High Precision Geoid Determination of Austria Using Heterogeneous Data

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Abstract. The first Austrian geoid computed by Sünkel in 1987, was an astrogeodetic geoid. Later on as a reasonable amount of gravity data was available, a gravimetric geoid has been computed by Kühtreiber in 1998. While the astrogeodetic geoid computation in principle provides absolute geoid values, the astrogeodetic method can only provide relative geoid heights. Nevertheless satellite observations (GPS) are used in both cases to fit the geoid. Since heterogeneous measurements (deflections of the vertical, gravity measurements and satellite measurements) have different sensibilities in terms of the spectral band of the gravity potential, a combination of all measurements in one computation process using the least squares collocation technique yields best geoid results. The presented Austrian geoid is based on the combination of different data types. The combined geoid solution (using deflections of the vertical and gravity anomalies) is influenced by the amount and distribution of the data as well as the weights of the different observations types. Investigations to determine the best possible combination are shown in this work. The precision of the Austrian geoid has improved to the cm-level by combining heterogeneous data.

1 Data

1.1 Digital Height Model

The Austrian height data used for the present investigation was generated by the Federal Office of Metrology and Surveying. The data is based on grids, with different resolutions determined photogrammetrically. The grid distance of these grids varies from 30 x 30 m for rough topography to 160 x 160 m in flat areas. From these base models, a digital terrain model was derived with the uniform resolution of 1”40625 x 2”34375, or equivalently, 44 m x 49 m, see Graf (1996). The big improvement in the quality of the height model is shown in Fig. (1).

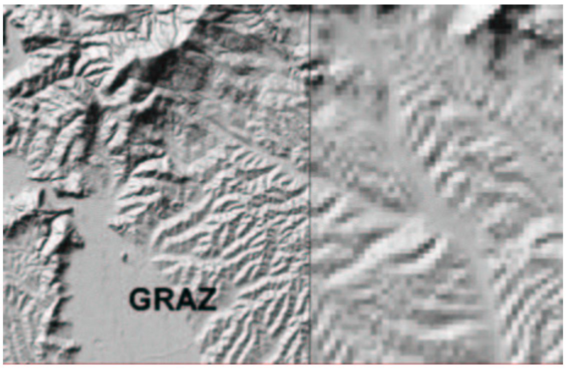

Figure 1. Comparison of the new height model (left side, resolution 1”40625 x 2”34375) and the old height model (right side, resolution 11”25 x 18”75)

For the neighboring countries, height data in different resolution and quality were provided. All height information was used to supplement the Austrian height model in the uniform resolution of 1”40625 x 2”34375. A detailed report of the development of this height model is found in Graf (1996).

1.2 Density

As the gravity signal has a close relation to the density of the underlying masses the density plays an important role in all geodetic and geophysical processing of gravity field quantities. A reasonable density model has to be introduced. Different approaches can be used.

One possible approximation which can be used is a surface density model by assuming that the density of the related grid cells is constant from the surface of the topography down to the crust-mantle boundary (Moho). Such a digital surface density model was used for the geoid computation of Austria in 1987. In

Walach (1987) the history of the mean surface density model of Austria is given. Due to history the resolution of the model is inhomogeneous which can be recognized in Fig. 2.

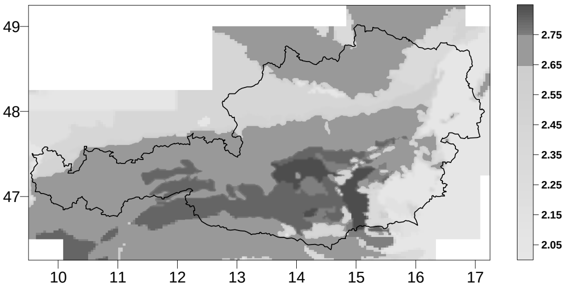

Figure 2. Digital surface density model of Austria

Investigations by Kühtreiber (1998 a) showed that the combination of the high resolution digital terrain model (1”40625 x 2”34375) with the above described surface density model (resolution $1.5' \times 2.5'$ or worse) gives no better results compared to the case of using a constant density value of $2.67\,\mathrm{g/cm^3}$ which is used for the present computations.

