Math 496-004 **Assignment # 1** **27 Aug 08**

Due: Sept 3rd

1. Do Exercise S-3.74 (b), that is, Show that $f(x) = 1/x$ is continuous in $1 \leq x \leq 3$.
2. For S a subset of $\mathbb{R}^n$, define

$$S^{\perp} = \{x \in \mathbb{R}^n : x \cdot s = 0 \text{ for all } s \in S\}.$$

   (a) Show that $S^{\perp}$ is a subspace of $\mathbb{R}^n$.
   (b) Show that $S^{\perp} = (\operatorname{span} S)^{\perp}$.
   (c) Show that $S \subseteq (S^{\perp})^{\perp}$.
   (d) Show there is a nonempty set S with $S \neq (S^{\perp})^{\perp}$.

3. Show that, for all $x, y \in \mathbb{R}^n$, $\big|\|x\| - \|y\|\big| \leq \|x - y\|$.
4. For what vectors $x, y \in \mathbb{R}^n$ is Cauchy's inequality an equality? That is, $|x \cdot y| = \|x\| \, \|y\|$. HINT: Examine the proof of Cauchy's inequality.