. 6 1. Evaluate $\lim_{t\to 0} \dfrac{t \sin t}{1 - \cos t}$.

Solution. If we substitute in $t = 0$, we get $0/0$, so we can use L'Hopital's Rule.

$$\lim_{t\to 0} \frac{t \sin t}{1 - \cos t} = \lim_{t\to 0} \frac{1 \cdot \sin t + t \cos t}{\sin t}$$

and this is still of the form $0/0$, so we use L'Hopital's Rule again,

$$\begin{aligned} &= \lim_{t\to 0} \frac{\cos t + 1 \cdot \cos t - t \sin t}{\cos t} \\ &= \frac{1 + 1 - 0}{1} = 2. \end{aligned}$$

. 4 2. Find $y(t)$ if $\dfrac{dy}{dt} = \sin t - 2t^2, \qquad y(0) = 5.$

Solution. Since

$$\frac{d}{dt}\left(-\cos t\right) = \sin t, \qquad \frac{d}{dt}\left(\frac{2}{3}t^3\right) = 2t^2,$$

the general antiderivative of $\sin t - 2t^2$ is $y(t) = -\cos t + 2/3t^3 + C$. To find C, we use

$$5 = y(0) = -\cos 0 + \frac{2}{3}0^2 + C = -1 + C$$

and so $C = 6$. Thus, the answer is $y(t) = -\cos t + 2/3t^3 + 6$.