Poisson Statistics

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In this experiment, poisson statistics are going to be examined by using randomness property of radioactive decay.

1. INTRODUCTION

Radioactive decay is a random process in which the emission of radiation depends on the number of atoms that can decay and a probability function that is characteristic their natural lifetimes. The detection of particles is random and any two measurements of the particles over equal periods of time will most likely be different. For large numbers, the difference between the measurements will be a small percentage. The probability of detecting a specific number of events for a given measurement is given by the standard normal or Gaussian distribution as was studied in the experiment on statistical analysis. However, for the measurement of a small number of events, the probability distribution for detecting a specific number of events is different and is given by a Poisson distribution.

For rare events, the average number detected might also be much less than one, The Poisson distribution applies to these measurements and is useful for determining the probability of detecting a single event or more than one event in the same period. The Poisson distribution is a special case of the binomial distribution, similar to the Gaussian distribution being a special case.

The Poisson Distribution is given by

$$P(\mu, n) = \frac{\mu^n e^{-\mu}}{n!}$$

where $P(\mu, n)$ is the normalized probability that in a given time interval n events will be observed and μ is the average number of events that are observed when many samples are taken. Probability function is normalized, so we know that

$$\sum_{n=0}^{N} P(n) = 1$$

We can also show that the standard deviation of the Poisson distribution is equal to the square root of the its mean $\sigma^2 = \mu$.

If the mean μ becomes larger and Poisson distribution approaches to Gaussian distribution as we know as

$$P_G(\mu, \sigma, x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

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We can also calculate the probability of observing n counts during a time interval t as

$$P(\mu, n) = P(\alpha, t, n) = \frac{(\alpha t)^n e^{-\alpha t}}{n!}$$

2. EXPERIMENTAL SETUP

In this experiment,

- Geiger Counter with a Scaler
- Sample Holder
- Various Gamma-ray Sources
- Lead Absorbents
- Chart Recorder

are used to study Poisson statistics.

3. DATA AND ERROR ANALYSIS

In first step of the experiment, we determined the limit potential V to work the Poisson statistics. From 300 V to 500 V, we increased the voltage by step size 20 V. At the end, we thought 380 V is the best value to work with by using fitting Figure 1.

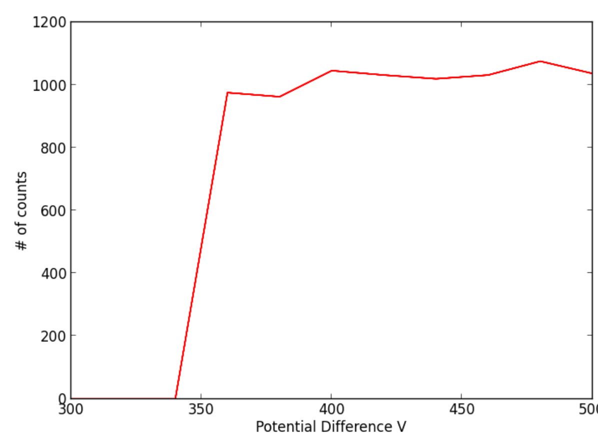

FIG. 1: The determation graph of the threshold potential

After finding the potential value, there are two steps in this experiment.In first place, there also two steps in here. We took 100 data for each from Greiger Counter for 10 seconds near and far, and 1 second near and far at position the sensor of radioactive decay. Thus, we have 4 data set and calculated χ^2, mean and standard deviation of each set by using Appendix of the course book and Python programming language.

