4180/5180: Reinforcement Learning and Sequential Decision Making (Spring 2025) Robert Platt
Northeastern University Due Jan 30, 2025

Exercise 2: Markov Decision Processes (MDPs)

Please remember the following policies:

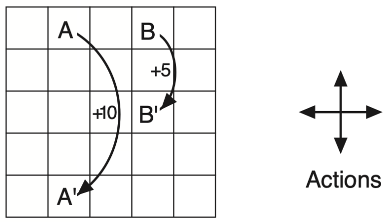
- Exercise due at **11:59 PM EST Jan 30, 2025**.
- Submissions should be made electronically on Canvas. Please ensure that your solutions for both the written and programming parts are present. You can upload multiple files in a single submission, or you can zip them into a single file. You can make as many submissions as you wish, but only the latest one will be considered.
- For **Written** questions, solutions may be handwritten or typeset. If you write your answers by hand and submit images/scans of them, please please ensure legibility and order them correctly in a single PDF file.
- The PDF file should also include the figures from the **Plot** questions.
- For both **Plot** and **Code** questions, submit your source code in Jupyter Notebook (.ipynb file) along with reasonable comments of your implementation. Please make sure the code runs correctly.
- You are welcome to discuss these problems with other students in the class, but you must understand and write up the solution and code yourself. Also, you *must* list the names of all those (if any) with whom you discussed your answers at the top of your PDF solutions page.
- Each exercise may be handed in up to two days late (24-hour period), penalized by 10% per day late. Submissions later than two days will not be accepted.
- Contact the teaching staff if there are medical or other extenuating circumstances that we should be aware of.
- **Notations: RL2e is short for the reinforcement learning book 2nd edition. x.x means the Exercise x.x in the book.**

1. **2 point.** *Formulating an MDP.*
   Written: It is instructive to formally define an MDP at least once, in particular, the dynamics function. Consider the four-rooms domain from Ex0.
   (a) What are the state and action spaces S, A?
   (b) Consider the dynamics function $p(s', r|s, a)$. Approximately how many rows in this conditional probability table have non-zero probability?
   (c) **Code:** Provide codes to generate the full table of non-zero $p(s', r|s, a)$ values. Notes:
      - Please include both the code and the generated table of values.
      - Recall: The goal state is at $(10, 10)$. Any action from there teleports the agent back to $(0, 0)$.

2. **2 point.** (RL2e 3.8, 3.9) *Discounted return.*
   Written:
   (a) Suppose $\gamma = 0.5$ and the following sequence of rewards is received:
   $R_1 = -1, R_2 = 2, R_3 = 4, R_4 = 3, R_5 = 2$
   with $T = 5$. What are $G_0, G_1, \ldots, G_5$?
   (b) Suppose $\gamma = 0.9$ and the reward sequence is $R_1 = 2$ followed by an infinite sequence of '8's.
   What are G_1 and G_0?

3. **1 point.** (RL2e 3.6, 3.7) *The RL objective.*
   Written: Imagine that you are designing a robot to run a maze. You decide to give it a reward of +1 for escaping from the maze and a reward of zero at all other times. The task seems to break down naturally into episodes, where each episode corresponds to a single run through the maze and ends when agent escapes from the maze. The goal is to maximize expected total reward (Equation 3.7). There is no discounting, i.e. $\gamma = 1$. After running the learning agent for a while, you find that it is showing no improvement in escaping from the maze. What is going wrong? Have you effectively communicated to the agent what you want it to achieve?

4. **1 point.** **[5180]** (RL2e 3.15, 3.16) *Modifying the reward function.*
Written: In the gridworld example (Figure 3.2 in RL2e, see below), rewards are positive for goals, negative for running into the edge of the world, and zero the rest of the time. Are the signs of these rewards important, or only the intervals between them? Prove, using Equation 3.8, that adding a constant c to all the rewards adds a constant, v_c, to the values of all states, and thus does not affect the relative values of any states under any policies. What is v_c in terms of c and γ?



3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

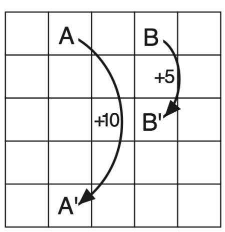
Figure 3.2: Gridworld example: exceptional reward dynamics (left) and state-value function for the equiprobable random policy (right).

5. **2 point.** (RL2e 3.14) *Bellman equation.*
Written:

   (a) The Bellman equation (Equation 3.14) must hold for each state for the value function v_π shown in Figure 3.2 (right) (see above) of Example 3.5. Show numerically that this equation holds for the state positioned one bin to the right of the central state, valued at $+0.4$, with respect to its four neighboring states, valued at $+1.9, +0.7, -0.4, -0.6$. (These numbers are accurate only to one decimal place.) Note that Figure 3.2 (right) (see above) is the value function for the equiprobable random policy, $\gamma = 0.9$.

   (b) The Bellman equation holds for *all* policies, including optimal policies. Consider v_* and π_* shown in Figure 3.5 (middle, right respectively) (See below). Similar to the previous part, show numerically that the Bellman equation holds for for the state positioned one bin to the right of the central state, valued at $+16.0$, with respect to its four neighboring states, for the optimal policy π_* shown in Figure 3.5 (right) (see below).

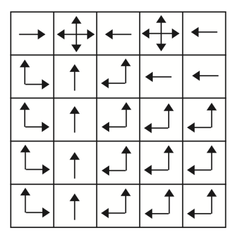
If the Bellman equation is unclear to you, you should practice this question again for other states.



Gridworld

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

v_*



π_*

Figure 3.5: Optimal solutions to the gridworld example.

6. **2 points.** *Solving for the value function.*
Written: Another way to solve for the value function is to write out the Bellman equation for each state, view it as a system of linear equations, and then solve for the unknowns (the value of each state). Consider a particularly simple MDP, the 2-state recycling robot in Example 3.3.

   (a) Expand the Bellman equation for the 2 states in the recycling robot, for an arbitrary policy $\pi(a|s)$, discount factor γ, and domain parameters $\alpha, \beta, r_{\texttt{search}}, r_{\texttt{wait}}$ as described in the example.

   (b) You should now have two linear equations involving two unknowns, $v(\texttt{high})$ and $v(\texttt{low})$, as well as involving the policy $\pi(a|s)$, γ, and the domain parameters. Let $\alpha = 0.8, \beta = 0.6, \gamma = 0.9, r_{\texttt{search}} = 10, r_{\texttt{wait}} = 3$.
   Consider the policy $\pi(\texttt{search} \,|\, \texttt{high}) = 1$, $\pi(\texttt{wait} \,|\, \texttt{low}) = 0.25$, and $\pi(\texttt{recharge} \,|\, \texttt{low}) = 0.75$.
   Find the value function for this policy, i.e., solve the equations for the values of $v(\texttt{high})$ and $v(\texttt{low})$.
   Check that your solution satisfies the Bellman equation.