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1. Use the properties of limits to decide if the following limit exists. If does exist, find its value.

$$\lim_{x \rightarrow +\infty} \frac{x^2 - 2x - 8}{5x^2 + 10x}$$

*Solution.* First, we compute

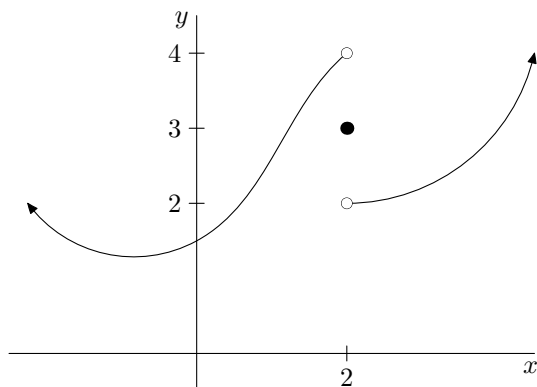
$$\lim_{x \rightarrow +\infty} \frac{x^2 - 2x - 8}{5x^2 + 10x} = \lim_{x \rightarrow +\infty} \frac{1 - 2/x - 8/x^2}{5 + 10/x}$$

Now, as  $x \rightarrow +\infty$ ,  $1/x$  and  $1/x^2$  go to zero, so

$$\lim_{x \rightarrow +\infty} \frac{1 - 2/x - 8/x^2}{5 + 10/x} = \lim_{x \rightarrow +\infty} \frac{1 - 0 - 0}{5 + 0} = \frac{1}{5}.$$

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2. Using the graph below, find  $\lim_{x \rightarrow 2^-} f(x)$ ,  $\lim_{x \rightarrow 2^+} f(x)$ ,  $\lim_{x \rightarrow 2} f(x)$ , and  $f(2)$ , if they exist.



*Solution.* We have

$$\lim_{x \rightarrow 2^-} f(x) = 4, \quad \lim_{x \rightarrow 2^+} f(x) = 2, \quad \lim_{x \rightarrow 2} f(x) \text{ does not exist}, \quad f(2) = 3.$$