UNIVERSITY OF KHARTOUM

GeiodApp: A Unified Framework for Geoid Computations

by

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Declaration of Authorship

I, Mohamed Yousif and Mohamed Jaafar, declare that this thesis titled, 'GeoidApp: A Unified Framework for Geoid Computations' and the work presented in it are my own. I confirm that:

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- I have acknowledged all main sources of help.
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"The gravity field over land areas on Earth is less well known than is that of Venus"

McKenzie, 1994

UNIVERSITY OF KHARTOUM

Abstract

Faculty of Engineering
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Doctor of Philosophy

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The main objective of this study is to evaluate new GOCE missions on Sudan. We have evaluated 5 models; EGM2008, EIGEN-6C4, GECO, ITU_GGC16, and ITU_GRACE16. We have compared our evaluation results with two published works (Abdalla, 2009, Godah and Krynski, 2015). We reported an accuracy of $0.356m$ which surpasses any previously conducted works on Sudan. Our results are also comparable to the corrected results of (Abdalla, 2009, Godah and Krynski, 2015). Thus, a new datum for Sudan using our proposed models can have an accuracy of $0.10m$ $0.20m$.

For our computations we developed GeoidApp an efficient software application to compute the geoid height and other geopotential functionals. The efficiency and ease of use of GeoidApp can lead into substantial improvements in geoid determination from GGM. We have compared GeoidApp with available libraries for geoid computations–specially the official ICGEM software. We found that our software is better than the available ones in terms of efficiency, customizations, and ease of use. GeoidApp will accelerate the task of choosing GGM by automating the whole process of choosing the best model.

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We would like to express our thanks for our supervisor Ahmed Abdalla for everything he has done for us. None of this work would ever happened without him. He was always with us with his deep insights and suggestions, and most of all, his time we would really have had horrible nights. You passion about science and research has inspired us all. It was a privilige and huge honor to work with you.
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Last but not least, we would like to thank our families; parents, brothers and sisters for their great love and trust.

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Abbreviations

GGM	**G**lobal **G**eopotentail **M**odel
BVP	**B**oundary **V**alue **P**roblem
GBVP	**G**eodetic **B**oundary **V**alue **P**roblem
GPS	**G**lobal **P**ositioning **S**ystem
GNSS	**G**lobal **N**avigation **S**atellite **S**ystem
CHAMP	**CHA**llenging **M**ini-satellite **P**ayload
GOCE	**G**ravity Field and Steady State **Ocean** **C**irculation **E**xplorer
GRACE	**G**ravity **R**ecovery and **C**limate **E**xperience
GRS80	**G**eodetic **R**eference **S**ystem 1980
WGS84	**W**orld **G**eodetic **S**ystem 1984
EGM2008	**E**arth **G**ravitational **F**ield 2008
NASA	**N**atioanl **A**eronautics and **S**pace **A**dministration
ALF	**A**ssociated **L**egendre **F**unctions

Physical Constants

Earth Gravitation constant	GM	3.986004418e+14
Semi-major axis	a	6378137.0 m
Semi-minor axis	b 6356752.314245m	
Inverse Flattening	$1/f$	298.257223563
Angular velocity	$7292115 \times 10\mathrm{e} - 11$	$rads^{-1}$

To our families and friends

Chapter 1

Introduction

Adam (1967) was the first to attempt to compute a geoid for Sudan. Due to lack of data from neighboring countries and large un-surveyed areas in Northwest and Southwest parts of Sudan, Adam found that the information was insufficient to determine an accurate geoid for Sudan and recommended to fill the gabs over there Abdalla (2009).

Historically, Sudan has three gravimetric geoid models. The first one (Geoid91) was computed by Fashir (1991) in 1991 using Geodetic reference System GRS80 Moritz (1980). The free-air co-geoid was computed from the combination of surface gravity data using a modified Stokes's kernel and GODDARD EARTH MODEL (GEM-T1) Fashir (1991). GEM-T1, a satellite-only model, is complete to degree and order of 36 Marsh and Center (1987). The accuracy of their datum was 1.6m Fashir (1991), Godah and Krynski (2015). The second model 'KTH-SDG08' was computed in 2008 using optimum least-squares modification of Stoke's kernels, which is widely known as KTH method Abdalla (2009). EIGEN-GRACE02S satellite-only model was adopted for KTH-SDG08 final computation at spherical harmonic degree and order 120. Without any corrections, they reported an accuracy of 5.86m. More recently, (Godah and Krynski, 2015) has proposed a new gravimetric geoid for Sudan SUD-GM2014. They used GO_CONS_GCF_2_TIM_R4 (TIM-R4) of 250 degree Pail et al. (2011). The accuracy of their chosen model was 0.644m without any corrections. It was dropped to 0.306m after '7-parameter fitting', which is lower than that reported by (Abdalla, 2009). We can naively conclude that their choice of GO_CONS_GCF_2_TIM_R4 (TIM-R4) did not make any contribution to the derived geoid. However, we think that the lack, and the bad distribution (in-homogeneities lead to more errors). Another observation, the higher degrees does not always mean a better results.
We have tested five models on different degrees. Three of our models were tested up to degree and order 2190. For those models we chose an step of roughly 5 degrees which

gave us a 500 variants of each model to evaluate on our local data. For the other small models (ITU_GGC16, ITU_GRACE16), we have tested each degree starting from the 10^{th} degree. The best result was $0.365m$ reported on ITU_GGC16 without any corrections. We propose to use either ITU_GGC16 or GECO for any future works of geoid determination in Sudan, as the difference between them is $0.131mm$.

1.1 Objectives of the thesis

The main objective of this thesis is to evaluate recent GRACE/GOCE models over Sudan using asterogeodetic data and GPS/Leveling data. For that purpose we have developed our own software to compute the geoid height, or more generally *geopotentail functionals*. In particular, it can be used to compute "geoid height", "geoid anomaly", and "geoid disturbance". There are available software but the lack many features, which led us, eventually, to develop our own implementation

- We want it to be very generic.
- Easy to use. Computations of geopotential functionals should be much easier than the available
- Very efficient. In terms of the stability of solution in higher degrees, and also in the running time for our chosen algorithms.

Our contribution can be summarized as follows

- Proposing new GGMs for datum computations in Sudan (ITU_GGC16, and GECO)
- GeoidApp: the software that we use to compute, automate, and evaluate the results of our models

1.2 Thesis structure

Following this introduction, the thesis is divided into five chapters

- Chapter 2
  It introduces the theory behind the use of GGM in the development of spherical harmonics series.

- Chapter 3
  Highlights the input data and it also shows their accuracy as it is reported by their corresponding authors.

- Chapter 4
  In chapter 4 we will study the computation of geoid height from GGM. We start from the parsing of GGM raw data up-to choosing of different algorithms, and discussing some numerical tricks for enhancing the computations performance.

- Chapter 5
  Shows our result on different datasets. It also compares our results with available similar published works.

- Chapter 6
  Our suggestions, recommendations and conclusion remarks.

Chapter 2

Geoid Determination from GGM

In this chapter we will present the mathematical and physical interpretations of computing the geoid from GGMs. Our task to compute the gravitational field in outer Earth without knowing the density structure of the Earth, only with the knowledge of the potential on the boundary. This kind of problem is called “Boundary Value Problem” or BVP. In our case, even the shape of that body e.g., the Earth must be considered unknown. Which leads us to a special type of BVP called “Geodetic Boundary Value Problem” or GBVP.

2.1 Boundary Value Problem

Earlier, we have introduced Poisson's equation 3.2, but without any further details about it. For the sake of convenience, we will write the equation again

$$\triangle W = -4\pi G\varrho$$

This equation is a general case of a more familiar equation in geodesy called 'Laplace's equation'. Laplace's equation is a special kind of Poisson equation, it can be derived by setting the ϱ to zero outside the masses

$$\triangle W = 0 \tag{2.1}$$

In a compact form

$$\triangle W = \begin{cases} -4\pi G\varrho & \text{inside, Poisson} \\ 0 & \text{outside, Laplace} \end{cases} \tag{2.2}$$

2.1.1 Existence and Uniqueness

The first step after developing our BVP is to prove their *existence* and *uniqueness*. We basically aim to prove that our BVP has a solution, and it is unique.

Uniqueness . Assuming the BVP is not unique, and we were able to find two different solutions W_1 and W_2. And let us say that their different is called U. That is $W_1 - W_1 = U$. Using Green's 1^{st} identity we can prove the existence of our BVP.
So either U, or its normal derivative is zero on the boundary and since the integrand is always positive, ∇U must be zero. And that prove the uniqueness of our 1^{st} BVP. For interested readers in the development and prove for the rest of BVPs refer to Sneeuw (2006).
We have proved that the solution of BVP is unique, but we did not derive that solution yet.

2.2 Solving Laplace's equation

We will simply start by solving Laplace's equation $\triangle W = 0$ in the Cartesian case, and then we will solve the problem in the spherical case. Both solutions will lead into a series of orthogonal base functions that can be solved by 1) Fourier series in the case of Cartesian solution, and 2) spherical harmonics in the case of spherical solution. The former solution (Cartesian one) is generally used in regional application, while the latter (spherical) serves more as a global solution. Hence the use of GGM.

2.2.1 Cartesian coordinates

Our task is to solve $\triangle W(x, y, z) = 0$ for $z > 0$. We start our solution by separating the variables

$$\triangle W(x, y, z) = \triangle f(x)\triangle g(y)\triangle h(z) = 0 \tag{2.3}$$

remember that $\triangle f$ is just a short hand notation for $\nabla \cdot \nabla f$

$$\triangle f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Applying the chain rule gives us

$$f''gh + fg''h + fgh'' \tag{2.4}$$

we substitute the second partial derivative symbol by $''$ notation, just for clarification. Dividing by fgh will lead us to

$$\frac{f''}{f} + \frac{g''}{g} + \frac{h''}{h} = 0 \tag{2.5}$$

The separation of variables lead to separate differential equations

$$\frac{f''}{f} = -n^2 \qquad : f'' + n^2 f = 0 \tag{2.6}$$

$$\frac{g''}{g} = -m^2 \qquad : f'' + m^2 g = 0 \tag{2.7}$$

$$\frac{h''}{h} = n^2 + m^2 \qquad : h'' - (n^2 + m^2) = 0 \tag{2.8}$$

We can easily solve these equation (they are second order ODEs), each part will have two solutions

$$\begin{aligned} f_1(x) &= \cos nx & f_2(x) &= \sin nx \\ g_1(y) &= \cos my & g_2(y) &= \sin my \\ h_1(z) &= e^{-\sqrt{n^2+m^2 z}} & h_2(z) &= e^{n^2+m^2 z} \end{aligned}$$

The general solution is a combination of all possible solutions. That is, for each n and m we get a new solution. However, due to the regularity condition, we discard all terms with amplifying upward continuation (For further discussions cf. Sneeuw (2006), Hofmann-Wellenhof and Moritz (2006)). That leads us to

$$W(x,y,z) = \sum_{m=0}^{\infty}\sum_{m=0}^{\infty}(p_{nm}\cos nx \cos my + q_{nm}\cos nx \sin my +$$
$$r_{nm}\sin nx \cos my + s_{nm}\sin nx \sin my) \quad (2.9)$$

Now we have to develop our BVP in terms of 2D Fourier series

$$\frac{\partial W(x,y,z)}{\partial x}\Big|_{z=0} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(p_{nm}\cos nx \cos my +$$
$$q_{nm}\cos nx \sin my + r_{nm}\sin nx \cos my + s_{nm}\sin nx \sin my) \quad (2.10)$$

We have solved the Laplace's equation in the sphere case. With slight modifications, we can derive Laplace's equation in the spherical case. Separation of variables gives us

$$\triangle(r,\phi,\lambda) = r^2\frac{f''}{f} + 2r\frac{f'}{f} + \frac{\triangle_S Y}{Y} = 0 \quad (2.11)$$

Again, that leads to the known solutions

$$\begin{aligned} f_1(r) &= r^{-(l+1)} & f_2 &= r^l \\ h_1(\lambda) &= \cos m\lambda & h_2(\lambda) &= \sin m\lambda \\ g_1(\theta) &= P_{lm}(\cos\theta) & g_2(\theta) &= Q_{lm}(\cos\theta) \end{aligned}$$

That will leave us with two different types of base functions. **Solid and spherical harmonic functions**. Solid spherical harmonics are

$$\begin{Bmatrix} r^{-(l+1)} \\ r^l \end{Bmatrix} = P_{lm}(\cos\theta)\begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \quad (2.12)$$

Where surface spherical harmonics are

$$Y_{lm}(\phi,\lambda) = P_{lm}(\cos\theta)\begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \quad (2.13)$$

Where l is the degree of the spherical harmonic, and n is the order. This inconsistency in notations is a bit tricky, but it is the one adopted by ICGEM, so we decided to go with it. In other literature, degree and order, of the model are ofter represented by n and m respectively.
Fourier's series cannot be used in spherical coordinates, so another types of base functions should be introduced. Hence the use of spherical harmonics.

$$\begin{aligned} W(r,\phi,\lambda) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} & P_{lm}(\sin\theta)(a_{lm}\cos m\lambda + b_{lm}\sin m\lambda)r^{-(l+1)} \\ & + P_{lm}(\cos\theta)(c_{lm}\cos m\lambda + d_{lm}\sin m\lambda)r^{l} \end{aligned}$$

Our solution should follow the general form as follows

$$W(r,\phi,\lambda) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\sin\theta)(a_{lm}\cos m\lambda + b_{lm}\sin m\lambda)R^{-(l+1)} \tag{2.14}$$

Comparing our solution with the general form yields

$$W(r,\phi,\lambda) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\sin\theta)(u_{lm}\cos m\lambda + u_{lm}\sin m\lambda)\left(\frac{R}{r}\right)^{l+1} \tag{2.15}$$

2.2.2 Legendre Polynomials

We need Legendre polynomials as a base function to solve our BVP. For interested readers, a thorough discussion of Legendre polynomials can be found at Mezian (2015). We will begin our discussion of Legendre polynomials by *analytical recipe*. In particular Rodrigues and Ferrers formulas. The only different between them is that Ferres' is a general form of Rodrigues. Rodrigues works on zonal areas. That is the order (or m) is zero. We begin our derivation by assuming that $t = \cos\theta$

$$P_l = \frac{d^l(t^2-1)^l}{2^l\ l!\ dt^l} \qquad \text{(Rodrigues)} \tag{2.16}$$

$$P_{lm} = \frac{(1-t^2)^{\frac{m}{2}} d^m P_l(t)}{dt^m} \qquad \text{(Ferres)} \tag{2.17}$$

Associated Legendre Polynomials are then provided by this equation

TABLE 2.1: first 3 values for Legendre Polynomials

l	m	$P_{lm}(t)$
0	0	1
1	0	t
	1	$\sqrt{1-t^2}$
2	0	$\frac{1}{2}(3t^2-1)$
	1	$3t\sqrt{1-t^2}$
	2	$3(1-t^2)$
3	0	$\frac{1}{2}(5t^2-3)$
	1	$\frac{3}{2}(5t^2-1)\sqrt{1-t^2}$
	2	$15(1-t^2)t$
	3	$15(1-t^2)^{3/2}$

$$P_l^m(t) = \frac{(1-t)^{m/2}\ d^m\ P_l(t)}{t} \tag{2.18}$$

Note that $P_l^0(t) = P_l(t)$. Plugging Legendre function to our previous equation result in

$$W(r,\phi,\lambda) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} \bar{P_{lm}}(\sin\theta)(a_{lm}\cos m\lambda + b_{lm}\sin m\lambda) \tag{2.19}$$

Combining the above steps, and solving the potential for the Earth yields

$$W(r,\phi,\lambda) = \frac{GM}{r}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^{l+1}\sum_{m=0}^{l} \bar{P_{lm}}(\sin\theta)(\bar{C_{lm}}\cos m\lambda + \bar{S_{lm}}\sin m\lambda) \tag{2.20}$$

In Table 2.1 we present the first three degrees of Legendre polynomials. Note that there is a different solution for each degree.

