Math 107-250/350 Answers to Homework #10 (the last!) 13 Apr '07

. 5 1. Find the equation of the plane such that every point on the plane is equidistant from the points $P(1,1,0)$ and $Q(3,5,-2)$.

HINT: The midpoint of $\overline{PQ}$ is in this plane. Now find a normal vector. Recall that we found in class the equation of plane of all points equidistant from $(0,0,0)$ and $(0,3,0)$.

Solution. The midpoint of $\overline{PQ}$ is $R(2,3,-1)$.

We claim that if S is any point on the plane, then $SR \perp PQ$. Consider the triangles $\triangle SRQ$ and $\triangle SRP$ have a common side, $\overline{SR}$, and the sides $\overline{RP}$ and $\overline{RQ}$ are equal (since R is the midpoint of $\overline{PQ}$), and the sides $\overline{SP}$ and $\overline{SQ}$ are equal (since S is equidistant from P and Q). Thus, by Side-Side-Side congruance, the triangles are congruent. In particular the angles $\angle PRS$ and $\angle QRS$ are equal. Since $\angle PRS + \angle QRS$ equals $180°$, both $\angle PRS$ and $\angle QRS$ must be right angles.

This shows that $\overrightarrow{PQ}$ is a normal vector for the plane. Note $\overrightarrow{PQ} = 2\vec{\imath}+4\vec{\jmath}-2\vec{k}$. So the equation of the plane is

$$\begin{aligned} 2x+4y-2z &= 2\cdot 2+4\cdot 3-2\cdot(-1), \\ 2x+4y-2z &= 18 \\ x+2y-z &= 9 \end{aligned}$$