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Global height system unification with GOCE: a simulation study on the indirect bias term in the GBVP approach

Christian Gerlach · Reiner Rummel

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Abstract One of the main objectives of ESA's Gravity Field and Steady-State Ocean Circulation mission GOCE (Gravity field and steady-state ocean circulation mission, 1999) is to allow global unification of height systems by directly providing potential differences between benchmarks in different height datum zones. In other words, GOCE provides a globally consistent and unbiased geoid. If this information is combined with ellipsoidal (derived from geodetic space techniques) and physical heights (derived from leveling/gravimetry) at the same benchmarks, datum offsets between the datum zones can be determined and all zones unified. The expected accuracy of GOCE is around 2–3 cm up to spherical harmonic degree $n_{\max} \approx 200$. The omission error above this degree amounts to about 30 cm which cannot be neglected. Therefore, terrestrial residual gravity anomalies are necessary to evaluate the medium and short wavelengths of the geoid, i.e. one has to solve the Geodetic Boundary Value Problem (GBVP). The theory of height unification by the GBVP approach is well developed, see e.g. Colombo (A World Vertical Network. Report 296, Department of Geodetic Science and Surveying, 1980) or Rummel and Teunissen (Bull Geod 62:477–498, 1988). Thereby, it must be considered that terrestrial gravity anomalies referring to different datum zones are biased due to the respective datum offsets. Consequently, the height reference surface of a specific datum zone deviates from the unbiased geoid not only due to its own datum offset (*direct bias term*) but is also indirectly affected by the integration of biased gravity anomalies. The latter effect is called the *indirect bias term* and it considerably complicates the adjustment model for global height unification. If no satellite based gravity model is employed, this error amounts to about the same size as the datum offsets, i.e. 1–2 m globally. We show that this value decreases if a satellite-only gravity model is used. Specifically for GOCE with $n_{\max} \approx 200$, the error can be expected not to exceed the level of 1 cm, allowing the effect to be neglected in practical height unification. The results are supported by recent findings by Gatti et al. (J Geod, 2012).

Keywords GOCE · GBVP · Height unification · Indirect bias term

1 Introduction

Classically, height systems are based on spirit leveling in combination with gravimetry along the leveling lines. From this, potential differences can be derived as basic observations in a leveling network. As only potential differences can be determined, there is a datum defect which is fixed by assigning a potential (or height) value to a datum point. The choice of both the datum point and the value is in principle arbitrary. But given the intention of deriving heights above sea level, datum points are usually chosen close to tide gauges, i.e. close to mean sea level (MSL), and the adopted potential (or height) value at a datum point is chosen such that the zero level corresponds to MSL. The potential difference between any point P in the network and the datum point O is the geopotential number

$$C_{PO} = W_O - W_P, \tag{1}$$

where W_O and W_P are gravity potential values at points O and P, respectively. The geopotential number can be transformed

C. Gerlach (✉)
Bavarian Academy of Sciences and Humanities, Munich, Germany
e-mail: gerlach@keg.badw.de

R. Rummel
Technische Universität München, Munich, Germany

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into a height value, i.e. into metric units, by division by a suitable gravity value. Depending on the gravity value chosen, different types of heights can be defined like orthometric or normal heights (Heiskanen and Moritz 1967). In case of orthometric heights, the vertical reference surface is an equipotential surface. Figure 1 explains the geometry under the assumption that the datum point is located at MSL which is adopted as the zero level surface in the corresponding datum zone; this assumption on datum choice is used throughout the text.

Globally, a large number of independent height datums exists. They all refer to different equipotential surfaces and in many cases height values of different national networks show jumps along the border lines. It is the aim of height system unification to refer all of these different height systems to one common datum. Globally, consistent height information will eliminate complications in civil constructions associated with linking data from different height zones and it will facilitate interpretation of sea level records at globally distributed tide gauges.

The height datum problem has been treated extensively in the geodetic literature, e.g. in Colombo (1980), Rummel and Teunissen (1988), Xu (1992) or Rapp and Balasubramania (1992). An overview of the development of different approaches is given in Sansò and Venuti (2002). In principle, there are three methods available for the connection of different datum zones (Rummel 2001), namely

1. direct connection by leveling and gravimetry,
2. oceanographic approaches and
3. indirect connection by solving the geodetic boundary value problem.

Approach (1) is straight forward, but can only be used on the continents. Therefore, it is not suitable for global height unification.

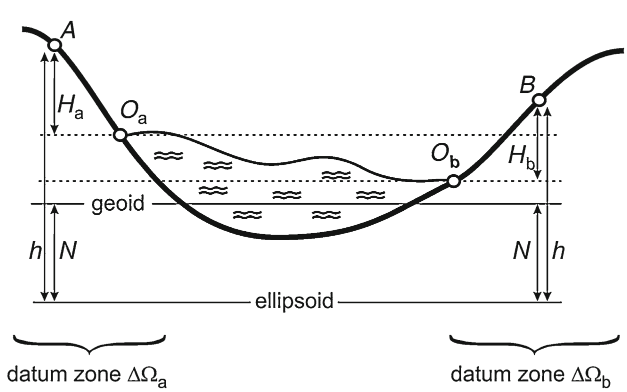

Fig. 1 Relation between geoid heights N, ellipsoidal heights h and orthometric heights H_a and H_b (referring to datum zones $\Delta\Omega_a$ and $\Delta\Omega_b$) given at two benchmarks A and B. The dashed lines represent equipotential surfaces through datum points O_a and O_b; N_a and N_b are the corresponding heights of these surfaces above the ellipsoid

Oceanographic methods (2) make use of the mean dynamic topography (MDT), i.e. the height of the mean sea surface above the geoid. If the MDT would be precisely known at tide gauges (co-located at datum points), then the MDT difference between different gauges would correspond to the height offset between the datum zones. Information on MDT can come from different sources, like ocean models (steric or dynamic leveling) or a combination of satellite altimetry and a precise gravimetric geoid (altimetric leveling). The accuracy of oceanographic methods depends on data availability and quality as well as on the spatial resolution of the ocean model in use. The altimetric method, which does not require oceanographic data, faces problems at the coastline because of the low quality of altimetric sea surface heights close to the coastline, see Dufau et al. (2011) or Gommenginger et al. (2011). But the coastline actually is where tide gauges are located. In addition, one has to take care of different spectral resolutions of a geoid model and altimetric sea surfaces.

