Math 896-503 **Assignment # 5** **16 June 09**

Due: June 18th

1. A unit speed parametrization of a circle may be written

$$c(s) = \mathbf{c} + r\cos s/r\mathbf{e}_1 + r\sin s/r\mathbf{e}_2.$$

   where $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit length orthogonal vectors.

   If f is a unit-speed curve with $\kappa(0) \neq 0$, prove that there is one and only one circle c which approximates f near $f(0)$ in the sense that $c(0) = f(0)$, $c'(0) = f'(0)$, and $c''(0) = f''(0)$. Show that c lies in the osculating plane of f at $f(0)$ and find its center and radius.

2. Suppose that $f : I \to \mathbb{R}^3$ and a reparametrization $g = f \circ \alpha : J \to \mathbb{R}^3$ are both unit-speed curves.

   (a) Show there is $t_0 \in \mathbb{R}$ such that for all $t \in J$, $\alpha(t) = \pm t + t_0$.

   (b) If T_g, N_g, B_g is the Frenet frame field for g, and κ_g and τ_g are the curvature and torsion functions for g, then

$$T_g = \pm T \circ \alpha, N_g = N \circ \alpha, B_g = \pm B \circ \alpha, \kappa_g = \kappa \circ \alpha, \tau_g = \tau \circ \alpha.$$