

Due **Tuesday** March 27, at the start of the recitation.

Explain your reasoning. A correct answer poorly explained will not get full marks.

5

1. We approximate $\int_0^{\pi/2} \cos(\sqrt{x}) dx$ using a power series.
 - (a) Find a Taylor polynomial, $p(x)$, that approximates $f(x) = \cos(\sqrt{x})$ to within $2 \cdot 10^{-4}$ i.e., so that $|p(x) - f(x)| \leq 2 \cdot 10^{-4}$, for all x in $[0, \pi/2]$.
 - (b) If $P(t) = \int_0^t p(x) dx$, then the polynomial $P(t)$ approximates $F(t) = \int_0^t \cos(\sqrt{x}) dx$. Find $P(t)$ and give a reasonable bound for error $|P(t) - F(t)|$ for t in $[0, \pi/2]$.
Hint: You can write $P(t) - F(t)$ as an integral of $p(x) - f(x)$.
 - (c) Find $P(\pi/2)$ and an error bound for how well it approximates $\int_0^{\pi/2} \cos(\sqrt{x}) dx$.