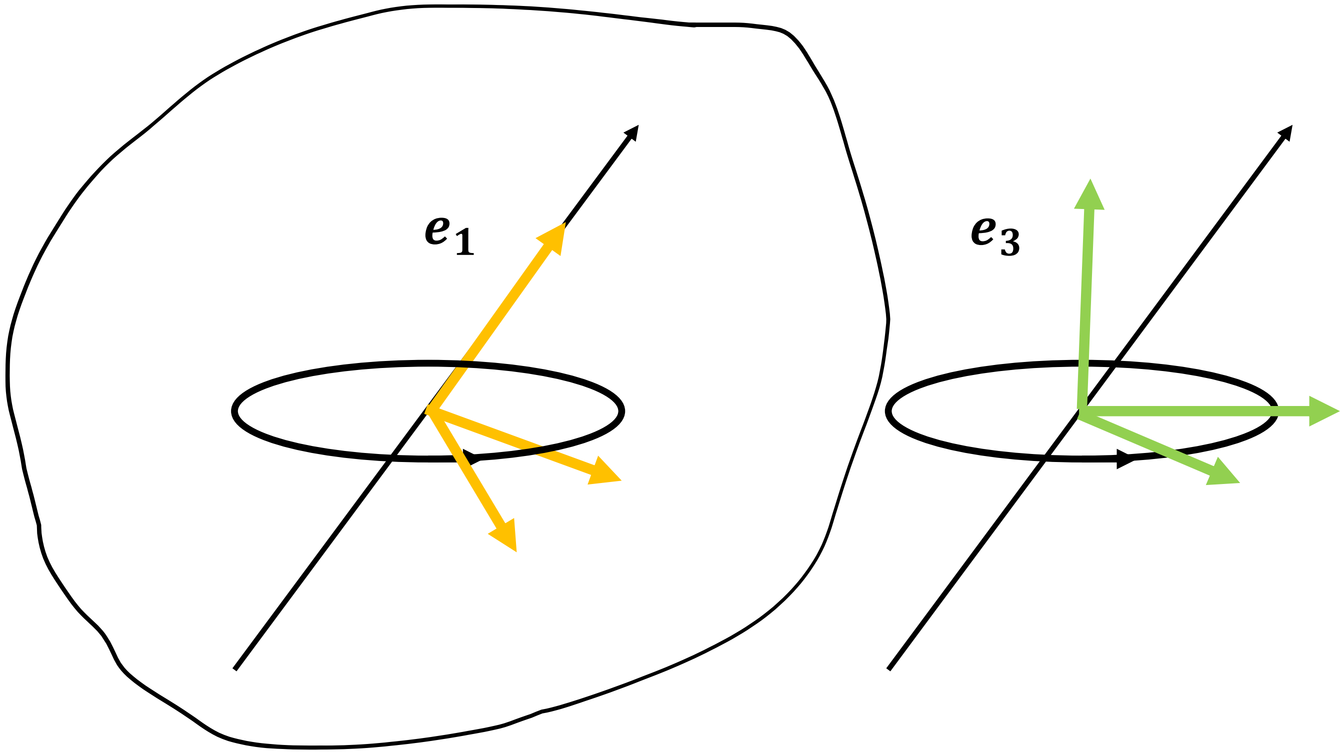
$$\widehat{\nabla u}\begin{bmatrix}\hat{\xi}\\ \hat{\nu}\\ 0\end{bmatrix}=\begin{bmatrix}\alpha & \hat{\beta} & \hat{\zeta}\\ \hat{\gamma} & \alpha & \hat{\varepsilon}\\ 0 & 0 & \lambda_R\end{bmatrix}\begin{bmatrix}\hat{\xi}\\ \hat{\nu}\\ 0\end{bmatrix}=\begin{bmatrix}\alpha\hat{\xi}+\hat{\beta}\hat{\nu}\\ \hat{\gamma}\hat{\xi}+\alpha\hat{\nu}\\ 0\end{bmatrix}$$



$$\overline{\nabla u}\begin{bmatrix}0\\ \xi\\ \nu\end{bmatrix}=\begin{bmatrix}\lambda_R & \varepsilon & \zeta\\ 0 & \alpha & \beta\\ 0 & \gamma & \alpha\end{bmatrix}\begin{bmatrix}0\\ \xi\\ \nu\end{bmatrix}=\begin{bmatrix}\varepsilon\xi+\zeta\nu\\ \alpha\xi+\beta\nu\\ \gamma\xi+\alpha\nu\end{bmatrix}$$



For consistency – go with Euler!



Stability analysis of NSE

- Navier-Stokes equations

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p - \upsilon \Delta u = f$$
$$\nabla \cdot u = 0$$

- Perturbation equation, adjoint equation, vorticity equation

$$\frac{\partial u'}{\partial t} + (u \cdot \nabla)u' + (u' \cdot \nabla)U + \nabla p' - \upsilon \Delta u' = P(\cdot), \qquad \nabla \cdot u' = 0$$
$$-\frac{\partial \varphi}{\partial t} - (u \cdot \nabla)\varphi + \nabla U^T \varphi + \nabla \theta - \upsilon \Delta \varphi = M(\cdot), \qquad \nabla \cdot \varphi = 0$$
$$\frac{\partial \omega}{\partial t} + (u \cdot \nabla)\omega - (\omega \cdot \nabla)u - \upsilon \Delta \omega = \Omega(\cdot), \qquad \nabla \cdot \omega = 0$$

Stability analysis of NSE

- All have the same type of stability property ($\phi = \varphi, u', \omega$)

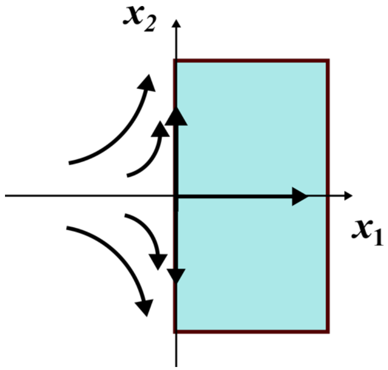
$$\frac{1}{2}\frac{d}{dt}\|\phi\|^2 \pm \int_\Omega \phi^T \nabla u\, \phi\, dx = -\upsilon\|\nabla\phi\|^2 + S(\phi)$$

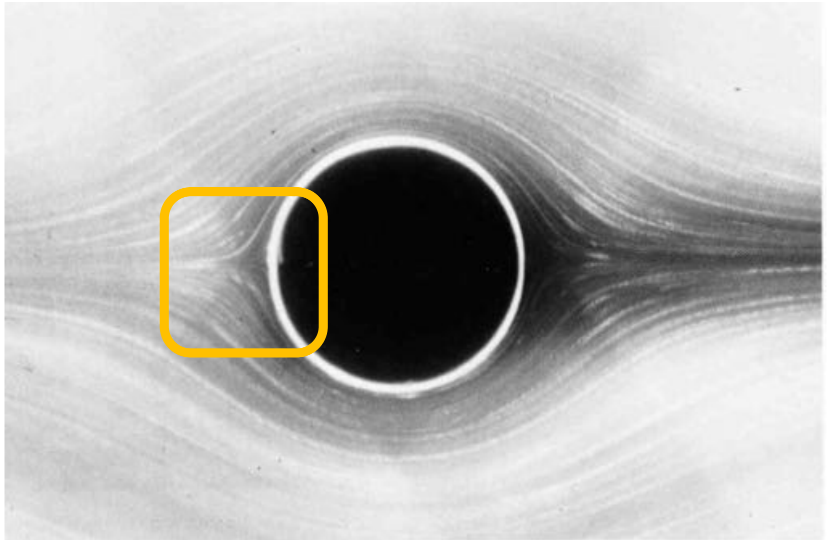
- The key term which determines the stability is the integral

$$\int_\Omega \phi^T \nabla u\, \phi\, dx = \int_\Omega \phi^T (\nabla u_{sym} + \nabla u_{skew} + \nabla u_{nn})\, \phi\, dx = \int_\Omega \phi^T (\nabla u_{sym} + \nabla u_{nn})\, \phi\, dx$$

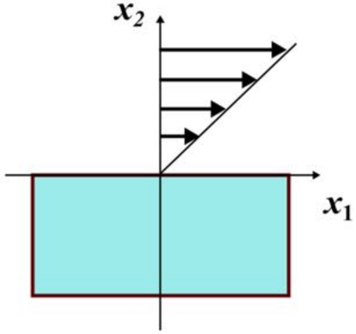
- Rigid body rotational flow is stable (stable vortices)
- Shear flow is linearly unstable (e.g. shear layer roll-up)
- Straining flow exponentially unstable (e.g. vortex stretching)

