Due: Wednesday, December 5th

1. Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Q} \\ \frac{1}{q^3} & \text{if } x = \frac{p}{q} \text{ in lowest terms and } q > 0. \end{cases}$$

   Show that $f'(\sqrt{3})$ exists and is zero.

   HINT: Exercise 2.2.H could be useful.

2. (June 1998 Qual) Suppose that $f : [0, +\infty) \to \mathbb{R}$ is twice differentiable on $[0, +\infty)$ and satisifies $f(0) = 0$, $f'(0) = 1$, and $f''(x) \leq 0$ for all x.
   (a) Prove that $f(x) \leq x$ for all $x > 0$.
   (b) Prove that $f(x)/x$ is decreasing on $[0, +\infty)$.

3. Do Exercise 6.2.E in the text.

4. Do Exercise 6.2.I in the text.

5. Suppose that $f : [a, b] \to \mathbb{R}$ is integrable and $g : [a, b] \to \mathbb{R}$ has $g(x) = f(x)$ for all x in $[a, b]$ except $c_0, \ldots, c_n$. Prove that g is integrable and $\displaystyle\int_a^b g(x)\,dx = \int_a^b f(x)\,dx$.