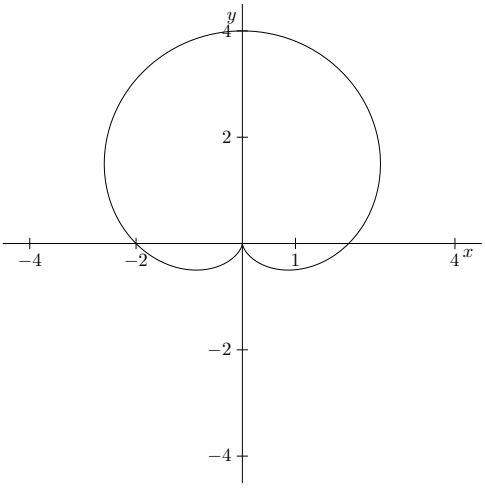
. 10 1. Graph the polar equation $r = 2+2\sin\theta$ on the (polar) axes below. List any symmetries that it displays, giving algebraic justifications for your answers.

Solution.



This is a cardioid with the main bulb on the upper half of the plane. To determine the types of symmetry, we first replace θ with $-\theta$ and obtain $2 + 2\sin\theta \longrightarrow 2 + 2\sin(-\theta) = 2 + 2\sin\theta = r, \neq -r$. Thus we have x-axis symmetry but not symmetry about the y-axis. Also, if we replace r with $-r$, we get $r = 2+2\sin\theta \longrightarrow -r = -(2+2\sin\theta) = -2-2\sin\theta \neq 2+2\sin\theta$, which suggests that the graph does not have symmetry about the origin.