Charge to Mass Ratio of the Electron

Adnan Basar
Phys 442.01
2010205108
(Dated: March 15, 2013)

This experiment uses the study of the motion of an electron that moves perpendicular to a magnetic field and measures the charge-to-mass ratio of electron.

I. INTRODUCTION

The charge to mass ratio of an electron is measured from observing the trajectories of electrons in a magnetic field. When an electron moves in an electrostatic field from point 1 to point 2 with electric potential difference V between them, the electron will gain a kinetic energy (K) given by

$$K = \frac{mu^2}{2} = eV$$

where e = charge of the electron, m = mass of the electron, u = velocity of the electron.

When a charged particle of charge q with velocity u moves through a magnetic field with magnetic field strength B, it experiences a force ("Lorentz force") given by

$$F = q(uxB)$$

If B is perpendicular to u then the electron will move in a circle whose plane is also perpendicular to B, and the magnetic force (Lorentz force) provides the centripetal force for this circular motion, i.e.

$$\frac{mu^2}{r} = euB$$

where m = mass of electron r = radius of circular path.

Solve Eq. (3) for u and substitute into Eq. (1) to obtain an expression for the charge-to-mass ratio of the electron

$$\frac{e}{m} = \frac{2V}{(Br)^2}$$

So if we know the magnitude of the magnetic field, the potential difference V and the radius of the path r, we can calculate the charge-to-mass ratio of the electron. For electrons (or any given kind of particle) the left hand side is a constant. For example, if we raise the energy of the electrons injected into the magnetic field region (by increasing the accelerating potential V) , and we desire to maintain an orbit with the same radius, then this equation informs us we would have to increase the magnitude of the magnetic field. Alternatively, if we want to increase the radius of the circular orbit we would have to decrease the magnitude of the magnetic field. The experimental problem is to generate a uniform magnetic field.

The setup consists of a fine beam tube and a pair of Helmholtz coils to produce the magnetic field. The field strength will be determined using the expression

$$B = \frac{8\mu_o IN}{\sqrt{125}r_c}$$

where $\mu_0 = 1.257x10^{-6}$, $N = 130$ turns and $r_c = 15$ cm.

The experiment involves injecting electrons into a magnetic field generated by Helmholtz coils, which provide an approximately uniform magnetic field close to the center, but there is some non-uniformity which may affect the measurement.

II. EXPERIMENTAL SETUP

In this experiment, the following apparatus are used:

- Fine Beam Tube Set (including the Helmholtz coils)

- DC Power Supply (0-300 V)
- Filament Power Supply (6.3 V AC, 5 A)
- AC Ammeter (0-3 A)
- DC Power Supply (0-5 A)
- DC Ammeter (0-5 A)
- DC Voltmeter (0-300 V)
- Connecting Leads

III. DATA AND ANALYSIS

For analysis part, there are some experimental data for both fixed voltage and magnetic field. And our known errors are $\sigma_V = 10V$, $\sigma_r = 0.4cm$ and $\sigma_I = 0.1A$

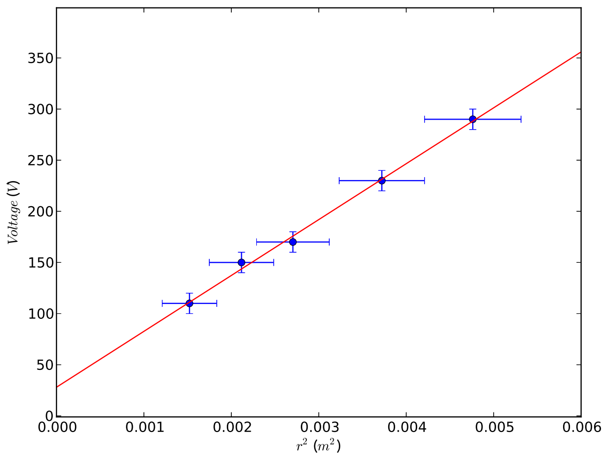
TABLE I: Experimental Data for fixed magnetic field B at $(1A)$

