

Not to be handed in

1. Suppose that  $\mathbf{0} = (0, 0)$ ,  $r > 0$  and  $f = (f_1, f_2)$  mapping  $B_r(\mathbf{0})$  into  $\mathbb{R}^2$  is  $C^1$ . Prove that if  $df_{\mathbf{0}}$  is invertible and  $f^{-1} = (g_1, g_2)$  is the local inverse, then

$$D_1(g_1)(f(\mathbf{0})) = \frac{D_2(f_2)(\mathbf{0})}{Jf_{\mathbf{0}}}, \quad D_2(g_1)(f(\mathbf{0})) = \frac{-D_2(f_1)(\mathbf{0})}{Jf_{\mathbf{0}}},$$

and

$$D_1(g_2)(f(\mathbf{0})) = \frac{-D_1(f_2)(\mathbf{0})}{Jf_{\mathbf{0}}}, \quad D_2(g_2)(f(\mathbf{0})) = \frac{D_1(f_1)(\mathbf{0})}{Jf_{\mathbf{0}}},$$

where  $Jf_{\mathbf{0}}$  is the determinant of the  $2 \times 2$  matrix for  $df_{\mathbf{0}}$ .