

MATH 104      HOUR 2 Review

1. Find  $\frac{dy}{dx}$  (You need not simplify):

(a)  $y = \frac{3x^4 + 5x - 1}{2e^{(x^3+x)}}$

(b)  $y = e^{(x^2+4)} + 3\ln(3x^5 - 5x + 2)$

(c)  $y = \ln\left(\frac{e^{2x}}{1 + e^{-x}}\right)$

(d)  $y = \ln\left(e^{5x}\sqrt{e^{2x-1}}\right)$

(e)  $y = \frac{4e^{x^2-1}}{2x^5 + 4}$

(f)  $y = e^{-2x+4} + 3\ln(2x^4 - 5)$

2. Solve the following equations for  $x$ :

(a)  $7 + 3\ln(2x^2 - 1) = 13$

(b)  $(9)^{x^2} = (27)^{x+4}$

(c)  $5 + 3\ln(2x^2 - 1) = 8$

(d)  $(25)^{x^2} = (125)^{x+4}$

3. (a) Find an equation of the tangent line to the graph of the curve  $y = f(x) = \sqrt{8x^3 + 17}$  at the point  $(1, 5)$ .

(b) Find an equation of the tangent line to the graph of the curve  $y = f(x) = \ln x^3 - 6x^2$  at the point  $(1, -6)$ .

(c) Find an equation of the tangent line to the graph of  $y = \frac{(x+1)^2}{e^{x^2}}$  at  $x = 1$ .

(d) Find an equation of the tangent line to the graph of the curve  $y = f(x) = \sqrt{4x^3 + 17}$  at the point  $(2, 7)$ .

4. (a) How much money should be deposited into an account paying 5.75 per cent interest compounded quarterly in order to have a balance of \$12,000 seven years from now? (Round off your answer to the nearest penny).

(b) How much money should be deposited into an account paying 6.25 per cent interest compounded semi-annually in order to have a balance of \$13,000 six years from now? (Round off your answer to the nearest penny).

5. How long does it take for \$2500 to triple in value at an annual interest rate of 6.35 per cent, compounded continuously?

6. Ruth invested \$20,000 in a mutual fund on March 1, 1997. On March 1, 2006, her investment was worth \$34,500. What was the annual rate of growth of this fund assuming continuous compounding?

8. What interest rate is required for an investment of \$5500 to grow to \$9500 in 8 years if interest is compounded continuously?

9. Find the half life of a radioactive material if after one year 92 per cent of the initial amount remains.

10. Find the derivatives of the functions:

(a)  $f(x) = \ln\left(\frac{e^{x^2}}{1+e^{-3x}}\right)$

(b)  $f(x) = \ln\left(e^{3x}\sqrt{e^{5x^2-1}}\right)$

(c)  $f(x) = (x^2 - 2)^3 e^{(x^2+5)}$

(d)  $f(x) = (\ln|x^2 - 5x|) \left(3\sqrt{x-7}\right)$

11. Find the half life of a radioactive material if after one year 99.57 per cent of the initial amount remains.

12. If a radioactive substance has a half life of 23 years, how much of a 40 gram mass will remain after 38 years? (round off your answer to the nearest hundredth of a gram).

13. Let  $y = f(x) = 3x^5 - 20x^3$  for  $-\infty < x < +\infty$ .

(a) Find all critical numbers of  $f$ .

(b) List the open intervals on which  $f$  is a decreasing function. (Hint: Chart  $f'(x)$ ).

(c) List the open intervals on which  $f$  is an increasing function.

14. Let  $y = f(x) = x - 3\ln(4x - 6)$  for  $x > 1.50$

(a) Find all critical numbers of  $f$ .

(b) List the open intervals on which  $f$  is a decreasing function and the intervals on which  $f$  is an increasing function.

15. Suppose that the cost and revenue functions for a certain product are given by  $C(x) = 5.4x - .0003x^2$  and  $R(x) = 9.7x - .002x^2$ , respectively. Determine where the profit function is increasing.

16. Let  $y = f(x)$  be a function such that  $f'(x) = x^2(x-4)^2(x+7)(6-x)$  for all  $x \in (\infty, +\infty)$ .

(a) List the critical numbers of  $f$ .

(b) Chart  $f'(x)$ .

(c) List the open interval(s) on which  $f$  is an increasing function.

(d) List the number(s)  $x$  at which  $f$  has a relative maximum.