Voltage (V)	r (cm)
110	3.8
140	4.5
180	5.1
220	6.2
280	7.0

With the values of Table I, we obtained Figure I by using the straight line fit to $y = mx + n$. The slope m is $\frac{V}{r^2}$.Thus, $\frac{e}{m} = \frac{2m}{B^2}$. By using error propagation, we will obtain:

$$\sigma_{\frac{e}{m}}{}^2 = \left(\frac{\partial}{\partial m}\left(\frac{2m}{B^2}\right)\right)^2 \sigma_m{}^2 \quad + \quad \left(\frac{\partial}{\partial B}\left(\frac{2m}{B^2}\right)\right)^2 \sigma_B{}^2 = \left(\frac{2m}{B^2}\right)^2 \sigma_m{}^2 + \left(\frac{2m}{B^2}\right)^2 \sigma_m{}^2 = \left(\frac{-4m}{B^3}\right)^2 \sigma_B{}^2$$

FIG. 1: Voltage vs. r^2



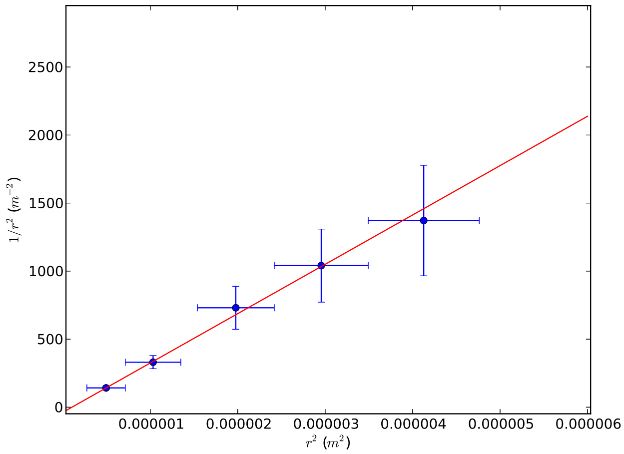
With the values of Table II, we obtained Figure I by using the straight line fit to $y = mx + n$. The slope m is $\frac{1}{Br^2}$.Thus, $\frac{e}{m} = 2mV$. By using error propagation, we will obtain:

$$\sigma_{\frac{e}{m}}{}^2 = \left(\frac{\partial}{\partial m}(2mV)\right)^2 \sigma_m{}^2 \quad + \quad \left(\frac{\partial}{\partial V}(2mV)\right)^2 \sigma_V{}^2 = (2V)^2 \sigma_m{}^2 + (2m)^2 \sigma_B{}^2$$

TABLE II: Experimental Data for voltage V at $(250V)$

Current (A)	r (cm)
0.9	8.2
1.3	5.3
1.8	3.5
2.2	3.3
2.6	2.9

FIG. 2: $1/r^2$ vs B^2



IV. RESULTS

By using two error propagations, we will obtain two proper $\frac{e}{m}$ ratio:

TABLE III: Experimental Data for voltage V at $(250V)$

Current (A)	r (cm)
0.9	8.2
1.3	5.3
1.8	3.5
2.2	3.3
2.6	2.9

The real value of electron mass ratio is $1.76x10^{11}$ C/kg

- Part I
  $\frac{error}{sigma} = \frac{|(1.79-1.76)x10^{11}|}{3.60x10^{10}} = 0.07$ true value is in 1σ range.
- Part II

  $\frac{error}{sigma} = \frac{|(1.81-1.76)x10^{11}|}{2.74x10^{10}} = 0.18$ true value is in 1σ range.

When we think thoroughly what can cause the experiment to have error is fundamentally instrumental errors (especially Voltmeter) and we were given.

