UNIVERSITE PARIS – CERGY

CY Tech. Département Mathématiques

Option INGENIERIE FINANCIERE. Option ACTUARIAT.

MODEL CALIBRATION AND SIMULATION

TP1 Calibration de volatilité implicite dans le modèle de Black et Scholes. Smile de volatilité.

Adnane EL KASMI

TP1

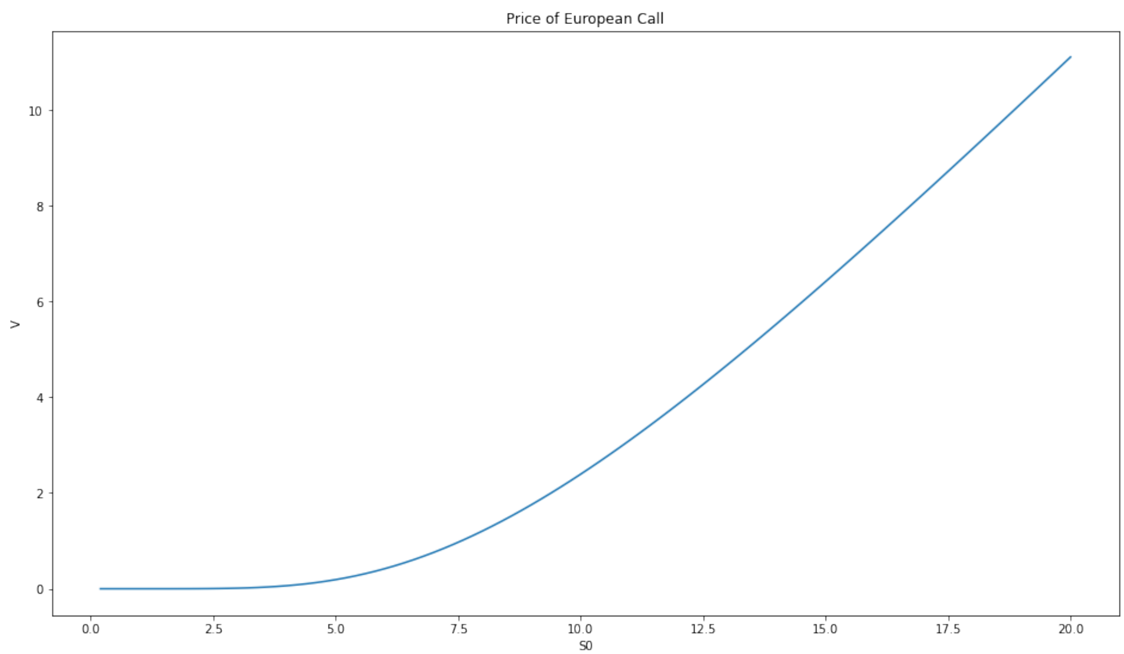
[1]:
```python
################### Partie I ###################

#-------------------------------#
# Chargement des modules Python #
#-------------------------------#

import math as mt
import random as rm
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from pylab import *
from mpl_toolkits.mplot3d import Axes3D
from copy import copy, deepcopy

#-------------------#
# Valeurs initiales #
#-------------------#

t=0
K=10
T=1
r=0.1
sigma=0.5
S0=0.2

#-----------------#
# Données tableau #
#-----------------#

K_liste=[5125,5225,5325,5425,5525,5625,5725,5825]
M_liste=[475,405,340,280.5,226,179.5,139,105]

#-----------------------------------#
# Fonction de répartition loi N(0,1) #
#-----------------------------------#
```