1.3 Gravity

The gravity data base at the Institute of Geodesy in Graz was established by Kraiger and Kühtreiber during the last 18 years. The database includes Austrian data and data from the neighboring countries.

Gravity measurements within Austria have been provided by the Institute for Meteorology and Geophysics (University of Vienna), the Institute of Geophysics (Mining University Leoben), the Federal Office of Metrology and Surveying, the OMV and the Institute of Geophysics (Technical University Clausthal). At the moment about 86000 gravity observations are stored. For the area outside Austria the following countries contributed by providing gravity data in the different formats: Switzerland, Italy, Slovenia, Hungary, Germany (Bavaria, Baden-Württemberg) and the Czech Republic. The kind cooperation with all the above mentioned institutions has to be acknowledged.

All gravity measurements are stored in an uniform system for position, height and gravity. Regarding position the local Austrian coordinate system based on the Bessel ellipsoid was chosen. The height system used is the Austrian orthometric height system based on the tide gage Triest. All gravity measurements refer to the system IGSN71.

For the following computations only a subset of all the gravity measurements is used. The uniform

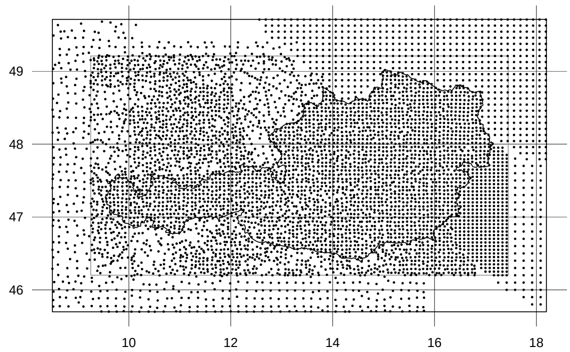

Figure 3. Gravity data set 6×6 km

distribution of gravity measurements used for the geoid computation is fixed by the mean distance of gravity measurements in mountainous areas. Figure 3 shows the selected 5796 gravity data points. Gravity measurements were selected with a mean distance of 6 km for a rectangle of $46.20 \leq \phi \leq 49.21$ and $9.25 \leq \lambda \leq 17.25$. Additional data with a mean distance of 12 km were added around this inner rectangle up to the maximum limits of $45.70 \leq \phi \leq 49.70$ and $8.50 \leq \lambda \leq 18.20$.

1.4 Deflections of the Vertical

For the geoid determination in 1987 a set of more than 680 points was used. This data set shows a quite homogeneous distribution. Since that time no further deflection data were added to the data base in Graz.

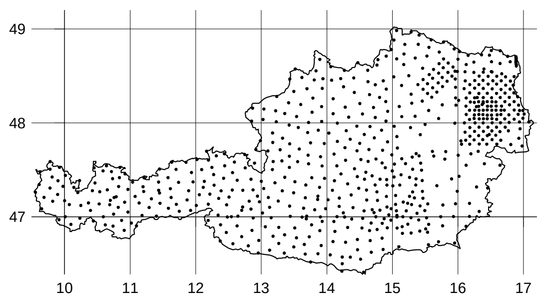

Figure 4. Deflections of the vertical

The deflections refer to the Bessel ellipsoid and to the datum of the Military Geographic Institute. Figure 4 shows the data set of deflections used in the following.

1.5 GPS/levelling

Figure 5 shows the distribution of the available GPS/levelling points, which can be used for scaling as well as for checking the geoid. The points shown are points of the AGREF-net of the Federal Office of Metrology and Surveying. The geocentric coordinates are referred to ITRF94. The orthometric heights refer to UELN98. The distribution of the points is not quite homogeneous as most of the stations are located in the eastern part of Austria. Only few stations are located at the mountainous western part of Austria.

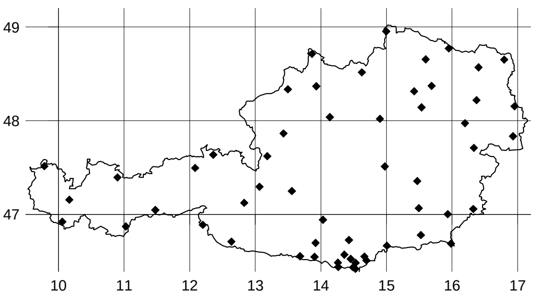

Figure 5. GPS/levelling points

2 Remove-Restore

The geoid computation is done by the well-known remove-restore procedure. The basic idea behind it is to take advantage of the fact that parts of the gravitational potential can be approximated by existing models. The long wavelength part is known through a given earth's gravitational model which is expressed in terms of a spherical harmonic expansion. The short wavelength part is a function of the mass (density) distribution of the topography and can be approximated by a DTM/DDM.

