The Austrian Geoid 2007










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Abstract

In the framework of the project “The Austrian Geoid 2007“ (GEOnAUT), funded by the Austrian Research Promotion Agency (Forschungsförderungsgesellschaft – FFG), a new Austrian geoid solution has been computed. Compared to the official Austrian geoid model, the accuracy could be significantly improved mainly due to the substantially enhanced quality of the input data. A new digital terrain model (DTM) has been assembled as a combination of highly accurate regional DTMs of Austria and Switzerland, complemented by data of the Shuttle Radar Topography Mission (SRTM) in the neighbouring countries. In addition to a thoroughly validated data base of gravity anomalies and deflections of the vertical, new measurements of deflections of the vertical in the South-East of Austria as well as GPS/levelling information have been incorporated. Finally, these terrestrial data have been combined with global gravity field information represented by a recent GRACE gravity field model, leading to a significantly improved representation of the long to medium wavelengths of the solution. Several strategies for the optimum combination of different (global and local) data types, including optimum weighting issues, have been investigated. For the final geoid solution, the Least Squares Collocation (LSC) technique, representing the most frequently used approach, has been selected. The new geoid solution, including covariance information, has been thoroughly validated both internally and externally.

Keywords: Geoid, Least Squares Collocation, Global gravity model, Digital terrain model, Gravity anomaly, GPS, Levelling

Kurzfassung

Im Rahmen des Austrian Space Applications Programme (ASAP), Phase 3, gefördert durch die Österreichische Forschungsförderungsgesellschaft m.b.H. (FFG), wurde eine Neuberechnung des österreichischen Geoids (Projekt GEOnAUT) realisiert. Dieses Projekt wurde gemeinsam von den Instituten für Navigation und Satellitengeodäsie (Projektleitung) und für Numerische Mathematik der TU Graz durchgeführt. Das Bundesamt für Eich- und Vermessungswesen (BEV) wirkte als beratender Partner mit und stellte Daten zur Verfügung.

Hauptziel von GEOnAUT war die Berechnung einer Geoidlösung für Österreich als Kombination einerseits aus terrestrischen lokalen Schwerefeldbeobachtungen (Schwereanomalien, Lotabweichungen, „direkten“ Geoidbeobachtungen als Differenz zwischen mittels GPS gemessenen geometrischen Höhen und aus dem Präzisionsnivellement erhaltenen orthometrischen Höhen in identischen Punkten) und andererseits aus einem globalen Schwerefeldmodell basierend auf der Satellitenschwerefeldmission GRACE. Das globale Schwerefeldmodell trägt primär die langwellige Schwerefeldinformation und ermöglicht die Lagerung der lokalen Lösung in einem globalen Bezugsrahmen.

Im Rahmen des Projektes wurde die Datenbank der lokalen Schwerefelddaten erweitert, validiert, homogenisiert und durch Neumessung von ca. 15 Lotabweichungspunkten ergänzt. Letztlich wurden ca. 14000 Schwereanomalien, 672 Lotabweichungspaare und 161 GPS/Nivellementpunkte verwendet. Hinsichtlich der globalen Komponente wurde das GRACE-Schwerefeldmodell EIGEN-GL04S verwendet. Weiters wurde ein digitales Geländemodell für Zentraleuropa

als Kombination der hochauflösenden Geländemodelle von Österreich und der Schweiz (DHM25), sowie einem Geländeoberflächenmodell, abgeleitet aus Daten der Space-Shuttle-Topografiemission SRTM, in den Nachbarländern erstellt.
Methodologisch wurden alternative Berechnungsansätze zur optimalen Kombination dieser unterschiedlichen Datentypen, wie z.B. Reihenentwicklungen basierend auf harmonischen Basisfunktionen, Multi-Resolution Analysis unter Verwendung sphärischer Wavelets und schnelle Randelementmethoden (Multipolmethode, ACA, H-Matrizen) untersucht, sowie das funktionale Konzept der Standardmethode der Kollokation (Least Squares Collocation, LSC) erweitert.
Zur Berechnung der finalen Geoidlösung wurde letztlich die LSC-Methode verwendet. Besonderes Augenmerk wurde dabei auf die optimale relative Gewichtung der einzelnen Datentypen gelegt. Die Geoidlösung sowie die zugehörige geschätzte Genauigkeitsinformation wurden durch das Bundesamt für Eich- und Vermessungswesen evaluiert. Die (externe) Genauigkeit dieser Lösung beträgt 2–3 cm. Verglichen mit dem bisherigen offiziellen österreichischen Geoid, stellt dies eine signifikante Verbesserung dar. Dies ist hauptsächlich auf die wesentlich bessere Qualität der Eingangsdaten, sowohl hinsichtlich der Schweredatenbank und des digitalen Höhenmodells, aber auch auf die genauere Repräsentation der langwelligen Komponente aufgrund des globalen GRACE-Modells zurückzuführen.
Zukünftiges Verbesserungspotential besteht vor allem in den Grenzregionen, da die verfügbare Datenquantität und -qualität in manchen Nachbarländern unzureichend ist. Aus wissenschaftlicher Sicht stellen die theoretischen Weiterentwicklungen von Methoden zur optimalen Kombination von lokaler und globaler Schwerefeldinformation sowie deren praktische Umsetzung ein interessantes Feld für zukünftige Forschungsaufgaben dar.

