Math 106, Sect. 550 **Answers to Quiz # 2** **31 Aug '06**

. 5 1. Find the limit $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$, making sure to state what limit theorems you are using.

Solution. We can factor the numerator as $x - 4 = (\sqrt{x}-2)(\sqrt{x}+2)$. So

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \lim_{x\to 4}(\sqrt{x}+2).$$

By the sum and power rules for limits, $\lim_{x\to 4}(\sqrt{x}+2) = \sqrt{\lim_{x\to 4} x} + 2 = 4$.

. 5 2. Does the limit $\lim_{x\to 0} \dfrac{x^2}{|x|}$ exist? Explain why or why not.

Solution. The limit does exist. Recall that $|x| = x$ for $x \geq 0$ and $|x| = -x$ for $x \leq 0$. So $\lim_{x\to 0^+} \dfrac{x^2}{|x|} = \lim_{x\to 0^+} x = 0$ and $\lim_{x\to 0^-} \dfrac{x^2}{|x|} = \lim_{x\to 0^-} -x = 0$. Since both one-sided limits exist and are equal, the limit exists and is 0.