6、电磁波辐射.

真空麦氏方程.

$$\begin{cases} \nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{B}=\mu_0\vec{J}+\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \end{cases}$$

$\because \nabla\cdot\vec{B}=0$

$\therefore \nabla\times\vec{A}=\vec{B}$

但 $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\neq 0$

$\therefore \vec{E}\neq\nabla\varphi$.

但 $\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}$

$=\nabla\times\vec{E}+\frac{\partial}{\partial t}\nabla\times\vec{A}$

$=\nabla\times(\vec{E}+\frac{\partial\vec{A}}{\partial t})=0$

$\therefore \vec{E}+\frac{\partial\vec{A}}{\partial t}=-\nabla\varphi$

$\therefore \vec{E}=-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}$.

规范变换:

$\vec{A}'=A+\nabla\lambda$, $\varphi'=\varphi-\frac{\partial\lambda}{\partial t}$

$\Rightarrow \vec{B}'=\nabla\times\vec{A}'=B$

$\vec{E}'=-\nabla\varphi+\nabla\frac{\partial\lambda}{\partial t}-\frac{\partial\vec{A}}{\partial t}-\nabla\frac{\partial\lambda}{\partial t}=-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}=\vec{E}$.

又所以自由度冗余，通过取规范来给出限制

①、库仑规范，$\nabla\cdot\vec{A}=0$

$$\vec{E}=\underline{\underline{-\nabla\varphi}}-\frac{\partial\vec{A}}{\partial t}.$$

无旋（有源） （涡旋）（无源）

$$-\nabla\cdot\vec{E}=\nabla^2\varphi+\frac{\partial}{\partial t}\nabla\cdot\vec{A}=-\frac{1}{\varepsilon_0}\rho$$

$$\nabla^2\varphi=-\frac{\rho}{\varepsilon_0}$$

$$\nabla\times(\nabla\times\vec{A})=\mu_0\vec{J}-\mu_0\varepsilon_0\nabla\frac{\partial\varphi}{\partial t}-\mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}.$$

$$\left(\nabla^2\vec{A}-\mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}\right)-\nabla\left(\nabla\cdot\vec{A}+\mu_0\varepsilon_0\frac{\partial\varphi}{\partial t}\right)=-\mu_0\vec{J}$$

$$\nabla\cdot\vec{A}=0.$$

$$\therefore\ \nabla^2\vec{A}-\mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}-\mu_0\varepsilon_0\frac{\partial\nabla\varphi}{\partial t}=-\mu_0\vec{J}$$

② ~~洛伦兹~~ Lorentz规范：

$$\nabla\cdot\vec{A}+\mu_0\varepsilon_0\frac{\partial\varphi}{\partial t}=0$$

$$\therefore\ \nabla^2\vec{A}-\mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{J}$$

$$\nabla^2\varphi+\frac{\partial}{\partial t}\nabla\cdot\vec{A}=-\frac{1}{\varepsilon_0}\rho$$

$$\therefore\ \nabla^2\varphi-\mu_0\varepsilon_0\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon_0}$$

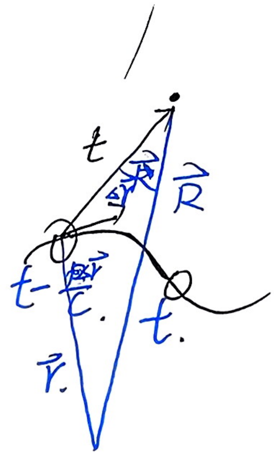
$$\Box^2=\nabla^2-\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}.$$

$$\therefore\ \begin{cases}\Box^2\varphi=-\frac{\rho}{\varepsilon_0}\\ \Box^2\vec{A}=-\mu_0\vec{J}\end{cases}$$

6.2 推迟势

$\therefore \varphi(\vec{x},t)=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}',t-\frac{|\Delta r|}{c})}{|\Delta r|}dV.$

$\vec{A}(\vec{x},t)=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}',t-\frac{|\Delta r|}{c})}{|\Delta r|}dV.$



φ和$\vec{A}$满足 Lorentz规范.

$\therefore\begin{cases}\vec{B}=\nabla\times\vec{A},\\ \vec{E}=-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}.\end{cases}$

6.3. 辐射.

