

Due: Wednesday, February 20th

1. Do Exercise 7.2.M in the text.
2. (a) Do Exercise 7.3.F in the text.
(b) Let V be a normed vector space and $A \subset V$. For a fixed $v \in V$, call $b \in A$ a *best approximation to v in A* if $\|v - b\| = \inf\{\|v - a\| : a \in A\}$. Prove that if the norm of V is strictly convex, A is convex and has a best approximation to some $v \in V$, then that best approximation is unique.
3. Fix a convex set A in an inner product space V . Define $\Phi : V \rightarrow A$ by mapping v to the unique best approximation to v in A . Show that, for all $v, w \in V$,

$$\|\Phi(v) - \Phi(w)\| \leq \|v - w\|.$$

HINT: : A hint is available on the course webpage.

4. Do Exercise 7.5.D in the text.
5. Do Exercise 7.6.E in the text. (Don't do this from scratch!)