Math 106, Sect. 350 **Answers to Quiz # 9** **9 Nov '06**

. 6 1. Evaluate $\lim\limits_{x\to\infty} x^2e^{-x}$.

Solution. We can rewrite x^2e^{-x} as $\frac{x^2}{e^x}$ and then letting x approach ∞ we find that the limit is of the indeterminate form $\frac{\infty}{\infty}$. Thus, we can apply L'Hôpitals Rule, to get

$$\lim_{x\to\infty} x^2e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x}.$$

As x goes to ∞, again we find that the limit is the indeterminate form $\frac{\infty}{\infty}$. Thus, we can apply L'Hôpitals Rule a second time and find

$$\lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x}.$$

Finally, letting x approach ∞ here, the limit equals $\frac{2}{\infty} = 0$. Therefore, $\lim\limits_{x\to\infty} x^2e^{-x} = 0$

. 4 2. Find the most general antiderivative (indefinite integral) of $7\sin\frac{\theta}{3}$, that is, $\displaystyle\int 7\sin\frac{\theta}{3}\,d\theta$. Check your answer by differentiation.

Solution.

$$\int 7\sin\left(\frac{\theta}{3}\right)d\theta = -21\cos\left(\frac{\theta}{3}\right) + C.$$

Now we check this using differentiation:

$$\frac{d}{dx} - 21\cos\frac{\theta}{3} + C = -21\left(-\sin\left(\frac{\theta}{3}\right)\right)\cdot(\frac{1}{3}) + 0 \qquad = 7\sin\left(\frac{\theta}{3}\right)$$