2.3 Application of spherical harmonics in deriving geopotential functionals

We have used Brums formula as to compute the geoid height Eq. 2.21.

$$N = \frac{T}{\gamma} \tag{2.21}$$

where γ is the normal gravity, T is the disturbance potential. The disturbance potential T in spherical harmonics is given by 2.22

$$T(r,\phi,\lambda) = \frac{GM}{r}\sum_{l=0}^{l_{max}}\left(\frac{R}{r}\right)^{l}\sum_{m=0}^{l}\bar{P_{lm}}(\sin\theta)\left[\bar{C_{lm}}\cos m\lambda + \bar{S_{lm}}\sin m\lambda\right] \tag{2.22}$$

The geoid height is then can be computed as follows

$$N(r,\phi,\lambda) = \frac{GM}{r\ \gamma}\sum_{l=0}^{l_{max}}\left(\frac{R}{r}\right)^{l}\sum_{m=0}^{l}\bar{P_{lm}}\sin\theta)\left[(\bar{C_{lm}}\cos m\lambda + \bar{S_{lm}}\sin m\lambda\right] \tag{2.23}$$

The last piece of our puzzle is the normal gravity γ. We have used Somigliana-Pizzetti normal gravity formula as shown in Eq. 2.24.

$$\gamma(\varphi) = \frac{a\gamma_a\cos^2\phi + b\gamma_b\sin^2\phi}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} \tag{2.24}$$

2.4 Summary

We aimed at this chapter to introduce a sufficient mathematical derivation for geopotential functionals (geoid height being the example used). Our goal was never to show complete derivation, but rather a very simple and intuitive representation of geopotential functionals. A clear and detailed derivation of the aforementioned topics can often be found in (Hofmann-Wellenhof and Moritz, 2006, Sneeuw, 2006, Vanicek, 1976). We derived the geoptential of the Earth in terms of cartesian coordinates and spherical coordinates, using spherical harmonics for the latter. Our goal of that, was to help the reader to familiarize himself with the use of spherical harmonics in geodesy context. We have derived the geoid height–one of several geopotential functionals, but arguably the most important one. The notation was a common problem, we found different notations in the literature. We followed the notations that are used by ICGEM website.

Chapter 3

Input Data

In this chapter we will introduce our datasets. The source of the data, its acquisition date will also be introduced. Our selected GGM will also be introduced.

3.1 Astrogeodetic data

Adam (1967) during his studies at Cornell University has conducted a study to compute the geoid in Sudan. His data consists of 46 points for different locations in Sudan. Due to the lack of data from neighboring countries, and the huge gaps between measurements inside. He suggested to fill the gaps to have a reliable datum. For a detailed discussion about the attempts to compute datums for Sudan, reader is referred to (Abdalla et al., 2012).

We have mentioned that Adam (1967) used an astro-geodetic observation, hence their datum is astro-geodetic datum. Abdalla (2009) used gravity data to compute gravimetric datum for Sudan. The astro-geodetic datum, or *geodetic datum* is a geoid computed by astronomically determined deflection of the vertical, while the gravimetric geoid is a geoid referred to the geocentric datum Vanïček (1975), hence the use of satellite data. A good discussion about the differences between the geoid types can be found in Vanïček (1975). A sample of this data is provided in Table 3.1. While the full data is provided in the appendix. The distribution of (Adam, 1967) work is introduced in Figure 3.1.

3.2 GPS/Leveling Data

GPS/leveling data are used to evaluate GGMs results by deriving the geoid height from them. The data were collected over Khartoum area.

Table 3.1: Astro-geodetic data for latitude longitude and geoid height

station	ϕ°	λ°	*geoid height* (m)
zv	22.1686	31.489	10
12	20.1361	30.662	9.556
28	18.4728	30.840	10.198
43	17.0511	31.272	10.199
53	14.4962	30.251	14.180
70	13.8316	29.654	15.833
76	13.2329	30.110	15.354
79	12.8660	29.956	15.647
80	12.7763	30.853	14.453
85	11.6038	30.411	15.387

GPS/Leveling data are two separate heights systems (orthometric and ellipsoidal height) for the same points (in terms of latitude and longitude). GPS data is ellipsoidal height data that is computed by means of GPS/GNSS systems. Leveling data on the other hand, is the orthometric height that is computed using spirit leveling. The geoid height can then be computed using Eq. (3.1)

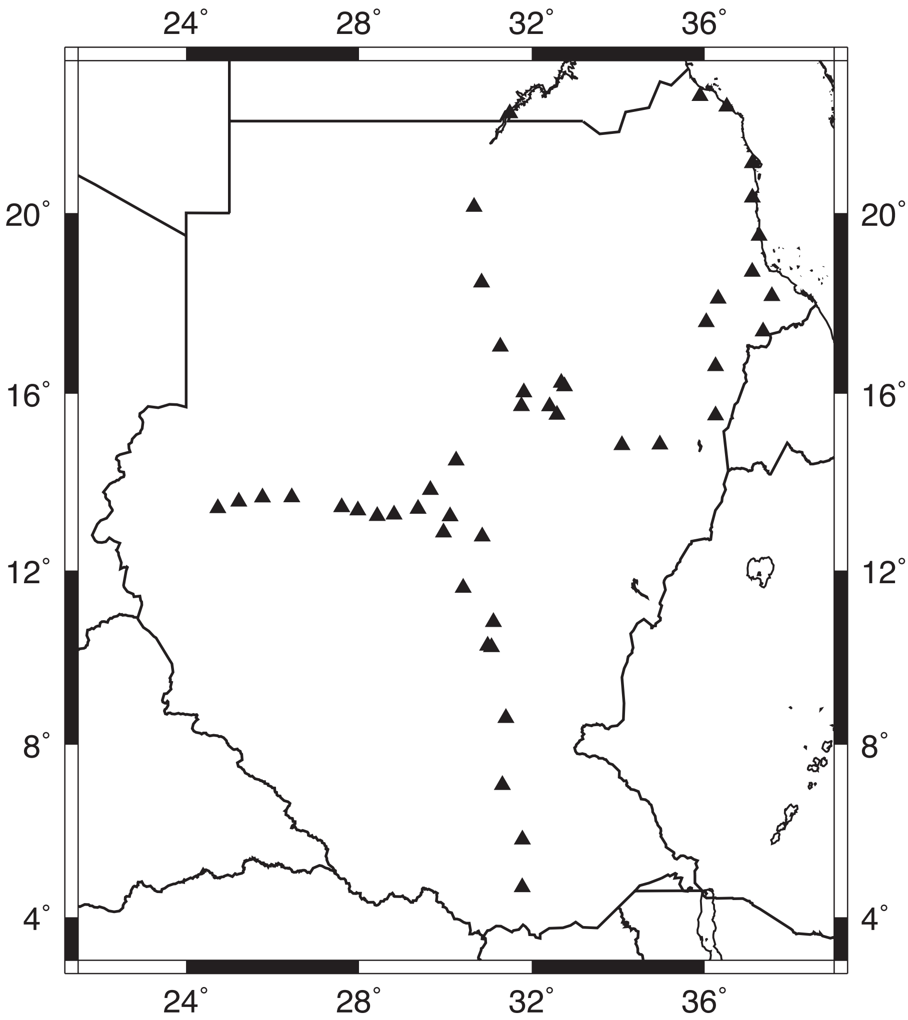
$$N = h - H \tag{3.1}$$

The GPS measurements were performed using dual frequency GPS receivers LEICA 1200, LEICA RS500 and Trimple 5700, and choke rings antennas ASH701945E_M from Ashtech, LEICA AT504 used from Abdalla (2009) work. The accuracy in leveling data is a bit problem because some of them were taken from a geodetic network of 3^{rd} order, which drastically affect our evaluations. The distribution of GPS/leveling results are shown in Figure 3.2.

3.3 Global Geopotential Models

A Global Geopotential Model is a mathematical function approximates the real gravity potential of the Earth. From such an approximation, all related gravity field functionals can be computed (e.g., gravity potential, gravity vector). However, other gravity field functionals e.g., *geoid height, gravity anomaly and gravity disturbance, and the second radial derivative* cannot be computed without a defined reference system (Geodetic Reference System 1980). As the centrifugal part can be modeled easily and accurately. It is usually beneficial to select a global geopotential model (and degree) that is best fit to the local gravity field as base for a regional gravimetric model. This will reduce the

FIGURE 3.1: Distribution of (Adam, 1967) points in Sudan. It is clear the huge gap that Adam (1967) suggested to be filled



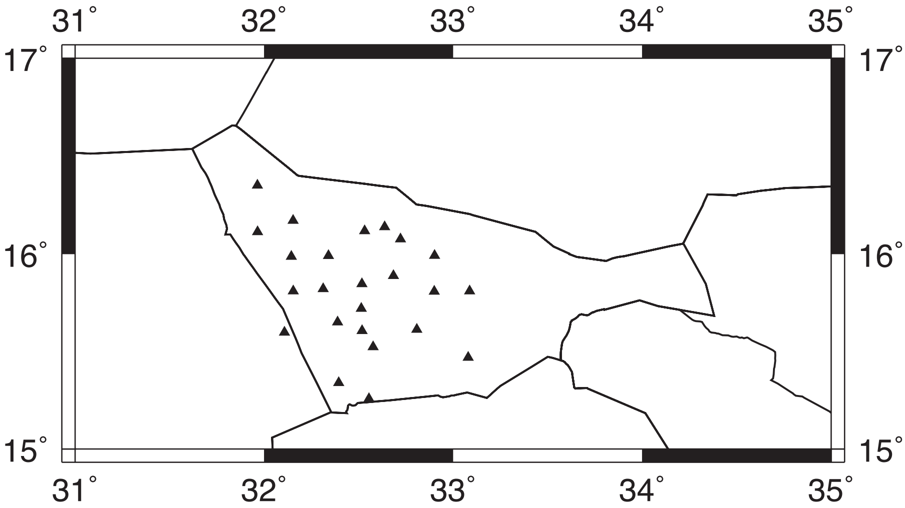
error by Stokes' formula by reducing the amount of the geoid contribution to the total error.

3.3.1 Satellite-only models

They are solely derived from the analysis of the satellite orbit. They use laser and Doppler measurements to track the satellites **?** Historically, these models were known to have a low precision due to

1. the inability to track complete satellite orbits using ground-based stations
2. imprecise modeling of atmospheric drag attraction
3. the incomplete sampling of the global gravity due to the limited number of satellites

FIGURE 3.2: Distribution of GPS/leveling data over Khartoum



These issues were fixed by the new dedicated satellite missions in the earlier 2000s Rummel et al. (2002). In our study, we used two satellite-only models ITU_GCC16 and ITU_GRACE16 of 280, 180 degree and order respectively.

3.3.2 Combined Models

These models are combined from different source. A GGM is combined with land and shiptrack gravity observations, and marine gravity anomalies Amos and Featherstone (2003). It is often that combined models have higher degrees than satellite-only models. Even with the added source of e.g., terrestrial data, combined models are also limited in their precision because of their use of satellite-only models. In our study we have tested three different combined models namely EGM2008, EIGEN-6C4 (2014), and GECO (2015). They are all up to degree 2190, and we tested them up-to their maximum degree. One source of error caused by the combination of different regional (local) datums and the offset between them. Heck (1990) reported that the magnitude of errors due to the inconsistency of regional datums with global datum is underestimated. The source of error was due to the simplified free-air reduction procedure and of different kinds of height system. Heck (1990) in their work have show that the corresponding systematic errors in gravity anomalies are maximum in mid-latitudes. The error due to datums inconsistency is a systematic error.

