Math 106-350/550 Answers to Homework #11 1 Dec '06

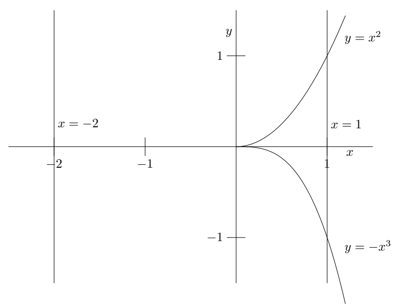
. 10 1. Consider the region bounded by $y = x^2$, $y = -x^3$ and $x = 1$, revolved around the vertical axis $x = -2$. (Ignore the region bounded only by $y = x^2$ and $y = -x^3$ between $x = 0$ and $x = -1$.)

(a) Sketch the region, including intersection points and the axis of revolution.

(b) Set up (but do not evaluate) the integral(s) for the volume of revolution using the washer method.

(c) Set up (but do not evaluate) the integral(s) for the volume of revolution using the shell method.

Solution. Here is the sketch:



We found the intersection points by observing that $y = x^2$ and $y = -x^3$ both pass through $(0, 0)$ and that when $x = 1$, then $y = x^2$ has $y = 1$ and $y = -x^3$ has $y = -1$.

For part (b), we are using horizontal slices with y ranging from -1 to 1. Notice that for y between -1 and 0, the bounding curves are $y = -x^3$ and $x = 1$, while for y between 0 and 1, the bounding curves are $y = x^2$ and $x = 1$. Thus, we need two different slices and two different integrals.

Since we are using horizontal slices, we need solve for x in $y = x^2$, giving $x = \sqrt{y}$, and in $y = -x^3$, giving $x = -y^{1/3}$ (we used $(-y)^{1/3} = -y^{1/3}$ in solving for x).

For y from -1 to 0, the left endpoint is $(-y^{1/3}, y)$ and the right endpoint is $(1, y)$. Thus, the inside radius, the distance from $(-y^{1/3}, y)$ to $(-2, y)$ is $-y^{1/3} - (-2) = -y^{1/3} + 2$, and the outside radius, the distance from $(1, y)$ to $(-2, y)$, is 3. Thus, the integral for this part of the volume is

$$\int_{-1}^{0} \pi(9 - (2 - y^{1/3})^2)\, dy.$$

For y from 0 to 1, the left endpoint is $(\sqrt{y}, y)$ and the right endpoint is $(1, y)$. Thus, the inside radius, the distance from $(\sqrt{y}, y)$ to $(-2, y)$

is $\sqrt{y} - (-2) = \sqrt{y} + 2$, and the outside radius, the distance from $(1, y)$ to $(-2, y)$, is 3. Thus, the integral for this part of the volume is

$$\int_0^1 \pi(9 - (\sqrt{y} + 2)^2)\, dy.$$

Thus, the total volume, using the washer method is

$$\int_{-1}^0 \pi(9 - (2 - y^{1/3})^2)\, dy + \int_0^1 \pi(9 - (\sqrt{y} + 2)^2)\, dy.$$

For (c), we use a vertical slice with x ranging from 0 to 1. Since the bounding curves for the slice are always $y = x^2$ and $y = -x^3$, we only need once slice and one integral.

The upper endpoint is (x, x^2) and lower endpoint is $(x, -x^3)$. So, the height of the shell is $x^2 - (-x^3) = x^2 + x^3$ and radius of the shell is distance from the axis to the slice, $x - (-2) = x + 2$. Thus, the total volume, using the shell method, is

$$\int_0^1 2\pi(x + 2)(x^2 + x^3)\, dx.$$