Method (3) makes use of the basic relation

$$h - N = H \tag{2}$$

between ellipsoidal heights h, orthometric heights H and geoid heights N. The relation can in similar form be expressed in terms of normal heights and height anomalies, but in order to keep the discussion simple, we will stick to the geoid case throughout the text. Figure 1 shows the basic geometry. Given are two datum zones $\Delta\Omega_a$ and $\Delta\Omega_b$ related to datum points O_a and O_b, respectively, and one leveling benchmark per zone (stations A and B in Fig. 1). Let us assume the geoid is solely derived from satellite gravimetry (neglecting for a moment the corresponding omission error), then the geoid heights $N(A)$ and $N(B)$ can be computed at both benchmarks independently from any terrestrial data and, therefore, also independent from vertical datum definitions. If the three-dimensional geocentric coordinates of the benchmarks have been determined from space techniques (GNSS, SLR, etc.), then ellipsoidal heights $h(A)$ and $h(B)$ are available and orthometric heights $H(A)$ and $H(B)$ (referring to the geoid) can be derived according to Eq. (2). In practise, orthometric heights of different datum zones are derived from the combination of leveling and gravimetry. Such heights are connected to the datum points of the respective datum zones (zones $\Delta\Omega_a$ and $\Delta\Omega_b$ in Fig. 1) and, therefore, refer to the equipotential surfaces through the corresponding datum points rather than to the geoid (see Fig. 1). Due to the datum offsets between these surfaces and the geoid, the orthometric heights are biased and consequently are labeled $H_a(A)$ and $H_b(B)$. Comparison of $H_a(A)$ and $H_b(B)$ with $H(A)$ and $H(B)$ reveals the datum offset between datum zones $\Delta\Omega_a$ and $\Delta\Omega_b$ and both zones can be unified. The approach can also be applied in relative mode, where potential or height differences between benchmarks are compared, rather than absolute values. Of course, only relative datum offsets between datum zones can be derived in this case (one zone would have to be selected as reference zone to which all others are linked), but this is sufficient for practical needs.

One of the mission objectives of GOCE is to contribute to the global unification of height systems (ESA 1999), by providing a high resolution geoid at the cm-level, solely derived from satellite data. The accuracy of geoid heights derived from GOCE at the end of the mission life time can be expected to be around 2–3 cm up to spherical harmonic degree 200 (Pail, personal communication). Therefore, global height unification should in principle be possible at the same level of accuracy. Unfortunately, the omission error at degree 200 still amounts to about 30 cm on global average, which is too high to be neglected. Therefore, the omission part, i.e. the residual geoid height above the resolution of GOCE, must be determined from terrestrial gravity anomalies. As will be outlined in Sect. 2, this is exactly what complicates height unification, because the terrestrial anomalies are not only derived from measured gravity g_P (at surface point P) and normal gravity γ_0 (computed at the surface of the ellipsoid), but also from orthometric heights (for the purpose of gravity reduction from the surface to the geoid) according to

$$\Delta g_l = g_P - \frac{\partial g}{\partial h} H_l - \gamma_0, \tag{3}$$

where index l indicates relation to a datum zone $\Delta\Omega_l$. As the orthometric height H_l is biased due to the unknown offset between the height reference surface through O_l and the geoid, so will the gravity anomaly Δg_l be biased (see also Heck 1990). Assuming a maximum datum offset ΔH_l in the order of 1–2 m, gravity anomalies will be biased by

$$\frac{\partial g}{\partial h} \Delta H_l \approx 0.3086 \text{ mGal/m} \cdot \Delta H_l < 0.6 \text{ mGal}. \tag{4}$$

This systematic distortion is constant within each datum zone and it enters the solution of the GBVP, therefore, indirectly also affecting height datum unification. As discussed in Sect. 2, this indirect bias term can be estimated in a least-squares adjustment model (see also the simulation study by Xu 1992). Amos and Featherstone (2009) have performed unification of different local datums in New Zealand, taking care of the indirect bias term in an iterative manner. They combined terrestrial gravity data with a hybrid satellite gravity model consisting of GGM02S (Tapley et al. 2005) and EGM96 (Lemoine et al. 1998). Within each iteration step gravity anomalies were corrected for datum offsets and iteration was continued until convergence of the offsets was achieved, i.e. until all datums were unified. Offsets of up to 58 cm could be determined this way with an average accuracy of 8 cm. In a recent study, Gatti et al. (2012) have estimated the quality of height system unification from a combination of GOCE and EGM2008 (Pavlis et al. 2012), the latter being affected by unknown datum offsets in terrestrial gravity data. By means of the Schwarz inequality they derive an upper bound for the indirect bias term in the spectral domain. For an example of a global patch of 11 large datum zones (covering both continents and oceans) and assuming a maximum degree of 250 for the GOCE model they come up with an estimate of 4.2 mm. Finally, their accuracy estimate for global height system unification from a combination of GOCE (up to degree 250) and EGM2008 is at the level of 4 cm, indicating that the unification can be performed without explicitly considering the indirect bias term.

It is the intention of our paper to explicitly evaluate the magnitude of the indirect bias term and to investigate under which conditions it can be neglected—especially considering the case when a GOCE gravity field is used to model the long wavelengths of the geoid up to maximum degree $n_{\max} = 200$. In Sect. 2, we review the theory of height unification by the GBVP approach and discuss the indirect bias term. In Sect. 3, several simulations are carried out to investigate the size of the indirect bias term. This comprises a simplified one-dimensional case (Sect. 3.1) as well as a realistic scenario adapted to the case of the well-known datum offsets in Europe (Sect. 3.2).