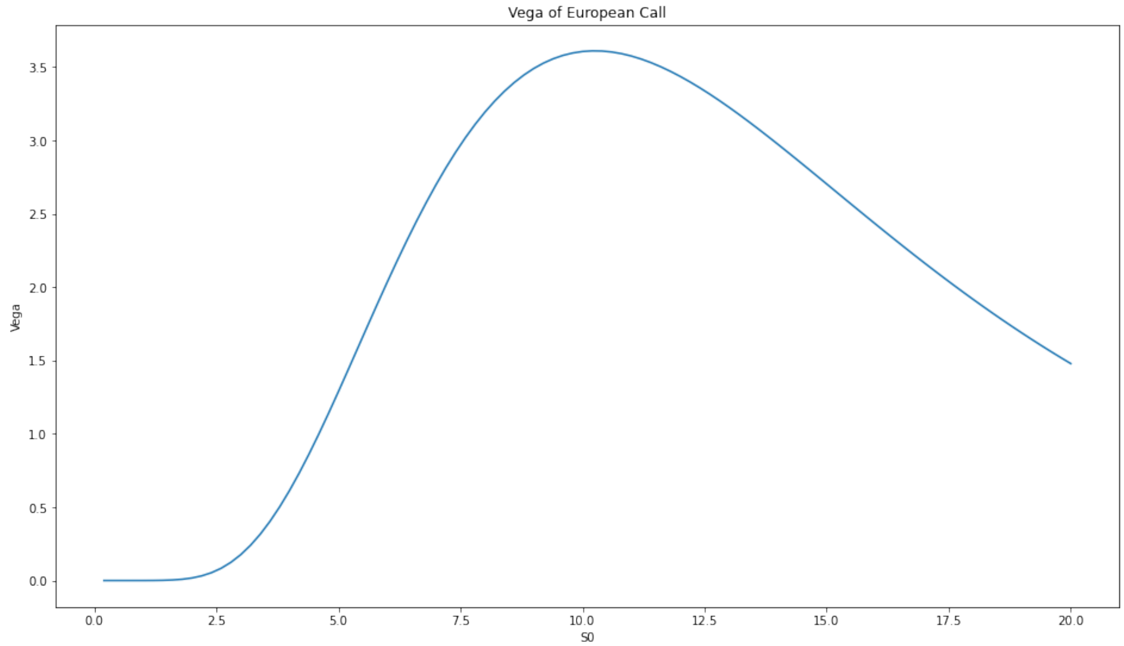
```python
def repartition(x):
    return (1/2)*(1+mt.erf(x/mt.sqrt(2)))

#--------------------#
# Fonction d1 et d2 #
#--------------------#

def d1(t, S, K, T, r, sigma):
    return (mt.log(S/K)+(r+(1/2)*mt.pow(sigma,2))*(T-t))/(sigma*mt.sqrt(T-t))

def d2(t, S, K, T, r, sigma):
    return (mt.log(S/K)+(r-(1/2)*mt.pow(sigma,2))*(T-t))/(sigma*mt.sqrt(T-t))

#-------------------------------#
# Fonction Call Black & Scholes #
#-------------------------------#

def Call_BS(t, S, K, T, r, sigma):
    if t==T:
        return max(S-K, 0)
    else:
        return S*repartition(d1(t, S, K, T, r, sigma)) - K*mt.
↪exp(-r*(T-t))*repartition(d2(t, S, K, T, r, sigma))

#-------------------------------#
# Fonction Vega Black & Scholes #
#-------------------------------#

def Vega_BS(t, S, K, T, r, sigma):
    return ((S*mt.sqrt(T-t))/mt.sqrt(2*np.pi))*mt.exp(-(mt.pow(d1(t, S, K, T,␣
↪r, sigma),2)/2))
```

[2]:
```python
#-------------------------------#
#            Tests              #
#-------------------------------#

S0_liste = []
Call_Test = []
Vega_Test = []

for i in range(1, 101):
    S0_liste.append(S0*i)
    Call_Test.append(Call_BS(t, S0_liste[i-1], K, T, r, sigma))
    Vega_Test.append(Vega_BS(t, S0_liste[i-1], K, T, r, sigma))
```

[3]:

```
#------------------------------#
#            Figures           #
#------------------------------#

plt.rcParams["figure.figsize"]=[16,9]
plt.plot(S0_liste, Call_Test)
plt.xlabel("S0")
plt.ylabel("V")
plt.title("Price of European Call")
plt.show()

plt.plot(S0_liste, Vega_Test)
plt.xlabel("S0")
plt.ylabel("Vega")
plt.title("Vega of European Call")
plt.show()
```