Schlüsselwörter: Geoid, Least Squares Collocation, Globales Schwerefeldmodell, Digitales Geländemodell, Schwereanomalie, GPS, Präzisionsnivellement

1. Introduction

The first determination of the Austrian geoid was realized as a pure astrogeodetic solution in 1987 using about 650 observations of deflections of the vertical ([4], [26]). In 1998, with the availability of a sufficient amount of gravity anomaly data for the Austrian territory and neighbouring countries, a first gravimetric geoid was computed ([15]). Consequently, in the frame of the project "Austrian Geoid 2000", a combined high-precision geoid solution, based on about 5800 gravity anomaly data, 650 deflections of the vertical, and complemented by about 100 GPS/levelling observations, was processed ([16], [5]). Although the overall quality of this solution was very good, local distortions of a separate astrogeodetic solution in the Eastern part of Austria as well as long-wavelength error structures showed up.

This fact led to the project "The Austrian Geoid 2007" (GEOnAUT), initiated by the Institute of Navigation and Satellite Geodesy (INAS), Graz University of Technology, which was performed in the frame of the Austrian Space Applications Programme (ASAP), Phase 3, funded by the Austrian Research Promotion Agency (FFG). This project has been performed in the years 2006 and 2007 as a joint effort of the INAS, the Institute of Computational Mathematics, Graz University of Technology, as well as the Federal Office of Metrology and Surveying (Bundesamt für Eich- und Vermessungswesen – BEV) as data provider, consultant, and external evaluator of the output products. The main objective of GEOnAUT was the recomputation of a combined geoid model for the Austrian territory, incorporating on the one hand terrestrial gravity data (gravity anomalies, deflections of the vertical, GPS/levelling observations) and on the other hand satellite data from the dedicated gravity field mission GRACE ([11]) to stabilize the solution particularly concerning its medium and long wavelength content. For the local refinement, the gravity field and height data bases have been extended, reprocessed, and (re-)evaluated by a joint effort of INAS and BEV.

The first steps were the pre-processing of the terrestrial gravity data, the assembling of an enhanced digital terrain model, and the computation of a global satellite gravity model mainly from GRACE data by numerical integration applied to the observational equations of both GRACE K-band range and range-rate data ([11]). In the near future, also the incorporation of data from the satellite gravity gradiometry mission GOCE ([7]) is envisaged, yielding a substantially higher spatial resolution than GRACE. In parallel, several methods for the optimum combination of these different data types have been investigated, analyzed, assessed, and compared:

- tailored series expansions (based on spherical harmonic (SH) base functions; [27]);
- multiresolution analysis and spherical wavelet techniques (e.g., [25]);
- Least Squares Collocation (LSC; [20]);
- fast multipoles approach and algebraic approximation methods (e.g., [21], [24]).

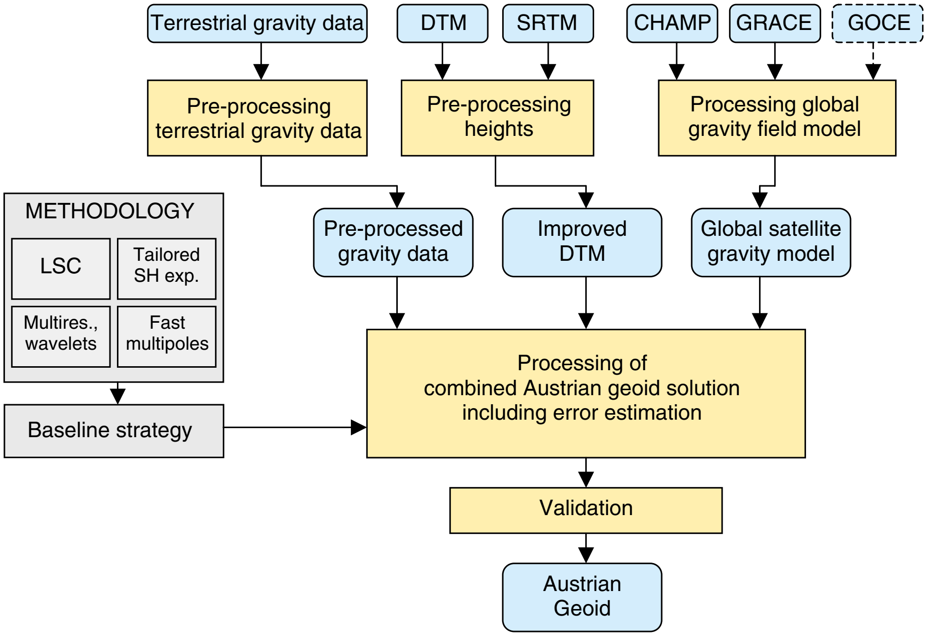


Fig. 1: GEOnAUT architecture, processing and data flow.

After a thorough assessment of these methods, finally the LSC approach turned out to be the currently most mature technique to be applied for the final Austrian geoid solution. This final solution was validated internally by applying several strategies as described in this paper, and externally by the BEV. The final output product, a new Austrian geoid solution, is complemented by corresponding error estimates.

Fig.1 shows the overall architecture of the GEOnAUT project including the main components and the data flow.