2 The geodetic boundary value problem approach

The GBVP approach to height unification was intensively discussed and tested in the literature. Here we follow closely the derivation of Rummel and Teunissen (1988). Given a surface point P and its vertical projections P_0 on the equipotential surface at sea level (through datum point O) and P_0' on the ellipsoid (see Fig. 2), the gravity potential W_0 on the equipotential surface through O can be written as

$$W_0 = U_0 + \frac{\partial U}{\partial n} N + T, \tag{5}$$

where U_0 is the normal potential on the surface of the ellipsoid, $\partial U/\partial n$ its vertical derivative, N the height of P_0 above the ellipsoid and T the disturbing potential $T = W - U$. Replacing, in the above equation, the vertical derivative by normal gravity, i.e. $\partial U/\partial n = -\gamma$, and setting

$$\Delta W_0 = W_0 - U_0 \tag{6}$$

we get Bruns's equation

$$N = \frac{T - \Delta W_0}{\gamma}. \tag{7}$$

According to the classical definition (Heiskanen and Moritz 1967) the normal potential value U_0 on the surface of the ellipsoid equals the value of gravity potential W_0 on the geoid. In theory, the solution of the GBVP in linearized form leads to $\Delta W_0 = 0$ (see Heiskanen and Moritz 1967) and Bruns's

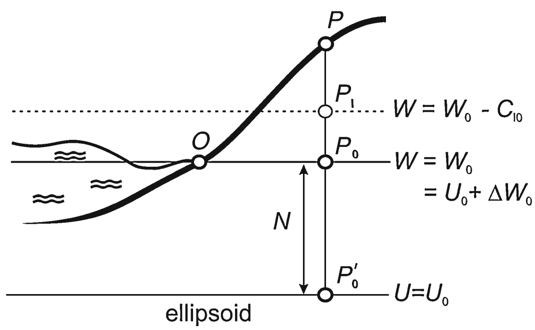

Fig. 2 Relations between points on different reference levels/surfaces

equation simplifies to $N = T/\gamma$. In reality, we do a priori not know any point laying on the geoid (such that we can put a reference marker on the geoid in order to physically realize the vertical reference frame). Therefore, we will always have to consider an unknown offset term ΔW_0. For this aspect, we refer to Sacerdote and Sansò (2001). Besides this globally constant offset, different height datums refer to even different equipotential surfaces, i.e. there exist additional biases which vary from datum zone to datum zone.

Let us assume there are $L+1$ different height datum zones $\Delta\Omega_l$, $l = \{0, 1, \ldots, L\}$, referring to different tide gauges and, therefore, to different reference levels. Let one of the datums ($\Delta\Omega_{l=0}$) be selected as reference datum. Then height unification aims at linking all other datum zones to $\Delta\Omega_0$. The difference between gravity potential W_0 refering to the selected reference datum and gravity potential W_l of any of the other datums is

$$C_{l0} = W_0 - W_l. \tag{8}$$

Combining this with equations (5) and (6), we get

$$\Delta W_0 - C_{l0} = W_l - U_0 = -\gamma N_l + T, \tag{9}$$

where N_l is the height of point P_l above the ellipsoid (see Fig. 2). Consequently, Bruns's equation (7) takes the form

$$N_l = \frac{T - \Delta W_0 + C_{l0}}{\gamma}. \tag{10}$$

Equation (9) is the linear observation equation of potential anomalies. Analogously, the linear model for gravity anomalies in spherical approximation is (see Rummel and Teunissen 1988)

$$\Delta g_l = \frac{\partial\gamma}{\partial n} N_l - \frac{\partial T}{\partial n} = -\frac{2\gamma}{r} N_l - \frac{\partial T}{\partial r}. \tag{11}$$

Inserting (10) into (11) leads to the *fundamental equation of physical geodesy*, which serves as boundary condition in solving the GBVP:

$$\Delta g_l = \frac{2}{r}\Delta W_0 - \frac{2}{r} C_{l0} - \left(\frac{2}{r} + \frac{\partial}{\partial r}\right) T. \tag{12}$$

Thereby, Δg_l are the biased gravity anomalies which are derived from terrestrial data and

$$\Delta g = \Delta g_l + \frac{2}{r} C_{l0} \tag{13}$$

are the unbiased anomalies which are corrected for the datum offset of datum zone l and, therefore, refer to the reference datum zone $\Delta\Omega_0$. Prior to height unification, the datum offsets C_{l0} are unknown and only the biased anomalies Δg_l can be used to determine geoid heights from terrestrial data. The solution of the GBVP for points P in datum zone k is given by

$$T(P) = \frac{\delta(GM)}{r} + \frac{R}{4\pi} \iint_{\Omega} St(\psi_{PQ}) \left(\Delta g_l + \frac{2}{R} C_{l0}\right) \mathrm{d}\Omega_Q \tag{14}$$

and after insertion into Bruns's equation by

$$N_k(P) = \frac{\delta(GM)}{R\gamma} - \frac{\Delta W_0}{\gamma} + \frac{C_{k0}}{\gamma} + \ldots + \frac{R}{4\pi\gamma} \iint_{\Omega} St(\psi_{PQ}) \left(\Delta g_l + \frac{2}{R} C_{l0}\right) \mathrm{d}\Omega_Q. \tag{15}$$

Thereby, $\delta(GM)$ is the unknown error of the adopted value of the geocentric mass constant GM. Combining the first two terms on the right hand side to the global constant offset term N_0, setting $C_{k0}/\gamma = N_{k0}$ and separating the Stokes integral into two parts N^{Stokes} and N^{ind}, where the first one takes care of the global integration of terrestrial (biased) gravity anomalies Δg_l, we can write Eq. (15) in the form

$$N_k(P) = N_0 + N_{k0} + N^{\text{Stokes}} + N^{\text{ind}}, \tag{16}$$

with

$$N^{\text{Stokes}} = \frac{R}{4\pi\gamma} \iint_{\Omega} St(\psi_{PQ})\, \Delta g_l \, \mathrm{d}\Omega_Q \tag{17}$$

and

$$N^{\text{ind}} = \frac{1}{2\pi} \iint_{\Omega} St(\psi_{PQ})\, N_{l0} \, \mathrm{d}\Omega_Q. \tag{18}$$

Assuming the bias N_{l0} to be constant in each datum zone, we can write

$$N^{\text{ind}} = \frac{1}{2\pi} \sum_{l=1}^{L} N_{l0} \iint_{\Delta\Omega_l} St(\psi_{PQ_l})\, \mathrm{d}\Omega_{Q_l} \doteq \sum_{l=1}^{L} N_{l0} S_l, \tag{19}$$

where S_l represents the integration of Stokes's function over the area of datum zone l. Note that summation starts at $l = 1$, because the bias of datum zone $\Delta\Omega_0$ is zero by definition. Equation (15) shows that the offset term C_{l0} (or N_{l0}) enters twice into the solution of the GBVP:

1. In datum zone k, it directly enters as N_{k0}; this is called the **direct bias term**; it is important to note that the geoid height in datum zone k is only affected by the bias from its own datum zone via the direct bias term.
2. In addition, the offset shows up in N^{ind}; this part is called the **indirect bias term**; since the Stokes integration is performed globally, the geoid height in datum zone k is affected by offsets from all datum zones. This is clearly shown in Eq. (19) where summation is performed over all datum zones globally.

