1. (a)
$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{fX}{Z} \\ \frac{fY}{Z} \end{bmatrix}$$
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{10\times 3}{1} \\ \frac{10\times 2}{1} \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$$

(b) Image plane behind the center reversed the Image, but for in front of the center of projection it won't. therefore we prefer in front of the center of projection. In other model, we will use lens to fix it.

(c) The projection become smaller when the focal length gets bigger.
The projection become bigger when the focal length gets smaller.

(d) $(1, 1) \longrightarrow (1, 1, 1)$
2D 2DH
another corresponds point $(2, 2, 2)$

(e) $(1, 1, 2) \longrightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$
2DH 2D

(f) $(1, 1, 0)$ point at infinity. It represents direction instead of position.

(g) Because the characteristic of Homogeneous coordinate is the equations for perspective projection to the image plane are non-linear when express in non-homogeneous coordinates, but are linear in homogeneous in Homogeneous coordinate.

(h) $$\underset{3\times4}{M} = \underset{3\times3}{k}\,[\,\underset{3\times3}{I} \mid \underset{3\times1}{0}\,]$$

(i). $$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

2DH: $$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

2D : $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{u}{w} \\ \frac{v}{w} \end{bmatrix} = \begin{bmatrix} \frac{9}{5} \\ \frac{23}{5} \end{bmatrix}$$

2. (a)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

(d)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -tx \\ 0 & 1 & -ty \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 2 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 2 - \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 - \frac{2}{\sqrt{2}} \\ 1 \end{bmatrix}$$

Scanned by CamScanner

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2-\sqrt{2} \end{bmatrix}$$

(e) $M' = T \cdot R \cdot M$

(f) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \iff p' = \left[\begin{array}{c|c} S & 0 \\ \hline 0 & 1 \end{array}\right] p$

M is a scaling Matrix.

(g) $M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \iff p' = \left[\begin{array}{c|c} I & t \\ \hline 0 & 1 \end{array}\right] p$

M is a scaling Matrix.

(h) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$M \cdot M^{-1} = I$

M is a inverse Matrix.

(i). $M = R_{(45)} \, T_{(1,2)}$

$M^{-1} = (R_{(45)} T_{(1,2)})^{-1} = T^{-1}_{(1,2)} \, M^{-1}_{(45)}$

(j). $M = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\qquad M \cdot M^{\perp} = 0$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow x + 3y = 0$$

assume $x = 1$ $\quad M^{\perp}$ can be $\begin{bmatrix} 1 \\ -\frac{1}{3} \end{bmatrix}$

(k). direction of the $(2,5)$ is $(2,5,0)$

vector $(1,3)$ in 2DH is $(1,3,1)$

projection vector can be $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 1 \end{bmatrix}$

2D: $v = \begin{bmatrix} \frac{3}{1} \\ \frac{8}{1} \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

3. (a) Because the projection process is transfer from 3D to 2D image. It need different coordinate to store and balance the measure of the matrix.

(b) $M_{cw} = \tilde{R}^{-1} \tilde{T}^{-1} = \left[\begin{array}{c|c} R^T & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & -T \\ \hline 0 & 1 \end{array}\right]$

$$= \left[\begin{array}{c|c} R^T & -R^T T \\ \hline 0 & 1 \end{array}\right]$$

(c.) $R^* = \begin{bmatrix} \hat{x}^T \\ \hat{y}^T \\ \hat{z}^T \end{bmatrix}$

(d) $R^* = R^T \quad T^* = -R^T T$

(e) $M_{i \leftarrow c} = (M_{c \leftarrow i})^{-1} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & V_u & v_0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} k_u & 0 & 512 \\ 0 & k_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$

(f) R^* and T^* is external it means the parameter of rotation and translation.

k^* is intrinsic parmeter. which contains f, k_u, k_v, u_0, u_v.

(g) 2D skew ~~...~~ can make model more accurate.

(h) Radial lens distortion

$$p^{(i)} = \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} k^* [R^* | T^*] \, p^{(w)}$$

λ is scale but not constant.

It will made camera model more scale away from the center.

(i) M_∞ is a good approximation to M, when depth variation is small compare to distance from Object.

an affine camera is a arbitrary. It's worse than the weak-perspective camera, and it's not a real camera which is combination of affine fraction and projection. It's a less useful model.

4. (a) $L(p)$ = power of light per surface area reflected from surface. (surface radiance).

$E(p)$ = power of light per surface area recorded at each pixel. (image irradiance)

(b) $E(p) = L(p) \frac{\pi}{4} (\frac{d}{f}) (\cos\alpha)^4$

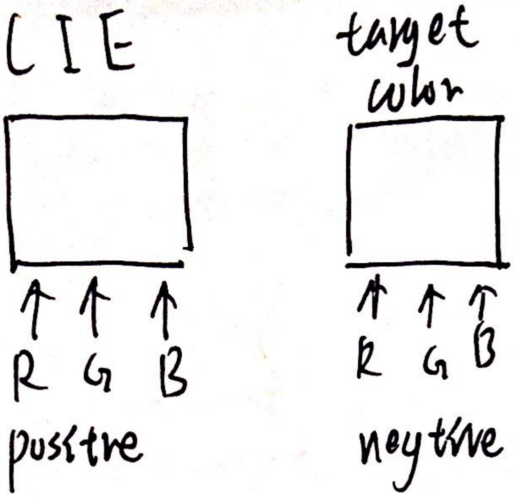
(c) The albedo of a surface is the fraction of the incident sunlight that the surface reflects. Radiation that is not reflected is absorbed by the surface.

(d) Because use three primary color can represent all the color. And this is the color model for human vision.

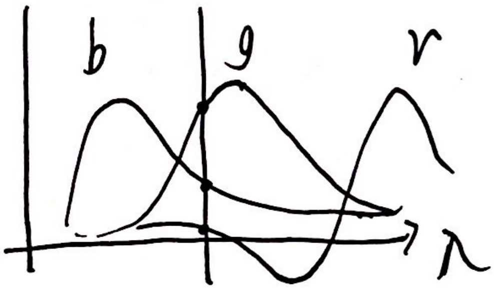
(e) The color along the line is gray, from (0,0,0) to (1,1,1), the gray color. changes gradualy from dark to light.

(f) The way is to compare R, G, B in CIE RGB model with the real world target color.

CIE



then use the three curve of R, G, B, in the table to represent the color. which covert them in to X, Y, Z.



(g) Luminance component Y means the black and white information. It can be used to display the black and white TV information. Also, It can be used to ~~ant~~ analysis. Because black and white information is surffcent ~~th~~ and easiler to anlyze.

(h) The natrual way use distance to judge the ~~differe~~ difference, it is different from the human vision result. However, the Lab ED is more reperesentative of perception distance.