Math 826 **Assignment # 6** **27 Feb 08**

Due: **Monday**, March 10th

1. (a) Suppose that $A \subset \mathbb{R}$ and a is a limit point of A. If a sequence of functions $f_n : A \to \mathbb{R}$ converge uniformly to $f : A \to \mathbb{R}$ and, for each n, $\lim_{x\to a} f_n(x)$ exists, then

$$\lim_{n\to\infty} \lim_{x\to a} f_n(x) = \lim_{x\to a} f(x).$$

   (b) Show that the conclusion can be false if the convergence of the f_n is not uniform.

   (c) If $\sum_{k=1}^{\infty} a_k$ converges, find $\lim_{x\to 1^-} \sum_{k=1}^{\infty} a_k x^k$.

2. Do Exercise 8.3.C in the text.

3. Suppose, for functions $f_k : A \to \mathbb{R}$, we know that $\sum_{k=1}^{\infty} f_k(x)$ converges uniformly and absolutely on $A \subset \mathbb{R}$. Does it follows that $\sum_{k=1}^{\infty} |f_k(x)|$ converges uniformly on A?

4. (January 2003 Qual)

   (a) Assume that $\sum_{k=1}^{\infty} a_k$ is a convergent series of nonnegative real numbers. Prove that the series $\sum_{k=1}^{\infty} a_k^x$ converges uniformly on $[1, +\infty)$.

   (b) Prove: the series $\displaystyle\sum_{k=0}^{\infty} \frac{x^3}{(1+x^3)^k}$ converges uniformly on $[a, b]$ for every $0 < a < b$ but the convergence is not uniform on $[0, b]$ for any $b > 0$.

5. Do Exercise 8.5.H in the text. Stated slightly more precisely,

   (a) Compute $f(x) = \sum_{n=0}^{\infty} (n+1)x^n$.

   (b) Compute $\sum_{n=0}^{\infty} \frac{n}{3^n}$. Justify your method.

   (c) Would the substitution of $x = -1$ in your formula from part (a) be justified?