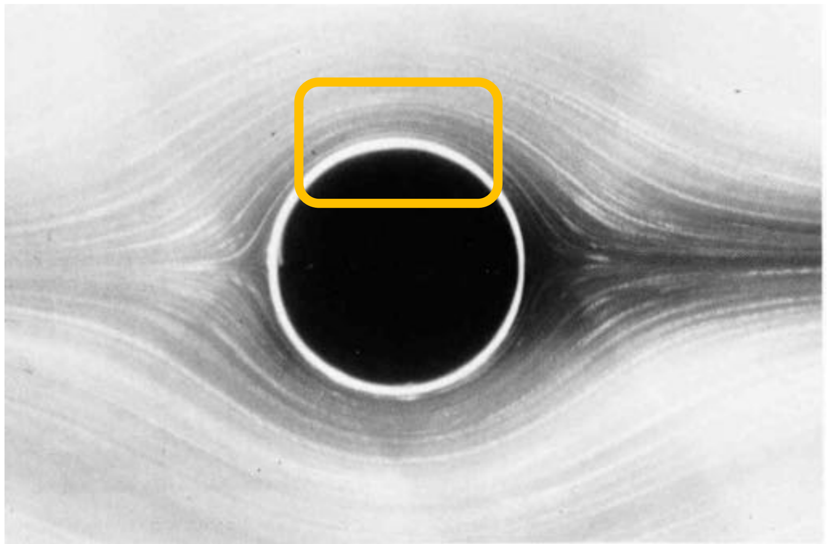
Cylinder (Re = 0.16) – attachment point



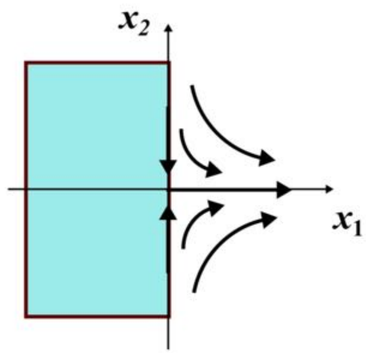


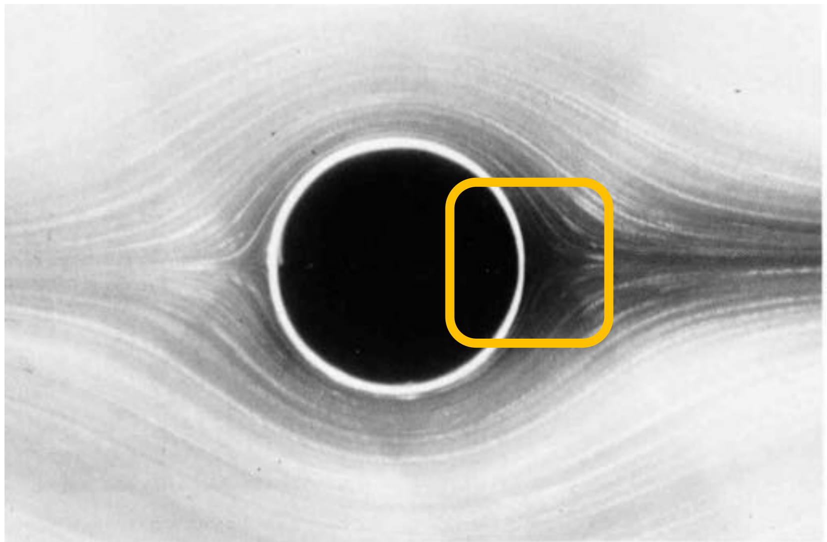
Cylinder (Re = 0.16) – boundary layer



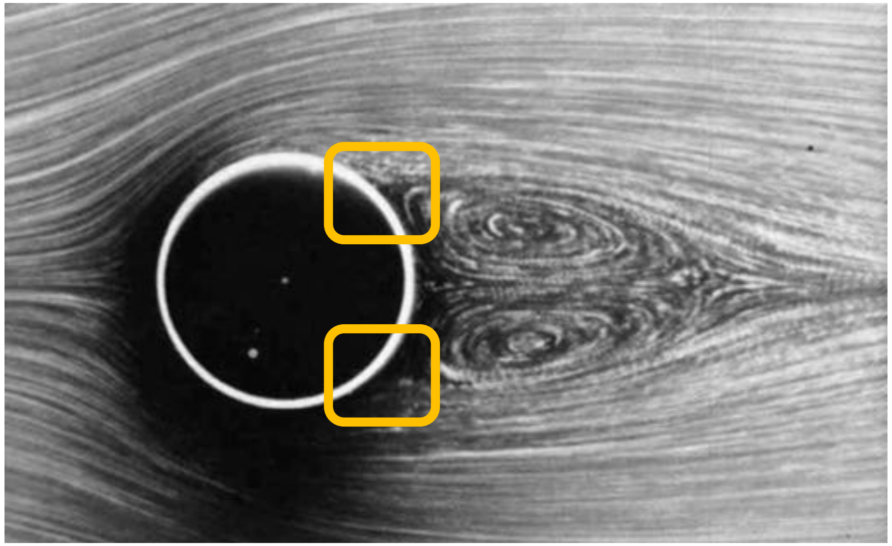


Cylinder (Re = 0.16) – separation point

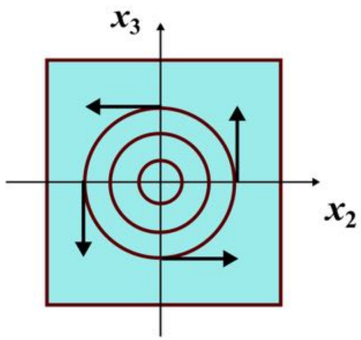


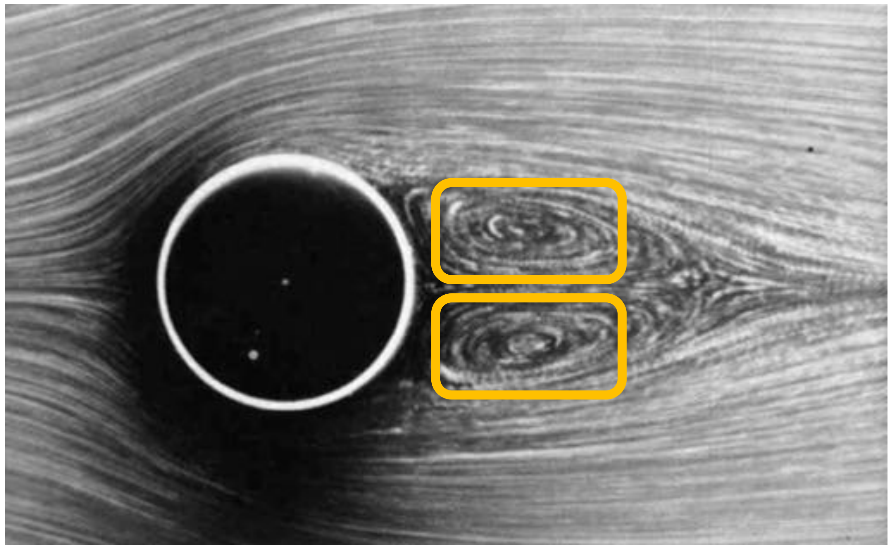


Cylinder (Re = 26) – 2 separation points



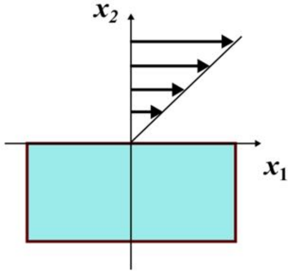
Cylinder (Re = 26) – 2 vortices

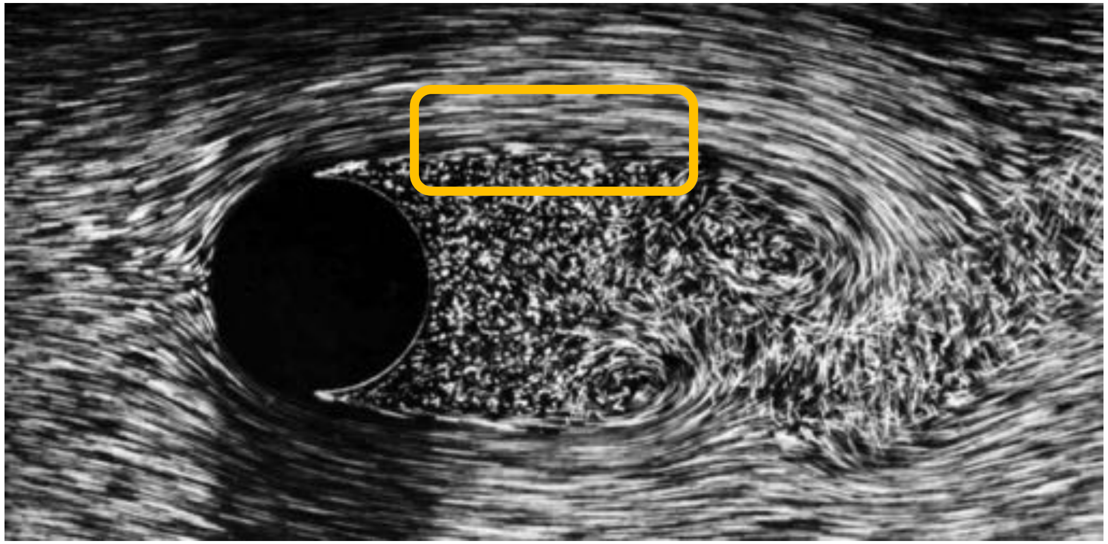




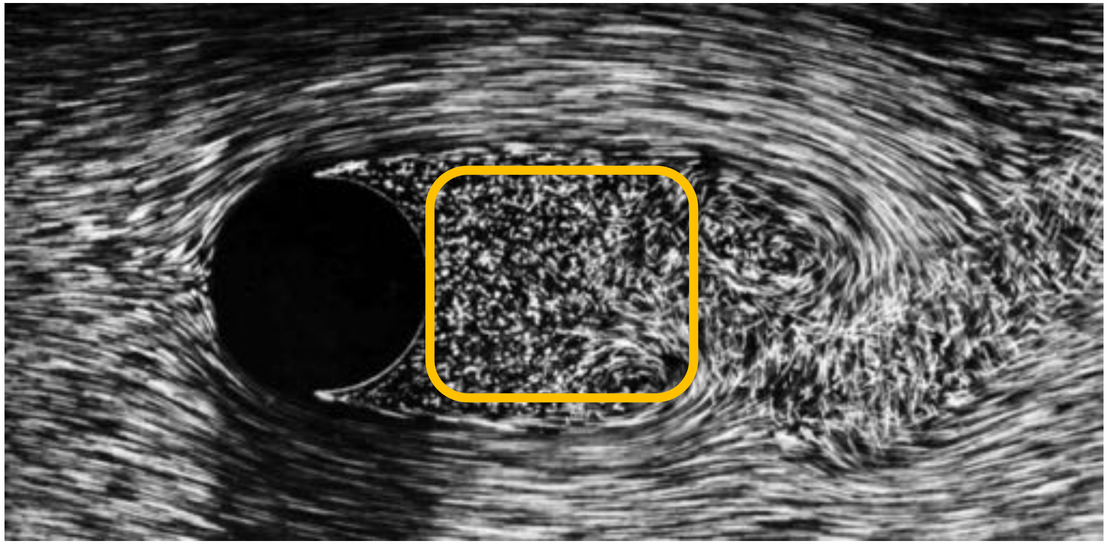
Cylinder (Re = 300) – Karman vortex street

Cylinder (Re = 2000) – shear layer

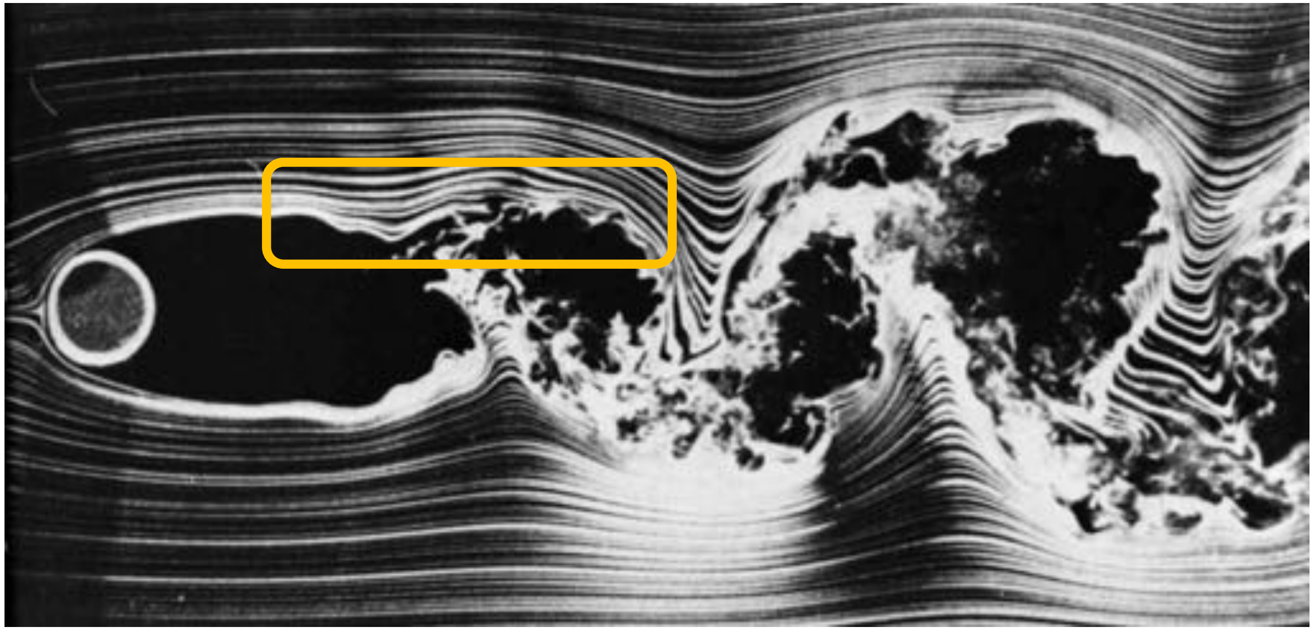




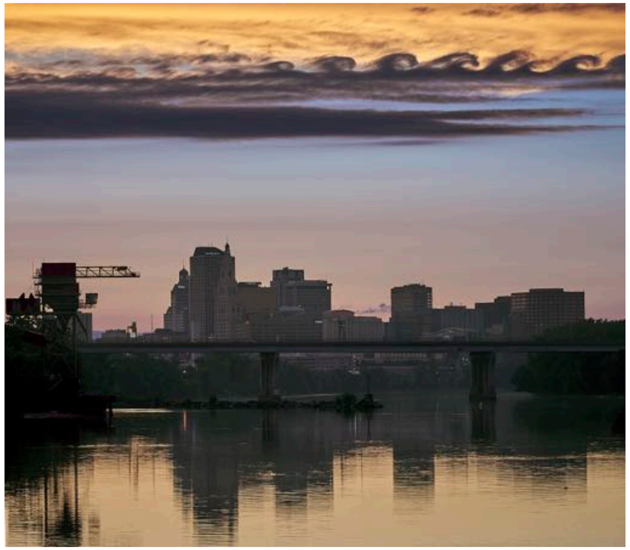
Cylinder (Re = 2000) – 3D turbulent wake



Re = 10 000 – turbulent shear layers

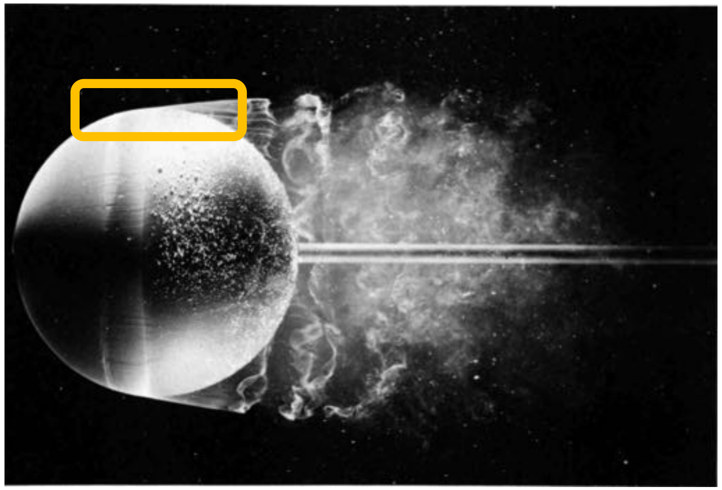


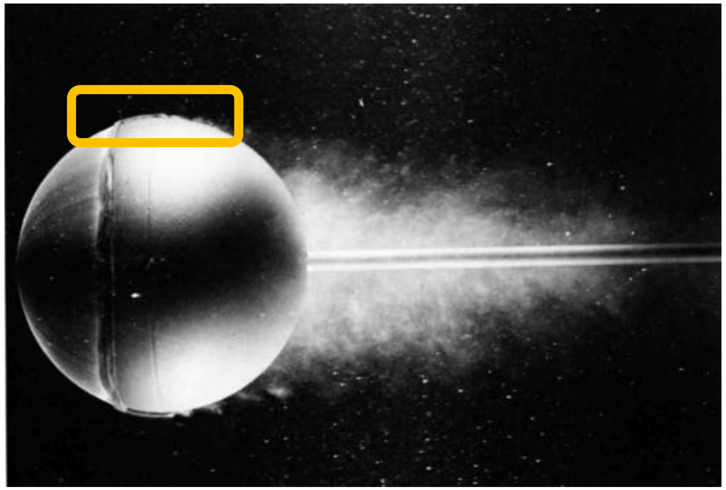
Kelvin-Helmholtz shear layer instability



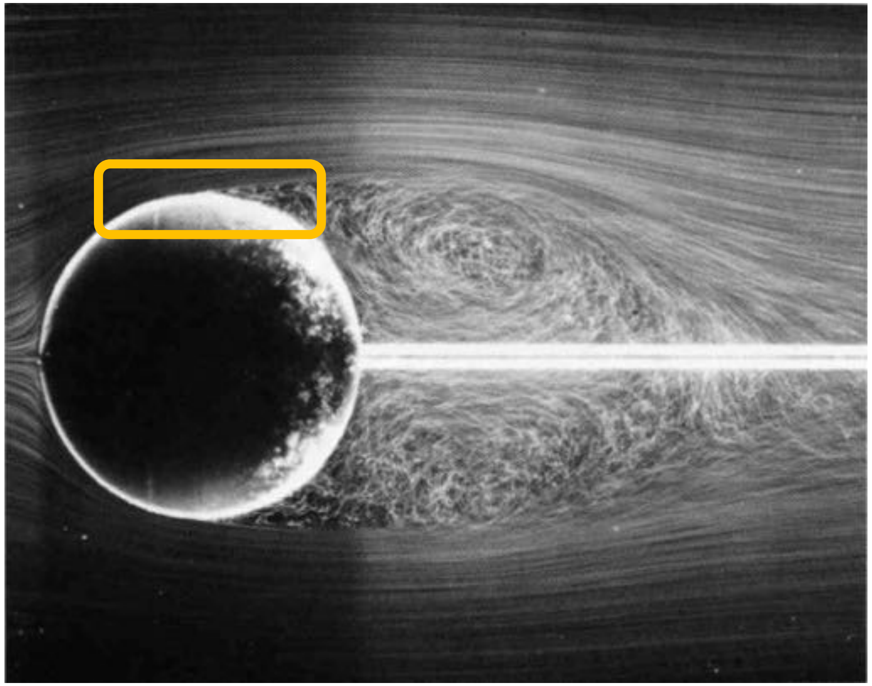
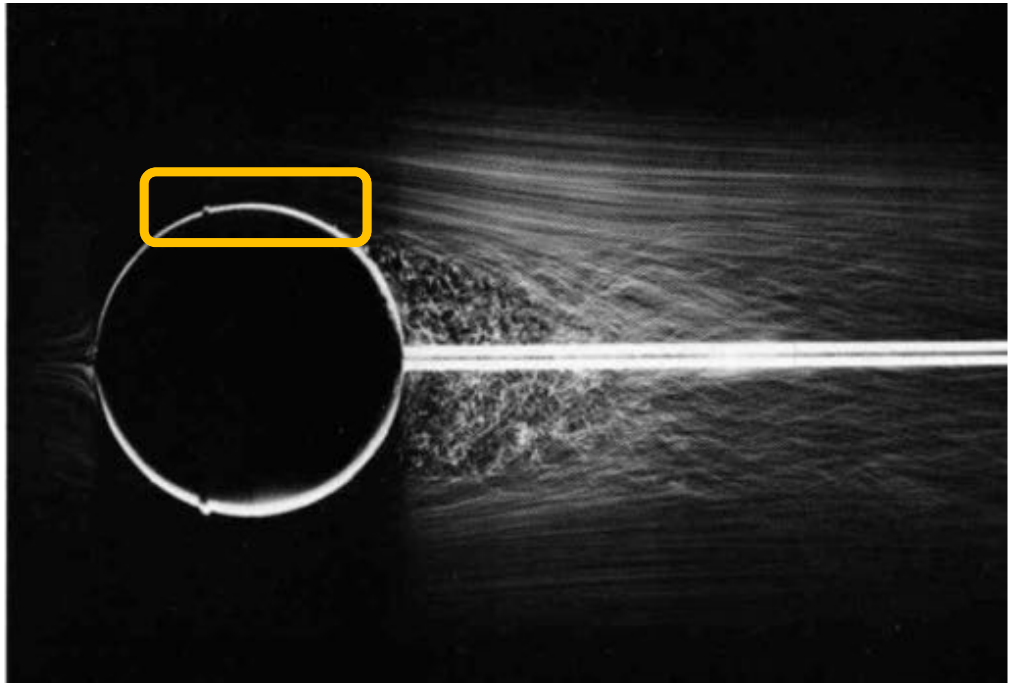
[Wikipedia]