Assuming that $N_k(P)$ can be derived independently from GNSS observations at leveled benchmarks, Eq. (16) can be rearranged in form of an observation equation for determining the unknown offsets by least-squares adjustment:

$$N_k(P) - N^{\text{Stokes}} = N_0 + N_{k0} + \sum_{l=1}^{L} N_{l0}\, S_l. \tag{20}$$

The quantities on the left hand side are observed or computed from observations. On the right hand side appear the unknowns, i.e. the global offset term N_0 as well as the relative offsets N_{k0} and N_{l0}. This adjustment model was set up by Rummel and Teunissen (1988) and tested with simulated data by Xu (1992). The indirect bias term clearly complicates the adjustment, because it requires to integrate, for each observation point P, Stokes's function separately over all datum zones; this also requires to reconstruct all datum zones; in addition, the model assumes that there is just one constant offset in each zone, while in practise distortions of large leveling networks can easily reach decimeters within a datum zone. But these distortions are caused by systematic measurement errors and are, therefore, distinct from the problem of datum choice. Therefore, they will not be discussed in this paper.

The indirect bias term can only be avoided, if the disturbing potential $T(P)$ is not derived from (biased) terrestrial data (according to Eq. (14)), but if it is derived from satellite methods. Assuming that $T(P)$ could solely be derived from a satellite gravimetry mission like GOCE, then the indirect bias term would disappear. In reality, due to the limited spectral/spatial resolution of satellite methods, only the low frequency portion of T up to a certain maximum degree $n_{\max}$ can be derived from satellite methods. Even if T is derived from GOCE, $n_{\max}$ will be limited to around 200–230. The remaining omission part above this degree still amounts to some decimeters and cannot be neglected. We may write

$$T(P) = T^{\text{GOCE}} + T^{\text{res}}, \tag{21}$$

with the omission part

$$T^{\text{res}} = \frac{R}{4\pi} \iint_{\Omega} St^{\text{res}}(\psi_{PQ})\, \Delta g_l^{\text{res}}\, d\Omega_Q \tag{22}$$

derived from residual (biased) terrestrial gravity anomalies. Since only the residual part of the terrestrial data above degree $n_{\max}$ needs to be integrated, we may also replace the original Stokes kernel by the residual kernel St^{res} which contains only degrees $n > n_{\max}$. It holds

$$St^{\text{res}}(\psi) = St(\psi) - St^{n_{\max}}(\psi). \tag{23}$$

The low frequency component $St^{n_{\max}}$ can be computed according to Heiskanen and Moritz (1967) by

$$St^{n_{\max}}(\psi) = \sum_{n=2}^{n_{\max}} \frac{2n+1}{n-1} P_n(\cos\psi), \tag{24}$$

where $P_n(\cos\psi)$ are the Legendre polynomials.

Based on known datum offsets $C_{l0} = \gamma N_{l0}$ the effect of the indirect bias term on geoid heights at any point P can be evaluated from Eq. (18). Evaluations of N^{ind} have been performed, e.g., by Denker (2001) or Gerlach (2001) in order to quantify possible remaining error contributions in regional quasigeoid computations. Both studied the hypothetic effect of known datum offsets in Europe (see Sect. 3.2), when combining the accordingly biased terrestrial gravity anomalies with EGM96 and found error structures with amplitudes of some few centimeters. Similar simulations are carried out in the following section focusing on the exploitation of GOCE gravity models.

3 Simulation studies

We now want to quantify the size of the indirect bias term in the solution of the GBVP employing both, a high degree satellite gravity model and (biased) terrestrial gravity anomalies. For a general treatment on the order of magnitude of the effect, we have carried out a one-dimensional simulation (Sect. 3.1). In Sect. 3.2, the evaluation is applied to the case of Europe, where known datum offsets are used to quantify the indirect bias term.

3.1 One-dimensional example

If we assume a circular symmetric datum zone and compute the effect of its datum offset on a geoid height at the center of the circle, the order of magnitude of the indirect bias term can be tested almost analytically. In such an isotropic case, integration over azimuth can be performed analytically and leaves us with only one dimension, i.e. the spherical distance. The offset zones $\Delta\Omega_l$ are annular regions, with polar radius ψ ranging between ψ_l and $\psi_l + \Delta\psi_l$. Thereby, ψ_l is the spherical distance between the computation point P and the l-th datum zone $\Delta\Omega_l$ and $\Delta\psi_l$ is the width of the datum zone (see Fig. 3, where there is only one datum zone $\Delta\Omega_{(l=1)}$ and computation point P is in the origin). If $\psi_l = 0$, the annulus

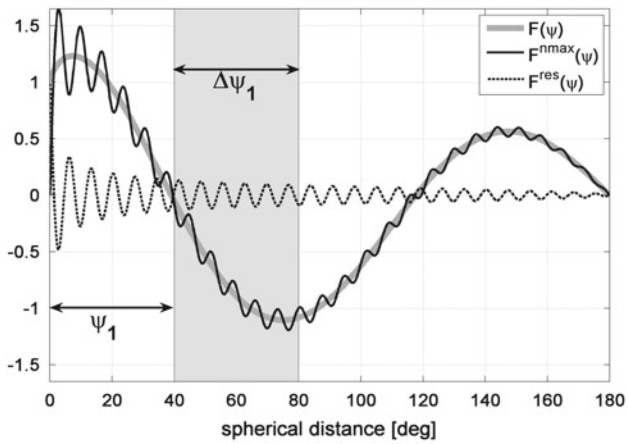

Fig. 3 Function $F(\psi)$, its spherical harmonic approximation $F^{n_{\max}}(\psi)$ up to maximum degree $n_{\max}$ (in this example $n_{\max} = 50$) and the high frequency residual $F^{\text{res}}(\psi)$. The *gray zone* marks a datum zone of width $\Delta\psi_1$ in a distance of ψ_1 from the origin/computation point

becomes a spherical cap. Introducing function

$$F(\psi) = \frac{1}{2}\, St(\psi) \sin\psi \tag{25}$$

(see Heiskanen and Moritz 1967, p. 69) and setting according to Eq. (23)

$$F^{\text{res}}(\psi) = F(\psi) - \frac{1}{2}\, St^{n_{\max}}(\psi) \sin\psi \tag{26}$$

we find

$$N^{\text{ind}} = 2 \sum_{l=1}^{L} N_{l0} \int_{\psi=\psi_l}^{\psi_l+\Delta\psi_l} F^{\text{res}}(\psi_{PQ_l})\, \mathrm{d}\psi. \tag{27}$$

This is our basic equation for the one dimensional experiment. Since this equation is linear in the datum offset N_{l0}, computations are carried out for $N_{l0} = 1$ m, for one of the l at a time. The results can then simply be scaled by multiplication with the actual datum offset of interest. It is worthwhile to remind that according to the magnitude of MDT, global datum offsets are assumed not to exceed the order of 1–2 m. Thus, the unscaled results are almost the upper bound of what one can expect.

