(a). 1) find corner matrix in local neighborhood.
2) find eignvalue of matrix
3) check that $ev_1, ev_2 > \tau$

corner : more than one directions
edge : only one direction.
Only significent difference will be count a corner.

(b.) PCA: find direction to minimal projection.
find additional direction subject to being perpendicular to all previous directions.

(c). $$\sum_{i=1}^{9} P_i P_i^T = \begin{bmatrix} x_i^2 & x_i y_i \\ y_i x_i & y_i^2 \end{bmatrix} = \begin{bmatrix} 1+1+1+1 & 1+2+3 \\ 1+2+3 & \begin{matrix}1+4+9+16+1 \\ +4+9\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 6 & 44 \end{bmatrix}$$

(d). direction are eigenvalues of correlation matrix which is the projections proportional eigenvalue.
It means, when $\lambda_1 \cdot \lambda_2 > \tau$, there is a corner.

(e) non-maximum suppsion:

1). compute λ_1, λ_2 for all location
2) sort pixels based on λ_1, λ_2
3). Start from the top, select strongest corner
4). detect corners in vicinits of selected corner.

(f). Harris corner detection

$$C(G) = \det(G) - k\, tr^2(G) \quad k \in [0, 0.5]$$

Let $k=0$, we can avoid compute eignvalue.
use $\det(G)$ to do pure corner detection.
In this condition, $C(G) = \lambda_1 \cdot \lambda_2$.

(g) $E(p) = \sum_{i=0}^{n} \left((p_i - p) \cdot \nabla I(p_i) \right)^2$

location the corner $p^* = \underset{p}{\operatorname{argmin}} E(p) \Rightarrow$ solve $\nabla E(p) = 0$

therefore: $\underbrace{\sum_i \nabla I(p_i) \nabla I(p_i)^T p_i}_{V} = \sum_i \underbrace{\nabla I(p_i) \nabla I(p_i)^T}_{C:\ \text{correlation matrix}} \cdot p$

therefore: $p = C^{-1} \cdot V = C^{-1} \sum_i \nabla I(p_i) \nabla I(p_i)^T p_i$

(C is nonsingular, because $\lambda_1 \lambda_2 > \tau$)

project gradients onto edge. hypothesis and chose p with minimal projection

(h) HOG (histogram of Gradient)
- take window
- split into blocks
- create histogram of gradient of orientations in each blocks
- concatenate histogram

requirements from a good characterization of feature points.

1). translation invarient 2) rotation invarient
3) scale invarient 4). illumination invarient

(i) SIFT features are formed by computing the gradient around the detected key point, and weighted by Gaussian fall-off function, then compute the weighted gradient orientation histogram in each subregion. using trilinear interpolation.

2. (a) The problem is the size of parameter space not determined. It can be infinite. So we can transfer in to distance and angle.

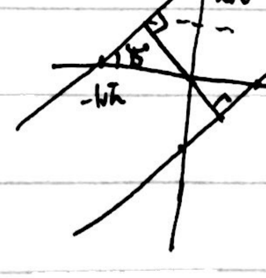
(b) $a \cdot x + b \cdot y + c = 0$ $\cancel{y = -\frac{a}{b}x - \frac{c}{b}}$

$\cancel{-\frac{c}{b} = 10\sqrt{2}}$

$\cancel{10\sqrt{2}b + c = 0}$

$\cancel{-10\sqrt{2}a + c = 0}$

$\cancel{-5\sqrt{2}a + 5\sqrt{2}b + c = 0}$



$(0, 10\sqrt{2})$
$(-10\sqrt{2}, 0)$
$(-5\sqrt{2}, 5\sqrt{2})$
$(0, -10\sqrt{2})$
$(10\sqrt{2}, 0)$
$(5\sqrt{2}, -5\sqrt{2})$

① $\begin{cases} a = 1 \\ b = -1 \\ c = 10\sqrt{2} \end{cases}$ ② $\begin{cases} a = 1 \\ b = -1 \\ c = -10\sqrt{2} \end{cases}$

