Math 496-004 **Assignment # 9** **29 Oct 08**

Due: Nov 5th

1. Do problem II.7.10 in Edwards (page 141), as corrected below. That is, classify the critical point $(-1, \pi/2, \pi)$ of $f(x, y, z) = x \sin z - z \sin y$.

2. Do problem 8.82 (b) in Schaums (page 205). That is, Examine $w = x^2 + y^2 + z^2 - 6xy + 8xz - 10yz$ for maxima and minima.

3. Do problem II.8.4 in Edwards (page 158). That is, Consider $f : \mathbb{R}^3 \to \mathbb{R}$ given by

$$f(x, y, z) = x^2 + 4y^2 + z^2 + 2xz + (x^2 + y^2 + z^2) \cos xyz.$$

   You may assume $\mathbf{0} = (0, 0, 0)$ is a critical point of f.

   (a) Show that $q(x, y, z) = 2x^2 + 5y^2 + 2z^2 + 2xz$ is the quadratic form of f at $\mathbf{0}$, (that is, $d^{(2)} f_{\mathbf{0}}$) by substituting the expansion $\cos t = 1 - t^2/2 + O(t^4)$, collecting all second degree terms and *verifying* the appropriate condition on the remainder $R(x, y, z)$. State the uniqueness theorem you apply.

   (b) Write down the symmetric 3×3 matrix A such that

$$q(x, y, z) = \begin{bmatrix} x & y & z \end{bmatrix} A \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

   By calculating appropriate determinants of submatrices of A, determine the behavior of f at $\mathbf{0}$.

   (c) Find the eigenvalues of q. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be the associated eigenvectors (do not solve for them). Find the matrix of q with respect to this basis [Edwards says something more complicated, but this is what he means]. Then give a geometric description of the surface $2x^2 + 5y^2 + 2z^2 + 2xz = 1$.