Math 496-004 **Assignment # 8** **22 Oct 08**

Due: Oct 29th

1. Do problem II.6.10 in Edwards (page 128). That is, Let $f(x) = x\arctan(x) - \sin^2 x$. Assuming that order 6 Taylor polynomials at 0 for $\arctan(x)$ and $\sin^2 x$ are
$$x - \frac{x^3}{3} + \frac{x^5}{5} + O(x^7), \qquad x^2 - \frac{x^4}{3} + \frac{2}{45}x^6 + O(x^8),$$
respectively, prove that
$$f(x) = \frac{7}{45}x^6 + O(x^8).$$
Show, using the work in class [proof of Theorem 6.3] that f has a local minimum at 0.

2. (a) For a differentiable $g : \mathbb{R} \to \mathbb{R}$, prove that if g has only one critical point, a local minimum at $a \in \mathbb{R}$, then g has a global minimum at a.
   (b) The previous part is not true in several variables. Consider $f(x, y) = e^{3x} + y^3 - 3ye^x + 1$.
      i. Prove that $(0, 1)$ is a local minimum.
         HINT: express $f(x, y)$ in terms of $a = e^x - 1$ and $b = y - 1$.
      ii. Show that $(0, 1)$ is the only critical point.
      iii. Is $(0, 1)$ a global minimum?
   (c) Can you explain (briefly) what goes wrong in trying to extend your proof of part (a) to the function in part (b)?

3. Do problem II.7.6 in Edwards (page 141). That is, let $f(x, y) = e^{xy}\sin(x+y)$. Multiply together the Taylor expansions
$$e^{xy} = 1 + xy + \frac{x^2y^2}{2} + R(x, y)$$
and
$$\sin(x + y) = (x + y) - \frac{(x + y)^3}{6} + S(x, y)$$
and apply Theorem 7.4 to show that the order 3 Taylor polynomial for f at 0 is
$$x + y + \frac{-x^3 + 3x^2y + 3xy^2 - y^3}{6}.$$

4. Do problem II.7.10 in Edwards (page 141). That is, classify the critical point $(-1, \pi/2, 0)$ of $f(x, y, z) = x\sin z - z\sin y$.