Re = 30 000- turbulent boundary layer

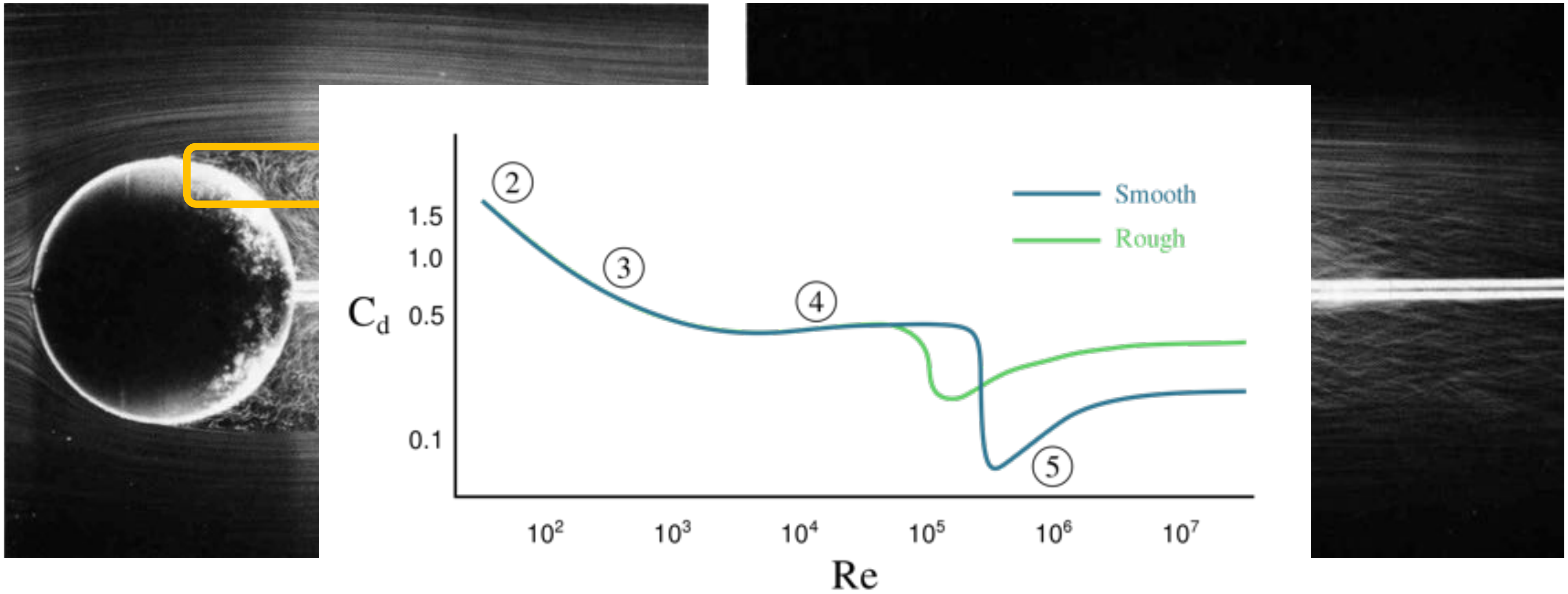




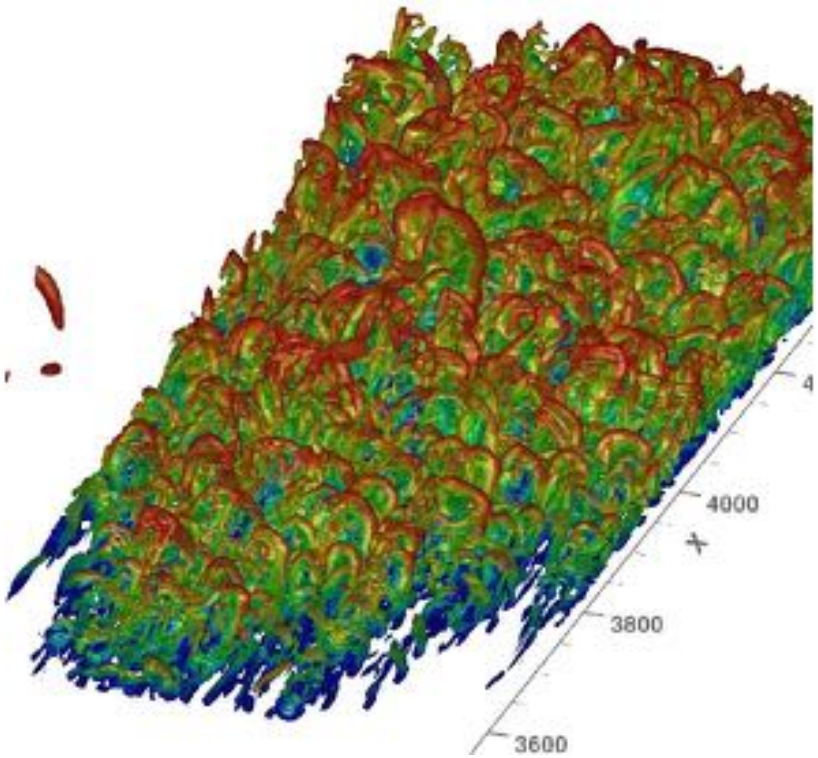
Sphere: Re = 15 000 vs 30 000
turbulent boundary layers (drag crisis)

Sphere: Re = 15 000 vs 30 000
turbulent boundary layers (drag crisis)



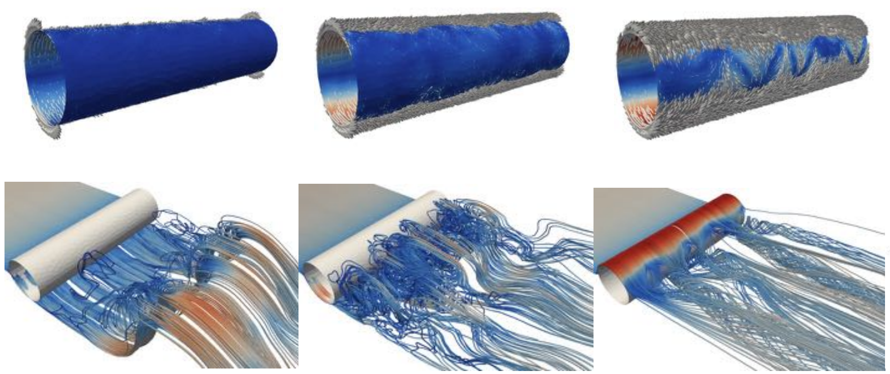
Turbulent boundary layer

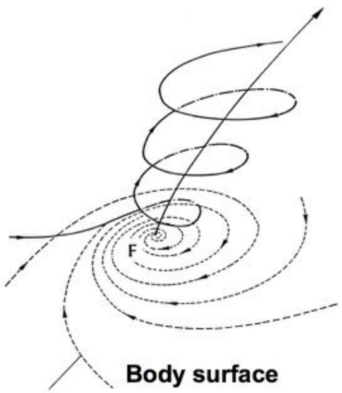


- Simulation tbl [Schlatter et al.]

Turbulent separation by a slip bc

- For high Re: model effect of turbulent boundary layers as a slip boundary condition (zero skin friction)
- Effectively computing dissipative Euler solutions by GLS
- Example: high Re flow past circular cylinder (drag crisis)
- Resolution of d'Alembert's paradox:
  potential flow unstable in 3D

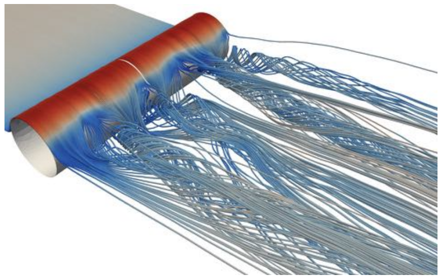




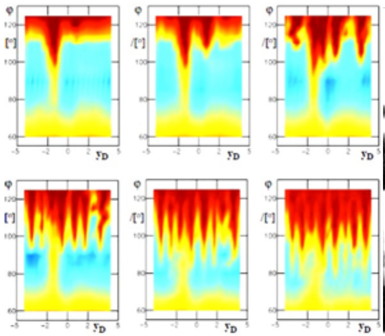
Turbulent separation by a slip bc

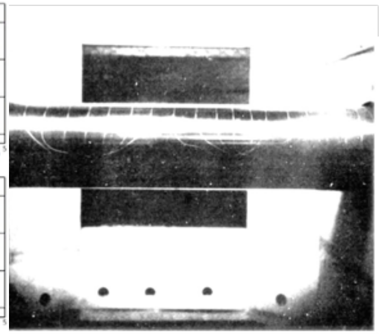
Consistent with experiments:

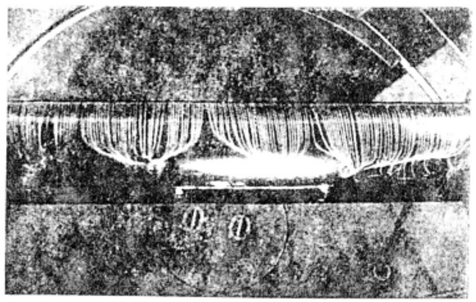
- Drag crisis: drag drops by a factor 4
- Streamwise vortices form stable cells in zig zag pattern
- Cell diameter ~ cylinder diameter



[Gölling DLR 2001] [Humphreys JFM 1960] [Korotkin 1976]

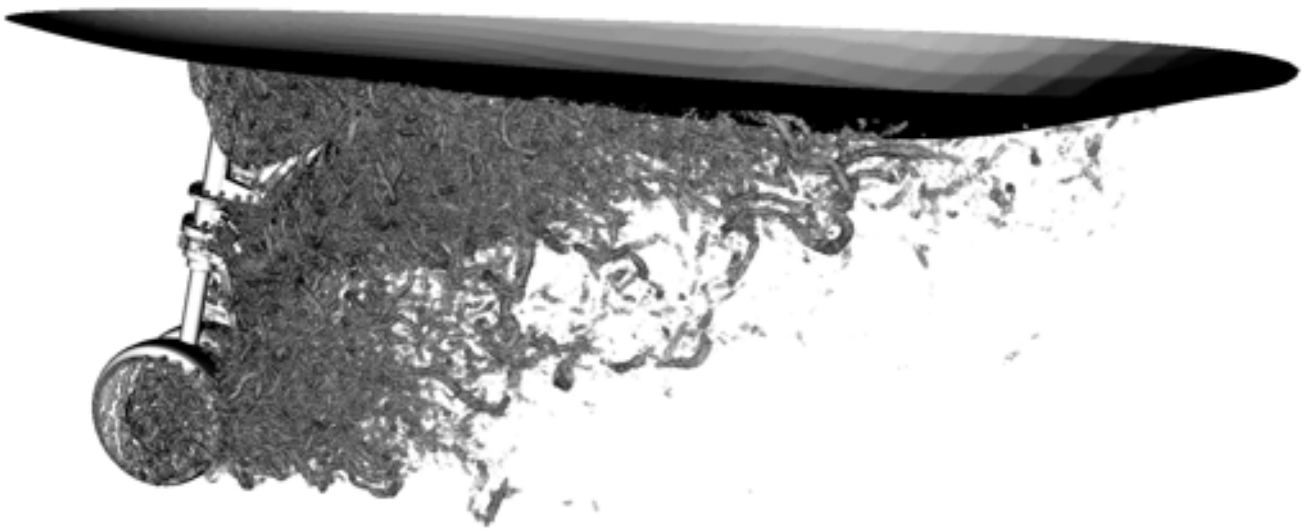






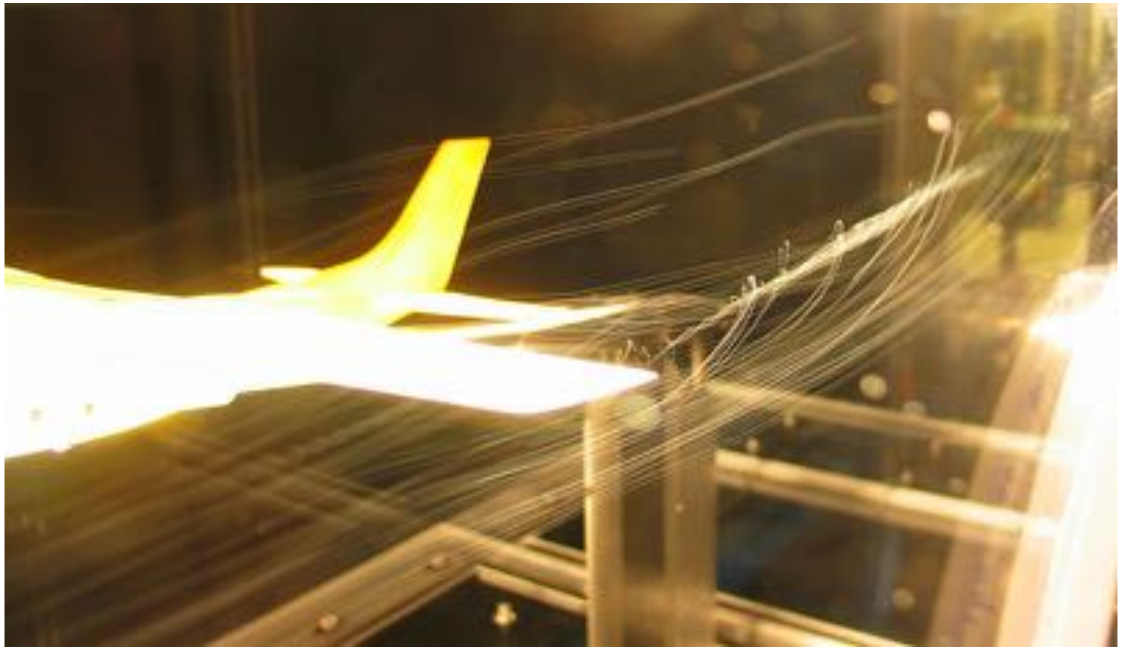
Simulation of airflow past landing gear

Turbulent vortices (lambda2 criterion)



