

UNIVERSITY OF KHARTOUM

GeiodApp: A Unified Framework for Geoid Computations

by

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Declaration of Authorship

I, Mohamed Yousif and Mohamed Jaafar, declare that this thesis titled, ‘GeoidApp: A Unified Framework for Geoid Computations’ and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Abstract

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Figure of the earth remains one of the most crucial aspects of geodesy. Modern studies of the figure of the earth have started with Gauss ? who call it ‘geoid’. The geoid according to Gauss is the surface that approximates the sea level. Another approach of describing the figure of the earth was proposed by Molodnskey ?. His proposal was to treat the earth’s figure as ‘a boundary value problem’. We’re interested in the later approach as we believe that it’s economically feasible, yet accurate enough.

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor...

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Abbreviations

GGM	G lobal G eopotential M odel
BVP	B oundary V alue P roblem
GBVP	G eodetic B oundary V alue P roblem
GPS	G lobal P ositioning S ystem
GNSS	G lobal N avigation S atellite S ystem

Physical Constants

$$\text{Speed of Light } c = 2.997\,924\,58 \times 10^8 \text{ ms}^{-\text{s}} \text{ (exact)}$$

Symbols

a	distance	m
P	power	W (Js^{-1})
ω	angular frequency	rads^{-1}
σ	standard deviation	meters

For/Dedicated to/To my...

Chapter 1

Introduction

Historically Sudan has two gravimetric geoid models ?. The first one (Geoid91) was computed by [Fashir \(1991\)](#) in 1991 using Geodetic reference System GRS80. The free-air co-geoid was computed from the combination of surface gravity data using a modified Stokes's kernel and GODDARD EARTH MODEL (GEM-T1) [Fashir \(1991\)](#). GEM-T1, a satellite-only model, is complete to degree and order of 36 ?. The second model is known as 'KTH-SDG08' was computed in 2008 using optimum least-squares modification of Stoke's kernels, which is widely known as KTH method [Abdalla \(2009\)](#). EIGEN-GRACE02S satellite-only model was adopted for KTHSDG08 final computation at spherical harmonic degree and order 120. Adam was the first to attempt to compute the geoid for Sudan in 1967 ?. Due to lack of data from neighboring countries and large un-surveyed areas in Northwest and Southwest parts of Sudan, Adam found that the information was insufficient to determine the accurate geoid in Sudan and recommended to fill the gabs over there [Abdalla \(2009\)](#).

1.1 Objectives of the thesis

The main objective of this thesis is to evaluate recent grace/goce models over Sudan using asterogeodetic data and GPS/Leveling data. As a result we also propose a general software framework for various geoid components computations. In particular, it can be used to compute "geoid-height", "geoid-undulation", and "geoid-disturbance". To the best of our knowledge, we could not find a software for geoid computations that is

- Supported. Most of geoid computations libraries are no longer supported, their links are dead.

- Strong back-end, and simple front-end. It is common to use a low-level programming language in the calculation of geoid components e.g. geoid height. Low-level languages are very fast compared to high-level languages, but they are much harder. We built a framework on top of C/C++ libraries, with a very simple interface, in particular Java and Matlab.
- Customizable. Because we know that users need to explore different models with different degree. We offer them just that. GeoidApp is designed such that the user can easily modify any parameter.

In particular our contribution is

- Evaluate recent Grace/Goce money in Sudan
- GeoidApp: A unified software framework for geoid computations

1.2 Thesis structure

Following this introduction, the thesis is divided into five chapters

- Chapter 2
Glances the theory behind GGMs, and summarizes dedicated satellites missions.
- Chapter 3
Details about the input data
- Chapter 4
Methodology

Chapter 2

Data Acquisition

In this chapter we will discuss our dataset and we will compare between them in terms of

Chapter 3

Input Data

In this chapter we will introduce our datasets that we use to evaluate recent Goce/Grace models in Sudan.

3.1 Astrogeodetic data

In 1967 [Adam \(1967\)](#) during his studies at Cornell University has conducted a study to compute the geoid in Sudan. His data consists of 46 points for different locations in Sudan. Due to the lack of data from neighboring countries, and the huge gaps between measurements inside. He suggested to fill the gaps to have a reliable datum. For a detailed discussion about the attempts to compute datums for Sudan, reader is referred to ([Abdalla et al., 2012](#)).

