. 10

1. Evaluate the integral $\displaystyle\int \frac{v^2}{(1-v^2)^{1/2}}\,dv$.

   Recall that $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ and that $\sin 2\theta = 2\sin\theta\cos\theta$.

   Solution. We set $v = \sin\theta$; then $dv = \cos\theta\,d\theta$ and $\cos\theta = \sqrt{1-v^2}$, according to the table of substitutions given in class.
   Using this substitutions, we get

$$\begin{aligned}
\int \frac{v^2}{(1-v^2)^{1/2}}\,dv &= \int \frac{\sin^2\theta\cos\theta}{\cos\theta}\,d\theta \\
&= \int \sin^2\theta\,d\theta \\
&= \int \frac{1-\cos 2\theta}{2}\,d\theta \\
&= \frac{1}{2}\theta - \frac{\sin 2\theta}{4} + C \\
&= \frac{1}{2}\arcsin v - \frac{\sin\theta\cos\theta}{2} + C \\
&= \frac{1}{2}\arcsin v - \frac{v\sqrt{1-v^2}}{2} + C
\end{aligned}$$