Math 496-004 **Assignment # 5** **24 Sept 08**

Due: Oct 1

1. For functions $f, g : \mathbb{R} \to \mathbb{R}^3$, verify that
$$\frac{d}{dt}(f \times g) = \frac{df}{dt} \times g + f \times \frac{dg}{dt}.$$

2. Let $D \subset \mathbb{R}^n$ be open and for a function $f : D \to \mathbb{R}$, suppose there is a constant M so that, for $i = 1, 2, \ldots, n$, and all points $p \in D$, we have
$$\left\| \frac{\partial f}{\partial x_i}(p) \right\| \leq M.$$
Prove that f is continuous at every point of D.

   Hint: : Write the change in f using partial derivatives, along the lines of the proof of the Differentiability Theorem.

   It is *not* true that f is differentiable at every point of D; the function in Question 3 of the last assignment is a counterexample.

3. Do Exercise S-7.73, that is, If $\mathbf{r} = \mathbf{a}\cos\omega t + \mathbf{b}\sin\omega t$, where $\mathbf{a}$ and $\mathbf{b}$ are any constant non-collinear vectors and ω is a constant scalar, prove that
$$\text{(a)}\mathbf{r} \times \frac{d\mathbf{r}}{dt} = \omega(\mathbf{a} \times \mathbf{b}), \qquad \text{(b)}\frac{d^2\mathbf{r}}{dt^2} + \omega^2\mathbf{r} = \mathbf{0}.$$

4. Do Exercise S-7.96 (b), that is, Prove that the radius of curvature at any point of the plane curve $y = f(x)$, $z = 0$, where $f(x)$ is [twice] differentiable, is given by
$$\rho = \left| \frac{(1 + (y')^2)^{3/2}}{y''} \right|.$$