

100 Points, 50 minutes; Show all your work!

This is the actual test I used in Feb 2005; Questions 5(b) isn't relevant now.

- 30 1. Showing all steps, evaluate each of the following integrals:
- $$(a) \int x\sqrt{3x+1} dx, \quad (b) \int \frac{1}{3+12x^2} dx, \quad (c) \int_1^3 \frac{x}{4+x^2} dx$$
- 22 2. (a) A conical tank (with pointy end down) has a depth of 30 feet and width across the top of 20 feet. It contains water to a depth of 10 feet. Set up (**but do not evaluate**) an integral for the work to **fill up** the tank from a water source 20 feet below the bottom of the tank. (As usual, the pump is at the bottom of the tank.)
- (b) A dam is in the shape of a trapezoid with height 100 feet. The width at the top is 40 feet and the width at the bottom is 60 feet. Set up (**but do not evaluate**) an integral for the maximum hydrostatic force the wall would need to withstand.
- 18 3. Let R be the region bounded by $y = x^3$ and $y = 4x$, $x > 0$. Sketch the region R and find the volume of the solid formed by revolving R about the axis $x = -1$.
- 15 4. A cookie at room temperature (70°F) is placed in a 350°F oven. After 2 minutes, the temperature of the cookie is 85°F . The cookie is baked when it reaches a temperature of 160°F . When will the cookie be baked?
- 15 5. (a) Evaluate $\sin(\tan^{-1}(x))$.
- (b) If $f(x) = 2x^3 + 2x$, find the derivative of f^{-1} at $a = 4$. (You don't need to check that f^{-1} exists.)

Answers: 1 (a) $\frac{2}{45}(3x+1)^{5/2} - \frac{2}{27}(3x+1)^{3/2} + C$ (b) $\frac{1}{6} \arctan(2x) + C$, (c) $\frac{\ln 13 - \ln 5}{2} = \ln(\sqrt{13/5})$. 2 (a) $\int_{10}^{30} 62.4\pi \left(\frac{x}{3}\right)^2 (20+x) dx$, (b) $\int_0^{100} 62.4x \left(40 + \frac{x}{5}\right) dx$. 3. $248\pi/15$
 4. 14.08 minutes 5. (a) $x/\sqrt{1+x^2}$ (b) The question isn't relevant, but $1/8$.