Figure 3 shows the integrand function $F^{\text{res}}(\psi)$ along with functions $F(\psi)$ and $F^{n_{\max}}(\psi)$. For the simulation we assume only one datum zone ($l = 1$) of width $\Delta\psi_1$ at a distance ψ_1 from the computation point (gray zone in Fig. 3). Since we keep the size of the offset fixed to $N_{10} = 1$ m throughout this section, there are only three independent parameters: (1) the width $\Delta\psi_1$ of the datum zone, (2) the distance ψ_1 of the datum zone from the computation point and (3) the maximum degree $n_{\max}$ of the underlying (unbiased) satellite-only gravity model (SGM).

Before satellite missions GRACE (Tapley et al. 2004) and GOCE, the maximum degree of SGMs was limited to about $n_{\max} \approx 40$–70 (see e.g., Tapley et al. 1996; Schwintzer et al. 1997; Reigber et al. 2003) The cumulative geoid error reaches the centimeter level somewhere between degree 100 and 130 in case of GRACE and around degree 200 for GOCE. Accordingly, the simulations in the following paragraphs employ the three cases $n_{\max} = \{50, 100, 200\}$ which can roughly be identified with height system unification using SGMs before GRACE/GOCE, SGMs from GRACE and SGMs from GOCE, respectively.

Variable size of datum zone ($\psi_1 = 0°$): First, we assume the computation point to be located in the center of the datum zone. Figure 4 shows the size of N^{ind} as a function of the size of the datum zone for different maximum degrees of the underlying SGM. Due to the $n_{\max}$ oscillations of the kernel function (see Fig. 3) also N^{ind} is oscillating. The oscillations and to some extent also the magnitude of the effect can be damped by a suitable modification of the integration kernel. We did not apply any such modification, thus the results can be assumed a worst case scenario. This also holds for the two dimensional example in the next section. For $n_{\max} = 50$ the effect in Fig. 4 reaches a maximum of about 2.3 cm; this relates to the use of a SGM before the times of GRACE and GOCE and can also be identified with computation of regional geoid models based on EGM96, assuming that datum offsets (because they were unknown) could not be corrected during the preparation of EGM96. This corresponds to the computations carried out by Denker (2001) and Gerlach (2001) and the order of magnitude of the effect fits to their findings. In case of GRACE ($n_{\max} = 100$), the maximum value is about 1.2 cm and for GOCE ($n_{\max} = 200$) the curve does not exceed 0.7 cm.

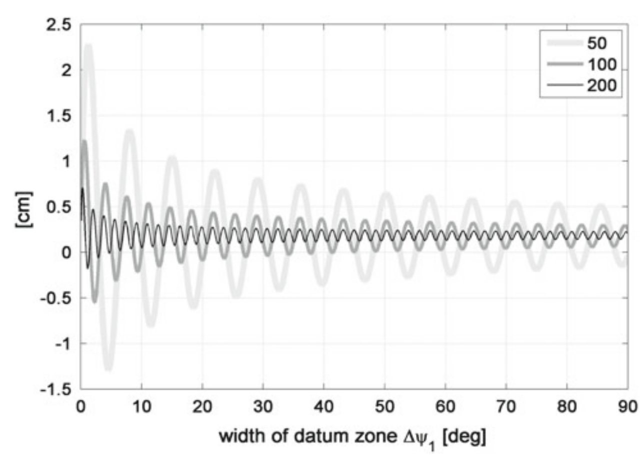

Fig. 4 Indirect bias term for different maximum degrees of the underlying satellite gravity model (SGM) as function of the datum zone size $\Delta\psi_1$ (computation point in the center of the datum zone)

Variable separation from offset zone ($\Delta\psi_1 = 30°$): In this example, we have kept the size of the datum zone fixed at $\Delta\psi_1 = 30°$ and vary the distance between computation point and offset zone. This shows the influence of remote offset zones on a computation point outside this zone. The size of the zone in ψ-direction roughly corresponds to the size of Europe and can be compared to the two dimensional example in Sect. 3.2. The magnitude of the effect shown in Fig. 5 corresponds to the first example, GOCE now providing even smaller numbers. It is obvious that the magnitude of the effect decreases with increasing distance from the offset zone. In case of GOCE, the effect does not exceed 2 mm after about 4° distance and 1 mm after about 20°.

Summary—upper bound for the indirect bias term: Extending the examples of the two proceeding paragraphs, several cases with various combinations of ψ_1 and $\Delta\psi_1$ can be analyzed for different maximum degrees $n_{\max}$. In order to condense the results in just one graph we decided to eliminate one of the parameters. Since the amplitude of N^{ind} decreases for increasing distance ψ_1, it is not essential for our purpose to show this behavior. Instead, from the function of variable separation ψ_1 (corresponding to the example shown in Fig. 5) only the largest absolute value is considered and drawn into a graph of variable size $\Delta\psi_1$ (corresponding to the example shown in Fig. 4). The result is shown in Fig. 6 and represents an upper bound for the size of the indirect bias term N^{ind} for offset zones of variable size.

