Math 106, Sect. 350 — Answers to Quiz # 3 — 7 Sept '06

. 6 1. Find $\lim \dfrac{x^2+4x-21}{x^2-x-6}$ as (a) $x \to -2^+$, (b) $x \to -2^-$, (c) $x \to 3$.

Solution. Notice that

$$\frac{x^2+4x-21}{x^2-x-6} = \frac{(x-3)(x+7)}{(x-3)(x+2)}.$$

For $x \to -2^+$, substituting $x=-2$ gives $-25/0$ and since the numerator is not zero, the limit is $\pm\infty$. Notice that the numerator is negative $(-5\cdot 5)$, and the denominator is negative $(-5\cdot\text{positive})$, so we have $\displaystyle\lim_{x\to -2^+} \frac{x^2+4x-21}{x^2-x-6} = +\infty$.

For $x \to -2^-$, we already know this is a vertical asymptote from part (a). The numerator is negative $(-5\cdot 5)$, and the denominator is postive $(-5\cdot\text{negative})$, so we have $\displaystyle\lim_{x\to -2^+} \frac{x^2+4x-21}{x^2-x-6} = -\infty$.

For $x \to 3$, since $x \neq 3$, we have

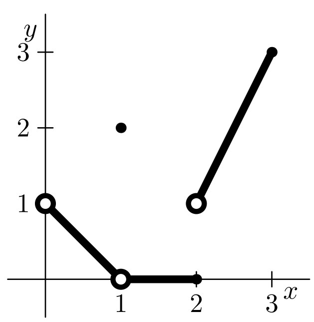
$$\lim_{x\to 3} \frac{x^2+4x-21}{x^2-x-6} = \lim_{x\to 3} \frac{x+7}{x+2} = 2.$$

. 4 2. Answer the following questions for the function $y = f(x)$ graphed below.

(a) Does $f(1)$ exist? If so, what is it?

(b) Does $\lim_{x\to 1} f(x)$ exist? If so, what is it?

(c) Is $f(x)$ continuous at $x = 1$? (Explain your answer.)



Solution. (a) Yes, $f(1) = 2$. (b) Yes, $\lim_{x\to 1} f(x) = 0$. (c) No, since to be continuous at $x = 1$ we must have $\lim_{x\to 1} f(x) = f(1)$.

The original graph for Question 2 did not clearly indicate the value of the function on the interval $(1, 2)$. If you answered 2(b) by saying that the function was not defined on $(1, 2)$ and hence the two-sided limit did not exist, you should get full marks. The graph above has been clarified to fit with the the original intent of the question.