**Math 107, Sect. 250** **Answers to Quiz # 7** **8 Mar '07**

. 5 1. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-7)^n}{n+11^n}$ converge absolutely, converge conditionally, or diverge?

*Solution.* Let $u_n = \left|\dfrac{(-7)^n}{n+11^n}\right| = \dfrac{7^n}{n+11^n}$. We use the comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{11^n}$, since

$$\frac{7^n}{n+11^n} < \frac{7^n}{11^n}.$$

Since $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{11^n}$ is a geometric series with ratio $7/11 < 1$, it converges and so $\displaystyle\sum_{n=1}^{\infty} \frac{(-7)^n}{n+11^n}$ converges absolutely.

. 5 2. Determine the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2n}$.

*Solution.* Let $a_n = \dfrac{x^n}{2n}$. Then

$$\frac{|a_{n+1}|}{|a_n|} = \frac{2n|x|^{n+1}}{(2n+2)|x|^n} = \frac{2n}{2n+2}|x|.$$

Now,

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{2n}{2n+2}|x| = |x|$$

Thus, the series converges (absolutely) for $|x| < 1$ and diverges for $|x| > 1$. At $x = 1$, the series is $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$, which diverges, being a multiple of the harmonic series. At $x = -1$, the series is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$, which converges by the alternating series test.

Thus, the interval of convergence is $[-1, 1)$.