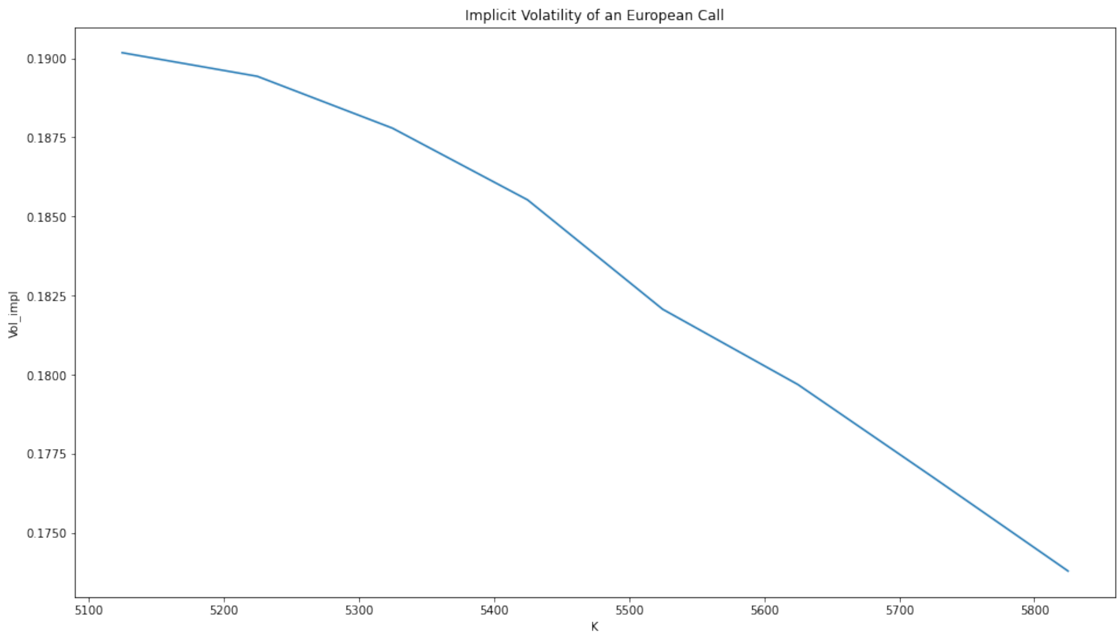




[4]:
```
#-------------------#
# Valeurs initiales #
#-------------------#

t=0
r=0.05
epsilon=mt.pow(10,-3)
T=4/12
S0= 5430.3
volatilite_implicite_2D=[]
sigma_liste=[]

#-------------------#
#     Données       #
#-------------------#

M_liste=[475,405,340,280.5,226,179.5,139,105]
T_liste = np.linspace(0.05, 3, 8)
K_liste=[5125,5225,5325,5425,5525,5625,5725,5825]

#-------------------#
#     Fonction F    #
#-------------------#

def F(V_M, t, S, K, T, r, sigma):
    return Call_BS(t, S, K, T, r, sigma) - V_M
```

```
#-------------------------------#
#     Volatilité implicite 2D    #
#-------------------------------#

#n=1

for i in range(len(M_liste)):
    if ((M_liste[i] < S0) and (M_liste[i] >= max(S0-K_liste[i]*mt.exp(-r*T),␣
↪0))):
        sigma_liste.append(mt.sqrt(2*abs(mt.log(S0/K_liste[i]) + r*T)/T))

        while abs(F(M_liste[i], t, S0, K_liste[i], T, r, sigma_liste[-1])) >␣
↪epsilon:
            sigma_liste.append(sigma_liste[-1] - F(M_liste[i], t, S0,␣
↪K_liste[i], T, r, sigma_liste[-1])/Vega_BS(t, S0, K_liste[i], T, r,␣
↪sigma_liste[-1]))

        volatilite_implicite_2D.append(sigma_liste[-1])
    else:
        volatilite_implicite_2D.append(0)
```

[5]:
```
#------------------------------#
#            Figure             #
#------------------------------#

plt.rcParams["figure.figsize"]=[16,9]
plt.plot(K_liste, volatilite_implicite_2D)
plt.xlabel("K")
plt.ylabel("Vol_impl")
plt.title("Implicit Volatility of an European Call")
plt.show()
```