3.3.3 Tailored Models

A combination of the aforementioned models designed (tailored) for a specific area. That actually against the term global in GGM. None of these models were tested in our

Table 3.2: GPS/Leveling data for Khartoum area

id	ϕ°	λ°	*geoid height* (m)
1	16.1719143	32.15278395	3.573
2	15.822078561111	32.312747819444	2.998
3	15.889747230556	32.683920419444	3.619
4	16.076534180556	32.721614461111	3.119
5	15.810990861111	33.086944869444	2.175
6	15.809063111111	32.899570888889	2.285
7	15.613491838889	32.808035077778	2.02
8	16.351884494444	31.964771408333	4.174
9	15.847591619444	32.517391980556	3.078
10	16.118530788889	32.531269330556	2.263
11	15.99188345	32.3404394	3.21
12	15.810295125	32.154373358333	3.21
13	16.11349295	31.965449913889	3.197
14	15.992628230556	32.9012736	3.254
15	15.987956075	32.144070430556	3.447
16	15.469951816667	33.079683027778	2.297
17	15.651378211111	32.388177761111	2.81
18	15.721159511111	32.514142069444	2.655
19	16.139034477778	32.637590772222	2.4199
20	15.607008355556	32.518298555556	2.6729
21	15.599259138889	32.107760216667	2.5378
22	15.524018605556	32.576891125	2.5993
23	15.258628422222	32.554645944444	2.2311
24	15.340130033333	32.394058058333	2.6918

experiments.

3.3.4 Difficulties in modeling the Earth from the space

Rummel et al. (2002) in their work reported that there are two problems in efficiently model the Earth using satellite observations. In particular they report that 1) satellites can be tracked from the ground only over short intervals and, as a consequence, the gravity signal printed onto the orbit can only be extracted where it produces an orbit signal of large size such as at or close to orbit resonances, and 2) satellite motion is not determined by gravitation alone but disturbed by several types of surface forces of non-gravitational origin. The main objectives of the new satellite missions is to tackle these two fundamental problems.

3.4 New Satellite Missions

Because of the mentioned limitations of satellite missions, new dedicated satellite missions was launched in early 2000s to tackle those problems. There are three satellite missions launched for that purpose

3.4.1 CHAMP

CHAMP **CHA**llenging **M**inisatellite **P**ayload Reigber et al. (2006) is a German small satellite mission for geoscientific and atmospheric research and applications, managed by GFZ. The orbit of CHAMP is almost circular (inclination 87degree) which gives it an advantage of getting homogeneous and complete global coverage. With an altitude of 454 km, it guarantees that the satellite can work in severe atmospheric conditions. It also has a direct on-board measurement to account for the non-gravitational orbit perturbations. . CHAMP uses satellite-to-satellite tracking working on high-to-low mode, i.e., GNSS satellites are tracking the spaceship position, and it uses an accelerometer to compute the geoid functionals (cf. REIGBER et al. (1999) for detailed discussions about CHAMP mission).
Clearly, the aforementioned issues with previous satellite mission was resolved in the new dedicated satellite mission. A detailed description of CHAMP, and other new satellite missions e.g., GOCE and GRACE will be presented in Table 3.3.

3.4.2 GRACE

Gravity **R**ecovery and **C**limate **E**xperiment Schmidt et al. (2006) is the second satellite mission that was launched on March 2002. GRACE consists of two identical satellites separated by 220 km from each others, and at 500 km above the Earth surface. Unlike CHAMP, GRACE is uses satelite-to-satellite tracking to precise position determination, but it uses low-low mode to compute the gravity functionals i.e., the gravity is measured by the change of the distance between the twin satellites. Areas of slightly stronger gravity (greater masses) will affect the leading satellite first. An accelerometer will be used to measure the non-gravitational acceleration–common errors due to atmospheric attraction–such that only the accelerations caused by the gravity are measured.
It is common to find a model with data from different satellite mission–each one of them contribute to specific range of spherical harmonics degrees.

3.4.3 GOCE

The last mission in the new gravity measurement satellites era is GOCE Floberghagen et al. (2011). **G**ravity Field and Steady-State **O**cean **C**irculation **E**xplorer. It is specifically designed for the determination of the stationary gravity field geoid and gravity anomalies to high accuracy and spatial resolution Rummel et al. (2002). As reported by (**?**)rummer), GOCE is the only satellite mission that was able to solve the problem of gravity observations from satellites that was introduced in Section 3.3.4. In summary, GOCE has tackled the classic problems of using satellite to compute the gravity as follow

1. Uninterrupted tracking in three spatial dimensions
2. Measurement or compensation of the effect of non-gravitational forces
3. Orbit altitude as low as possible

3.5 Our models

For our study we have tested five models. Two of them are satellite-only models (ITU_GCC16, and ITU_GRACE16), three are combined models (EGM2008, EIGEN-6C4, and GECO). The choice of these models are based on their release date. The choice of the number of models is totally arbitrary.

1. ITU_GGC16 (Akyilmaz et al., 2016). It was released on 2016 with degree and order up-to 280.
2. ITU_GRACE16 (Akyilmaz et al., 2016b). It was also released on 2016, with degree and order of 180.
3. EGM2008 (Pavlis et al., 2012). The popular new version of EGM series, the previous ones were EGM84 and EGM96. We use this one for up-to degree of 2190. Released on 2008.
4. EIGEN-6C4 (Forste et al., 2015). Composed of data from different EIGEN series missions, in particular it uses data from EIGEN*6C, EIGEN6C2, and EIGEN6C3. It was released on 2014, and it is the last mission of EIGEN series. The model is up-to to degree and order of 2190.
5. GECO (Gilardoni et al., 2015). It was released 2015, and it is the latest models of high o/d models (those above 2190 d/o). It combined data from GOCE mission as well as EGM2008.

Table 3.3: Comparsion of new dedicated satellite missions

mission	*inclination angle* °	*altitude (km)*	*release date*	status
CHAMP	87.2777	454		stopped-2010
GRACE	89	500		still operating
GOCE	96.7	268		still operating

Table 3.4: Breakthrough in satellite observations

Mission	*Technology*	mode	*missions*
SST-hl	Accelerometer	high-low	CHAMP
SST-ll	inter-satellite link	low-low	GRACE
SST-ll	Gradiometer	low-low	GOCE

3.6 Chapter summary

In this chapter we have introduced the new method of computing the geoid, or more generally measuring the surface of the Earth. While in the classic conservative approach to problems of physical geodesy as described by **?**. The advantage of this approach is that the geoid as a reference surface has a very simple definition in terms of the physically meaningful and geodetically important, Potential V. The disadvantage, however, is that the potential V inside the earth depends on the density ϱ because of the Poisson's equation Eq. (3.2).

$$\triangle W = -4\pi G\varrho \tag{3.2}$$

The mathematical aspects of GGM are reserved for the next chapter. It is clearly impossible that we cannot measure the density at each point in the earth, hence there should be some approximations. The other, or the modern approach is what proposed by Molodensky in 1945. He was able to show that the physical surface of the earth can be determined from geodetic measurements alone, without the need of the density of earth's crust. That means the concept of the geoid should be abandoned. This disadvantage is that the concept of the reference surface–which was easily defined by the geoid–will be more abstract, and the mathematical method will be more difficult.
The development of satellite missions (that uses Molodensky's theory) was going such that with every new model there is a possible increase in degrees, and hence the accuracy. Table 3.4 summarizes the major technical details regarding the new dedicated satellite missions. Table 3.5 summarizes the models that were used in our study.

Table 3.5: Summary of GGM that were used in our study

Model Name	Type	Max Degree °	*Tested models* (m)
ITU_GGC16	satellite-only	280	270
ITU_GRACE16	satellite-only	180	170
EGM2008	combined	2190	500
EIGEN-6C4	satellite-only	2190	200
GECO	combined	2190	500

Chapter 4

Geoid Height Determination Using GeoidApp

4.1 Introduction

In this chapter we will discuss the computation of the geoid height and the validation of our results based on local terrestrial data. High resolution models are required to convert GPS leveling data (ellipsoidal height) into orthometric height. For evaluation purposes we have two terrestrial data 1) Astrogeodetic data provided by Adam (1967), and 2) GPS leveling data by Abdalla (2009). Our results show higher degrees will often result in a lower standard deviation, i.e., better results. But that should does not always holds true, hence the our evaluation was done using different degrees.

Combined models e.g., EGM2008, EIGEN-6C4, and GECO have a very similar trend–it is expected because both EIGEN-6C4 and GECO share some degrees with EGM2008. Interestingly, ITU_GGC16, the best model with an error of $0.3653m$ in 149 degree, has the worst results for degrees ¿ 154. Our comparsion with the previous works (Abdalla, 2009, Fashir, 1991, Godah and Krynski, 2015) shows that our choosen models has surpassed them all. For recent works (Abdalla, 2009, Godah and Krynski, 2015), we show that our model can contribute to their suggested datums of to 40% (37% for (Abdalla, 2009), and 43 % for (Godah and Krynski, 2015)).

4.2 Computing the Geoid

We developed a software application called GeoidApp for this task. GeoidApp is a huge suite of applications to not only compute geoid height (in addition to gravity anomaly and gravity disturbance), but it also has analysis and interactive visualization features to help the users get the most out of it. For that reason we will discuss the core features about GeoidApp. To compute the geoid height you need to account for these details

- The data. GGMs are provided as *.gfc files from ICGEM. They contains in addition to the coefficients text contents about the authors, acknowledgments, etc. They also include model parameters, such as the maximum degree, the model name, the GM, and a few others. The idea is to store the coefficients in an array (for computations efficiency).
- Helper functions for Associated Legendre Polynomials. That could tricky, solving Legendre function is non-trivial. The recursive version of Legendre function is suitable for programming environment.
- The main function for geoid computations. You have to choose whether you want to work on point mode, grid mode.

4.2.1 Working with GGMs data

Typical GGMs provided by ICGEM are raw and need to be parsed to extract the model's coefficients and other useful informations about the model. When we say a 'patter', or 'template' we mean that there are certain keywords in the *.gfc files, and it is available in all models.

4.2.1.1 Parsing the data

A typical model from ICGEM would consist of the following

- General information about the mission.
- A header with structured details about the models, and their parameters
- The coefficients part. Which is the core part of the *.gfc files

Model's summary . It includes summary about the mission its release date, mission time, etc. There is also a link for detailed informations about the mission. You can safely ignore this part of of the *.gfc file. This part of *.gfc files does not follow any pattern, you cannot build a generic tool to extract informations from it.

The header . This summarizes the previous in a template way. It will begin with the keyword *original header*. Below it, there is the name of the model, and the release date, mission duration. It also has information about GM but you can also ignore it here as it will be provided later in more structured way. A good way to think of *.gfc files is xml format. xml files are constructed as `<begin_of_tag> some_data </end_of_tag>`. For our work, we treated *.gfc files as follows

```
<begin_of_head>
   <original header>
   "product_type": "gravity_field" /* In our case we need this */,
   "model_name": "model_name",
   "radius": "6378136.30" /* It varies over models */,
   "earth_gravity_constant": "398600.4415D+09" /* It varies over models */,
   "max_degree": "n",
   "errors": "formal", /* It could take other values */,
   "tide_system": "tide_free" /* other types are available */,
   </original header>
</end_of_head>
```

Note that we use those keywords to store these variables with their corresponding values in a data structure (a dictionary, or hash table is suitable for this purpose).

The last part of parsing GGM files is extracting the model coefficients C_{nm}, S_{nm}. We want to construct a $(n \times n)$ matrix, where n denotes the maximum degree of our model. For each raw of our matrix will correspond to the degree of the model. Each will have no-zero elements as the index of of that raw. The first raw will always have value 1, and the rest of it are zeros. The second raw will depend on an argument that is specified by the user *subtract_normal_field*. In our evaluations we always set such that the "normal_field" will be subtracted from coefficients $c_{nm}(1:9, 1:9)$. It's important to note that indexing for global geopotential models starts from zero as oppose of one. You should account for that if your are using programming language that uses one as a base for indexing e.g., MATLAB/Octave or Julia.

$$C_{nm} = \begin{bmatrix} 1 & \dots & \dots & N_{max} \\ 0 & 0 & \dots & N_{max} \\ \vdots & \vdots & \ddots & N_{max} \\ -4.84e-04 & -3.98e-10 & 2.43e-06 & N_{max} \end{bmatrix}$$

For the S_{nm} part there the first column of it, i.e., the value of S_{nm} coefficients are always zero. That is $S_{nm}[:, 0] = 0$. Unlike C_{nm} the number of non-zero elements for each raw in S_{nm} equals the index of that raw minus 1.

$$S_{nm} = \begin{bmatrix} 0 & \dots & \dots & N_{max} \\ 0 & 0 & \dots & N_{max} \\ \vdots & \vdots & \ddots & N_{max} \\ 0 & 1.42e-09 & -1.40e-06 & N_{max} \end{bmatrix}$$

The same was applied to construct $e_{C_{nm}}, e_{S_{nm}}$ the std values for C_{nm}, S_{nm}, respectively.

4.3 Software Implementation

4.3.1 Associated Legendre Function

Solving Legendre function is not easy. It is always good to build on top of others projects, and reference them as needed. We found a few libraries that solve Legendre function. One of them is legendre function in provided by MATLAB. This function does not come with the standard version of MATLAB, so we avoided it. The implementation of it does not also meet with our use case. We have also found other implementation of Legendre that was written in either C++ or Java, both of them was not used during the development of GeoidApp. Another very recent implementation of legendre function was that of NumPy, a popular numerical library for Python. As the time of writing our code, NumPy's legendre was not yet implemented.

We had to either translate libraries from other languages, or modify that version of MATLAB. We decided to write our own implementation of Legendre. In both case we had modify something anyway, we also want to ensure that the implementation of our function should be compatible with other parts of our application. Legendre function should also be well optimized, the most expensive part of geoid computations is the computing legendre polynomials for each degree, not to forget that the computation increase with the increase in degrees. Another criteria is the stability of the solutions in high order and degrees of the model. However, when evaluated increasingly close to the poles, the ultra-high degree and order (e.g. 2700) ALFs range over thousands of orders of magnitude. This causes existing recursion techniques for computing values of individual ALFs and their derivatives to fail Holmes and Featherstone (2002). We followed Holmes and Featherstone (2002) implementation to make our Legendre function.

4.3.2 Geoid height function

We will begin by discussing various implementation techniques regarding computing the geoid. We assume that the reader is familiar with the mathematical details about computing the geoid. Hence, our discussion here will be about the efficiency, and our design decision for GeoidApp. We will begin our discussion by an overview of the existing libraries and tools to compute the geoid, and compare them–in terms of design and efficiency–with our work. We have tried to contact the authors of EGMLab, but the did not respond, neither we did able to find any valid link for it. Kiamehr and Eshagh (2008) a model reader was used to parse the *.gfc files. However, it was not clear whether their model directly handles *.gfc files from ICGEM, or a the user must have to do preprocessing. What they reported however, is converting coefficients to column shape. The result column(s) is then by the GM (as a fixed value here, but they also said that GM might be dynamically changed.) Their model, which is a critical part in our comparison only works within maximum degree and order of 720 (based on EGM2008). They also reported that the use of Horner and Clenshaw algorithms algorithm will expand the maximum degree and order up to 2170 as shown in Holmes and Featherstone (2002).

