Math 106, Fall '06 **Pendumonium** **Due 11/14/06**

Calculus Student
Avery Hall
Lincoln, NE 68588

Suluclac Clock Company
1230 Horology Drive
Lincoln, NE 68599

Dear Calculus Student,

I am writing to you because your name was suggested to me by your calculus professor. He gave you a glowing reference. I work here at the Suluclac Clock Company and we have some questions that have arisen in our most recent clock designs.

To give you some background, we build a line of clocks called "Bernoulli"s which are large, upright, pendulum clocks. They all use the same mechanism, but the length of the pendulum and the size of the cabinet and clock face vary. The most fundamental fact you'll need to know is the relationship between the period T of a pendulum (the time for one full swing and back) and its effective length ℓ, which is

$$T^2 = 4\pi^2 \ell / g.$$

In the formula g is the acceleration due to gravity. The Suluclac Clock Company uses SI units throughout the company, so lengths are measured in meters, time is measured in seconds, and $g = 9.81\,\mathrm{m/s^2}$.

My first question is the following. The Jacob Bernoulli is one of our largest clocks and has a pendulum with a standard effective length of 1.5 meters. The clock is provided with an adjustment wheel with which the effective length of the pendulum can be changed by a small amount. As the length changes a marker moves along a scale on the pendulum. We would like to know how to mark that scale in units of seconds per day. (Fred down in the Research department is convinced the scale will be linear and that there's some fancy way to work it out using calculus. I'd appreciate an explanation of this technique.)

Secondly I need some help with the design of a sensor used to measure the amplitude of the pendulum's swing. This sensor is used to activate a protective mechanism if the pendulum is swinging too much, and to activate a warning that the clock needs winding if the swing is too small. It measures the amplitude indirectly, by measuring the speed of the pendulum at the bottom of its arc. To help us calibrate the sensor we would like you to find out the relationship between this speed and the amplitude. You can assume that the angle the pendulum makes with the vertical is a sinusoidal function of time (this is true to a very high degree of accuracy according to the standard clockmaker's literature).

After you have solved these problems I would like a report from you describing your results and also the techniques you used. You should include justifications and explanations together with the appropriate equations, diagrams, etc. You should write your report to be read by a reasonably technically literate audience (I've taken calculus myself).

Yours sincerely,

M. Remontoire
Managing Director, Suluclac Clock Company

Overview The intent of projects is to expose you to mathematics as you might meet it in 'the real world', i.e., working as a team. Your group must understand the problem; translate it into mathematics; learn, read about, or develop mathematical methods to find the answer; show that the answer is correct; translate the mathematical answer back into the original problem and, finally, explain the significance of the translated answer. Projects are easier than 'real world' problems, in that we make sure that the problem can be solved using the methods of this course. (You may need to look up information in your text.)

Preparing formal reports is an important job skill for mathematicians, scientists, and engineers. For example, the Columbia Investigation Board, in its report on the causes of the Columbia space shuttle accident, wrote:

> *"During its investigation, the board was surprised to receive [PowerPoint] slides from NASA officials in place of technical reports. The board views the endemic use of PowerPoint briefing slides instead of technical papers as an illustration of the problematic methods of technical communication at NASA."*

The project is the solution to an open-ended multistep problem, formally presented. It will probably require several meetings for your group to find a solution to the problem and to present that solution clearly and understandably. Everyone in the group should contribute to the project.

Project Report. Your group should write up a short paper explaining the problem and the mathematics you used to solve it, and then discussing the significance of your solution. Your paper should be a a grammatically correct, organized discussion of the problem, with an introduction and a conclusion. It should conform to proper English usage (yes, spelling counts) and should include appropriate diagrams and/or graphs, clearly labeled. You should show enough relevant calculations to justify your answers but not so much as to obscure the calculations' purpose.

Explain your results and conclusions, pointing out both strengths and weaknesses of your analysis. Assume that your reader is someone who took a calculus class course a while ago and does not remember all of the details. Be sure to avoid plagiarism.

Group Structure. To help the group function smoothly, you might want to consider assigning each person a role, such as:

Convenor	Arrange times and places for meetings
Chair	Ensure everyone is involved and understands the ideas discussed
Reporter	Jot down ideas and suggestions as they are discussed
Scribe	Prepare first draft of final reportt