

5

1. Using a suitable substitution, evaluate $\int t(1+t)^{1/4} dt$.

Be prepared to take several tries to find the right substitution; you may want to look at what we did in class on Wednesday or the “challenge integrals” on the course website: <http://www.math.unl.edu/~adonsig1/107.html>.

Solution. The simplest substitution is $u = 1 - t$, so that $t = 1 - u$ and $du = dt$. Using this,

$$\begin{aligned} \int t(1+t)^{1/4} dt &= \int (u-1)u^{1/4} du \\ &= \int u^{5/4} - u^{1/4} du \\ &= \frac{4u^{9/4}}{9} - \frac{4u^{5/4}}{5} + C \\ &= \frac{4u^{5/4}45}{9u-5} + C \\ &= \frac{4(1+t)^{5/4}}{45}(9(1+t)-5) + C \\ &= \frac{4(1+t)^{5/4}}{45}(5t-4) + C \end{aligned}$$