Due: Nov 12th

1. Do Problem 11.55 in Schaum's (page 298). For x and y positive, prove that there is $\theta \in (0,1)$ so that
$$\ln \frac{x+y}{2} = \frac{x+y-2}{2+\theta(x+y-2)}.$$
HINT: Use Taylor's Theorem with linear remainder term.

2. (a) Do Problem III.3.2(b) in Edwards (page 194). That is, Show that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x,y) = (e^x \cos y, e^x \sin y)$ is locally invertible at every point in $\mathbb{R}^2$.
   (b) Show that f does not have a global inverse.

3. Consider $S : \mathbb{R}^3 \to \mathbb{R}^3$ given by
$$S(r,\phi,\theta) = (r \sin\phi\cos\theta, r\sin\phi\sin\theta, r\cos\phi).$$
   (a) Compute dS as a point (r,ϕ,θ) and show $JS_{(r,\phi,\theta)} = r^2 \sin\phi$.
   (b) At which points in $\mathbb{R}^3$ is S locally invertible? What is the corresponding condition on a point in the range of S?

4. Suppose that $\mathbf{0} = (0,0)$, $r > 0$ and $f = (f_1, f_2)$ mapping $B_r(\mathbf{0})$ into $\mathbb{R}^2$ is C^1. Prove that if $df_{\mathbf{0}}$ is invertible and $f^{-1} = (g_1, g_2)$ is the local inverse, then
$$D_1(g_1)(f(\mathbf{0})) = \frac{D_2(f_2)(\mathbf{0})}{Jf_{\mathbf{0}}}, \qquad D_2(g_1)(f(\mathbf{0})) = \frac{-D_2(f_1)(\mathbf{0})}{Jf_{\mathbf{0}}},$$
and
$$D_1(g_2)(f(\mathbf{0})) = \frac{-D_1(f_2)(\mathbf{0})}{Jf_{\mathbf{0}}}, \qquad D_2(g_2)(f(\mathbf{0})) = \frac{D_1(f_1)(\mathbf{0})}{Jf_{\mathbf{0}}}.$$