The way the remove-restore technique is applied is the following. In the remove step residual gravity anomalies Δg_{RES} are computed by

$$\Delta g_{RES} = \Delta g - \Delta g_{EGM} - \Delta g_{DTM}\,. \qquad (1)$$

The two effects removed from the gravity anomalies Δg are, Δg_{EGM} the long wavelength part of the gravity anomalies and Δg_{DTM} the short to medium wavelength part of the gravity anomalies. By this remove step we gain Δg_{RES} which represent a smooth field with only local-to-regional structures.

Here the adapted EGM96 (Abd-Elmotaal and Kühtreiber, 2001) was used to compute the long wavelength part in the remove-restore procedure. For the short to medium wavelengthes a topographic isostatic reduction was performed using the adapted technique and a detailed height model with the resolution $11''25 \times 18''75$. For the isostatic model an Airy-Heiskanen approach with a standard constant density of 2.67 g/cm^3, a normal crustal thickness T of 30 km and a density contrast of 0.4 g/cm^3 was used. Table 1 shows the statistics for the reduction process. Figure 6 shows the reduced gravity anomaly field.

Table 1. Gravity reduction using the standard density of 2.67 g/cm^3 and the adapted geopotential model EGM96. Anomalies are given in mgal. Statistics based on 5796 points. (6×6 km set)

	min	max	mean	std.dev.
Δg_F	-154.1	187.2	9.8	± 42.2
Δg_{EGM96}	-204.3	224.0	-1.1	± 47.6
Δg_{iso}	-72.0	85.4	0.6	± 23.6

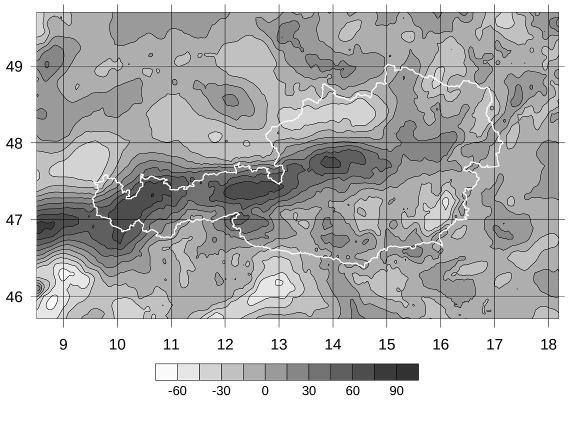

Figure 6. Gravity Anomalies after removing the short and long wavelength part of the gravitational potential (contour interval 7.5 mgal).

After the remove-step geoid heights N_{RES} are estimated from the residual gravity anomalies Δg_{RES}. In the following the estimation was done by collocation. Details on the used covariance function are given in sec. 3.

Finally the effects removed are restored again in terms of geoidal heights to N_{RES}

$$N = N_{RES} + N_{EGM} + \delta N_{DTM} \,. \tag{2}$$

Here δN_{DTM} is nothing else but the indirect effect which takes into account that removing the masses has changed the potential. N_{EGM} is computed using the spherical harmonic expansion of the earth's gravitational model.

This technique is commonly applied in local gravity field determination. Early computations done by this method are e.g. Sünkel (1983), Forsberg (1984).

3 Covariance Function

The well-known Tscherning-Rapp covariance function model was used for the following LSC solutions. The global covariance function of the gravity anomalies $C_g(P,Q)$ given by Tscherning and Rapp (1974, p. 29) is written as

$$C_g(P,Q) = A \sum_{n=3}^{\infty} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos\psi)\,, \tag{3}$$

where $P_n(\cos\psi)$ denotes the Legendre polynomial of degree n, ψ is the spherical distance between P and Q and A, B and s are the model parameters. A closed expression for (3) is available in (ibid., p. 45).

