Math 107, Sect. 350 **Answers to Quiz # 5** **20 Feb '07**

. 5 1. Find the center of mass of a thin plate covering the region enclosed by the parabola $y = 2x^2$ and the line $y = 2$ if the density function is $\delta(x) = 3x^2$.

Solution. Note that, the mass distribution is symmetric about the y-axis, so $\bar{x} = 0$. **The typical vertical strip has**
center of mass : $(\tilde{x}, \tilde{y}) = (x, 1 + x^2)$
width : dx
area : $dA = (2 - 2x^2)dx$
mass : $dm = (3x^2)(2 - 2x^2)dx = 6(x^2 - x^4)dx$
The moment of the strip about the x-axis is

$$\tilde{y}\, dm = 6(1 + x^2)(x^2 - x^4)dx$$

The moment of the region about the x-axis is

$$M_x = \int \tilde{y}\, dm = \int_{-1}^{1} 6(1 + x^2)(x^2 - x^4)\, dx = \frac{16}{7}$$

Also,

$$M = \int dm = \int_{-1}^{1} (3x^2)(2x^2 - 2x^4)\, dx = \frac{8}{5}$$

Therefore,

$$\bar{y} = \frac{M_x}{M} = \frac{16}{7} . \frac{8}{5} = \frac{10}{7}$$

Hence, **the plate's center of mass is** $(\bar{x}, \bar{y}) = (0, \dfrac{10}{7})$

. 5 2. Determine if $a_n = \dfrac{\sin(n)}{n^2}$ converges or diverges. If it converges, find its limit.

Solution. Note that

$$\frac{-1}{n^2} \leq \frac{\sin(n)}{n^2} \leq \frac{1}{n^2}$$

for every natural number n and

$$\lim_{n \to \infty} \frac{-1}{n^2} = \lim_{n \to \infty} \frac{1}{n^2} = 0\,.$$

Therefore, by the Sandwich Theorem, $\displaystyle\lim_{n \to \infty} \frac{\sin(n)}{n^2} = 0$ as well.