Solenoid electron lenses

Fundamentals and design

Anton Douginets   Andrii Yanovets

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Structure

Motivation

- Magnetic lenses - important components of particle accelerators, microscopes
- Electromagnet solenoids - physical foundation of more advanced magnetic lenses
- Design constraints:
  - Power consumption
  - Physical size, material usage
  - Characteristic parameters for interaction with other machine components

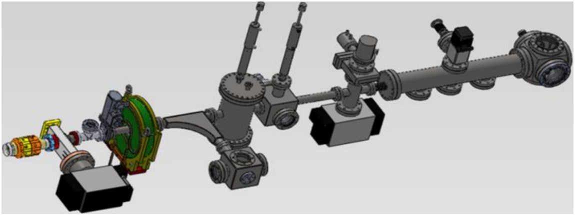


Figure: Schematic[1] of AREAL, an electron bunch-research oriented linac[2]

[1] **Grigoryan2014**.
[2] **Grigoryan2011**.

Structure

Electron optics

Definition

Classic optics	Electron optics
•Light ray - photons	•Electron beam - electrons
•Can pass through optically transparent solids	•Gets absorbed/loses energy due to interactions with atoms in media
•Bends due to refractive index difference between media	•Bends due to Coulomb and Lorentz forces in the presense of external EM-fields

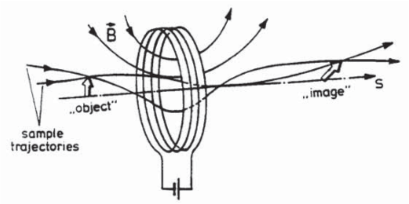
Magnet lenses

Requirements

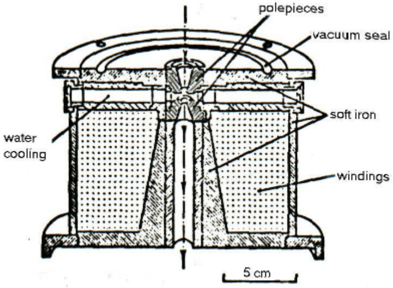
A lens must have the following properties:

- deflection increases with increasing deviation of the beam from the optic axis;
- electron energy should not change, or change negligebly;
- symmetry of deflection on all sides of the optical axis;
- methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses
Solenoids



(a) A solenoid with S as optical axis[1]



(b) Cross-section of a magnetic lens[2]

[1]**a˙optics**.
[2]**Egerton**.

Magnet lenses

Solenoids

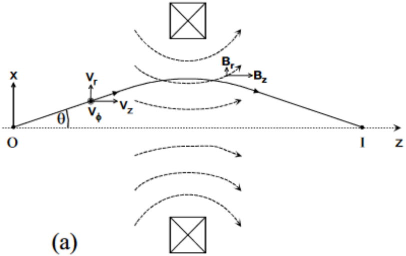


Figure: A solenoid cross-section along the optic axis[1]

$$F_\varphi = e\left(B_z v_r - v_z B_r\right) \quad (1)$$

$$F_r = -e\left(v_z B_z\right) \quad (2)$$

$$F_z = e\left(v_\varphi B_r\right) \quad (3)$$

$$\gamma m\left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \quad (4)$$

$$\gamma m\frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \quad (5)$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \quad (6)$$

[1] **Egerton**.

Magnet lenses

Electron path equations

$$B_z\left(z,r\right)=\sum_n\frac{(-1)^n}{n!n!}\left(\frac{r}{2}\right)^{2n}\frac{\partial^{2n}B_{z,\,axis}}{\partial z^{2n}}\qquad(7)$$

$$B_r\left(z,r\right)=\sum_n\frac{(-1)^n}{n!\left(n-1\right)!}\left(\frac{r}{2}\right)^{2n-1}\frac{\partial^{2n-1}B_{z,\,axis}}{\partial z^{2n-1}}\qquad(8)$$

• From (5) follows:

$$\dot{\varphi}=\frac{e}{2\gamma m}B_z\qquad(9)$$

• From (4) and (6) follows:

$$\ddot{r}=-\left(\frac{e}{2\gamma m}\right)^2 rB_z^2\qquad(10)$$

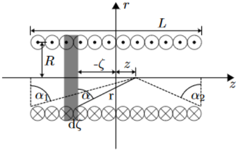
$$\ddot{z}=-\left(\frac{e}{2\gamma m}\right)^2 r^2B_zB_z'\qquad(11)$$

$$\ddot{r}=r''\dot{z}^2\approx r''\left(\beta c\right)^2\Rightarrow r''=\left(\frac{e}{2p_z}\right)^2 rB_z^2\qquad(12)$$

