Math 496-004 **Assignment # 13 (the last)** **3 Dec 08**

Due: Dec 10th

1. Do Problem IV.5.6 in Edwards (page 264). That is, Let R be the solid torus in $\mathbb{R}^3$ obtained by revolving about the z-axis the disc $(y-a)^2 + z^2 \leq b^2$ in the yz-plane. Define a mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$T \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} (a + w\cos v)\cos u \\ (a + w\cos v)\sin u \\ w\sin v \end{bmatrix}.$$

   Note that T maps the parallepiped $[0, 2\pi] \times [0, 2\pi] \times [0, b]$ in (u, v, w)-space onto R. Apply the change of variables formula to compute the volume of R.

2. Do Problem 9.57 in Schaum's (page 228). That is, If R is the region $x^2 + xy + y^2 \leq 1$, prove that

$$\iint_R \exp(-(x^2 + xy + y^2))\, dx\, dy = \frac{2\pi}{e\sqrt{3}}(e-1).$$

   HINT: Let $x = u\cos\alpha - v\sin\alpha$, $y = u\sin\alpha + v\cos\alpha$ and choose α to eliminate the xy term in the integrand. Then let $u = a\rho\cos\phi$, $v = b\rho\sin\phi$, where a and b are appropriately chosen

   In other words, rotate the coordinate axes so they are the principal axes of the ellipse $x^2 + xy + y^2 = 1$ and then stretch the coordinate axes to turn the ellipse into a circle.

3. Do Problem II.5.6 in Edwards (page 116). That is, Show that the maximum value of $f(\mathbf{x}) = x_1^2 x_2^2 \cdots x_n^2$ on the sphere $S^{n-1} = \{\mathbf{x} \in \mathbb{R}^n : |\mathbf{x}| = 1\}$ is $(1/n)^n$.

   Remark: This shows that $(x_1^2 x_2^2 \cdots x_n^2)^{1/n} \leq 1/n$ if $\mathbf{x} \in S^{n-1}$. Given positive numbers $a_1, \ldots, a_n$, define, for $i = 1, \cdots, n$,

$$x_i = \frac{a_i^{1/2}}{(a_1 + \cdots + a_n)^{1/2}}.$$

   Then $(x_1, \ldots, x_n) \in S^{n-1}$ and so

$$\left(\frac{a_1 \cdots a_n}{(a_1 + \cdots + a_n)^n} \right)^{1/n} \leq \frac{1}{n}.$$

   Clearing fractions shows that the *geometric mean* of n positive numbers is no greater than their *arithmetic mean*.