We have mentioned that [Adam \(1967\)](#) used an astro-geodetic observation, hence their datum is astro-geodetic datum. [Abdalla \(2009\)](#) used gravity data to compute gravimetric datum for Sudan. The astro-geodetic datum, or *geodetic datum* is a geoid computed by astronomically determined deflection of the vertical, while the gravimetric geoid is a geoid referred to the geocentric datum, hence the use of satellite data. A good discussion about the differences between the geoid types can be found in [Vaneek \(1975\)](#). A sample of this data is provided in Table 3.1. While the full data is provided in the appendix. The distribution of ([Adam, 1967](#)) work is introduced in Figure 3.2.

3.2 GPS/Leveling Data

GPS/leveling data are used to evaluate GGMs results by deriving the geoid height from them. The data were collected over Khartoum area.

TABLE 3.1: Astro-geodetic data for latitude longitude and geoid height

<i>station</i>	ϕ°	λ°	<i>geoid height (m)</i>
zv	22.1686	31.489	10
12	20.1361	30.662	9.556
28	18.4728	30.840	10.198
43	17.0511	31.272	10.199
53	14.4962	30.251	14.180
70	13.8316	29.654	15.833
76	13.2329	30.110	15.354
79	12.8660	29.956	15.647
80	12.7763	30.853	14.453
85	11.6038	30.411	15.387

GPS/Leveling data are two separate heights systems (orthometric and ellipsoidal height) for the same points (in terms of latitude and longitude). GPS data is ellipsoidal height data that is computed by means of GPS/GNSS systems. Leveling data on the other hand, is the orthometric height that is computed using spirit leveling. The geoid height can then be computed using Eq. (??)