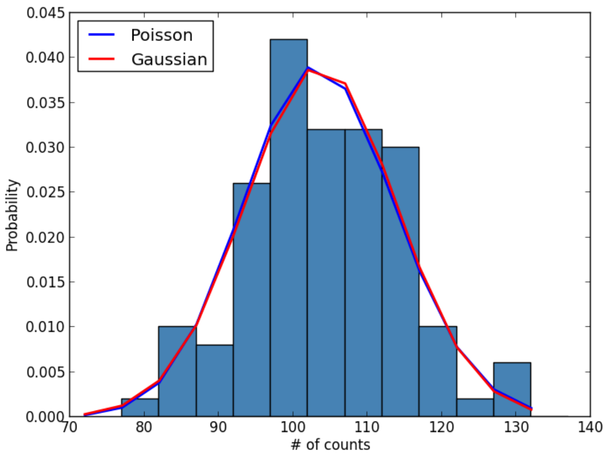

FIG. 2: 10 seconds from closer bin size=5

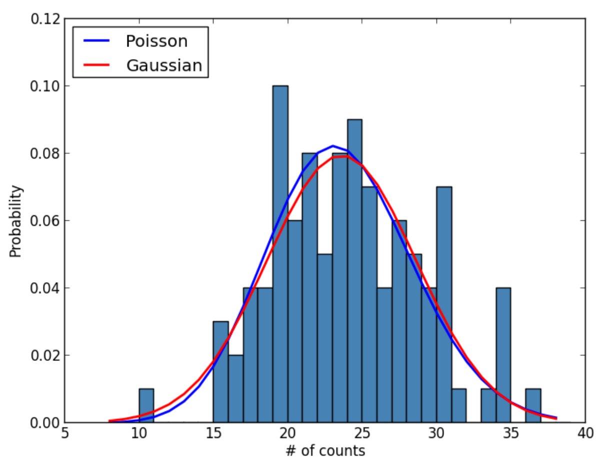

FIG. 3: 10 seconds from further bin size=1

number of counts	Mean (μ)	Standard Deviation (σ)
10 Sec. Close	103.68	10.18
10 Sec. Far	23.58	5.01
1 Sec. Close	10.18	2.82
1 Sec. Far	2.43	1.72

TABLE I: Mean and Standard Deviation for 4 data sets.

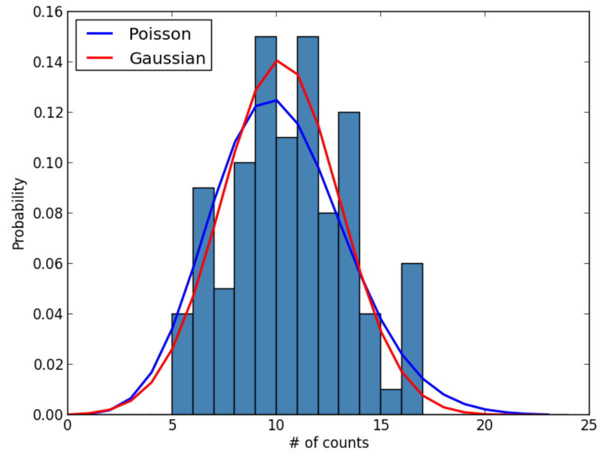

FIG. 4: 1 second from closer bin size=1

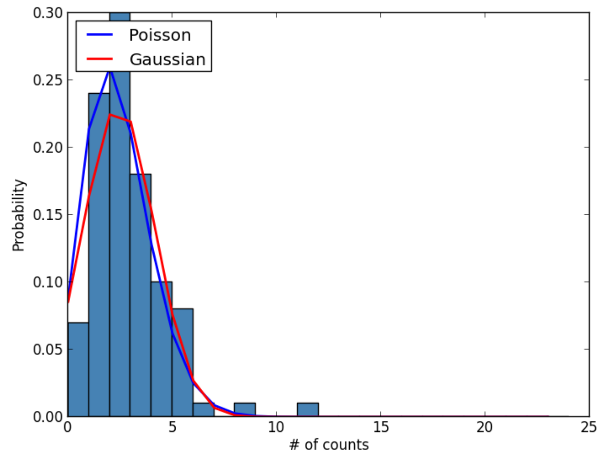

FIG. 5: 1 second from further bin size=1

By using $\chi^2 = \sum \frac{(y_i - f(x))^2}{\sigma_i^2}$ calculation, we found χ^2 and $\frac{\chi^2}{dof}$ where dof (degree of freedom) is data-1 for Poisson and data-2 for Gaussian.

