Math 106-350/550 **Answers to Homework #3** **8 Sept '06**

· 4 1. At what points is $f(x) = \dfrac{1}{\sqrt{x^2 - 4}}$ continuous?

Solution. First, we find the domain of $f(x)$. In order for the square root to be defined, we need $x^2 - 4 \geq 0$. If $x^2 - 4 = 0$, then $\sqrt{x^2 - 4} = 0$ and $f(x)$ is undefined. So to have $f(x)$ defined, we must have $x^2 - 4 > 0$, i.e., $x < -2$ or $x > 2$.

The next question is if $f(x)$ is continuous on its domain, or if there are other points that must be excluded. Since $x^2 - 4$ is a polynomial, it is continuous everywhere and by the power property of continuous functions, $\sqrt{x^2 - 4}$ is continuous everywhere it is defined. By the quotient property, $1/\sqrt{x^2 - 4}$ is continuous everywhere the denominator is defined. The conclusion is that $f(x)$ is continuous on its domain. Thus, $f(x)$ is continuous for all x with eitther $x < -2$ or $x > 2$.

· 6 2. For the function $f(t) = t^3 + 1$, find the equation of the tangent line through the point $(3/2, 35/8)$.

Solution. To find the slope of the tangent line, use a limit of slopes of secant lines. Let $h \neq 0$. Let $P = (3/2, 35/8)$ and $Q = (3/2 + h, (3/2 + h)^3 + 1)$. Then the slope of the tangent line is

$$\lim_{h \to 0} \frac{(3/2 + h)^3 + 1) - (35/8)}{3/2 + h - 3/2}.$$

Before computing the limit, we first simplify this fraction

$$\begin{aligned}
\frac{(\frac{3}{2} + h)^3 + 1 - \frac{35}{8}}{\frac{3}{2} + h - \frac{3}{2}} &= \frac{\frac{27}{8} + \frac{27}{4}h + \frac{9}{2}h^2 + h^3 + 1 - \frac{35}{8}}{h} \\
&= \frac{\frac{27}{4}h + \frac{9}{2}h^2 + h^3}{h} \\
&= \frac{27}{4} + \frac{9}{2}h + h^2
\end{aligned}$$

Thus,

$$\lim_{h \to 0} \frac{(3/2 + h)^3 + 1) - (35/8)}{3/2 + h - 3/2} = \lim_{h \to 0} \frac{27}{4} + \frac{9}{2}h + h^2 = \frac{27}{4}.$$

So the tangent line has slope $27/4$ and goes through the point $(3/2, 35/8)$. Using the slope-point equation of a line, we have

$$y - \frac{35}{8} = \frac{27}{4}\left(x - \frac{3}{2}\right)$$

Multiplying both sides by 8 and then distributing gives

$$\begin{aligned}
8y - 35 &= 54x - 81 \\
-54x + 8y + 56 &= 0 \\
27x - 4y - 23 &= 0
\end{aligned}$$

It is also acceptable to give the equation as $y = 27x/4 - 23/4$.