Math 104-004 **Answers to Quiz # 8b** **2 Nov 07**

. 4 1. Find the absolute minimum of $f(x) = 3x^2 - 6x + 3$ on $[-1, 4]$ and the x-value where it occurs.

Solution. First, $f'(x) = 6x - 6$ and so $f'(x) = 0$ when $x = 1$. Notice that $f'(x)$ is never undefined.

To see if this is a minimum, we must evaluate $f(x)$ at $x = 1$ and the endpoints: $f(-1) = 3 + 6 + 3 = 12$, $f(1) = 3 - 6 + 3 = 0$, and $f(4) = 48 - 24 + 3 = 27$. Thus, the minimum is 0 at $x = 1$.

. 6 2. Find positive numbers x and y with $x + y = 24$ so that xy^2 is as large as possible.

Solution. First, notice that $x = 24 - y$ and the function to be maximized is

$$f(y) = (24 - y)y^2 = 24y^2 - y^3.$$

Thus, $f'(y) = 48y - 3y^2 = 3y(16 - y)$. So the critical numbers are $y = 0$ and $y = 16$.

To see where we have an absolute max, we can chart f'. Using $f'(1) = 3 \cdot 15 > 0$, we have that f increases on $(0, 16)$. Using $f'(17) = 51 \cdot (-1) < 0$, we have the f decreases on $(16, +\infty)$. Thus f has a relative max at $y = 16$ and because we are only interested in positive y, this is an absolute max.

The other way to see that the absolute max is at $x = 16$ is to notice that for $x = 24 - y$ to be positive, y must be at most 24. Thus, we are looking for the absolute max on the interval $[0, 24]$. Trying the endpoints and the critical numbers, we have $f(0) = 0$, $f(16) = 24 \cdot 16^2 - 16^3 = 2048$, $f'(24) = 0$. So the absolute max is at $y = 24$.

Either way, the choice of numbers to maximize xy^2 is $y = 16$ and $x = 24 - 16 = 8$.