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1. Evaluate the following:

(a) $\displaystyle\int_3^5 \left(y^2 + 3y + \frac{1}{y}\right)\, dy,$

(b) $\displaystyle\int_1^2 \frac{\ln x}{x}\, dx.$

Solution. For (a), we have

$$\begin{aligned}\int_3^5 \left(y^2 + 3y + \frac{1}{y}\right)\, dy &= \frac{y^3}{3} + \frac{3y^2}{2} + \ln|y|\Big|_{y=3}^{y=5} \\ &= \left(\frac{5^3}{3} + \frac{3\cdot 5^2}{2} + \ln 5\right) - \left(\frac{3^3}{3} + \frac{3\cdot 3^2}{2} + \ln 3\right) \\ &= \frac{170}{3} + \ln 5 - \ln 3\end{aligned}$$

Note that (b) is Problem 25 from Section 7.4, a homework problem. We use the substituion $u = \ln x$ so $du = \frac{1}{x}\, dx$. If $x = 1$, $u = \ln 1 = 0$ and if $x = 2$, $u = \ln 2$. Thus,

$$\begin{aligned}\int_1^2 \frac{\ln x}{x}\, dx &= \int_0^{\ln 2} u\, du \\ &= \frac{u^2}{2}\Big|_{u=0}^{u=\ln 2} \\ &= \frac{(\ln 2)^2}{2}.\end{aligned}$$