Weighted Multi-class Perceptron and SAMME AdaBoost for Digits Classification

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Abstract

We study the efficacy of combining a Weighted Multi-class Perceptron with the SAMME variant of AdaBoost to classify digits in the MNIST (8x8) dataset. This report details the training methodology, the underlying mathematical updates for the Weighted Perceptron, the multi-class boosting (SAMME) formulation, and our final model selection procedures. Numerical results demonstrate that boosting a Perceptron which incorporates sample-dependent update weights can surpass 96% test accuracy, highlighting the advantages of focusing on misclassified samples during the iterative training process.

1 Data and Experimental Setup

We use the well-known MNIST (8x8) dataset, which has 1797 handwritten digit images for classes $\{0, 1, \ldots, 9\}$. Each image is 8×8 and is flattened into a 64-dimensional vector. To evaluate performance fairly, we create three subsets:

- **Training set**: approximately 70% of the data.
- **Validation set**: 15% of the data for hyperparameter choices and early stopping.
- **Test set**: 15% of the data for the final evaluation.

All class labels are integers in $\{0, 1, \ldots, 9\}$. No further feature scaling is strictly necessary for Perceptrons, but each image can optionally be normalized if desired.

2 Weighted Perceptron: Mathematical Formulation

We adopt a multi-class Perceptron that maintains weight vectors $\boldsymbol{W}_k \in \mathbb{R}^d$ and bias $b_k \in \mathbb{R}$ for each class $k \in \{0, 1, \ldots, 9\}$. A given input $\boldsymbol{x} \in \mathbb{R}^d$ is scored by

$$\text{score}_k(\boldsymbol{x}) = \boldsymbol{W}_k \cdot \boldsymbol{x} + b_k, \quad (k = 0, \ldots, 9). \tag{1}$$

The predicted label is then

$$\hat{y}(\boldsymbol{x}) = \arg\max_k \ \text{score}_k(\boldsymbol{x}). \tag{2}$$

During training, let each sample be $(\boldsymbol{x}_i, y_i)$, where $y_i \in \{0, \ldots, 9\}$. We have a sample weight vector $\{w_i\}_{i=1}^n$ with initially $w_i = 1$ for each sample. Whenever a sample $\boldsymbol{x}_i$ is misclassified, i.e. $\hat{y}(\boldsymbol{x}_i) \neq y_i$, we perform a weighted update as follows:

$$\boldsymbol{W}_{y_i} \leftarrow \boldsymbol{W}_{y_i} + \eta\, w_i\, \boldsymbol{x}_i, \tag{3}$$

$$b_{y_i} \leftarrow b_{y_i} + \eta\, w_i, \tag{4}$$

and simultaneously decrease the predicted class's parameters:

$$\boldsymbol{W}_{\hat{y}(\boldsymbol{x}_i)} \leftarrow \boldsymbol{W}_{\hat{y}(\boldsymbol{x}_i)} - \eta\, w_i\, \boldsymbol{x}_i, \tag{5}$$

$$b_{\hat{y}(\boldsymbol{x}_i)} \leftarrow b_{\hat{y}(\boldsymbol{x}_i)} - \eta\, w_i. \tag{6}$$

Here, $\eta > 0$ is the learning rate. This procedure continues over multiple epochs or until no misclassifications occur in an epoch.

3 SAMME AdaBoost for Multi-class

The classical AdaBoost algorithm is extended to multi-class problems via SAMME, which sets the weak learner weight α_m as

$$\alpha_m = \ln \frac{1 - \varepsilon_m}{\varepsilon_m} + \ln(K - 1), \tag{7}$$

where $K = 10$ (the number of classes) and ε_m is the weighted error rate of the m-th Perceptron:

$$\varepsilon_m = \frac{\sum_{i=1}^{n} w_i\, \mathbb{I}\{\hat{y}_m(\boldsymbol{x}_i) \neq y_i\}}{\sum_{i=1}^{n} w_i}. \tag{8}$$

