

CERI- 8104/CIVL-8126 –
Data Analysis in Geophysics

Determining geohydrological parameters from hydraulic pump tests (60 points)

Submit a short report with 4 figures documenting your results! Submit your MATLAB code along with a PDF of the report. The latter needs to be generated with latex.

Required submission files:

- 1) PDF document that includes 4 figures:
 - a. One figure with three subplots that shows the data on different scales!
 - b. Three figures (one for each drawdown measurement) of data and best non-linear LSQ-fit with respective values in title or legend.
- 2) MATLAB code used to solve each task below.



Fig. 1: Deep geothermal well in Nevada used to determine reservoir parameters during pump tests.

“The goal of a pump test, as in any aquifer test, is to estimate hydraulic properties of an aquifer system. For the pumped aquifer, one seeks to determine **transmissivity**, hydraulic conductivity (horizontal and vertical) and **storativity** (storage coefficient). In layered systems, one also uses pump tests to estimate the properties of aquitards (vertical hydraulic conductivity and specific storage). Pump tests can identify and locate recharge and no-flow boundaries that may limit the lateral extent of aquifers as well.”

Diagram illustrating a well in an unconfined aquifer with a leak to a lower aquifer. The diagram shows a well with discharge Q , a leak at depth d , and a lower aquifer at depth b . The water table height is $h(r,t)$ and the initial height is h_0 . The leak is characterized by length l and leak coefficient K_z . The lower aquifer has properties T , S , and K_z/K_r . The diagram is labeled "aquiclude" for the upper and lower layers and "aquifer" for the middle layer.

$$s = h_0 - h$$

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + \frac{K_z}{K_r} \frac{\partial^2 s}{\partial z^2} = \frac{S}{T} \frac{\partial s}{\partial t}$$

Storativity or the storage coefficient is the volume of water released from storage per unit decline in hydraulic head in the aquifer, per unit area of the aquifer. Storativity is a dimensionless quantity, and is always greater than 0.

Hydraulic Diffusivity: Similar to thermal diffusivity, hydraulic diffusivity is proportional to the speed at which a finite pressure pulse will propagate through the system (compare to rate of heat transfer). It is defined as the ratio of hydraulic conductivity to specific storage,

or, equivalently, of transmissivity to the storage coefficient (T/S). Hydraulic diffusivity is sometimes also determined from seismicity migration characteristics.

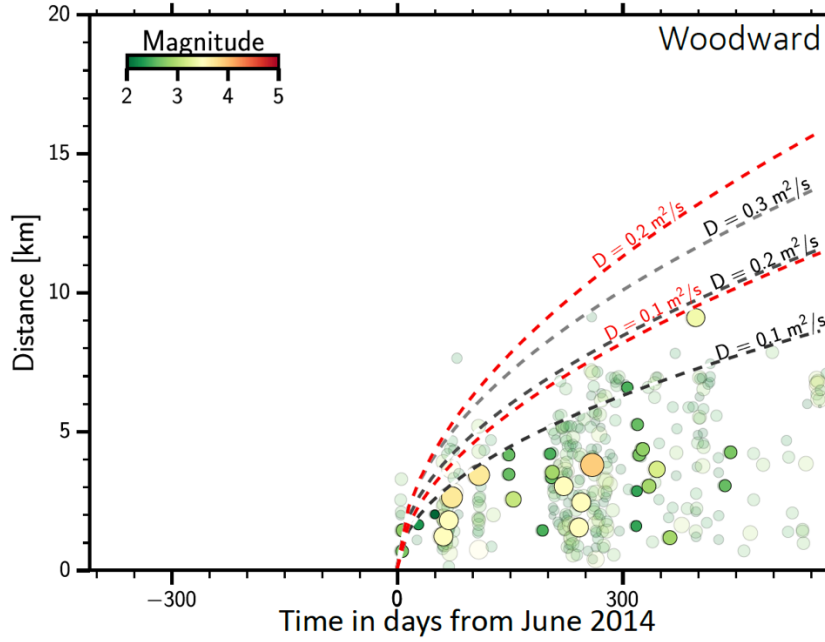


Fig. 3: Hydraulic diffusivity from earthquake migration in space and time (from Goebel et al. 2017).

I. Theoretical Background

Consider the 1D diffusion equation in the presence of fluid injection/production, $Q(t)$ (e.g. Hsieh & Bredehoeft, 1981):

$$T_r \frac{\partial^2 h}{\partial r^2} = S \frac{\partial h}{\partial t} - Q(t) \quad (1)$$

where h is the vertically averaged buildup of hydraulic head above the initial head, T_r is the principal value of the transmissivity tensor, S is the storage coefficient, and $Q(t)$ is the variable injection or production rate.

For constant pump rates, $Q(t)=Q=\text{const.}$, a solution for pressure build-up (or draw down) - assuming radial flow in a vertical confined aquifer - can be found as:

$$\Delta h = h_0 - h = \frac{Q}{4\pi T} W(u), \quad (2)$$

where Q is the constant pumping rate [L^3/T], h is the hydraulic head [L], h_0 is the initial hydraulic head [L], $h_0 - h$ is the drawdown [L], T is the aquifer transmissivity [L^2/T], and $W(u)$ is the well function. The latter is also referred to as the exponential integral expressed through:

$$W(u) = Ei(u) = - \int_{-u}^{\infty} \frac{e^{-t}}{t} dt = \int_{-\infty}^u \frac{e^t}{t} dt \quad (3)$$

Solutions for the well function can be determined from evaluating rapidly converging infinite series. Such solutions are tabulated for various values of u which is defined as:

$$u = \frac{r^2 S}{4Tt} \quad (4)$$

The most convenient way to arrive at a reasonably exact solution is by using MATLAB's implementation of the exponential integral. In MATLAB the syntax for using Equation 3 is simply:

```
>> expint(u)
```

Equation 2 is commonly referred to as Theis's solution and the simplicity of the expression allows for a straight-forward inversion of geohydrological parameters (i.e. storage coefficient, S and transmissivity, T) for homogeneous aquifers. We will use non-linear least-squares in MATLAB to do the inversion. Based on these S and T other hydrogeological parameters such as diffusivity and permeability can be determined if aquifer thickness and fluid properties are known.

