A. 1. $2A - B = 2\begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}-2\\-1\\0\end{bmatrix}$

2. $\|A\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

unit vector for X axis $a = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$\cos\theta = \dfrac{a \cdot A}{\|A\| \cdot \|a\|} = \dfrac{1+0+0}{\sqrt{14} \cdot 1} = \dfrac{\sqrt{14}}{14}$

$\theta = \arccos\theta \dfrac{\sqrt{14}}{14}$

3. unit vector in the direction $A = \dfrac{A}{\|A\|} = \begin{bmatrix}\frac{\sqrt{14}}{14}\\ \frac{\sqrt{14}}{7}\\ \frac{3\sqrt{14}}{14}\end{bmatrix}$

4. $A = \dfrac{A}{\|A\|}(\cos\alpha + \cos\beta + \cos c)$

$\alpha = \cos\alpha = \dfrac{\sqrt{14}}{14}$ $\beta = \cos\beta = 2\dfrac{\sqrt{14}}{14} = \dfrac{\sqrt{14}}{7}$

$c = \cos c = 3 \cdot \dfrac{\sqrt{14}}{14} = \dfrac{3\sqrt{14}}{14}$

5. $A \cdot B = 1\times4 + 2\times5 + 3\times6 = 32$

$B \cdot A = 4\times1 + 5\times2 + 6\times3 = 32$

6. $\cos(A\cdot B) = \dfrac{A \cdot B}{\|A\| \cdot \|B\|} = \dfrac{32}{\sqrt{14}\cdot\sqrt{77}} = \dfrac{32}{\sqrt{1078}}$

$= \dfrac{16\sqrt{1078}}{539}$

7. $A \cdot A_\perp = 0$

if $A_\perp = (a, b, c)$

then $a + 2b + 3c = 0$

one of $A_\perp$ should be $(2, 1, -1)$

8. $A \times B = (1, 2, 3) \times (4, 5, 6)$

$= (2\times6 - 3\times5,\ 3\times4 - 1\times6,\ 1\times5 - 2\times4)$

$= (-3, 6, -3)$

$B \times A = (4, 5, 6) \times (1, 2, 3)$

$= (5\times3 - 6\times2,\ 6\times1 - 4\times3,\ 4\times2 - 5\times1)$

$= (3, -6, 3)$

9. if a vector $D \perp$ to both A and B

then $\begin{cases} x + 2y + 3z = 0 \\ 4x + 5y + 6z = 0 \end{cases}$

then D can be $(1, -2, 1)$

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10. Determinant = $\begin{vmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{vmatrix}$

$= 1\times5\times3 + 4\times1\times3 + -1\times2\times6 - 1\times1\times6$

$- 4\times2\times3 - -1\times5\times3$

$= 15 + 12 - 12 - 6 - 24 + 15$

$= 0$

A, B, C are not linear dependency.

11. $A^T B = [1, 2, 3] \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$= [4, 10, 18]$

~~$AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [4, 5, 6]$~~

$B^T = [4, 5, 6]$

Because A colums not equal to B^T rows,

AB^T can't be implement.

B. 1.

$$2A - B = 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 2 \\ 9 & 12 & -3 \end{bmatrix}$$

2. $$A \cdot B = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3. $$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T A^T = (AB)^T$$

4. $$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix}$$

$$= 2 + 0 + 60 - 15 + 8 + 6$$
$$= 61$$

$$|B| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= 1 \cancel{4} - 24 - 4 - 8 - 4 - 3$$
$$= -46$$

5. in matrix B, row2 and row3.

$$B_{row2} \cdot B_{row3} = 2 \times 3 + 1 \times -2 + -4 \times 1$$
$$= 6 - 2 - 4 = 0$$

they form an orthogonal set

6. $$A^{-1} = \begin{bmatrix} \frac{-13}{55} & \frac{13}{55} & \frac{12}{55} \\ \frac{4}{55} & \frac{-1}{55} & \frac{9}{55} \\ \frac{4}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{2}{21} & \frac{3}{14} \\ \frac{1}{3} & \frac{1}{21} & \frac{-3}{21} \\ \frac{1}{6} & \frac{-4}{21} & \frac{1}{14} \end{bmatrix}$$

C. 1.

$$\det(A - \lambda I) = 0$$

$$= \left| \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda_1 = 4 \quad \lambda_2 = -1$$

$$(A - \lambda I) X = 0$$

$$\lambda = 4 \quad \begin{bmatrix} 1-4 & 2 \\ 3 & 2-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$X = \begin{bmatrix} \frac{2}{3} \\ 1 \end{bmatrix}$$

$\lambda = -1$ $\quad \begin{bmatrix} 1+1 & 2 \\ 3 & 2+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$V = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

2. $V = \begin{bmatrix} -1 & \frac{2}{3} \\ 1 & 1 \end{bmatrix}$ $\quad V^{-1} = \begin{bmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{3}{5} \end{bmatrix}$

$V^{-1}AV = \begin{bmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & \frac{2}{3} \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$

3. $\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ 1 \end{bmatrix} = \frac{1}{3}$

4. $[2, 1] \begin{bmatrix} -1 \\ 2 \end{bmatrix} = -2+2 = 0$

5. They are orthogonal matrex

D. 1. $f'(x) = 2x$

$f''(x) = 2$

2. $\dfrac{\partial g}{\partial x} = 2x$

$\dfrac{\partial g}{\partial y} = 2y$

3. $\nabla g(x,y) = \left(\dfrac{\partial g}{\partial x}, \dfrac{\partial g}{\partial y}\right) = (2x, 2y)$

4.