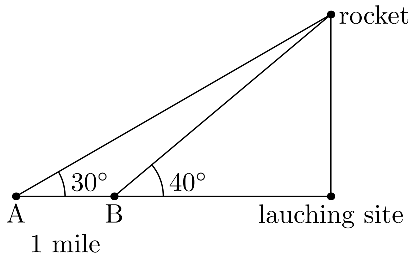
Math 106-350/550 — Answers to Homework #1 — 24 Aug '06

6 1. A rocket takes off vertically. Two observers, A and B, are standing a mile apart. At a certain moment, the angle of elevation for the rocket is 30° for observer A and 40° for observer B. What is the height of the rocket?

Solution. First, we assume that the rocket is not between the observers. (It is reasonable to assume that it is, but we don't solve that problem here.) Draw a diagram. Because the angle of elevation for A is smaller than for B, A must be further away from the lauching site.



Let x be the distance from the launching site to B and h be the height of the rocket. Using right triangles, we have the equations:

$$\tan 30° = \frac{h}{x+1}, \qquad \tan 40° = \frac{h}{x}.$$

We solve this pair of equations for x and h. Using the second equation, $h = x \tan 40°$. Substituting this into the first equation gives

$$\tan 30° = \frac{x \tan 40°}{x+1}$$

which we can rearrange to give

$$\frac{\tan 30°}{\tan 40°} = \frac{x}{x+1}.$$

Let $r = \tan 30° / \tan 40°$, so the equation becomes $r = x/(x+1)$. Clearing fractions, we have $r(x+1) = x$. Solving for x gives $r = x - rx$, then $r = x(1-r)$, and finally

$$x = \frac{r}{1-r}.$$

Using a calculator, $r = 0.688059$, so $x = 2.20574$, and so $h = 2.20574 \tan(2\pi/9) = 1.85083$. Thus, the height of the rocket is 1.85 miles.

. 4 2. If $\displaystyle\lim_{x\to 1} \frac{f(x)-3}{x-1} = 3$, find $\displaystyle\lim_{x\to 1} f(x)$.

Solution. Although it seems silly to say it, one of the key points in this problem is that, as x approaches 1, $x-1$ approaches 0, i.e., $\lim_{x\to 1} x-1 = 0$. The only way the fraction $(f(x)-3)/(x-1)$ can approach 3 when the denominator is going to zero is for the numerator also to go to zero.

In symbols, we can write

$$\begin{aligned}
\lim_{x\to 1} f(x) - 3 &= \lim_{x\to 1} \frac{f(x)-3}{x-1}(x-1) && \text{since } x-1 \neq 0 \\
&= \lim_{x\to 1} \frac{f(x)-3}{x-1} \lim_{x\to 1}(x-1) && \text{by the product limit law} \\
&= 3 \cdot 0 = 0.
\end{aligned}$$

Using the sum limit law, $\lim_{x\to 1} f(x) = 3$.