Math 325-001 **Assignment # 6** **25 Feb 09**

Due: March 4th

1. Prove that a bounded nonincreasing sequence (a_n) converges.

2. Suppose that $S \subseteq \mathbb{R}$ is not empty. Prove that there is a nondecreasing sequence (s_n) of element of S so that $\lim\limits_{n\to\infty} s_n = \sup S$.

   NOTE: Your proof should have at least two cases: $\sup S \in \mathbb{R}$ and $\sup S = +\infty$.

3. Let $s_1 = 1$ and $s_{n+1} = (s_n + 1)/3$ for $n \geq 1$.

   (a) Find s_2, s_3, and s_4.

   (b) Use induction to show that $s_n > 1/2$ for all n.

   (c) Show that (s_n) is a nonincreasing sequence.

   (d) Show that $\lim_{n\to\infty} s_n$ exists and find $\lim_{n\to\infty} s_n$.

4. No proofs are required in this question. For each of the sequences below, answer the following questions:

   (a) Give an example of a monotone subsequence,

   (b) Give the set of subsequential limits,

   (c) Give the lim sup and the lim inf.

   (d) Does it converge? diverge to $+\infty$? diverge to $-\infty$?

$$a_n = (-1)^n, \qquad b_n = \frac{1}{n}, \qquad c_n = n^2, \qquad d_n = \frac{6n+4}{7n-3}$$