Implicit – Explicit (IMEX)

temporal integrators

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Here are some standard <mark>2nd order</mark> IMEX schemes for solving ODEs of the form:

$$u'(t) = \underbrace{f(u,t)}_{\substack{\text{not stiff} \\ \text{(explicit)} \\ \color{red}{\text{advection}}}} + \underbrace{g(u,t)}_{\substack{\text{stiff} \\ \text{(implicit)} \\ \color{red}{\text{diffusion}}}}$$

Often $g(u,t) = Lu$ is linear to avoid non-linear equations in implicit solve

$$\textcircled{1}$$

Multistep   AB2 + CN

Adams - Bashorh + Crauh- Nicolson
(implicit midpoint)

$$U^{n+1} = U^n + \frac{\tau}{2}\left(3f(U^n,t^n) - f(u^{n-1},t^{n-1})\right)$$

$$\underset{\text{L-stable}}{\underline{\underline{\text{not}}}} \quad + \frac{\tau}{2}\left(g(U^n,t^n) + g(U^{n+1},t^{n+1})\right)$$

$$\underset{\text{implicit}}{\uparrow}$$

Backwards Differentiation

Semi-implicit BDF = SBDF2

$$U^{n+1} = \frac{4}{3}U^n - \frac{1}{3}U^{n-1} + \frac{2}{3}\tau g(U^{n+1},t^{n+1})$$

$$+ \frac{2}{3}\tau\left[f(U^n,t^n) - f(U^{n-1},t^{n-1})\right]$$

BDF2 is L-stable $\underset{\text{(good for diffusion)}}{}$ $\begin{pmatrix}8.3.2 \text{ in}\\ \text{Le Veque}\end{pmatrix}$ ②

Runge–Kutta IMEX <mark>L-stable</mark>

$$U^{n+1/2,\ast} = U^n + \tau\left(\left(1 + \frac{\sqrt{2}}{2}\right)g\left(U^{n+1/2,\ast}, t^n + \frac{\tau}{2}\right)\right.$$

$$\left. - \left(\frac{1+\sqrt{2}}{2}\right)g\left(U^n, t^n\right)\right)$$

$$+ \frac{\tau}{2}f\left(U^n, t^n\right)$$

(Predictor to midpoint)

$$U^{n+1} = U^n + \left(\left(1 + \frac{\sqrt{2}}{2}\right)g\left(U^{n+1}, t^{n+1}\right)\right.$$

(corrector) $- (1+\sqrt{2})g\left(U^{n+1/2,\ast}, t^n + \frac{\tau}{2}\right)$

$$\left. + \left(1 + \frac{\sqrt{2}}{2}\right)g\left(U^n, t^n\right)\right)$$

explicit
midpoint $\left\{ + \tau f\left(U^{n+1/2,\ast}, t^n + \tau/2\right)\right.$ ③