Math 106, Sect. 350 — Answers to Quiz # 4 — 14 Sept '06

. 4 1. Find the derivative of $f(x) = 2x^2 + \dfrac{1}{\sqrt{x}}$ and find the slope of the tangent line at $x = 1$. (Use the rules!)

Solution. We rewrite $f(x)$ as $2x^2 + x^{-1/2}$. Using the sum rule and the power rule,

$$f'(x) = 4x + \frac{-1}{2}x^{-3/2} = 4x - \frac{1}{2x^{3/2}}.$$

Then

$$f'(1) = 4 - \frac{1}{2} = \frac{7}{2}.$$

. 6 2. Using the definition of the derivative, differentiate $f(x) = \dfrac{x^2-1}{x}$.

Solution. Starting from the definition,

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{(x+h)^2-1}{x+h} - \frac{x^2-1}{x}}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{x^2+2xh+h^2-1}{x+h} - \frac{x^2-1}{x}\right)
\end{aligned}$$

The common denominator is $x(x+h)$, so we have

$$\begin{aligned}
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{(x^2+2xh+h^2-1)x - (x^2-1)(x+h)}{x(x+h)}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{x^3+2x^2h+h^2x-x-(x^3-x+hx^2-h)}{x(x+h)}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{hx^2+h^2x+h}{x(x+h)}\right) \\
&= \lim_{h\to 0} \frac{x^2+hx+1}{x(x+h)} \\
&= \frac{x^2+1}{x^2}
\end{aligned}$$