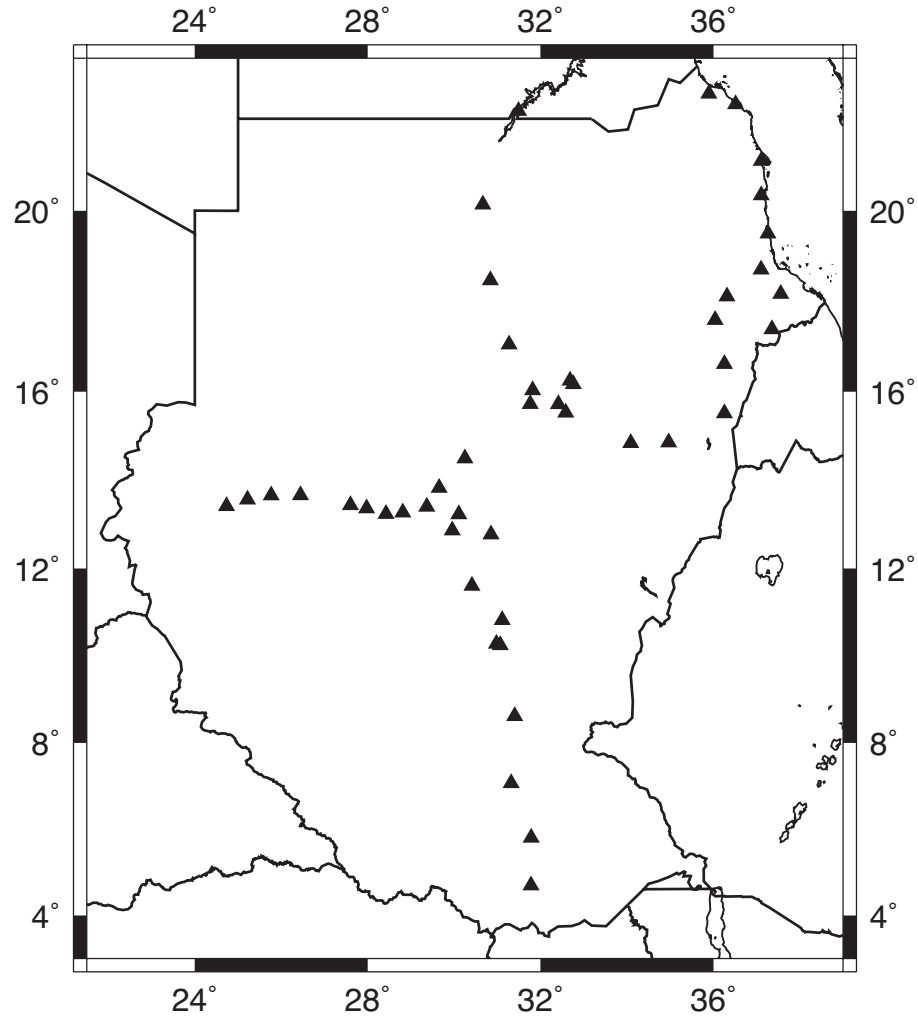
$$N = h - H \quad (3.1)$$

The GPS measurements were performed using dual frequency GPS receivers LEICA 1200, LEICA RS500 and Trimble 5700, and choke rings antennas ASH701945E_M from Ashtech, LEICA AT504 used from Abdalla (2009) work. The accuracy in leveling data is a bit problem because some of them were taken from a geodetic network of 3rd order, which drastically affect our evaluations. The distribution of GPS/leveling results are shown in Figure ??.

3.3 Global Geopotential Models

A global Geopotential Model is a mathematical function approximates the real gravity potential of the Earth. From such an approximation, all related gravity field functionals can be computed (e.g., gravity potential, gravity vector). However, other gravity field functionals e.g., *geoid height*, *gravity anomaly* and *gravity disturbance*, and the *second radial derivative* cannot be computed without a defined reference system (Geodetic Reference System 1980). As the centrifugal part can be modeled easily and accurately. It is usually beneficial to select a global geopotential model (and degree) that is best fit to the local gravity field as base for a regional gravimetric model. This will reduce the

FIGURE 3.1: Distribution of (Adam, 1967) points in Sudan. It is clear the huge gap that Adam (1967) suggested to be filled



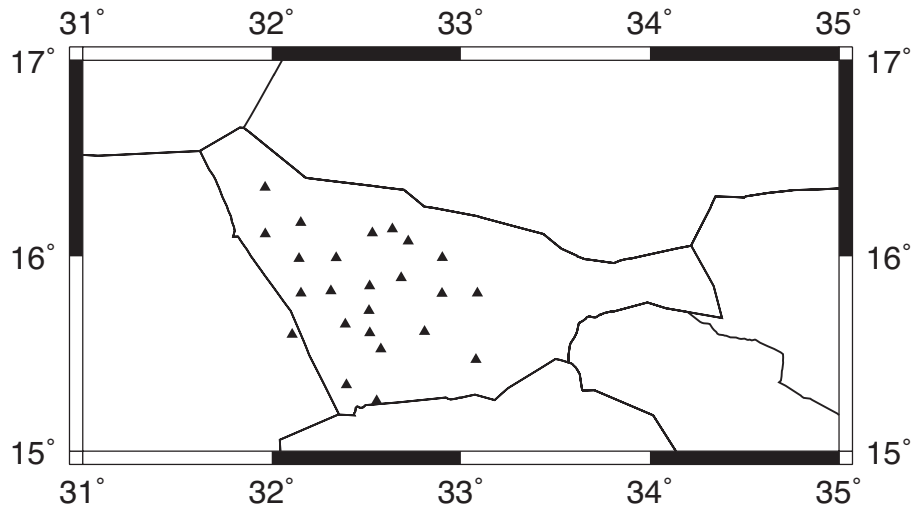
error by Stokes' formula by reducing the amount of the geoid contribution to the total error.

3.3.1 Satellite-only models

they are derived solely from the analysis of the satellite orbit. They use laser and Doppler measurements to track the satellites ? Historically, these models were known to have a low precision due to

1. the inability to track complete satellite orbits using ground-based stations
2. imprecise modeling of atmospheric drag attraction
3. the incomplete sampling of the global gravity due to the limited number of satellites

FIGURE 3.2: Distribution of GPS/leveling data over Khartoum



These issues were fixed by the new dedicated satellite missions in the earlier 2000s ?. In our study, we used two satellite-only models ITU_GCC16 and ITU_GRACE16 of 280, 180 degree and order respectively.