2. Data

2.1 Gravity data base

The original gravity data set is composed of about 122000 points in Austria and its neighbourhood with a quite inhomogeneous data distribution. However, LSC results become more stable if the input data is rather homogeneous, and for the geoid computation a smaller number of gravity data is by far sufficient. Based on numerical case studies concerning the prediction error of different input data distributions, finally an average distance among the gravity anomaly observations of 4 km has been chosen and the number of gravity data has been reduced accordingly, using as the primary selection criterion the accuracy information assigned to the data sets. This procedure resulted in 14001 stations (5036 stations in Austria, 8965 stations in the neighbouring countries) and the data distribution shown in Fig. 2 a. While the distribution in the Austrian territory is very homogeneous, there are still regions in the neighbourhood of Austria with potential for further improvement. The data distribution in the Czech Republic as well as in Slovakia is too sparse. Concerning Germany, the data distribution should be denser in the region near the border between Upper Austria and Bavaria. Another problematic region is the northern part of Slovenia where accurate gravity data is heavily missing.

The internal consistency of this gravity anomaly data set has been validated by separating this data set into two sub-sets, using one of them as the input of a LSC procedure, predicting gravity anomalies at the stations of the complementary data set, and, finally, comparing the residuals between predicted and measured values.

2.2 Deflections of the vertical data base

During the evaluation of the former Austrian geoid solution ([16]), the comparison of the astro-geodetic with the gravimetric geoid revealed quite large discrepancies especially in the South-East of Austria. According to [16], the sparse distribution of deflections of the vertical in this region is mainly responsible for these differences.

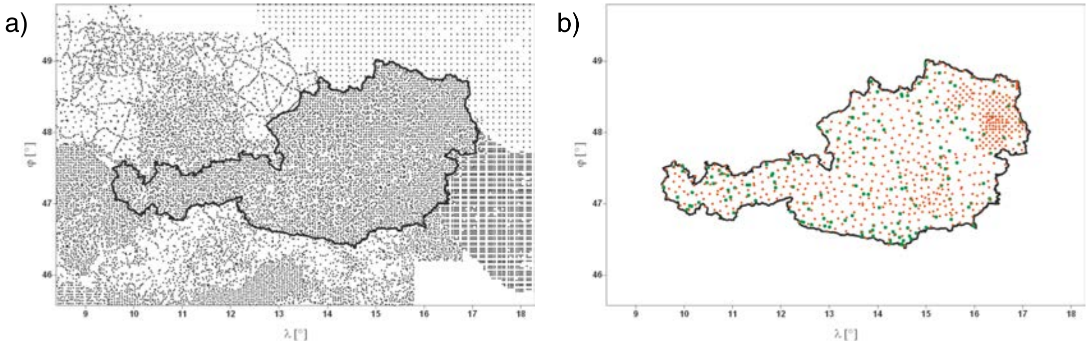


Fig. 2: *a) Data set of gravity anomalies reduced to approximately 4 × 4 km average distance (14001 stations); b) deflections of the vertical (red; 672 stations), and GPS/levelling observations (green; 161 stations).*

Several numerical case studies based on LSC have been conducted for this region. Indeed, the results indicate that an insufficient distribution of measured deflections of the vertical, but also erroneous measurements, are responsible for these inconsistencies ([28], [29]). Therefore, additional stations of deflections of the vertical were selected to be measured in order to densify the distribution and also to evaluate the values of already existing suspicious observations ([30]). The new measurements have been performed using the system ICARUS ([1]), which represents an online observation system for rapid and easy determination of the direction of the plumbline in terms of astronomical latitude and longitude as needed for the computation of deflections of the vertical. This software package, which was developed at the Geodesy and Geodynamics Lab (GGL) at ETH Zurich and was kindly provided for a period of 2 months in September and October 2006, is complemented by a theodolite, which is driven by servo motors, a GPS receiver which is used for time synchronization and event marking, and a notebook with the driver software installed. At 12 stations new deflections of the vertical were measured, and additionally, at 3 already existing stations (where the data base values have been identified to be suspicious) the results could be significantly improved. A detailed documentation of the ICARUS measurement campaign and the results can be found in [28]. In total, now the data base consists of 672 deflections of the vertical (cf. Fig. 2 b) with an assumed accuracy of 0.8 to 1 arcsec.

2.3 Digital Terrain Model (DTM)

The current Austrian digital height model, provided by the BEV, has a uniform resolution of 1.40625″ (in latitude) × 2.34375″ (in longitude), or, equivalently, approximately 44 × 49 m. It is originally based on a photogrammetrically determined digital terrain model ([12], [10]) and refers to the Austrian system MGI. For the purpose of this project, the DTM (and consistently also all other data sets, e.g., the gravity data base) was transformed to the WGS84 reference frame. For the border region of Switzerland, six blocks of the Swiss high-resolution DHM25 were kindly provided by the Swiss Federal Office of Topography (swisstopo). This DTM has a uniform resolution of 1″ × 1″, refers to ETRS89 (horizontal) and LN02 (vertical), and, thus, was transformed to WGS84 as well. After a resampling of the DHM25 to the Austrian grid spacing and a thorough analysis and comparison of the two regional DTMs in the overlapping regions, no significant offsets and systematic discrepancies could be detected. Finally, the two data sets have been merged, using DHM25 in the Swiss territory and the Austrian DTM in the Austrian territory. Since no high-resolution digital terrain information was available for the other neighbouring countries, data from the Shuttle Radar Topography Mission (SRTM; [8]) has been used. The output of this mission is a 3″ × 3″ consistent digital surface model of the Earth from 56° S to 60° N. Due to the fact that SRTM does not provide the height of the terrain, but surface heights (the radar beam of SRTM is reflected by natural and artificial objects such as trees and buildings), a correction had to be applied using Corine Land Cover (CLC90; [6]) data. Further, data gaps, so called voids, which occur in regions where the radar reflection was too week to be measured, e.g., mountainous areas, had to be filled by applying refined interpolation strategies. Details on the processing of SRTM data can be found in [19].