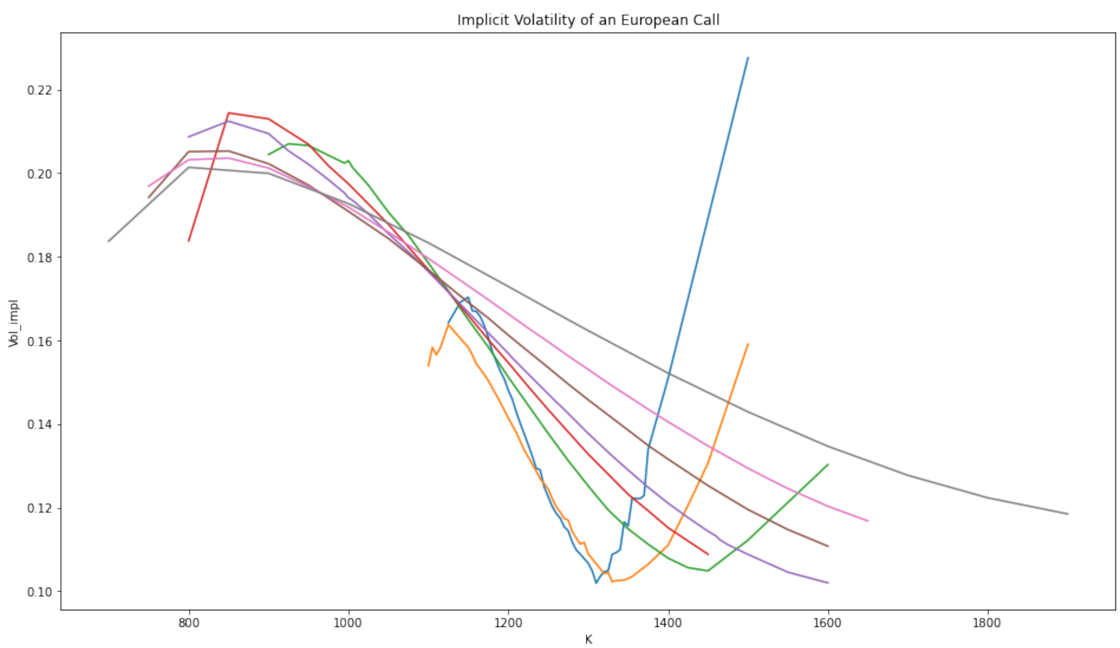


[6]:
```
################### Partie II ###################



#-------------------------------#
#       Importer la data        #
#-------------------------------#

data = pd.read_csv("Calibration_TP1_data.csv", sep =',')

#-------------------#
# Valeurs initiales #
#-------------------#

r = 0.046
q = 0.02
S0 = 1260.36

V_Call_Market = []
V_Put_Market = []
S0_discounted = []


for i in range(len(data[' T'])):
    V_Call_Market.append((data['Cb'][i] + data['Ca'][i])/2)
    V_Put_Market.append((data['Pb'][i] + data['Pa'][i])/2)
    S0_discounted.append(S0*mt.exp(-q*data[' T'][i]))
```

```
data['V_Call_Market'] = V_Call_Market
data['V_Put_Market'] = V_Put_Market
data['S0_discounted'] = S0_discounted

T_liste_data = data[' T'].unique()

#--------------------#
#   Corps du code    #
#--------------------#

Matrix_vol_impl = []
K_Matrix = []
for j in range(len(T_liste_data)):
    volatilite_implicite=[]
    K_sub_list = []
    for i in range(len(data['V_Call_Market'])):
        if data[' T'][i] == T_liste_data[j]:
            if ((data['V_Call_Market'][i] < data['S0_discounted'][i]) and␣
 ↪(data['V_Call_Market'][i] >= max(data['S0_discounted'][i]-data['K'][i]*mt.
 ↪exp(-(data['r'][i]/100)*T_liste_data[j]), 0))):
                sigma_liste=[]
                sigma_liste.append(mt.sqrt(2*abs(mt.
 ↪log(data['S0_discounted'][i]/data['K'][i]) + (data['r'][i]/
 ↪100)*T_liste_data[j])/T_liste_data[j]))

                while abs(F(data['V_Call_Market'][i], t,␣
 ↪data['S0_discounted'][i], data['K'][i], T_liste_data[j], (data['r'][i]/100),␣
 ↪sigma_liste[-1])) > epsilon:
                    sigma_liste.append(sigma_liste[-1] -␣
 ↪F(data['V_Call_Market'][i], t, data['S0_discounted'][i], data['K'][i],␣
 ↪T_liste_data[j], (data['r'][i]/100), sigma_liste[-1])/Vega_BS(t,␣
 ↪data['S0_discounted'][i], data['K'][i], T_liste_data[j], (data['r'][i]/100),␣
 ↪sigma_liste[-1]))

                volatilite_implicite.append(sigma_liste[-1])
            else:
                volatilite_implicite.append(0)
            K_sub_list.append(data['K'][i])

    Matrix_vol_impl.append(volatilite_implicite)
    K_Matrix.append(K_sub_list)

Matrix_vol_impl_non_Null = []
K_Matrix_non_Null = []

for i in range(len(Matrix_vol_impl)):
```

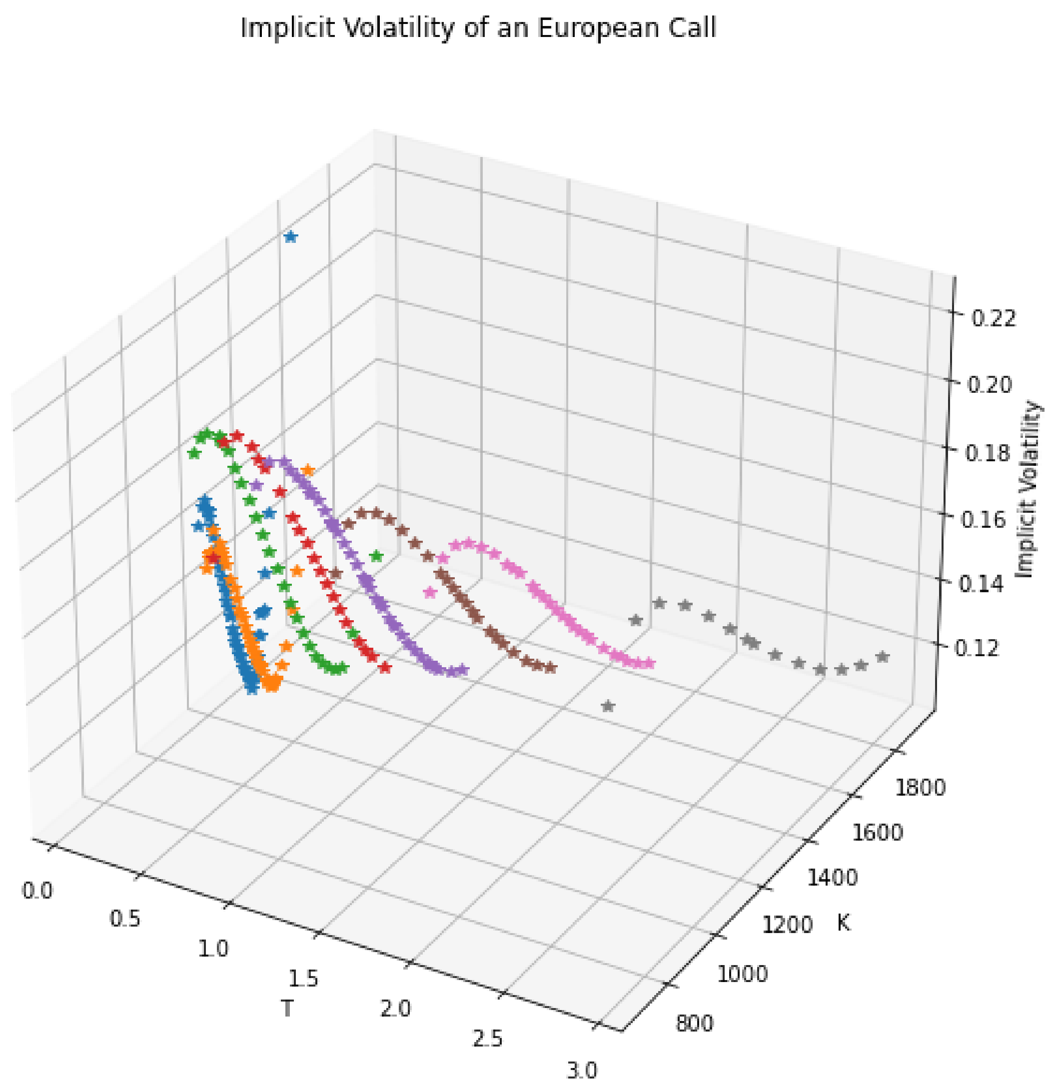
```
    Sigma_non_null = []
    K_non_Null = []
    for j in range(len(Matrix_vol_impl[i])):
        if Matrix_vol_impl[i][j] != 0 :
            Sigma_non_null.append(Matrix_vol_impl[i][j])
            K_non_Null.append(K_Matrix[i][j])

    Matrix_vol_impl_non_Null.append(Sigma_non_null)
    K_Matrix_non_Null.append(K_non_Null)
```

