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CAVENDISH EXPERIMENT

ABSTRACT:

It all begins with the story of the apple which fell onto the head of Sir Isaac NEWTON. Currently due to Einstein's theory of general relativity the concept of gravitation generates hot debates whether it is a force that attract the objects one into another or it just curves the spaces dependent on its mass and make the another object to come behind as if it is attracted. This debate goes beyond of our vision or our aim in the experiment; we try to concentrate on how much objects attract each other, in other words the constant in the gravitation force.

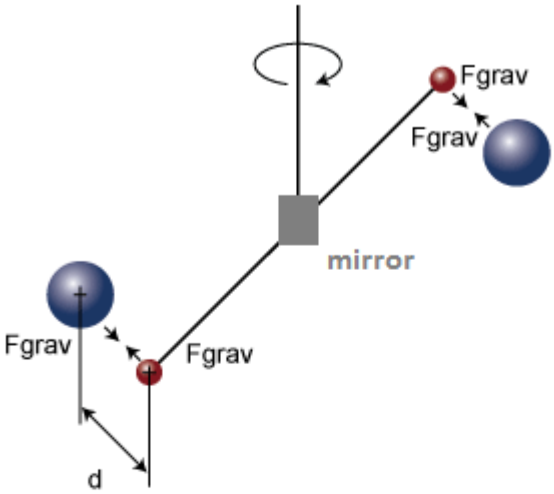
INTRODUCTION:

In 1798, an aristocrat English scientist Lord Henry CAVENDISH was able to measure Gravitational constant (G) not exactly but with a great accuracy. What he did is finding Gravitational constant (G) in Newton's law of universal gravitation.

$$F = G\frac{m_1 m_2}{r^2}$$

Newton showed the direct proportionality in the product of their masses and inverse proportionality in the distance between their centers, meanly he had not been able to estimate the constant G.

However, Cavendish estimated the constant (G) in the laboratory. Nowadays we also use similar setup to calculate (G). To grasp the experiment we have to understand the mechanism of the torsion balance. Torsion balance consists of two small masses, which are linked one to each other with light and rigid wire, and two big masses placed on the opposite sides of the smaller masses. The following figure [reference:1] demonstrates the torsion balance.



Due to gravitational force between the one pair of small and large masses, there exists a net torque:

$$\tau=2Fd$$

where $$F = G\frac{m_1 m_2}{r^2}$$

so this net torque and naturally opposing torque produced by the will result in a damped oscillation of the dumbbell. The equation of the motion is as the following;

$$I\frac{d^2\theta}{dt^2} + k\Theta = 0$$

where I is the moment of inertia of the dumbbell system and k is the torsion constant of the wire used. Observing the oscillations through the displacement of a laser beam which is reflected off the small mirror spotted in the middle of the dumbbell and falls on a scale at a distance L, the Universal Gravitation Constant can be determined as

$$G=\frac{\pi^2 b^2 dS}{MT^2 L}$$

where b is the distance between the adjacent small and large massesi M is the large mass, 2d is the length of the dumbbell, and S is the difference between the initial and the final equilibrium positions of the laser beam on the scale. T is the period of the oscillation determined by requiring time for one successive wavelength.

SETUP:

We have used such apparatus as :

- ✓ Low Power Laser
- ✓ Scale
- ✓ Cavendish Torsion Balance with large masses
- ✓ Ruler
- ✓ A computer with appropriate program

By the help of technology (computer), experiment become so much easier than Cavendish did in 1798. We turn on the laser and light beam reflected from the mirror leave a track on the scale. We figured out the track on the scale is stationary and we wrote down as initial equilibrium position. Then computer begins to sketch the graph voltage versus time, and as we expected we get a wave package seems like a sinusoidal wave. We took the data when the graph has a maximum or a minimum, we saved the voltage value at the same time the value on the scale. The graph we have seen in the computer screen, data and data anlaysis is attached.

DATA ANALYSIS:

$$\theta(t) = \theta_e + A\, e^{-bt} \cos(\omega_1 t + \delta)$$

(1)

Where Qe is the equilibrium angle, A the initial amplitude, 1/b the time for the amplitude to decay to 1/e of its initial value, w1 the oscillation frequency(2p/T) and d the phase of the oscillation at the time t=0.

