Math 826 **Assignment # 1** **16 Jan 08**

Due: Wednesday, January 23rd

1. Do Exercise 6.7.D in the handout.
2. Do Exercise 6.7.H in the handout.
3. For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x, i.e., the floor function, and let $((x)) = x - [x]$.

   (a) If f is continuous on $[a, b]$, show that $\displaystyle\sum_{a<n\leq b} f(n) = \int_a^b f(x)\, d[x]$. (Of course, n must be an integer.)

   (b) Prove that if f' is C^1 on $[a, b]$, then

   $$\sum_{a<n\leq b} f(n) = \int_a^b f(x)\, dx + \int_a^b f'(x)((x))\, dx + f(a)((a)) - f(b)((b)).$$

   Hint: Use integration by parts.

4. (June 2003 Qual)

   (a) Suppose f is a nonnegative, continuous function on $[a, b]$ and $\alpha : [a, b] \to \mathbb{R}$ is strictly increasing on $[a, b]$. Show that if $\int f\, d\alpha = 0$ then $f \equiv 0$ on $[a, b]$.

   (b) Let α be given by:

   $$\alpha(x) = \begin{cases} 0 & 0 \leq x < 1, \\ 2 & 1 \leq x < e, \\ 5 & e \leq x \leq \pi. \end{cases}$$

   Either directly or by the aid of a theorem, calculate the value of the integral $\int_0^\pi x^{100}\, d\alpha$, and show all the details in your calculation.

   Hint: For (b), use Exercise 6.7.H.