Math 106-350/550 - Analytic Geometry & Calculus I **1st Semester, '06-'07**

Sample Homework Problem

To show how solutions to homework should be written up, here is the solution to a somewhat complicated trigonometry problem, taken from the textbook used at UNL.

Problem: Find the area of a piece of land given in Figure 1. The vertexes of the quadrilateral are marked by dots.

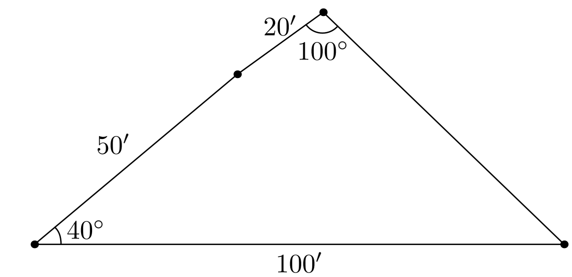

FIGURE 1. A quadrilateral piece of land.

Solution: We find the area by breaking the quadrilateral into two triangles as in Figure 2, and finding the area of each triangle.

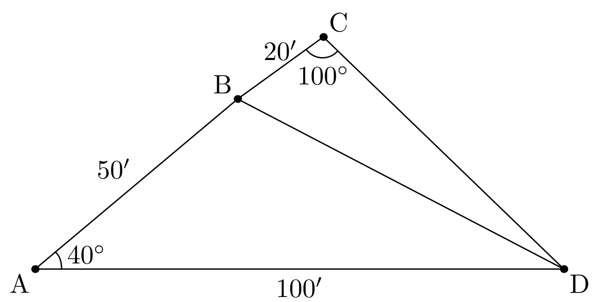

FIGURE 2. Split into two triangles.

To find the area of $\triangle ABD$, consider $\overline{AD}$ to be the base. Then the altitude is given in Figure 3.

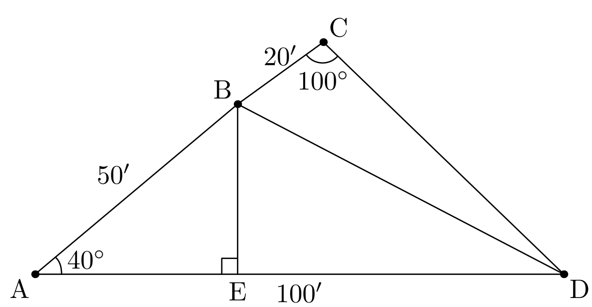

FIGURE 3. The altitude of $\triangle ABD$.

In the right triangle $\triangle ABE$, we have that $\overline{BE}$ is $\sin 40^\circ \overline{AB}$. Thus,

$$\begin{aligned}\triangle ABD &= \frac{1}{2}\sin 40^\circ\, \overline{AB}\cdot\overline{AD}\\ &= \frac{1}{2}\sin(2\pi/9)\, 50\cdot 100\\ &= 1606.9690.\end{aligned}$$

To find the area of $\triangle BCD$, consider $\overline{BD}$ to be the base. We have to find both $\overline{BD}$ and the altitude, which means finding the angle $\angle CBD$.

To find $\overline{BD}$, apply the law of cosines to $\triangle ABD$ to get

$$\begin{aligned}\overline{BD}^2 &= \overline{AD}^2 + \overline{AB}^2 - 2\overline{AD}\cdot\overline{AB}\cos 40^\circ\\ &= 50^2 + 100^2 - 2\cdot 50\cdot 100\cdot\cos(2\pi/9)\\ &= 4839.55557.\end{aligned}$$

Thus, $\overline{BD} = 69.56691$.

To find $\angle CBD$, we first find $\angle CDB$ by applying the law of sines to $\triangle BCD$. Let $\alpha = \angle CDB$.

$$\frac{\sin\alpha}{\overline{BC}} = \frac{\sin 100^\circ}{\overline{BD}}.$$

Thus,

$$\sin\alpha = \frac{\overline{BC}\sin 100^\circ}{\overline{BD}} = \frac{20\sin(5\pi/9)}{69.56691} = 0.2831253.$$

So $\alpha = \sin(0.2831253) = 16.44682^\circ$ or 0.2870512 radians. Thus, $\angle CBD = 180 - 100 - 16.44682 = 63.55318^\circ$.

As in $\triangle ABD$, we can drop a perpendicular from C to $\overline{BD}$ to get a right triangle. As before, this shows that the altitude of $\triangle BCD$ is $\sin(63.56^\circ)20$. Hence the area of $\triangle BCD$ is

$$1/2\sin(63.56^\circ)20\cdot 69.56691 = 622.9019.$$

Thus, the total area of the quadrilateral is the sum $1606.9690 + 622.9019 = 2229.8719$.

An Aside. In order to draw the figures, I had to compute $\angle ABD$. Call it β. Using the law of sines in $\triangle ABD$, we have

$$\frac{\sin\beta}{\overline{AD}} = \frac{\sin\angle DAB}{\overline{BD}}.$$

Solving for $\sin\beta$, we have

$$\sin\beta = .924$$

and $\arcsin(.925) = 67.5^\circ$.

On the other hand, the diagram (or the law of sines) shows that $\angle ADB < \angle DAB = 40^\circ$. So the sum of the three interior angles of $\triangle ADB$ is less than 180°, which is impossible.

Where is the mistake?

Hint #1: Draw a graph with $y = \sin x$ and $y = .925$.

Hint #2: Read the part on inverse trig functions in Section 1.5 of the text.