

Due: Wednesday, November 7th

1. Suppose that $A \subset \mathbb{R}^n$ and $B \subset \mathbb{R}^m$ are open. Show that the set

$$A \times B = \{(x_1, \dots, x_{n+m}) : (x_1, \dots, x_n) \in A, (x_{n+1}, \dots, x_{n+m}) \in B\}$$

is open in \mathbb{R}^{m+n} .

2. Exercise 5.2.H in the text.
3. Exercise 5.3.E in the text.
4. Exercise 5.3.G in the text.
5. Exercise 5.3.L in the text.
6. (A problem from the January 2007 Analysis Qualifying Exam.) Let f be a positive continuous function defined on \mathbb{R} such that $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$. Show that f attains its maximum value, that is, there is $b \in \mathbb{R}$ so that $f(b) = \sup f(\mathbb{R})$.