Math 896-503 **Assignment # 7** **18 June 09**

Due: June 22nd

1. (a) For a differentiable $g : \mathbb{R} \to \mathbb{R}$, prove that if g has only one critical point, a local minimum at $a \in \mathbb{R}$, then g has a global minimum at a.

   (b) The previous part is not true in several variables. Consider $f(x, y) = e^{3x} + y^3 - 3ye^x + 1$.

      i. Prove that $(0, 1)$ is a local minimum.
      HINT: express $f(x, y)$ in terms of $a = e^x - 1$ and $b = y - 1$.

      ii. Show that $(0, 1)$ is the only critical point.

      iii. Is $(0, 1)$ a global minimum?

   (c) Can you explain (briefly) what goes wrong in trying to extend your proof of part (a) to the function in part (b)?