Math 325-001 **Assignment # 1** **14 Jan 09**

Due: Jan 23rd

1. Prove that if $n+1$ integers are selected between 1 and $2n$, then the selection includes at least one pair of consecutive integers. Give a selection of n integers between 1 and $2n$ that does not contain a pair of consecutive integers. Hint: Pigeonhole.

2. What is the minimum number of points to be selected from the interior of an equilateral triangle of side length two, to ensure that there are two of them with distance less than one? (Do I have to say "prove your answer"?)

3. Do Exercise 0.3.B(b) in the background chapter. That is, Prove that $\log_{10} 3$ is irrational. Hint: If $x = \log_{10} 3$, first show $x > 0$ and then consider $10^x = 3$. Remember 3 is a prime.

4. Prove, using induction, for all integers $n \geq 2$, that

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

5. Consider the sequence (a_n) defined by, for each $n \in \mathbf{N}$,

$$a_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)}.$$

   (a) Conjecture a formula for a_n in terms of n (no summations).

   (b) Prove, using induction, that your formula is correct.

6. Show that a convex polygon with n sides has $\dfrac{n(n-3)}{2}$ diagonals, for $n \geq 3$.