Note that Fig. 6 is drawn in semi-logarithmic scale. In addition to the previous examples, also the case without use of any SGM is included, i.e. $n_{\max} = 2$. In the latter case, the effect amounts to the same size as the datum offset itself. The case $n_{\max} = 50$ (before GRACE/GOCE), gives values in the range of some few centimeters, the maximum being

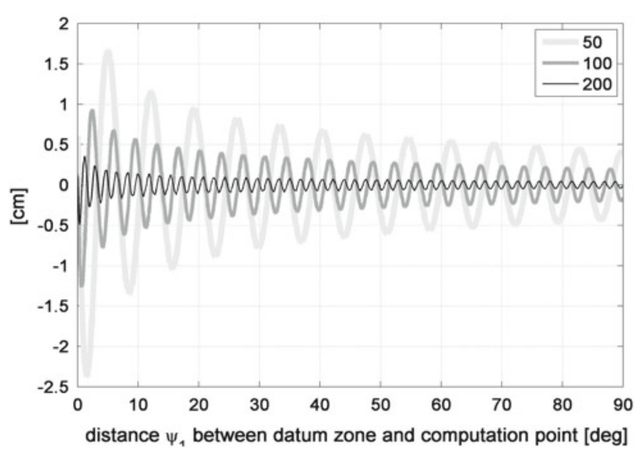

Fig. 5 Indirect bias term for different maximum degrees of the SGM as function of the distance ψ_1 between offset zone and computation point (datum zone width $\Delta\psi_1 = 30°$)

around 3.5 cm. Making use of a GRACE-only model leaves an indirect bias term in the range of about 1 cm and a maximum of 1.8 cm. In case of GOCE, the indirect bias always stays below the centimeter, with an average value of 5 mm and a maximum of 9 mm.

3.2 Two-dimensional example: Europe

Propagation of datum offsets derived from EVRF2007: As a realistic example, known datum offsets in Europe were used to compute the size of N^{ind}. The offsets are shown in Fig. 7. The numbers are differences between heights of the

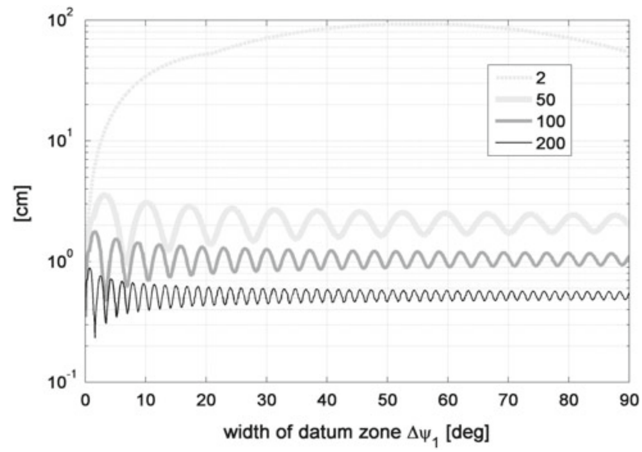

Fig. 6 Upper bound for the indirect bias term for different maximum degrees of the underlying potential model as function of the datum zone size ψ_1

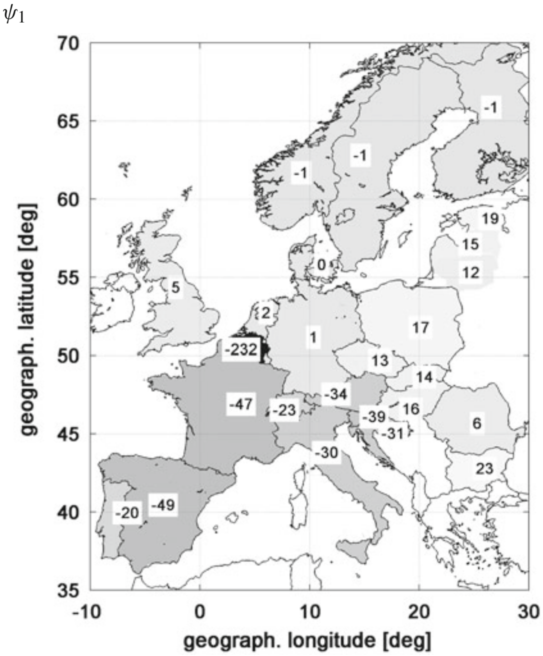

Fig. 7 Offsets between heights systems in Europe. Numbers are adopted from the description of the European Vertical Reference Frame 2007 on http://www.bkg.bund.de

