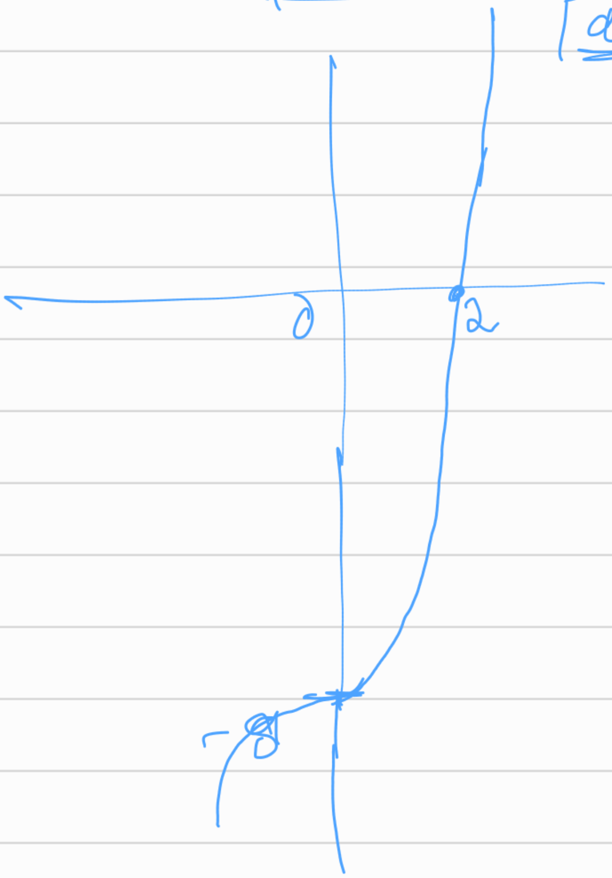
$x = (a)^{1/3}$

$x^3 = a$

$f(x) = x^3 - a.$

$\boxed{a = 8}$

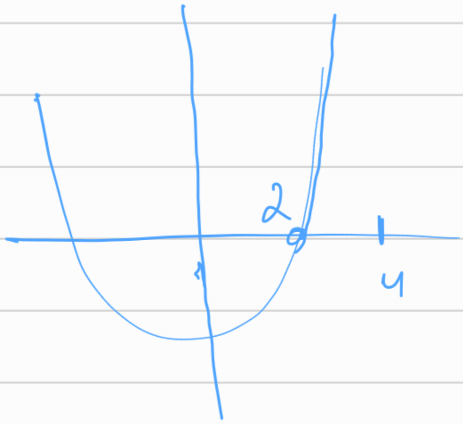


$x = a^2$

$x^2 = a$

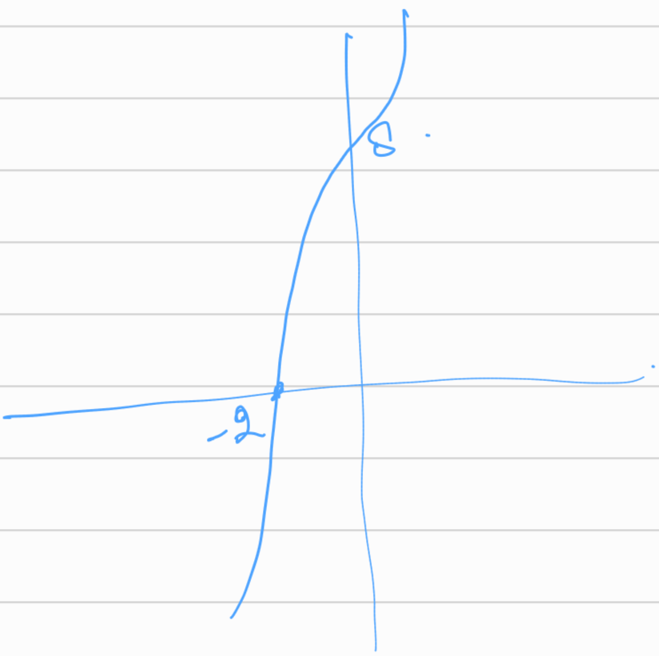
$f(x) = x^2 - a.$

$a = 4$



$x^2 + a$ doesn't exist in real plane

$f(x) = x^3 + 8.$



$\rightarrow \quad f(x) = x^3 - a . \qquad\qquad \infty \leq a \leq \infty$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}.$$

$$= x_n - \left[\frac{x_n^3 - a}{3x_n^2}\right]$$

$$x_{n+1} = \frac{3x_n^3 - x_n^3 + a}{3x_n^2}.$$

$$\boxed{x_{n+1} = \frac{1}{3}\left[2x_n + \frac{a}{x_n^2}\right]}.$$

$\rightarrow$ Convergence rate.

$$\rightarrow . \quad x_n = x(1+\delta). \qquad \rightarrow \frac{x_n}{x} = (1+\delta)$$

Since $x_{n+1} = \frac{1}{3}\left[2x_n + \frac{a}{x_n^2}\right].$

$$\frac{x_{n+1}}{x} = \frac{1}{3}\left(2\frac{x_n}{x} + \frac{a}{x_n^2 x}\right)$$

we know $x^3 = a.$

$$\left(\frac{a}{x}\right) = x^2. \qquad \downarrow \frac{x^2}{x_n^2}$$

$$\Rightarrow \quad 1 + \delta_{n+1} = \frac{1}{3}\left(2(1+\delta_n) + \frac{1}{(1+\delta_n)^2}\right)$$

$$1+\delta_{n+1} = \frac{1}{3}\left(2(1+\delta_n) + \frac{1}{(1+\delta_n)^2} \right).$$

Taylor expand $\underbrace{1+\delta_n}_{x})^{-2}$

$$= 1 - 2x + 3x^2 - 4x^3 + (O^4).$$

$$\Rightarrow \cancel{1}+\delta_{n+1} = \frac{1}{3}\left(2+\cancel{2\delta_n} + 1 - \cancel{2\delta_n} + 3\delta_n^2 - 4\delta_n^3 + O^4 \right)$$

$$\delta_{n+1} = \delta_n^2 - 4\delta_n^3 + O(\delta_n^4).$$

$$\delta_{n+1} = \delta_n^2 + O(\delta_n^3).$$

quadratic.