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Gyro-Bias Estimation Filter Design for the Stabilization Accuracy Enhancement of Two Axes Gimbaled Sighting Systems

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Abstract: In this study the development of a bias estimation technique for the gyroscopes used in stabilization of two axes gimbaled sighting systems is presented. Due to their size and cost limitations, generally, two axes gyroscopes with high bias values are used on such systems. These high bias values cause the sight-line to drift from the target direction and result in a need for frequent corrections from the operator. The proposed technique uses the aiding from the inertial navigation system (INS) of the host vehicle and the kinematical constraints to enable the complete attitude solution of the sighting system with only using a two axes gyroscope. This full attitude solution is used in designing an extended Kalman filter which estimates the gyroscope biases. As for the simulations, a dynamical model of the sighting system, carried on a host land vehicle, is developed and the performance improvement resulting from the bias corrections is demonstrated with simulations using this model.

1. INTRODUCTION

The sighting systems such as optical and radar missile seekers, aerial surveillance cameras, gun platforms and tank sighting systems require line of sight stabilization with high accuracies (Dudzik 1993, Merhav 1996). These systems are usually stabilized by isolating them from the vibrations and maneuvers of the host vehicle using two axes gimbaled platforms. The gyroscopes on the gimbals measure the angular rates of the gimbals with respect to the inertial frame and the sensors like resolvers, potentiometers or encoders measure the angular positions of the gimbals with respect to each other and with respect to the host vehicle. Over the years the emphasis has been mainly conducted in the performance improvement of these systems by focusing on the control design techniques (Moorty 2002, Chen 2000).

The bias, or drift, and noise characteristics of the gyroscopes are the most important parameters that affect the stabilization performance. In the stabilization mode large drift values on the gyroscope outputs cause the sight-line to drift from the target. This can be compensated with frequent corrections from the operator, but leads to the inefficient usage of the system. Also, in order to prevent the high frequency oscillations (jitter) during the operation the gyroscopes with low noise values are needed.

Due to physical size and cost restrictions usually two axes *Dynamically Tuned Gyroscopes* (*DTG*) are used for line of sight stabilization systems (Merhav 1996). Although these sensors are generally selected to have low noise characteristics to satisfy the low jitter requirements within the desired bandwidth they may have considerably high bias values.

In this paper a bias estimation technique is developed for the gyroscopes used on two axes sighting systems. The technique assumes the presence of an inertial navigation system (INS) on the host vehicle as the aiding source. The angular rate and attitude measurements of this INS are used as the aiding information to the attitude and bias estimation filter. Actually, in order to achieve the full attitude solution, all three components of the inertial angular velocity are necessary. However, in this study, the missing third axis angular velocity is calculated in the designed filter using the information coming from the INS together with the kinematical relations between the two axes sighting system and the host vehicle.

Attitude estimation techniques are used extensively for aircraft and satellite applications (Lefferts 1982, Kingston 2004). In the majority of these applications nonlinear differential attitude equations of the vehicle are formulated and solved in time by using the angular rate information obtained from a three axes gyroscope. The attitude representation can be formulated in terms of quaternions, Euler angles or Direction Cosine Matrices (DCM) (Savage 1998, Titterton 2004). This mathematical model of the attitude is aided by sensors such as star trackers, global positioning systems (GPS), magnetometers by using extended Kalman filtering techniques. Although the main goal of these techniques is to get accurate and continuous estimates of the attitude of the vehicle gyro-biases can also be estimated by appropriately augmenting the states of the Kalman filter (Brown 1997, Gelb 2001, Savage 2000). In the present study, in order to enhance the conventional stabilization application done with low cost two axes gyroscopes, the pre-mentioned attitude estimation techniques are modified and shown to be applied on stabilization study of general class of two axes systems.

The general organization of the paper is as follows; in Section 2 the kinematical relations are derived for the two axes sighting system and the carrying host vehicle. In section 3, the derivation of the attitude filter including the gyroscope bias states is presented. In Section 4, the dynamic equations of the sighting system are given. The stabilization controller design and the simulation model of the sighting system with the closed loop control and the bias estimation filter are presented in Section 5. Consequently, the results of the gyroscope bias estimation and its enhancement on the closed loop stabilization performance are given in Section 6.

2. KINEMATICS OF THE TWO AXES SYSTEM

In this study the sighting system is assumed to be mounted on a revolving base which has a single rotational degree of freedom in azimuth with respect to the host vehicle. In order to construct the kinematical model of the two axes sighting system and the host vehicle, the definitions of the reference frames, the relative distances and the transformations between them are defined in Fig. 1.

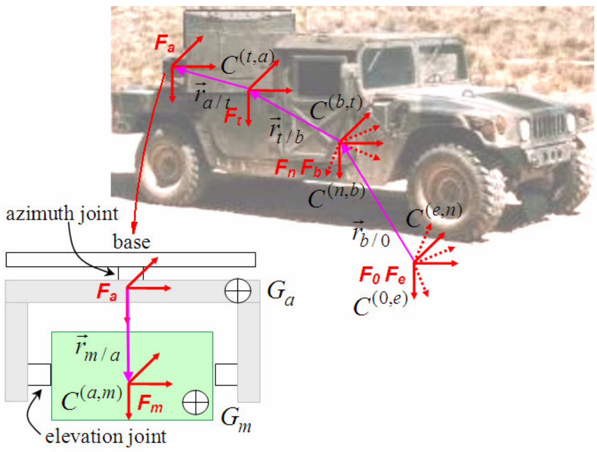

Fig. 1. The definitions of the reference frames, the relative distances and the transformation matrices

The transformation matrices in Fig. 1 can be defined as

$C^{(0,e)} = R_3(\Omega_e t)$, $C^{(e,n)} = R_3(\lambda)R_2(-L-\pi/2)$,

$C^{(n,b)} = R_3(\psi)R_2(\theta)R_1(\phi)$, $C^{(b,t)} = R_3(\psi_t)$,

$C^{(t,m)} = R_3(\psi_m)R_3(\theta_m)$.