辐射: 脱离源点向外传播的场

$\nabla\vec{J}+\frac{\partial\rho}{\partial t}=0.$

$\vec{J}=\vec{J}(\vec{r}')e^{-i\omega t}$, $\rho=\rho(\vec{r}')e^{-i\omega t}.$

推迟$\vec{J}=\vec{J}(\vec{r}')\cdot e^{i(k|\Delta r|-\omega t)}$, $k=\frac{\omega}{c}.$

$\therefore \vec{A}(\vec{x},t)=\frac{\mu_0}{4\pi}e^{-i\omega t}\int\frac{\vec{J}(\vec{r}')e^{ik|\Delta r|}}{|\Delta r|}\cdot dV$

$\therefore \vec{A}(\vec{x})=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')\cdot e^{ik|\Delta r|}}{|\Delta r|}dV.$

$\nabla\vec{J}(\vec{r}')=i\omega\rho(\vec{r}')$

$\therefore \vec{B}=\nabla\times\vec{A}.$

在线源区中, $\vec{J}=0$

$\therefore \nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}=-\mu_0\varepsilon_0 i\omega\vec{E}.$

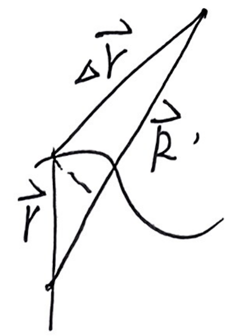
$\therefore \vec{E}=\frac{ic}{k}\nabla\times\vec{B}.$

$\Delta\vec{r} = \vec{R} - \vec{r}$.

$|\Delta\vec{r}|^2 = \vec{R}^2 + \vec{r}^2 - 2\vec{R}\vec{r}$

$\approx \vec{R}^2 - 2\vec{R}\vec{r}$.

$\Delta r = |\vec{R}|\sqrt{1 - 2\frac{|\vec{r}|\cos\theta}{|\vec{R}|}}$.



$\sqrt{1+x} \approx 1 + \frac{1}{2}x$

$\therefore \Delta r = |R|\left(1 - \frac{1}{2}\cdot 2\frac{|r|}{R}\cos\theta\right)$

$= |R| - |r|\cdot e\cos\theta$

$= R - \vec{e}_R\cdot\vec{r}$

①辐射区，$\vec{R} \gg \lambda$.

②小区域辐射，$|\vec{r}| \ll R$

~~③~~ $|\vec{r}| \ll \lambda$.

$\therefore \vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r})e^{ik(R - \vec{e}_R\cdot\vec{r})}}{R - \vec{e}_R\cdot\vec{r}}\,dV$.

$\approx \frac{\mu_0}{4\pi R}\cdot\int_V \vec{J}(\vec{r})\cdot e^{ik(R - \vec{e}_R\cdot\vec{r})}\,dV$.

$= \frac{\mu_0 e^{ikR}}{4\pi R}\int \vec{J}(\vec{r})\cdot e^{-ik(\vec{e}_R\cdot\vec{r})}\,dV$.

$k = \frac{2\pi}{\lambda}$, $\left(\frac{\vec{e}_R\cdot\vec{r}}{\lambda} \ll 1\right)$ $e^{-\frac{i2\pi}{2\pi}\left(\frac{\vec{e}_R\cdot\vec{r}}{\lambda}\right)} \approx \left(1 - \frac{i2\pi}{2\pi\lambda}\cdot\vec{e}_R\cdot\vec{r} + \cdots\right)$

$\therefore \vec{A}^{(0)} = \frac{\mu_0 e^{ikR}}{4\pi R}\int \vec{J}(\vec{r})\,dV$.

