Math 106, Sect. 350 — Answers to Quiz # 3 — 25 Jan '07

. 10 1. Evaluate $\int \frac{5x^2 - 10x + 17}{(x-1)(x^2-2x+5)}\,dx$ using partial fraction decomposition.

Solution. Notice that $x^2 - 2x + 5 = (x^2 - 2x + 1) + 4 = (x-1)^2 + 4$ has no real roots, the denominator is already factored. Thus, the form of the partial fractions decomposition is

$$\frac{5x^2 - 10x + 17}{(x-1)(x^2-2x+5)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 - 2x + 5}.$$

Multiplying both sides by $(x-1)(x^2-2x+5)$, we get

$$5x^2 - 10x + 17 = A(x^2 - 2x + 5) + (Bx + C)(x - 1).$$

Setting $x = 1$, we get

$$\begin{aligned} 5 \cdot 1^2 - 10 \cdot 1 + 17 &= A(1^2 - 2 \cdot 1 + 5) \\ 12 &= 4A \end{aligned}$$

and so $A = 3$.
Setting $x = 0$, we get $17 = 5A + C(-1)$ and, since $A = 3$, $C = 5A - 17 = -2$.
Finally, we let $x = -1$ to get

$$\begin{aligned} 5 + 10 + 17 &= 10A + (-B + C)(-2) \\ 32 &= 10(3) + (-B - 2)(-2) \\ 2 - 4 &= -B \end{aligned}$$

and so $B = 2$.
Thus, we have

$$\begin{aligned} \int \frac{5x^2 - 10x + 17}{(x-1)(x^2-2x+5)}\,dx &= \int \frac{3}{x-1} + \frac{2x-2}{x^2-2x+5}\,dx \\ &= 3\ln|x-1| + \int \frac{1}{u}\,du \qquad u = 2x^2 - 2x + 5,\ du = 2x - 2\,dx \\ &= 3\ln|x-1| + \ln|u| + C \\ &= 3\ln|x-1| + \ln|x^2 - 2x + 5| + C \end{aligned}$$