Math 106, Sect. 550 **Answers to Quiz # 3** **7 Sept '06**

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1. Evaluate $\lim\limits_{x\to 0^+} \dfrac{x^2-1}{x^3+x^2}$.

   Solution. Notice that substituting $x = 0$ into the fraction, we get $-1/0$, so the limit must be $\pm\infty$. To find out the sign, we need to see if $(x^2-1)/(x^3+x^2)$ is positive or negative as x approaches 0 from above.
   If x is a bit bigger than zero, then x^2-1 is negative and x^3+x^2 is positive. Thus, the fraction is negative and so the limit is $-\infty$.
   Alternatively, you can simplify the fraction before working out the sign,

   $$\lim_{x\to 0^+} \frac{x^2-1}{x^3+x^2} = \lim_{x\to 0^+} \frac{(x-1)(x+1)}{x^2(x+1)} = \lim_{x\to 0^+} \frac{x-1}{x^2}.$$

   Now we can do the same kind of sign analysis as before, noticing that $x-1$ is negative and x^2 is positive to again conclude that the limit is $-\infty$.

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2. Let $f(x) = \begin{cases} 1+\frac{x}{2} & \text{if } x > 0 \\ x & \text{if } x \le 0 \end{cases}$

   For which values of x is $f(x)$ continuous? What types of discontinuities does $f(x)$ have, if any? Explain.

   Solution. Our function is continuous when $x \neq 0$. For $x < 0$, it is a polynomial, so it is continuous at every $x < 0$. For $x > 0$, it is a polynomial, so it is continuous at every $x > 0$.
   At $x = 0$ it has a jump discontinuity. One way to see this is by computing the one-sided limits

   $$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 + \frac{x}{2} = 1, \qquad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0.$$

   This is perhaps best illustrated with a picture: