Math 825 **Assignment # 5** **26 Sept 07**

Due: Friday, October 5th

1. Exercise 3.2.K in the text.
2. Exercise 3.2.N in the text.
3. Suppose (a_n) is a strictly decreasing positive sequence, i.e., $0 < a_{n+1} < a_n$.
   (a) Suppose that (g_k) is a strictly increasing sequence of integers and there is a constant C so that for $k = 2, 3, \ldots$, $g_{k+1} - g_k \leq C(g_k - g_{k-1})$. Prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{k=1}^{\infty} (g_{k+1} - g_k) a_{g_k}$ converges.

   (b) By a suitable choice of (g_k), prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} 2^n a_{2^n}$ converges

   (c) Similarly, prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} n a_{n^2}$ converges.
4. Suppose (a_n) is a decreasing positive sequence, i.e., $0 < a_{n+1} \leq a_n$.

   (a) Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} n a_n = 0$.

   (b) Give a sequence (a_n) as above so that $\lim_{n \to \infty} n a_n = 0$ but $\sum_{n=1}^{\infty} a_n$ diverges.
5. Exercise 3.4.G in the text.