An interesting point from their work is their investigation on Clenshaw and Horner algorithm using A Java program on official ICGEM website. They mentioned that in order to use a model with a degree up to 2160, the result will take at least an hour! In our work we show that we can compute geoid height (or other geoid components) within a few minutes (6 minutes being the highest observed time for computing the geoid for all Earth with models up-to d/o of 2190, and step size of 0.1°). In terms of efficiency, clearly that our work surpasses Kiamehr and Eshagh (2008) work by many order of magnitudes. Also, because of Clenshaw and Horner, our work is able to account for any future models, as long as they are within 2170 range, which is not going to be exceeded too soon. In terms of software design, Kiamehr and Eshagh (2008) works in point-wise mode as well as in area mode. Our work does also provide that feature i.e., you can compute the geoid height for a point, as well as a grid. It is also important to mention that Kiamehr and Eshagh (2008) in their work report that their results surpassed that of official EGM96 results. Which indicates the same for our work.

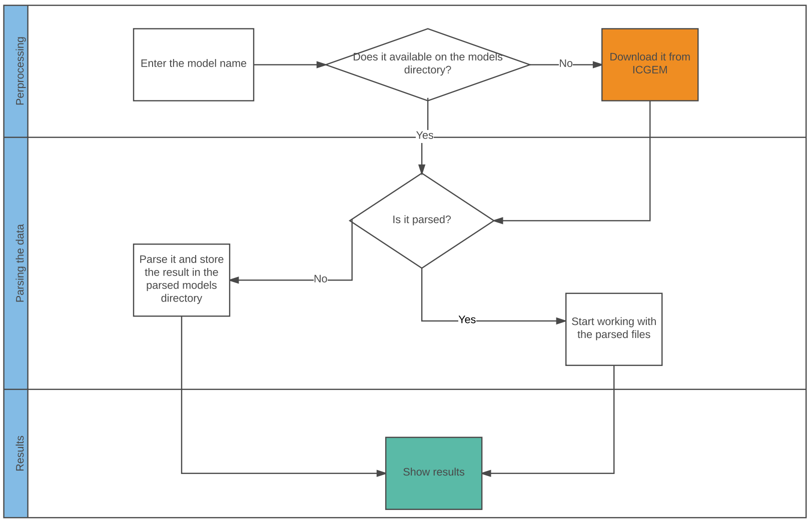
The other work we have considered is the official ICGEM calculation service available at ICGEM. The implementation details are hidden. They have a nice JavaScript front-end, that works not only as a wrapper for a low level language (C or C++), but also serves well their purpose of running the application in the browser. They have many application in addition to geoid height. We were inspired by many of their implementations, namely "functional", "tide_system" keywords. Their results are in a grid format. You cannot use their results in any sort of evaluations, unless you made a separate interpolation function for that purpose. You cannot also run multiple instances of ICGEM service simultaneously (3 instances simultaneously is the maximum allowed). But that should not be a problem. Our work not only does allow you to work in both point-wise and grid mode, but it also gives you a powerful application with no limitations in terms of usage or in computations. Our work also come with a tool to automatically download

models from ICGEM website, so if you use it without specifying the model name, it will download the latest and the greatest model. We also have an interactive visualization tool so that we will make the process of understanding the data more easier. Another feature of our work is the automation of degree computations of the models. This feature is specially important in the case of the evaluations, where researchers often need to work with different models at different degree to find the best one. This task could be a problem if you manually tune the models. Another feature that is unique in our work, is that we try to automate the process of using the model by comparing it with freely available terrestrial data the is geographically near to the user's input data (based on their latitude and longitude). If the user has provided an evaluation data (GPS/leveling data) that will ease and automate the process, so that the output will be the best model with the best degree. The output will also contain a file with a top ten models with their degrees based on std criteria.
The last similar work is GeographicLib which is a small C++ classes for various geodesy computations e.g., conversion between UTM and geodetic coordinates (latitude, longitude). What is similar to our work is their geoid class implementation. They only used 3 models, EGM84, EGM96, and EGM2008. You can see that it is very limited and also too outdated. Another limitation is that this library does only work in the point-wise mode. That will severely limit the use of it in large project where you have several points.

4.4 GeoidApp

GeoidApp consists of mainly 4 parts. A model reader to parse the *.gfc files and store the results into the appropriate data structure. Depending on the model–models of degrees and order of 2190 their size exceeds 200 MB–took upto several minutes to be parsed. Never the time has exceeded 6 minutes. We have decided to compute the geoid hight for a grid as oppose of computing it from the beginning for each point. For point wise mode we have implemented an interpolation functions so that the user can choose whatever mode they wish. However, a native point-wise program is available for use. From computational point of view, we found that the use of grids is much faster than computing for each point whenever the user requested a new results. This actually the core programs of GeoidApp, and they are typically found (or one of them,) in the previous works Kiamehr and Eshagh (2008), Drewes et al. (n.d.), ?.

FIGURE 4.1: GeoidApp workflow summary



4.4.1 Automations in GeoidApp

A shell script was integrated into our main program suite to automate the process of the computation. When you run the application, you are only prompted to enter the name of you model. They are many other arguments but they are not required e.g., the number of the degrees to be computed, and their range. Because, when the user enter the name of the model we will check it against the directory where we store our *.gfc files. If the model exists, we will tell the user that this model is available. Then, we will prompt it whether to proceed with it, or use another model. If he decided to go with that model we perform another check to make sure with this model was already parsed or not. If it was parsed we will tell the user that their model was already parsed, and we will ask them whether to proceed with it or not. We make all of these checking, because these operations are not trivial. If the model does not exist in the user directory we will automatically download it from official ICGEM website, and then start the process on it. Table 4.1 shows a diagram about GeoidApp workflow.

GeoidApp also consists of advanced data analysis tools to help the users get the best out of their models. For each parsed model, GeoidApp creates a directory for that model and stores a log file that logs the result e.g., std and difference, and a *.csv files to store the results in a way that can easily be parsed later. GeoidApp also provides figures for each model being parsed. Another distinct feature is the interactive visualization for the result. This feature comes with the online version of GeoidApp .
The website of GeoidApp has many resources that cover different aspect of GeoidApp. A comprehensive documentation about the application, interactive visualizations, case studies results will all be presented at the website.

Chapter 5

Results and Discussions

We have evaluated different models in two types of datasets. The first data collected by (Adam, 1967) consists of 46 points. They use astrogeodetic observation to compute the geoid height. GPS/leveling data consists of 24 points collected during 2005-2008. We evaluated our models in different range of degrees (step of 5 degrees for EGM2008 and GECO, step of one degree for ITU_GGC16 and ITU_GRACE16). We used standard deviation as a measure of accuracy.

5.1 Evaluation on astrogeodetic data

Our evaluation of GGM using astrogedetic data shows very interesting results. There is a huge difference between GGM results and data of (Adam, 1967). Clearly there is a problem with (**?**) works. For the five that were tested, the best model has standard deviation of 5.867 m. Which is really very good. From our study on Adam (1967) we find that lowest degrees result on average in relatively good std compared with high order degrees. However, the trend of std with the increase in degrees shows clearly that, while some very lower degree (less than 20) are having the best score, but the general trend is that the error is reduced with increase of degrees. A table summarizes our results for each model showing the best five degrees for each model is presented in the below tables.

ITU_GRACE16 has the best score of 5.8679769m on degree 12. ITU_GGC16 has its best score on 12 degree also with 5.8679787m. The other models were close to each others of EGM2008 having its best score on 14 degree with std of 6.57m, interestingly GECO does also has its best score on 14 degree with standard deviation of 6.579m. EIGEN-6C4 has std of 6.718 on 1729 degree. In our results we found that it is often the case that near degrees have quite some similar std. To Have a very clear representation

TABLE 5.1: Top 5 degrees for model ITU_GGC16 on Adam (1967)

degree	std σ [m]	difference
12	5.86797	-10.7197
13	6.232	-10.0005
14	6.575	-9.9920
15	6.62	9.8420
11	6.74	-10.6067
121	6.758	-11.0029

TABLE 5.2: Top 5 degrees for model EGM2008 on Adam (1967)

degree	std σ [m]	difference
14	6.57797646	10.01617876
1722	6.72616231	11.10208562
1726	6.72836585	-11.10328207
1595	6.73249422	-11.10455418
1731	6.73263864	-11.10572496

TABLE 5.3: Top 5 degrees for EIGEN-6C4 on Adam (1967)

degree	std σ [m]	difference
1729	6.71829049	-11.0980582
1718	6.72207977	-11.10011044
1609	6.72228078	-11.09995271
1598	6.72349438	-11.09978464
1872	6.72480567	-11.10075124

TABLE 5.4: Top 5 degrees for model GECO on Adam (1967)

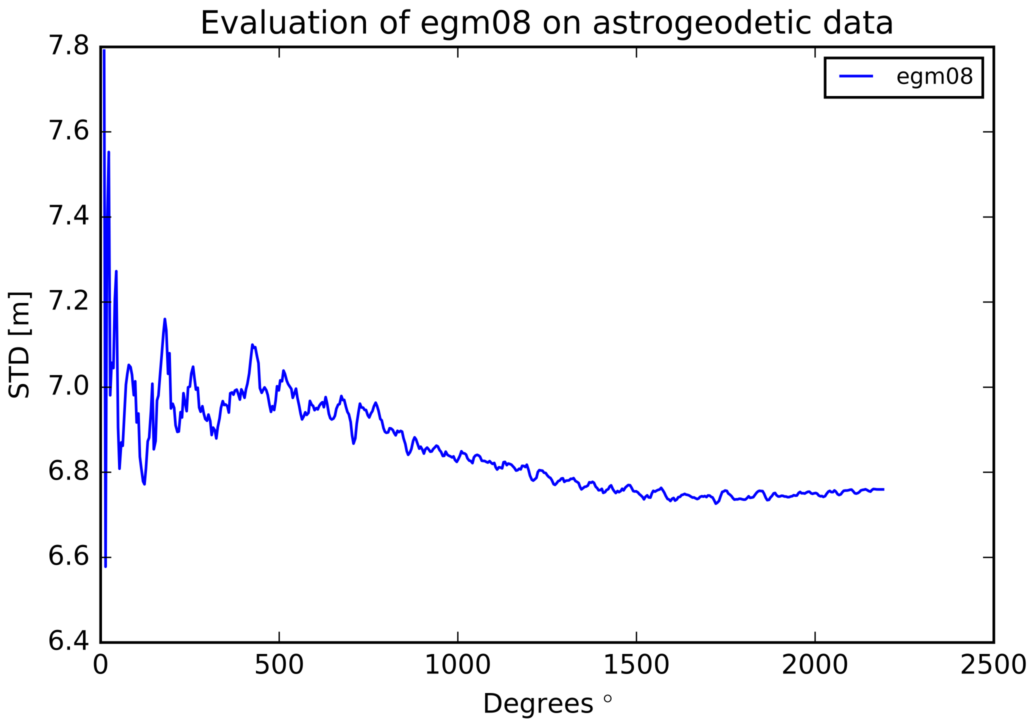
degree	std σ [m]	difference
14	6.579	-10.016
1722	6.7273	-11.102
1726	6.7295	-11.104
1595	6.7335	-11.105
1731	6.7342	-11.106

of our best model we decided to use as several degrees as possible. For ITU_GGC16 and ITU_GRACE16 we have tested each degree starting from the 10^{th} degree. For the other models (those of 2190), we have tested 500 degree for EGM2008, and GECO and 200 models from EIGEN-6C4. To the best of our knowledge, no such exhaustive testing was done for GGM before.

TABLE 5.5: Top 5 degrees for ITU_GRACE16 on Adam (1967)

degree	std σ [m]	difference
12	5.8679769	-10.7196877
13	6.23255147	-10.000530
14	6.57485872	-9.992074
15	6.61925941	-9.84207
120	6.73941682	-10.9837

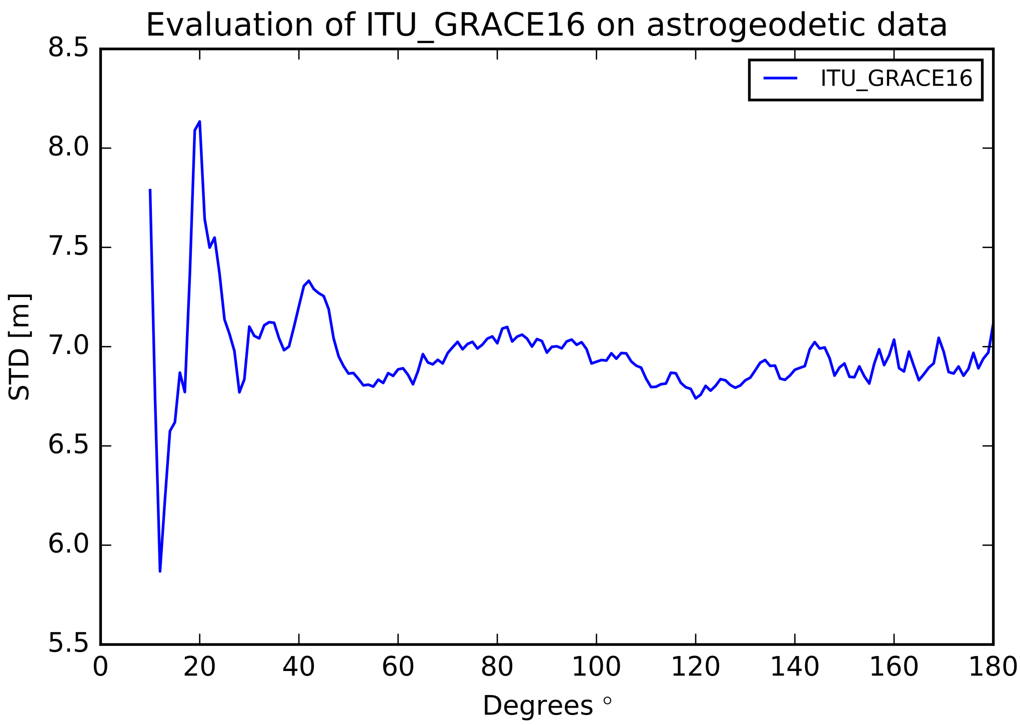
FIGURE 5.1: Evaluation of EGM2008 on Adam (1967)