3.3.2 Combined Models

These models are combined from different source. A GGM is combined with land and shiptrack gravity observations, and marine gravity anomalies [Amos and Featherstone \(2003\)](#). It is often that combined models have higher degrees than satellite-only models. Even with the added source of e.g., terrestrial data, combined models are also limited in their precision because of their use of satellite-only models. In our study we have tested three different combined models namely EGM2008, EIGEN-6C4 (2014), and GECO (2015). They are all up to degree 2190, and we tested them up-to their maximum degree. One source of error caused by the combination of different regional (local) datums and the offset between them. [Heck \(1990\)](#) reported that the magnitude of errors due to the inconsistency of regional datums with global datum is underestimated. The source of error was due to the simplified free-air reduction procedure and of different kinds of height system. [Heck \(1990\)](#) in their work have show that the corresponding systematic errors in gravity anomalies are maximum in mid-latitudes. The error due to datums inconsistency is a systematic error.

3.3.3 Tailored Models

A combination of the aforementioned models designed (tailored) for a specific area. That actually against the term global in GGM. None of these models were tested in our

TABLE 3.2: GPS/Leveling data for Khartoum area

id	ϕ°	λ°	$geoid\ height\ (m)$
1	16.1719143	32.15278395	3.573
2	15.822078561111	32.312747819444	2.998
3	15.889747230556	32.683920419444	3.619
4	16.076534180556	32.721614461111	3.119
5	15.810990861111	33.086944869444	2.175
6	15.809063111111	32.899570888889	2.285
7	15.613491838889	32.808035077778	2.02
8	16.351884494444	31.964771408333	4.174
9	15.847591619444	32.517391980556	3.078
10	16.118530788889	32.531269330556	2.263
11	15.99188345	32.3404394	3.21
12	15.810295125	32.154373358333	3.21
13	16.11349295	31.965449913889	3.197
14	15.992628230556	32.9012736	3.254
15	15.987956075	32.144070430556	3.447
16	15.469951816667	33.079683027778	2.297
17	15.651378211111	32.388177761111	2.81
18	15.721159511111	32.514142069444	2.655
19	16.139034477778	32.637590772222	2.4199
20	15.607008355556	32.518298555556	2.6729
21	15.599259138889	32.107760216667	2.5378
22	15.524018605556	32.576891125	2.5993
23	15.258628422222	32.554645944444	2.2311
24	15.340130033333	32.394058058333	2.6918

experiments.

3.4 New Satellite Missions

Because of the mentioned limitations of satellite missions, new dedicated satellite missions were launched in early 2000s to tackle those problems. There are three satellite missions launched for that purpose

3.4.1 CHAMP

CHAMP **CH**allenging **M**inisatellite **P**ayload is a German small satellite mission for geoscientific and atmospheric research and applications, managed by GFZ. The orbit of CHAMP is almost circular (inclination 87degree) which gives it an advantage of getting homogeneous and complete global coverage. With an altitude of 454 km, it guarantees

that the satellite can work in severe atmospheric conditions. It also has a direct on-board measurement to account for the non-gravitational orbit perturbations. . CHAMP uses satellite-to-satellite tracking working on high-to-low mode, i.e., GNSS satellites are tracking the spaceship position, and it uses an accelerometer to compute the geoid functionals (cf. REIGBER et al. (1999) for detailed discussions about CHAMP mission). Clearly, the aforementioned issues with previous satellite mission was resolved in the new dedicated satellite mission. A detailed description of CHAMP, and other new satellite missions e.g., GOCE and GRACE will be presented in Table ??.

3.4.2 GRACE

Gravity **R**ecovery and **C**limate **E**xperiment is the second satellite mission that was launched on March 2002. GRACE consists of two identical satellites separated by 220 km from each others, and at 500 km above the Earth surface. Unlike CHAMP, GRACE is uses satellite-to-satellite tracking to precise position determination, but it uses low-low mode to compute the gravity functionals i.e., the gravity is measured by the change of the distance between the twin satellites. Areas of slightly stronger gravity (greater masses) will affect the leading satellite first. An accelerometer will be used to measure the non-gravitational acceleration—common errors due to atmospheric attraction—such that only the accelerations caused by the gravity are measured.

It is common to find a model with data from different satellite mission—each one of them contribute to specific range of spherical harmonics degrees.

