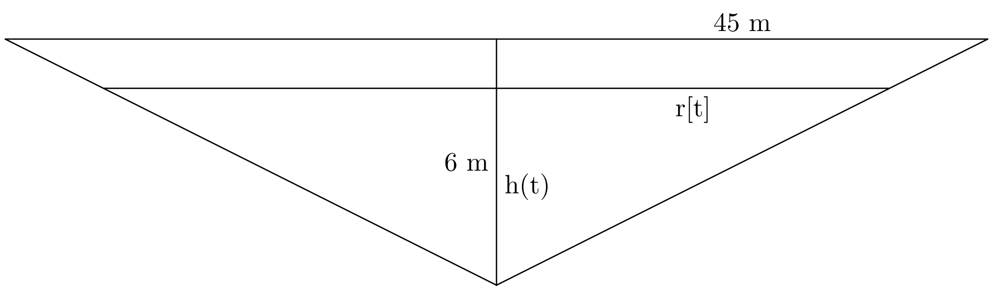
1. Water is flowing at the rate of 50 m^3/min from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

   (a) How fast (in centimeters per minute) is the water level falling when the water is 5 m deep?

   (b) How fast (in centimeters per minute) is the radius of the water's surface changing then?

   Be sure to draw a relevant picture and define your variables (in a sentence or two).

   Solution. Let t be time (in minutes), $h(t)$ the height of the water, and $r(t)$ the radius of the water's surface, (both in meters), as in Figure 1. Let $V(t)$ be the volume of the water, in cubic meters.



We are told that

$$\frac{dV}{dt} = -50. \tag{1}$$

By similar triangles, we know

$$\frac{h}{r} = \frac{6}{45}$$

and so $h = 2r/15$. In particular, when $h = 5$ m, $r = 37.5$ m.
Since the formula for the volume of a cone $\Pi/3r^2h$, we have

$$V = \frac{\Pi}{3}r^2\left(\frac{2r}{15}\right) = \frac{2\Pi}{45}r^3.$$

Since V and r are both functions of t, we have

$$\frac{dV}{dt} = \frac{2\Pi}{45}\left(3r^2\frac{dr}{dt}\right)$$

Solving for $\frac{dr}{dt}$ substituting in (1) and $r = 37.5$, we have

$$\frac{dr}{dt} = \frac{15}{2\Pi r^2}\frac{dV}{dt} = \frac{15}{2\Pi(37.5)^2}(-50) = \frac{-4}{15\Pi} = -0.08488.$$

Notice that the units for this number are meters per minute. To convert to centimeters per minute, we multiply by 100, giving the answer to part b) to be -8.488 cm/min.