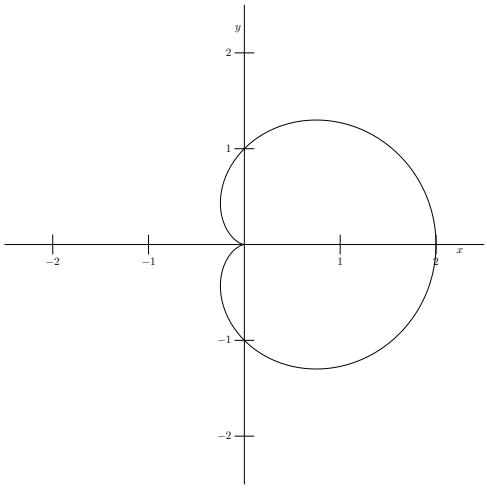
. 10 1. Graph the polar equation $r = 1+\cos\theta$ on the (polar) axes below. List any symmetries that it displays, giving algebraic justifications for your answers.

*Solution.*




This is a cardioid with the main bulb on the right half of the plane. To determine the types of symmetry, we first replace $\theta$ with $-\theta$ and obtain $1 + \cos\theta \longrightarrow 1 + \cos(-\theta) = 1 + \cos\theta = r, \neq -r$. Thus we have $x$-axis symmetry but not symmetry about the $y$-axis. Also, if we replace $r$ with $-r$, we get $r = 1 + \cos\theta \longrightarrow -r = -(1 + \cos\theta) = -1 - \cos\theta \neq 1 + \cos\theta$, which suggests that the graph does not have symmetry about the origin.