Delayed separation and vortex structures

[https://en.wikipedia.org/wiki/File:Airplane_vortex_edit.jpg]



[https://sv.m.wikipedia.org/wiki/Fil:Cessna_182_model-wingtip-vortex.jpg]

Delayed separation - downwash

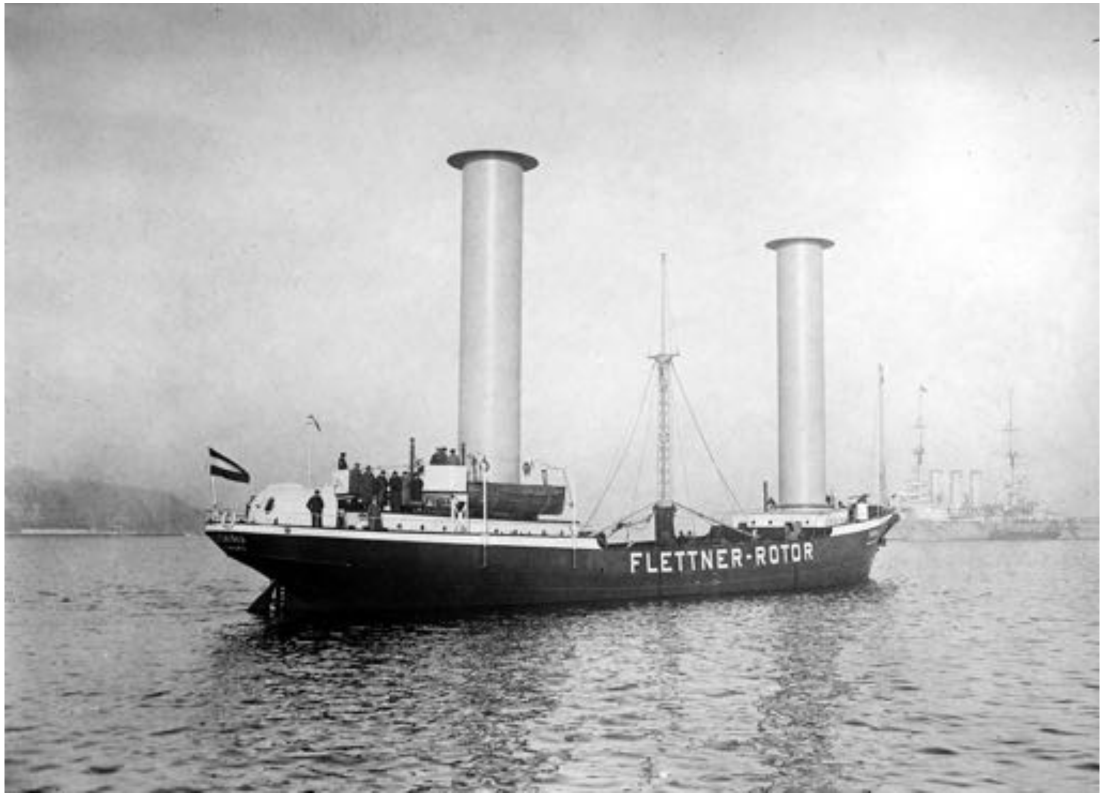
[https://i.redd.it/7yj9h0x2h9f61.jpg]

[https://www.grc.nasa.gov/www/k-12/airplane/downwash.html]

Magnus effect – downwash through rotation

[https://en.wikipedia.org/wiki/Magnus_effect#/media/File:Magnus-anim-canette.gif]

Buckau- Flettner rotor ship 1924



[https://en.wikipedia.org/wiki/Flettner_rotor#/media/File:Buckau_Flettner_Rotor_Ship_LOC_37764u.jpg]

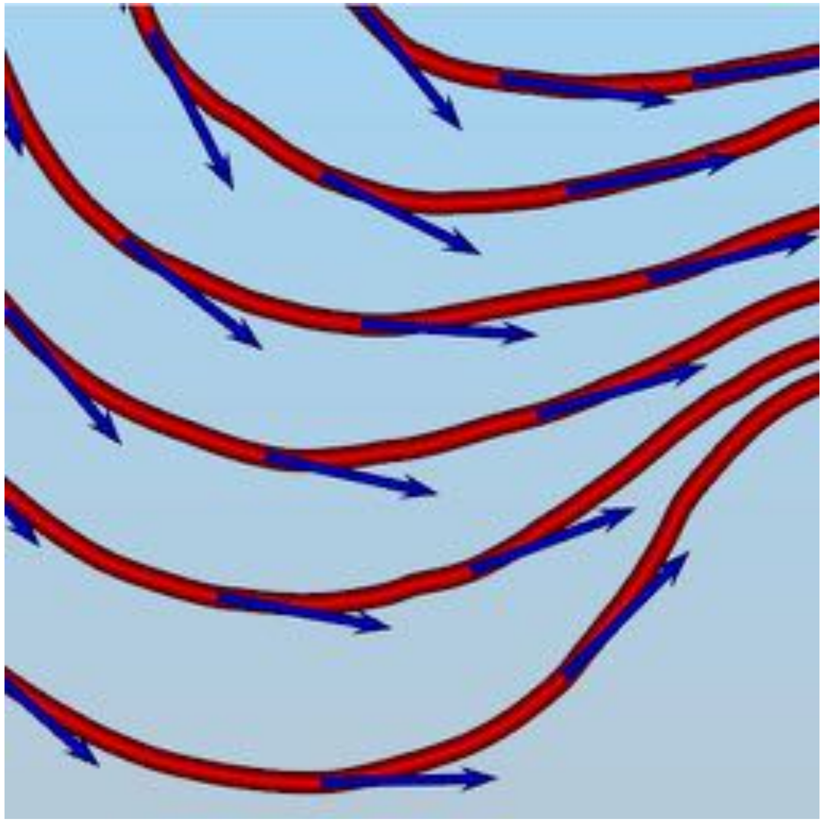
Grace- Flettner rotor ship 2019

Grace - Flettner rotor ship 2019

Flow visualization: pathlines

[https://en.wikipedia.org/wiki/Streamlines,_streaklines,_and_pathlines#/media/File:Kaberneeme_campfire_site.jpg]

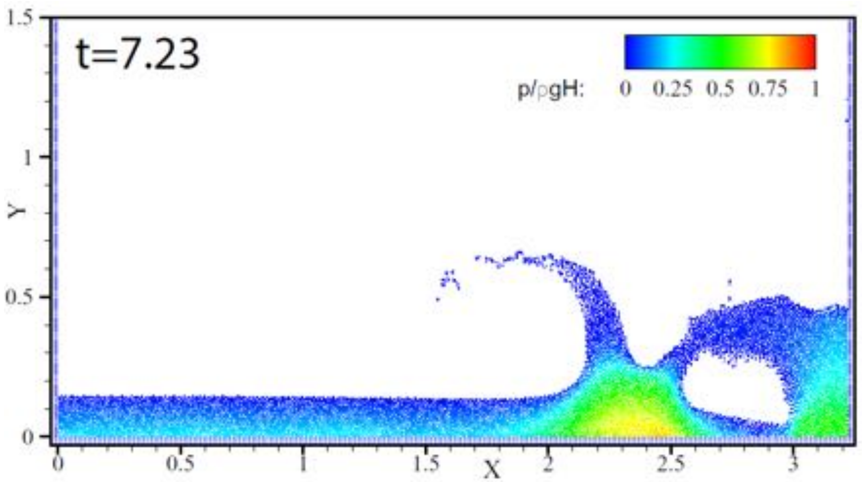
Flow visualization: streamlines



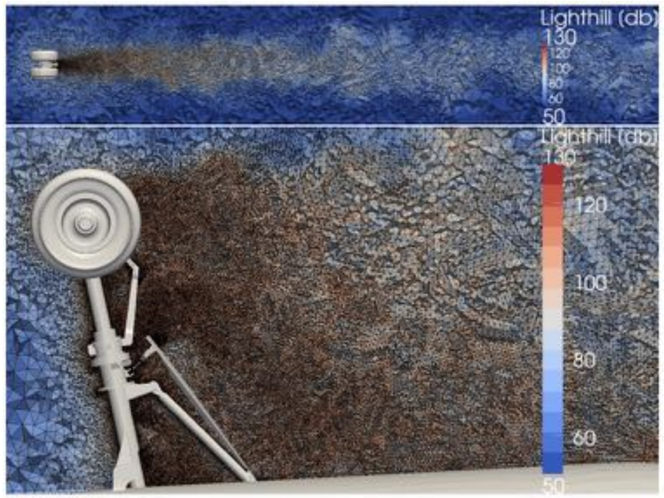
https://www3.nd.edu/~cwang11/2dflowvis.html

Representation of fluid flow

- Pathlines vs Streamlines
- Particles vs mesh/fixed coordinate system
- Lagrangian vs Eulerian representation
- Smooth particle hydrodynamics vs Finite element method

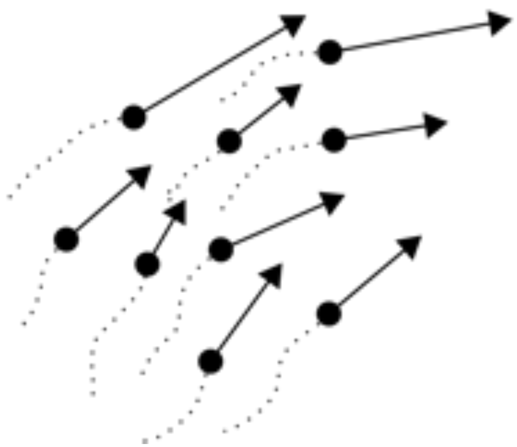


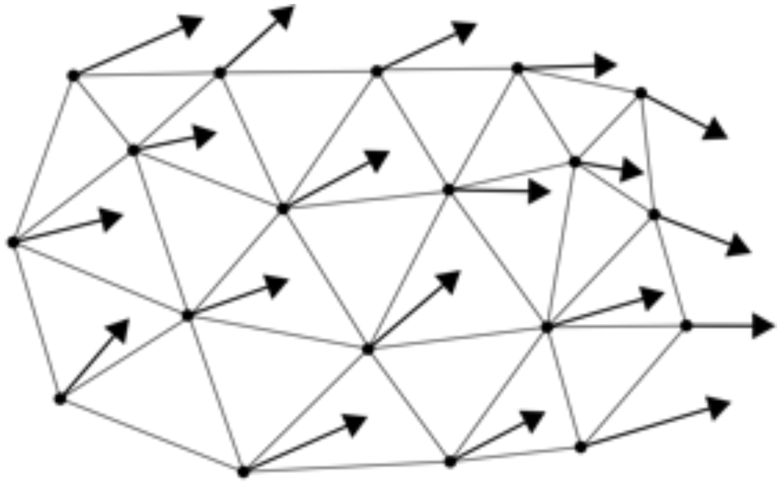
[https://www3.nd.edu/~cwang11/2dflowvis.html]



Representation of fluid flow

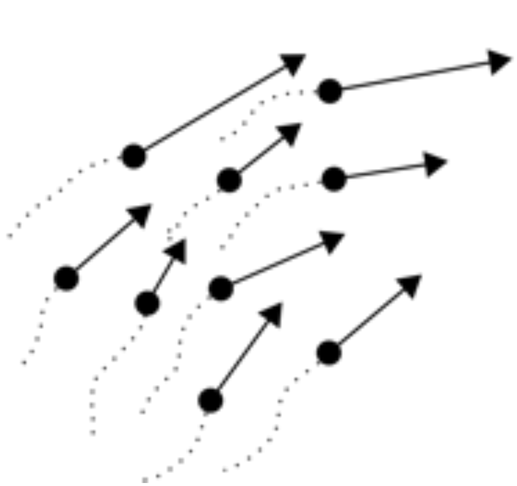
- Pathlines vs Streamlines
- Particles vs mesh/fixed coordinate system
- Lagrangian vs Eulerian representation
- Smooth particle hydrodynamics vs Finite element method

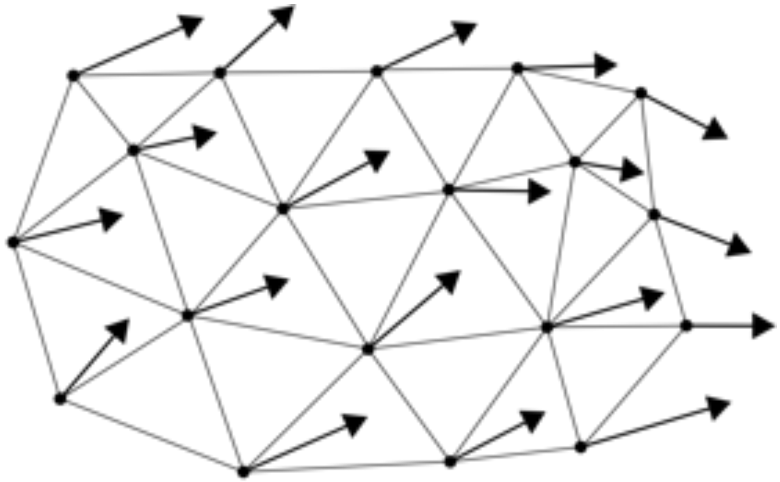




Representation of fluid flow

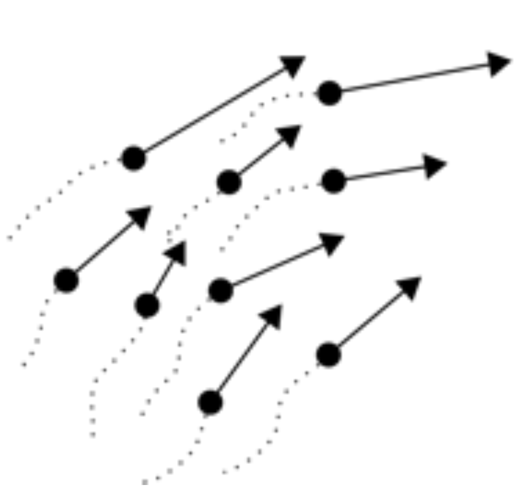
- Lagrangian representation: moving particle position $X(t)$, $X(0) = X_0$
- Eulerian representation: fixed position x, velocity $u(x, t)$

