Due: June 15th

1. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Prove that f is continuous at $(0,0)$.
   Hint: First show that $|xy| \leq x^2 + y^2$ for all $x, y \in \mathbb{R}$.

   (b) For each direction vector $v \in \mathbb{R}^2$, show that $D_v f$ exists and compute it.

   (c) Show that f is **not** differentiable at $(0,0)$.
   Hint: If it was, we could compute the directional derivatives using the partial derivatives at $(0,0)$. Compare these formulae to your answer to part (b).

2. Do Problem 3.14 (page 89) in Edwards. That is, the following example illustrates the hazards of denoting functions by real variables. Let $w = f(x,y,z)$ and $z = g(x,y)$. Then

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial x} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial x},$$

   since $\partial x/\partial x = 1$ and $\partial y/\partial x = 0$. Hence $\partial w/\partial x\, \partial z/\partial x = 0$. But if $w = x + y + z$ and $z = x + y$, then $\partial w/\partial z = \partial z/\partial x = 1$, so we have $1 = 0$. Where is the mistake?