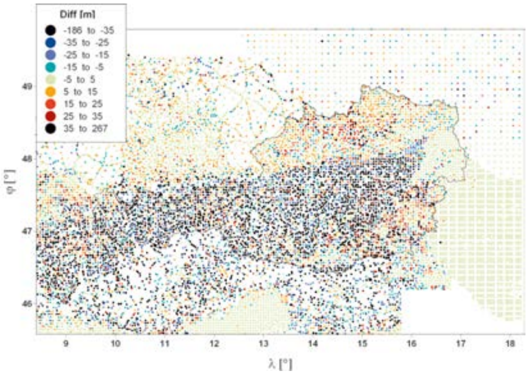


Fig. 3: *Differences between the station heights of the gravity observation points and the height model used for the former geoid computation ([16], [12]).*

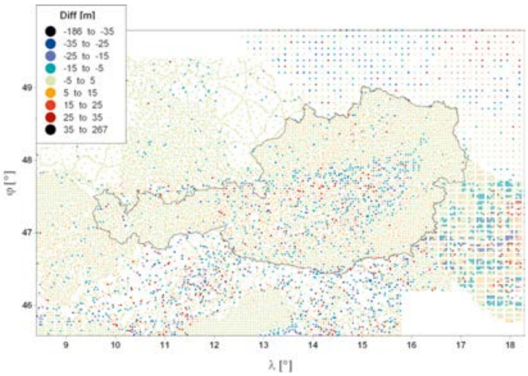


Fig. 4: *Differences between the station heights of the gravity observation points and the newly established height model used for GEOnAUT.*

Fig. 3 and 4 demonstrate the substantial improvement of the newly generated height data base compared to the height model formerly used ([16], [17]). Figure 3 shows the deviations of the orthometric height information assigned to the gravity field data points (cf. section 2.1) from the heights used for the former geoid solution, while Fig. 4 shows the differences to the new height model. Concerning the former model, large discrepancies appear especially in the Alpine region. In contrast, for most of the Austrian gravity data points the height difference between the height of the gravity point and the newly established DTM is small. The degree of improvement of the new model is smaller in the neighbouring countries, which is mainly due to the lower quality of the station heights of the gravity data points (e.g., Slovenia, Northern Italy, Czech Republic and Slovakia), i.e., it can safely be assumed that the accuracy of the SRTM heights incorporated in the new DTM is better than the given station heights. The only country where the differences turned out to be slightly worse was Hungary. However, this effect has been neglected due to its relative small impact. During the following computation process, all gravity field observations showing a difference larger than 35 m between station height and DTM height have been omitted.

2.4 GPS/levelling data base

In total, 161 stations where both highly precise ellipsoidal heights h_{ell} and orthometric heights H resulting from precise spirit levelling (using surface gravity measurements) are available (cf. Fig. 2 b), have been used to derive direct observations of geoid heights N by

$$N = h_{ell} - H. \tag{1}$$

Note that if GPS/levelling points are included in the geoid solution, it is more appropriate to use the term "transformation surface" instead of geoid, because the resulting surface is, depending on the weight assigned to these GPS/levelling observations relative to the gravity field data, more or less forced towards these observations, introducing potential (systematic) errors into the "geoid" related to the height systems, e.g., due to the age and history of the measurements, and the GPS measurement configuration. In this case, the geoid is not a physically defined surface any longer. Therefore, great care was taken during the geoid computation to cross-validate these GPS/levelling observations among each other and with the gravity data.

2.5 Global gravity field model

One task of the GEOnAUT project was the processing of a global gravity field model, parameterized in terms of spherical harmonic coefficients, from data of the satellite mission GRACE. Several global gravity field solutions, based on ranges and range rates of the K-band microwave system, as well as precise orbit information, could be determined. Corresponding error estimates in terms of variance-covariance matrices could be derived. However, for the sake of comparability with other geoid solutions such as the European geoid model ([2], [3]), finally it was decided to use the official GRACE model EIGEN-GL04S ([9]) complete to a harmonic degree/order 70 for the long-wavelength part of the new Austrian geoid solution. It should be emphasized that EIGEN-GL04S is a gravity field model which has been derived solely from satellite data, and, thus, is not biased by terrestrial gravity field observations. Based on several numerical simulations and the analysis of the variance-covariance structure, the maximum degree of resolution of 70 turned out to be a reasonable compromise between the requirement of still high-accurate long-wavelength information and the spatial limitation of the small area of interest.

3. Geoid Computation

3.1 Remove-restore procedure

In the frame of this project, in addition to the investigation of alternative techniques to LSC such as tailored series expansion techniques ([27]), multi-resolution analysis applying spherical wavelets (e.g., [25]), or fast multipole and algebraic approximation techniques (e.g., [21], [24]), also an adaptation of the functional model of LSC for the purpose of a direct incorporation of global gravity field models and, thus, an optimum combination of local and global data was investigated ([22]). Since it turned out that the strict formulation fails when applied practically due to numerical problems during the inversion of the large combined systems resulting from very large correlations among the local and global data types, an extended remove-restore technique has been developed, where the global covariance information of the global model is adequately incorporated.