The local covariance function of gravity anomalies $C(P,Q)$ given by Tscherning-Rapp can be defined as

$$C(P,Q) = A \sum_{NN+1}^{\infty} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos\psi)\,. \tag{4}$$

Modelling the covariance function means in practice fitting the empirically determined covariance function (through its three essential parameters; the variance $C_\circ$, the correlation length ξ and the variance of the horizontal gradient $G_{\circ H}$) to the covariance function model. Hence the four parameters A, B, NN and s are to be determined through this fitting procedure. A simple fitting of the empirical covariance function was done using COVAXN-Subroutine (Tscherning, 1976).

The essential parameters of the empirical covariance parameters for 2489 gravity stations in Austria are $740.47\,\mathrm{mgal}^2$ for the variance C_0 and 43.5 km for the correlation length ξ. The value for the horizontal gradient $G_{\circ H}$ was roughly estimated by $100\,\mathrm{E}^2$.

For the fixed value $B = 24$ the following Tscherning-Rapp covariance function model parameters were fitted: $s = 0.997065$, $A = 746.002\,\mathrm{mgal}^2$ and $NN = 76$. The parameters were used for the astrogeodetic, the gravimetric as well as the combined geoid solution.

4 Astrogeodetic Solution

The astrogeodetic solution by collocation is based on 659 deflections of the vertical uniformly distributed over Austria (see sec. 1.4). After removing the long

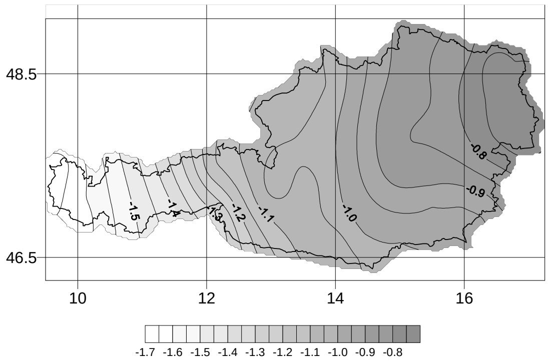

Figure 7. Difference in geoid heights given in m, for the astrogeodetic geoid solution ($N_{2002astro}$) and the GPS/levelling geoid. Contour interval = 5 cm.

and short wavelength effect of the gravitational potential from the observations a geoid was estimated by LSC. Figure 7 shows the difference between the geoid solution by LSC and the GPS/levelling derived geoid. The contour plot is based on the differences

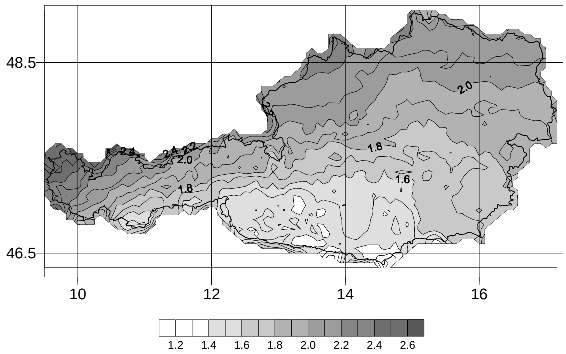

Figure 8. Difference N_{1987} - $N_{2002astro}$ in m

given at 48 GPS/levelling points. The differences show a long wavelength character with a west-east trend of 1 m.

After the new astrogeodetic solution ($N_{2002astro}$) has been fitted to all GPS/levelling points using collocation the new astrogeodetic solution is compared to the Austrian geoid solution of 1987 (N_{1987}) which is based on the same data set of deflections (see Fig. 8). The big offset of 1.2 to 2.6 m can be explained by the fact that the solution of 1987 has been scaled by Doppler/levelling points, which provide an accuracy of about 1 to 2 m only. Besides the big north-south trend we notice short wavelength variations which can be explained by the better height information used in the new solution.

The comparison of the new astrogeodetic solution and the geoid provided by the Federal Office of Metrology and Surveying N_{BEV} (Erker, 1987) is shown in Fig. 9. The differences in this case are smaller . They range from -0.4 to 0.6 m. The astrogeodetic solution N_{BEV} is also an astrogeodetic solution which is based on the same amount of deflections of the vertical.

