Math 496-004 **Assignment # 3** **10 Sept 08**

Due: Sept 17th

1. Suppose $F, G : \mathbb{R}^n \to \mathbb{R}$ satisfy $\lim_{x \to a} F(x) = L$ and $\lim_{x \to a} G(x) = M$. Prove that
$$\lim_{x \to a} F(x)G(x) = LM.$$
Hint: Look at the proof given in class of the analogous result for sums.

2. Let $S^* \subset \mathbb{R}^n$ be the set of all limit points of $S \subset \mathbb{R}^n$. Show that $S \cup S^*$ is closed, i.e., contains all of its limit points.

3. Consider the function $F : \mathbb{R}^3 \to \mathbb{R}$ given by $F(x, y, z) = x^2 + y^2 + z^2$.
   (a) Find the differential of F at $a = (3, 2, 6)$, dF_a, which is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.
   (b) Using the differential, find an approximate value for $3.02^2 + 1.97^2 + 5.98^2$.

4. If $F : \mathbb{R}^n \to \mathbb{R}^m$ is linear, show that, for each point $a \in \mathbb{R}^n$, F is differentiable at a and the differential dF_a equals F.

5. Suppose that $F : \mathbb{R}^n \to \mathbb{R}^m$ and $G : \mathbb{R}^n \to \mathbb{R}^k$ are both differentiable at $a \in \mathbb{R}^n$. If $H : \mathbb{R}^n \to \mathbb{R}^{m+k}$ is given by $H(x) = (F(x), G(x))$ then show directly from the definition that H is differentiable at a.