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1. Find the following limit:

$$\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}$$

Solution.

$$\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1},$$

and by multiplying by the conjugate of $\sqrt{x^2 + 8} - 3$ we get

$$\lim_{x \rightarrow -1} \frac{(\sqrt{x^2 + 8} - 3)(\sqrt{x^2 + 8} + 3)}{(x + 1)(\sqrt{x^2 + 8} + 3)}$$

which expanded out is

$$\lim_{x \rightarrow -1} \frac{x^2 + 8 - 9}{(x + 1)(\sqrt{x^2 + 8} + 3)} = \lim_{x \rightarrow -1} \frac{x^2 - 1}{(x + 1)(\sqrt{x^2 + 8} + 3)}$$

By simplifying the numerator we get "two perfect squares and a minus sign," so we can factor the numerator to $(x + 1)(x - 1)$.

$$\lim_{x \rightarrow -1} \frac{(x - 1)(x + 1)}{(x + 1)(\sqrt{x^2 + 8} + 3)} \lim_{x \rightarrow -1} \frac{x - 1}{\sqrt{x^2 + 8} + 3} = \frac{(-1) - 1}{\sqrt{(-1)^2 + 8} + 3} = \frac{-1}{3},$$

where we've used the limit laws to evaluate the limit.