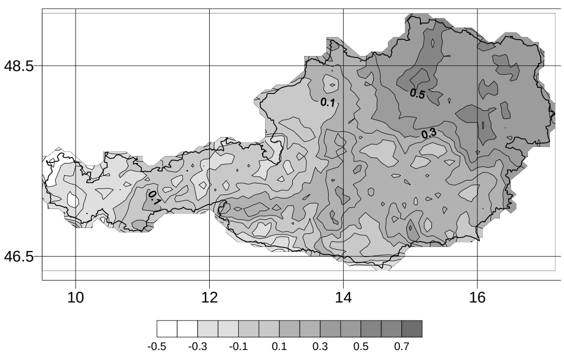

Figure 9. Difference $N_{2002astro} - N_{BEV}$ in m

5 Gravimetric Solution

The gravimetric solution by collocation is based on the gravity anomaly data set shown in sec. 1.3. Figure 10 shows the difference between the LSC geoid solution and the GPS/levelling derived geoid. The contour plot is based on the differences given at 48 GPS/levelling points. The differences are in the order of the astrogeodetic geoid result (see Fig. 7). The differences show a high order polynomial trend with a west-east gradient of about 0.8 m.

Of particular interest is how the new gravimetric solution $N_{2002grav}$ fits the new astrogeodetic solution $N_{2002astro}$. Figure 11 shows the difference between

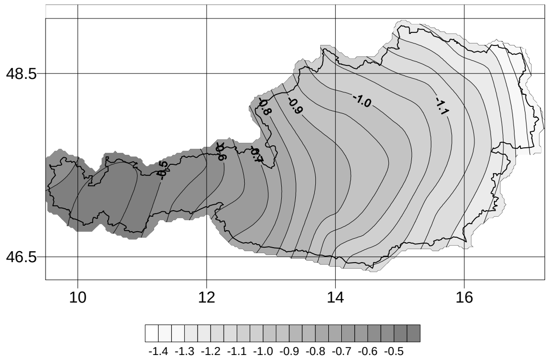

Figure 10. Difference in geoid heights given in m, for the gravimetric geoid solution ($N_{2002grav}$, based on DATA1) and the GPS/levelling geoid. Contour interval = 5 cm.

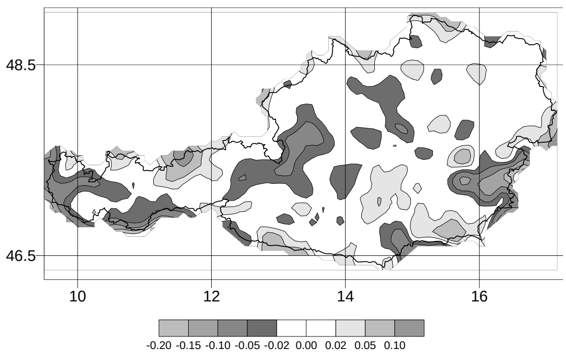

Figure 11. Difference gravimetric minus astrogeodetic solution given in cm.

the two solutions. The differences are most of the time less than $\pm$ 10 centimeter. It has to be noted that unequally spaced contour intervals were chosen to figure out three different categories, namely ± 2 cm, ± 5 cm and ± 10 cm. The white pattern shows all points were the two solutions are identical within ± 2 cm. Around 50% of the points fulfill this criterion. If we compare the two solutions and choose the category ± 5 around 75% of the differences are within this range. A closer look to the differences reveals that the bigger differences show no correlation with topography. For instance big differences near the Austrian border reflect the fact that the astrogeodetic solution is a local solution based on deflections points inside Austria only, while the gravimetric solution was computed on a more regional basis. The biggest difference is located in the eastern part of Austria.

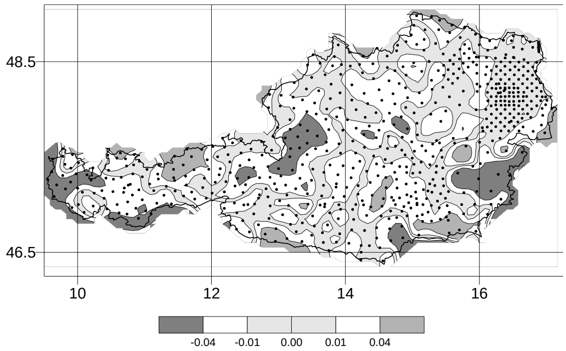

Figure 12. Difference astrogeodetic minus gravimetric solution given in cm.