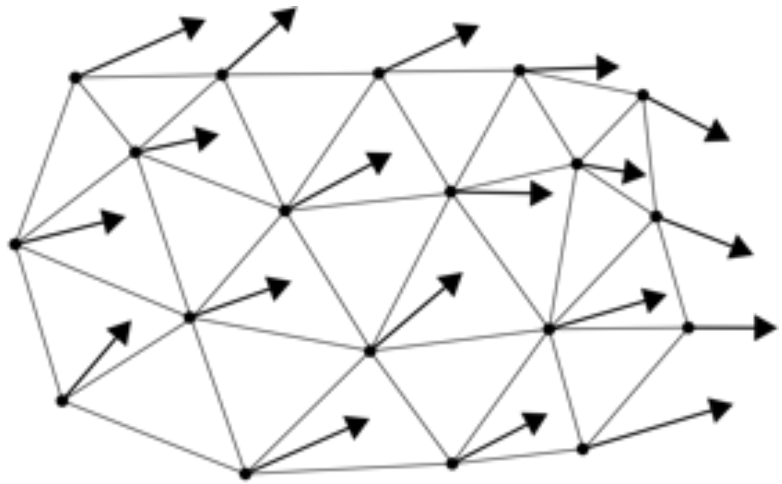




Representation of fluid flow

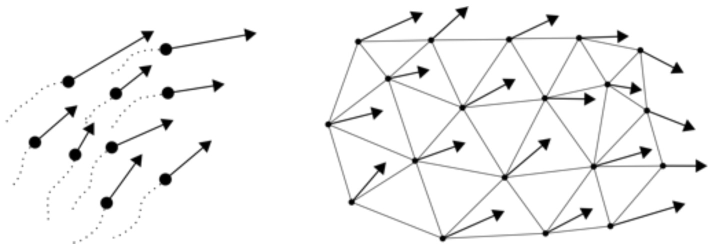
- Lagrangian representation: moving particle position $X(t)$, $X(0) = X_0$
- Eulerian representation: fixed position x, velocity $u(x, t)$
- $u(X(t), t) = \frac{dX}{dt}$





Representation of fluid flow

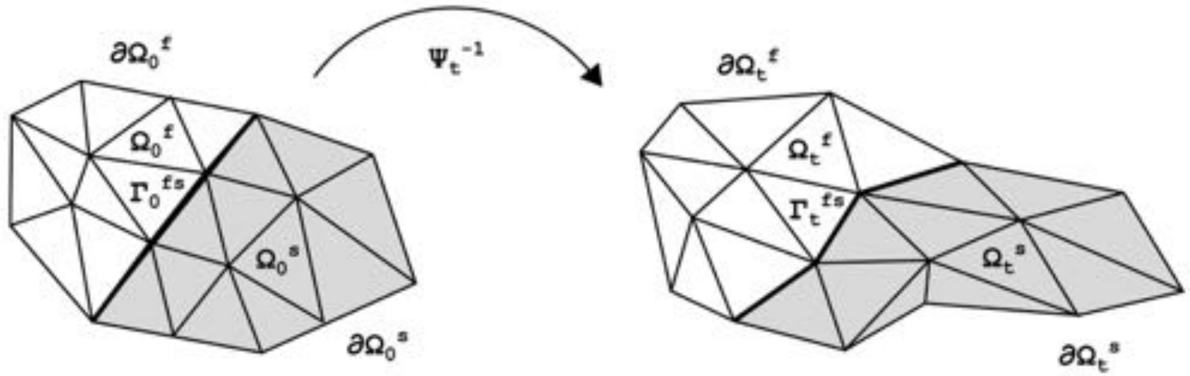
- Lagrangian representation: moving particle position $X(t)$, $X(0) = X_0$
- Eulerian representation: fixed position x, velocity $u(x, t)$
- $u(X(t), t) = \frac{dX}{dt}$
- $\frac{Du}{Dt} = \left(\frac{dX}{dt} \cdot \nabla\right) u + \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} + (u \cdot \nabla) u$
- Acceleration along particle path



Fluid-structure interaction (FSI)

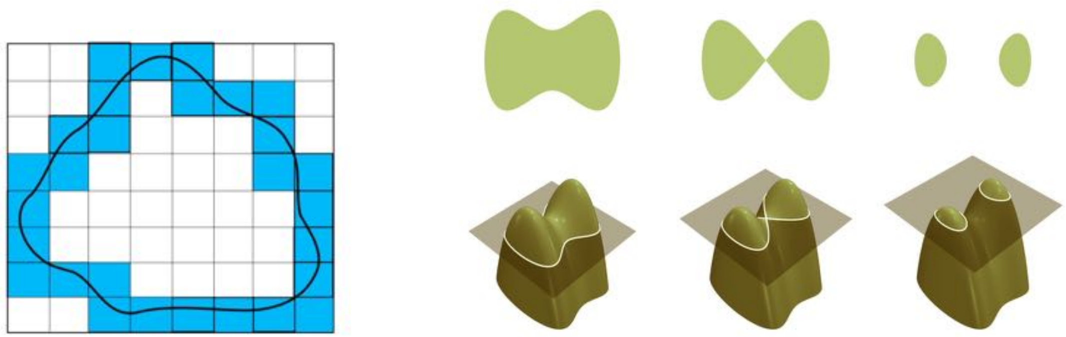
Interface tracking

- Mesh follow deformation
- Explicit interface representation



Interface capturing

- Mesh fixed
- Implicit interface representation



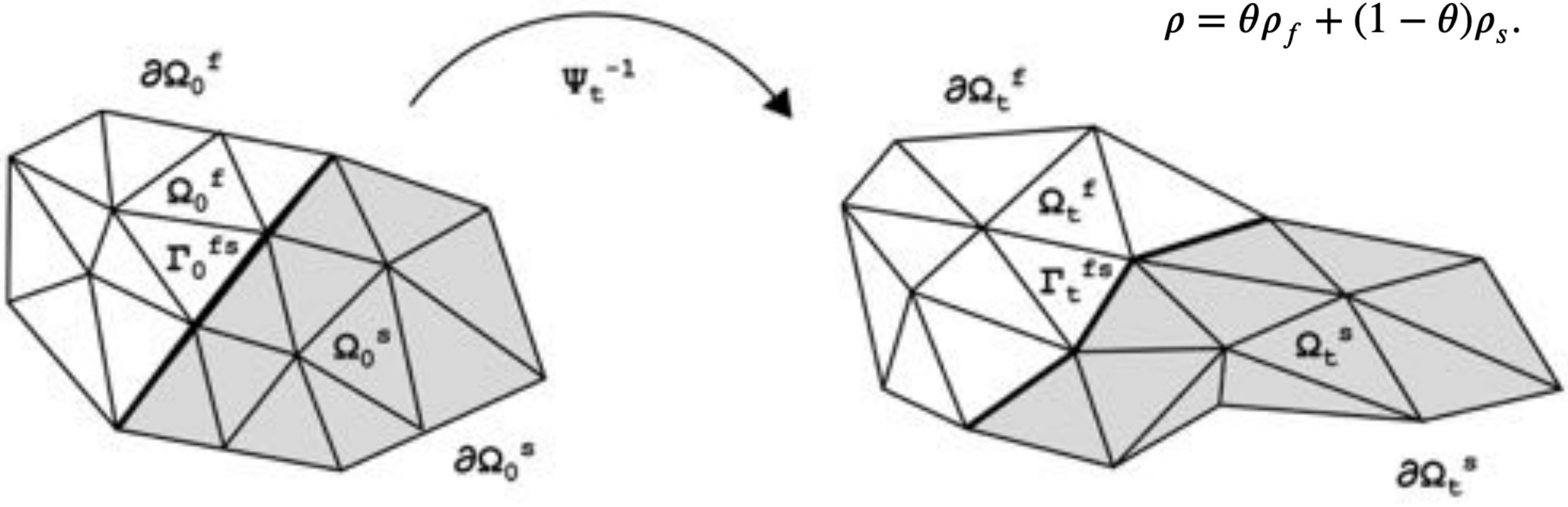
[https://en.wikipedia.org/wiki/Level-set_method#/media/File:Level_set_method.png]

Unified continuum fluid-structure interaction

$$\rho(\dot{\mathbf{u}} + ((\mathbf{u} - \mathbf{m}) \cdot \nabla)\mathbf{u}) + \nabla p - \nabla \cdot \boldsymbol{\tau} = \rho \mathbf{f}, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$
$$\nabla \cdot \mathbf{u} = 0, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$

$$\theta(\mathbf{x}, t) = \begin{cases} 1, & x \in \Omega_t^f, \\ 0, & x \in \Omega_t^s, \end{cases}$$

$$\boldsymbol{\tau} = \theta \boldsymbol{\tau}_f + (1 - \theta)\boldsymbol{\tau}_s,$$
$$\rho = \theta \rho_f + (1 - \theta)\rho_s.$$



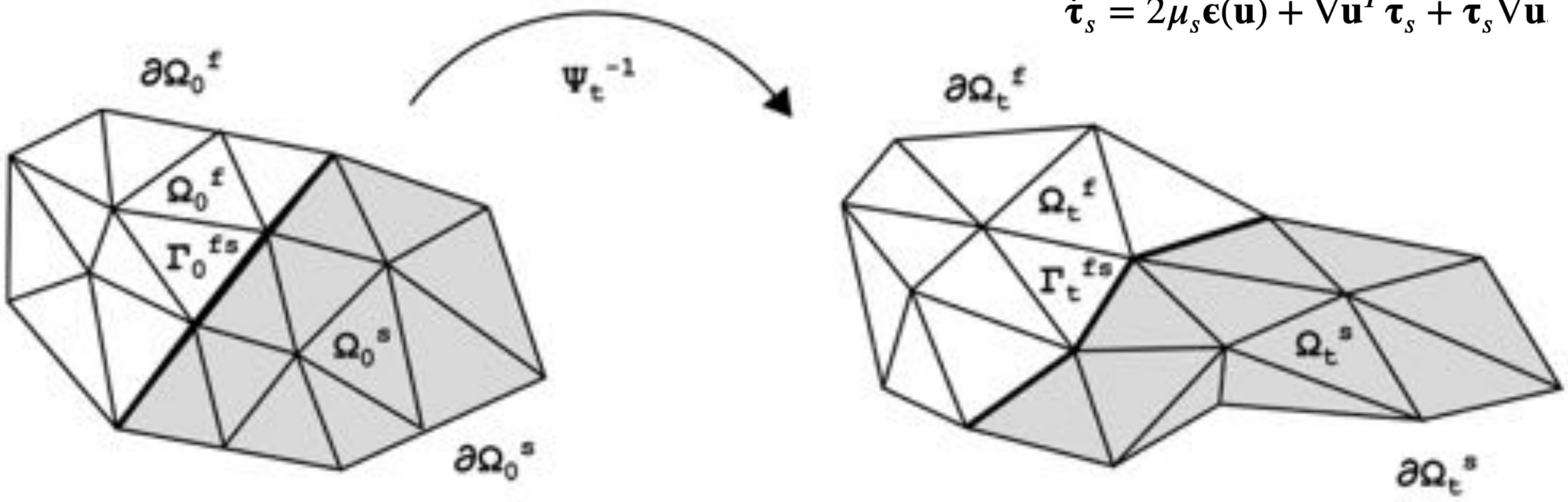
Unified continuum fluid-structure interaction

$$\rho(\dot{\mathbf{u}} + ((\mathbf{u} - \mathbf{m}) \cdot \nabla)\mathbf{u}) + \nabla p - \nabla \cdot \boldsymbol{\tau} = \rho \mathbf{f}, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$

$$\nabla \cdot \mathbf{u} = 0, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$

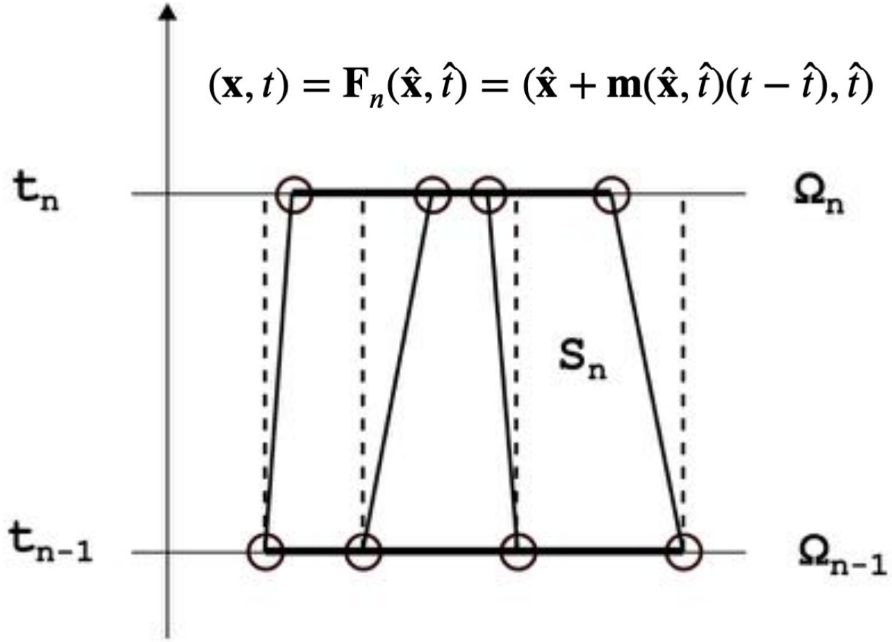
$$\boldsymbol{\tau}_f = 2\mu_f \boldsymbol{\epsilon}(\mathbf{u}), \quad \boldsymbol{\epsilon}(\mathbf{u}) = \frac{1}{2}(\nabla u + \nabla u^T)$$

$$\dot{\boldsymbol{\tau}}_s = 2\mu_s \boldsymbol{\epsilon}(\mathbf{u}) + \nabla \mathbf{u}^T \boldsymbol{\tau}_s + \boldsymbol{\tau}_s \nabla \mathbf{u}$$



