Exam logic and set theory

Anna Szczepańska-Álvarez

1. Determine which of the following formulae are tautologies, which are contradictory and which are satisfiable

(a) $\neg((p \wedge \neg p) \rightarrow q)$

(b) $\neg p \wedge \neg(p \rightarrow q)$

(c) $((p \rightarrow q) \rightarrow p) \rightarrow p$

(d) $(p \vee \neg q) \rightarrow \neg(q \wedge \neg p)$

2. The following statements about sets are false. Give a counterexample to each statement

(a) $A \cup B = A \cup C \rightarrow B = C$

(b) $A \subseteq B \cup C \rightarrow A \subseteq B \vee A \subseteq C$

3. Check if the following inference rules are valid.

(a) $\dfrac{p \rightarrow q,\ \neg q \vee r,\ \neg r}{\neg p}$

(b) $\dfrac{p \rightarrow q,\ p \vee \neg r,\ \neg r}{\neg q \vee r}$

(c) $\dfrac{\neg p \rightarrow \neg q,\ q,\ \neg(p \wedge \neg r)}{r}$

(d) $\dfrac{((p \wedge q) \rightarrow r),\ \neg(p \rightarrow r)}{q \rightarrow r}$

4. Find the truth set of each of these predicates where the domain is the set of integers.

(a) $P(x) : x^2 < 3$

(b) $Q(x) : x^2 > x$

(c) $R(x) : 2x + 1 = 0$

(d) $P(x) : x^3 \geq 1$