

Due: Jan 28th

1. Suppose we have functions $f : A \rightarrow B$ and $g : B \rightarrow C$. If $D \subset C$, show that $(g \circ f)^{-1}(D) = f^{-1}(g^{-1}(D))$.
2. Do Exercise 0.3.J in the background chapter. That is, Consider the following “proof” by induction. We will argue that all students receive the same mark in calculus. Let $P(n)$ be the statement that every set of n students receives the same mark. This is evidently valid for $n = 1$. Now look at larger n . Suppose that $P(n - 1)$ is true. Given a group of n people, apply the induction hypothesis to all but the last person in the group. The students in this smaller group all have the same mark. Now repeat this argument with all but the first person. Combining these two facts, we find that all n students have the same mark. By induction, all students have the same mark.

This is patently absurd, and you are undoubtedly ready to refute this by saying that Paul has a much lower mark than Mary. But you must find the mistake in the induction argument, not just in the conclusion.

HINT: The mistake is not $P(1)$, and $P(73)$ does imply $P(74)$.

3. Prove that for all integers $n \geq 8$, there are non-negative integers a and b with $n = 3a + 5b$.
4. EXTRA CREDIT: For $n \in \mathbf{N}$, show that $\frac{n}{(n+1)(2n+1)} < \sum_{k=n+1}^{2n} \frac{1}{k^2} < \frac{1}{2n}$.