Math 325-001 **Assignment # 8** **25 Mar 09**

Due: April 1st

1. For each of the following, decide if the series converges or diverges, proving your answer.

$$\text{a)} \sum_{n=2}^{\infty} \frac{1}{(n+(-1)^n)^2}, \qquad \text{b)} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}-\sqrt{n}}, \qquad \text{c)} \sum_{n=1}^{\infty} \frac{2^n n!}{n^n}.$$

2. Prove that the Harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n}$, diverges by comparing it to the series: $\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{32}+\cdots$.

3. Show that $\sum_{n=2}^{\infty} 1/(n(\ln n)^p)$ converges if and only if $p > 1$.
   HINT: Show that $\int_3^n 1/(x(\ln x)^p)\,dx = \int_{\ln 3}^{\ln n} u^{-p}\,du$.

4. Suppose a_n and b_n are nonnegative for all n.
   (a) Show that if $\limsup_{n\to\infty} \frac{a_n}{b_n} < \infty$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
   (b) Show that if $\liminf_{n\to\infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

5. (a) Prove that if $x, y \geq 0$, then $\sqrt{xy} \leq x + y$.
   (b) Prove that if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent series of nonnegative numbers, then $\sum_{n=1}^{\infty} \sqrt{a_n b_n}$ converges.

6. EXTRA CREDIT: For both of the following, decide if the series converges or diverges, proving your answer.

$$\text{a)} \sum_{n=1}^{\infty} \frac{1}{(\ln n)^{\ln n}}, \qquad \text{b)} \sum_{n=1}^{\infty} \frac{1}{(\ln n)^{\ln(\ln n)}}.$$