Math 107 - Analytic Geometry & Calculus I **2nd Semester, '06-'07**

Group Project

The Good Earth

Due: Tuesday, April 3 in class

Introduction. Soil temperature, obviously, depends on where on Earth your soil is, what time of year it is, how deep under the surface the soil is, as well as such things as the type of soil and whether it is wet or dry. Accurate models of soil temperature are important for, among other things, climate modeling, predicting crop growth, or even just deciding the most cost-efficient depth for the pipes in heat pump or how deep water pipes should be to avoid freezing. In this project, we derive some simple models of soil temperature, based on Newton's law of cooling. We will ignore the daily variation in surface temperatures, since this only affects the topmost layer of soil, and concentrate on seasonal variations.

Once you get to a depth of about 30 feet, the soil temperature doesn't vary with the time of year, but instead more or less corresponds to groundwater temperature. Sometimes, this is called the *mean earth temperature*. For Lincoln, the mean earth temperature is about 51° F. It varies across the country, from below 40 in northern Minnesota to over 75 in Florida and southern California.

The National Climatic Data Center reports[1] that, for Lincoln, the mean annual (air) temperature is $51°F$, while the mean daily (air) temperature for each month is:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp (° F)	22.4	28.3	39.4	51.2	62.0	72.7	77.8	75.4	66.0	53.5	38.1	26.5

As a function of depth. First, we fix time and consider soil temperature as a function of depth. Using Newton's law of cooling, set up initial value problems for the soil temperature as a function of depth, assuming the surface temperature is at the extreme high and low temperatures in the above table and the temperature at a depth of 30 feet should be within a degree or so the equilibrium temperature of 51° F.

Solve these initial value problems and, based on the one for January, decide how deep water pipes should be buried. Is it reasonable to assume that the coldest sustained temperature in Lincoln is 22.4° F, based on your personal experience? By sustained, we mean a temperature that could last for a week or so. If not, decide what would be a reasonable coldest sustained temperature, solve that initial value problem, and compare it to the results using the mean daily temperature for January. You should justify your choice for the coldest sustained temperature.

As a function of time. Second, we fix the depth in the soil and consider soil temperature as a function of time of year. Let t be the time of year, where t runs from 0 to 12, with $t = 0$ to $t = 1$ corresponding to January, and so on.

We need a continuous function for the surface temperature. Find a function of the form

$$S(t) = 51 + A\cos\left(\frac{\pi}{6}(t + C)\right)$$

that is a reasonable approximation for the mean daily temperatures given above. Plot your function and the data points above on a graph and explain why you think your chosen function is reasonable.

Next, set up a differential equation, using Newton's law of cooling, for the soil temperature at some fixed depth, with the "environmental" temperature equal to $S(t)$. Notice that, because the environmental temperature is not constant, this DE is not separable and you cannot solve it exactly. Instead, find approximate solutions using Euler's method.

You have to learn what Euler's method is. Your report should have a brief discussion of Euler's method (in your own words), including the sources you used to learn about it and what "step size" means in Euler's method. Your calculator or a mathematical software package can find approximate solutions using Euler's

[1] Data from `http://lwf.ncdc.noaa.gov/oa/climate/online/ccd/meantemp.html`.

method (maple, matlab, and mathematica can all do this, for example). Your report should have an appendix explaining the commands you used, either on your calculator or in the software package.

Your differential equation should have a proportionality constant, k, in it. For various values of k, give a graph of the approximate solution and $S(t)$ on the same axes. What does a large or small value of k signify, in terms of the physical situation? In particular, what is k if the depth is zero, if the depth is about 30 ft, and how should k change as you increase the depth?

Finally, based on your graphs, how do the maximum and minimum soil temperature change as the depth changes? Can you characterize where the maximum and minimum are?

Project Advice

Overview. The project is the solution to an open-ended multistep problem, formally presented. It will probably require several meetings for your group to find a solution to the problem and to present that solution clearly and understandably. Everyone in the group should contribute to the project.

The intent of projects is to expose you to mathematics as you might meet it in the real world, i.e., working as a team. Your group must understand the problem; translate it into mathematics; learn, read about, or develop mathematical methods to find the answer; show that the answer is correct; translate the mathematical answer back into the original problem and, finally, explain the significance of the translated answer. Projects are easier than real world problems, in that we make sure that the problem can be solved using the methods of this course. You may need to learn some new information to do the project.

Project Report. Your group should write up a short paper explaining the problem and the mathematics you used to solve it, and then discussing the significance of your solution. Your paper should be a grammatically correct, organized discussion of the problem, with an introduction and a conclusion. While you should answer the specific questions asked in the project, your report should *not* be a disconnected set of answers but a connected narrative with transitions. It should conform to proper English usage (yes, spelling counts) and should include appropriate diagrams and/or graphs, clearly labeled. You should show enough relevant calculations to justify your answers but not so much as to obscure the calculations' purpose. Your report should be typed, but it is fine to leave blank spaces and write the equations in.

Explain your results and conclusions, pointing out both strengths and weaknesses of your analysis. Assume that your reader is someone who took a calculus class course a while ago and does not remember all of the details. Be sure to avoid plagiarism.

Preparing formal reports is an important job skill for mathematicians, scientists, and engineers. For example, the Columbia Investigation Board, in its report on the causes of the Columbia space shuttle accident, wrote:

> During its investigation, the board was surprised to receive [PowerPoint] slides from NASA officials in place of technical reports. The board views the endemic use of PowerPoint briefing slides instead of technical papers as an illustration of the problematic methods of technical communication at NASA.

Group Structure. To help your group function smoothly, you might want to consider assigning each person a role, such as:

Convener: Arrange times and places for meetings
Chair: Ensure everyone is involved and understands the ideas discussed
Reporter: Jot down ideas and suggestions as they are discussed
Scribe: Prepare first draft of final report