Here, $\Omega_e = (7.29)10^{-5}$ rad/sec is the earth rotation rate and $\Omega_e t$ defines the orientation of the rotating earth reference frame (F_e) with respect to the inertial reference frame (F_0), where t is time in seconds. L,λ are the longitude and latitude of the body position and they define the orientation of the navigation frame (F_n) with respect to the rotating earth reference frame, ψ,θ,ϕ are the Euler angles defining the orientation of the host vehicle with respect to the navigation frame, ψ_t is the azimuth angle of the mentioned revolving base with respect to the body fixed reference frame (F_b) and ψ_m, θ_m are the azimuth and elevation angles of the sighting system with respect to the revolving base fixed reference frame (F_t). Also, $R_i(\delta)$ is the rotation matrix constructed for the rotation around the i^{th} axis with the rotation angle δ.

In order to ease the derivations of the equations of motion the earth rate and the transport rates, which are dependent on $\dot{L},\dot{\lambda}$, are neglected. Hence, the inertial frame (F_0) is taken to be equal to the navigation frame (F_n). The earth rate and transport rates are integrated only in the nonlinear attitude calculations as explained in Section 3.1. In all of the derivations it is assumed that there are not boresight angles between the body and revolving base, between the revolving base and base frame of the sighting system and between the base frame and moving gimbals of the sighting system. It is obvious that in practice such deviations will exist, but measuring or estimating them once they can be included into the equations as constant transformation matrices.

In order to analyze the angular motion of the sighting system with respect to F_0 the angular velocity and acceleration expressions of the body, the revolving base and the sighting system with respect to F_0 should be derived. So, the angular velocity and acceleration of the body at the body fixed reference frame can be written as

$$\bar{\omega}_{b/0}^{(b)} = p\bar{u}_1 + q\bar{u}_2 + r\bar{u}_3 \tag{1.a}$$

$$\bar{\alpha}_{b/0}^{(b)} = \dot{p}\bar{u}_1 + \dot{q}\bar{u}_2 + \dot{r}\bar{u}_3 \tag{1.b}$$

Here, p,q,r are the body angular velocity components and $\bar{u}_1,\bar{u}_2,\bar{u}_3$ are the column representations of the unit vectors along each axis. Also, the angular velocity and acceleration of the revolving base expressed at the revolving base frame ($\bar{\omega}_{t/0}^{(t)},\bar{\alpha}_{t/0}^{(t)}$) and the angular velocity and acceleration of the sighting system outer (azimuth) frame expressed at the sighting system outer frame ($\bar{\omega}_{a/0}^{(a)},\bar{\alpha}_{a/0}^{(a)}$) are

$$\bar{\omega}_{t/0}^{(t)} = \bar{\omega}_{t/b}^{(t)} + \bar{\omega}_{b/0}^{(t)} = \dot{\psi}_t\bar{u}_3 + R_3(-\psi_t)\bar{\omega}_{b/0}^{(b)} \tag{2.a}$$

$$\bar{\alpha}_{t/0}^{(t)} = \ddot{\psi}_t\bar{u}_3 - \dot{\psi}_t\tilde{u}_3 R_3(-\psi_t)\bar{\omega}_{b/0}^{(b)} + R_3(-\psi_t)\bar{\alpha}_{b/0}^{(b)} \tag{2.b}$$

$$\bar{\omega}_{a/0}^{(a)} = \bar{\omega}_{a/t}^{(a)} + \bar{\omega}_{t/0}^{(a)} = \dot{\psi}_m\bar{u}_3 + R_3(-\psi_m)\bar{\omega}_{t/0}^{(t)} \tag{2.c}$$

$$\bar{\alpha}_{a/0}^{(a)} = \ddot{\psi}_m\bar{u}_3 - \dot{\psi}_m\tilde{u}_3 R_3(-\psi_m)\bar{\omega}_{t/0}^{(t)} + R_3(-\psi_m)\bar{\alpha}_{t/0}^{(t)} \tag{2.d}$$

Here, $\tilde{u}_i$ is the skew symmetric matrix form of the column representation of the unit vector $\bar{u}_i$ and used for matrix representation of the cross product. Thus, the angular velocity and acceleration of the sighting system expressed at the sighting system frame ($\bar{\omega}_{m/0}^{(m)},\bar{\alpha}_{m/0}^{(m)}$) are

$$\begin{aligned}\bar{\omega}_{m/0}^{(m)} &= \bar{\omega}_{m/a}^{(m)} + \bar{\omega}_{a/0}^{(m)} = \dot{\theta}_m\bar{u}_2 + \dot{\psi}_m R_2(-\theta_m)\bar{u}_3 \\ &\quad + R_2(-\theta_m)R_3(-\psi_m)\bar{\omega}_{t/0}^{(t)}\end{aligned} \tag{3.a}$$

$$\bar{\alpha}_{m/0}^{(m)} = \ddot{\theta}_m\bar{u}_2 + \ddot{\psi}_m\Delta_1\bar{\alpha}_{m/0}^{(m)} + \Delta_2\bar{\alpha}_{m/0}^{(m)} \tag{3.b}$$

Here, $\Delta_1\bar{\alpha}_{m/0}^{(m)},\Delta_2\bar{\alpha}_{m/0}^{(m)}$ are functions of $\theta_m,\psi_m,\dot{\theta}_m,\dot{\psi}_m,\psi_t,\dot{\psi}_t$.

