Math 106, Sect. 350 **Answers to Quiz # 8** **2 Nov '06**

. 10 1. You need to fence off a rectangular region of land using a fence along three sides and a straight river for the fourth side. If the total region to be enclosed is 1800 square feet, what dimensions for the rectangle use the least amount of fencing?

Solution. Let w be the length of the rectangular region and let l be the length. We assume that one of the sides of length l is along the river. We are minimizing the total length of fencing, call it S, and $S = 2w + l$.

The area of the region is $A = wl$ which we are given is 1800, so $l = \frac{1800}{w}$. Substituting $l = \frac{1800}{w}$ into the formula for S gives

$$S = 2w + \frac{1800}{w}.$$

To see the domain, first $w > 0$. Second, we can make w as big as we like (but l then is very small). So we have domain of w in $(0, +\infty)$.

To minimize S we will take the first derivative and find any critical points. The derivative is $S' = 2 - 1800w^{-2}$ which we can solve for zero by doing the following steps:

$$\begin{aligned}
0 &= 2 - 1800w^{-2} \\
0 &= w^{-2}(2w^2 - 1800) \\
0 &= 2w^2 - 1800 \\
2w^2 &= 1800 \\
w^2 &= 900 \\
w &= 30
\end{aligned}$$

To see that this is a minimum and not a maximum you can either look at the sign of S', use the second derivative test, or notice that as $w \to 0$ or $w \to +\infty$, we get $S \to +\infty$.

Substituting back, we get $l = \frac{1800}{30} = 60$.

So the dimension of the rectangle are 60' for the sides parallel to the river and 30' on the sides perpendicular to the river.