[7]:
```
#------------------------------------------------------------------#
# L'ensemble des courbes de (K, sigma_implicite) sur un même plan #
#------------------------------------------------------------------#

plt.rcParams["figure.figsize"]=[16,9]
for i in range(len(T_liste_data)):
    plt.plot(K_Matrix_non_Null[i], Matrix_vol_impl_non_Null[i])
plt.xlabel("K")
plt.ylabel("Vol_impl")
plt.title("Implicit Volatility of an European Call")
plt.show()
```

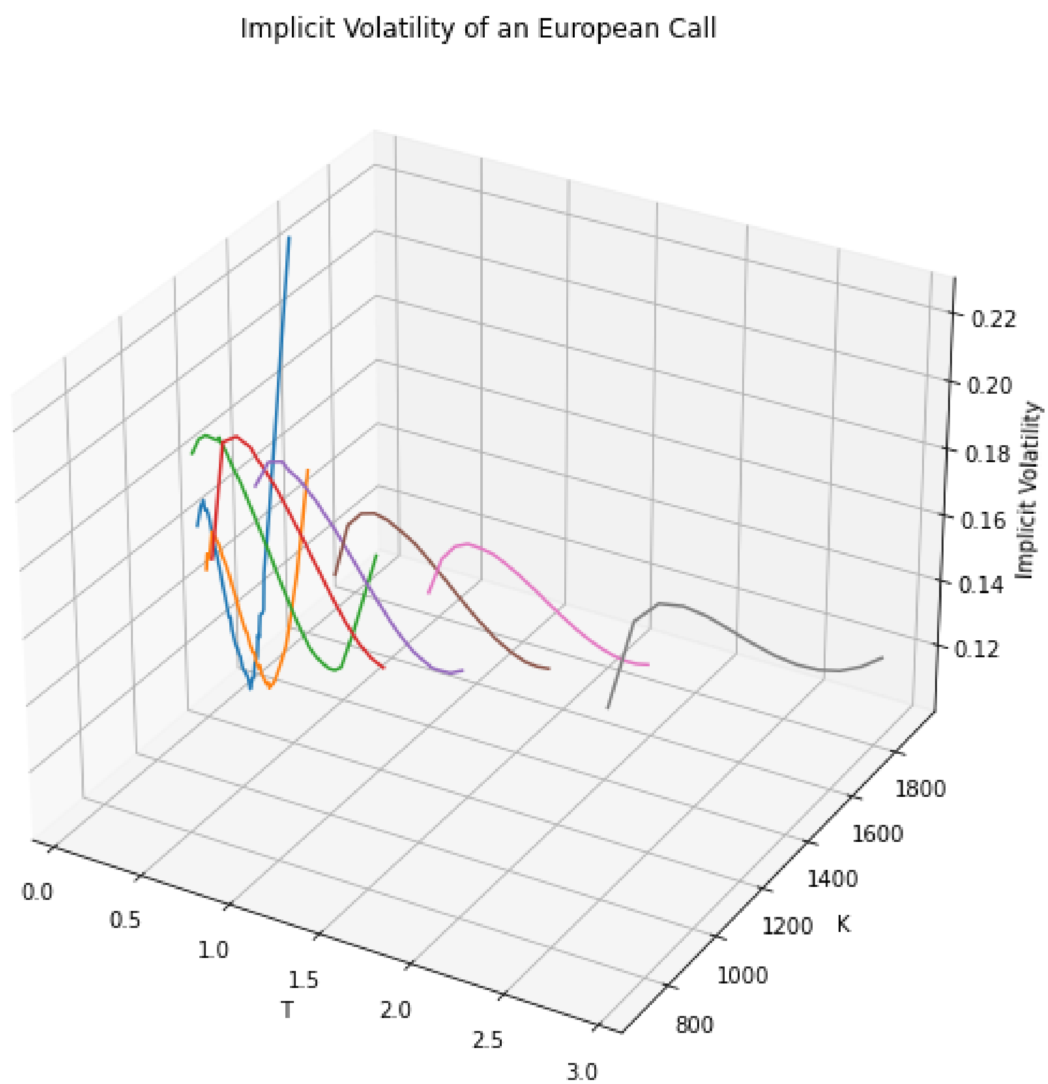


[8]:
```
x=[1 for k in range((len(K_Matrix_non_Null[0])))]
y=K_Matrix_non_Null[0]

fig = plt.figure()
```

```
ax = fig.add_subplot(111,projection='3d')
for j in range(len(K_Matrix_non_Null)):
    x=[T_liste_data[j] for p in range(len(K_Matrix_non_Null[j]))]
    y=K_Matrix_non_Null[j]

    ax.plot(x,y,Matrix_vol_impl_non_Null[j],"*")
ax.set_xlabel('T')
ax.set_ylabel('K')
ax.set_zlabel('Implicit Volatility')
plt.title("Implicit Volatility of an European Call")
plt.show()
```