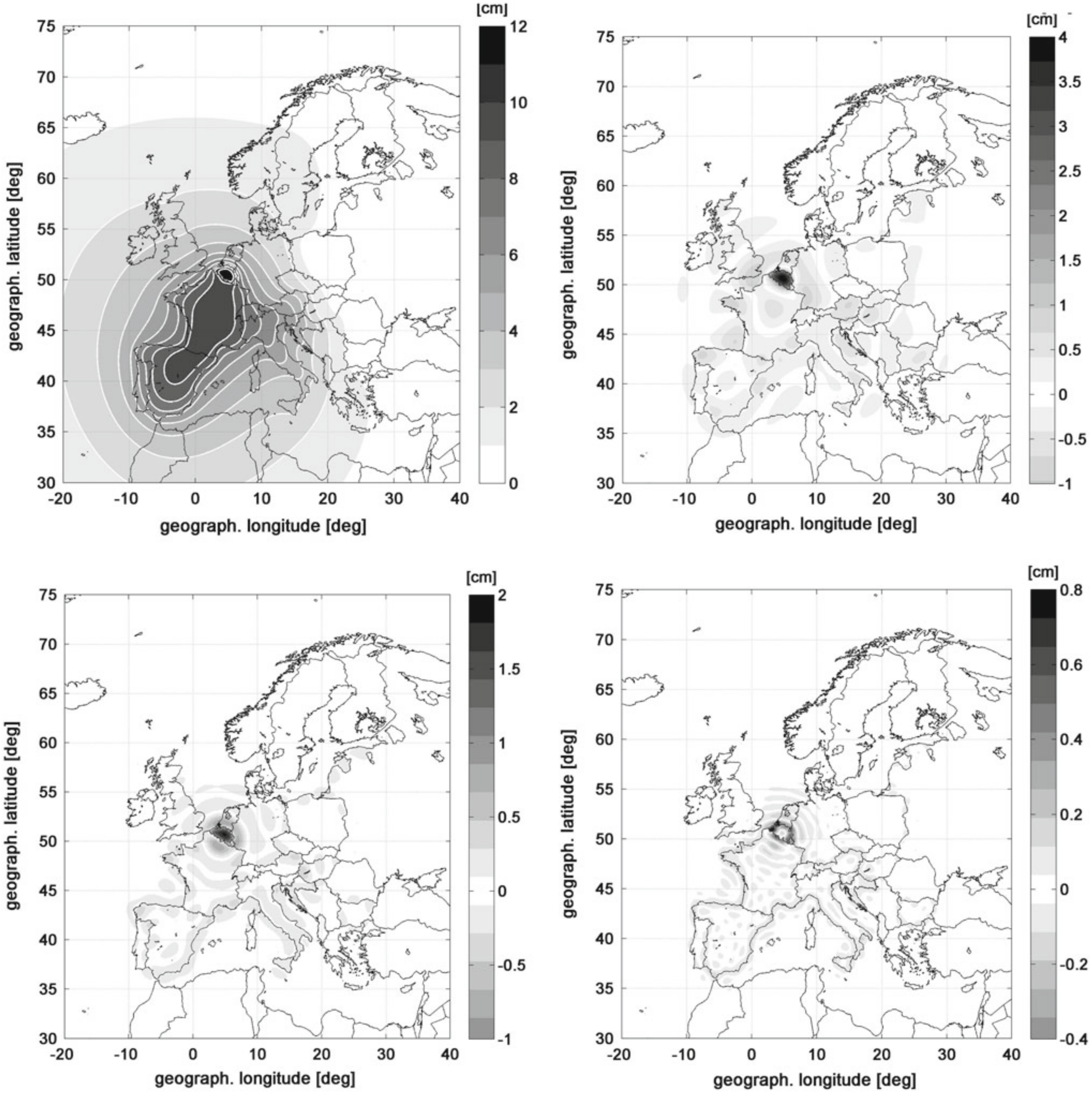

Fig. 8 Propagation of datum offsets (Fig. 7) to N^{ind} using the original Stokes kernel (*upper left*) as well as residual kernels St^{res} above degree $n_{\max}$ for: $n_{\max} = 50$ (*upper right*), $n_{\max} = 100$ (*lower left*) and $n_{\max} = 200$ (*lower right*)

European Vertical Reference Frame2007 (Sacher et al. 2009) and heights in the respective national height systems; they are published along with the description of European height reference systems on http://www.bkg.bund.de. The numbers contain differences due to relative datum offsets as well as due to the different type of heights being used. But for our experiment we just adopt the numbers as pure datum offsets.

According to the datum offsets shown in Fig. 7, the largest effects can be expected in Belgium and in the area around Spain, Italy and France. This fits to the results in Fig. 8, where the effect reaches a maximum of about 12 cm when no SGM is used.

Relating again the different maximum degrees of the SGM to GRACE/GOCE, we find that without GRACE or GOCE the effect amounts up to 4 cm in Belgium and values in the order of about 0.5–1 cm in the rest of Europe. Making use of a GRACE based SGM, the amplitude goes down to about 2 cm in Belgium and stays below 5 mm in the rest of Europe. In case of GOCE, N^{ind} stays below the centimeter with amplitudes of about 1 mm in most of Europe. The results correspond well with the upper bounds given in Fig. 6; thereby, it must be considering that the latter figure is derived for a standard datum offset of 1 m.

The case of Europe surely is not an extreme example, because most datum offsets are in the range between −50 and +20 cm, with an average magnitude of 17 cm. Only Belgium shows a significantly larger value of −2.32 m because the datum is not referred to MSL (as is the case for other European countries), but to mean low water (epoch 1878–1885) at the tide gauge in Ostend (see http://www.crs-geo.eu). But even though the amplitude of the offset in Belgium is huge, the country is small and one cannot expect a significant contribution from the indirect bias term at larger distances.

Modified European example: Both, the relatively small average datum offset of 17 cm and the small extension of datum zones with large offsets (Belgium in this case) make Europe a moderate example and one might argue that this is not representative on the global scale. One could, e.g., assume that the whole European datum area is shifted with respect to a globally unified datum and that this shift could amount to about 1–2 m. Therefore, the European example was modified in order to generate a datum zone with larger average amplitudes. Figure 9 shows the modified datum offsets, which were derived from the numbers of the first example by simply adding 1.5 m to all offsets. This way an offset zone is generated with an average amplitude of 1.4 m. The effect of these offsets on N^{ind} are shown in Fig. 10. If no

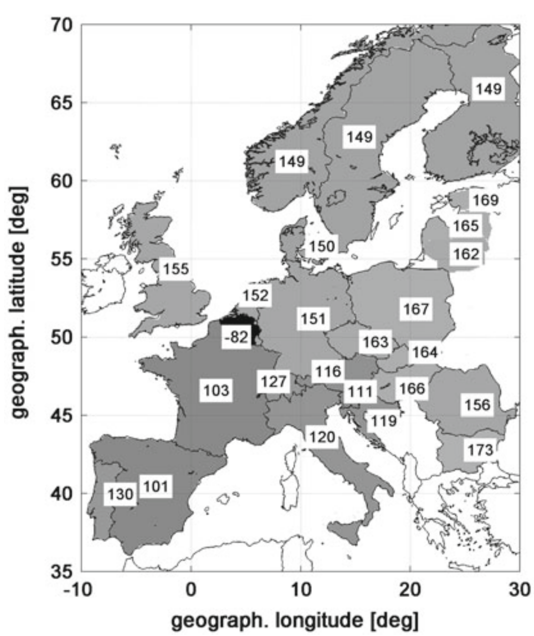

Fig. 9 Offsets between heights systems in Europe. The numbers in Fig. 7 are increased by 1.5 m

SGM is employed (upper left panel), the indirect bias term shows a long wavelength structure similar to the situation shown in Fig. 8, but due to the increased offsets it amounts to almost 60 cm in the present case. When using a SGM up to $n_{\max} = 50$ (upper right panel), N^{ind} is dominated by mid-wavelength oscillations with amplitudes up to 3 cm. In case of GRACE (lower left panel), the wavelengths get shorter and the amplitudes smaller. Still they can reach values of up to 2 cm, especially along the border of the datum zone. For GOCE, as can be expected, the wavelengths get even shorter and the amplitudes smaller. Now N^{ind} safely stays below the centimeter throughout the whole area of investigation. Also the correlation with the coast lines gets more pronounced. Once more it should be mentioned that the oscillations visible in all figures are caused by the sharp spectral cut-off of the Stokes kernel above the maximum degree of the SGM. The effect can be damped by introducing a suitable kernel modification. Since we did not introduce any kind of kernel modification, our results can be regarded a worst case scenario.

