9 July 09

1. Let $\alpha = \sum f_i \, dx_i$ be a closed C^∞ 1-form on $\mathbb{R}^n$. Define a function g on $\mathbb{R}^n$ by setting $g(\mathbf{x})$ equal to

$$\int_0^{x_1} f_1(t, x_2, x_3, \ldots, x_n) \, dt + \int_0^{x_2} f_2(0, t, x_3, \ldots, x_n) \, dt$$

$$+ \int_0^{x_3} f_3(0, 0, t, x_4, \ldots, x_n) \, dt + \cdots + \int_0^{x_n} f_n(0, \ldots, 0, t) \, dt.$$

   Show that $dg = \alpha$.

   HINT: Don't forget to use $d\alpha = 0$, use the fundamental theorem of calculus, use differentiation under the integral sign, and the chain rule.