### Clicker Questions

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#### Review of Limits

If a function y = f(x) is not defined at x = a, then

- a)  $\lim_{x\to a} f(x)$  cannot exist.
- b)  $\lim_{x\to a} f(x)$  could be zero.
- c)  $\lim_{x\to a} f(x)$  must approach  $\infty$
- d) None of the above are true.



#### Derivative and Limits

Recall that we defined the *instantaneous velocity* as the limit of the average rate of change of position.

Can the average rate of change on an interval  $\left[1,2\right]$  equal the instantaneous velocity at t=1?

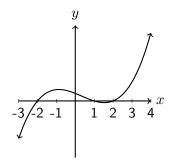
- a) Yes
- b) No



#### Order derivative values

For the function g(x) shown below, arrange the following numbers in increasing order.

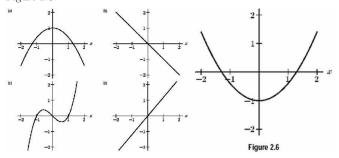
- a) 0
- **b)** g'(-2)
- c) g'(0)
- d) g'(1)
- e) g'(3)





### Derivative Function I

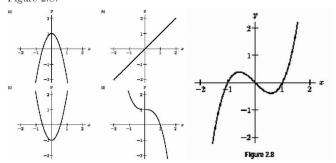
1. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.6?





### Derivative Function II

2. Which of the following graphs is the graph of the derivative of the function shown in Figure 2.8?





#### Derivative Rules I

13. If 
$$f(x) = x^2 + \frac{3}{x}$$
, then what is  $f'(x)$ ?

(a) 
$$2x - 3x^{-2}$$

(b) 
$$2x + 3x^{-1}$$

(c) 
$$2x - 3x^2$$

(d) 
$$x^2 - 3x^{-1}$$



#### Derivative Rules II

If 
$$f(x) = \pi^2$$
, then what is  $f'(x)$ ?

- (a)  $2\pi$
- (b)  $\pi^2$
- (c) 0
- (d) 2



#### Derivative Rules III

The derivative of the function  $f(x) = e^{x+2}$  is

- a)  $(x+2)e^{x+1}$
- b)  $e^2e^x$
- c)  $e^2$
- d) 0
- e) Cannot be determined from what we know



# The 10th derivative of $\sin x$ is

- (a)  $\sin x$
- (b)  $\cos x$
- (c)  $-\sin x$
- (d)  $-\cos x$



#### Product Rule I

1. 
$$\frac{d}{dx}(x^2e^x) =$$

- (a)  $2xe^x$
- (b)  $x^2 e^x$
- (c)  $2xe^x + x^2e^{x-1}$
- (d)  $2xe^x + x^2e^x$



#### Product Rule II

- 5. When differentiating a constant multiple of a function (like  $3x^2$ ) the Constant Multiple Rule tells us  $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$  and the Product Rule says  $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x) + f(x)\frac{d}{dx}c$ . Do these two rules agree?
  - (a) Yes, they agree, and I am very confident.
  - (b) Yes, they agree, but I am not very confident.
  - (c) No, they do not agree, but I am not very confident.
  - (d) No, they do not agree, and I am very confident.



### Quotient Rule I

11. 
$$\frac{d}{dt} \frac{\sqrt{t}}{t^2 + 1} =$$

$$(a) \quad \frac{\frac{1}{2}t^{-1/2} - 2t}{(t^2 + 1)^2}$$

$$(b) \quad \frac{\frac{1}{2}t^{-1/2}t^2 - 2t\sqrt{t}}{(t^2 + 1)^2}$$

$$(c) \quad \frac{\frac{1}{2}t^{-1/2}(t^2 + 1) - 2t\sqrt{t}}{(t^2 + 1)^2}$$

(d)  $\frac{t^{-1/2}(t^2+1)-2t\sqrt{t}}{(t^2+1)^2}$ 



#### Product Rule III

12. If 
$$f(3) = 2$$
,  $f'(3) = 4$ ,  $g(3) = 1$ ,  $g'(3) = 3$ , and  $h(x) = f(x)g(x)$ , then what is  $h'(3)$ ?

- (a) 2
- (b) 10
- (c) 11
- (d) 12
- (e) 14



#### Product Rule IV

14. If 
$$h = \frac{ab^2e^b}{c^3}$$
 then what is  $\frac{dh}{db}$ ?

$$(a) \frac{2abe^b}{c^3}$$

(b) 
$$\frac{2abe^b}{3c^2}$$

$$\left(\mathbf{c}\right) \ \frac{2abe^b + ab^2e^b}{c^3}$$

$$\left(\mathbf{d}\right) \ \frac{2abe^bc^3 - 3c^2ab^2e^b}{c^6}$$



#### Chain Rule I

3. 
$$\frac{d}{dx}\sqrt{1-x} =$$

(a) 
$$\frac{1}{2}(1-x)^{-1/2}$$

(b) 
$$-\frac{1}{2}(1-x)^{-1/2}$$

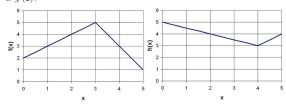
(c) 
$$-(1-x)^{-1/2}$$

(d) 
$$-\frac{1}{2}(1-x)^{1/2}$$



#### Chain Rule II

12. The functions f(x) and h(x) are plotted below. The function g(x) = f(h(x)). What is g'(2)?



- (a)  $g'(2) = -\frac{1}{2}$
- (b) g'(2) = 1
- (c) g'(2) = 3
- (d) g'(2) = 4
- (e) g'(2) is undefined



