. 5 1. (a) Find r so that the geometric series $1 + r + r^2 + \cdots$ converges and has sum 5.

(b) Find b so that $1 + e^b + e^{2b} + e^{3b} + \cdots$ converges and has sum 9.

Solution. For (a), we know, from the formula for the sum of a geometric series, that

$$1 + r + r^2 + \cdots = \frac{1}{1-r}.$$

Thus, we want $(1-r)^{-1} = 5$, and so $1 - r = 1/5$ giving $r = 4/5$.

For (b), we use the same formula for the sum of a geometric series, to obtain

$$1 + e^b + e^{2b} + e^{3b} + \cdots = \frac{1}{1-e^b}.$$

Now, we have

$$\frac{1}{1-e^b} = 9$$

and so $1 - e^b = 1/9$, so $e^b = 8/9$ and $b = \ln(8/9) = -0.117783$.