Math 107-250/350 - Analytic Geometry & Calculus I **2nd Semester, '06-'07**

Improper Integral Example

Evaluate $\displaystyle\int_{-1}^{1} |x|^{-1/2}\,dx$.

Because $|x|^{1/2}$ has a vertical asymptote at $x = 0$, we split the integral into two limits of proper integrals, namely

$$\int_{-1}^{1} |x|^{-1/2}\,dx = \lim_{d\to 0^-} \int_{-1}^{d} |x|^{-1/2}\,dx + \lim_{g\to 0^+} \int_{g}^{1} |x|^{-1/2}\,dx.$$

For the first term, we have

$$\begin{aligned}\lim_{d\to 0^-} \int_{-1}^{d} |x|^{-1/2}\,dx &= \lim_{d\to 0^-} \int_{-1}^{d} (-x)^{-1/2}\,dx \\ &= \lim_{d\to 0^-} -\left(2(-x)^{1/2}\Big|_{-1}^{d}\right) \\ &= -\left(\lim_{d\to 0^-} 2\sqrt{-d} - 2\right) = 2.\end{aligned}$$

For the second term, we have

$$\begin{aligned}\lim_{g\to 0^+} \int_{g}^{1} |x|^{-1/2}\,dx &= \lim_{g\to 0^+} \int_{g}^{1} x^{-1/2}\,dx \\ &= \lim_{g\to 0^+} 2x^{1/2}\Big|_{g}^{1} \\ &= -\lim_{g\to 0^+} 2 - 2\sqrt{g} = 2.\end{aligned}$$

Adding the terms together,

$$\int_{-1}^{1} |x|^{-1/2}\,dx = 4.$$