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1. Evaluate the following:

$$(a) \int_1^4 \left(x^3 + 5x + \frac{1}{x} \right) dx,$$

$$(b) \int_{-1}^0 y(2y^2 - 3)^5 dy.$$

Solution. For (a), we have

$$\begin{aligned} \int_1^4 \left(x^3 + 5x + \frac{1}{x} \right) dx &= \frac{x^4}{4} + \frac{5x^2}{2} + \ln|x| \Big|_{x=1}^{x=4} \\ &= \left(\frac{4^4}{4} + \frac{5 \cdot 4^2}{2} + \ln 4 \right) - \left(\frac{1}{4} + \frac{5 \cdot 1}{2} + \ln 1 \right) \\ &= \frac{405}{4} + \ln 4 \end{aligned}$$

Note that (b) is Problem 21 from Section 7.4, a homework problem. We use the substitution $u = 2y^2 - 3$ so $du = 4y dy$ and hence $y dy = du/4$. If $x = -1$, $u = -1$ and if $x = 0$, $u = -3$. Thus,

$$\begin{aligned} \int_{-1}^0 y(2y^2 - 3)^5 dy &= \int_{-1}^{-3} \frac{u^5}{4} du \\ &= - \int_{-3}^{-1} \frac{u^5}{4} du \\ &= - \left(\frac{u^6}{24} \Big|_{u=-3}^{u=-1} \right) \\ &= - \left(\frac{1^6}{24} - \frac{3^6}{24} \right) \\ &= \frac{91}{3} \end{aligned}$$