

Proving Trigonometric Formulas using Euler's Formula: $e^{ix} = \cos(x) + i \cdot \sin(x)$

$$\sin^2(x) + \cos^2(x) = 1$$

$$e^{ix} = \cos x + i \cdot \sin x$$

$$e^{-ix} = e^{i(-x)} = \cos(-x) + i \cdot \sin(-x) = \cos(x) - i \cdot \sin(x)$$

$$e^{ix} \cdot e^{-ix} = (\cos x + i \cdot \sin x) (\cos x - i \sin x)$$

$$1 = \cos^2 x - i \cos x \sin x + i \sin x \cos x - i^2 \sin^2 x$$

$$1 = \cos^2(x) + \sin^2(x)$$

(a)

$$\vec{P}(x,y) = P(x,y) \hat{i}$$

$$\oint_C \vec{P} \cdot d\vec{r}$$

$$\oint_C P(x,y) dx = \int_a^b P(x, y_1(x)) dx - \int_a^b P(x, y_2(x)) dx$$

$$= \int_a^b (P(x, y_1(x)) - P(x, y_2(x))) dx = - \int_a^b (P(x, y_2(x)) - P(x, y_1(x))) dx$$

$$= - \int_a^b P(x,y) \Big|_{y=y_1(x)}^{y=y_2(x)} dx = - \int_a^b \left(\frac{\partial P}{\partial y} \right) dy dx$$

(b)

Probability Lesson #22
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Uniform Distribution

$$X \sim \text{Unif}(a,b)$$

$$(b-a)h = 1$$

$$\Rightarrow h = \frac{1}{b-a}$$

$$f_X(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

$$F(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ 1, & \text{if } x > b \end{cases}$$

$$E(X) = \frac{b+a}{2}; \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

(c)

f continuous on $[a,b]$

Fund. theorem of calculus

$$\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) dt = f(x)$$

- Every cont. f has an antiderivative $F(x)$
- Connection between derivatives/integration

$$F(x) = \int_a^x f(t) dt, \text{ where } x \text{ in } [a,b]$$

$$\frac{d}{dx} \left(\int_a^x \cos^2 t \ln(t - \sqrt{t}) dt \right) = \frac{\cos^2 x}{\ln(x - \sqrt{x})}$$

(d)