Show your work and explain your reasoning!

1. Find the total mass and the center of mass for the region bounded by the graphs of $f(x) = 4 - x^2$ and $g(x) = x + 2$, if the density if $\delta(x, y) = x^2$.

2. Determine if $a_n = \dfrac{\cos(n\pi)}{n^2}$ converges or diverges. If it converges, find the limit.

3. Determine if $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ converges or diverges.

4. Determine if the following series converge or diverge and, if one converges, find its sum.
$$\text{a)} \sum_{n=1}^{\infty} \frac{5(2^n)}{3^n}, \qquad \text{b)} \sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^3 - 2n^2 + 1}.$$

5. Determine whether or not the following series converge or diverge:
$$\text{a)} \sum_{n=1}^{\infty} (-1)^n \ln(n), \qquad \text{b)} \sum_{k=2}^{\infty} \frac{3}{k(\ln k)^2}$$

6. Estimate the error in approxmating $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5}{n^3}$ by $S_{10} = \dfrac{5}{1} - \dfrac{5}{8} + \cdots + \dfrac{5}{10^3}$.

7. Find the radius of convergence and the interval of convergence for $\displaystyle\sum_{k=1}^{\infty} \frac{k}{4^k} x^k$.

8. Find a Taylor series for $f(x) = x^2 \cos \sqrt{x}$ centered at $c = 0$. You can give either the whole series or the first four nonzero terms.

9. Find a Taylor series for $g(x) = \dfrac{2}{4 + x}$ centered at $c = 0$ and determine its radius of convergence.

10. Using an appropriate Taylor series, solve each of the following problems:

    (a) Evaluate $\displaystyle\lim_{x \to 0} \frac{e^{-2x} - 1}{x}$.

    (b) Find $e^{.4}$ to 10^{-5}.

    Answers: 1. mass is 63/20, center of mass is (-8/7,78/49); 2. converges to 0; 3. diverges; 4. a. converges, sum is 10, b. diverges; 5. a. diverges, b. converges; 6. 0.00376; 7. radius 4, interval (-4,4); 8. $\sum_{k=1}^{\infty} (-1)^k/(2k)! x^{k+2}$; 9. $\sum_{k=1}^{\infty} (-1)^k / 24^k x^k$; radius is 4. 10. a. 2 b. 1.49182.

    Complete solutions are on the course webpage.