$\frac{\partial \vartheta}{\partial t}\Big|_{t=0} = 0$ by this differentiation we can find that $\delta = -b/w1$ and when we put this δ on the (1) equation we can find that;

$$\theta(t) = \theta_e + A\, e^{-bt} [\cos(\omega_1 t) + b/\omega_1 \sin(\omega_1 t)].$$

(2)

Let $t_n = (n-1)T/2$ and $\theta_n = \theta(t_n)$ and $A = \theta_1 - \theta_e$ then the equation becomes;

$$\theta_n = \theta_e + (\theta_1 - \theta_e)(e^{-(n-1)bT/2})\,(-1)^{n-1} \qquad (3)$$

since $\omega_1 t_n = (n-1)\pi$. The factor $e^{-bT/2}$ occurs so often in the formulas below that it is convenient to define a separate symbol for it; let's call it x ($x \equiv e^{-bT/2}$). With this definition, Equation 3 becomes

$$\theta_n - \theta_e = (-x)^{n-1} (\theta_1 - \theta_e) \qquad (4)$$

In free decay, x can be measured using any two adjacent turning points:

$$x = -(\theta_{n+1} - \theta_e)/(\theta_n - \theta_e)$$

(5)

By using three adjacent turning points, only differences in the turning point angles need be measured. Using Equation 4 twice, we find

$$x = -(\theta_{n+2} - \theta_{n+1})/(\theta_{n+1} - \theta_n)$$

(6)

If an odd number N of adjacent turning points are measured, multiple use of Equation 4 gives

$$x = 1 - (\theta_1-\theta_N)/(\theta_1-\theta_2+\theta_3-\theta_4+...-\theta_{N-1}) \qquad (7)$$

Let $\pm\theta_D$ be the change in the equilibrium angle when the large masses are rotated from the center position to either of the (symmetrically located) extreme positions. Suppose at time t=0 (a turning point if the balance is oscillating) the large masses are rotated to the extreme position where the new equilibrium angle is $\theta_e-\theta_D$. Then from Equation 2, the time dependence of the boom angle is

$$\theta(t) = (\theta_e-\theta_D) + (\theta_1-(\theta_e-\theta_D))\ e^{-bt}\ [\cos(\omega_1 t\ )+b/\omega_1\ \sin(\omega_1 t)] \qquad (8)$$

where θ_1 is the angle of the boom at t=0 (the first turning point). The boom angle at the second turning point is

$$\theta_2 = (\theta_e-\theta_D)\ -x\ (\theta_1-(\theta_e-\theta_D)) \qquad (9)$$

At the second turning point (t=T/2) the large masses are quickly rotated so that the new balance equilibrium angle becomes $\theta_e+\theta_D$. Thus, the boom angle at the third turning point (t=T) is

$$\theta_3 = (\theta_e+\theta_D)\ -x\ (\theta_2-(\theta_e+\theta_D))$$

(10)

Each pair of adjacent turning points can be used to measure θ_D from which the gravitational constant G can be determined. It can be seen from Equations 9 and 10 that the solution for θ_D in terms of the two turning points θ_n and θ_{n+1} is

$$\theta_D = (-1)^n[(\theta_{n+1} - \theta_e) + x(\theta_n - \theta_e)]/(1+x)$$

(11)

The equilibrium angle θ_e can be eliminated from the measurement process if the results of two measurements of θ_D using three adjacent turning points are averaged:

$$\theta_D = (-1)^n[x\,\theta_n + (1-x)\,\theta_{n+1} - \theta_{n+2}] / [2\,(1+x)]$$

(12)

To reduce errors, the results of an odd number N of adjacent turning points can be averaged:

$$\theta_D = [(1-x)(\theta_1 - \theta_2 + \theta_3 - \theta_4 + \ldots \theta_N) - \theta_1 + x\,\theta_N] / [(N-1)(1+x)]$$

(13)

We measured the distance between the scale and mirror which is 204cm. So, we can calculate the boom angle θ from the position data using the approximation for small angles.

$$\sin\vartheta \approx \tan\vartheta \approx \vartheta$$

L=204cm and $\vartheta_n \approx \tan\vartheta_n = \frac{S_n}{L}$

There are 50 datas, and we were recording minimums and maximums, therefore I can say there are 25 maximums and 25 minimums. Between two adjacent peaks period exists, there are 25 periods, we had better get average of it to use in the following calculations.