TABLE IV: Fixed Current

$\frac{e}{m}$	$\sigma_{\frac{e}{m}}$	m	σ_m	B	σ_B
$1.79x10^{11}$	$3.90x10^{10}$	$5.46x10^{4}$	$3.87x10^{3}$	$7.81x10^{-4}$	$7.81x10^{-5}$

TABLE V: Fixed Voltage

$\frac{e}{m}$	$\sigma_{\frac{e}{m}}$	m	σ_m	V	σ_V
$1.81x10^{11}$	$2.74x10^{10}$	$3.62x10^{8}$	$5.29x10^{7}$	250	10

Acknowledgments

I would like to thank (Associate Professor) V.Erkcan Ozcan and my partner Kadir Simsek for their contributions, and also to the teaching assistant Serhat Istin for his guidance during this experiment.

V. REFERENCES

- E. Gulmez, "Advanced Physics Experiment", Istanbul, Bogazici University Publication, 1999

VI. APPENDIX

Python code for part I:

```
from pylab import *


radius = array([ 3.9, 4.6, 5.2, 6.1, 6.9 ]) *1e-2
sigma_radius = 0.004

Voltage = array([ 110, 150, 170, 230, 290 ])
sigma_Voltage = 10




std_radius = 2*radius*sigma_radius
std_Voltage = ones(len(Voltage))*sigma_Voltage


radius=radius**2



x = radius
sx = std_radius
y = Voltage
sy = std_Voltage



S   = sum(1 / sy**2)
```

```
Sx  = sum(x / sy**2)
Sy  = sum(y / sy**2)
Sxx = sum(x**2 / sy**2)
Sxy = sum(x*y / sy**2)

delta = S*Sxx - Sx**2

n = (Sxx*Sy - Sx*Sxy) / delta
m = (S*Sxy - Sx*Sy) / delta

sn = sqrt(Sxx / delta)
sm = sqrt(S / delta)



errorbar(x,y,xerr=sx,yerr=sy,fmt='o')

xx = arange(0,6e-3,1e-5)
yy = n + m*xx

plot(xx, yy, 'r-')

xlabel('$r^{2}$ ($m^{2}$)',fontname='Ubuntu')
ylabel('$Voltage$ ($V$)',fontname='Ubuntu')


show()

print m
print sm

b=7.81e-4
sb=7.81e-5

print 2*m/(b)**2
print sqrt((4/(b**4))*((sm)**2) +  (16* (m**2) /(b**6))*((sb)**2)   )
```

Python code for part II:

```
from pylab import *


radius = array([ 8.4, 5.5, 3.7, 3.1, 2.7 ])*1e-2
sigma_radius = 0.004

current = array([ 0.9, 1.3, 1.8, 2.2, 2.6 ])
sigma_Current = 0.1
M_field = current*((8*1.26*1e-6*130)/(sqrt(125)*(0.15)))
sigma_M_field = sigma_Current*((8*1.26*1e-6*130)/(sqrt(125)*(0.15)))

std_radius = 2*sigma_radius/radius**3
std_M_field = 2*sigma_M_field*2*M_field


radius=1/(radius**2)
M_field=M_field**2
```

```
y = radius
sy = std_radius
x = M_field
sx = std_M_field



S   = sum(1 / sy**2)
Sx  = sum(x / sy**2)
Sy  = sum(y / sy**2)
Sxx = sum(x**2 / sy**2)
Sxy = sum(x*y / sy**2)

delta = S*Sxx - Sx**2

n = (Sxx*Sy - Sx*Sxy) / delta
m = (S*Sxy - Sx*Sy) / delta

sn = sqrt(Sxx / delta)
sm = sqrt(S / delta)

errorbar(x,y,xerr=sx,yerr=sy,fmt='o')

xx = arange(0,6e-6,1e-7)
yy = n + m*xx

plot(xx, yy, 'r-')

xlabel('$B^{2}$ ($T^{2}$)',fontname='Ubuntu')
ylabel('$1/r^{2}$ ($m^{-2}$)',fontname='Ubuntu')


show()

print m
print sm

V=250
sv=10

print 2*m*V
print sqrt(4*V**2*sm**2+4*m**2*sv**2)
```