In most of the two axes gimbaled systems either two single axis gyroscopes or a single two axes gyroscope are used. The sensing axes of the gyroscopes are aligned with the axes of the system at which the stabilization will be pursued and the roll motion is not measured. However, as explained in detail below, the first component of $\overline{\omega}_{m/0}^{(m)}$ should be known to calculate the attitude of the sighting system and to construct the attitude filter. Defining $\overline{\omega}_{m/0}^{(m)} = [p_m \, q_m \, r_m]^{\mathrm{T}}$ and making the necessary manipulations on (3.a), the roll angular velocity of the inner (elevation) frame of the sighting system assembly (p_m) can be calculated in terms of the known variables $p, q, r_m \;\; \psi_t, \psi_m, \theta_m$:

$$p_m = [\cos(\psi_t + \psi_m)p + \sin(\psi_t + \psi_m)q]/\cos(\theta_m) - r_m \tan(\theta_m) \tag{4}$$

The position vector defining the position of the centre of gravity of the sighting system base frame with respect to the inertial frame is

$$\vec{r}_{G_a/0} = \vec{r}_{G_a/a} + \vec{r}_{a/t} + \vec{r}_{t/b} + \vec{r}_{b/0} \tag{5}$$

The frame dependent second derivation (D_0^2) of the position vector will lead to the linear acceleration of the centre of gravity of the base frame of the sighting system assembly with respect to the inertial frame ($\vec{a}_{G_a}$):

$$\vec{a}_{G_a} = D_0^2 \vec{r}_{G_a/0} = D_0^2 (\vec{r}_{G_a/a} + \vec{r}_{a/t} + \vec{r}_{t/b} + \vec{r}_{b/0}) \tag{6}$$

Defining $a_{b/0}^{(b)} = [a_x \, a_y \, a_z]^{\mathrm{T}}$ and assuming the body, revolving base and the sighting system are rigid bodies, the linear acceleration of the centre of gravity of the base frame of the sighting system assembly with respect to the inertial frame expressed at the sighting system base frame can be defined as

$$\begin{aligned} \overline{a}_{G_a}^{(a)} &= \overline{a}_a^{(a)} + \left[\tilde{\alpha}_{a/0}^{(a)} + (\tilde{\omega}_{a/0}^{(a)})^2\right] \overline{r}_{G_a/a}^{(a)} \\ \overline{a}_a^{(a)} &= C^{(a,b)} \overline{a}_{b/0}^{(b)} + C^{(a,b)} \left[\tilde{\alpha}_{b/0}^{(b)} + (\tilde{\omega}_{b/0}^{(b)})^2\right] \overline{r}_{t/b}^{(b)} \\ &\quad + C^{(a,t)} \left[\tilde{\alpha}_{t/0}^{(t)} + (\tilde{\omega}_{t/0}^{(t)})^2\right] \overline{r}_{a/t}^{(t)} \end{aligned} \tag{7}$$

Substituting (2) into (7) $\overline{a}_{G_a}^{(a)}$ is found:

$$\overline{a}_{G_a}^{(a)} = \ddot{\psi}_m \tilde{u}_3 \overline{r}_{G_a/a}^{(a)} + \overline{a}_a^{(a)} + \left[\Delta\tilde{\alpha}_{a/0}^{(a)} + (\tilde{\omega}_{a/0}^{(a)})^2\right] \overline{r}_{G_a/a}^{(a)} \tag{8}$$

Similarly the linear acceleration vector defining the position of the center of gravity of the inner (elevation) frame of the sighting system assembly with respect to the inertial frame ($\vec{a}_{G_m}$) is:

$$\vec{a}_{G_m} = D_0^2 \vec{r}_{G_m/0} = D_0^2 (\vec{r}_{G_m/m} + \vec{r}_{m/a} + \vec{r}_{a/t} + \vec{r}_{t/b} + \vec{r}_{b/0}) \tag{9}$$

Again with rigid body assumption the linear acceleration of the center of gravity of the inner frame of the sighting system assembly with respect to the inertial frame expressed at the sighting system inner frame can be found:

$$\begin{aligned} \overline{a}_{G_m}^{(m)} &= C^{(m,a)} \left[\overline{a}_a^{(a)} + \left[\tilde{\alpha}_{a/0}^{(a)} + (\tilde{\omega}_{a/0}^{(a)})^2\right] \overline{r}_{m/a}^{(a)}\right] \\ &\quad + \left[\tilde{\alpha}_{m/0}^{(m)} + (\tilde{\omega}_{m/0}^{(m)})^2\right] \overline{r}_{G_m/m}^{(m)} \end{aligned} \tag{10}$$

Plugging (2) and (3) into (10) $\overline{a}_{G_m}^{(m)}$ is found:

$$\begin{aligned} \overline{a}_{G_m}^{(m)} &= \ddot{\psi}_m (R_2(-\theta_m) \tilde{u}_3 \overline{r}_{m/a}^{(a)} + \Delta_1 \tilde{\alpha}_{m/0}^{(m)} \overline{r}_{G_m/m}^{(m)}) \\ &\quad + \ddot{\theta}_m \tilde{u}_2 \overline{r}_{G_m/m}^{(m)} + \left[\Delta_2 \tilde{\alpha}_{m/0}^{(m)} + (\tilde{\omega}_{m/0}^{(m)})^2\right] \overline{r}_{G_m/m}^{(m)} \\ &\quad + R_2(-\theta_m) \left[\overline{a}_a^{(a)} + \left[\Delta\tilde{\alpha}_{a/0}^{(a)} + (\tilde{\omega}_{a/0}^{(a)})^2\right] \overline{r}_{m/a}^{(a)}\right] \end{aligned} \tag{11}$$

3. THE ATTITUDE FILTER DESIGN

In this section the attitude filter will be designed to estimate the biases of the gyroscopes of the sighting system assembly. The filter will use the attitude relations derived below as the mathematical process model. The measurement model is obtained from the aiding provided by the host vehicle INS and the encoders mounted on the revolving base and the moving frames of the sighting system.