Space-time finite element approximation

$$\hat{V}_{h,n}^{(k,m)}(\hat{S}_n) = \{\hat{v}(\mathbf{x},t) \,:\, \hat{v} \in V_h^{(k)}(\Omega_{n-1}) \text{ for each } t \in I_n, \text{ and } \hat{v} \in P^m(I_n) \text{ for each } \mathbf{x}_{n-1} \in \Omega_{n-1}\}$$



$$V_{h,n}^{(k,m)}(S_n) = \{v(\mathbf{x},t) = \hat{v}(\mathbf{F}_n^{-1}(\mathbf{x},t),t) \,:\, \hat{v} \in \hat{V}_{h,n}^{(k,m)}(\hat{S}_n)\}$$

$$W_h^{(k,m)}(Q) = \{v \,:\, v|_{S_n} \in V_{h,n}^{(k,m)}(S_n)\}$$

$$V_h^{(k,m)}(Q) = \{v \in W_h^{(k,m)}(Q) \,:\, v \in C(\bar{Q})\}$$

Space-time finite element approximation

find $\mathbf{U} \in [V_{h,g_D}^{(1,1)}(Q)]^d$ and $P \in W_h^{(1,0)}(Q)$, such that

$$(\rho(\dot{\mathbf{U}} + ((\mathbf{U} - \mathbf{M}) \cdot \nabla)\mathbf{U}), \mathbf{v})_Q + (\mu_f \boldsymbol{\epsilon}(\mathbf{U}), \boldsymbol{\epsilon}(\mathbf{v}))_{Q_f} + (\mathbf{T}, \nabla \mathbf{v})_{Q_s} - (P, \nabla \cdot \mathbf{v})_Q + (q, \nabla \cdot \mathbf{U})_Q$$
$$+(\mathbf{g}_N, \mathbf{v})_{\partial Q_N} + SD_{\boldsymbol{\delta}}(\mathbf{U}, \mathbf{M}, P; \mathbf{v}, q) = (\rho \mathbf{f}, \mathbf{v})_Q,$$

for all $\mathbf{v} \in [W_{h,0}^{(1,0)}(Q)]^d$ and $q \in W_h^{(1,0)}(Q)$.

$$W_h^{(k,m)}(Q) = \{v : v|_{S_n} \in V_{h,n}^{(k,m)}(S_n)\}$$

$$V_h^{(k,m)}(Q) = \{v \in W_h^{(k,m)}(Q) : v \in C(\bar{Q})\}$$

Mesh smoothing – elastic analogy

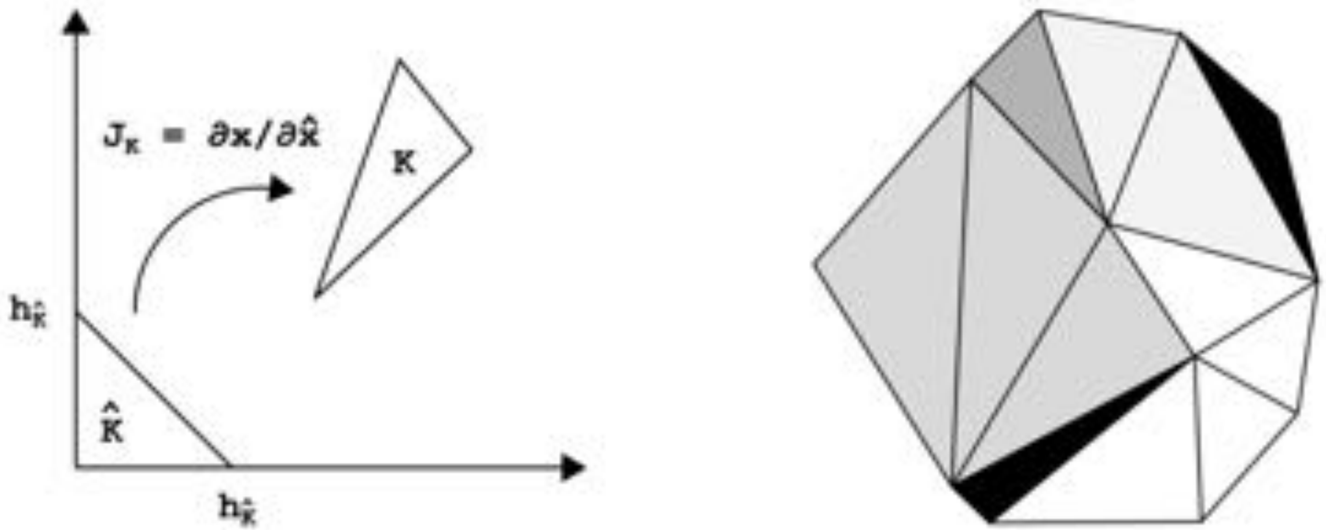
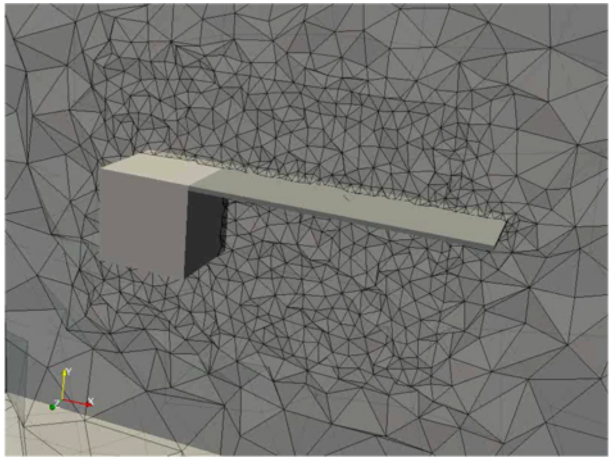


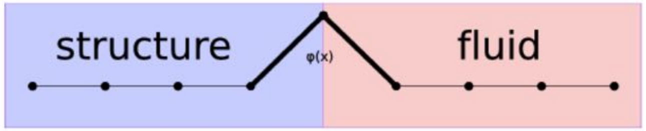
FIGURE 3 Map from a reference triangle $\hat{K}$ to an element K and its associated Jacobian $\mathbf{J}_K$ (left), and a triangle mesh with each element K coloured based on its quality measure $Q(K)$, with darker colours for higher values (right).

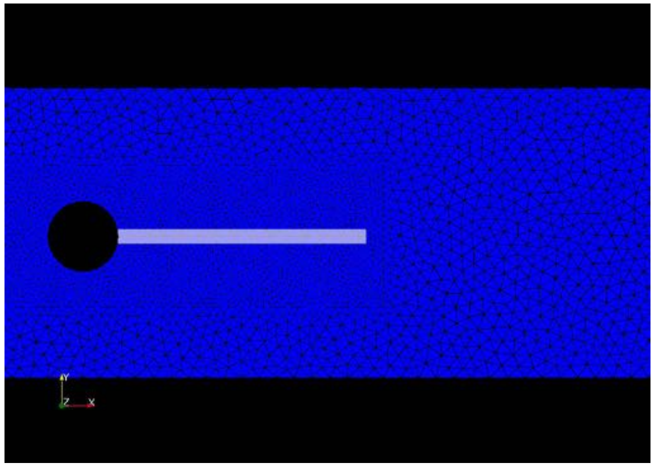
Unified Continuum fluid-structure interaction



ALE-FEM method

- Conforming fluid-solid mesh
- Mesh smoothing





[J. Hoffman, J. Jansson, M. Stöckli, M3AS, Vol.21(3), 2011.]

Contact model

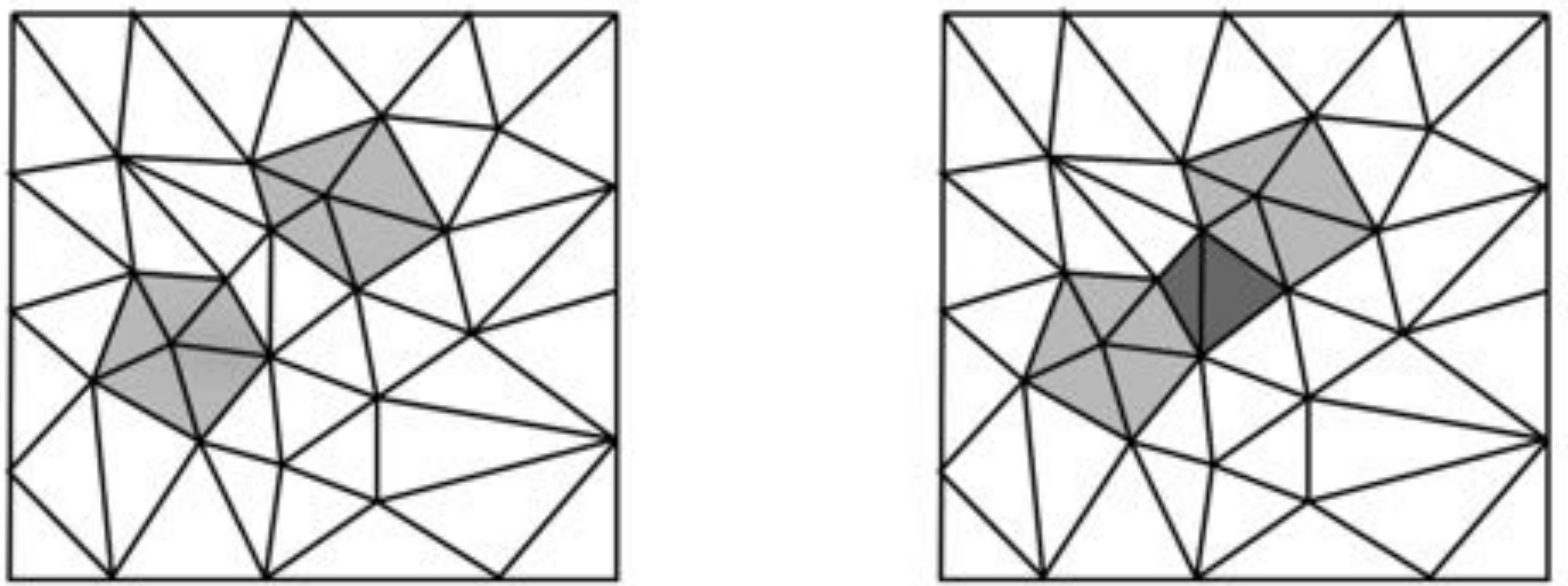
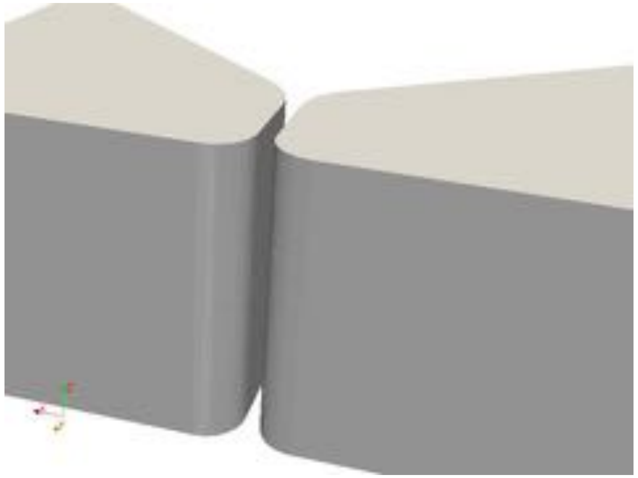


FIGURE 4 Illustration of the UC-FSI contact model, where collision is detected between two structure domains marked by a light shade of grey (left), which then activates a phase change in the contact region marked in a darker shade of grey (right).

Unified Continuum contact model

[Spühler, Degirmenci, Jansson, Hoffman, KTH PhD thesis, 2018]

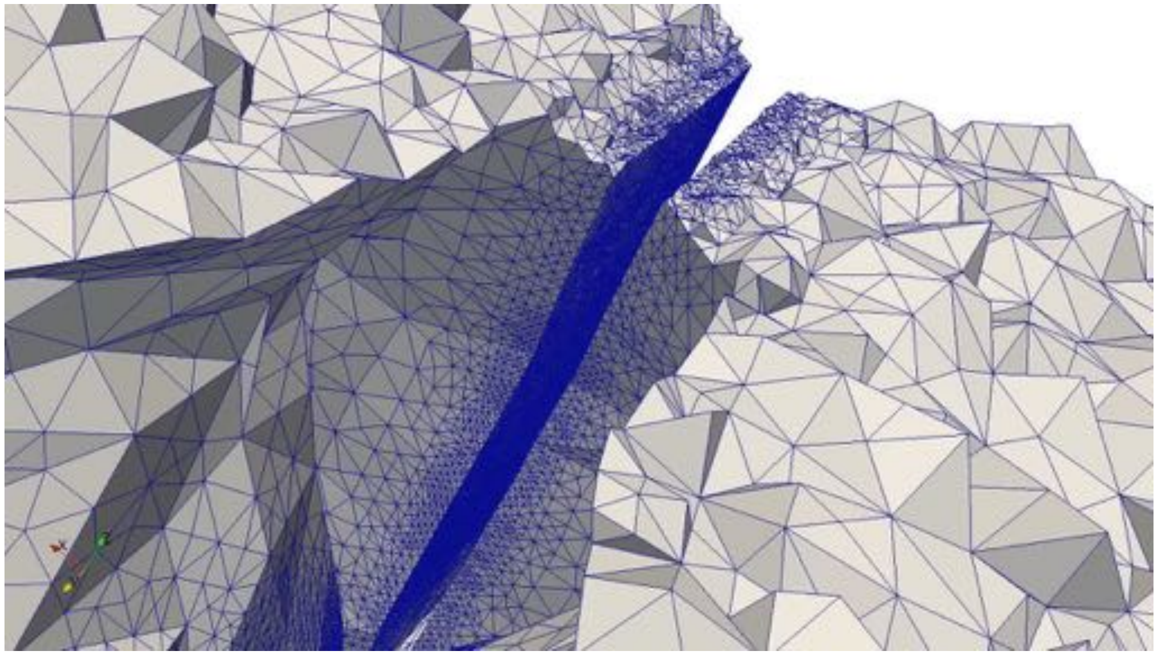


Algorithm 1 *Contact algorithm*

1. *Mark all cells K as non-contact.*
2. *Solve the Eikonal equation $|\nabla D| = 1$ using a artificial viscosity stabilized cG(1) method with $D = 0$ on the boundary Γ and in the structure subdomain Ω_s for the distance $D = D(x)$.*
3. *Compute $|\nabla D|$, and define the medial axis M as: $M = \{x \mid |\nabla D(x)| \leq \gamma\}$, with the threshold parameter $\gamma < 1$.*
4. *Define the contact medial axis: $\hat{M} = \{x \mid x \in M, x \notin \Omega_s, D(x) < \alpha\hat{h}\}$, with $\hat{h}$ the minimum cell size in the mesh.*
5. *Solve the Eikonal equation $|\nabla D_{\hat{M}}| = 1$ using an artificial viscosity stabilized cG(1) method with $D_{\hat{M}} = 0$ on the contact medial axis $\hat{M}$ for the distance from $\hat{M}$ $D_{\hat{M}} = D_{\hat{M}}(x)$.*
6. *Mark all fluid cells as contact which fulfill: $C = \{x \mid D_{\hat{M}} \leq \beta\hat{h}\}$*

