X-ray Scattering: The Duane-Hunt Displacement Law

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1 Introduction

When energetic electrons strikes to chemical elements with high atomic number (Z), x-rays are produced, emitted. Generally as a material of target; tungsten(74), molybdenum(42), or rhodium(45) is used. Higher atomic number means higher energetic x-rays. There are two dominant ways of producing x-rays; first is bremsstrahlung and the second is characteristic x-rays.

Bremsstrahlung means "breaking radiation". Bremsstrahlung originally comes from German to describe the radiation which is emitted when electrons are decelerated or even "braked" in other words "stopped". When electrons sent to the metal material, without any collision with inner electrons of metal, the bombarding electrons go into orbit of nucleus and decelerates. Change in the energy because of deceleration gives off electromagnetic radiation. If the energy of bombarding electrons is high enough, that electromagnetic radiation is in the x-ray region of electromagnetic spectrum. Bremsstrahlung have a continuous spectrum. The process can be well understood by Figure 1. INSERT THE FIGURE Bremsstrahlung.jpg HERE

Characteristic x-rays are emitted when the bombarding electrons collide with inner shell of the atom and eject an electron from there. The quick filling of that hole by electrons dropping down

from higher levels give rise to sharply characteristic x-rays. Characteristic x-rays have discrete line spectrum. Characteristic x-ray photons named with the shell of the dropping down electrons. The process can be well understand and the name of the photons can be seen from Figure 2. INSERT THE SECOND 07-02.jpeg FIGURE HERE

From Bremsstrahlung process we can find Planck's constant using the fact that there is a minimum wavelength of x-ray radiation for a given accelerating potential. We know that the energy of the electrons is supplied by the accelerating potential applied on tube. Therefore we can write;

$$\frac{\hbar c}{\lambda_{min}} = \hbar \nu^{max} = qV_{acc} \tag{1.1}$$

This is so-called Duane-Hunt Displacement law which is named after William Duane and Franklin Hun in 1915.

In order to determine the wavelength of x-rays we can use the Bragg's law which is found by William Lawrence Bragg and William Henry Bragg in 1913.

$$2d \sin\theta = m\lambda \tag{1.2}$$

where d is separation between lattice points, m is order of diffraction and λ is wavelength of x-rays and θ is angle of scatter between x-rays and NaCl target. X-rays scatter from NaCl crystal target will form maxima at points that satisfy the Bragg's law. The equation can be well understood with the help of Figure 3. INSERT bragglaw(1).png FIGURE HERE. The path that is taken by the wave which is represented with blue line is grater than the red one's path. If the extra path is equal to the λ then the light will form a maxima, if that path is equal to $\frac{\lambda}{2}$ then the light will form minima. Extra path is $2d \sin\theta$.

X-rays have many application areas in today's world, such as x-ray crystallography, mammography, medical CT, airport security.

2 Experimental Setup

2.1 Apparatus

The equipment is as the following;
***NaCl Crystal,**
***X-ray Tube,**
***Molybdenum Target,**
***X-ray Detector,**
***Computer.**

2.2 Procedure

By using computer program set the applied voltage V_{acc} to $35kV$, in order to get full spectrum, and increasing the angle of deviation of NaCl crystal by 0.1 degree, find the rate which is Impulse/second. After you collect the data from 35 kV, save it as a text file.
Then repeat the same procedure by increasing the applied voltage with little amount. As in our case, selected applied voltages are 15 kV, 17 kV, 20 kV, 22 kV, 25 kV, 27 kV, 30 kV.

3 Data and Analysis

The text files which contains data about crystal angle and rate at mentioned voltages is used to obtain scatter-graph on excel.

Table 1: Theoretical θ Values θ_{teo}

θ_{teo} (°)
$\theta_{\beta 1}^{teo} = 6.434$
$\theta_{\alpha 1}^{teo} = 7.222$
$\theta_{\beta 2}^{teo} = 12.95$
$\theta_{\alpha 2}^{teo} = 14.56$
$\theta_{\beta 3}^{teo} = 19.64$
$\theta_{\alpha 3}^{teo} = 22.16$

From 35 kV scatter-graph, I found experimental θ values where characteristic x-rays occur. Please see Appendix B for details. The electron transition to lower atomic levels in heavy atoms have energy regions in x-ray region of the electromagnetic spectrum.

The electron transition we are interested in is K_α and K_β, which are from $n = 2$ to $n = 1$ and $n = 3$ to $n = 1$, respectively. Energies for molybdenum target is as following for each electron transition;

$$E_{K\alpha} = 17426.82$$

and

$$E_{K\beta} = 19608.3eV.$$

With the help of equation

$$\frac{hc}{\lambda} = E \tag{1.3}$$

I found corresponding wavelengths;

$$\lambda_\alpha = 0.709A^\circ$$

and

$$\lambda_\beta = 0.632A^\circ.$$

With the equation (1.2) Bragg's Law, we can find theoretical θ values θ_{teo} for each order of diffraction, where second subscript signifies in which order of diffraction we are.