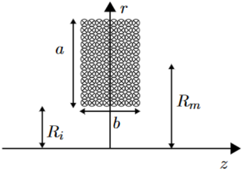
$$-\frac{r'}{r}=\frac{1}{f}\underset{integrate\,(12)}{\Rightarrow}\frac{1}{f}=\left(\frac{e}{2p_z}\right)^2\int_{-\infty}^{\infty}B_z^2dz:=\left(\frac{e}{2p_z}\right)^2F_2\qquad(13)$$

Magnet lenses

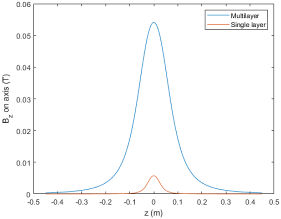
Magnetic field of solenoid



(a) Single-wind solenoid[1]



(b) Solenoid with multilayred windings[1]



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 nI}{2}\left(\frac{\triangle z}{\sqrt{R^2 + \triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2 + \triangle z^{*2}}}\right) \tag{14}$$

$$\triangle z = z - L/2 \tag{15}$$

Approximate field of the solenoid (b)[1]:

$$B_z\,(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}}\right) \tag{16}$$

$$Rc = R_{sq} + c,\; R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right),\; c^2 = \frac{b^2 - a^2}{12} \tag{17}$$

[1] **Disser**.

Lens impefections

There are 3 main limitations to consider when designing a solenoid lens:

	Source:
•Chromatic aberration	Spread of electron energies
•RMS Emittance growth	Spread of eletron coordinates in position-and-momentum phase space
•Spherical aberration	Real lens $\rightarrow$ different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc}\sqrt{\left\langle x^2 \right\rangle \left\langle \tilde{p}_x^2 \right\rangle - \langle x\tilde{p}_x \rangle^2} = \frac{1}{mc}\left(\frac{e^2\sigma^4}{3\sqrt{2}p_{z,0}}F_3 + \frac{e^4\sigma^4}{24\sqrt{2}p_{z,0}^3}F_4\right) \quad (18)$$

Chromatic aberration:

$$r_c \approx \alpha f\,(\triangle E_0/E_0) \approx \alpha C_c\,(\triangle E_0/E_0) \quad (19)$$

Lens impefections

Spherical aberration[1]

$$\triangle f = c \cdot x^2 \tag{20}$$

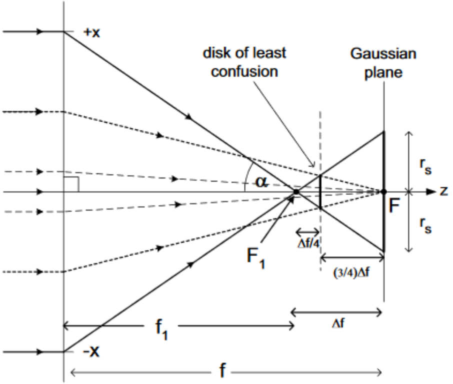
$$x \approx f \cdot tan(\alpha) \tag{21}$$

$$r_s = \triangle f \cdot tan(\alpha) \approx c \cdot f^2 tan(\alpha)^3 \tag{22}$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \tag{23}$$

$$F_3 = -\int \frac{B_z B_z''}{2} dz \tag{24}$$

$$F_4 = \int B_z^4 dz \tag{25}$$



$$C_s = \frac{e}{96 m\tilde{U}} \int \left(\frac{2e}{m\tilde{U}} B_z^4 + 5 \left(B_z' \right)^2 - B_z B_z'' \right) R^4 dz \tag{26}$$

$$= \frac{e^2 R^4}{4 p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4 \tag{27}$$

Figure: Illustration of beam radius change due to spherical aberration

[1] **Egerton**.

Structure

Project task

Simple solenoid lens design:

- Monochromatic e beam, fixed beam radius R
- Target FWHM, peak B_z, f
- Optimize geometry, current for minimal spheric aberration

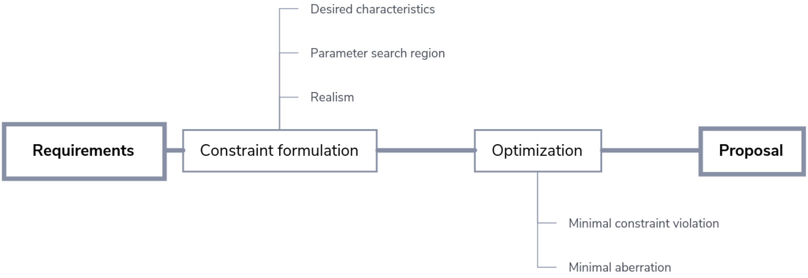


Figure: Generalized design process

Solenoid model

- Rectangular cross-section solenoid[1]

Two-loop field approximation:

$$B_z(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{(z^2+Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2+Rc^{*2})^{3/2}}\right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2 - a^2}{12},$$

$$R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right).$$

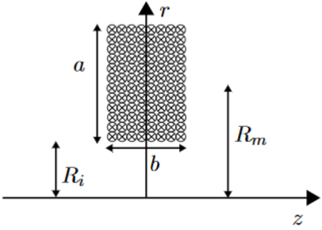


Figure: Solenoid geometry:
R_m - mean radius
a - transverse width
b - axial length

Parameters: geometry, scaling factor $N \cdot I$ [Ampere-Turns]

[1] **Disser**.