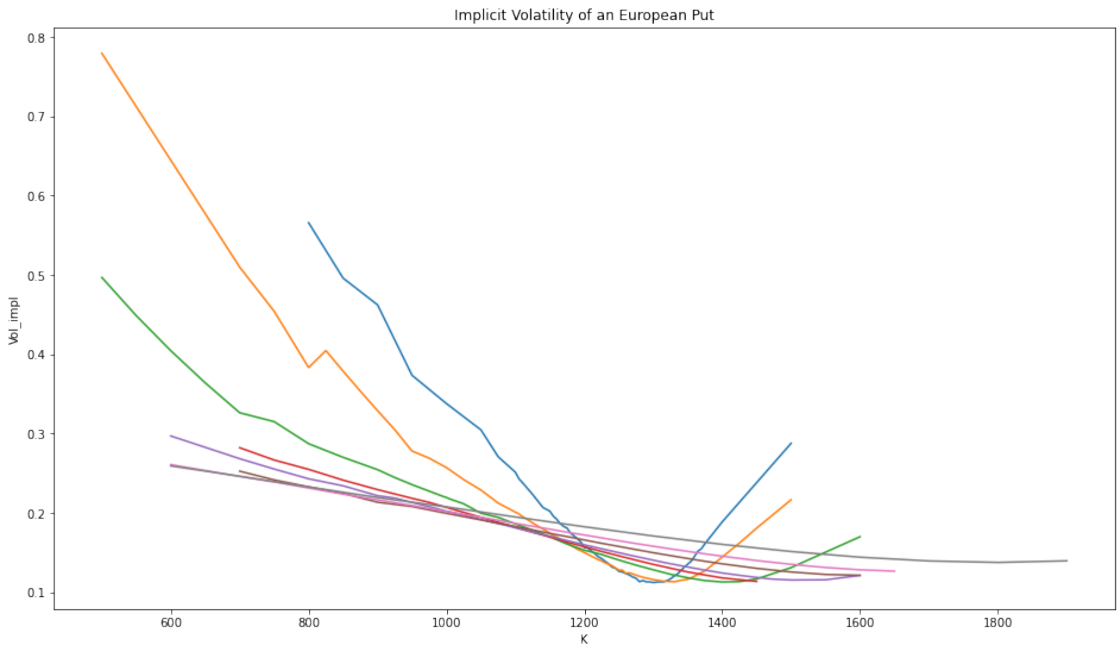
[9]:
```
fig = plt.figure()
ax = fig.add_subplot(111,projection='3d')
for j in range(len(K_Matrix_non_Null)):
    x=[T_liste_data[j] for p in range(len(K_Matrix_non_Null[j]))]
    y=K_Matrix_non_Null[j]

    ax.plot(x,y,Matrix_vol_impl_non_Null[j])
ax.set_xlabel('T')
ax.set_ylabel('K')
ax.set_zlabel('Implicit Volatility')
plt.title("Implicit Volatility of an European Call")
plt.show()
```



