Math 107, Sect. 350 **Answers to Quiz # 1** **11 Jan '07**

. 5 1. Find the total area between the graph of $y = x^3 - 4x$ and the x-axis over $[0, 3]$.

Solution. To find the total area, we need to find the roots of $x^3 - 4x$.

$$\begin{aligned} x^3 - 4x &= 0 \\ x(x^2 - 4) &= 0 \\ x(x+2)(x-2)0 \end{aligned}$$

By putting $x = 0$ $x = -2$ and $x = 2$ into $x^3 - 4x$, we see all three roots are valid. By graphing or putting $x = 1$ and $x = 5/2$ into $x^3 - 4x$, we see $y = x^3 - 4x$ is below the x-axis on $(0, 2)$ and above it on $(2, 3)$. So we'll have to split the integral into two parts: one from 0 to 2 and the other from 2 to 3.

Thus, the total area is

$$\int_0^2 -(x^3 - 4x)\,dx + \int_2^3 x^3 - 4x\,dx.$$

Notice that

$$\int x^3 - 4x\,dx = \frac{x^4}{4} - 2x^2 + C$$

and so the total area is

$$\begin{aligned} \int_0^2 -(x^3 - 4x)\,dx + \int_2^3 x^3 - 4x\,dx &= -\frac{x^4}{4} + 2x^2\Big|_0^2 + \frac{x^4}{4} - 2x^2\Big|_2^3 \\ &= (-4 + 8) - 0 + \left(\frac{81}{4} - 18\right) - (4 - 8) = \frac{41}{4}. \end{aligned}$$

. 5 2. Evaluate $\displaystyle\int \frac{3}{x^2} e^{1/x} \sin(1 + e^{1/x})\,dx$.

Solution. Making the substitution $u = 1 + e^{1/x}$, we have $du = e^{1/x}\left(\frac{-1}{x^2}\right)dx$, and so

$$\int \frac{3}{x^2} e^{1/x} \sin(1 + e^{1/x})\,dx = -3 \int \sin u\,du = 3\cos u + C = 3\cos(1 + e^{1/x}) + C$$