number of counts	χ^2 for Gaussian	χ^2/dof for Gaussian
10 Sec. Close	0.0281	0.0025
10 Sec. Far	0.331	0.011
1 Sec. Close	0.253	0.011
1 Sec. Far	103.37	4.698

TABLE II: χ^2 for Gaussian

In second part, we have chart recorder and it draws

number of counts	χ^2 for Poisson	χ^2/dof for Poisson
10 Sec. Close	0.0281	0.00234
10 Sec. Far	0.391	0.0130
1 Sec. Close	0.214	0.0093
1 Sec. Far	2.646	0.115

TABLE III: χ^2 for Poisson

events and time as 1mm=1 sec. By binning our data for n=0 and n=1, we have two figures below:

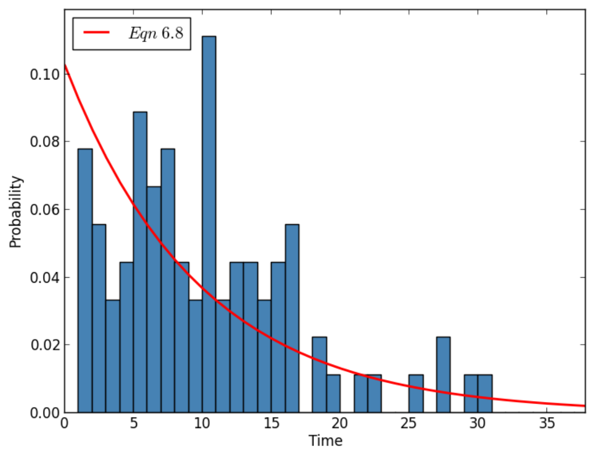

FIG. 6: n=0 Graph

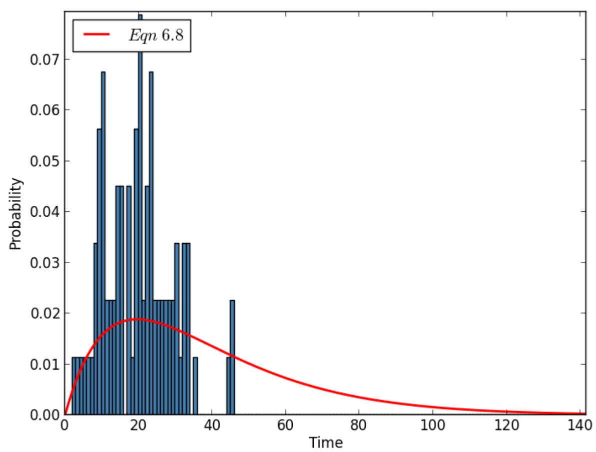

FIG. 7: n=1 Graph

where Equation 6.8 is

$$P_q(n+1,t) = \frac{(\alpha t)^n e^{-\alpha t}\alpha}{n!}$$

and $\alpha = \frac{total\ number\ of\ events}{total\ time}$.

Again χ^2 calculations give:

n	Mean (μ)	Standard Deviation (σ)	α
0	9.744	6.642	0.102
1	19.505	9.347	0.051

TABLE IV: Mean and Standard Deviation for 4 data sets.

n	χ^2 for Eqn 6.8	χ^2/dof for Eqn 6.8
0	0.600	0.0006
1	0.741	0.0007

TABLE V: χ^2 for Eqn. 6.8

4. CONCLUSIONS

We have seen two examples of Poisson processes , and have analyzed the effectiveness of the Poisson distribution as a model for these processes. For the radioactive decay, we measured the distribution of emission events for various expected values. We found the Poisson distribution to be a good fit for the data. However, Table I and II shows that in bigger μ values,χ^2 are smaller.

In second part,ee have found the Poisson distribution to be an accurate description for discrete events occurring on a continuous scale.

5. REFERENCES

- E. Gulmez, Advanced Physics Experiment, Istanbul, Bogazici University Publication, 1999
- http://web.mit.edu/8.13/www/experiments.shtml

Acknowledgments

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