2 介质中的电磁场

2.1 诱导偶极子，~~□~~ 介质中的电场

电中性的"分子在电场作用下，正负中心分开，形成诱导电偶极子.

$\vec{p} = \alpha \vec{E}$,

~~对于各向导体~~异性材料. α 是极化张量

对于电介质，材料的分子形成了许多朝着同一方向的~~□~~诱导偶极子

∴ 电极化强度矢量：$\vec{P} = \dfrac{\sum \vec{p}_i}{\Delta V}$

极化的电介质产生的电势：

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{P}dV \cdot \hat{\Delta r}}{|\Delta r|^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_V \vec{P}\cdot\nabla\left(\frac{1}{\Delta r}\right)\cdot dV.$$

$$\nabla\cdot\left(\vec{P}\frac{1}{\Delta r}\right) = \vec{P}\cdot\nabla\left(\frac{1}{\Delta r}\right) + \frac{1}{\Delta r}(\nabla\cdot\vec{P})$$

$$\therefore V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \nabla\left(\frac{\vec{P}}{\Delta r}\right) dV - \frac{1}{4\pi\varepsilon_0}\int_V \frac{1}{\Delta r}(\nabla\cdot\vec{P})\cdot dV.$$

$$= \frac{1}{4\pi\varepsilon_0}\oint_{\partial V}\frac{\vec{P}}{\Delta r} d\vec{S} + \frac{1}{4\pi\varepsilon_0}\int_V \frac{-(\nabla\vec{P})}{\Delta r} dV.$$

$$\vec{P}d\vec{S} = \vec{P}\cdot\vec{n}\cdot dS = dQ = \sigma_p dS, \quad -\nabla\vec{P}dV = dQ = \rho dV$$

∴ 第一项为表面电荷的电势，($\sigma_p = \vec{P}\cdot\vec{n}$)

第二项为体电荷 $\rho = -\nabla\vec{P}$ 的电势

由电荷守恒：表面束缚电荷与体内的相等，符号相反

$$\int\rho dV + \oint_{\partial V}\sigma dS \Longrightarrow \int\rho dV + \int\vec{P}\cdot d\vec{S} = 0$$

$$\rho + \nabla\vec{P} = 0.$$

将非束缚电荷称为自由电荷 ρ_f

在电介质中,总电荷 $\rho = \rho_f + \rho_b$

$\therefore \varepsilon_0 \nabla \cdot \vec{E} = \rho = \rho_f + \rho_b = -\nabla \cdot \vec{P} + \rho_f$

$\therefore \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$.

定义电位移矢量:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}.$$

$$\therefore \nabla \cdot \vec{D} = \rho_f.$$

对于线性电介质:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$, χ_e 为电极化率

$\therefore \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (1+\chi_e) \vec{E}$, $\varepsilon_r = 1+\chi_e$, $\varepsilon = \varepsilon_0 \varepsilon_r$.

$\therefore \vec{D} = \varepsilon \vec{E}$. (相对介电常数) (介电常数)

对非均匀介质

$\vec{P} = \chi_m \vec{E}$, χ_m 为电极化率张量

2.2 介质中的磁场

分子电流给出介质中的磁偶极子:

$$\vec{m} = I \cdot \vec{S},$$

在外电磁场存在下,介质被磁化,$\vec{m}$ 有定向性,考虑磁化强度

$$\vec{M} = \frac{\sum \vec{m}}{\Delta V},$$ 为单位体积中磁偶极子

单个磁偶极子的矢势:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \cdot \frac{\vec{m} \times \vec{r}}{|\Delta r|^2}$$

$\therefore$ 总矢势: $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M} \times \Delta \vec{r}}{|\Delta r|^2} dV$.