Field integrals

For an axial beam, only the on-axis B_z is of significance[1]. The field's optical properties are described in terms of:

$$F_1 = \int B_z dz \qquad F_3 = \int -\frac{B_z'' \cdot B_z}{2} dz$$

$$F_2 = \int B_z^2 dz \qquad F_4 = \int B_z^4 dz$$

whereas the integration domain is $(-\infty, \infty)$.

[1] **Disser**.

Solenoid characteristics

- Peak $B_z = B_z(0)$;
- Effective field length $\rightleftarrows$ FWHM
- Focal length:

$$f = \left(\frac{2p_z}{e}\right)^2 \frac{1}{F_2}$$

- Aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4$$

Considerations:

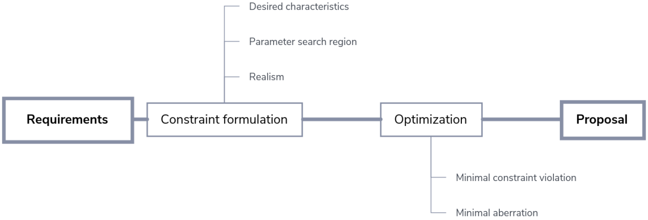
- Geometry - size, material usage
- Scaling factor - power, material usage
- f, FWHM, c_s: interaction with other components, fitness to purpose

Optimization

- Constrained Trust Region algorithm[1]:
  Iterative gradient search process; minimize merit function $m(p)$ of parameters $p = (s, R_m, a, b)$:

  $$m(p) = c_s(p) + k \cdot |C(p)|_2, \quad \Delta p \leq R_T;$$

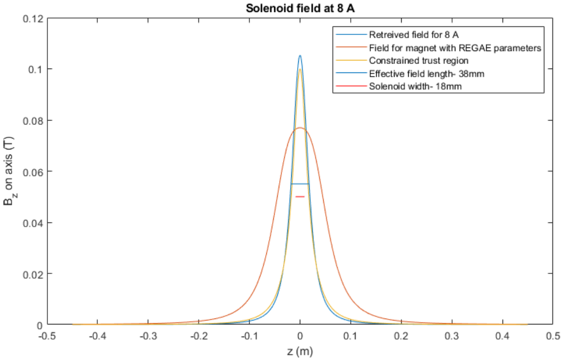
  $C(p) = (C_1, C_2, ..., C_n)(p)$ - measure of constraint violation;
- Interior Point Algorithm[2]:
  Solves a sequence of approximate optimization problems, by transforming the original problem $\underset{x}{min}\, f(x)\,,\; h(x) = 0 \wedge g(x) \leq 0$ into $\underset{x}{min}\, f(x) = \underset{x,s}{min}\, f(x) - \mu \sum_i ln(s_i)\,,\; h(x) = 0 \wedge g(x) + s = 0.$



[1] **scipyctr**.
[2] **sqp**.

Interior point Algorithm results

Retrieved parameters

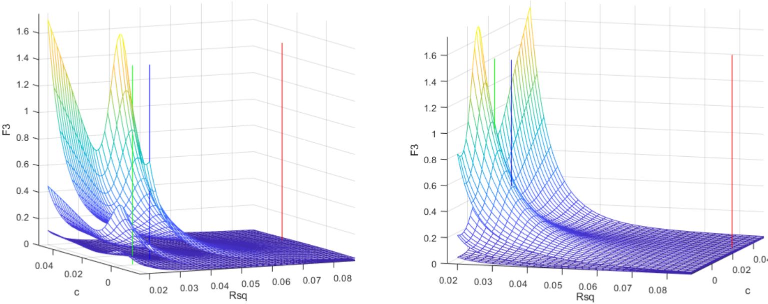


	Height a mm	Width b mm	Max field B_z mT	F. length f cm	Spherical ab. C_s	RMS emi. $\epsilon_{n.rms}$
Opt. parameters	17.6	17.6	105	50	$1.7e-9m$	$3.4e-10m$
REGAE[1]	99.5	41.8	79	30.5	$6.3e-11m$	$7.9e-11m$

[1] **Disser**.

Interior point Algorithm results

F_3 Integral



(a) **(b)**

Figure: F_3 integral

Perspective

Subtitle

Summary

Subtitle

Thank you for your attention!

References I