

## Math 825 & 826 - Mathematical Analysis I & II

2007-2008

### Approximate Syllabus

You may also want to look at listing of topics (and old qualifying exams) on the department's webpage. The following outline may be modified.

**Introduction—1 week** development of analysis; role of proofs; review of proof techniques

**Real Numbers—3 weeks** infinite decimals; limits and their properties; upper and lower bounds; subsequences; Cauchy sequences

**Series—2 weeks** convergence and convergence tests; absolute and conditional convergence

**Topology of  $\mathbb{R}^n$ —2 weeks** convergence and completeness; open and closed sets; compactness and the Heine-Borel Theorem

**Functions—2/3 weeks** continuous functions and their properties; compactness and extreme values; uniform continuity; Intermediate Value Theorem

**Calculus—3/4 weeks** differentiable functions; the Mean Value Theorem; Riemann integration; Fundamental Theorem of Calculus; Stirling's formula; Riemann-Stieltjes integration

**Cardinality—1 week** countable sets; diagonal arguments; Schroeder-Bernstein Theorem

**Christmas break**

**Normed Vector Spaces—3 weeks** examples and topology; inner product spaces; orthonormal sets and orthogonal expansions

**Limits of Functions—3 weeks** uniform vs. pointwise convergence; properties of uniform convergence; series of functions and power series

**Metric Spaces—3 weeks** compactness in terms of open covers and the Borel-Lebesgue Theorem; Baire category; completion of a metric space

**Approximation by Polynomials—3 weeks** Taylor series; Weierstrass's Theorem; characterizing best approximations; Chebyshev polynomials

**Fourier Series & Approximation—3 weeks, or as time allows** orthogonality relations; least squares approximation; Riemann-Lebesgue Lemma; pointwise convergence of Fourier series; Gibbs's phenomenon; Cesaro summation of Fourier series