Table 2: Comparison of θ_{teo} and θ_{exp}

θ_{teo} (°)	θ_{exp} (°)
$\theta^{teo}_{\beta 1} = 6.434$	$\theta^{exp}_{\beta 1} = 6.5$
$\theta^{teo}_{\alpha 1} = 7.222$	$\theta^{exp}_{\alpha 1} = 7.4$
$\theta^{teo}_{\beta 2} = 12.95$	$\theta^{exp}_{\beta 2} = 13.1$
$\theta^{teo}_{\alpha 2} = 14.56$	$\theta^{exp}_{\alpha 2} = 14.7$
$\theta^{teo}_{\beta 3} = 19.64$	$\theta^{exp}_{\beta 3} = 19.8$
$\theta^{teo}_{\alpha 3} = 22.16$	$\theta^{exp}_{\alpha 3} = 22.2$

Now, I can compare it with experimental θ values which are obtain through scatter-graph on excel. We know that characteristic x-rays occur at peak values, so with that knowledge we can find θ_{exp}. We can make comparison of and θ_{teo} with θ_{exp} .

Table 3: Corrected θ Values θ_{corr}

θ_{corr} ($^\circ$)
$\theta_{\beta 1}^{corr} = 6.358$
$\theta_{\alpha 1}^{corr} = 7.259$
$\theta_{\beta 2}^{corr} = 12.97$
$\theta_{\alpha 2}^{corr} = 14.57$
$\theta_{\beta 3}^{corr} = 19.68$
$\theta_{\alpha 3}^{corr} = 22.09$

By applying least-square fit method to the line

$$\theta_{teo} = p0 \times \theta_{exp} + p1$$

I get the following for slope

$$p0 = 1.002 \pm 0.007036$$

and for y-intercept

$$p1 = -0.1549 \pm 0.1063.$$

See Appendix A for details. By this line-fit, I corrected my θ_{exp} values. I find the following values for $\theta corr$

Table 4: Minimum Angle, Minimum Wavelength, Maximum Frequency for Corresponding Applied Voltages Table

$V_{acc}(kV)$	$\theta_{min}(^\circ)$	$\lambda_{min}(A^\circ)$	$\nu_{max}(10^{18}Hz)$
15	8.470	0.831	3.61
17	7.492	0.735	4.08
20	6.362	0.625	4.80
22	5.783	0.568	5.28
25	5.087	0.500	6.00
27	4.625	0.455	6.59
30	3.969	0.390	7.59

Then, for each different voltage at my voltage set, I find the the wavelength λ_{min} values. But in order to find λ_{min} values, I firstly find θ_{min} values. Then used the Bragg's Law - equation 1.2- to find λ_{min}. In Bragg's law, following constants are used $d = 2.82A^\circ$ as separation between lattice points of NaCl, and diffraction order

$m = 1$. **Calculation of** θ_{min} **is done by least-square fit method to the first order portion of the full spectrum. For details, please see Appendix B. After,** θ_{min} **and** λ_{min}**, I find frequency** ν_{max} **with equation** $c = \lambda \times \nu$ **where c is speed of light which is equal to** $3 \times 10^8 m/sec$.

Later, I applied least-square fit method to the line

$$V_{acc} = po\nu_{max} + p1.$$

I find following values for slope and y-intercept, p0 and p1 respectively.

$$p0 = 3.832 \times 10^{-15} \pm 2.9 \times 10^{-20} \qquad p1 = 1513 \pm 0.1617.$$

For details, please refer to Appendix C.

4 Results

The slope of V_{acc} **vs.** ν_{max} **graph gives us** $\frac{h}{q}$**. We know this from Duane-Hunt Displacement law, equation (1.1). When we divide the equation by q, we get** $V_{acc} = \frac{h}{q}\nu_{max}$**. From this formula, we can find experimental Planck's constant by multiplying the slope with elementary charge q which** $q = 1.6022765 \times 10^{-19} coulomb$**. My experimental Planck's constant is**

$$h = 6.140 \times 10^{-34} \triangle h = 4.65 \times 10^{-39}.$$

I can calculate the error with formula

$$\frac{v_{experimental} - v_{theoretical}}{v_{theoretical}} \times 100.$$

Theoretical Planck's constant is $h = 6.62606957 \times 10^{-34}$ **and my error is**

$$\epsilon = 7\%$$

My error is small, which is expected since the experiment conducts in X-ray tube, which is an isolated environment, therefore; there is not too much cause for systematic error.

5 Acknowledgements

I would like to thank to Saime Sarikaya for her help about finding θ_{min}, and wavelengths.

6 Appendix A

INSERT thetalarson.pdf TO HERE. There is no error on theoretical values of θ. As a error on my experimental value, which is systematic error, I take $0.1°$.

7 Appendix B

With my data when $V_{acc} = 35kV$ I draw a graph with Excel which is shown in following figure. The points where peaks occurs give us the θ_{gm}^{exp} where $g = \alpha, \beta$ and $m = 1, 2, 3$. INSERT THE Kitap1.xlsx TO HEREE

For each V_{acc} I find the θ_{min} values, by applying Excel's fit tool to the first portion of full spectrum INSERT all-the-exam-graphs-for-each-v.xlsx to here

8 Appendix C

INSERT sonfitoldumha.pdf TO HERE. Here is my line-fit for V_{acc} and ν_{max}. While obtaining these graph through ROOT, I assumed that there is no error on V_{acc}.

9 References

For first figure->http://www4.nau.edu/microanalysis/Microprobe-SEM/Signals.html
For second figure->http://whs.wsd.wednet.edu/faculty/busse/mathhomepage/busse
For third figure->http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/bragg.html
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/xrayc.html