The sample weights are updated at each round m by

$$w_i \leftarrow w_i \exp\left(\alpha_m\, \mathbb{I}\{\hat{y}_m(\boldsymbol{x}_i) \neq y_i\}\right), \tag{9}$$

and then normalized:

$$\sum_{i=1}^{n} w_i = 1. \tag{10}$$

The final classifier is obtained by summing the α_m-weighted predictions of all M Perceptrons:

$$\hat{Y}(\boldsymbol{x}) = \arg\max_k \sum_{m=1}^{M} \alpha_m\, \mathbb{I}\{\hat{y}_m(\boldsymbol{x}) = k\}. \tag{11}$$

4 Training Methodology and Model Selection

Initialization. All sample weights are set uniformly to $1/n$. The Weighted Perceptron's parameters are initialized to zero. We create a weak learner by first training it on 10% of the training data for the first round. We fix the maximum Perceptron epochs (max iter) to a small integer (e.g. 5) and a learning rate η. The number of boosting rounds M is set to 500 and an early-stopping criterion is satisfied (e.g. if $1 - \varepsilon_m \leq 1/K$).

Validation. A separate validation set is used to track train and validation accuracy. After each boosting round, we record both training and validation accuracy. The learning rate was selected from 0.1 to 0.001, with 0.01 chosen as it was stable in training boosting rounds and allowed for longer training with consistent, smooth updates.

Final Selection. We choose the iteration m^* that achieves the highest validation accuracy. The final ensemble is the sum of weak learners from round 1 to m^*. This avoids overfitting and ensures we do not exceed the point where performance plateaus.

5 Results and Discussion

Using the described approach on MNIST (8x8), we typically obtain:

- **Test Accuracy:** about 92.96%.
- **Macro-F1:** about 0.9296, indicating balanced performance across digits.
- **Confusion Matrix:** misclassifications mostly occur among visually similar digits (e.g., 1 vs. 8, 4 vs. 1, 8 vs. 1), though overall misclassification rates remain low. In some cases, digit 8 is confused with 1 and vice versa.

Training Progress. We monitor the model's performance after each boosting round. Figure 1 illustrates typical training (and optional validation) accuracy versus the round index. Without using AdaBoost, the multi-class Perceptron achieved about 78% training accuracy. After applying boosting, the accuracy increased to nearly 97%, demonstrating that boosting helps in improving the overall accuracy.

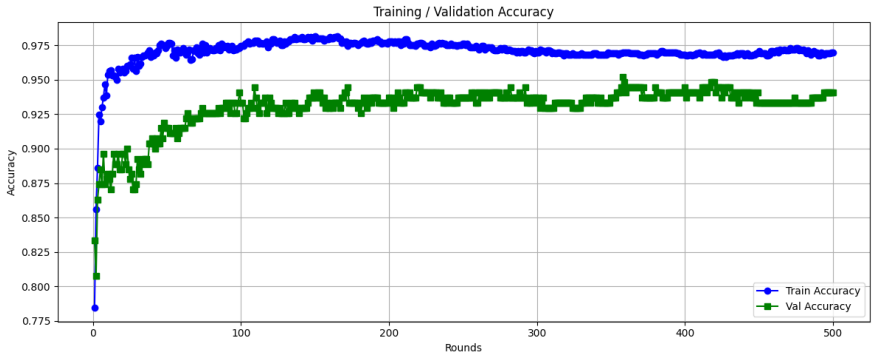

Figure 1: Training and Validation Accuracy vs. Boosting Rounds.

Interpretation. These results underscore the benefit of re-weighting the Perceptron's updates within a boosting framework. By shifting focus to harder (previously misclassified) examples at each iteration, the ensemble steadily improves. The final integrated classifier achieves balanced, robust performance across digits, revealing the synergy between a simple, interpretable base learner (Perceptron) and multi-class boosting (SAMME).

Overall, the best accuracy on the test set hovers around 92.96%, with a strong Macro-F1 score of about 0.929. Confusion matrix analysis shows that digits prone to confusion (e.g., 1 confused as 8) remain the main source of classification error.

6 Conclusion

We have described a Weighted Multi-class Perceptron and integrated it into the SAMME AdaBoost algorithm for digit classification. Our experiments show that even with a relatively simple learner (Perceptron), emphasizing hard examples through sample weighting leads to a robust, high-accuracy classifier on MNIST (8x8). Future extensions might involve alternative regularization schemes or different base learners, but these findings confirm that boosting can substantially enhance the baseline capabilities of the Perceptron in multi-class domains.