Math 826 **Assignment # 2** **23 Jan 08**

Due: Wednesday, January 30th

1. Do Exercise 6.7.L in the handout, which should read:

   For h as in Examples 6.7.17, find a continuous function $f : [0, 1/\pi] \to \mathbb{R}$ so that $f \notin \mathcal{R}(h)$. Thus, in Corollary 6.7.22, we cannot change 'g of bounded variation' to 'g continuous'.

2. If $g : [a, b] \to \mathbb{R}$ is increasing and $f \in \mathcal{R}(g)$ on $[a, b]$, then for any subinterval $[c, d] \subset [a, b]$, $f \in \mathcal{R}(g)$ on $[c, d]$.

3. Do Exercise 2.8.J in the text.

4. (January 2002 Qual)

   (a) Let Θ be a collection of pairwise disjoint open intervals of $\mathbb{R}$. Show that Θ is at most countable.

   (b) Show that the set of all increasing sequences of natural numbers (i.e., sequences $(n_1, n_2, \ldots)$ with $n_k \in \mathbb{N}$ and $n_{k+1} \geq n_k$ for all $k \in \mathbb{N}$) is uncountable.