Math 107-250/350 - Analytic Geometry & Calculus I **2nd Semester, '06-'07**

Arctan Examples

Evaluate $\displaystyle\int \frac{2}{x^2+6x+15}\,dx$.

Notice that

$$x^2+6x+15 = x^2+6x+9+6 = (x+3)^2+6$$

so the quadratic denominator is irreducible. Then we rewrite the fraction as

$$\frac{2}{x^2+6x+15} = \frac{2/6}{\left(\frac{x+3}{\sqrt{6}}\right)^2+1}$$

Using this and the substitution $u=(x+3)/\sqrt{6}$, we obtain

$$\begin{aligned}\int \frac{2}{x^2+6x+15}\,dx &= \int \frac{1/3}{\left(\frac{x+3}{\sqrt{6}}\right)^2+1}\,dx \\ &= \int \frac{\sqrt{6}/3}{u^2+1}\,du \\ &= \frac{\sqrt{6}}{3}\arctan u + C \\ &= \frac{\sqrt{6}}{3}\arctan\left(\frac{x+3}{\sqrt{6}}\right)+C\end{aligned}$$

Evaluate $\displaystyle\int \frac{5}{x^2-4x+8}\,dx$.

Notice that

$$x^2-4x+8 = x^2-4x+4+4 = (x-2)^2+4$$

so the quadratic denominator is irreducible. Then we rewrite the fraction as

$$\frac{5}{x^2-4x+8} = \frac{5/4}{\left(\frac{x-2}{2}\right)^2+1}$$

Using this and the substitution $u=(x-2)/2$, we obtain

$$\begin{aligned}\int \frac{5}{x^2-4x+8}\,dx &= \int \frac{5/4}{\left(\frac{x-2}{2}\right)^2+1}\,dx \\ &= \int \frac{5/2}{u^2+1}\,du \\ &= \frac{5}{2}\arctan u + C \\ &= \frac{5}{2}\arctan\left(\frac{x-2}{2}\right)+C\end{aligned}$$