Math 496-004 **Assignment # 4** **17 Sept 08**

Due: Sept 23rd

1. Suppose $F : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $F(x, y, z) = (x^2y + (z/y), z^2x - y)$.
   (a) Find $dF_{(3,2,4)}$.
   (b) Using the differential, approximate $F(3.01, 2.08, 3.98)$.

2. Suppose $F : \mathbb{R}^3 \to \mathbb{R}$ is given by $F(x, y, z) = \dfrac{x^2y^{3/2}z}{z+1}$. Note that $F(5, 4, 1) = 100$.
   (a) If we change x to 5.03 and y to 3.92, then how much should we change z in order to keep the value of F equal to 100?
   (b) Suppose we want to increase the value of F but can only change one of the independent variables. Which variable should we change to get the biggest change in the value of F for the smallest change in the independent variable?

   Of course, you should justify your answers to each of these questions.

3. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Prove that f is continuous at $(0, 0)$.
   Hint: First show that $|xy| \leq x^2 + y^2$ for all $x, y \in \mathbb{R}$.
   (b) For each direction vector $v \in \mathbb{R}^2$, show that $D_v f$ exists and compute it.
   (c) Show that f is **not** differentiable at $(0, 0)$.
   Hint: If it was, we could compute the directional derivatives using the partial derivatives at $(0, 0)$. Compare these formulae to your answer to part (b).

4. Consider $f : \mathbb{R} \to \mathbb{R}^n$ with $df_t \neq 0$ for all $t \in \mathbb{R}$. Suppose $p \in \mathbb{R}^n$. If there is $t \in \mathbb{R}$ so that $q = f(t)$ the point of the image closest to p, show that $p - q$ is orthogonal to $df_t(1)$.

   Here, "point of the image closest to p" means that $\|p - f(t)\| \leq \|p - f(s)\|$ for all $s \in \mathbb{R}$.

   Hint: Differentiate $\phi(s) = \|p - f(s)\|^2$. Note that ϕ maps $\mathbb{R}$ into $\mathbb{R}$.