5.2 Evaluation of GPS/Leveling data

The second dataset that we used is GPS/Leveling data. It consists of 24 GPS/leveling data that were collected from 2005-2008 from different geodetic networks ranging from 1^{st} order networks to 3^{rd} order networks. Our evaluation on GPS/leveling data shows that the std error has dropped from 5.86m to 0.365349m. ITU_GRACE16 had a very strange trend in degrees ¿ 150. The reason for that is due to the emission error that takes place in the higher degree of the satellite model. Another observation is that in all of our models, the best 5 models are dominant by the higher degrees, 150 for ITU_GCC16 and (669, 665, and 674) for EGM2008. For EIGEN-6C4 there is 667, 327, and 678. GECO had a near result to that recorded by ITU_GCC16 (the difference is 0.000131m), has three high degree models in its 5 best models, 669, 674, and 263. The definition of high order models is quite arbitrary, but use the score that degrees ¿ 100

FIGURE 5.2: Evaluation of ITU_GRACE16 on Adam (1967)



is called high degree models, else are low degree models. In our best 20 models (based on std) for all models, 18 of the 20 are classified as high degree models which is 90% of the total 20 models. We conclude that using high order/degree models often leads to a better results. It's true for the global representation of the gravitational functionals that "the *higher* is the *better*", but that does not always hold true for the local case. Hence, the evaluation of different degrees of the model on local data. We evaluated three high models together (EGM2008, EIGEN-6C-4, and GECO), the results on the lower degrees (¡ 900) seems to be a bit noisy (a few peaks occur), but for the higher degrees the results were very stable. GECO and EIGEN-6C4 has always surpassed those of EGM2008, even though some of their degrees are derived from EGM2008. There a constant difference between EGM2008 and GECO and EIGEN-6C4 results, they all have the same trend. The magnitude of the difference between GECO and EIGEN-6C4 is constant, which lead us to believe that there were a systematic error(s) in EGM2008 and they were resolved in the latter models.

5.3 Summary

In this chapter we have shown our results on two different datasets. Our evaluation on astrogeodetic data shows that there is a huge offset between the two results. The std

Table 5.6: Top 5 degrees for model ITU_GRACE on GPS/leveling data

degree	std σ [m]	difference
46	0.367751	0.571191
152	0.369840	0.417293
149	0.373809	0.127576
29	0.3739635	-0.027817
30	0.374076	-1.040373

Table 5.7: Top 5 degrees for model ITU_GGC on GPS/leveling data

degree	std σ [m]	difference
149	0.365349	0.330661
150	0.366355	0.354195
46	0.367751	0.571312
148	0.368081	0.300463
152	0.368113	0.509021

Table 5.8: Top 5 degrees for model EGM2008 on GPS/leveling data

degree	std σ [m]	difference
669	0.373804	0.487649
665	0.373929	0.486516
149	0.375045	0.407975
674	0.376249	0.486830
31	0.376251	1.316226

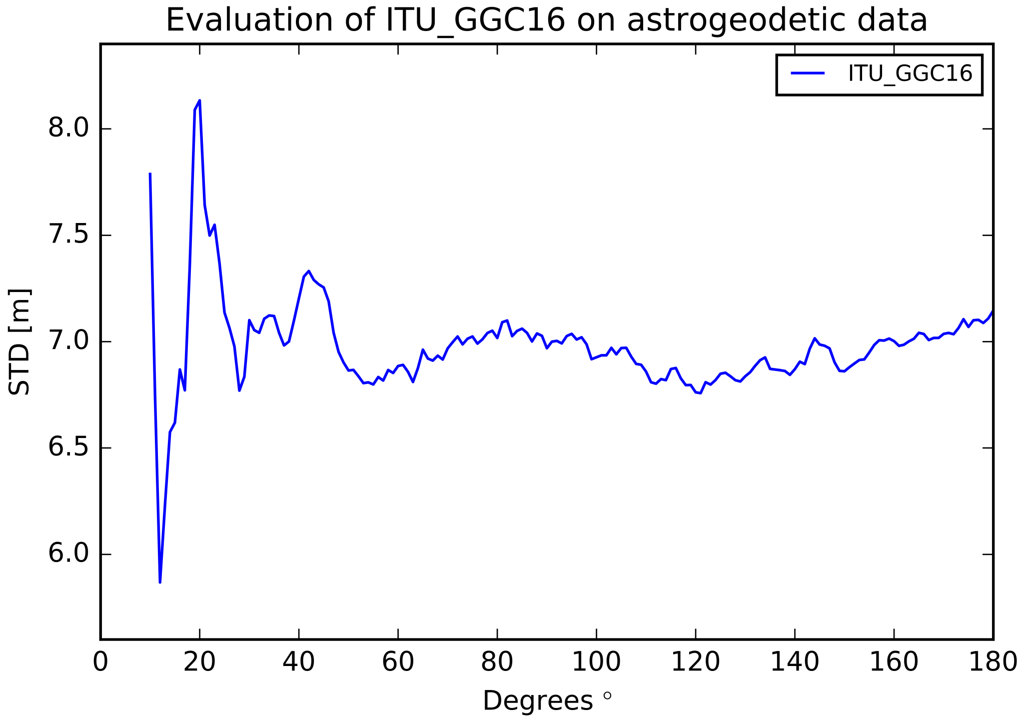
Table 5.9: Top 5 degrees for model EIGEN-6C4 on GPS/leveling data

degree	std σ [m]	difference
667	0.368126	0.371325
152	0.368243	0.460720
327	0.3713845	0.400191
678	0.372534	0.372325
261	0.372603	0.352102

Table 5.10: Top 5 degrees for model GECO on GPS/leveling data

degree	std σ [m]	difference
149	0.365480	0.304553
669	0.369906	0.379220
665	0.370001	0.378087
263	0.370542	0.367333
674	0.372194	0.378396

Figure 5.3: Evaluation of ITU_GGC16 on Adam (1967)



error was in order of meters (5 meters the lowest). A plot for the new
Our evaluation on GPS/leveling data shows a huge increase in the accuracy, as the std was dropped to 0.35m.

Figure 5.4: Evaluation of GECO on Adam (1967)

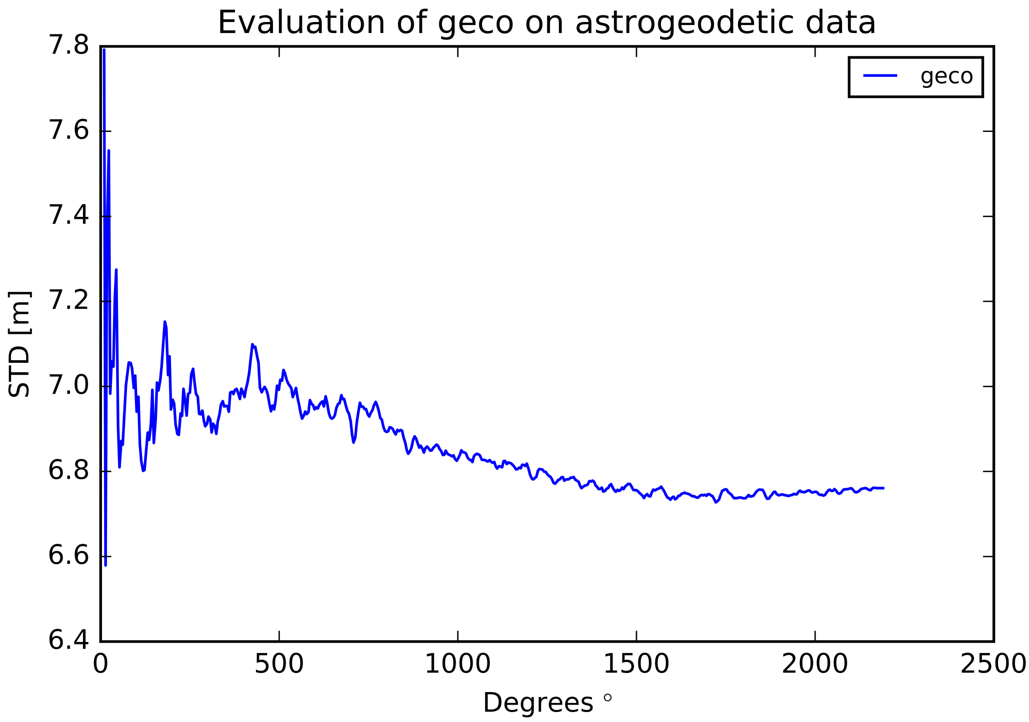


Figure 5.5: Evaluation of EIGEN-6C4 on Adam (1967)