$\int \vec{J}(\vec{r})\,dV = \int \frac{dQ}{dt}\cdot d\vec{l} = \int d\vec{l}\cdot\vec{v}\,dQ = \dot{\vec{q}}$

$\left(\frac{d}{dt}\int d\vec{l}\cdot dQ\right) = \frac{d}{dt}[q\cdot\vec{l}]$

$\therefore \vec{A}^{(0)} = \frac{\mu_0 e^{ikR}}{4\pi R} \dot{p}$，$p = qd$ 为电偶极，$\vec{A}^{(0)}$ 为电偶极辐射

$$\vec{B}^{(0)} = \nabla \times \vec{A}^{(0)} = \frac{\mu_0}{4\pi}\left(\nabla \times \frac{e^{ikR}}{R}\right) \times \dot{p}$$

$$= \frac{ik\mu_0 e^{ikR}}{4\pi R} \vec{e}_R \times \dot{p}$$

$$= \frac{k e^{ikR} \mu_0}{4\pi R \omega} \vec{e}_R \times i\omega \dot{p}$$

$$= \frac{e^{ikR}}{4\pi \varepsilon_0 c^3 R} \ddot{p} \times \vec{e}_R$$

$\left\{ \begin{array}{l} \nabla \to ik\vec{e}_R \\ \frac{\partial}{\partial t} \to -i\omega \end{array} \right.$

$$E = \frac{ic}{k} \nabla \times \vec{B} = \frac{ic}{k}(ik\vec{e}_R) \times \vec{B}$$

$$= cB \times \vec{e}_R$$

$$\langle S \rangle = \frac{1}{2} Re(\vec{E}^* \times \vec{H}) = \frac{c}{2\mu_0} Re[(B^* \times e_R) \times B]$$

磁偶极和电四极：

$$\vec{A}^{(1)} = \frac{\mu_0 e^{ikR}}{4\pi R} \int \vec{J}(\vec{r}) \cdot (-ik \cdot \vec{e}_R \cdot \vec{r})$$

$$= \frac{-ik\mu_0 e^{ikR}}{4\pi R} \int (\vec{J}(\vec{r})\vec{r}) \cdot \vec{e}_R$$

$$\vec{r}\vec{J}(\vec{r}) = \frac{1}{2}(\vec{r}\vec{J} + \vec{J}\vec{r}) + \frac{1}{2}(\vec{r}\vec{J} - \vec{J}\vec{r})$$

$$\vec{A}^{(1)} = \frac{-ik\mu_0 e^{ikR}}{4\pi R}\left[-\vec{e}_R \times \vec{m} + \frac{1}{2}\int [(e_R \cdot \vec{r})\vec{J} + (e_R \cdot \vec{J})\vec{r}] dV\right]$$

$$\frac{d\vec{r}\vec{r}}{dt} = \vec{r}\vec{v} + \vec{v}\vec{r}$$

6.3

$\therefore \vec{A}(\vec{x}) = \frac{-ik\mu_0 e^{ikR}}{4\pi R}\left(-\vec{e}_R\times\vec{m} + \frac{1}{6}\vec{e}_R\cdot\dot{\vec{\mathcal{D}}}\right)$

$\nabla \to ik e_R$

磁偶极：

$$\vec{A}_m(\vec{x}) = \frac{ik\mu_0 e^{ikR}}{4\pi R}(\vec{e}_R\times\vec{m})$$

$$\vec{B} = \nabla\times\vec{A}_m(\vec{x}) = \frac{\cancel{i}k^2\mu_0 e^{ikR}}{4\pi R}(\vec{e}_R\times\vec{m})\times\vec{e}_R.$$

$$\vec{B} = \cancel{\ominus}\frac{k^2\mu_0}{\omega^2}\frac{e^{ikR}}{4\pi R}(\vec{e}_R\times\ddot{\vec{m}})\times\vec{e}_R.$$

$$= \frac{\mu_0 e^{ikR}}{4\pi c^2 R}(\ddot{\vec{m}}\times\vec{e}_R)\times\vec{e}_R$$

$$\vec{E} = c\vec{B}\times\vec{e}_R = \frac{-\mu_0 e^{ikR}}{4\pi cR}(\ddot{\vec{m}}\times\vec{e}_R)$$

电四极

$$\vec{A}(\vec{x}) = -\frac{ik\mu_0 e^{ikR}}{24\pi R}e_R\cdot\dot{\vec{\mathcal{D}}}$$

$$\vec{B} = ik\vec{e}_R\times(\vec{e}_R\cdot\dot{\vec{\mathcal{D}}}) = \frac{e^{ikR}}{24\pi\varepsilon_0 c^3 R}\dddot{\vec{\mathcal{D}}}\times\vec{e}_R$$

$$\vec{E} = c\vec{B}\times\vec{e}_R$$