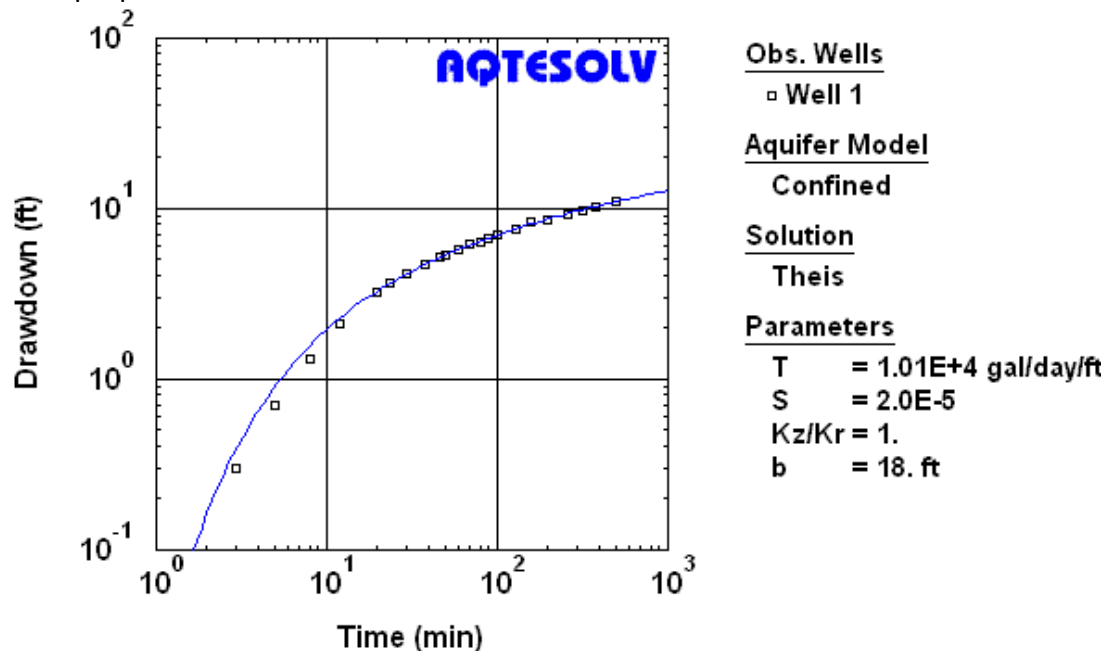


Fig. 4: Example of drawdown data, functional fitting and resulting geohydrological parameters. (from <http://www.aqtesolv.com/aquifer-tests>)

II. Activity – Part 1 – Data inspection

Note that the provided MATLAB template contains some guidance for the analysis but a few lines of codes will have to be added in some places.

- i) Download the data file (draw_down_data.csv) and MATLAB template from Canvas. Load the data using `readtable()` and inspect the data.
- ii) What are the different columns? What units are the data in? Do you have to convert the data to SI units?
- iii) Create three scatter plots on linear, log-log and semi-log scales!

III. Part 2 – Functional fitting and parameter inversion

- i) Write a new function called “well_fct” that requires 4 inputs: the time vector, distance, flow rate and $[S, T]$, i.e. transmissivity and storage and one output: hydraulic head change, h . The function should be an implementation of Eq. 2 and 4 (Theis’s solution). You can take a look at the bottom of the provided MATLAB template to find some inspiration about how to do this.
- ii) Look at lines 49 and 51 in the provided script. Note that we now have a function with known parameters (Q , and r) and two additional parameters (S and T) that we are inverting for. To navigate the set of parameters, we created an additional anonymous function called “modelFct” that handles the function in and outputs. Complete line 49 to 51 and invert for S and T !

The pumping rate in this example is $Q = 96,000 \text{ ft}^3/\text{dy}$, which you need to convert to SI units.

The radius, r , should be varied between $r = 200, 400$ and 800 ft .

- iii) What are the values of storage coefficient, transmissivity and hydraulic diffusivity?
- iv) Plot your solution together with the data on double-log scales!

- v) Repeat for all three distances at which the drawdown behavior was measured (200, 400 and 800 ft.)! How do storage, transmissivity and hydraulic diffusivity compare? Discuss your observation!

IV. Part 3 – Outlier removal, statistical testing

- i) Inspect your solution and determine the robustness of the results! You can use bootstrap resampling, R^2 -values, confidence limits on the inverted parameters or anything else that we discussed in class to make sure your results are robust.
- ii) Determine at what distances the data has issues and remove outliers if needed.
- iii) Redo the analysis without outliers and compare the respective hydrogeological parameters! Discuss your observations in light of expected results for a single pumping test and three associated draw-down measures!
- iv) Inspect lines 54 to 58 to see how to do a constraint non-linear least squares inversion (look for the MATLAB function: `lsqcurvefit`). Try to change the initial guess and the range of the constraint inversion. How robust is `lsqcurvefit` vs. `nlinfit`?