Math 826 **Assignment # 7** **26 Mar 08**

Due: April 2nd

1. Do Exercise 9.1.E in the text.
2. Do Exercise 9.1.J in the text.
3. Do Exercise 9.1.M in the text, which should read:
   (a) Give an example of a decreasing sequence of closed balls in a complete metric space with empty intersection. Compare with Exercise 7.2.J.
   HINT: Use a metric on $\mathbb{N}$ topologically equivalent to the discrete metric so that $\{n \geq k\}$ are closed balls.
   (b) Show that a metric space (M, d) is complete if and only if every *decreasing* sequence of closed balls with radii going to zero has a nonempty intersection.
4. Do Exercise 9.2.E in the text.
5. (June 1998 Qual) Do only part (b) of this question; part (a) is included only for your reference.
   (a) Define the term *compact set* in a metric space.
   (b) Let (X, ρ) be a metric space and (x_n) a sequence in X that converges to $a \in X$. Prove directly from the (open cover) definition of compactness that $K := \{a\} \cup \{x_n : n \geq 1\}$ is compact.