3.4.3 GOCE

Chapter 4

Geoid Height Computation

In this chapter we will discuss the computation of the geoid height and the validation of our results based on local terrestrial data. High resolution models are required to convert GPS leveling data (ellipsoidal height) into orthometric height. As we mentioned in the previous chapter, we have two terrestrial data 1) Astrogeodetic data provided by [Adam \(1967\)](#), and 2) GPS leveling data by ?. Our results show that high degrees will often result in a lower standard deviation. Combined models e.g., EGM2008, EIGEN-6C4, and Geco have a very similar trend—it is expected because both EIGEN-6C4 and Geco share some degrees with EGM2008. Interestingly, ITU GCC has shown a peak in higher degrees (above 150), we believe that is one of the problems of satellite-only models in high degrees.

4.1 Historical Background about Geoid Computations in Sudan

There are several attempts was done through the years to compute the geoid for Sudan. The first was committed by [Adam \(1967\)](#) in 1967. He was done it during his MSc in Cornell University. He used astrogeodetic data i.e., astrogeodetic geoid, it's common to use astrogeodetic observations when there is a lack of data. Even with that, Osman recommended to fill the gaps due to large un-surveyed areas (largely on Northwest and Southwest parts of Sudan), and lack of data from neighboring countries. He used Clarke 1880 as a reference ellipsoid which is reasonable because the well-known WGS wasn't established at that time. Another attempt was done by [Fashir \(1991\)](#) in 1991. Unlike Osman's geoid, he used a gravimetric data i.e., a gravimetric geoid. He covered a grid of $(5^\circ \leq \phi \leq 22^\circ, 22^\circ \leq \lambda \leq 38^\circ)$. Fashir introduced the use of GGMs in computing the long wavelength components of the Earth gravity. He used Goddard Earth Model

(GEM-T1) with a modified Stoke's kernel to compute the geoid height. More recently [Abdalla \(2009\)](#) proposed a new gravimetric datum for Sudan KTH-SDG08. Ahmed's model was based on the new dedicated satellite missions (Goce/Grace, and CHAMP). New satellite mission have an improved results over the previous satellite missions, thanks to the gradiometer and the precise positioning system from GNSS. The new gravimetric geoid model (KTH-SDG08) has been determined over the whole country of Sudan at 5 x 5 grid for area ($4^\circ \leq \phi \leq 23^\circ$, $22^\circ \leq \lambda \leq 38^\circ$). The optimum method provides the best agreement with GPS/levelling estimated to 29 cm while the agreement for the relative geoid heights to 0.493 ppm [Abdalla \(2009\)](#). Ahmed used GRACE02S gravitational model (for the long wavelength part) and 30x 30 SRTM DEM (for the short wavelength part of Earth's gravity field), beside the terrestrial data for the medium wavelength part. For a fair comparison, we compared only the results of [Abdalla \(2009\)](#) with our results. It clearly shows that the std has dropped from 0.576 (in KTH-SDG08) to 0.349 in our case. We gained a 40% accuracy without any terrestrial observations. We wanted to compare our results with that of [Fashir \(1991\)](#), but as indicated by [Abdalla \(2009\)](#) Doppler data of [Fashir \(1991\)](#) is not available. From [Abdalla \(2009\)](#), Fashirs model looks smoother than KTH-SDG08. The drastic values of the geoidal height in the north-west corner and south-east corner are 14 m and -10 m for Fashirs model while 20 m and -14 m for KTH-SDG08, respectively. The fitting with the reference ellipsoid is similar from the northwest to south-east. Fashirs model covers the reference ellipsoid over a large area (approximately 75%). On the contrary KTH-SDG08 model apparently keeps the same fitting with reference ellipsoid as in the original area before resizing.

4.2 Computing the Geoid

We developed a software application called GeoidApp for this task. GeoidApp is a huge suite of applications to not only compute geoid height (in addition to gravity anomaly and gravity disturbance), but it also has analysis and interactive visualization features to help the users get the most out of it. For that reason we will discuss the core features about GeoidApp. To compute the geoid height you need to account for these details

- The data. GGMs are provided as *.gfc files from ICGEM. They contains in addition to the coefficients text contents about the authors, acknowledgments, etc. They also include model parameters, such as the maximum degree, the model name, the GM, and a few others. The idea is to store the coefficients in an array (for computations efficiency).

- Helper functions for Associated Legendre Polynomials. That could be tricky, solving Legendre function is non-trivial. The recursive version of Legendre function is suitable for programming environment.
- The main function for geoid computations. You have to choose whether you want to work on point mode, grid mode.