$x - y + 10\sqrt{2} = 0$ points: $(0, 10\sqrt{2}), (-5\sqrt{2}, 5\sqrt{2})$

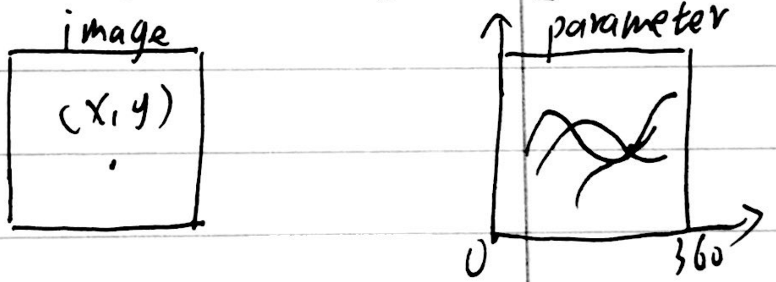
$x - y - 10\sqrt{2} = 0$ $(0, -10\sqrt{2}), (5\sqrt{2}, -5\sqrt{2})$

satisfy the equation.

(c) The vote for a point (x, y):

$$d = x\cos\theta + y\sin\theta$$

scan $\theta \in [0, 360]$ and compute d



It scan all the θ range, and comes to different lines.

(d) when different lines which represent points in image, has a intersection point. the distance and angle of that point, represent the line.

(e) big bin: It take fewer vote but much faster. The accuracy is bad.

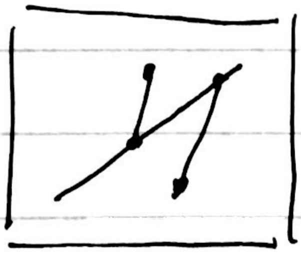
small bin: more vote and slower than big bin. And it might give no intersection.

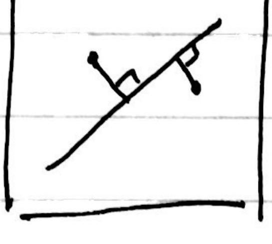
(f) if the normal at each voting points is known, To be more efficent, instead of $\theta \in [0, 180]$, take $\theta \in [\theta_{min}, \theta_{max}]$

(g) For each voting plane $\underline{r}$, for each x, y, scan θ and compute $\underline{a, b}$. So we vote for (a, b, r). we need 3 dimensions.

3. (a) $y = ax + b$, the disadvantge of this equation is the geomatric distance between the prediction and the real point are not minimized.

When the distance is large, but the vertical distances from the point to the predicted line is small. It can't be fitted accurately.





Good fitting

(b) $n = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $d = 2$ $l: x + 2y - 2 = 0$

$$l^T \cdot x = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} [x, y, 1] = 0$$

$l = [\,1 \quad 2 \quad -2\,]$

(c) $E(l) = \sum_{i=1}^{n} (l^T x_i)^2$

$= l^T \underbrace{\left(\sum_{i=1}^{n} x_i x_i^T\right)}_{C} l$

$E(l) = l^T \cdot C \cdot l$

$l^* = \underset{l}{\arg\min}\, E(l) \Rightarrow \nabla E(l) = 0 \Rightarrow C l = 0$

solution is eignvector belonging to zero eignvalue

(d) $C = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & n \end{bmatrix} = \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$

e) $ax^2 + bxy + cy^2 + dx + ey + f = 0$

satisfy : $b^2 - 4ac < 0$

guarantees the model will be an ellipse.

f). $\sum_{i=1}^{n} (l^T p_i)^2$ on ellipse $l^T p_i = 0$

points close to short axis are effect more

q_i (proportional) $\frac{d_i}{d_i + r_2}$

$d_1 > a_2$ $\quad r_2 > r_1$

therefore $\frac{d}{d+r_1} > \frac{d}{d+a_2}$

short axis affect more.

(g) geometric objective $E(l) = \sum_i \frac{|f(p_i, l)|}{|\nabla f(p_i, l)|}$

This formula is not a linear formula.

(h) Error Functional

$$E[\phi(s)] = \int (\alpha(s) \underbrace{E_{continuity} + \beta(s) E_{curvature}}_{internal} + \underbrace{r(s) E_{image}}_{external}) \, ds$$

objective

$$E_{continuity} = \left| \frac{\partial \phi}{\partial s} \right|^2$$

$$E_{curvature} = \left| \frac{\partial^2 \phi}{\partial s^2} \right|^2 \quad E_{image} = -|\nabla l|^2$$

(i) $E_{continuity} = \sum |p_i - p_{i-1}|^2$

$E_{curvature} = \sum |(p_{i+1} - p_i) - (p_i - p_{i-1})|^2$

(j) The continuity of active contours will be $|p_i - p_{i-1}| - d$ to allow for sharp corners.