同理：$\nabla \frac{1}{|\vec r|} = \frac{\hat{r}}{|\vec r|^2}$.

$$\therefore \int \frac{\vec m \times \hat r}{|\vec r|^2} dV = \int \vec m \times \nabla \frac{1}{|\vec r|} \cdot dV.$$

$$\nabla \times \left(\vec m \cdot \frac{1}{|\vec r|}\right) = \left(\nabla \cdot \frac{1}{\vec r}\right) \times \vec m + \frac{1}{\vec r} \cdot \nabla \times \vec m.$$

$$\vec m \times \nabla \frac{1}{|\vec r|} = \frac{1}{\vec r} \cdot \nabla \times \vec m - \nabla \times \left(\vec m \cdot \frac{1}{|\vec r|}\right)$$

$$\vec A(\vec r) = \frac{\mu_0}{4\pi} \int \frac{\nabla \times \vec m}{|\vec r|} dV + \frac{\mu_0}{4\pi} \int -\nabla \times \left(\frac{\vec m}{|\vec r|}\right) dV$$

$$= \frac{\mu_0}{4\pi} \cdot \int \frac{\nabla \times \vec m}{|\vec r|} \cdot dV + \frac{\mu_0}{4\pi} \oint_{\partial V} \frac{\vec m}{|\vec r|} \times d\vec S$$

$$\left(\int (\nabla \times \vec v) dV = -\oint_{\partial V} \cancel{\nabla \cdot (\nabla \times v) d} \; \vec v \times d\vec S \right)$$

$$\cancel{\nabla \cdot \vec v = \nabla \times \vec v}$$

∴第一项 $\nabla \times \vec m = \vec J_b$ 为体电流密度.

第二项 $\vec m \times \vec n = \vec K_b$ 为面电流密度.

$$\therefore \vec A(\vec r) = \frac{\mu_0}{4\pi} \left(\int_V \frac{\vec J_b}{|\vec r|} dV + \oint_{\partial V} \frac{\vec K_b}{|\vec r|} \cdot dS \right).$$

由 $\nabla \cdot (\nabla \times \vec m) = \nabla \cdot \vec J_b = 0$

∴ $\vec J_b$ 是稳恒电流.

在磁介质中：

总电流 $\vec J = \vec J_b + \vec J_f$.

$\frac{1}{\mu_0}\nabla\times\vec{B} = \vec{j}_b + \vec{j}_f = \vec{j}_f + \nabla\times\vec{M}$

$\therefore \nabla\times\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{j}_f$.

$\therefore$ 磁场 $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$.

$\therefore \nabla\times\vec{H} = \vec{j}_f$

在线性介质中：

$\vec{M} = \chi_m \vec{H}$

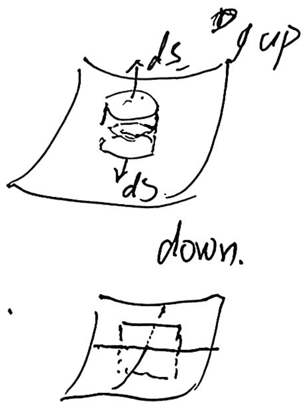
$\therefore \frac{\vec{B}}{\mu_0} - \vec{H} = \chi_m \vec{H}$，$\chi_m$为磁化率

$\vec{B} = \mu_0(1+\chi_m)\vec{H}$，

$\vec{B} = \mu\vec{H}$，μ为磁导率

2.3 ~~类~~边界条件：

~~普适边界~~：对静电场



$\oint_{\partial V}\vec{D}\cdot d\vec{S} = (\vec{D}_{up} - \vec{D}_{down})\cdot d\vec{S} = dQ_f$.

$\therefore (\vec{D}_{up} - \vec{D}_{down})\cdot\vec{n} = \sigma_f \overset{同理}{\Longrightarrow} (\vec{E}_{up} - \vec{E}_{down})\cdot\vec{n} = \frac{\sigma}{\varepsilon_0}$.

