Math 104-004 **Answers to Quiz # 6b** **12 Oct 07**

4 1. Find the derivative of $h(x) = \ln\big(\sqrt{e^{5x} + 4x}\big)$.

HINT: Simplify the function before differentiating.

Solution. This question is similar to questions 8 and 10 on page 260.
First, rewrite $h(x)$ as

$$h(x) = \ln\big((e^{5x} + 4x)^{1/2}\big) = \frac{1}{2}\ln\big(e^{5x} + 4x\big)$$

using the property $\ln(a^r) = r\ln(a)$.
Using the rule for the derivative of $\ln(f(x))$ and then the rule for the derivative of $e^{f(x)}$, we get

$$h'(x) = \frac{1}{2}\frac{e^{5x}5 + 4}{(e^{5x} + 4x)} = \frac{e^{5x}5 + 4}{2(e^{5x} + 4x)}.$$

If you don't simplify the function first, then using the rule of the derivative of $\ln(f(x))$ and then the chain rule, and then the rule for the derivative of $e^{f(x)}$, we get

$$\begin{aligned} h'(x) &= \frac{\frac{1}{2}\big(e^{5x} + 4x\big)^{-1/2} \cdot (e^{5x}5 + 4)}{(e^{5x} + 4x)^{1/2}} \\ &= \frac{5e^{5x} + 4}{2(e^{5x} + 4x)^{1/2} \cdot (e^{5x} + 4x)^{1/2}} \\ &= \frac{5e^{5x} + 4}{2(e^{5x} + 4x)}. \end{aligned}$$

6 2. For the function $f(x) = x^3 + 6x^2 - 36x + 2$, find its critical numbers, the open intervals where it is increasing, and the open intervals where it is decreasing.

Solution. The derivative of $f(x)$ is $f'(x) = 3x^2 + 12x - 36 = 3(x^2 + 4x - 12) = 3(x + 6)(x - 2)$. Thus, the critical numbers of $f(x)$ are -6 and 2.
By drawing the number line and choosing points in each of the intervals $(-\infty, -6)$, $(-6, 2)$ and $(2, +\infty)$, we can see if the function is increasing or decreasing.
For $(-\infty, -6)$, we pick $x = -7$ and get $f'(-7) = 3(-7+6)(-7-2) = 3(-1)(-9) > 0$. For $(-6, 2)$, we pick $x = 0$ and get $f'(0) = 3(6)(-2) < 0$. For $(2, +\infty)$, we pick $x = 3$ and $f'(4) = 3(3 + 5)(3 - 2) = 3 \cdot 8 \cdot 1 > 0$.
By the test for increasing/decreasing, $f(x)$ is increasing on $(-\infty, -6)$ and $(2, +\infty)$, while it is decreasing on $(-6, 2)$.