

- 5 1. Suppose that  $f(x)$ ,  $g(x)$  and their derivatives have the following values:

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	1	3	4	2
3	0	2	1	8

Find the derivative of  $f(g(x))$  with respect to  $x$  at  $x = 3$ .

*Solution.* Using the chain rule,  $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$ .  
Substituting 3 for  $x$ , we have

$$\left. \frac{d}{dx}(f(g(x))) \right|_{x=3} = f'(g(3)) \cdot g'(3) = f'(2) \cdot 8 = 4 \cdot 8 = 32.$$

- 5 2. Find the equation for the line tangent to the following curve at the point defined by the given value of  $t$ .

$$x = 3 \cos t, \quad y = -3 \sin t, \quad t = \pi/3$$

*Solution.* Since the slope of a tangent line is equal to the derivative  $\frac{dy}{dx}$ , we use the parametric formula to find  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3 \cos t}{-3 \sin t} = \cot t$$

When  $t = \pi/3$ , the slope is equal to  $\cot(\pi/3) = \sqrt{3}/3$ .

The point on the curve defined by  $t = \pi/3$  is

$$(x, y) = (3 \cos(\pi/3), -3 \sin(\pi/3)) = (3/2, -3\sqrt{3}/2)$$

Therefore, the equation of the tangent line through the point  $(3/2, -3\sqrt{3}/2)$  with the slope  $\sqrt{3}/3$  is

$$y - (-3\sqrt{3}/2) = (\sqrt{3}/3)(x - 3/2)$$

which simplifies to  $y = (\sqrt{3}/3)x - 2\sqrt{3}$ .