Math 106-350/550 Answers to Homework #7 20 Oct '06

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1. Define $f(x) = ax^2 + bx + c$, where $a \neq 0$. Find the intervals where $f(x)$ is increasing or decreasing. Does $f(x)$ have an absolute maximum or absolute minimum on $(-\infty, +\infty)$? If so, where are they?

   HINT: you may want to consider the cases $a > 0$ and $a < 0$ separately. Don't be intimidated by the parameters.

   Solution. First, we do what we would do for any other function, find the critical points and work out the sign of the derivative on each interval. Notice that $f'(x) = 2ax + b$. Since $f'(x)$ is never undefined, there is only one critical point, $x = -b/(2a)$.

   To work out the sign of the derivative, we need to split into two cases, based on whether $a > 0$ or $a < 0$.

   Case 1: $a > 0$.

   If $a < 0$, then $f'(x)$ is a line with a negative slope, so $f'(x)$ will be positive for $x < -b/(2a)$ and negative for $x > -b/(2a)$. This shows that $f(x)$ is increasing on $(-\infty, -b/(2a))$ and decreasing on $(-b/(2a), +\infty)$. Moreover, since $f'(x)$ goes from positive to negative at $x = -b/(2a)$, $f(x)$ has a local maximum there.

   Since $f(x)$ is a parabola opening down, it has no absolute minimum and the absolute maximum is at $x = -b/(2a)$.

   Case 2: $a < 0$.

   If $a > 0$, then $f'(x)$ is a line with a positive slope, so $f'(x)$ will be negative for $x < -b/(2a)$ and positive for $x > -b/(2a)$. This shows that $f(x)$ is decreasing on $(-\infty, -b/(2a))$ and increasing on $(-b/(2a), +\infty)$. Moreover, since $f'(x)$ goes from negative to positive at $x = -b/(2a)$, $f(x)$ has a local minimum there.

   Since $f(x)$ is a parabola opening up, it has no absolute maximum and the absolute minimum is at $x = -b/(2a)$.