In order to get an explanation for the discrepancies between the two solutions, the difference is plotted once again in combination with the used set of deflection points (cf. Fig. 12). The light grey pattern is used for areas where the difference between the gravimetric and astrogeodetic solution is within $\pm$ 1 cm. In the north-eastern part of Austria a larger area exists which fulfills the criterion. Within this area we recognize the densest coverage with deflection points. In contradictory the dark grey parts mark regions where the difference between the two solutions are bigger than -4 cm. In the south-east of Austria (east part of Styria) the difference is bigger than -15 cm. In this area only a very sparse distribution of deflection points can be recognized. A similar conclusion holds for the areas with a difference bigger than 4 cm (middle grey pattern). Figure 12 shows clearly that the difference depends on the distribution of the deflection points. The more homogeneous and dense the distribution of deflection points the better the two solutions fit. Of course differences are also dependent on the smoothness of the residual gravitational potential. Therefore we can draw the following conclusion. In order to get a precise geoid solution, the residual gravitational field should be as smooth as possible and the used measurements should homogeneously cover the region.

6 Combined Solution

The first combined solution was done by Kühtreiber (1999). This combination of the gravimetric and astrogeodetic geoid was done by computing a simple arithmetic mean of the astrogeodetic and the gravimetric solution. In order to take advantage of collocation for combining gravity anomalies and deflections of the vertical in one estimation process investigations concerning the weighting of the gravity anomalies versus the deflections of the vertical are needed.

An important point concerning a reasonable combination of deflections of the vertical and gravity anomalies in a collocation process is the definition of the error variances for the different data types. A case study was performed by the following approach. Geoid heights were computed by a combination of deflections of the vertical and gravity anomalies. Three different cases, each with a different error variance of Δg, but all with equal error variances for the deflections of the vertical, are considered. Each of the combined solution is compared to the pure astrogeodetic or pure gravimetric geoid solution (see Fig. 13).

Let us consider the starting configuration. The error variance of Δg was given as 0.3 mgal while the error variances of the deflections ξ and η were given as $0.2''$ and $0.3''$ respectively. Comparing the combined solution with the astrogeodetic solution shows more or less a big west-east trend with regions which don't fit the trend at all (e.g. eastern part of Austria). The difference between the combined solution and the gravimetric solution is a pure trend, no deviation from the trend is visible. The conclusions we can draw from the first pair of plots in Fig. 13 are:

- the error variance of 0.3 mgal for the gravity anomalies is too small. Hence the deflections of the vertical don't contribute to the solution on a local basis

- big discrepancies between the astrogeodetic and combined solutions appear in regions with sparse distributed deflections in all three cases

Increasing the error variance of the gravity anomalies should down-weight the part of the gravity anomalies in the combined solution, while the deflections should contribute more to the solution with the same error variances. We would expect that the difference between the astrogeodetic and the combined solution becomes more a pure trend, while the difference between the gravimetric and the combined solution becomes more irregular. The second and third pairs of plots in Fig. 13 prove this consideration.

The poor results of the astrogeodetic solution in the east of Styria exists in all solutions. Even a very high weight of the deflections relative to the gravity