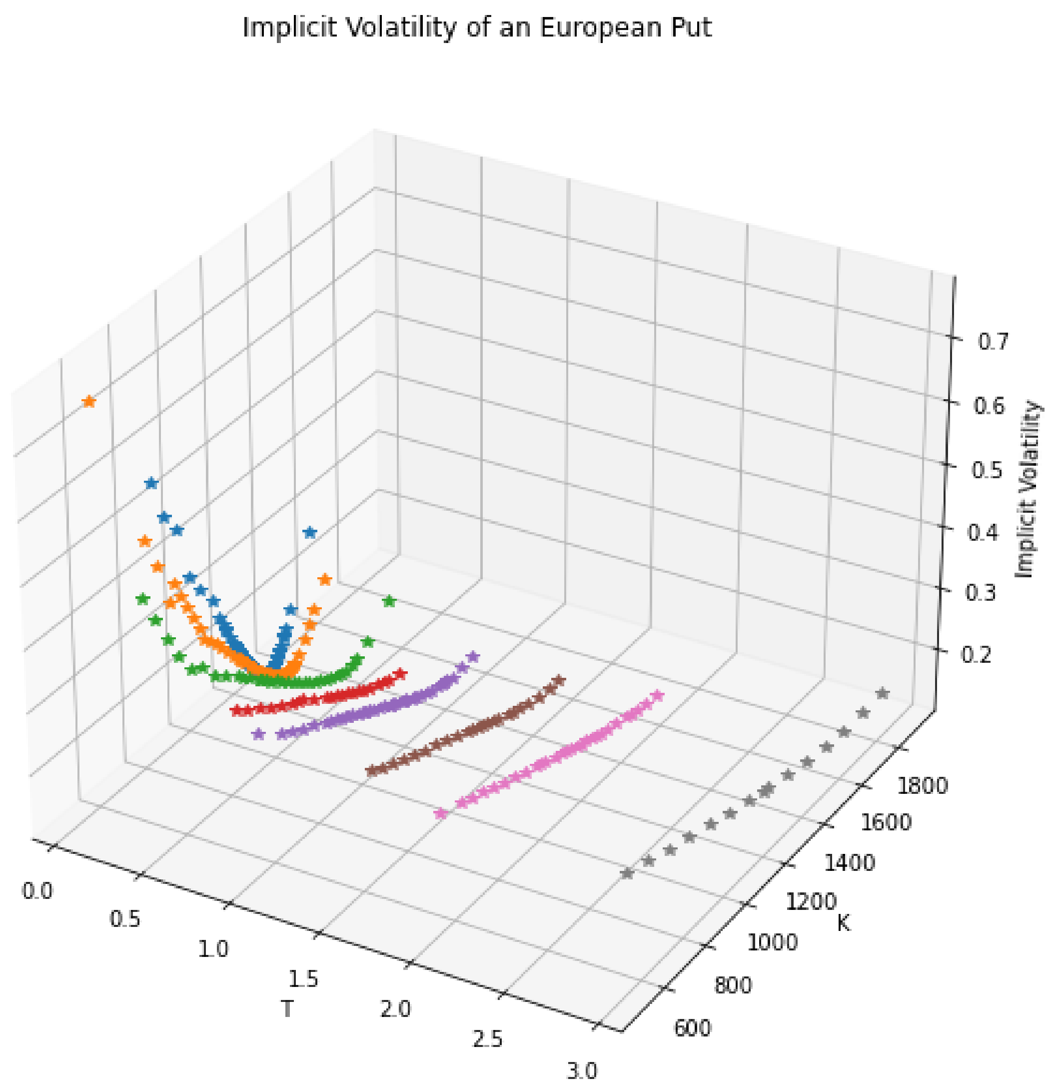
[12]:
```python
#-----------------------------------------------#
#       Avec la relation Call-Put parité        #
# en exprimmant V_Put_M en fonction de V_Call_M #
#-----------------------------------------------#

#-----------------------------------#
# Fonction Vega Put Black & Scholes #
# Sanchant que Vega_Put = Vega_Call #
#-----------------------------------#

def Put_BS(t, S, K, T, r, sigma):
    return Call_BS(t, S, K, T, r, sigma)-S+K*mt.exp(-r*(T-t))

#-----------------------------------------#
#   Fonction F2 (V_M c'est prix du marché)  #
#-----------------------------------------#

def F2(V_M, t, S, K, T, r, sigma):
    return Put_BS(t, S, K, T, r, sigma) - V_M

#------------------#
#   Corps du code  #
#------------------#

Matrix_vol_impl_Put1 = []
K_Matrix_Put1 = []

for j in range(len(T_liste_data)):
    volatilite_implicite=[]
    K_sub_list = []
    for i in range(len(data['V_Call_Market'])):
        if data[' T'][i] == T_liste_data[j]:
            if ((data['V_Put_Market'][i] < data['K'][i]*mt.exp(-(data['r'][i]/
↪100)*T_liste_data[j])) and (data['V_Put_Market'][i] >=␣
↪max(data['S0_discounted'][i]-data['K'][i]*mt.exp(-(data['r'][i]/
↪100)*T_liste_data[j]), 0) - data['S0_discounted'][i] + data['K'][i]*mt.
↪exp(-(data['r'][i]/100)*T_liste_data[j]))):
                sigma_liste=[]
                sigma_liste.append(mt.sqrt(2*abs(mt.
↪log(data['S0_discounted'][i]/data['K'][i]) + (data['r'][i]/
↪100)*T_liste_data[j])/T_liste_data[j]))

                n=1
                while abs(F2(data['V_Put_Market'][i], t,␣
↪data['S0_discounted'][i], data['K'][i], T_liste_data[j], (data['r'][i]/100),␣
↪sigma_liste[-1])) > epsilon:
```

```
                    sigma_liste.append(sigma_liste[-1] -␣
↪F2(data['V_Put_Market'][i], t, data['S0_discounted'][i], data['K'][i],␣
↪T_liste_data[j], (data['r'][i]/100), sigma_liste[-1])/Vega_BS(t,␣
↪data['S0_discounted'][i], data['K'][i], T_liste_data[j], (data['r'][i]/100),␣
↪sigma_liste[-1]))
                    n=n+1

                volatilite_implicite.append(sigma_liste[-1])
            else:
                volatilite_implicite.append(0)
            K_sub_list.append(data['K'][i])

    Matrix_vol_impl_Put1.append(volatilite_implicite)
    K_Matrix_Put1.append(K_sub_list)

Matrix_vol_impl_non_Null_Put1 = []
K_Matrix_non_Null_Put1 = []

for i in range(len(Matrix_vol_impl_Put1)):
    Sigma_non_null = []
    K_non_Null = []
    for j in range(len(Matrix_vol_impl_Put1[i])):
        if Matrix_vol_impl_Put1[i][j] != 0 :
            Sigma_non_null.append(Matrix_vol_impl_Put1[i][j])
            K_non_Null.append(K_Matrix_Put1[i][j])

    Matrix_vol_impl_non_Null_Put1.append(Sigma_non_null)
    K_Matrix_non_Null_Put1.append(K_non_Null)
```

