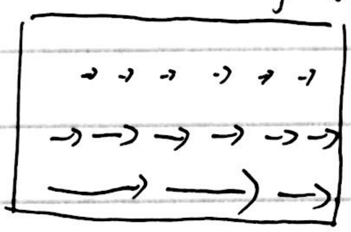
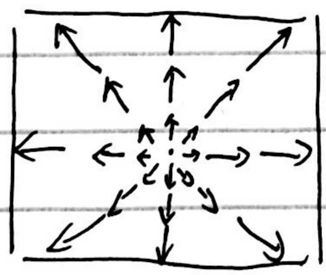
1. (a) 3D motion vector is in the real world. 2D motion vector is the projection of 3D motion vector. optical flow is what we observed 2D motion vector.

3D motion vector
↓ projection
2D motion vector
↓ distortion
observed 2D motion vector (optical flow)

(b) The objects close to camera will be bigger, the objects away from camera will be smaller. 3D motion is projected to the camera.



(c) The objects near aiming point will be smaller. The objects expand from the aiming point will be bigger.
It's a Radial motion field.



(d) $v = \frac{f}{Z} V - \frac{p}{Z} V_Z$ Z component: $v_Z = \frac{f}{Z}(V_Z Z - V_Z Z)$
$= 0$

(e) $V = W \times P + T$

$$V_x(\tau) = \frac{\tau_z x - \tau_x f}{Z}$$

$$V_x(\omega) = -\omega_y f + \omega_z y + \frac{\omega_x xy}{f} - \frac{\omega_y x^2}{f}$$

$$V_y(\tau) = \frac{\tau_z y - \tau_y f}{Z}$$

$$V_y(\omega) = \omega_x f - \omega_z x - \frac{\omega_y xy}{f} - \frac{\omega_x y^2}{f}$$

(f) with translational component in Z ($\tau_Z = 0$)

$$V_x = -\frac{\tau_x}{Z} f$$

$$V_y = -\frac{\tau_y}{Z} f$$

objects parallel to eachother, near is larger, away is small.

without translational component in Z ($\tau_Z \neq 0$)

$$V_x = \frac{\tau_Z}{Z}(x - x_0)$$

$$V_y = \frac{\tau_Z}{Z}(y - y_0)$$

radial motion field

$\tau_Z > 0$: focals of extendstion

$\tau_Z < 0$: focals of contraction

(g) $x_0 = \frac{\tau_x}{\tau_Z} f$

$y_0 = \frac{\tau_y}{\tau_Z} f$ ($\tau_Z \neq 0$)

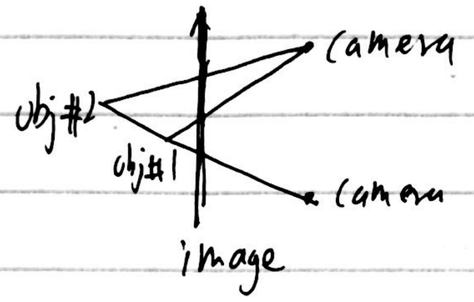
instaneous epipole is infinite ($\tau_Z = 0$)

(h) motion parallax: apparent motion of two instantaneous coindincident points.

relative motion field equations:
$$\begin{cases} \Delta V_x = V_x - \bar{V}_x \\ \Delta V_y = V_y - \bar{V}_y \end{cases}$$

$(V_x, V_y, \bar{V}_x, \bar{V}_y)$ get from:

$$\begin{cases} V_x = V_x^{(\tau)} + V_y^{(w)} \\ V_y = V_y^{(\tau)} + V_y^{(w)} \end{cases} \quad \begin{cases} \bar{V}_x = \bar{V}_x^{(\tau)} + V_y^{(w)} \\ \bar{V}_y = \bar{V}_y^{(\tau)} + V_y^{(w)} \end{cases}$$



2.(a) OFCE: $\frac{d}{dt} I(x(t), y(t), t) = 0$

$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \cdot \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}\right) + \frac{\partial I}{\partial t} = 0$$

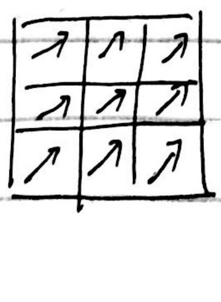
Aussume image brightness of object is constant. the Intesitive of $(x(t), y(t))$ is constant C.

(b) aperture problem: only observe the projection of 2D motion vector on ~~the~~ to Image Gradient.

$$\frac{\nabla I}{\|\nabla I\|} \cdot V = -\frac{I_t}{\|\nabla I\|}$$

Image gradient and the change of x,y overtime are hope to recover based on a single point.

(c) Assume constant spatial gradient in neighborhood. we can observed the local motion in local optical flow vectors.



(d) $E(v) = \sum_{(x,y)\in patch} (\nabla I(x,y) \cdot v + I_t)^2$

$v^* = \underset{v}{argmin}\ E(v) \Rightarrow \nabla E(v) = 0$

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix}$$

The parpuse of weighted block method is make center higher weighted.

we apply $w(x,y) = \frac{1}{\|(x,y)-(x_c,y_c)\|+1}$

(far from center, low weight)

(e) Affine motion make better assumption: assume local motion described by affine map.

$V(x,a) = \begin{bmatrix} a_1 + a_2 x + a_3 y \\ a_4 + a_5 x + a_6 y \end{bmatrix}$ $E(a) = \sum_{(x,y)\in patch} (I_x \cdot x_t + I_y y_t + I_t)^2$

$\nabla E(a) = 0 \Rightarrow \frac{\partial E}{\partial a_1} = 0 \dots \frac{\partial E}{\partial a_6} = 0$

Solve these, six equations to get parameters, and recompute the $V(x,a)$ to get the motion vector.

(f) Horn-Schunck: estimate all motion vector at the same time.

The advantages: ① all regularization

② give solution every where

Scanned by CamScanner

difficulty is noise will be more and more covered.

(g) Solution: start with guess for u, v (use L-K or affine flow) iterate to refine them.

$$u^{(n+1)} = u^{(n)} - \frac{(I_x\bar{u}^{(n)} + I_y\bar{v}^{(n)} + I_t) I_x}{I_x^2 + I_y^2 + \alpha^2}$$

$$v^{(n+1)} = \bar{v}^{(n)} - \frac{(I_x\bar{u}^{(n)} + I_y\bar{v}^{(n)} + I_t) I_y}{I_x^2 + I_y^2 + \alpha^2}$$

Stop: when $\max(u^{(n+1)} - u^{(n)} \,||\, v^{(n+1)} - v^{(n)}) < \tau$ stop

We use L-K or affine flow to initialize the first iteration.