The time rate of change of the direction cosine matrix (DCM) which relates the sighting system frame to the navigation frame is defined as: (Savage 1998, Titterton 2004)

$$\dot{C}^{(n,m)} = C^{(n,m)} \tilde{\omega}_{m/n}^{(m)} \tag{12}$$

Here, $\tilde{\omega}_{m/n}^{(m)}$ is the skew symmetric form of $\overline{\omega}_{m/n}^{(m)}$, which is the angular velocity vector of the sighting system inner frame with respect to navigation frame. This vector can also be expressed in terms of three angular velocity expressions:

$$\overline{\omega}_{m/n}^{(m)} = \overline{\omega}_{m/0}^{(m)} - \overline{\omega}_{n/e}^{(m)} - \overline{\omega}_{e/0}^{(m)} \tag{13}$$

Here, $\overline{\omega}_{e/0}^{(m)}$ represents the rotational rate of the earth with respect to the inertial frame (F_0), $\overline{\omega}_{n/e}^{(m)}$ represents the angular velocity of the navigation frame (F_n) with respect to the earth frame (F_e), i.e. the transport rate and $\overline{\omega}_{m/0}^{(m)}$ represents the angular velocity of the sighting system inner frame with respect to the inertial frame. Here, it should be noted that two components of $\overline{\omega}_{m/0}^{(m)}$ are measured by the gyroscopes on the sighting system inner frame and the third component is calculated by using (4).

3.1 The Linear Attitude Error Model

In this part the linear error model of the mechanization which will be utilized in the Kalman filter equations is presented. The estimated DCM, i.e. $\hat{C}^{(n,m)}$, can be represented in terms of the true DCM, i.e. $C^{(n,m)}$: (Savage 1998, Titterton 2004)

$$C^{(n,m)} = (I_{3x3} - \Psi) \hat{C}^{(n,m)} \tag{14}$$

Here Ψ is the skew-symmetric matrix form of the vector $d\overline{\psi} = [d\alpha \, d\beta \, d\gamma]^{\mathrm{T}}$ which defines the attitude errors and $I_{3x3} - \Psi$ represents a DCM which transforms the calculated navigation coordinate system to the true (error free) navigation coordinate system. It can be shown that the time rate of change of the attitude error can be expressed by the following matrix differential equation (Titterton 2004)

$$\dot{\Psi} = \Psi \tilde{\omega}_{n/0}^{(n)} - \tilde{\omega}_{n/0}^{(n)} \Psi + \delta\tilde{\omega}_{n/0}^{(n)} - \hat{C}^{(n,m)} \delta\tilde{\omega}_{m/0}^{(m)} \left(\hat{C}^{(n,m)}\right)^{\mathrm{T}} \tag{15}$$

Here $\delta\overline{\omega}_{m/0}^{(m)}$ represents the errors on the sighting system gyroscopes. For the application considered in this study, the first three terms on the right hand side of (15), which consist of the angular rate of the navigation frame, may be considered to be small when compared with the last term which is due to the gyroscope errors. With this simplification the linear attitude error dynamics can be written in the following vector form.

$$d\dot{\overline{\Psi}} = -\hat{C}^{(n,m)}\,\delta\overline{\omega}_{m/0}^{(m)} \tag{16}$$

The main purpose of the filter is to estimate the bias values on the gyroscopes. So, in addition to the linear attitude error model derived above the gyroscope errors must be modeled and integrated to the designed filter. Generally, the total error on gyroscopes can be modeled as the sum of the fixed (run-to-run) bias, the bias instability (in-run drift), the g-dependent bias and the noise terms:

$$\delta\overline{\omega}_{i/m}^{m}(t) = \overline{b}_f(t) + \overline{b}_{gm}(t) + \overline{w}_n(t) \tag{17}$$

Here the noise term $w_n(t)$ is assumed to be a white noise process with following spectral density

$$E\{\overline{w}_n(t)\overline{w}_n(t+\tau)\} = Q_w\delta(\tau) \tag{18}$$

The fixed (run-to-run) bias term is modeled as a random constant with σ_{bs}^2 initial variance.

$$\dot{b}_f(t) = 0 \text{ and } E\{\overline{b}(0)\overline{b}(0)^{\mathrm{T}}\} = \sigma_{bs}^2 I_{3x3} \tag{19}$$

The g-dependent bias term and bias instability term are modeled as a first order Gauss-Markov process.

$$\dot{\overline{b}}_{gm}(t) = (-1/\tau_{gm})I_{3X3}\overline{b}_{gm}(t) + \overline{w}_{gm}(t) \tag{20}$$

The process noise power spectral density matrix of this Gauss-Markov stochastic process is

$$E\{\overline{w}_{gm}(t)\overline{w}_{gm}(t+\tau)^{\mathrm{T}}\} = Q_{w,gm}\delta(\tau) \tag{21}$$

Here $Q_{w,gm} = (2\sigma_{gm}^2/\tau_{gm})I_{3x3}$ is the process noise power spectral density matrix and σ_{gm}^2 is the steady-state variance of the Gauss-Markov stochastic process. The attitude and gyroscope error models obtained above can be expressed in state-space form: $\delta\dot{\overline{x}}(t) = A(t)\delta\overline{x}(t) + B(t)\overline{w}(t)$. Here,

$\delta\overline{x}(t) = [d\overline{\Psi}(t)\ \overline{b}(t)]^{\mathrm{T}}$, $\overline{w}(t) = [\overline{w}_n(t)\ \overline{w}_{gm}(t)]^{\mathrm{T}}$ and

$$A(t) = \begin{bmatrix} 0_{3X3} & -\hat{C}^{(n,m)}(t) \\ 0_{3X3} & (-1/\tau_{gm})I_{3x3} \end{bmatrix},\ B(t) = \begin{bmatrix} -\hat{C}^{(n,m)}(t) & 0_{3x3} \\ 0_{3x3} & I_{3x3} \end{bmatrix}.$$

In order to reduce the number of total states the fixed bias term $\overline{b}_f$ and the in-run drift term $\overline{b}_{gm}(t)$ are considered as a single bias state vector $\overline{b}(t)$ in the filter design. However, it is obvious that $\overline{b}(t)$ is mostly dominated by $\overline{b}_f$.