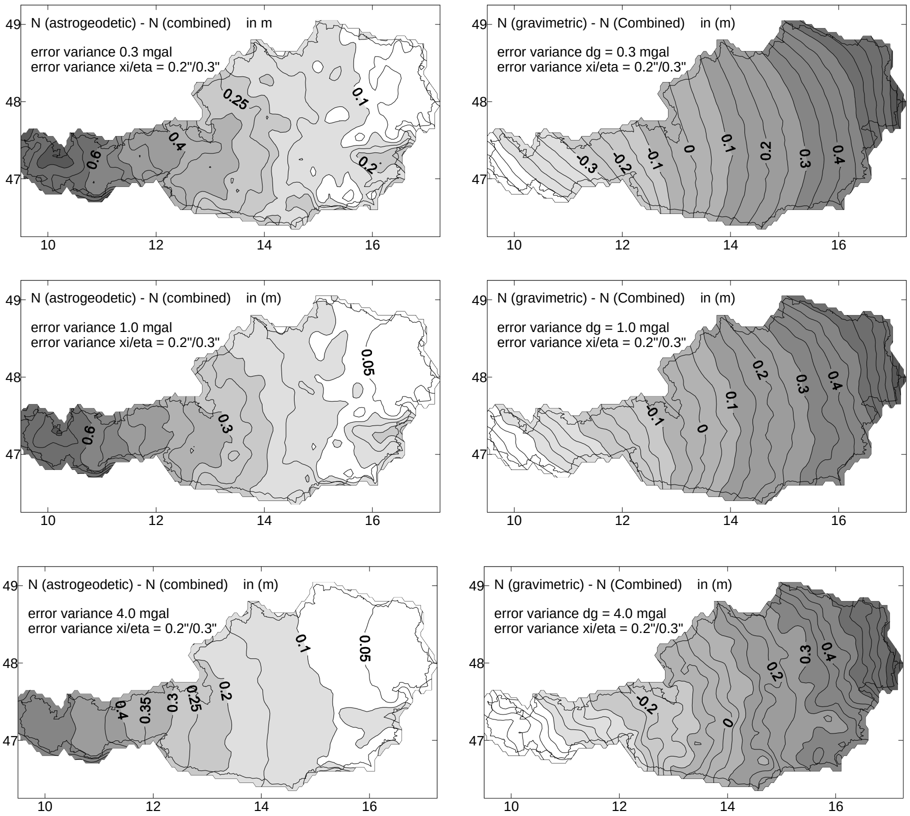

Figure 13. Changing the error variance of the gravity anomalies in the combined solution and comparing the solution to the astrogeodetic and gravimetric solution.

anomalies preserves the structure of the gravimetric geoid solution to some extent. This is clear as the deflections of the vertical are too sparse in this area to contribute to the combined solution.

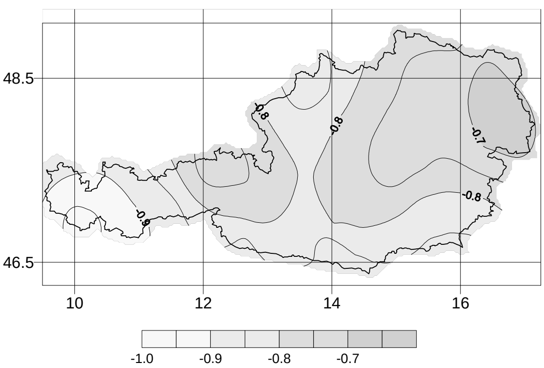

Figure 14. Difference in geoid heights given in m, for the combined geoid solution and the GPS/levelling geoid. Contour interval = 5 cm.

As a conclusion of this study, the error variances of the gravity anomalies and the deflections of the vertical were chosen as 1.5 mgal for Δg and 0.2", 0.3" for ξ, η, respectively.

Finally fig. 14 shows the difference between the combined LSC geoid solution and the GPS/levelling derived geoid. The contour plot is based on the differences given at 48 GPS/levelling points. The differences show by far the smoothest behaviour and the smallest range.

7 Restore and Scaling

The effect of the adapted EGM96 and the topographic-isostatic model were restored. The resulting geoid heights were scaled to 12 GPS/levelling points, while 38 GPS/levelling points were used for an external check. Table 2 shows the statistics of this check.

Table 2. Differences between the geoid solutions $N_{2002astro}$, $N_{2002gravi}$ and N_{2002} to N from GPS/Levelling given in cm. Statistics based on 38 external checking points

	min	max	std.dev.
$N_{2002astro}$	-12.7	10.6	± 4.4
$N_{2002gravi}$	-7.4	9.4	± 3.8
N_{2002}	-7.2	7.5	± 3.4

8 Conclusions

With the help of the adapted technique for estimation of the influence of the short and longwavelength part of the gravity field and the LSC approach combining gravity anomalies and deflections of the vertical a high precision geoid for Austria was estimated. The major achievements of the solution are:

- an excellent agreement of the astrogeodetic and gravimetric geoid solution

- a centimeter accuracy of the geoid solution as result of the combined solution

- a considerable improvement compared to the old solution in the eastern part of Styria and all areas in the vicinity to the Austrian border

- an external check with GPS/levelling points with a standard deviation of ± 3.4 cm for the differences

- a practical tool for providing levelling heights through GPS measurement.

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