$$T_{avg} = \frac{(205 + 212 + 223 + \cdots \ldots.)}{25} = 230\ second$$

$$sin\theta \approx tan\theta \approx \theta$$

L=204cm and $\theta_n \approx tan\theta_n = \frac{s_n}{L}$

x = 1 - $(\theta_1-\theta_N)/(\theta_1-\theta_2+\theta_3-\theta_4+\ldots-\theta_{N-1}) = 1 - \frac{0,00049-0,00050}{\sum\Delta\theta}$=1,003

$\theta_D = (-1)^n[(\theta_{n+1} + x\,\theta_n]/(1+x)$

$$\theta_{D1} = \frac{(-1)^1[\theta_2 + 1,003\times\theta_3]}{1 + 1,003}$$

rest showed in the excel sheet

$$\theta_D = \frac{1}{N}\sum_{i}^{N}\theta_{Di} = 0,00032$$

$$\delta\theta = \frac{1}{N-1}\sum_{i}^{N}(\theta_i - \vartheta')^2$$

$$\frac{1}{49}\sum_{i}^{50}(\theta_i - 0,00032)^2 = 0,09\times10^{-6}$$

$\delta x = \delta\theta\ (1-x)[(N-1)(1-x)^2 + 2x]^{1/2}/|\theta_1-\theta_N|$

$$1,56\times10^{-4}(1 - 1,003)[(50 - 1)(1 - 1,003)^2 + 2\times1,003)] = -0,9\times10^{-6}$$

$\delta\theta_{D_\theta}$ = δθ [(N-1)(1-x)^2 + 2x]$^{1/2}$/[(N-1)(1+x)]

we know $\delta\theta, N$ *and* x *. the answer is* $\delta\theta_D = 5\times 10^{-6}$

M = mass of each large sphere= 1,038 kg

m = mass of each small sphere= 0,014 kg

d = distance from the rotation axis to the center of the small sphere= 0,05 m

R = distance between the centers of the large and small spheres=

r= radius of the small sphere = 0,0065 m

m_b=mass= 0,0071 kg

l_b= length= 0,145 m

w_b=width= 0,0127 m

T=period=230 sec.

The moment of inertia of the small spheres;

$$I_s = 2(md^2 + 2/5mr^2) = 2.(0{,}014.0{,}0043 + \frac{2}{5}.0{,}014.3{,}6\times 10^{-5}) = 122{,}4\times 10^{-6}$$
$$I_b = m_b(l_b^2 + w_b^2)/12 = 0{,}0071(0{,}0145^2 + 0{,}127^2)/12 = 133\times 10^{-6}$$
$$I = I_s + I_b = 255{,}4\times 10^{-6}\,kg.m^2$$

Now , we can calculate the torsion constant K

$$K = (4\pi^2/T^2 + b^2)I = (\frac{4\pi^2}{230^2} + b^2)255{,}4\times 10^{-6} = 0{,}19\times 10^{-6} \text{ newton} - \text{meters / radian}$$

b is very small. We ignore it.

Now we can find G.

$$G = \frac{K\theta_D R^2}{2Mmd} = \frac{0,19\times10^{-6}\times0,00032\times(0,05)^2}{2\times(1,038)\times(0,014)\times(0,066)} = 7,96\times10^{-11}\ m^3kg^{-1}s^{-2}$$

Correction;

$G = K\,\theta_D\, R^2 / (2\,M\,[(m - m_h)(1-f_d) + m_b f_b]\,d$

$f_d = f = R^3 / [R^2 + (2d)^2]^{3/2}$

$$f_b = \frac{R}{d}\frac{R}{l_b}\left[\frac{1}{\sqrt{1+\left(\frac{x-d}{R}\right)^2}}\left(\frac{d\,(x-d)}{R^2} - 1\right)\right]_{x_I}^{x_H}$$

$$G = \frac{0,19\times10^{-6}\times0,00032\times0,0016}{0,0021} = 4,6\times10^{-11} m^3kg^{-1}s^{-2}$$

CONCLUSION:

G_T =6,673 x $10^{-11} m^3kg^{-1}s^{-2}$

We calculated G=$4,6\times10^{-11}m^3kg^{-1}s^{-2}$

$$Error = \frac{G_T - G_E}{G_T}\times100 = \frac{6,67\times10^{-11} - 4,6\times10^{-11}}{6,67\times10^{-11}} = 31\%$$

The error can be derived from too many reasons, first of all our masses, little vibrations caused from wind, temperature, etc. To minimize the error we need an isolated system.