Numerical PDEs

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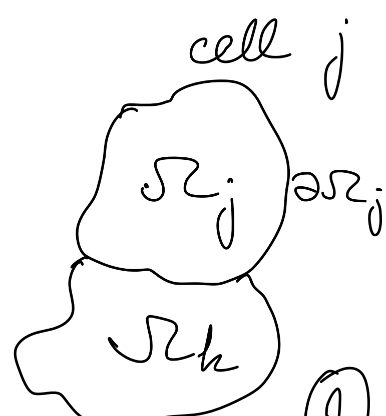
Finite Volume Methods

The bible on this topic is book "FVM for hyperbolic problems" R. LeVeque — freely available as PDF to you

Key idea: Break up domain into a grid of cells, and use as variables the average of u over each cell

$$\bar{u}_j = \frac{1}{|\Omega_j|} \int_{\Omega_j} u \, dr$$



Conservation law

$$\int_{\Omega_j} \frac{\partial u}{\partial t}\, dr = -\int_{\Omega_j} (\nabla \cdot \vec{f}\,)\, dr$$

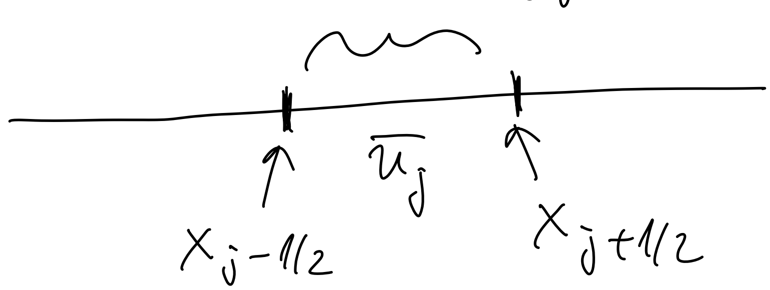
$$|\Omega_j| \frac{d}{dt} \bar{u}_j = -\int_{\partial \Omega_j} \vec{f}\, dA$$

$$\frac{d\bar{u}_j}{dt} = -\frac{1}{|\Omega_j|} \int_{\partial \Omega_j} \vec{f}\, dA$$

which is a system of ODEs

In 1D:

h (grid spacing)



In <u>1D adv-diff</u>:

$$h \cdot \frac{d}{dt} \bar{u}_j = -\left(f_{j+1/2} - f_{j-1/2} \right) =$$

$$- \left[a(x_{j+1/2}) u(x_{j+1/2}) - a(x_{j-1/2}) u(x_{j-1/2}) \right]$$
$$+ \left[d(x_{j+1/2}) u_x(x_{j+1/2}) - d(x_{j-1/2}) u_x(x_{j-1/2}) \right]$$

This is a weak form of PDE and not (yet) a discretization, i.e., it is exact.

To make it into a scheme we need to figure out the fluxes in terms of the cell averages.