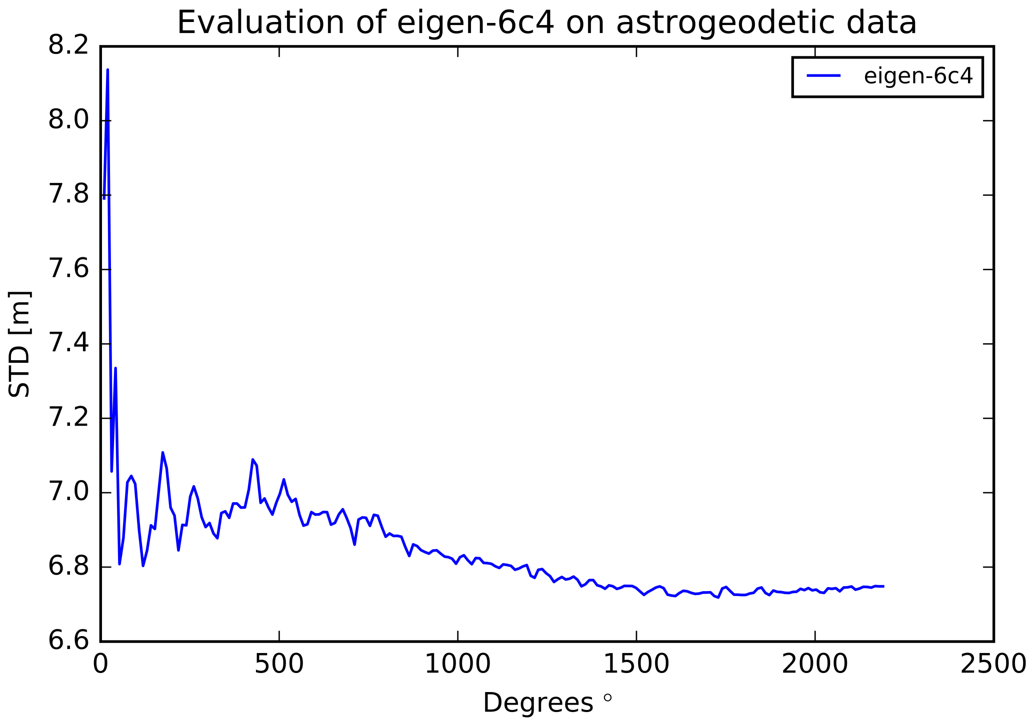


Figure 5.6: Our five models truncated to 180 degree

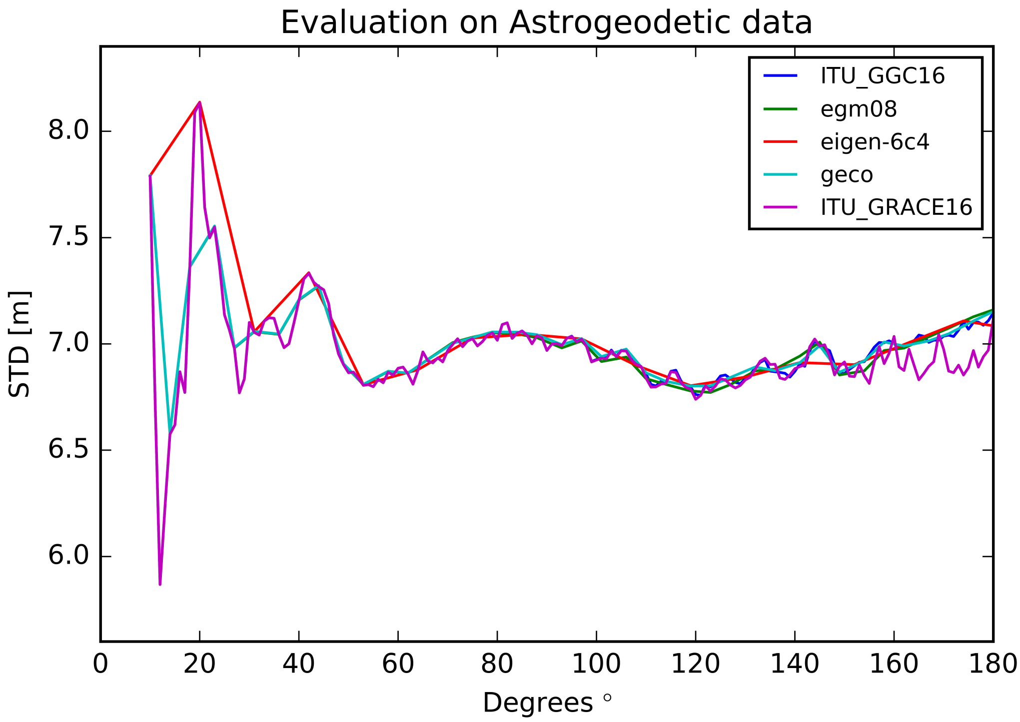


Figure 5.7: Evaluation of our GGM on GPS/leveling data

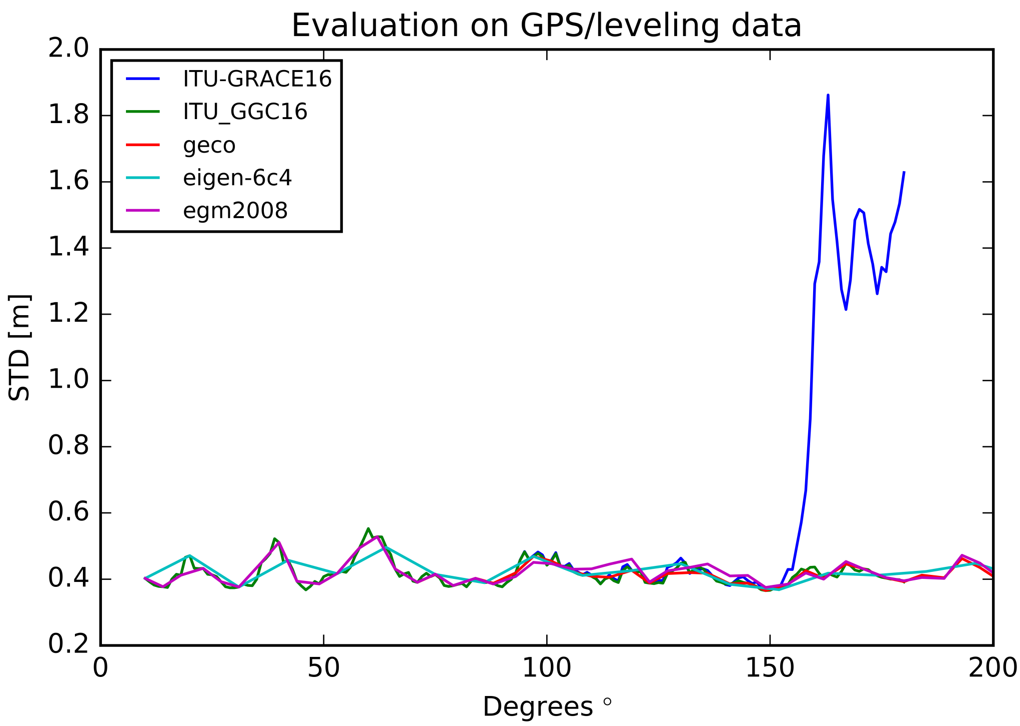


FIGURE 5.8: Evaluation of EGM2008 on GPS/leveling data

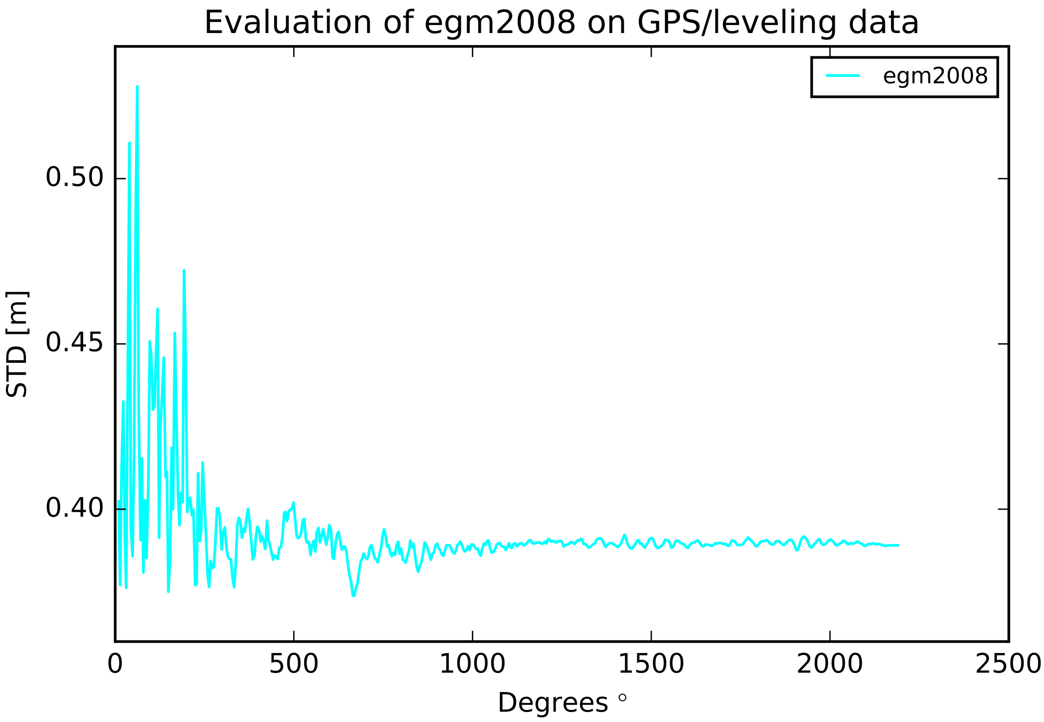


FIGURE 5.9: Evaluation of ITU_GRACE16 on GPS/leveling data

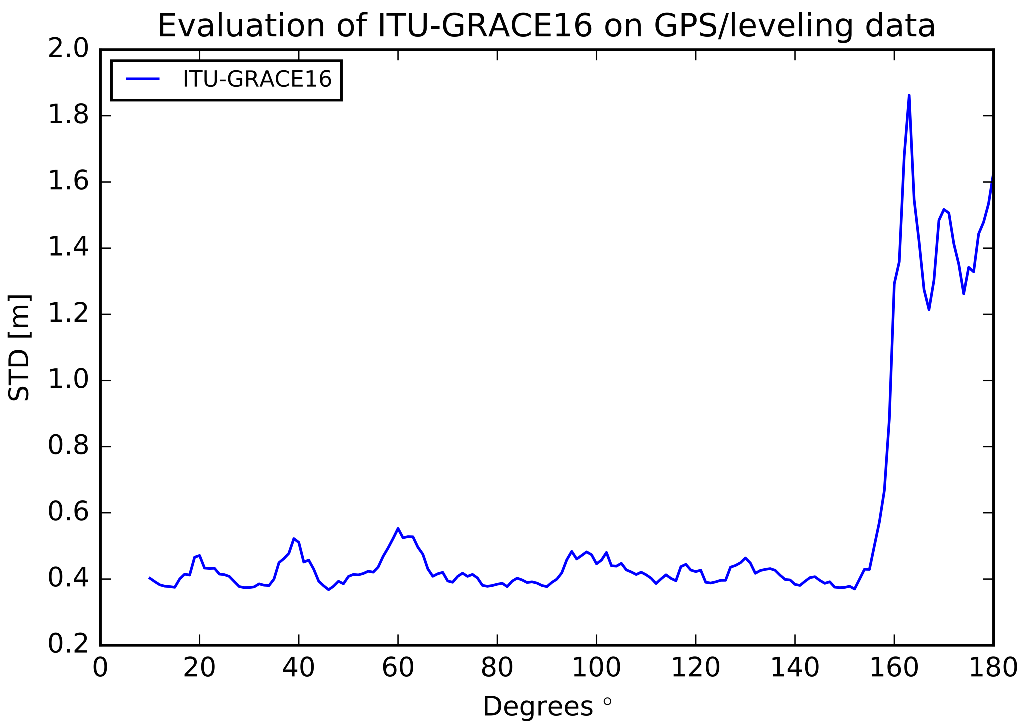


Figure 5.10: Evaluation of ITU_GGC16 on GPS/leveling data

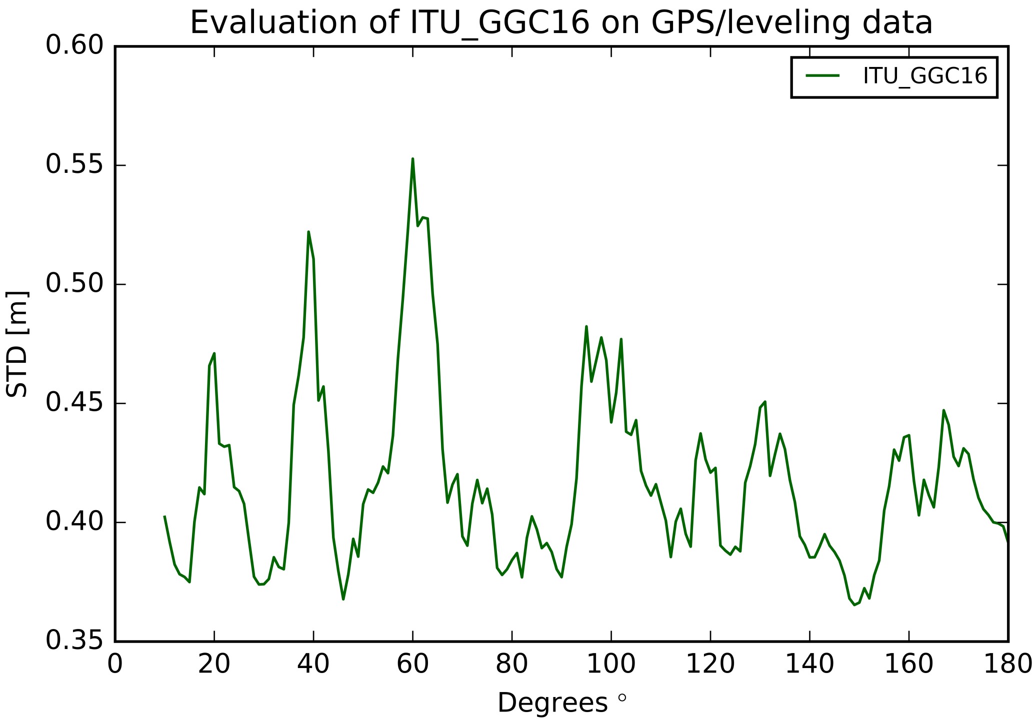


Figure 5.11: Evaluation of GECO on GPS/leveling data

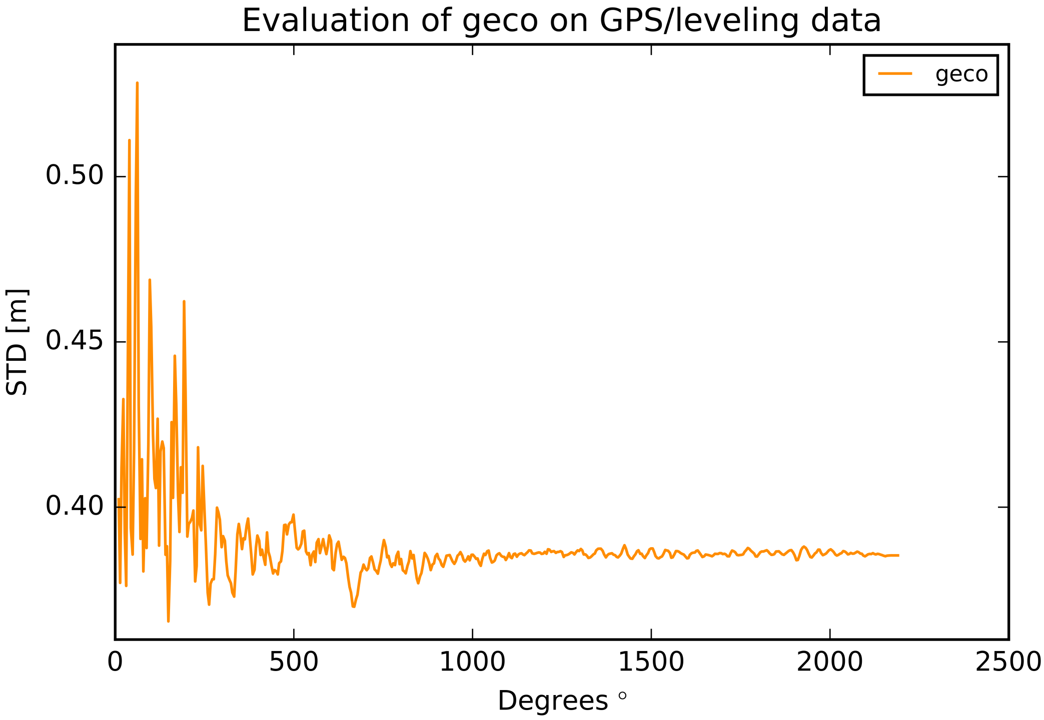


FIGURE 5.12: Evaluation EIGEN-6c4 on GPS/leveling data

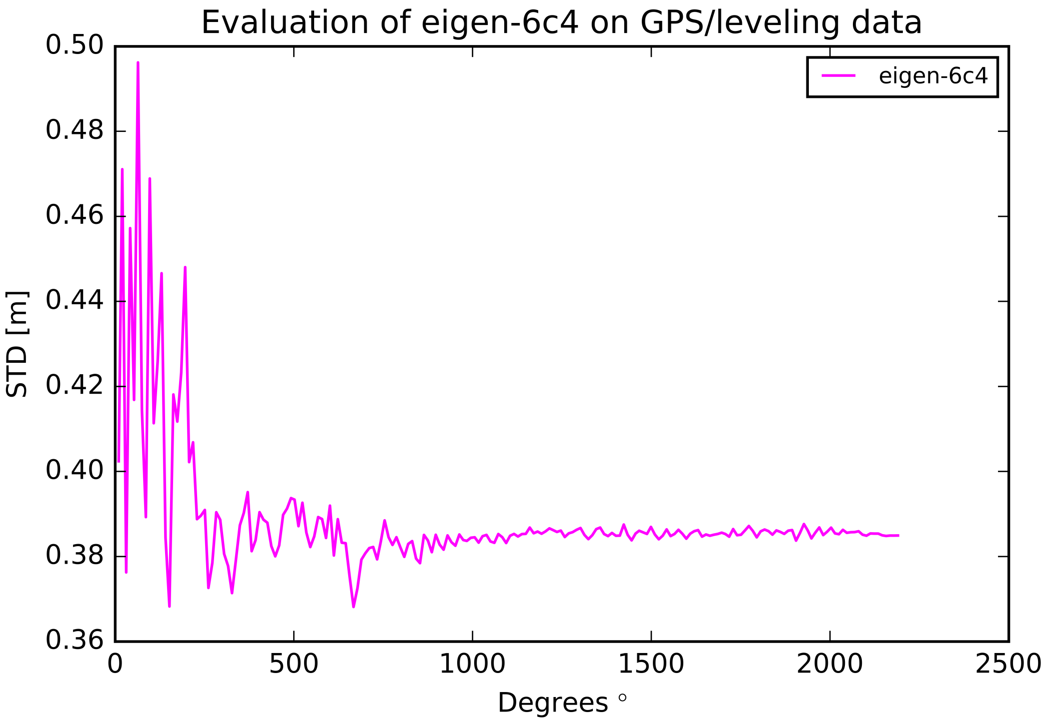


FIGURE 5.13: Evaluation of higher degrees of EGM2008, EIGEN-6C4 and GECO

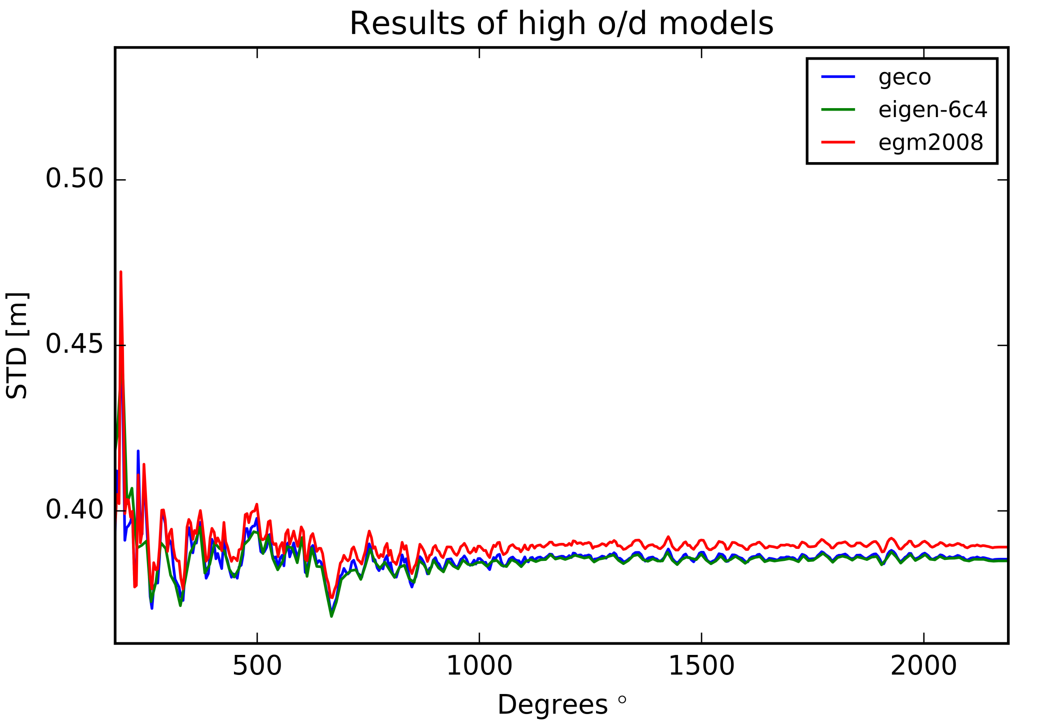
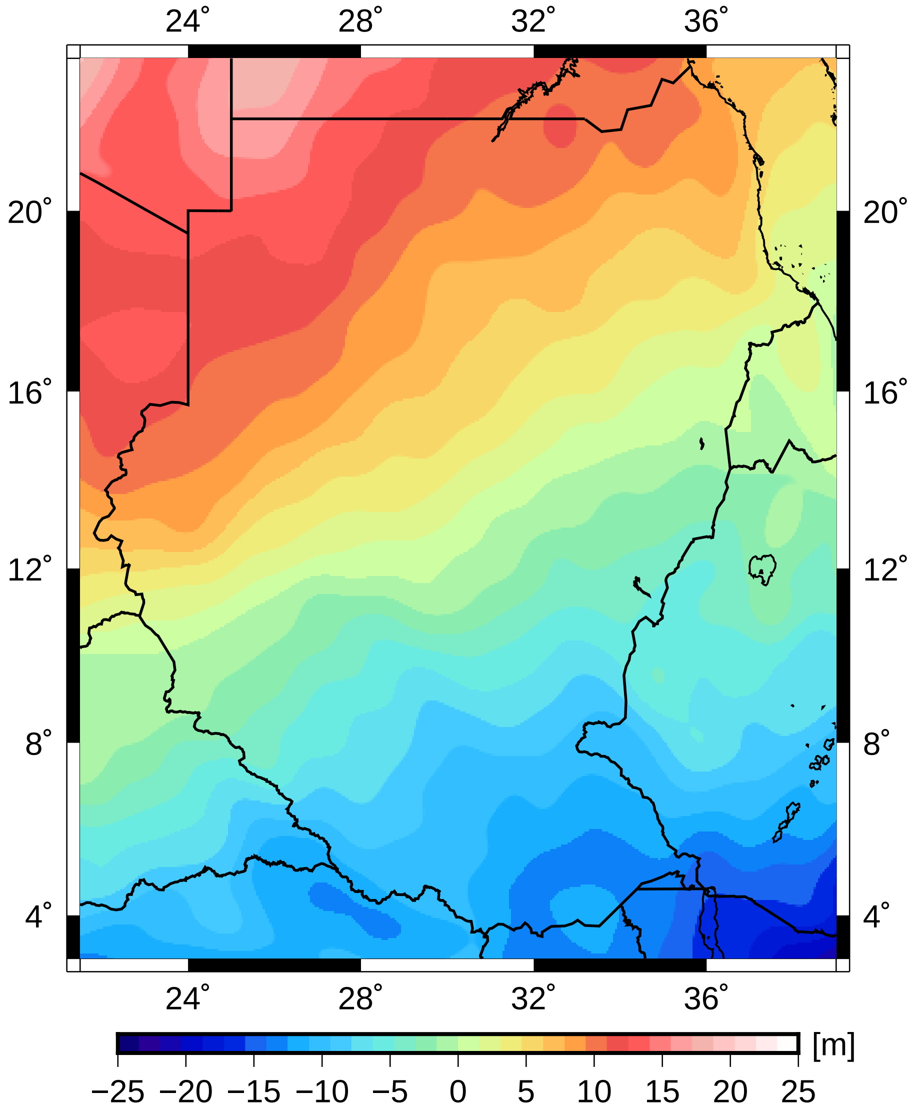


FIGURE 5.14: New Geoid for Sudan based on ITU_GGC16



Chapter 6

Conclusion and Recommendations

In this study we attempted to evaluate recent GRACE/GOCE models on Sudan. Our aim of this study was never to develop a new datum for Sudan, but rather to exhaustively try different models on our study area. Interestingly, our results–even without any corrections–are comparable to that reported by (Abdalla, 2009, Godah and Krynski, 2015).

- We begun our study by collecting terrestrial data for our study area
- GeoidApp: A software application that is mainly developed to help researchers evaluate different degrees of the model. The whole process is fully automated.
- The whole process of visualization, statistical analysis, reports are all handled within GeoidApp, and stored in their corresponding directories.

6.1 Recommendations

We believe that GGMs are the way to go for gravity field measurements. The traditional techniques for measuring gravity field has reached their intrinsic limitations. GGM provide global, regular and dense datasets of high and homogeneous quality. The use of GGM will lead, eventually, to the replace of expensive and time consuming spririt leveling, with the new fast, very cheap GPS/leveling. It will also contribute to the goal of unifying the regional datums.
The availability of the data is a huge problem, in terms of the large un-surveyed areas, and also the access of the data. We have contacted with GETECH to get a copy of their

data about Sudan, but they did not respond (as for the time we are writing this). We highly recommend to establish new GPS/leveling networks, that has more density and well distributed among Sudan. We do not recommend to use the dataset from (Adam, 1967) as it shows huge error compared to all of our models.

We, the authors of GeoidApp, are very welcome to the use of our application in any research. We believe that our work will help researcher around the world to experiment with many different degrees of their GGMs without the need to manually tune any thing. We also highly recommend researchers to work on "the unified datum"–geodetic main problem–which can only be solved by the means of GGM. The cost of GPS leveling, and their speed should encourage researchers and government agencies to work even harder towards 1-cm precision model.