4 Summary and conclusions

Height datums of different national or continent-wide vertical networks are tied to MSL at suitable tide gauge stations. Due to global variations in MDT, different height datums in general do not refer to the same equipotential surface. Therefore, datum offsets exist between different reference systems respectively datum zones. The global unification of height systems is desirable and it is one of the main goals of the GOCE satellite mission to contribute to this task. One of the methods for height system unification is based on the solution of the GBVP, which provides gravity potential values or geoid heights at the various datum points. If in addition (biased) geoid (or quasi-geoid) heights can be determined from a combination of orthometric (or normal) heights (referring to the respective datum zone) and ellipsoidal heights derived from geodetic space techniques, the datum offsets can be computed. If GOCE would provide the geoid without omission error, the problem could be solved easily. Unfortunately this is not the case and the residual geoid height (above the resolution of GOCE) must be derived from terrestrial data. However, the solution is affected by the indirect bias term, which results from the fact that terrestrial gravity anomalies themselves are affected by the unknown datum offsets. The height unification problem can be solved either by taking care of the indirect bias term in a least-squares adjustment as shown by Xu (1992) or in an iterative manner as done by Amos and Featherstone (2009). We have carried out several computations to estimate, how large the magnitude of the indirect bias term is, in case geoid computation is based on the combination of a high resolution satellite gravity model

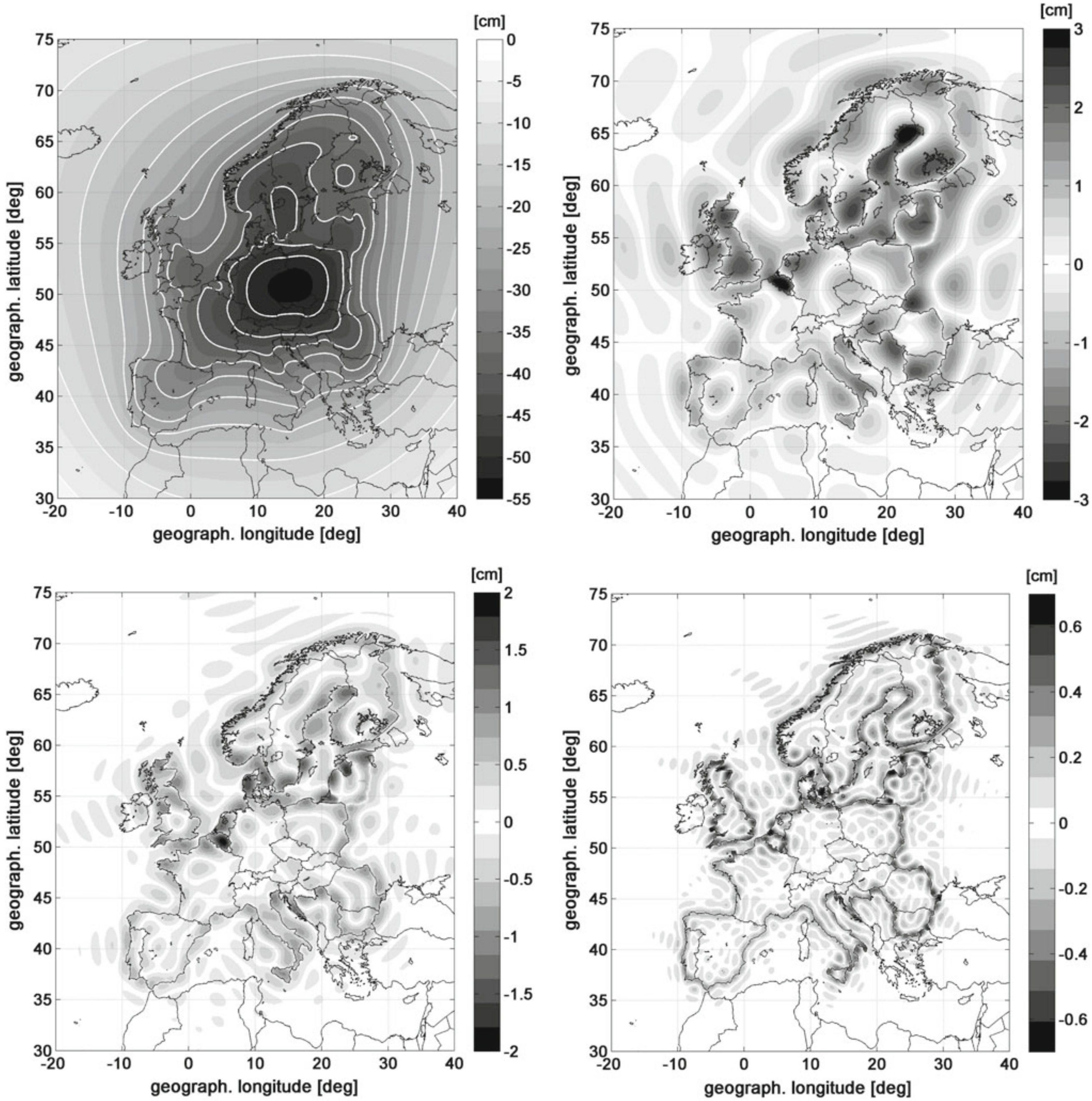

Fig. 10 Propagation of datum offsets (Fig. 9) to N^{ind} using the original Stokes kernel (*upper left*) as well as residual kernels St^{res} above degree $n_{\max}$ for: $n_{\max} = 50$ (*upper right*), $n_{\max} = 100$ (*lower left*) and $n_{\max} = 200$ (*lower right*)

(which is not affected by datum offsets) and residual terrestrial data (which are affected). We find that in case of GOCE, the indirect bias term stays below the level of 1 cm. This is below the expected commission error (about 2–3 cm) of GOCE at around spherical harmonic degree 200. Therefore, we conclude that the indirect bias term can be neglected when GOCE is used for height system unification. This conclusion is supported by recent findings by Gatti et al. (2012) who give an accuracy assessment for global height system unification for a combination of GOCE and EGM2008. They give an upper bound for the indirect bias term of 4.2 mm on global average derived in the spectral domain from a global patch of 11 large datum zones each having datum offsets in the order of 2 m (however they assume that the unbiased GOCE SGM is used even up to maximum degree $n_{\max} = 250$). Even though our result and the one by Gatti et al. (2012) are derived in different ways and assume different maximum degrees for the GOCE SGM, the conclusion is the same: one can fully exploit the high quality of GOCE for height system unification even when the indirect bias term is neglected. This simplifies height system unification enormously.

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