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Written exam at Numerical Calculus

a) $f(x) = 2x^2 - 10x + 8$

$f(0) = 2\cdot 0^2 - 10\cdot 0 + 8 = 8 > 0$
$f(3) = 2\cdot 3^2 - 10\cdot 3 + 8 = -4 < 0$
f is continuous
$\Rightarrow$ there $\exists\, x_0 \in (0,3)$ s.t. $f(x_0) = 0$

First Iteration

$a_0 = 0,\ b_0 = 3,\ c_0 = \dfrac{a_0\cdot f(b_0) - b_0\cdot f(a_0)}{f(b_0) - f(a_0)} = \dfrac{-3\cdot 8}{-4-8} = \dfrac{-24}{-12} = 2$

$f(c_0) = f(2) = 2\cdot 2^2 - 10\cdot 2 + 8 = -4$

$f(a_0)\cdot f(c_0) < 0 \Rightarrow a_1 = a_0,\ b_1 = c_0$

Second Iteration

$a_1 = 0,\ b_1 = 2,\ c_1 = \dfrac{a_1\cdot f(b_1) - b_1\cdot f(a_1)}{f(b_1) - f(a_1)} = \dfrac{-2\cdot 8}{-4-8} = \dfrac{16}{12} = \dfrac{4}{3}$

$f(c_1) = f\left(\frac{4}{3}\right) = 2\cdot\left(\frac{4}{3}\right)^2 - 10\cdot\frac{4}{3} + 8 = 2\cdot\frac{16}{9} - \frac{40}{3} + 8 = \dfrac{32 - 120 + 72}{9} = -\dfrac{16}{9}$

$f(a_1)\cdot f(c_1) < 0 \Rightarrow a_2 = a_1,\ b_2 = c_1$

b) We need to find the best fit for the line $y = ax + b$, where $x_0 = 0,\ x_1 = 1,\ x_2 = 3,\ f(x_0) = 2,\ f(x_1) = -1,\ f(x_2) = 11$ and $m = 2$.

The formulas for a and b are:

$$a = \frac{(m+1)\sum_{i=0}^{m} x_i f(x_i) - \sum_{i=0}^{m} x_i \sum_{i=0}^{m} f(x_i)}{(m+1)\cdot\sum_{i=0}^{m} x_i^2 - \left(\sum_{i=0}^{m} x_i\right)^2} = \frac{3\cdot\sum_{i=0}^{2} x_i f(x_i) - \sum_{i=0}^{2} x_i \sum_{i=0}^{2} f(x_i)}{3\sum_{i=0}^{2} x_i^2 - \left(\sum_{i=0}^{2} x_i\right)^2} =$$

$$= \frac{3\cdot(0\cdot 2+1\cdot(-1)+3\cdot 11)-(0+1+3)\cdot(2+(-1)+11)}{3\cdot(0^2+1^2+3^2)-(0+1+3)^2} = \frac{3\cdot 32-4\cdot 12}{3\cdot 10-4^2} = \frac{48}{14} = \frac{24}{7}$$

and

$$b = \frac{\sum_{i=0}^{m} x_i^2 \sum_{i=0}^{m} f(x_i) - \sum_{i=0}^{m} x_i\cdot f(x_i) \sum_{i=0}^{m} x_i}{(m+1)\sum_{i=0}^{m} x_i^2 - \left(\sum_{i=0}^{m} x_i\right)^2} = \frac{\sum_{i=0}^{2} x_i^2 \sum_{i=0}^{2} f(x_i) - \sum_{i=0}^{2} x_i\cdot f(x_i)\cdot \sum_{i=0}^{2} x_i}{3\cdot \sum_{i=0}^{2} x_i^2 - \left(\sum_{i=0}^{2} x_i\right)^2}$$

$$= \frac{(0^2+1^2+3^2)\cdot(2+(-1)+11) - (0\cdot 2+1\cdot(-1)+3\cdot 11)(0+1+3)}{3\cdot(0^2+1^2+3^2)-(0+1+3)^2} = \frac{10\cdot 12-32\cdot 4}{3\cdot 10-16} = \frac{-8}{14} = -\frac{4}{7}$$

So, the line that best fits our points is $y = \frac{24}{7}\cdot x - \frac{4}{7}$