The final geoid solution was computed using the remove-restore technique. The basic idea is to remove the long-wavelength gravity field effect represented by the global gravity field model, and the high-frequency signals, which are mainly related to topography, by a topographic-isostatic reduction. For the isostatic part, an Airy-Heiskanen model with a standard density of 2670 kg/m^3 was used, because previous studies ([14]) revealed that the use of a laterally variable (surface) density model cannot significantly improve the solution.

The remove step results in smoother signals of the form

$$\beta_{red} = \beta - \beta_{GRACE} - \beta_{TI} - \beta_{ind} \; , \tag{2}$$

where β can be any derived quantity of the gravity potential, e.g., gravity anomalies Δg, deflections of the vertical (ξ, η), or geoid heights N. Starting from the free-air quantity β, the following reductions are applied: global gravity field model reduction β_{GRACE}, topographic-isostatic reduction β_{TI}, including the indirect effect β_{ind}, which copes with the change in the potential due to the mass redistribution related to the topographic reduction β_{TI}. After the reduction procedure, β_{red} refers to the co-geoid ([13]).

This reduction procedure leads to a significantly smoother signal. As an example, Table 1 provides the statistics of the original free-air gravity field anomalies Δg and the reduced gravity anomalies after applying all reduction steps described by eq. (2).

Gravity signal [mGal]	min [mGal]	max [mGal]	std.dev. [mGal]
Δg	−155.60	200.86	42.32
$\Delta g - \Delta g_{Grace} - \Delta g_{TI} - \Delta g_{ind}$	−75.29	21.47	19.60

Table 1: Key statistical parameters of gravity anomalies before and after the reduction process.

The reduced gravity anomalies Δg_{red} have been used to derive an empirical covariance function and to adapt the parameters of the analytical Tscherning-Rapp covariance function model ([31]):

$$C(\Delta g_P, \Delta g_Q) = A \sum_{n=N+1}^{\infty} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos\psi), \tag{3}$$

where $P_n(\cos\psi)$ denotes the Legendre polynomials of degree n, ψ is the spherical distance between the stations P and Q, and A, B, s, and N are the four model parameters to be adjusted.

Fig. 5 shows the empirical covariance function (blue dots), as well as the adapted local Tscherning-Rapp model (red solid line).

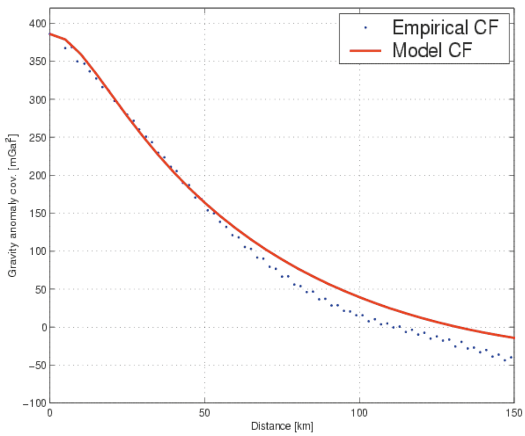


Fig. 5: Empirical covariance function (ECF) and model covariance function (MCF).

The variance of the residual gravity anomaly field is 385 mGal^2, the correlation length is about

42.6 km. The adjusted parameters of the local Tscherning-Rapp model are A = 332.453 $mGal^2$, B = 24, s = 0.998742, N = 70, which is in perfect correspondence with the maximum degree of the global gravity field reduction using the GRACE model EIGEN-GL04S.

Concerning the restore procedure, the correction terms of eq. (2) were evaluated for the predicted geoid heights, defined on a regular $0.05^\circ \times 0.05^\circ$ grid, and finally they were added to the LSC output.

3.2 Pure gravimetric solution

In a first step, the above described remove-restore concept was used to derive a pure gravimetric geoid, using the 14001 gravity anomaly observations as input to the LSC procedure. The geoid height was predicted for all 161 GPS/levelling points. The difference between measured and predicted geoid heights shows a long-wavelength structure. These systematic distortions are commonly attributed to inconsistencies in the datum, distortions of the orthometric height system, and systematic GPS errors. Partially, the differences can also be caused by a non-trivial kernel of the operator that maps height anomalies within a local area into gravity anomalies ([23]). In the case of a local data distribution, the null space comprises all non-zero harmonic functions that produce zero gravity anomalies over this restricted area. Thus, gravimetric height anomalies are non-unique.

In order to cope with this systematic term, a polynomial of degree 3 was fitted to the difference of predicted geoid heights and GPS/levelling heights, and the geoid height observations were reduced by this polynomial surface. This "correction term" is added back to the predicted geoid signals afterwards. As an alternative approach, the parameters of the polynomial surface can also be co-estimated in the frame of the LSC procedure using a more general formulation of LSC (e.g., [20]).

3.3 Combined Austrian geoid solution: case studies

When computing a combined geoid solution, one of the key issues is the optimum weighting among the different data types. Table 2 provides a summary of the input data used for the combined geoid solution, as well as their weights (in terms of an assumed accuracy σ).

Data type	number of obs.	$\sum$
Gravity anomaly Δg	14001	1 mGal
Vertical deflections (ξ, η)	672	0.3"
GPS/levelling obs. N	161	1 mm 5 mm 10 mm variable

Table 2: Input data for the Austrian geoid solution and assumed accuracies σ.

