MATH 104 HOUR 1 Review

You are not responsible for Problems 7(c), 9, and 18 on Test 1.

1. Let $f(x) = x^2 + 3x$ and $g(x) = 4x - 1$. Find:

(a) $f(g(x))$; (b) $g(4 + f(-2))$; (c) $f(1 + f(2))$

2. Evaluate the following limits:

(a) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$;
(b) $\lim_{x\to -2} \frac{4x^2+7x-2}{3x^2+17x+22}$;
(c) $\lim_{x\to 1} \frac{x-1}{x^2-1}$;
(d) $\lim_{x\to 3^+} \frac{2x+1}{3x-x^2}$;
(e) $\lim_{x\to +\infty} \frac{3x^2+10}{12x-7x^2}$;
(f) $\lim_{x\to -\infty} \left(10 + \frac{1-6x}{3x+12}\right)$;
(g) $\lim_{h\to 0} \frac{(2x+3h)^2-4x^2}{7h}$;

3. Find the value of k so that the function $f(x) = \begin{cases} 4x + 3k & \text{if } x < 2 \\ x^2 + 1 & \text{if } x \geq 2 \end{cases}$ is continuous.

4. Find the average rate of change of $f(x) = x^2 - x$ on the interval $[-4, 3]$.

5. Find the instantaneous rate of change of $f(x) = x^2 - x$ at $x = 2$.

6. Find the horizontal and vertical asymptotes for the following functions:

(a) $\frac{3x^2+10}{x^2-7x+12}$ (b) $\frac{x-6}{x(x^2-4)}$

(c) $\frac{x^2-9}{x-3}$ (d) $\frac{x+4}{x^2-16}$

7. Find $\frac{dy}{dx}$ for the following functions:

(a) $y = 4x^{-2} + \frac{x}{3} + \frac{3}{x}$ (b) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

(c) $y = \frac{2x+1}{5x+1}$ (d) $y = \frac{x^3+8x+1}{x}$

8. (a) Find an equation of the tangent line to the graph of $y = f(x) = 4x^3 - 8x + 1$ when $x = 1$.

9. If $f'(x) = \frac{1-3x}{x^2+1}$, find an equation of the tangent line to the graph of the curve at the point $(2, 3/4)$.

10. At what points on the graph of the function $y = f(x) = x^3 + 3x^2 - 24x + 10$ is the slope of the tangent line equal to 9?

11. Suppose that the supply equation for a certain commodity is $p = S(x) = 5 + .3x$ dollars and the demand equation is $p = D(x) = 40 - .2x$ dollars. Find the equilibrium point (x_0, y_0).

12. The total cost of producing x units of a certain product is $C(x) = 800 + 24x - .1x^2$ dollars.

(a) Find the **marginal cost** function.
(b) At what production level x does the marginal cost equal 14 dollars?
(c) Find the marginal cost when $x = 5$ units.
(d) Find the exact cost of the 6th unit.

13. A company which produces widgets has an initial investment of \$10000.00. If each widget costs \$21.50 to produce and can be sold at a price of \$30.65, find:

(a) the equation for the total cost $C(x)$ and the total revenue $R(x)$;
(b) the break-even point (the intersection of the cost and revenue functions); (Round off to the closest integer).
(c) How many widgets must be sold to yield a profit of \$8000.00 (Round off to the closest integer.)

14. A company invests \$100,000.00 for equipment to produce a new product. Each unit of the product costs \$11.40 and is sold for \$17.98. Let x be the number of units produced and sold. Find:

(a) The total cost function $C(x)$
(b) The total revenue function $R(x)$
(c) The total profit function $P(x)$.

15. The cost (in dollars) of producing x units of some product is given by $C(x) = 2000 + 35x - .02x^2$.

(a) Find the marginal cost function.
(b) Find the marginal cost when $x = 10$.
(c) Find the exact cost of the 11th unit.

16. Let $y = f(x) = \begin{cases} \frac{4x^2+x-5}{x-1} & \text{if } x \neq 1 \\ 8 & \text{if } x = 1 \end{cases}$

(a) Evaluate $\lim_{x \to 1} \dfrac{4x^2 + x - 5}{x - 1}$;

(b) Is $f(x)$ a continuous function at $x = 1$? Why or why not?

17. Write down an equation of a rational function (i.e., quotient of polynomials) which has vertical asymptotes at $x = 6$, $x = 0$, and $x = 21$ and a horizontal asymptote at $y = -29$.

18. Find $f'(2)$, if $f(x) = \dfrac{3 - 4x}{x^2 + 1}$.

19. Find the instantaneous rate of change of $g(x) = \sqrt{x}$ at $x = 4$.

20. Find k so that the line segment through $(3, -2)$ and $(k, 2)$ is
(a) parallel to the line $3x + 2y = 8$.
(b) perpendicular to the line $5x - 2y = 8$.