4.2.1 Working with GGMs data

Typical GGMs provided by ICGEM are raw and need to be parsed to extract the model's coefficients and other useful informations about the model. When we say a 'patter', or 'template' we mean that there are certain keywords in the *.gfc files, and it is available in all models.

4.2.1.1 Parsing the data

A typical model from ICGEM would consist of the following

- General information about the mission.
- A header with structured details about the models, and their parameters
- The coefficients part. Which is the core part of the *.gfc files

Model's summary . It includes summary about the mission its release date, mission time, etc. There is also a link for detailed informations about the mission. You can safely ignore this part of the *.gfc file. This part of *.gfc files does not follow any pattern, you cannot build a generic tool to extract informations from it.

The header . This summarizes the previous in a template way. It will begin with the keyword *original header*. Below it, there is the name of the model, and the release date, mission duration. It also has information about GM but you can also ignore it here as it will be provided later in more structured way. A good way to think of *.gfc files is xml format. xml files are constructed as `<begin_of_tag> some_data </end_of_tag>`. For our work, we treated *.gfc files as follow

```
<begin_of_head>
  <original header>
    "product_type": "gravity_field" /* In our case we need this */,
    "model_name": "model_name",
```

```

"radius": "6378136.30" /* It varies over models */,
"earth_gravity_constant": "398600.4415D+09" /* It varies over models */,
"max_degree": "n",
"errors": "formal", /* It could take other values */,
"tide_system": "tide_free" /* other types are available */,
</original header>
</end_of_head>

```

Note that we use those keywords to store these variables with their corresponding values in a data structure (a dictionary, or hash table is suitable for this purpose).

The last part of parsing global geopotential models is extracting the model coefficients C_{nm}, S_{nm} . We want to construct a $(n \times n)$ matrix, where n denotes the maximum degree of our model. For each row of our matrix will correspond to the degree of the model. Each will have no-zero elements as the index of of that row. The first row will always have value 1, and the rest of it are zeros. The second row will depend on an argument that is specified by the user *subtract_normal_field*. In our evaluations we always set such that the “normal_field” will be subtracted from coefficients $c_{nm}(1 : 9, 1 : 9)$. It’s important to note that indexing for global geopotential models starts from zero as oppose of one. You should account for that if your are using programming language that uses one as a base for indexing e.g., MATLAB/Octave or Julia.

$$C_{nm} = \begin{bmatrix} 1 & \dots & \dots & N_{max} \\ 0 & 0 & \dots & N_{max} \\ \vdots & \vdots & \ddots & N_{max} \\ -4.84e-04 & -3.98e-10 & 2.43e-06 & N_{max} \end{bmatrix}$$

For the S_{nm} part there the first column of it, i.e., the value of S_{nm} coefficients are always zero. That is $S_{nm}[:, 0] = 0$. Unlike C_{nm} the number of non-zero elements for each row in S_{nm} equals the index of that row minus 1.

$$S_{nm} = \begin{bmatrix} 0 & \dots & \dots & N_{max} \\ 0 & 0 & \dots & N_{max} \\ \vdots & \vdots & \ddots & N_{max} \\ 0 & 1.42e-09 & -1.40e-06 & N_{max} \end{bmatrix}$$

The same was applied to construct $e_{C_{nm}}, e_{S_{nm}}$ the std values for C_{nm}, S_{nm} , respectively.

4.3 Software Implementation

4.3.1 Associated Legendre Function

Solving Legendre function is not easy. It is always good to build on top of others projects, and reference them as needed. We found a few libraries that solve Legendre function. One of them is legendre function in provided by MATLAB. This function does not come with the standard version of MATLAB, so we avoided it. The implementation of it does not also meet with our use case. We have also found other implementation of Legendre that was written in either C++ or Java, both of them was not used during the development of GeoidApp. Another very recent implementation of legendre function was that of NumPy, a popular numerical library for Python. As the time of writing our code, NumPy's legendre was not yet implemented.

