. 10 1. Find the length of the curve given by the parametric equations $x(t) = t^3 + 1$ and $y(t) = 3t^2/2$ for $\sqrt{3} \le t \le 3$.

Solution. We have $x'(t) = 3t^2$ and $y'(t) = 3t$, so the length of the curve is

$$\begin{aligned}\int_{\sqrt{3}}^{3} \sqrt{(3t^2)^2 + (3t)^2}\, dt &= \int_{\sqrt{3}}^{3} \sqrt{9t^4 + 9t^2}\, dt \\ &= \int_{\sqrt{3}}^{3} 3t\sqrt{t^2+1}\, dt.\end{aligned}$$

Substituting $u = t^2 + 1$ we have $du = 2t\, dt$ and hence,

$$\begin{aligned}\int_{\sqrt{3}}^{3} 3t\sqrt{t^2+1}\, dt &= \int_{4}^{10} \frac{3}{2}\sqrt{u}\, du \\ &= u^{3/2}\Big|_4^{10} \\ &= 10^{3/2} - 4^{3/2}.\end{aligned}$$