By utilizing the difference between the sighting system attitude, calculated by using the INS and the encoder measurements and calculated by using the gyroscope outputs, the gyroscope biases can be estimated.

Let $C_{\mathrm{mea}}^{(n,m)}$ be the DCM between the sighting system frame and the navigation frame. Thus, it can be expressed in terms of the measurable quantities:

$$C_{\mathrm{mea}}^{(n,m)} = \hat{C}^{(n,b)}\left(\hat{C}^{(m,b)}\right)^{\mathrm{T}} \tag{22}$$

Here $\hat{C}^{(n,b)}$ is the DCM constructed from the body Euler angles measured by the INS on the host vehicle. Also, $\hat{C}^{(m,b)}$ is the DCM constructed from the sighting system elevation, azimuth and the revolving base encoders respectively:

$$\hat{C}^{(m,b)} = \hat{C}^{(m,a)}\hat{C}^{(a,t)}\hat{C}^{(t,b)} \tag{23}$$

The relation between the true $C^{(n,m)}$ and the calculated $\hat{C}^{(n,m)}$ was given in (16). Similarly the relation between the true $C^{(n,m)}$ and $C_{\mathrm{mea}}^{(n,m)}$ can be expressed as

$$C^{(n,m)} = (I - \Psi_{\mathrm{mea}})C_{\mathrm{mea}}^{(n,m)} \tag{24}$$

Thus, a measurement matrix (M) can be defined:

$$M = \hat{C}^{(n,m)}\left(C_{\mathrm{mea}}^{(n,m)}\right)^{\mathrm{T}} = (I-\Psi)^{-1}C^{(n,m)}\left(C^{(n,m)}\right)^{\mathrm{T}}(I + \Psi_{\mathrm{mea}}) \cong I + \Psi + \Psi_{\mathrm{mea}} \tag{25}$$

Thus, the measurement of tilt errors, i.e. $(d\gamma + d\gamma_{\mathrm{mea}})$, $(d\beta + d\beta_{\mathrm{mea}})$, $(d\alpha + d\alpha_{\mathrm{mea}})$ can be obtained from M:

$$\begin{bmatrix} d\alpha + d\alpha_{\mathrm{mea}} \\ d\beta + d\beta_{\mathrm{mea}} \\ d\gamma + d\gamma_{\mathrm{mea}} \end{bmatrix} = \begin{bmatrix} M(3,2) \\ M(1,3) \\ M(2,1) \end{bmatrix} = [I_{3x3}\ 0_{3x3}]\begin{bmatrix} d\overline{\Psi} \\ \overline{b}_{GM} \end{bmatrix} + \begin{bmatrix} d\alpha_{\mathrm{mea}} \\ d\beta_{\mathrm{mea}} \\ d\gamma_{\mathrm{mea}} \end{bmatrix} \tag{26}$$

This gives the measurement model as $\delta\overline{y} = H\delta\overline{x} + \overline{v}$, with $H = [I_{3x3}\ 0_{3x3}]$ and $\overline{v} = [d\alpha_{\mathrm{mea}}\ d\beta_{\mathrm{mea}}\ d\gamma_{\mathrm{mea}}]^{\mathrm{T}}$. Also, it is assumed that the measurement noise is a white noise with Gaussian distribution.

Using the constructed measurement model and the linear attitude model derived in Section 3.2 an extended Kalman filter is designed. Since the recursive equations for the Kalman filter are well known, and given in detail in references (Brown 1997, Gelb2001), they are not repeated here. The performance of the designed filter is verified on the closed loop operation for the stabilization of the sighting system. The results of the gyroscope bias estimation and its effect on the enhancement on the stabilization performance are shown in Section 6.

4. DYNAMICS OF THE TWO AXES SYSTEM

The purpose of designing the stabilization controller is to generate the necessary control torques in order to isolate the desired system from the base disturbances originating from the motion of the host vehicle. Hence, the system will be stabilized with respect to the inertial frame and kept in level position.

The dynamic equations of the sighting system are derived using Newton's 2nd law of motion. In the derivations the angular velocity, acceleration and translational acceleration expressions derived in Section 2 are used. Therefore, using the expressions for the linear accelerations ($\bar{a}_{G_a}^{(a)}, \bar{a}_{G_m}^{(m)}$) the Newton-Euler equations defining the dynamics of the outer (azimuth) and inner (elevation) frames of the sighting system assembly can be written:

$$m_a \bar{a}_{G_a}^{(a)} = \bar{F}_{a/m}^{(a)} + \bar{F}_{a/t}^{(a)} + m_a \bar{g}_a \tag{27.a}$$

$$J_{G_a} \bar{\alpha}_{a/0}^{(a)} + \tilde{\omega}_{a/0}^{(a)} J_{G_a} \bar{\omega}_{a/0}^{(a)} = \bar{M}_{a/m}^{(a)} + \bar{M}_{a/t}^{(a)} + \tilde{r}_{a/m}^{(a)} \bar{F}_{a/m}^{(a)} + \tilde{r}_{a/t}^{(a)} \bar{F}_{a/t}^{(a)} \tag{27.b}$$

$$m_m \bar{a}_{G_m}^{(m)} = \bar{F}_{m/a}^{(m)} + m_m \bar{g}_m \tag{27.c}$$

$$J_{G_m} \bar{\alpha}_{m/0}^{(m)} + \tilde{\omega}_{m/0}^{(m)} J_{G_m} \bar{\omega}_{m/0}^{(m)} = \bar{M}_{m/a}^{(m)} + \tilde{r}_{m/a}^{(m)} \bar{F}_{m/a}^{(m)} \tag{27.d}$$

