Math 107, Sect. 250 **Answers to Quiz # 1** **11 Jan '07**

. 5 1. Find the total area between the graph of $y = x - x^{1/3}$ and the x-axis over $[0, 4]$.

Solution. To find the total area, we need to find the roots of $x - x^{1/3}$.

$$\begin{aligned} x - x^{1/3} &= 0 \\ x &= x^{1/3} \\ x^3 &= x \qquad \text{cubing both sides may give spurious roots} \\ x^3 - x &= 0 \\ x(x^2 - 1) &= 0 \\ x(x+1)(x-1) &= 0 \end{aligned}$$

By putting $x = 0$ $x = -1$ and $x = 1$ into $x - x^{1/3}$, we see all three roots are valid. By graphing or putting $x = 1/2$ and $x = 2$ into $x - x^{1/3}$, we see $y = x - x^{1/3}$ is below the x-axis on $(0, 1)$ and above it on $(1, 4)$. So we'll have to split the integral into two parts: one from 0 to 1 and the other from 1 to 4.

Thus, the total area is

$$\int_0^1 -(x - x^{1/3})\,dx + \int_1^4 x - x^{1/3}\,dx.$$

Notice that

$$\int x - x^{1/3}\,dx = \frac{x^2}{2} - \frac{3x^{4/3}}{4} + C$$

and so the total area is

$$\begin{aligned} \int_0^1 -(x - x^{1/3})\,dx + \int_1^4 x - x^{1/3}\,dx &= -\frac{x^2}{2} + \frac{3x^{4/3}}{4}\bigg|_0^1 + \frac{x^2}{2} - \frac{3x^{4/3}}{4}\bigg|_1^4 \\ &= \left(-\frac{1}{2} + \frac{3}{4}\right) - 0 + \left(8 - 3 \cdot 4^{1/3}\right) - \left(\frac{1}{2} - \frac{3}{4}\right) \\ &= \frac{17}{2} - 3 \cdot 2^{2/3} \end{aligned}$$

. 5 2. Evaluate $\int \sin 2x e^{\sin^2 x}\,dx.$ REMEMBER $\sin 2x = 2 \sin x \cos x$.

Solution. Making the substitution $u = \sin x$, we have $du = \cos x dx$, and so $du = \cos x\,dx$. Using the trig identity and then making this substitution, we have

$$\begin{aligned} \int \sin 2x e^{\sin^2 x}\,dx &= \int 2 \sin x \cos x e^{\sin^2 x}\,dx \\ &= \int 2u e^{u^2}\,du \end{aligned}$$

and now substitute $v = u^2$ with $dv = 2u\,du$ to get

$$
\begin{aligned}
&= \int e^v \, dv \\
&= e^v + C \\
&= e^{u^2} + C \\
&= e^{\sin^2 x} + C
\end{aligned}
$$