FSI simulation of vocal folds

[N.C.Degirmenci, et al, Proc. Interspeech, 2017]



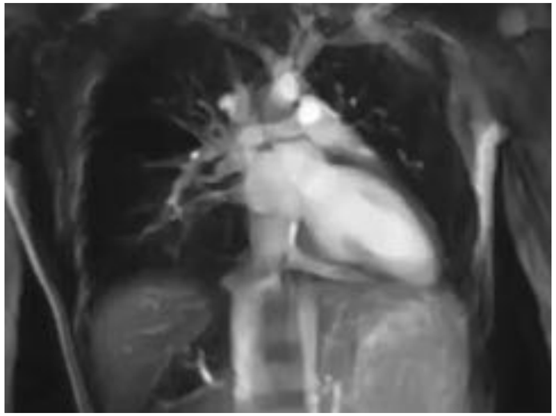
Navier-Stokes Brinkman model

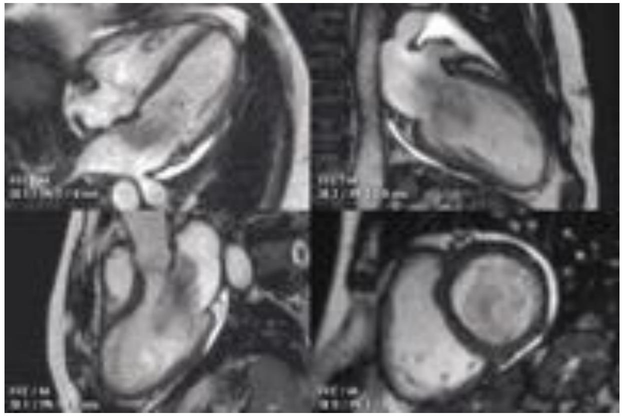
$$
\begin{aligned}
&\rho\left(\frac{\partial}{\partial t}\boldsymbol{u}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)-\nabla\cdot\boldsymbol{\sigma}(\boldsymbol{u},p)+\frac{\mu}{K}(\boldsymbol{u}-\boldsymbol{u}_s)=0 && \text{in } \mathbb{R}^+\times\Omega && (1)\\
&\nabla\cdot\boldsymbol{u}=0 && \text{in } \mathbb{R}^+\times\Omega && (2)\\
&\boldsymbol{u}=0 && \text{on } \Gamma_{\text{noslip}} && (3)\\
&\boldsymbol{\sigma}\boldsymbol{n}-\rho\beta(\boldsymbol{u}\cdot\boldsymbol{n})_{-}\boldsymbol{u}=\boldsymbol{h} && \text{on } \Gamma_{\text{outflow}} && (4)\\
&\boldsymbol{u}=\boldsymbol{g} && \text{on } \Gamma_{\text{inflow}} && (5)\\
&\boldsymbol{u}\big|_{t=0}=\boldsymbol{u}_0 && && (6)
\end{aligned}
$$

Here $p(\boldsymbol{x},t)$ and $\boldsymbol{u}(\boldsymbol{x},t)$ represent the fluid pressure and the flow velocity respectively, μ is the dynamic viscosity and ρ the density. The volume penalization term $\frac{\mu}{K(t,\boldsymbol{x})}\boldsymbol{u}(t,\boldsymbol{x})$ is commonly known as *Darcy drag* which is characterized by the permeability $K(t,\boldsymbol{x})$.

The left ventricle of the heart

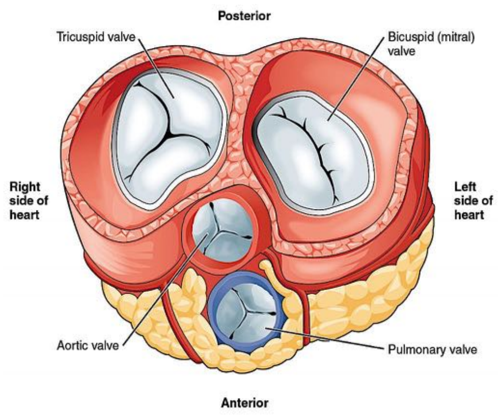
- The LV pumps the blood through the aorta and experiences the highest forces, and the most problems.
- In vivo imaging include ultrasound, MRI, CT.
- Tissue is easier to image than the blood flow, blood pressure.
- Two phases: **systole** (contract) and **diastole** (relax)

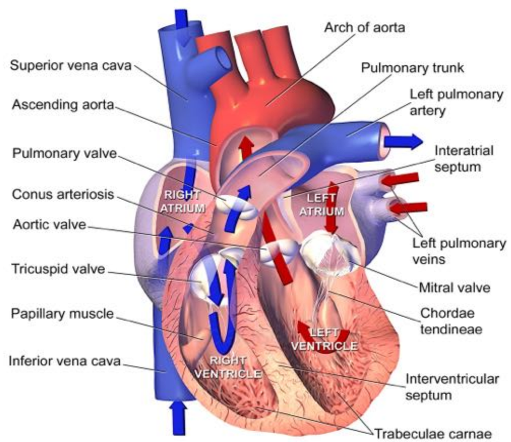




The left ventricle of the heart

- **Regurgitation** is the name for leaking heart valves.
- **Stenosis** is the term for a valve that doesn't open properly.
- **Thrombosis** is the formation of a blood clot, which may block the blood supply to cause a heart attack or stroke.

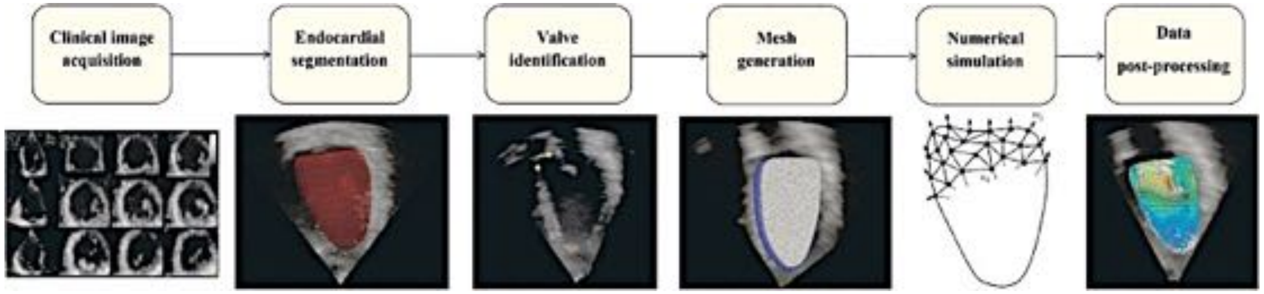




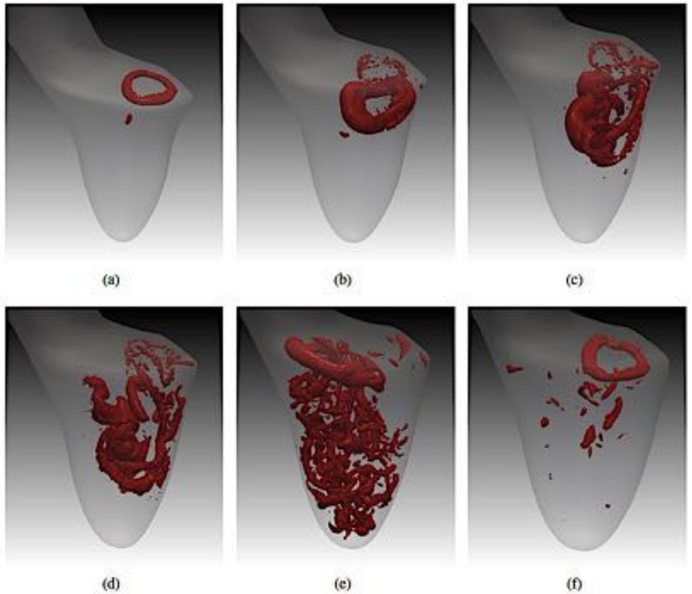
Sectional Anatomy of the Heart

Patient-specific LV blood flow simulation

- 4DTTE images, semi-automated segmentation
- Valves identified by sonographer
- Mesh generation by ANSA (BETA CAE Systems)
- >200 patients processed to date.

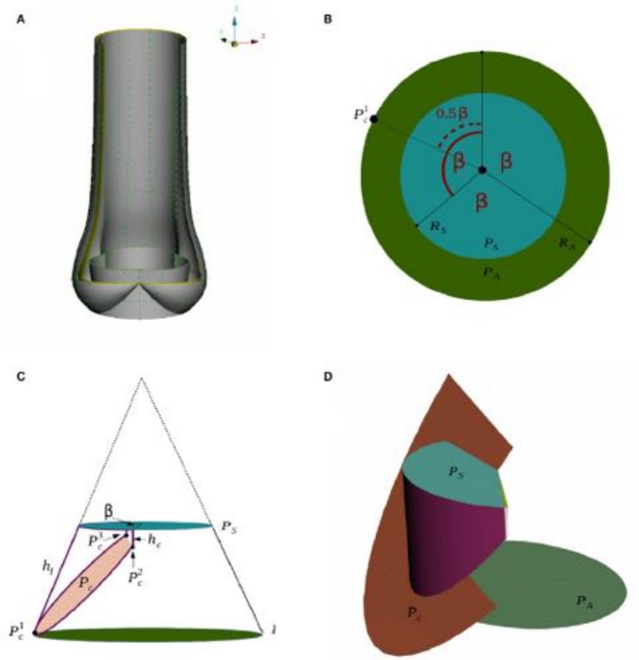


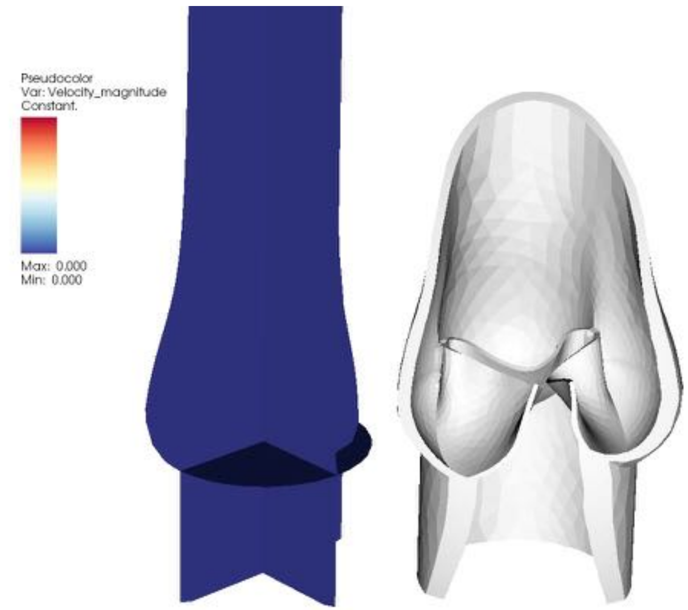
Patient-specific LV blood flow simulation



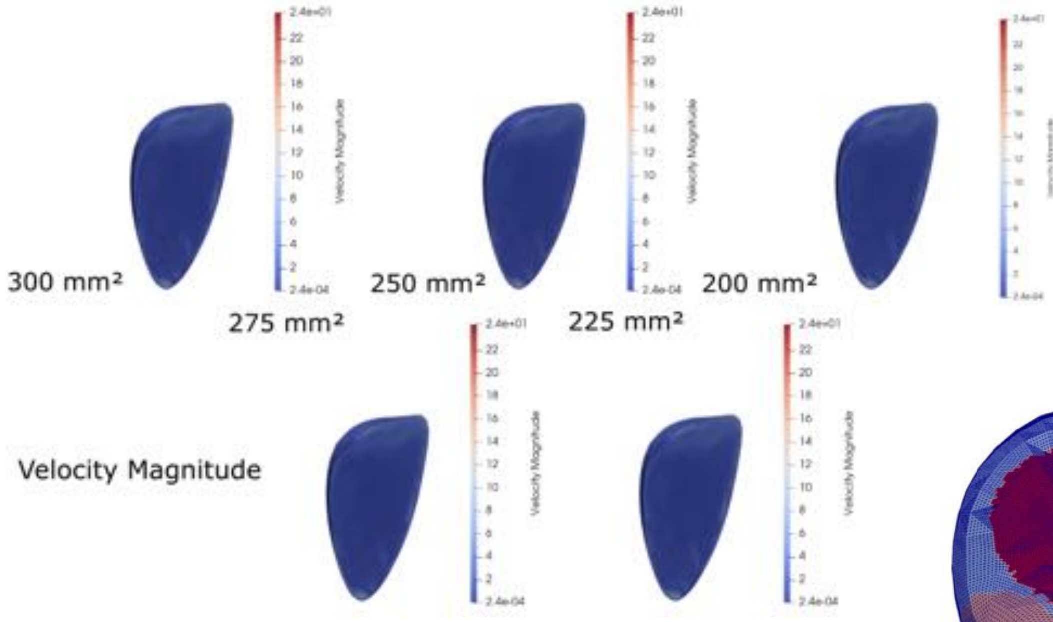
[Visualization by evolving lambda2 isosurfaces]

