. 4

1. Find the average rate of change of $y = \dfrac{1}{x+1}$ between $x = 1$ and $x = 4$.

   Solution. The average rate of change of $y = f(x)$ between $x = a$ and $x = b$ is

   $$\frac{f(b) - f(a)}{b - a}$$

   and so we have

   $$\frac{\frac{1}{4+1} - \frac{1}{1+1}}{4 - 1} = \frac{1}{3}\left(\frac{1}{5} - \frac{1}{2}\right) = \frac{1}{3}\frac{-3}{10} = \frac{-1}{10}$$

. 6

2. Find the value of the constant k so that the function $f(x) = \begin{cases} x^3 + k & \text{if } x \leq 3, \\ kx - 3 & \text{if } x > 3. \end{cases}$ is continuous.

   Solution. Because $x^3 + k$ and $kx - 3$ are polynomials, which are always continuous, the only possible problem is at $x = 3$. We have $f(3) = 27 + k$, so both one-sided limits must equal $27 + k$. Now,

   $$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x^3 + k = 27 + k,$$

   so there is no problem with this limit, but

   $$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} kx - 3 = 3k - 3,$$

   so $3k - 3 = 27 + k$. Thus, $2k = 30$ so $k = 15$ is the only value that makes the function continuous.