. 4

1. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converge absolutely, converge conditionally, or diverge?

   Solution.
   Let $u_n = \left|\dfrac{(-1)^{n+1}}{n^2}\right| = \dfrac{1}{n^2}$. Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with $p = 2$, it converges, so the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges absolutely.

. 6

2. Determine the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} e^{2n}x^n$.

   Solution. Let $a_n = e^{2n}x^n$. Then

$$\frac{|a_{n+1}|}{|a_n|} = \frac{e^{2n+2}|x|^{n+1}}{e^{2n}|x|^n} = e^2|x|.$$

   Thus,

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = e^2|x|$$

   Thus, the series converges (absolutely) for $e^2|x| < 1$ and diverges for $e^2|x| > 1$; that is, converges for $|x| < e^{-2}$ and diverges for $|x| > e^{-2}$.
   At $x = e^{-2}$, the series is $\displaystyle\sum_{n=1}^{\infty} e^{2n}e^{-2n} = \sum_{n=1}^{\infty} 1$, which diverges, as the terms do not go to zero. At $x = -e^{-2}$, the series is $\displaystyle\sum_{n=1}^{\infty} e^{2n}(-e^{-2})^n = \sum_{n=1}^{\infty}(-1)^n$, which diverges, as the terms do not go to zero.
   Thus, the interval of convergence is $(-e^{-2}, e^{-2})$.