While on the one hand the consistency of the Austrian geoid solution with the GPS/levelling observations is an important requirement for its applicability in practice, during this investigation it turned out that the gravity anomalies, which have been generally assumed to have an accuracy of 1 mGal, are not always totally consistent with these direct geoid height observations. In order to analyze this behaviour, three geoid height solutions have been computed with varying error assumptions for the geoid height measurements: 1 mm, 5 mm, and 10 mm. A lower accuracy assumed for the GPS/levelling points and, correspondingly, a higher relative weight in the combined geoid solution, means that the final result will be forced towards this input data type. Thus, the result will be rather a "transformation surface" than a physical geoid solution.

Exemplarily, Fig. 6 shows the geoid height residuals (deviations of the LSC predicted from the observed geoid heights) of the 161 stations, and the gravity anomaly residuals of the 5036 Austrian stations using a weight of 1 mm (top), 5 mm (centre), and 10 mm (bottom), respectively, for the GPS/levelling observations. Table 3 summarizes the main statistical parameters for these two solutions.

As expected, the 1 mm scenario shows a very good fit concerning the GPS/levelling points of less than $\sigma_N = 6$ mm. However, the rather large deviations in the gravity anomaly residuals, particularly in the neighbourhood of the GPS/levelling stations, indicate an inconsistency of these two input data sets. Vice versa, if a lower weight relative to the other data types is assigned to the GPS/levelling input data, a less constrained physical geoid solution results with smaller residuals and a smoother error behaviour in the gravity anomaly stations, but with deviations in the GPS/levelling points of $\sigma_N = 3.24$ cm for the 10 mm solution.

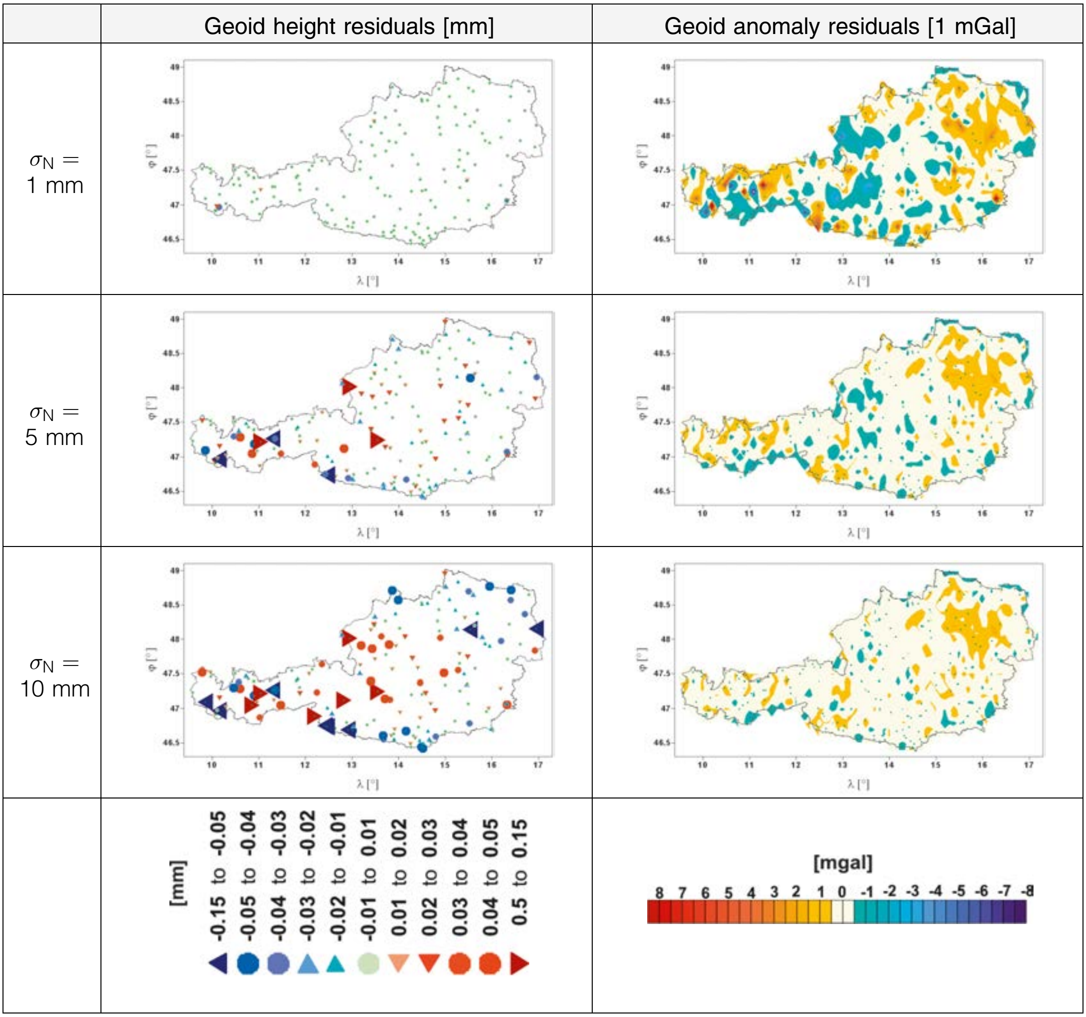


Fig. 6: Geoid height residuals (left) and gravity anomaly residuals (right) assuming various geoid accuracies for the GPS/levelling observations.

