**Math 106, Sect. 550** **Answers to Quiz # 8** **2 Nov '06**

. 10 1. You need to fence off a rectangular garden (with sides facing north, south, east and west) by using a wind-break fence on the north side costing \$20/foot and, on the other three sides, by a regular fence costing \$10/foot. If you have \$1320 available to pay for the fence, what are the dimensions and area of the largest garden you can enclose?

*Solution.* Let $w$ be the length of the east and west sides and let $l$ be the length of the north and south sides. We are maximizing area, call it $A$, and $A = wl$.

The total cost of the fence is $S = 20w + 30l$, which we are given is 1320, so $20w + 30l = 1320$. Solving for $w$, we have

$$l = \frac{1320 - 20w}{30} = 44 - \frac{2}{3}w.$$

Substituting this into the formula for $A$ gives

$$A = w\left(44 - \frac{2}{3}w\right) = 44w = \frac{2}{3}w^2.$$

To see the domain, first $w \geq 0$. Second, even if $l = 0$, then $20w = 1320$, so $w$ can be at most 66. So we have domain of $w$ is $[0, 66]$.

To maximize $A$ we will take the first derivative and find any critical points. The derivative is $A' = 44 - 4w/3$ which we can solve for zero by doing the following steps:

$$\begin{aligned} 0 &= 44 - \frac{4}{3}w \\ 44 &= \frac{4}{3}w \\ 33 &= w \end{aligned}$$

Since we have a closed finite interval, the global maximum for $A$ must occur at at least one of $w = 0$, $w = 33$ or $w = 66$. But, clearly, $w = 0$ and $w = 66$ give $A = 0$ and $w = 33$ gives $A = 726 > 0$. So the $w = 33$ is a global maximum.

Substituting back, we get $l = 44 - 2 \cdot 33/3 = 22$.

So the dimension of the rectangle are 33' for the east and west sides and 22' for the north and south sides, giving a total area of 726 square feet.