. 5 1. Find the area of the triangle determined by the points $P(-2,2,0)$, $Q(0,1,-1)$ and $R(-1,2,-2)$.

Solution. The area of the triangle with two sides $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is half the area of the parallelogram determined by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, which is $\overrightarrow{PQ} \times \overrightarrow{PR}$.
Note that $\overrightarrow{PQ} = \langle 2,-1,-1\rangle$ and $\overrightarrow{PR} = \langle 1,0,-2\rangle$.
Then, $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

So, the area of the triangle $= \dfrac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \dfrac{1}{2}\sqrt{4+9+1} = \dfrac{\sqrt{14}}{2}$.

. 5 2. Find parametric equations for the line through $\mathbf{Q}(-1,1,3)$ that is perpendicular to the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

Solution. We need a point on the line and a vector pointing along the line.
To find the vector we use $\mathbf{u} \times \mathbf{v}$, since it is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.
The vector is $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = -2i + 4j - 2k.$

Since the line is through $\mathbf{Q}(-1,1,3)$, the equations are

$$x = -1 - 2t,\ y = 1 + 4t,\ z = 3 - 2t$$

where $-\infty < t < \infty$