We would like to acknowledge the authors of NumPy Walt et al. (2011), SciPy McKinney (2010*a*), Pandas McKinney (2010*b*), Bokeh Team (2014), John W. Eaton and RikWehbring (2015), and Perez and Granger (2007).

Appendix A

Our Datasets

In this appendix we show the complete datasets that we used in our study. Table A.1 shows the full dataset of Adam (1967). The NaN value indicates a missing value.

A.1 Other models for Sudan using the rest of our GGM

We show different models for Sudan using our GGM. While ITU_GGC16 has scored the best result on our local data, the lack of the data, and the closeness of the results among the selected models make any of them a good candidate. The figures below are all purely from satellite observations, i.e., we did not use any local data to produce them. All of these models were evaluated using the maximum degree.
Figure A.1 clearly shows the lack of our local data, and in the inhomogeneity in their distribution.

On Fig. A.6, it is clear that ITU_GRACE16 has a different behavior than other models. They Akyilmaz et al. (2016b) reported that ITU_GRACE16 is not regularized or constrained in any way, the errors increase with degree.Hence, they don't recommend to use it in any work without smoothing.

Table A.1: Complete astrogeodetic data of Adam (1967)

latitutde	longitude	geoid height
22.169	31.489	10.000
20.136	30.662	9.556
18.473	30.840	10.198
17.051	31.272	10.199
14.496	30.251	14.180
13.832	29.654	15.833
13.233	30.110	15.354
12.866	29.956	15.647
12.776	30.853	14.453
11.604	30.411	15.387
10.823	31.113	14.851
10.279	30.980	15.346
10.247	31.067	15.188
8.619	31.402	15.332
7.071	31.319	13.970
5.805	31.783	11.396
4.701	31.779	14.180
14.840	34.088	12.449
15.511	36.253	12.683
16.615	36.254	12.745
17.593	36.038	11.665
18.710	37.102	6.847
19.501	37.260	6.133
20.337	37.107	7.051
21.087	37.107	6.440
22.314	36.513	9.930
22.541	35.894	12.432
15.526	32.582	NaN
18.172	37.559	7.384
18.114	36.312	13.107
17.388	37.355	11.026
15.721	31.761	11.939
16.027	31.817	11.713
15.716	32.414	11.318
16.242	32.681	10.689
16.164	32.756	10.631
14.858	34.963	12.301
13.404	29.368	16.498
13.273	28.811	17.360
13.239	28.424	17.432
13.368	27.976	17.699
13.439	27.602	18.221
13.664	26.446	19.149
13.660	25.768	20.917
13.566	25.221	22.669
13.417	24.733	24.250

Table A.2: Full GPS/leveling dataset of Khartoum (Abdalla, 2009)

latitude	longitude	geoid height
16.172	32.153	3.573
15.822	32.313	2.998
15.890	32.684	3.619
16.077	32.722	3.119
15.811	33.087	2.175
15.809	32.900	2.285
15.613	32.808	2.020
16.352	31.965	4.174
15.848	32.517	3.078
16.119	32.531	2.263
15.992	32.340	3.210
15.810	32.154	3.210
16.113	31.965	3.197
15.993	32.901	3.254
15.988	32.144	3.447
15.470	33.080	2.297
15.651	32.388	2.810
15.721	32.514	2.655
16.139	32.638	2.420
15.607	32.518	2.673
15.599	32.108	2.538
15.524	32.577	2.599
15.259	32.555	2.231
15.340	32.394	2.692

Figure A.1: Points distribution of Adam (1967)

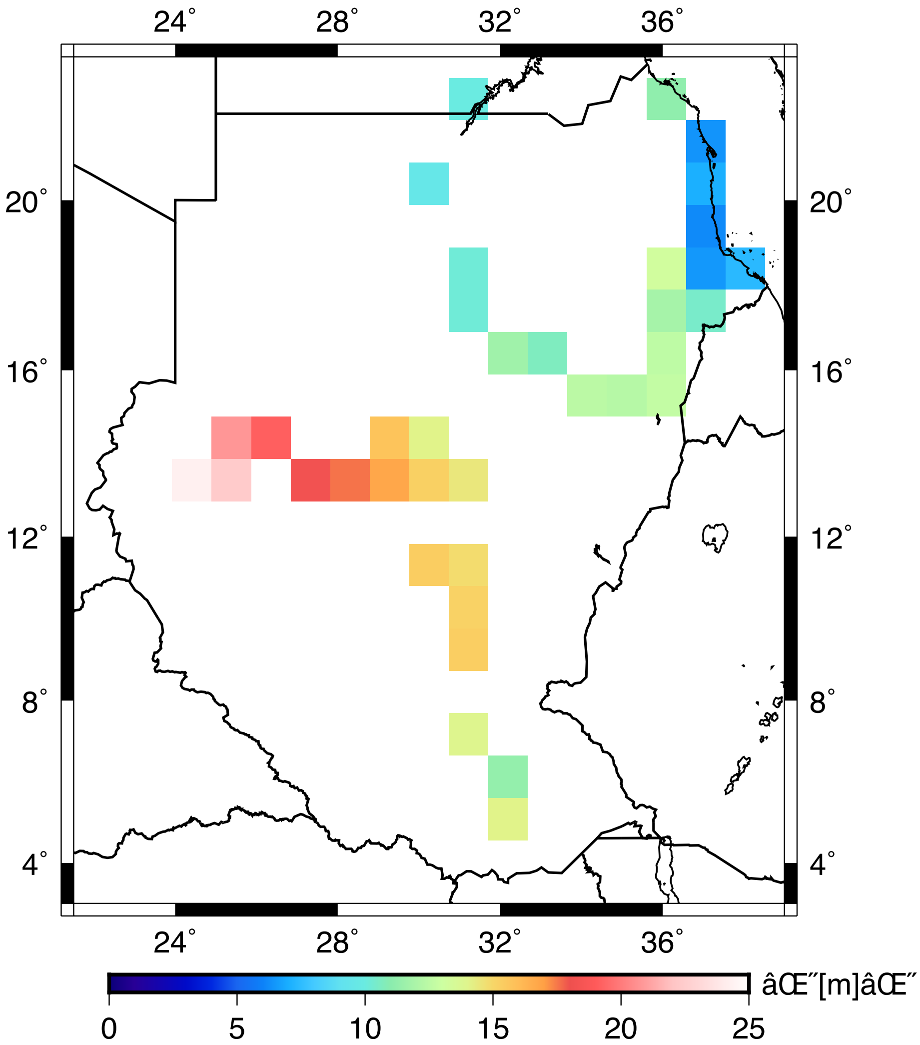


FIGURE A.2: Points distribution of Abdalla (2009)