Here, m_a, m_m are the masses of the outer and inner frames of the sighting system assembly, J_{G_a}, J_{G_m} are the inertia tensors defining the inertia components in the principal axis of the outer and inner frames of the sighting system assembly, g is the magnitude of the gravity vector and $\bar{g}_a = C^{(a,0)} g \bar{u}_3$, $\bar{g}_m = C^{(m,0)} g \bar{u}_3$ are the vectors defining the gravity vector components in the outer and inner frames of the sighting system assembly. Also, $\bar{F}_{t/a}^{(a)} = -\bar{F}_{a/t}^{(a)}$, $\bar{F}_{m/a}^{(a)} = -\bar{F}_{a/m}^{(a)}$ and $\bar{M}_{t/a}^{(a)} = -\bar{M}_{a/t}^{(a)}$, $\bar{M}_{m/a}^{(a)} = -\bar{M}_{a/m}^{(a)}$ are the reaction forces and moments at the outer frame joint (azimuth) and the inner frame joint (elevation) respectively. Here, it should be noted that 3rd component of $\bar{M}_{a/t}^{(a)}$ is composed of the control torque and the viscous friction on the azimuth joint of the sighting system assembly. Also, 2nd component of $\bar{M}_{a/m}^{(a)}$ is composed of the control torque and the viscous friction on the elevation joint of the sighting system assembly. They are expressed as

$\bar{M}_{a/t}^{(a)} = \left[M_{at1}\; M_{at2}\; \tau_a - b_a \dot{\psi}_a\right]^{\mathrm{T}}$, and,
$\bar{M}_{a/m}^{(a)} = \left[M_{am1}\; \tau_e - b_e \dot{\theta}_m\; M_{am3}\right]^{\mathrm{T}}$.

Here, τ_a, τ_e are the control torques generated to stabilize the sighting system and b_a, b_e are the viscous friction coefficients on the azimuth and elevation joints. Using these expressions the Newton-Euler equations, defined in (27), can be arranged as:

$$F \begin{bmatrix} \ddot{\psi}_m \\ \ddot{\theta}_m \end{bmatrix} + R_e \left[\bar{F}_{a/m}^{(a)}\; \bar{F}_{a/t}^{(a)}\; \bar{M}_r\right]^{\mathrm{T}} = D + G \begin{bmatrix} \tau_a \\ \tau_e \end{bmatrix} \tag{28}$$

Here, $\bar{M}_r = \left[M_{am1}\; M_{am3}\; M_{at1}\; M_{at2}\right]^{\mathrm{T}}$ is a vector composed of the reaction moment components on the azimuth and elevation joints of the sighting system. Note that F, R_e, D and G matrices are constructed by implementing algebraic manipulations on (27) and, due to the space limitations, they are not presented in their explicit form in this paper.

Hence, by using the instantaneous relative angular positions and velocities of the sighting system (ψ_m, θ_m, $\dot{\psi}_m, \dot{\theta}_m$), the base disturbance ($\bar{\omega}_{b/0}, \bar{\alpha}_{b/0}$, $\bar{a}_{b/0}, \bar{\omega}_{t/0}, \bar{\alpha}_{t/0}$) and the control torques (τ_a, τ_e), $\ddot{\psi}_m$, $\ddot{\theta}_m$, $\bar{F}_{a/m}^{(a)}$, $\bar{F}_{a/t}^{(a)}$ and $\bar{M}_r$ can be calculated. The open loop block diagram representation of the two axes sighting system dynamics is shown in Fig. 2.

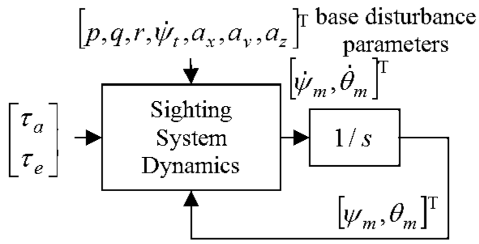

Fig. 2. Open loop block diagram for the two axes sighting system dynamics with base disturbance parameters

5. THE STABILIZATION CONTROLLER DESIGN

In this study, in order to reduce the design work and watch for the ease of the real-time controller implementation a linear controller is synthesized. For this purpose the nonlinear dynamics given in (28) is linearized at a desired point at which the global motion of the sighting system can nominally be represented.

The linearization point is chosen such that the relative angular accelerations and velocities of the sighting system assembly are all zero ($\ddot{\psi}_m = \ddot{\theta}_m = \dot{\psi}_m = \dot{\theta}_m = 0$). Also, at the linearization condition the host vehicle and the revolving base are both taken to be stationary. Thus, the base disturbance is not existent and $\bar{\omega}_{t/0}^{(t)} = \bar{\alpha}_{t/0}^{(t)} = \bar{a}_{b/0}^{(b)} = \bar{0}$. Furthermore, the host vehicle body is in level position with respect to the inertial frame ($\phi = \theta = 0$) and the azimuth and elevation joints are at their absolute zero positions ($\psi_m = \theta_m = 0$). Thus, using the specific values of the system, i.e. m_a, m_m, b_a, b_e, J_{G_a}, J_{G_m}, $\bar{r}_{G_a/a}^{(a)}$, $\bar{r}_{G_m/m}^{(m)}$, $\bar{r}_{t/b}^{(b)}$, $\bar{r}_{a/t}^{(t)}$, $\bar{r}_{m/a}^{(a)}$, and the linearization point conditions the control torques, the reaction forces and moments at the sighting system assembly joints can be calculated. Hence, using these values a linear model of the two axes sighting system dynamics can be written: $\dot{\bar{x}} = A_l \bar{x} + B_l \bar{u}$, $\bar{y} = C_l \bar{x}$.

