Math 104-004 **Answers to Quiz # 6p** **12 Oct 07**

. 4 1. Find the derivative of $g(x) = \ln\left((e^{x^2} + 5x)^{3/2}\right)$.

HINT: Simplify the function before differentiating.

Solution. This question is similar to questions 8 and 10 on page 260.
First, rewrite $g(x)$ as

$$g(x) = \frac{3}{2}\ln\bigl(e^{x^2} + 5x\bigr)$$

using the property $\ln(a^r) = r\ln(a)$.
Using the rule of the derivative of $\ln(f(x))$ and then the rule for the derivative of $e^{f(x)}$, we get

$$g'(x) = \frac{3}{2}\,\frac{e^{x^2}(2x) + 5}{(e^{x^2} + 5x)} = \frac{2xe^{x^2} + 5}{2(e^{x^2} + 5x)}.$$

If you don't simplify the function first, then using the rule for the derivative of $\ln(f(x))$ and then the chain rule, and then the rule for the derivative of $e^{f(x)}$, we get

$$\begin{aligned} g'(x) &= \frac{\frac{3}{2}\bigl(e^{x^2} + 5x\bigr)^{1/2} \cdot (e^{x^2}(2x) + 5)}{(e^{x^2} + 5x)^{1/2}} \\ &= \frac{2xe^{x^2} + 4}{2(e^{x^2} + 5x)^{1/2} \cdot (e^{x^2} + 5x)^{1/2}} \\ &= \frac{2xe^{x^2} + 4}{2(e^{x^2} + 5x)}. \end{aligned}$$

. 6 2. For the function $f(x) = x^3 + 3x^2 - 45x + 2$, find its critical numbers, the open intervals where it is increasing, and the open intervals where it is decreasing.

Solution. The derivative of $f(x)$ is $f'(x) = 3x^2 + 6x - 45 = 3(x^2 + 2x - 15) = 3(x + 5)(x - 3)$. Thus, the critical numbers of $f(x)$ are -5 and 3.
By drawing the number line and choosing points in each of the intervals $(-\infty, -5)$, $(-5, 3)$ and $(3, +\infty)$, we can see if the function is increasing or decreasing.
For $(-\infty, -5)$, we pick $x = -6$ and get $f'(-6) = 3(-6+5)(-6-3) = 3(-1)(-9) > 0$. For $(-5, 3)$, we pick $x = 0$ and get $f'(0) = 3(5)(-3) < 0$. For $(3, +\infty)$, we pick $x = 4$ and $f'(4) = 3(4 + 5)(4 - 3) = 3 \cdot 9 \cdot 1 > 0$.
By the test for increasing/decreasing, $f(x)$ is increasing on $(-\infty, -5)$ and $(3, +\infty)$, while it is decreasing on $(-5, 3)$.