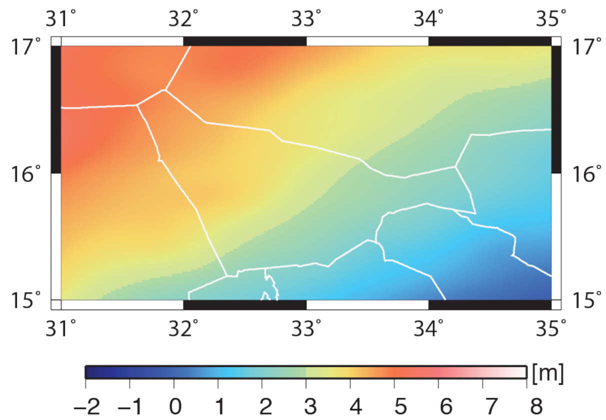


FIGURE A.3: New Geoid for Sudan based on EGM2008

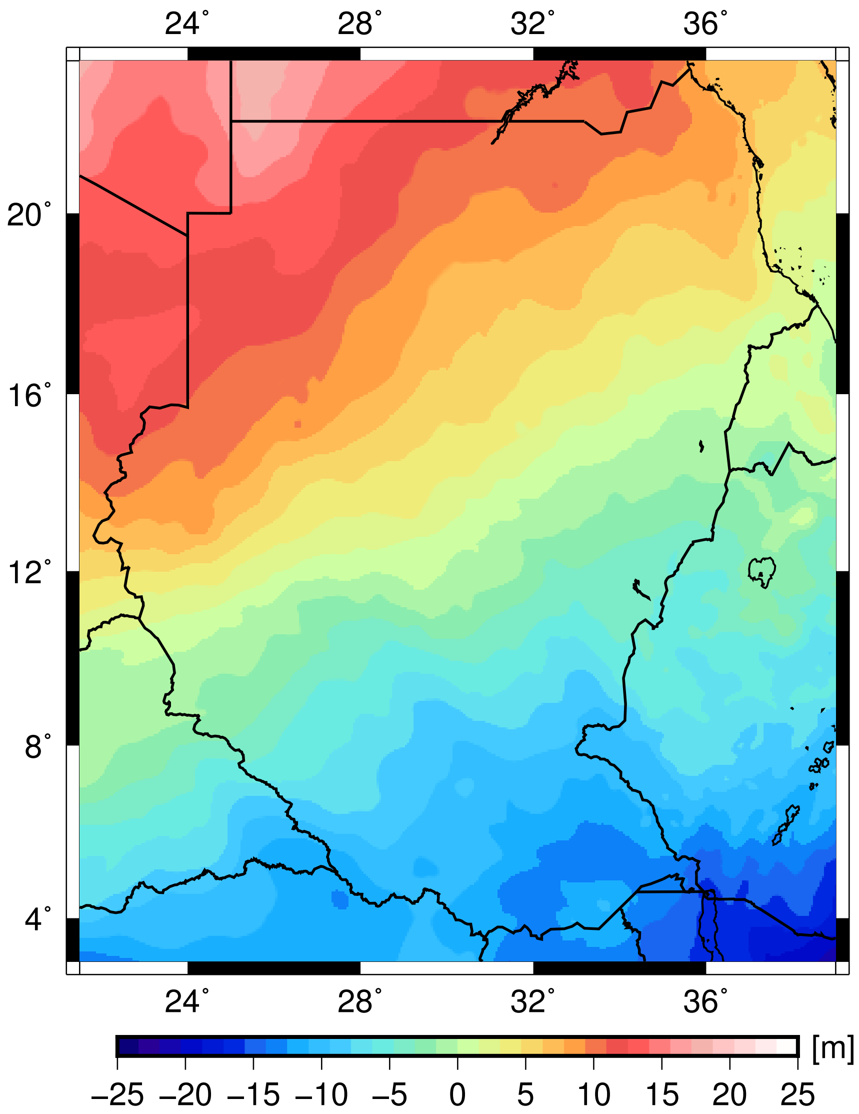


FIGURE A.4: New Geoid for Sudan based on EIGEN-6C4

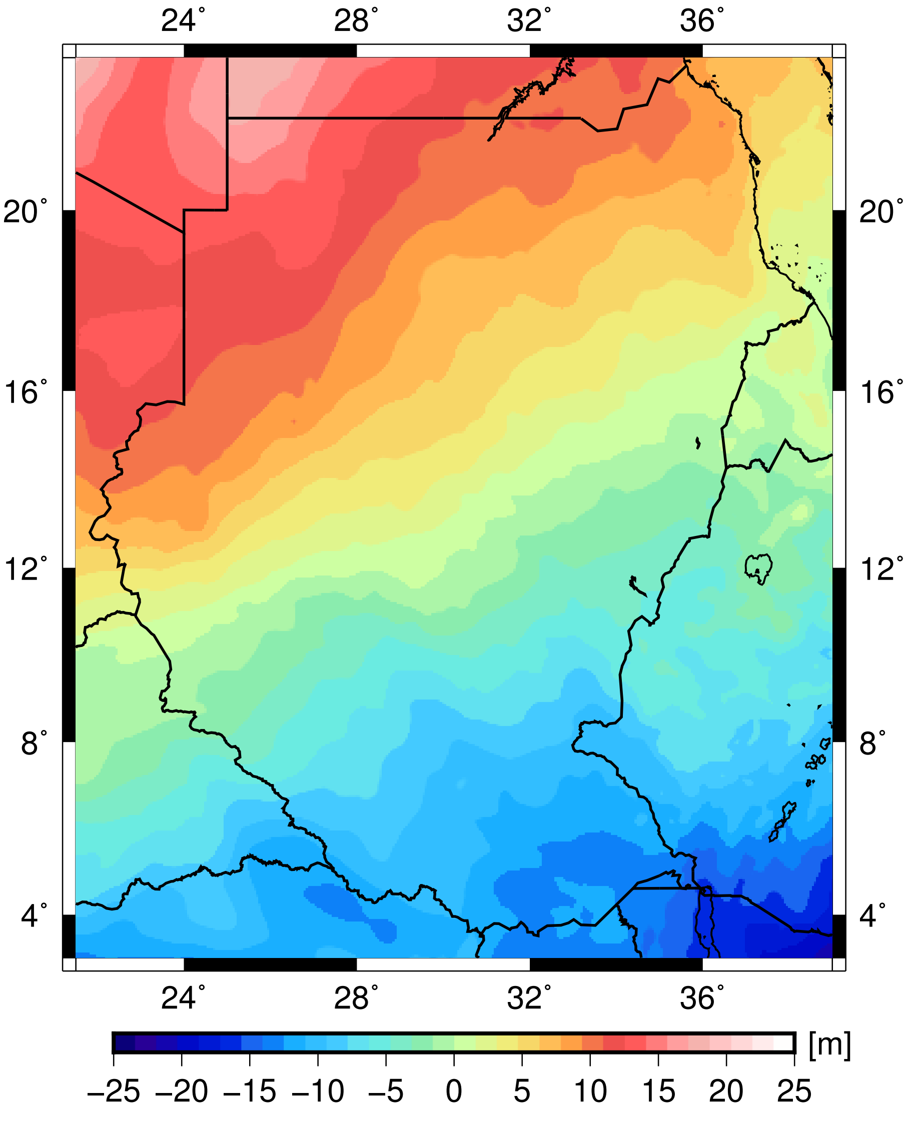


Figure A.5: New Geoid for Sudan based on GECO

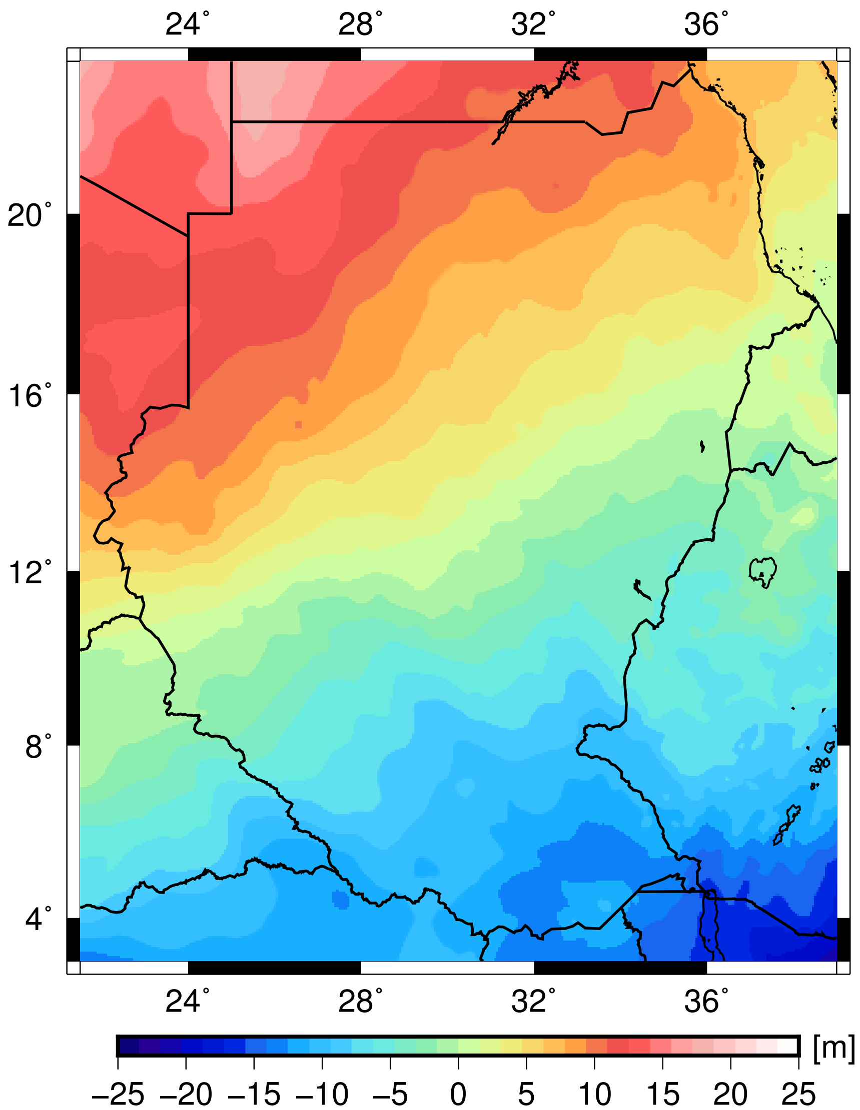
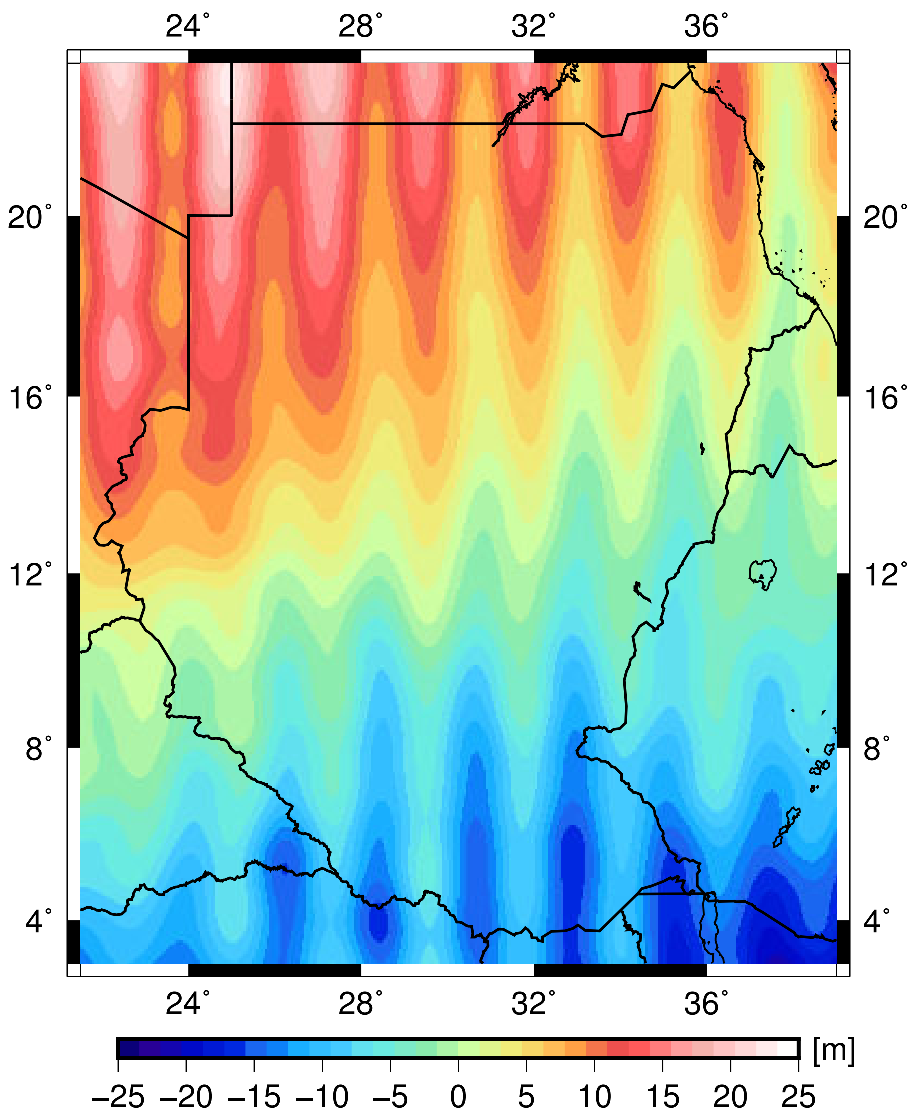


Figure A.6: New Geoid for Sudan based on ITU_GRACE16



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