Math 106-350/550 Answers to Homework #8 3 Nov '06

1. For each of the following formulas, say if it is right or wrong, clearly justifying your answer:

   (a) $\displaystyle\int \tan\theta \sec^2\theta \, d\theta = \frac{\sec^3\theta}{3} + C,$

   (b) $\displaystyle\int \tan\theta \sec^2\theta \, d\theta = \frac{1}{2}\tan^2\theta + C,$

   (c) $\displaystyle\int \tan\theta \sec^2\theta \, d\theta = \frac{1}{2}\sec^2\theta + C.$

   Solution. The key point is that the formula $\int f(\theta)\, d\theta = g(\theta) + C$ is right if and only if $g'(\theta) = f(\theta)$. So, for each formula, we take the derivative of the right hand side and see if it equals $\tan\theta\sec^2\theta$.

   For the first,

$$\frac{d}{d\theta}\left(\frac{\sec^3\theta}{3}\right) = \frac{1}{3}3\sec^2\theta(\sec\theta\tan\theta) = \sec^3\theta\tan\theta,$$

   which does not equal $\tan\theta\sec^2\theta$. (If it did, their ratio would be one, and canceling would show that $\sec\theta = 1$ for all θ.) So the first formula is wrong.

   For the second,

$$\frac{d}{d\theta}\left(\frac{1}{2}\tan^2\theta\right) = \frac{1}{2}2\tan\theta\sec^2\theta = \tan\theta\sec^2\theta,$$

   so the second formula is right.

   For the third,

$$\frac{d}{d\theta}\left(\frac{1}{2}\sec^2\theta\right) = \frac{1}{2}2\sec\theta(\sec\theta\tan\theta) = \tan\theta\sec^2\theta,$$

   so the third formula is also right.

   If you are curious why one function could have two different antiderivatives, notice that $\sec^2\theta = 1 + \tan^2\theta$. By changing the arbitrary constant in the second formula to be $C + 1$, we can use this trig identity to get the third. The difference between the two formulas is that when we solve for C using a given initial condition, the constants will be different, depending on which formula we use.