Math 104-004 **Answers to Quiz # 7b** **26 Oct 07**

. 6 1. Find all relative extrema of $f(x) = x^3 + 3x^2 - 9x + 5$ and the x-value where each occurs.

Solution. First, $f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$. Thus, $f'(x) = 0$ when $x = -3$ or when $x = 1$. Notice that $f'(x)$ is never undefined.

Charting $f'(x)$, we have $f'(-4) = 3(-1)(-5) > 0$, $f'(0) = -9 < 0$, and $f'(2) = 3(5)(1) > 0$. Since there is a sign change of $f'(x)$ at both $x = -3$ and at $x = 1$, both of these x-values give relative extrema.

As $f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5 = 32$, we have a relative extrema of 32 at $x = -3$. As $f(1) = (1)^3 + 3(1)^2 - 9(1) + 5 = 0$, we have a relative extrema of 0 at $x = 1$.

. 4 2. For the function $f(x) = x^3 - 3x^2 + 3x + 7$, find its inflection points, the open intervals where it is concave up, and the open intervals where it is concave down.

Solution. First, $f'(x) = 3x^2 - 6x + 3$ and so $f''(x) = 6x - 6$. Thus, $f''(x) = 0$ when $x = 1$.

Charting $f''(x)$, we have $f''(0) = -6 < 0$ and $f''(2) = 6(2) - 6 > 0$, so $f''(x)$ changes sign at $x = 1$. As $f(1) = 1 - 3 + 3 + 7 = 8$, we have that $(1, 8)$ is an inflection point. Since $f''(x)$ is negative on $(-\infty, 1)$, the graph is concave down on that interval. Since $f''(x)$ is positive on $(1, +\infty)$, the graph is concave up on that interval.