#### Inverse Functions I

The derivative of the function  $f(z) = \ln(z^2 + 1)$  is

- a)  $2z \ln(z^2 + 1)$
- b)  $\frac{2z}{z^2+1}$
- c)  $\frac{-1}{z^2+1}$
- d) None of the above



#### Inverse Functions II

10. If 
$$q = a^2 \ln(a^3 c \sin b + b^2 c)$$
, then  $\frac{dq}{db}$  is

$$(a) \frac{a^2}{a^3 c \sin b + b^2 c}$$

(b) 
$$\frac{a^5 c \cos b + 2a^2 bc}{a^3 c \sin b + b^2 c}$$

(c) 
$$\frac{a^3 c \cos b + 2bc}{a^3 c \sin b + b^2 c}$$

$$(d) \frac{6a^3\cos b + 4ab}{a^3c\sin b + b^2c}$$



### Implicit Differentiation

The derivative of the implicit function  $x^3 + y^3 - 9xy = 0$  is

a) 
$$\frac{dy}{dx} = \frac{-3x^2 - y}{y^2 - 3x}$$

$$b) \frac{dy}{dx} = \frac{x^2}{y^2 + 3x}$$

c) 
$$\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}$$

d) 
$$\frac{dy}{dx} = -\frac{x^2 + y}{y^2 + 3x}$$

e) None of the above



#### Local Linearization I

- 6. You wish to approximate  $\sqrt{9.3}$ . You know the equation of the line tangent to the graph of  $f(x) = \sqrt{x}$  where x = 9. What value do you put into the tangent line equation to approximate  $\sqrt{9.3}$ ?
  - (a)  $\sqrt{9.3}$
  - (b) 9
  - (c) 9.3
  - (d) 0.3



#### Local Linearization II

Suppose that f''(x) < 0 for x near a. Then the local linearization L(x) for y = f(x) at x = a is

- a) more than the true value (an over-estimate)
- b) less than the true value (an under-estimate)
- c) we cannot tell from the given information



#### Mean Value Theorem I

- 5. On a toll road a driver takes a time stamped toll-card from the starting booth and drives directly to the end of the toll section. After paying the required toll, the driver is surprised to receive a speeding ticket along with the toll receipt. Which of the following best describes the situation?
  - (a) The booth attendant does not have enough information to prove that the driver was speeding.
  - (b) The booth attendant can prove that the driver was speeding during his trip.
  - (c) The driver will get a ticket for a lower speed than his actual maximum speed.
  - (d) Both (b) and (c).



#### Mean Value Theorem II

- 8. Two racers start a race at the same moment and finish in a tie. Which of the following must be true?
  - (a) At some point during the race the two racers were not tied.
  - (b) The racers' speeds at the end of the race must have been exactly the same.
  - (c) The racers must have had the same speed at exactly the same time at some point in the race.
  - (d) The racers had to have the same speed at some moment, but not necessarily at exactly the same time.



### Derivatives and Graphing I

- 3. True or False: If x = p is not a local minimum or maximum of f, then x = p is not a critical point of f.
  - (a) True, and I am very confident
  - (b) True, but I am not very confident
  - (c) False, but I am not very confident
  - (d) False, and I am very confident



### Derivatives and Graphing II

- 6. Imagine that you are skydiving. The graph of your speed as a function of time from the time you jumped out of the plane to the time you achieved terminal velocity is
  - (a) increasing and concave up
  - (b) decreasing and concave up
  - (c) increasing and concave down
  - (d) decreasing and concave down



### Derivatives and Graphing III

**True or False:** If x = p is in the domain of y = f(x) not a local minimum or a local maximum, then it is **not** a critical point.

- a) True and I am very confident
- b) True but I am not very confident
- c) False but I am not very confident
- d) False and I am very confident



## Derivatives and Graphing IV

Let 
$$f(x)=ax+rac{b}{x}.$$
 Suppose that  $a$  and  $b$  are positive. What are the critical points of  $y=f(x)$ ?

- a) -b/a
- **b**) 0
- c)  $\pm \sqrt{b/a}$
- d)  $\pm \sqrt{-b/a}$
- e) No critical points



### Derivatives and Graphing V

**True or False:** A local maximum of y = f(x) only occurs at a point where f'(x) = 0.

- a) True and I am very confident
- b) True but I am not very confident
- c) False but I am not very confident
- d) False and I am very confident



### Global Optimzation I

- 9. Consider  $f(x) = -3x^2 + 12x + 7$  on the interval  $-2 \le x \le 4$ . Where does this function have its global minimum value?
  - (a) x = -2
  - (b) x = 0
  - (c) x = 2





### Global Optimzation II

3. Let f be a continuous function on the closed interval  $0 \le x \le 1$ . There exists a positive number A so that the graph of f can be drawn inside the rectangle  $0 \le x \le 1$ ,  $-A \le y \le A$ .

The above statement is:

- (a) Always true.
- (b) Sometimes true.
- (c) Not enough information.