$\nabla\times\vec{E} = 0$,

$\therefore E^{\parallel}_{up} = E^{\parallel}_{down}$

而 $\nabla\times\vec{D} = \nabla\times\vec{P} \Rightarrow D^{\parallel}_{up} - D^{\parallel}_{down} = P^{\parallel}_{up} - P^{\parallel}_{down}$

在线性介质中(均匀)，$\rho_b = -\nabla\cdot\vec{P} = -\nabla\cdot\left(\frac{\chi_e}{1+\chi_e}\vec{D}\right) = -\left(\frac{\chi_e}{1+\chi_e}\right)\rho_f$.

$\therefore \rho_f = 0 \Rightarrow \rho_b = 0$

由 $(\vec{D}_{up} - \vec{D}_{down})\cdot\vec{n} = \sigma_f \Rightarrow \varepsilon_{up}E^{\perp}_{up} - \varepsilon_{down}E^{\perp}_{down} = \sigma_f$.

在边界上：$E^{\perp}_{up} = -\frac{\partial V_{up}}{\partial n}$

$$= -\varepsilon_{up}\cdot\frac{\partial V_{up}}{\partial n} + \varepsilon_{down}\cdot\frac{\partial V_{down}}{\partial n} = \sigma_f.$$

$(V_{up} = V_{down}$，电势连续$)$.

对静磁场：

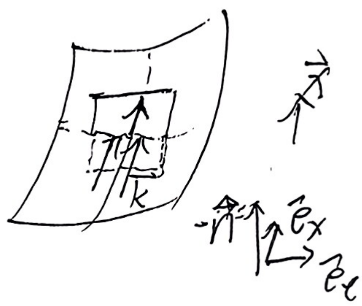
$$\vec{j}_b = \nabla\times\vec{M} = \nabla\times\left(\frac{\vec{B}}{\mu_0} - \vec{H}\right) = \nabla\times\left(\frac{\mu}{\mu_0}-1\right)\vec{H} = \left(\frac{\mu}{\mu_0}-1\right)\vec{j}_f$$

$\vec{j}_f = 0 \Rightarrow \vec{j}_b = 0$.

由 $\nabla\cdot\vec{B} = 0 \Rightarrow \vec{B}^{\perp}_{up} = \vec{B}^{\perp}_{down}$.

由 $\nabla\cdot\vec{H} = -\nabla\cdot\vec{M}$

$$H^{\perp}_{up} - H^{\perp}_{down} = \cancel{(B^{\perp}_{up} - B_{down}}\ -(M^{\perp}_{up} - M^{\perp}_{down}).$$



由安培环路定理：

$$(\vec{H}^{\parallel}_{up} - \vec{H}^{\parallel}_{down})\,d\vec{l} = \vec{K}_f\, dl\cdot\vec{e}_x\ ,\quad \hat{e}_x\cdot\vec{e}_l = 0.$$

$$\therefore \cancel{H^{\parallel}_{up} - H^{\parallel}_{down} = \vec{K}_f\cdot\vec{e}_x}$$

$$d\vec{l} = \vec{e}_x = \vec{n}\times\vec{e}_l.$$

$$\therefore \vec{K}_f\, dl\cdot\vec{e}_x = \vec{K}_f(\vec{n}\times\vec{e}_l)\,dl.$$

$$= \vec{K}_f\cdot(\vec{n}\times d\vec{l})$$

$$= (\vec{K}_f\times\vec{n})\,d\vec{l}.$$

$$\therefore \vec{H}^{\parallel}_{up} - \vec{H}^{\parallel}_{down} = \vec{K}_f\times\vec{n}.\ \xrightarrow{\text{同理}}\ \vec{B}^{\parallel}_{up} - \vec{B}^{\parallel}_{down} = \mu_0(\vec{K}\times\vec{n})$$

线性均匀介质中：

$$\frac{1}{\mu_{up}}\vec{B}^{\parallel}_{up} - \frac{1}{\mu_{down}}\vec{B}^{\parallel}_{down} = \vec{K}_f\times\vec{n}.$$