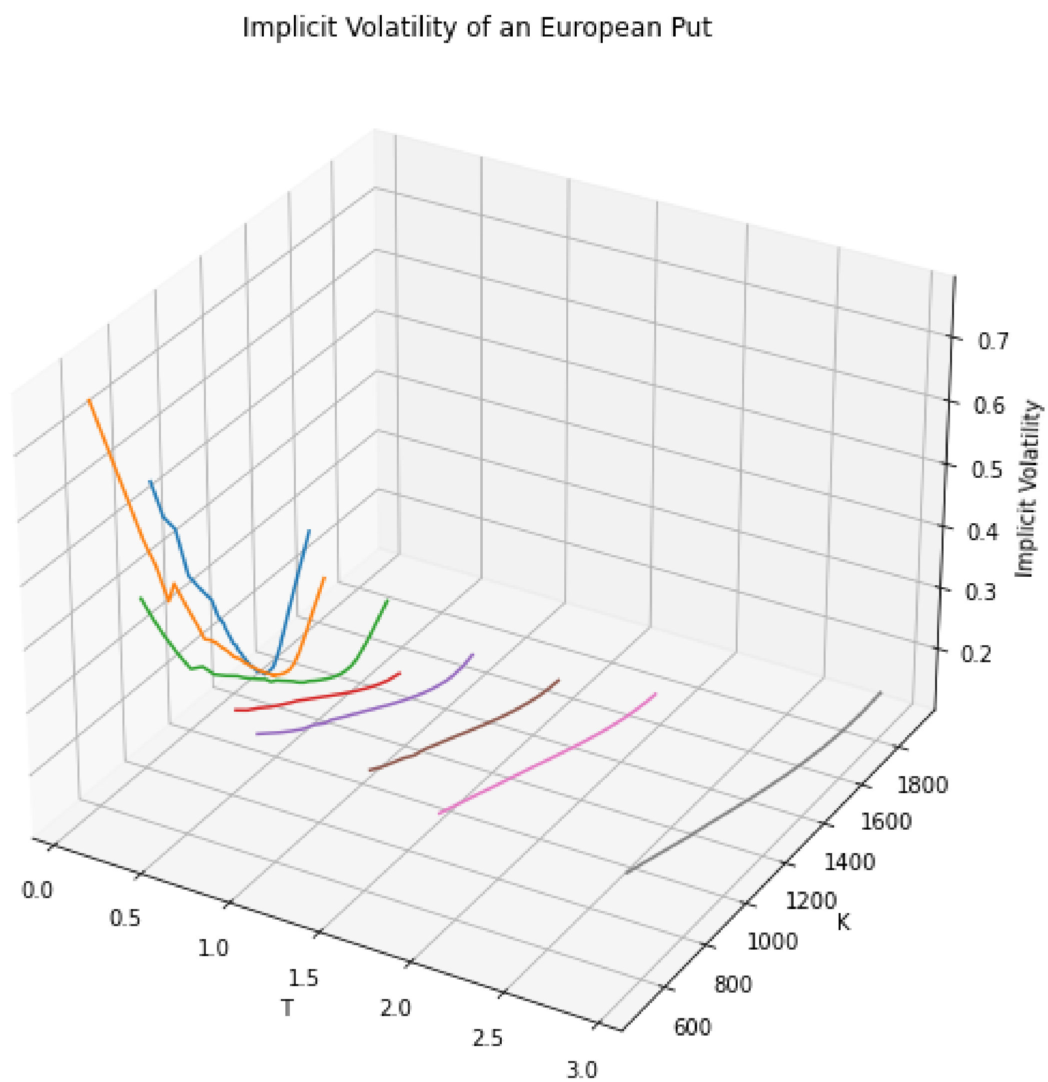
[13]:
```
#------------------------------------------------------------------#
# L'ensemble des courbes de (K, sigma_implicite) sur un même plan #
#------------------------------------------------------------------#

plt.rcParams["figure.figsize"]=[16,9]
for i in range(len(T_liste_data)):
    plt.plot(K_Matrix_non_Null_Put1[i], Matrix_vol_impl_non_Null_Put1[i])
plt.xlabel("K")
plt.ylabel("Vol_impl")
plt.title("Implicit Volatility of an European Put")
plt.show()
```



[14]:
```
x=[1 for k in range((len(K_Matrix_non_Null_Put1[0])))]
y=K_Matrix_non_Null_Put1[0]


fig = plt.figure()
ax = fig.add_subplot(111,projection='3d')
for j in range(len(K_Matrix_non_Null_Put1)):
    x=[T_liste_data[j] for p in range(len(K_Matrix_non_Null_Put1[j]))]
    y=K_Matrix_non_Null_Put1[j]

    ax.plot(x,y,Matrix_vol_impl_non_Null_Put1[j],"*")
ax.set_xlabel('T')
ax.set_ylabel('K')
ax.set_zlabel('Implicit Volatility')
plt.title("Implicit Volatility of an European Put")
plt.show()
```



[15]:
```
fig = plt.figure()
ax = fig.add_subplot(111,projection='3d')
for j in range(len(K_Matrix_non_Null_Put1)):
    x=[T_liste_data[j] for p in range(len(K_Matrix_non_Null_Put1[j]))]
    y=K_Matrix_non_Null_Put1[j]

    ax.plot(x,y,Matrix_vol_impl_non_Null_Put1[j])
ax.set_xlabel('T')
ax.set_ylabel('K')
ax.set_zlabel('Implicit Volatility')
plt.title("Implicit Volatility of an European Put")
plt.show()
```


Implicit Volatility of an European Put
Implicit Volatility
0.7
0.6
0.5
0.4
0.3
0.2
T
0.0
0.5
1.0
1.5
2.0
2.5
3.0
K
600
800
1000
1200
1400
1600
1800