Systole: FSI simulation of 3D aortic valves

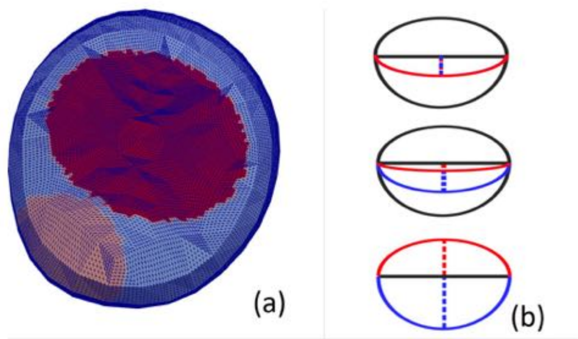




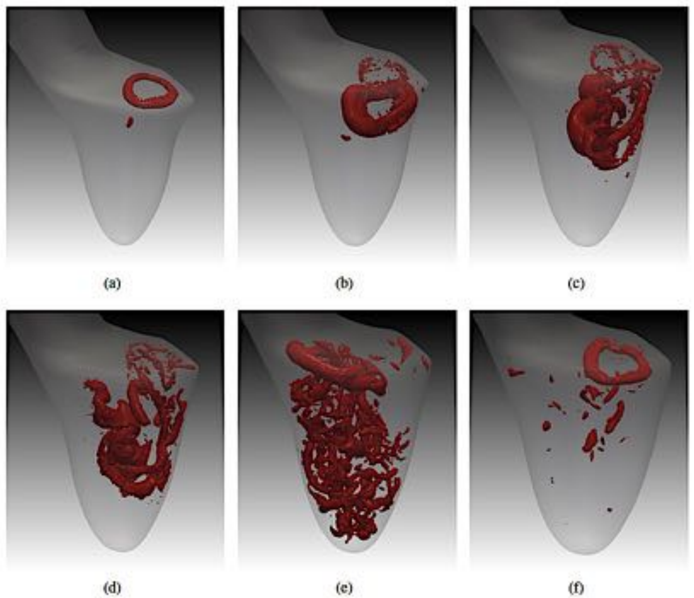
Diastole: 2D mitral valve model



[Velocity for different MV areas – modeling stenosis]

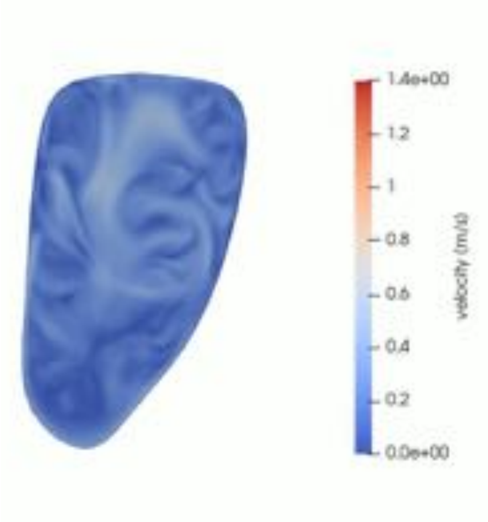


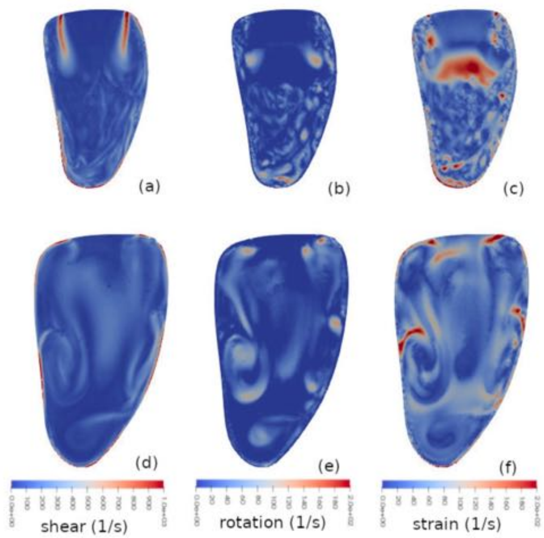
Analysis of LV turbulent flow structures



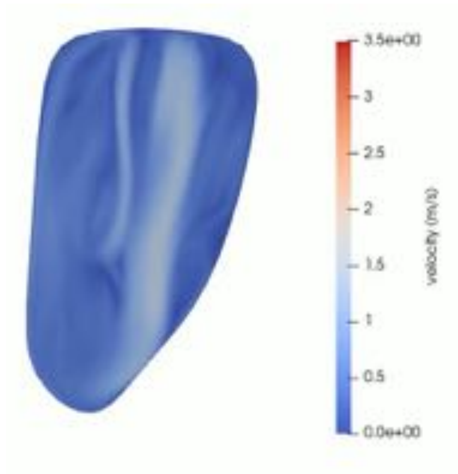
[Visualization by evolving lambda2 isosurfaces]

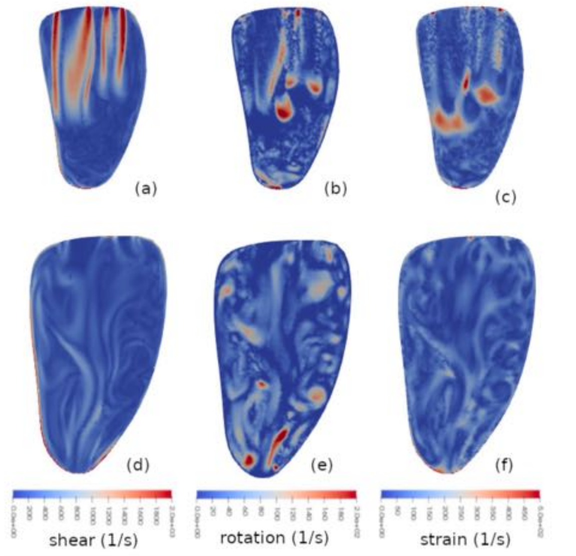
LV simulation (local Frobenius norms)



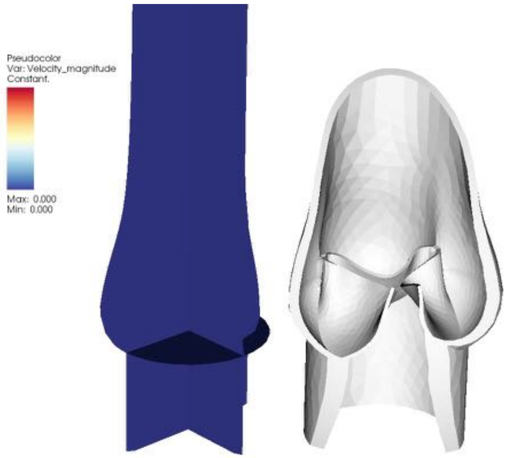


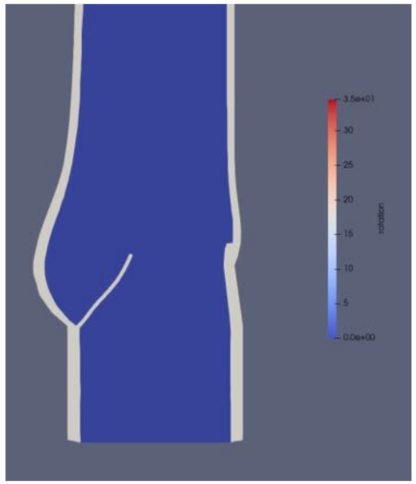
LV simulation (local Frobenius norms)



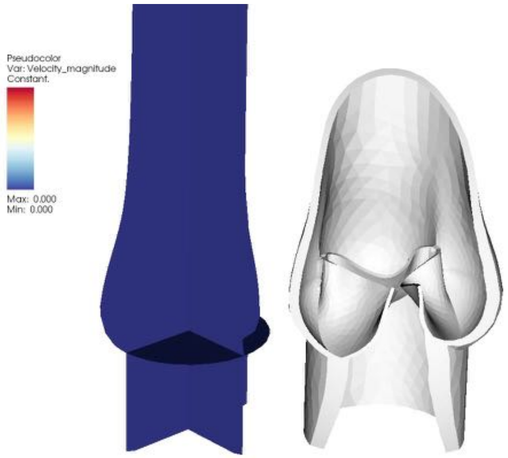


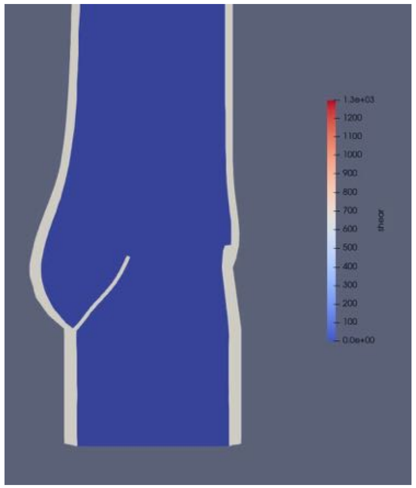
Aortic valve - rotation



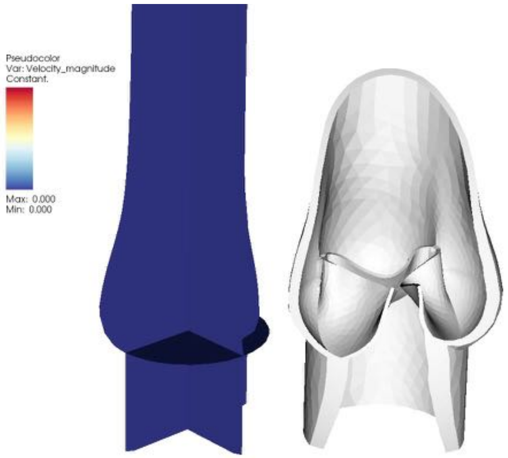


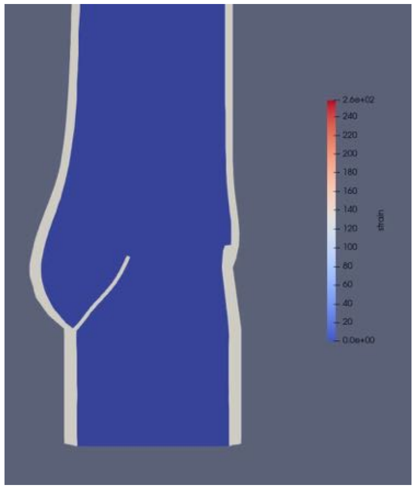
Aortic valve - shear



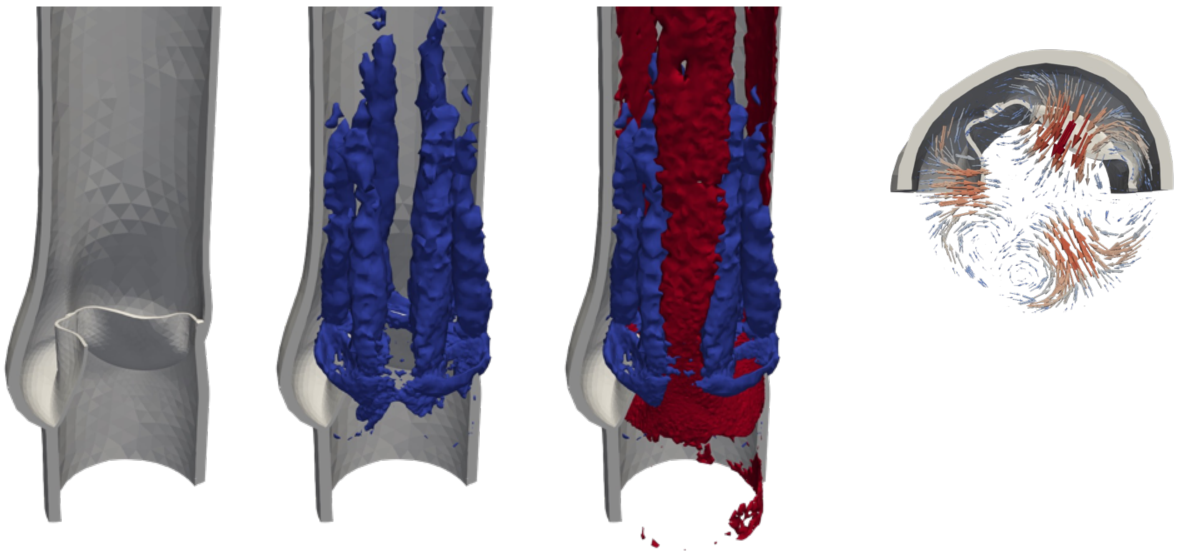


Aortic valve- strain





Aortic valve: rotation, rotation + shear



Models for shear-induced platelet activation

- The triple decomposition of the velocity gradient tensor separates the flow into the components of pure strain, rigid body rotation and shear.
- CFD models for shear-induced platelet activation are typically based on the strain rate tensor.
- The strain rate tensor does not distinguish between strain and shear flow.
- The triple decomposition offers a method to separate shear from strain, which can lead to more precise models for shear-induced platelet activation.

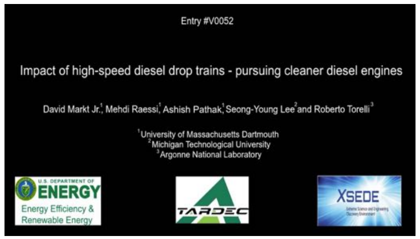
Nasa CFD Vision 2030

CFD TECHNOLOGY GAPS AND IMPEDIMENTS

- Effective Utilization of High-Performance Computing (HPC)
- Unsteady Turbulent Flow Simulations Including Transition to Turbulence and Separation
- Autonomous and Reliable CFD Simulation
- Knowledge Extraction and Visualization
- Multidisciplinary/Multiphysics Simulations and Frameworks

Multiphase flow

- https://gfm.aps.org/meetings/dfd-2020/5f5ec1d2199e4c091e67bd66



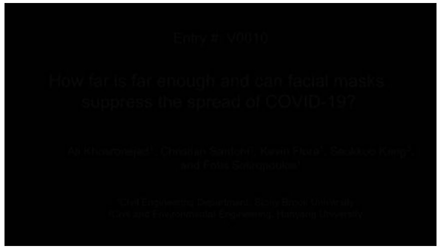
- https://gfm.aps.org/meetings/dfd-2020/5f5f6542199e4c091e67be2a

Multiphase flow

- https://gfm.aps.org/meetings/dfd-2020/5f5fc002199e4c091e67bf49

Gallery of fluid motion

Ex. COVID spread with and without mask: differential equations vs deep learning



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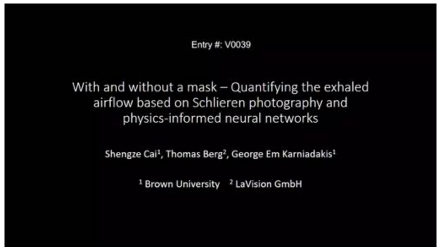

https://gfm.aps.org/meetings/dfd-2020/5f5e6ef8199e4c091e67bd37

Credits

Gallery of fluid motion (American Physical Society)

- https://gfm.aps.org

Album of fluid flow (Milton Van Dyke)

- https://en.wikipedia.org/wiki/An_Album_of_Fluid_Motion
- https://www.abebooks.com/9780915760022/Album-Fluid-Motion-Milton-Dyke-0915760029/plp

References

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