Implicit-Explicit (IMEX) temporal integrators

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Here are some standard 2nd order IMEX schemes for solving ODEs of the form:

$$u'(t) = \underbrace{f(u,t)}_{\substack{\text{not stiff} \\ \text{(explicit)} \\ \text{advection}}} + \underbrace{g(u,t)}_{\substack{\text{stiff} \\ \text{(implicit)} \\ \text{diffusion}}}$$

Often $g(u,t) = Lu$ is linear to avoid non-linear equations in implicit solve

Multistep AB2 + CN

Adams - Bashorth + Crank-Nicolson (implicit midpoint)

$$U^{n+1} = U^n + \frac{\tau}{2}\left(3f(U^n,t^n) - f(u^{n-1},t^{n-1})\right)$$

not L-stable

$$+ \frac{\tau}{2}\left(g(U^n,t^n) + g(\underset{\uparrow}{U}^{n+1},t^{n+1})\right)$$

implicit

Backwards Differentiation

Semi-implicit BDF = SBDF2

$$U^{n+1} = \frac{4}{3}U^n - \frac{1}{3}U^{n-1} + \frac{2}{3}\tau g(U^{n+1},t^{n+1})$$

$$+ \frac{2}{3}\tau\left[f(U^n,t^n) - f(U^{n-1},t^{n-1})\right]$$

BDF2 is L-stable (8.3.2 in Le Veque)
(good for diffusion)

Runge-Kutta IMEX L-stable

$$U^{n+1/2,*} = U^n + \tau\left(\left(1+\frac{\sqrt{2}}{2}\right) g\left(U^{n+1/2,*}, t^n + \frac{\tau}{2}\right) - \left(\frac{1+\sqrt{2}}{2}\right) g\left(U^n, t^n\right)\right) + \frac{\tau}{2} f\left(U^n, t^n\right)$$

(Predictor to midpoint)

$$U^{n+1} = U^n + \left(\left(1+\frac{\sqrt{2}}{2}\right) g\left(U^{n+1}, t^{n+1}\right) - \left(1+\sqrt{2}\right) g\left(U^{n+1/2,*}, t^n + \frac{\tau}{2}\right) + \left(1+\frac{\sqrt{2}}{2}\right) g\left(U^n, t^n\right)\right)$$

(corrector)

explicit midpoint $\left\{ + \tau f\left(U^{n+1/2,*}, t^n + \tau/2\right)\right.$