Math 107, Sect. 250 **Answers to Quiz # 2** **18 Jan '07**

. 5 1. Evaluate $\int x^2 \ln(x^2)\, dx$.

Solution. We use integration by parts. Let $u = \ln(x^2) = 2\ln(x)$ and $dv = x^2\, dx$; then $du = 2/x\, dx$ and $v = x^3/3$. Thus

$$\begin{aligned}
\int x^2 \ln(x^2)\, dx &= \frac{x^3}{3}\ln(x^2) - \int \frac{x^3}{3}\frac{2}{x}\, dx \\
&= \frac{x^3}{3}\ln(x^2) - \frac{2}{3}\int x^2\, dx \\
&= \frac{2x^3}{3}\ln(x) - \frac{2}{3}\left(\frac{x^3}{3}\right) + C \\
&= \frac{2x^3\ln(x)}{3} - \frac{2x^3}{9} + C
\end{aligned}$$

The simplification $\ln(x^2) = 2\ln(x)$ in the last line is optional.

. 5 2. Evaluate $\int_0^{\pi/2} \sqrt{1-\cos^2 t}\, dt$.

Solution. Using the trig identity $1 - \cos^2 t = \sin^2 t$, we have

$$\int_0^{\pi/2} \sqrt{1-\cos^2 t}\, dt = \int_0^{\pi/2} \sqrt{\sin^2 t}\, dt$$

and, since $\sin t \geq 0$ on $[0, \pi/2]$, this simplifies to

$$\begin{aligned}
&= \int_0^{\pi/2} \sin t\, dt \\
&= -\cos t\big|_0^{\pi/2} \\
&= -0 + 1 = 1
\end{aligned}$$