Here, $\bar{x} = \left[\psi_m, \theta_m, \dot{\psi}_m, \dot{\theta}_m\right]^{\mathrm{T}}$, $\bar{y} = \left[\dot{\psi}_m, \dot{\theta}_m\right]^{\mathrm{T}}$, $\bar{u} = \left[\tau_a, \tau_e\right]^{\mathrm{T}}$ and, A_l, B_l, C_l are (4x4), (4x2) and (2x4) real matrices respectively. Taking the first time derivative of the output equation the time rate of change of the output can be found:

$$\dot{\bar{y}} = A' \bar{x} + B' \bar{u}\,,\; A' = C_l A_l\,,\; B' = C_l B_l \tag{29}$$

Therefore, the commanded torques (τ_{ac}, τ_{ec}) with respect to the commanded accelerations ($\ddot{\psi}_{mc}, \ddot{\theta}_{mc}$) can be calculated:

$$\begin{bmatrix}\tau_{ac}\\ \tau_{ec}\end{bmatrix} = (B')^{-1}\left[\begin{bmatrix}\ddot{\psi}_{mc}\\ \ddot{\theta}_{mc}\end{bmatrix} - A'\left[\psi_m\ \theta_m\ \dot{\psi}_m\ \dot{\theta}_m\right]^{\mathrm{T}}\right] \quad (30)$$

The control task is to assign a stable dynamics to $\ddot{\psi}_{mc}$, $\ddot{\theta}_{mc}$ and guarantee the stability of $\dot{\psi}_m$, $\dot{\theta}_m$. For that purpose the following dynamic error model is proposed. The proper selection of the controller gain matrices K_p, K_i will lead to the stable control of $\dot{\psi}_m$ and $\dot{\theta}_m$.

$$\begin{bmatrix}\ddot{\psi}_{mc}\\ \ddot{\theta}_{mc}\end{bmatrix} = K_p\bar{e}(t) + K_i\int^{t}\bar{e}(\tau)d\tau\,,\ \bar{e}(t) = \begin{bmatrix}\dot{\psi}_{md}\\ \dot{\theta}_{md}\end{bmatrix} - \begin{bmatrix}\dot{\psi}_m\\ \dot{\theta}_m\end{bmatrix} \quad (31)$$

Here, note that, the torques, computed by the designed controller, are applied to the azimuth and elevation joints of the sighting system assembly. However, the stabilization is desired to be carried out with respect to the inertial frame F_0. Therefore, the task frame variables ($r_{md}=0$, $q_{md}=0$ and r_m, q_m) should be transformed to the joint frame variables ($\dot{\psi}_{md}, \dot{\theta}_{md}$ and $\dot{\psi}_m, \dot{\theta}_m$). This is done by using the angular velocity kinematics given in Section 2. Hence, $\bar{e}(t)$ is found:

$$\bar{e}(t) = \begin{bmatrix}\dot{\psi}_{md}-\dot{\psi}_m\\ \dot{\theta}_{md}-\dot{\theta}_m\end{bmatrix} = \begin{bmatrix}-r_m\sec(\theta_m)\\ -q_m\end{bmatrix} \quad (32)$$

Since the control action is decoupled by the inversion of the dynamics it is possible to assign diagonal controller gain matrices as $K_p = \mathrm{diag}(k_{pa}, k_{pe})$ and $K_i = \mathrm{diag}(k_{ia}, k_{ie})$. Therefore, the Laplace transform of (31) will lead to the following transfer functions:

$$\frac{\psi_m(s)}{\psi_{md}(s)} = \frac{k_{pa}s + k_{ia}}{s^2 + k_{pa}s + k_{ia}} \quad \text{and} \quad \frac{\theta_m(s)}{\theta_{md}(s)} = \frac{k_{pe}s + k_{ie}}{s^2 + k_{pe}s + k_{ie}}$$

Thus, $k_{pa} = k_{pe} = 2\xi\omega_n$ and $k_{ia} = k_{ie} = \omega_n^2$ can be chosen such that the natural frequency of the controlled closed loop will be ω_n and the damping ratio will be ξ. Here, considering the dynamic content of the base disturbance signals and corresponding desired stabilization performance $\omega_n = 100$Hz and $\xi = 0.8$ are chosen as the control design parameters.

6. RESULTS

To validate the performance of the gyroscope bias estimation filter developed in Section 3, a simulation environment is developed by using the dynamic model of the host vehicle and the sighting system assembly developed in Sections 2 and 4. The block diagram representation of the simulation model is shown in Fig. 3.

The error characteristics of the sensors used in this simulation study are selected in terms of their RMS values. The inertial navigation system on the host vehicle has roll (ϕ), pitch (θ) accuracies of 0.05° and the heading (ψ) accuracy is 0.2°. The INS angular rate (p,q,r) errors are 1°/hr fixed bias, 1°/hr bias instability modeled as a Gauss-Markov process with a correlation time of 60 seconds, and $0.1^\circ/\sqrt{\text{hr}}$ angle random walk. The sighting system pitch rate (q_m) and yaw rate (r_m) measurements have bias values of 100°/hr and angle random walk values of $0.15^\circ/\sqrt{\text{hr}}$. The bias instability model is a Gauss-Markov process with 30°/hr RMS and 100 seconds of correlation time. The revolving base, sighting system azimuth and elevation encoders have 0.05° noise levels.