GPS/levelling input accuracy	Geoid heights N [cm]			Gravity anomalies Δg [mGal]		
	min	max	σ_N	min	max	$\sigma_{\Delta g}$
1 mm	−3.56	2.38	0.57	−18.75	31.75	1.17
5 mm	−10.71	7.98	2.31	−6.60	5.24	0.75
10 mm	−13.20	9.93	3.24	−6.49	4.98	0.71
indiv.	−13.59	9.58	1.79	−7.56	8.77	0.89

Table 3: Minimum, maximum, and standard deviations σ of the 161 GPS/levelling stations and the 14001 gravity anomaly stations, respectively.

Another independent validation of the resulting solution is to include in the geoid computation only parts of the direct geoid height measurements (observation points), to predict geoid heights in the complementary geoid stations (validation points) and to compare them with the measured ones. A selection criterion could be the requirement that the geoid height differences in the observation stations (used as input in the geoid processing) and the independent validation stations are of comparable amplitude. Also the results of this numerical case study indicate an optimum weight for the GPS/levelling points in the order of 5 mm.

Concerning the Austrian border regions, substantially degraded geoid accuracies in many regions beyond the Austrian territory, e.g., in Czech Republic, Slovenia, and parts of Italy, can be observed. This is mainly due to the sparse input data distribution in these regions. In order to illustrate this problem, Fig. 7 shows the error estimates for the predicted gravity anomalies together with the gravity data distribution (black dots).

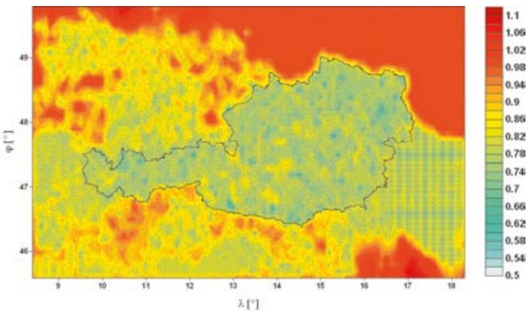


Fig. 7: Gravity anomaly error estimates [mGal], based on the $\sigma_N = 1\,mm$ configuration, and gravity data distribution (black dots).

As discussed above, for practical applications the geoid should be a transformation surface among the GPS/levelling points. However, such a solution evidently has to be constrained very strongly to these GPS/levelling observations and, thus, cannot be considered a free physical surface any longer. Therefore, a kind of compromise between these two contradicting requirements has been processed as a last scenario. Here, the geoid height residuals of the GPS/levelling stations for the 1 mm solution have been used to define individual weights for the GPS/levelling observations. Correspondingly, in those GPS/levelling stations which can be suspected to have a more degraded accuracy than initially assumed, the GPS/levelling observations are down-weighted and, thus, implicitly the surrounding gravity field observations obtain a higher relative weight. According to the residuals of the 1 mm solution (cf. Fig. 6), individual weights in the range of 1 mm to 17.8 mm have been assigned to the input GPS/levelling observations.

Fig. 8 (left) shows the LSC output in terms of the residuals of the 161 GPS/levelling points. As expected, the residuals are smaller than of the 5 mm scenario, but larger than of the 1 mm scenario displayed in Fig. 5. The result of the 1 mm scenario seems to be too optimistic and veils the fact that, especially in the western part of Austria, small inconsistencies of the gravity field and the GPS/levelling points appear. Fig. 8 (right) displays the corresponding gravity anomaly residuals of

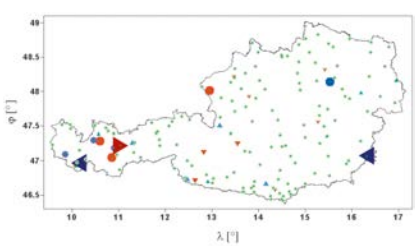
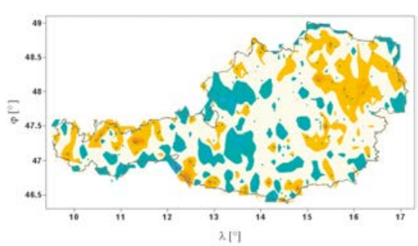
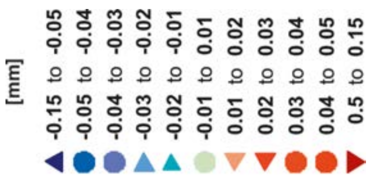
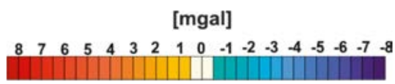
	Geoid height residuals [mm]	Geoid anomaly residuals [1 mGal]
$\sigma_N =$ indiv.		
		

Fig. 8: Geoid height residuals (left) and gravity anomaly residuals (right) using individual weights for the GPS/levelling observations.

the 14001 stations. The main statistical parameters of this solution are given in the fourth line ("indiv.") of Table 3.

Comparing the gravity anomaly residuals of the 1 mm and the individually weighted solutions, it can be concluded that gravity anomaly residuals larger than 10 mGal could be avoided in some critical regions (suspicious GPS/levelling observations) with the help of the weighting strategy.

