Math 107, Sect. 350 **Answers to Quiz # 6** **29 Feb '07**

. 5 1. Does $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n^2-4}}$ converge or diverge?

Solution. We apply the limit comparison test.
Let $a_n = \dfrac{3}{n\sqrt{n^2-4}}$, we expect a_n to behave like $\dfrac{3}{n\sqrt{n^2}} = \dfrac{3}{n^2}$, so we let $b_n = \dfrac{1}{n^2}$.
Since $\displaystyle\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n^2}{n\sqrt{n^2-4}} = \lim_{n\to\infty} \frac{3}{\sqrt{1-4/n^2}} = 3 > 0,$$

$\displaystyle\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n^2-4}}$ converges by the limit comparison test.

. 5 2. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)(n+3)}{(n+1)!}$ converge or diverge?

Solution. We apply the ratio test.
Let $a_n = \dfrac{(n+2)(n+3)}{(n+1)!}$, then $a_{n+1} = \dfrac{(n+3)(n+4)}{(n+2)!}$, and

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_{n+1}}{a_n} &= \lim_{n\to\infty} \frac{(n+3)(n+4)(n+1)!}{(n+2)(n+3)(n+2)!} \\
&= \lim_{n\to\infty} \frac{n+4}{(n+2)(n+2)} \\
&= \lim_{n\to\infty} \frac{n+4}{n^2+4n+4} \\
&= \lim_{n\to\infty} \frac{1+4/n}{n+4+4/n} = 0 < 1,
\end{aligned}$$

the series converges by the ratio test.