. 5

1. Warfarin is a drug used as a an anticoagulant. (It is also used in rat poison.) After administration of the drug is stopped, the quantity in a patient's body decreases at a rate proportional to the quanitity remaining, i.e., the differential equation is

$$\frac{dW}{dt} = kW.$$

A patient took an unknown dose of Warfarin at 8am on Monday. Testing indicates that at 10am on Tuesday, the patient contains 4.00mg of Warfarin and at 10am on Wednesday, it contains 2.55mg. What was the unknown dose of Warfarin?

Solution.   We are looking for a function $W(t)$, the amount of Warfarin in the patient's body, in milligrams, where t is hours since 8am Monday. The answer to the question will be $W(0)$. We know that $W(26) = 4.00$, $W(50) = 2.55$ and $W(t)$ satisfies the differential equation

$$\frac{dW}{dt} = kW,$$

for some value of k.

Separating variables in the DE,

$$\frac{1}{W} dW = k \, dt$$

$$\int \frac{1}{W} dW = \int k \, dt$$

$$\ln W = kt + C$$

$$W = e^{kt+C}$$

$$W = Ke^{kt} \text{ where } K = e^C$$

To find K and k, we need two equations, namely

$$4.00 = Ke^{26k}, \qquad 2.55 = Ke^{50k}$$

Dividing the first equation by the second, we have

$$\frac{4.00}{2.55} = \frac{Ke^{26k}}{Ke^{50k}} = e^{-24k}.$$

Thus, $-24k = \ln(1.569) = 0.450$ and so $k = -0.0188$. Substituting this value of k into the first equation, we have

$$4.00 = Ke^{(-0.0188)26} = Ke^{-0.0489} = K \cdot .613$$

Thus, $K = 4/.613 = 6.52$.
So $W(t) = 6.52e^{-0.0188k}$ and $W(0) = 6.52$. The patient took a dose of 6.52 mg of Warfarin at 8am on Monday.