3 1. Find $g'(x)$ if $g(x) = \dfrac{12}{x^2} - 4\sqrt{x}$. (Use the rules!)

Solution. Notice that $g(x) = 12x^{-2} - 4x^{1/2}$ and so, using the power rule,

$$g'(x) = -24x^{-3} - 4 \cdot \frac{1}{2}x^{-1/2} = -\frac{24}{x^3} - \frac{2}{\sqrt{x}}.$$

7 2. Using the definition, find the derivative of $p(t) = \dfrac{1}{2t^2}$. Find the equation of the tangent line to $y = p(t)$ at $t = 2$.

Solution. Using the definition, we have

$$\begin{aligned} p'(t) &= \lim_{h\to 0} \frac{p(t+h) - p(t)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{2(t+h)^2} - \frac{1}{2t^2}}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{2(t+h)^2} - \frac{1}{2t^2}\right) \end{aligned}$$

The common denominator is $2t^2(t+h)^2$, so have

$$\begin{aligned} &= \lim_{h\to 0} \frac{1}{h}\left(\frac{t^2}{2t^2(t+h)^2} - \frac{(t+h)^2}{2t^2(t+h)^2}\right) \\ &= \lim_{h\to 0} \frac{1}{h}\left(\frac{t^2 - (t^2 + 2th + h^2)}{2t^2(t+h)^2}\right) \\ &= \lim_{h\to 0} \frac{1}{h}\left(\frac{-2th - h^2}{2t^2(t+h)^2}\right) \\ &= \lim_{h\to 0} \left(\frac{-2t - h}{2t^2(t+h)^2}\right) \\ &= \frac{-2t}{2t^4} = \frac{-1}{t^3} \end{aligned}$$

Thus, $p'(2) = -1/8$ and since $p(2) = 1/8$, the tangent line has slope $-1/8$ and goes through $(2, 1/8)$. So it's equation is $y - 1/8 = -1/8(x-2)$, which simplifies to $y = -1/8x + 3/8$.