Fig. 9 shows the change of the geoid heights if individual weights are used for the GPS/levelling points instead of the 1 mm scenario. Similarly to the comparison of the predicted gravity anomalies, the differences are negligible in most areas. However, these differences reflect the fact that changes in a few GPS/levelling points affect the solution in their vicinity of several tens of kilometres extension. This has the effect that in the west of Austria the re-weighting of only 6 GPS/levelling points changes the whole geoid solution by several centimetres and underpins the importance of accurate GPS/levelling observations, because their influence will change the geoid (transformation surface) not only locally, but in a more extended area.

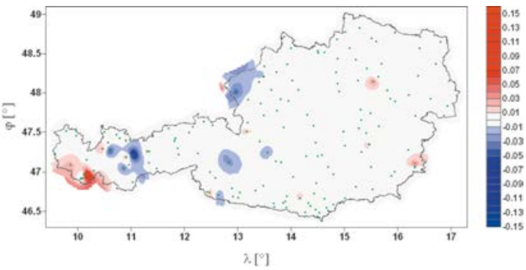


Fig. 9: Differences of geoid heights [m], based on the 1 mm configuration versus the individual weighting of the GPS/levelling configuration.

3.4 The Austrian Geoid 2007

On the basis of the numerical case studies described above, the weighted solution has been selected to represent the final result of this project. Fig. 10 shows the geoid after restoring the correction terms described in eq. (2). Fig. 11 displays the corresponding error estimates. In summary, the accuracy of this new solution can be estimated to be of the order of 2 to 3 cm with a significant degradation in the border regions due to the insufficient input data distribution. This error estimate is originally based on the formal errors of the LSC procedure, but has been re-scaled using the the standard deviation of the residuals in selected GPS/levelling control points, and thus can be considered as a realistic estimate for the total error of the solution.

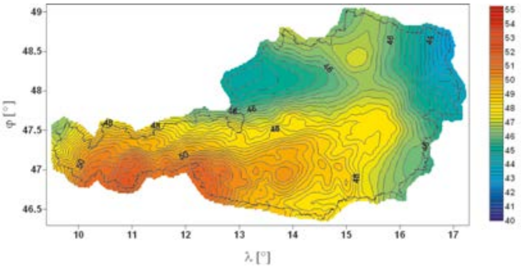


Fig. 10: The Austrian Geoid 2007 [m].

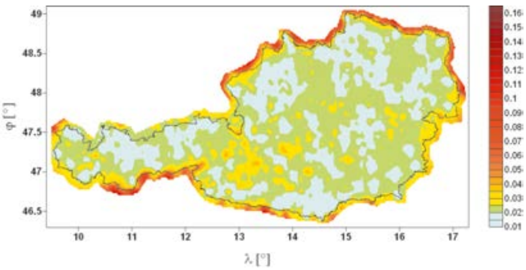


Fig. 11: The Austrian Geoid 2007: error estimates [m].

Fig. 12 shows the improvements of the new solution with respect to the currently official BEV geoid model, which is a purely astro-geodetic solution dating from 1987. In [4], pg. 25, the accuracy of this solution was specified to be in the order of ±35 cm in the region of Austria. These differences may be attributed to a lower quantity and quality of the Doppler/levelling data available at this time, which have been used to for the definition of the absolute vertical datum, as well as a lower quality of the digital terrain model.

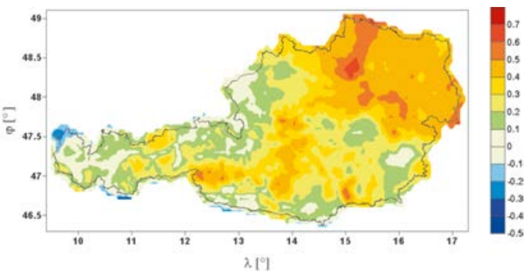


Fig. 12: Differences between the Austrian Geoid 2007 and the official BEV geoid [m].

4. Conclusions

In the frame of the GEOnAUT project, a new Austrian geoid solution was computed. Compared to the previous official Austrian geoid model, the accuracy and reliability could be significantly improved. This is mainly due to the substantially improved quality of the input data, which is particularly evident for the digital terrain model, which has been assembled as a combination of high-accuracy regional DTMs of Austria and Switzerland, complemented by SRTM data in the neighbouring countries, the incorporation of GPS/levelling information, as well as the inclusion of a GRACE global gravity field model. It turned out that the combination of long and short wavelengths using the GRACE model of spherical harmonics fits the Austrian territory very well.

While the Austrian gravity data base is homogeneous and accurate, several problems particularly in the border regions appear due to either sparse data distribution in the neighbouring countries or degraded accuracy of the gravity data. Thus, for future Austrian geoid solutions the effort to acquire gravity data from the neighbouring countries needs to be prolonged especially for the Czech Republic, Slovenia, and some parts of Italy.

An equally important aspect concerning future activities is a thorough validation and quality analysis of the GPS/levelling observations. This is inevitable in order to fulfil the needs of the geoid as transformation surface between the national height system and the GPS heights. Currently, several GPS/levelling observations can be suspected to be slightly inconsistent with the very homogeneous gravimetric data.

A thorough validation procedure has been applied to the new Austrian geoid solution, both internally during the individual processing steps, and also externally. In the near future, it will be also validated against the European geoid model ([2], [3]). In spite of the fact that there is potential for future improvement by including a larger number of GPS/levelling points of validated quality and improved gravimetric data of the neighbouring countries, it can be concluded that a substantially improved Austrian geoid 2007 could be achieved in the frame of this project.

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