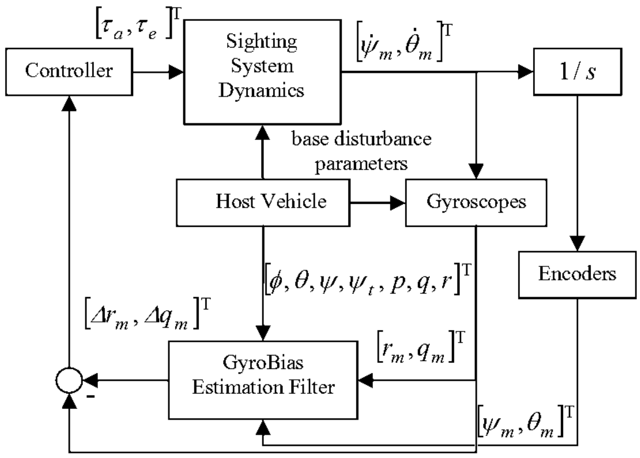

Fig. 3. Closed loop block diagram for the two axes sighting system, host vehicle, gyro-bias estimation filter and controller

In order to validate the closed loop integrated performance enhancement of the designed filter the disturbance signals are generated artificially by considering the dynamic content of the previously measured signals during the vehicle's motion on a rough terrain. The host vehicle's low frequency roll, pitch and yaw angular motions with respect to the ground are generated as:

$\phi = 1.5^\circ\sin(0.6t) + 1.0^\circ\sin(0.3t)$,

$\theta = -1.5^\circ\sin(0.6t) + 1.0^\circ\sin(0.3t)$,

$\psi = 1.5^\circ\sin(0.6t + \pi/3) + 1.0^\circ\sin(0.3t + \pi/3)$.

The revolving structure angular motion with respect to the vehicle body is generated as: $\psi_t = 0.1^\circ\sin(60t + \pi/4)$. Also, in order to represent the motion components that arise from the vehicle and sighting system structural modes $\Delta = 0.01^\circ\sin(101t) + 0.0001^\circ\sin(258t)$ is added on all of the defined angular motions of the base disturbance. The linear acceleration components of the base disturbance (a_x, a_y, a_z) are taken to be equal to zero.

Fig. 4 shows the results of a simulation when the controller is on but the biases are not estimated. It is seen that the sighting system azimuth and elevation angles drift 3-4 degrees in 2 minutes. In a normal operation, i.e. without the bias estimation filter proposed in this study, the biases should be corrected periodically by the user.

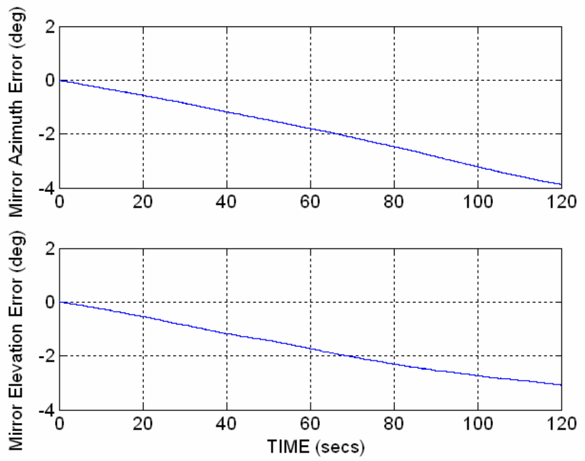

Fig. 4. Closed-loop control without gyro-bias estimation

Fig. 5 shows the result of a simulation when the controller is on and the sighting system gyroscope biases are estimated. It is seen that the drift of the sighting system azimuth and elevation angles are negligible. The gyro-biases which are estimated and fed back as corrections on the measured sighting system angular rates are shown in Fig. 6.

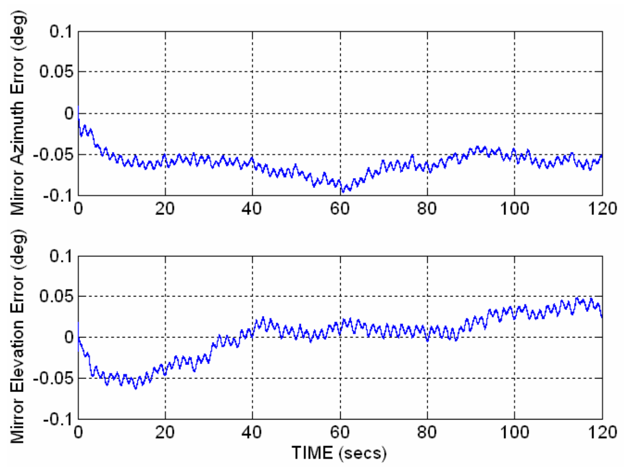

Fig. 5. Closed-loop control with gyro-bias estimation

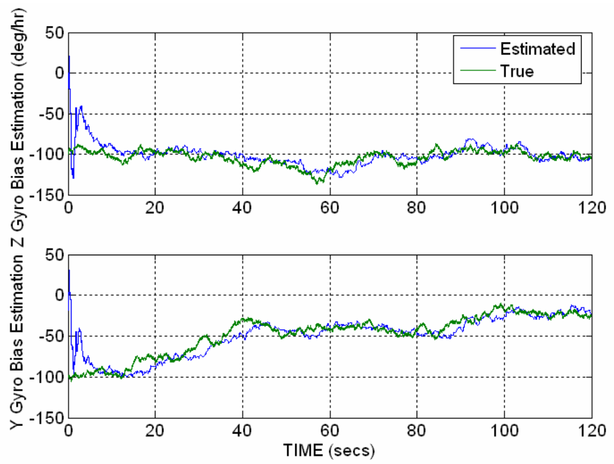

Fig. 6. Gyro-bias estimation with base disturbance

6. CONCLUSIONS

In this study, the positive effect of estimating the gyroscope biases of a two axes gimbaled system on the performance of closed loop stabilization is demonstrated. Throughout the study, since the attitude equations require all three angular rates, the missing roll rate is calculated by using the kinematical relations, the measurements of inertial navigation system on the host vehicle and the encoders on the system. It is shown by the simulations that the integration of the bias estimation filter into the feedback loop gradually enhances the stabilization quality and truly eliminates the need for the frequent sighting system angle drift corrections.

As for the future studies the effect of the possible errors originating from the sensor measurement synchronization, i.e. the encoders and the attitude measurements of the host vehicle INS, should be investigated for real time implementations. Furthermore, instead of computer generated base disturbance signals the stabilization performance can be investigated by using the real time data sampled from the host vehicle platforms.

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