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1. Evaluate $\int e^{3x} \sin x \, dx$.

Solution. We use integration by parts twice to build a loop. First, let $u = e^{3x}$, $dv = \sin x$, so that $du = 3e^{3x}$ and $v = -\cos x$, giving

$$\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3 \int e^{3x} \cos x \, dx$$

and now let $u = e^{3x}$, $dv = \cos x$, so that $du = 3e^{3x}$ and $v = \sin x$, giving

$$= -e^{3x} \cos x + 3e^{3x} \sin x - 9 \int e^{3x} \sin x \, dx$$

Letting $I = \int e^{3x} \sin x \, dx$, we have

$$I = -e^{3x} \cos x + 3e^{3x} \sin x - 9I$$

and so $10I = e^{3x}(3 \sin x - \cos x)$, giving

$$\int e^{3x} \sin x \, dx = \frac{e^{3x}}{10}(3 \sin x - \cos x).$$