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1. Find the area of the triangle whose vertices are given by A(0,0,0), B(2,3,-1) and C(3,-1,4).

Solution. Consider the vectors $\overrightarrow{AB} = \langle 2, 3, -1 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, 4 \rangle$. The area of the triangle \overrightarrow{ABC} is half the area of the parallelogram determined by the vectors $\overrightarrow{AB} = 2\vec{\imath} + 3\vec{\jmath} - 1\vec{k}$ and $\overrightarrow{AC} = 3\vec{\imath} - 1\vec{\jmath} + 4\vec{k}$.

Since $\overrightarrow{AB} \times \overrightarrow{AC} = 11(\vec{\imath} - \vec{\jmath} - \vec{k})$, we have:

Area
$$(ABC) = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{11\sqrt{3}}{2}$$
.

2. Find an equation of the line passing through the points P(1, 2, -1) and Q(5, -3, 4). Either parametric or vector form is fine.

Solution. First find a vector that is parallel to the given line. For example, choose the vector $\overrightarrow{PQ} = \langle 4, -5, 5 \rangle$. Pick either point to get the equations for the line:

$$\begin{cases} x = 1 + 4t \\ y = 2 - 5t \\ z = -1 + 5t \end{cases}, t \in \mathbb{R}.$$