We had to either translate libraries from other languages, or modify that version of MATLAB. We decided to write our own implementation of Legendre. In both case we had modify something anyway, we also want to ensure that the implementation of our function should be compatible with other parts of our application. Legendre function should also be well optimized, the most expensive part of geoid computations is the computing legendre polynomials for each degree, not to forget that the computation increase with the increase in degrees. Another criteria is the stability of the solutions in high order and degrees of the model. However, when evaluated increasingly close to the poles, the ultra-high degree and order (e.g. 2700) ALFs range over thousands of orders of magnitude. This causes existing recursion techniques for computing values of individual ALFs and their derivatives to fail [Holmes and Featherstone \(2002\)](#). We followed [Holmes and Featherstone \(2002\)](#) implementation to make our Legendre function.

4.3.2 Geoid height function

We will begin by discussing various implementation techniques regarding computing the geoid. We assume that the reader is familiar with the mathematical details about computing the geoid. Hence, our discussion here will be about the efficiency, and our design decision for GeoidApp. We will begin our discussion by an overview of the existing libraries and tools to compute the geoid, and compare them—in terms of design and efficiency—with our work. We have tried to contact the authors of EGMLab, but the did not respond, neither we did able to find any valid link for it. In [Kiamehr and Eshagh \(2008\)](#) they have a model reader, which was not very clear from their description how it was being implemented. Their model reader does basic things like converting the column coefficients into a matrix form, but they did not mention, and extracting the

model name. Then they multiply the parsed coefficients by the GM (as a fixed value here, but they also said that GM might be dynamically changed.) Their model, which is a critical part in our comparison only works within maximum degree and order of 720 (based on EGM2008). They also report the use of Horner and Clenshaw algorithms algorithm will expand the maximum degree and order up to 2170 as shown in [Holmes and Featherstone \(2002\)](#). Their investigation on Clenshaw and Horner algorithm using A Java program on official ICGEM website indicates that in order to use a model with a degree up to 2160, the result will take at least an hour! In our work we show that we can compute geoid height (or other geoid components) within a few minutes (particularly 6 minutes). In terms of efficiency, it is clear that our work surpasses [Kiamehr and Eshagh \(2008\)](#) work by many order of magnitude. Also, because of Clenshaw and Horner, our work is able to account for any future models, as long as they are within 2170 range, which is not going to be exceeded too soon. In terms of software design, [Kiamehr and Eshagh \(2008\)](#) works in point-wise mode as well as in area mode. Our work does also provide that feature i.e., you can compute the geoid height for a point, as well as a grid. It is also important to mention that [Kiamehr and Eshagh \(2008\)](#) in their work report that their results surpassed that of official EGM96 results. Which indicates the same for our work.

The other work we have considered is the official ICGEM calculation service available at [ICGEM](#). The implementation details are hidden. They have a nice JavaScript front-end, that works not only as a wrapper for a low level language (C or C++), but also serves well their purpose of running the application in the browser. They have many application in addition to geoid height. We were inspired by many of their implementations, namely “functional”, “tide_system” keywords. Their results are in a grid format. You cannot use their results in any sort of evaluations, unless you made a separate interpolation function for that purpose. You cannot also run multiple instances of ICGEM service simultaneously (3 instances simultaneously is the maximum allowed). But that should not be a problem. Our work not only does allow you to work in both point-wise and grid mode, but it also gives you a powerful application with no limitations in terms of usage or in computations. Our work also come with a tool to automatically download models from ICGEM website, so if you use it without specifying the model name, it will download the latest and the greatest model. We also have an interactive visualization tool so that we will make the process of understanding the data more easier. Another feature of our work is the automation of degree computations of the models. This feature is specially important in the case of the evaluations, where researchers often need to work with different models at different degree to find the best one. This task could be a problem if you manually tune the models. Another feature that is unique in our work, is that we try to automate the process of using the model by comparing it with freely available terrestrial data the is geographically near to the user’s input data (based on

their latitude and longitude). If the user has provided an evaluation data (GPS/leveling data) that will ease and automate the process, so that the output will be the best model with the best degree. The output will also contain a file with a top ten models with their degrees based on std criteria.

The last similar work is GeographicLib which is a small C++ classes for various geodesy computations e.g., conversion between UTM and geodetic coordinates (latitude, longitude). What is similar to our work is their geoid class implementation. They only used 3 models, EGM84, EGM96, and EGM2008. You can see that it is very limited and also too outdated. Another limitation is that this library does only work in the point-wise mode. That will severely limit the use of it in large project where you have several points.

Appendix A

An Appendix

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