

Due: June 10th

1. Consider the function $F : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by

$$F(x, y) = \begin{cases} \frac{2x^2y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

- (a) Show, for any straight line L through $(0, 0)$, the limit of F along the line L is 0.
- (b) Show that, for the function $\phi : \mathbb{R} \rightarrow \mathbb{R}^2 : t \mapsto (t, t^2)$, $\lim_{t \rightarrow 0} F(\phi(t)) = 1$.
- (c) Is it true that $\lim_{(x, y) \rightarrow (0, 0)} F(x, y) = 0$? Justify your answer.