FINAL EXAM REVIEW QUESTIONS–MATH 104

1. Find $\dfrac{dy}{dx}$ for the following expressions y (do not simplify):

(a) $y = \dfrac{x^2 - 8x}{3 - 2x}$ (b) $y = e^{x^2+1}\ln(x^2+4)$

(c) $y = e^{x^2+1}\ln(x^2+4)$ (d) $y = (6x + e^{-3x})^{10}$

2. Solve the following equation for x:

$3 + 2e^{4x-5} = 17$

3. Find an equation of the tangent line to the graph of the curve $y = f(x) = (x^2 - 7)^3$ at the point $(3, 8)$. Do the same for $y = f(x) = xe^{x^2}$ at $x = 1$.

4. Evaluate the following limits:

(a) $\lim_{x\to 1} \dfrac{2x^2 + 5x - 7}{3x^2 - 8x + 5}$ (b) $\lim_{x\to +\infty} \left(6 + \dfrac{2x^2 - 8}{3 - x^2}\right)$

5. Let $f(x) = x^2 \ln(x)$.

(a) Find all critical values of f. (b) Also determine all values x where the graph of f is concave up.

6. Let $y = f(x) = x^2 + 6x$.

(a) Find dy when $x = 2$ and $dx = .25$

(b) Find Δy when $x = 2$ and $\Delta x = .25$

7. Is the function $f(x) = \dfrac{e^x}{x}$ increasing or decreasing at $x = \frac{1}{2}$? Is f concave up or down at $x = 1$. Explain why.

8. Let $y = f(x)$ be a function such that $f''(x) = x^2(x+4)(x-2)$ for all $x \epsilon (-\infty, +\infty)$.

(a) List the open interval(s) where the graph of f is <u>concave up</u>.

(b) List the number(s) x where $(x, f(x))$ is a <u>point of inflection</u> on the graph of f.

9. How much money should Barbara invest on December 19, 2002 at an annual interest rate of 4.92 per cent, compounded continuously, in order to have \$42,500 on December 19, 2015? (Round off your answer to the nearest cent).

10. Assume that for some commodity, the <u>price elasticity of demand</u> E is given by the formula $E = E(p) = \dfrac{900 - 2p}{p}$, $0 < p < 450$ units.

Is the demand <u>elastic</u> or <u>inelastic</u> when $x = 250$ units? Explain why.

11. Given the cost function $C = C(x) = x^2 + 20x + 900$ dollars, use <u>calculus methods</u> to determine the number of units x that should be produced in order to <u>minimize</u> the <u>average cost</u> per unit.

12. A manufacturer can sell x widgets at a <u>price</u> of $90 - .05x$ dollars <u>each</u>. It costs the manufacturer $60x + 4500$ dollars to produce <u>all x of them</u>.

(a) Find the average cost per widget when $x = 350$.

(b) Find the revenue function $R(x)$.

(c) Find the value of x that will maximize the revenue function $R(x)$.

13. Kelly invested \$12,000 in a mutual fund on December 21, 1997. On December 21, 2007 her investment was worth \$26,500.

(a) What was the annual rate of growth of this investment, assuming continuous compounding?
(b) If this mutual fund continues to appreciate at the same rate, how much will her investment be worth on December 21, 2012?

14. If a material has a half-life of 17 years, how much of a 40 gram mass will remain after 42 years? (Round off your answer to the nearest hundredth of a gram).
15. Find the antiderivative:
$\int \left(6x^{-\frac{2}{5}} + \frac{1}{x^{10}}\right) dx$ 16. Evaluate the following definite integrals:

(a) $\int_0^2 \left[x^2 - e^{3x}\right] dx$ (b) $\int_1^3 \left(1 + \frac{1}{x}\right) dx$

17. If $\int_0^{10} f(x)\, dx = 12$ and $\int_4^{10} f(x)\, dx = -3$, evaluate the definite integral $\int_0^4 (5f(x) - 3x^2)\, dx$.

18. Let $p = D(q) = 40 - q^2$ dollars be the demand function and let $p = S(q) = 2q + 5$ be the supply function for some commodity.

(a) Find the equilibrium point (q_0, p_0).
(b)Find the Consumer Surplus and the Producer Surplus.

19. Use the substitution method to evaluate the definite integral:
$\int_0^2 \frac{x}{\sqrt{6x^2+1}}\, dx$. Clearly identify what substitution u you are using and show all your work.

20. Find the interest rate required for an investment of \$4500.00 to grow to \$8000.00 in 6 years if interest is compounded:
(a) continuously, (b) quarterly.

21. Let R be the region enclosed by the curves $y = 5 - x^2$ and $y = x + 3$.
(a) Sketch a graph of the region R.
(b) Express the area of the region R as a definite integral. (Do not evaluate this integral.)

22. Find the average value of the function $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ on the interval $[0, 4]$.

23. Find two nonnegative numbers x and y such that the sum $x + 4y$ is 100 and the sum of the squares of x and y is minimized.

24. A fence is to be built to enclose a rectangular area of $30,000$ square feet. If the cost of the sides facing north and south is \$4 per foot and the cost of the other two sides is \$ 6 per foot, find the cost of the least expensive fence.