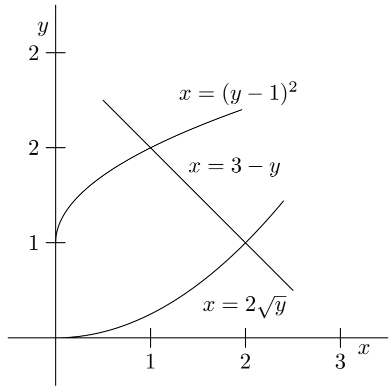
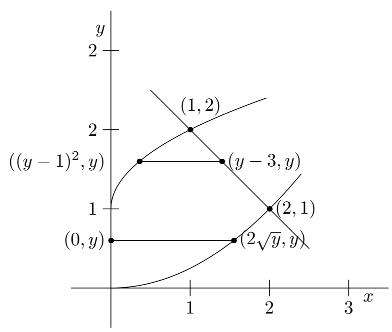
Math 106-350/550 Answers to Homework #10 10 Nov '06

. 10 1. Find the area of the region in the left quadrant bounded on the left by the y-axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y-1)^2$, and above right by the line $x = 3 - y$.



Solution. There are many ways to do this problem, but they all involve at least two integrals. We set this up using horizontal slices and two integrals.



Notice that the intersection points of $x = 2\sqrt{y}$ and $x = 3 - y$ is $(2,1)$ as can be checked by substituting the point into both curves. Similarly the intersection point of $x = (y-1)^2$ and $x = 3-y$ is $(1,2)$. For y from 0 to 1, the upper endpoint is $(2\sqrt{y}, y)$ and the lower endpoint is $(0, y)$, so the length is $2\sqrt{y}$. For y from 1 to 2, the upper endpoint is $(3-y, y)$ and the lower endpoint is $((y-1)^2, y)$, so the length is $3 - y - (y-1)^2 = 2 + y - y^2$. Thus, the total area is

$$\begin{aligned}
\int_0^1 2\sqrt{y}\,dy + \int_1^2 2 + y - y^2\,dy &= \frac{4}{3}y^{3/2}\Big|_0^1 + 2y + \frac{y^2}{2} - \frac{y^3}{3}\Big|_1^2 \\
&= \frac{4}{3} - 0 + \left(4 + \frac{4}{2} - \frac{8}{3}\right) - \left(2 + \frac{1}{2} - \frac{1}{3}\right) \\
&= 5/2
\end{aligned}$$