Solenoid electron lenses

Fundamentals and design

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Structure

Motivation

- ▶ Magnetic lenses - important components of particle accelerators, microscopes
- ▶ Electromagnet solenoids - physical foundation of more advanced magnetic lenses
- ▶ Design constraints:
  - ▶ Power consumption
  - ▶ Physical size, material usage
  - ▶ Characteristic parameters for interaction with other machine components
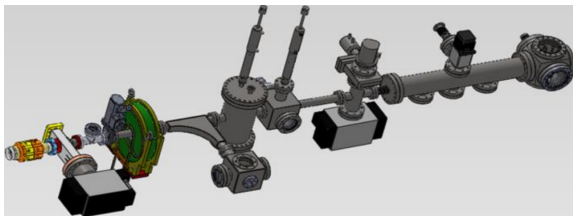

Figure: Schematic[1] of AREAL, an electron bunch-research oriented linac[2]

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[1] Grigoryan et al., "Status of AREAL RF Photogun Test Facility".

[2] Grigoryan et al., "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun".

Structure

Electron optics

Definition

Classic optics

- Light ray - photons

- Can pass through optically transparent solids

- Bends due to refractive index difference between media

Electron optics

- Electron beam - electrons

- Gets absorbed/loses energy due to interactions with atoms in media

- Bends due to Coulomb and Lorentz forces in the presense of external EM-fields
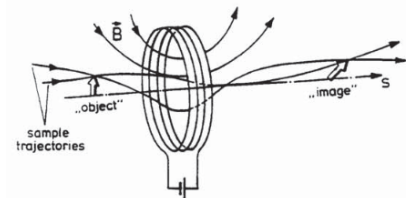
Magnet lenses
Requirements

A lens must have the following properties:

- ▶ deflection increases with increasing deviation of the beam from the optic axis;

- ▶ electron energy should not change, or change negligebly;

- ▶ symmetry of deflection on all sides of the optical axis;

- ▶ methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses
Solenoids

(a) A solenoid with S as optical axis[1]
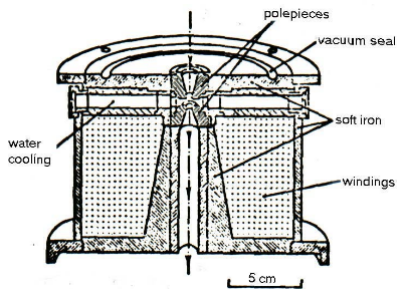
(b) Cross-section of a magnetic lens[2]

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[1]Rossbach and Schmüser, *Basic course on accelerator optics*.
[2]Egerton et al., *Physical principles of electron microscopy*.
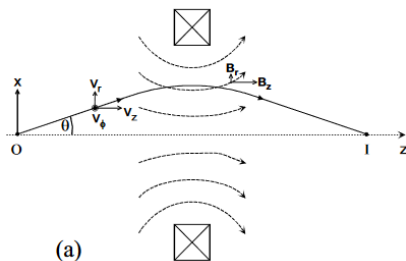
Magnet lenses
Solenoids

Figure: A solenoid cross-section along the optic axis[1]

$$F_\varphi = e\left(B_z v_r - v_z B_r\right) \quad (1)$$

$$F_r = -e\left(v_z B_z\right) \quad (2)$$

$$F_z = e\left(v_\varphi B_r\right) \quad (3)$$

$$\gamma m \left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \quad (4)$$

$$\gamma m \frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \quad (5)$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \quad (6)$$

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[1]Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses
Electron path equations

• From (5) follows:

$$\dot{\varphi} = \frac{e}{2\gamma m} B_z \qquad (9)$$

$$B_z\left(z, r\right) = \sum_n \frac{(-1)^n}{n! n!} \left(\frac{r}{2}\right)^{2n} \frac{\partial^{2n} B_{z,\,axis}}{\partial z^{2n}} \qquad (7)$$

• From (4) and (6) follows:

$$B_r\left(z, r\right) = \sum_n \frac{(-1)^n}{n!\,(n-1)!} \left(\frac{r}{2}\right)^{2n-1} \frac{\partial^{2n-1} B_{z,\,axis}}{\partial z^{2n-1}} \qquad (8)$$

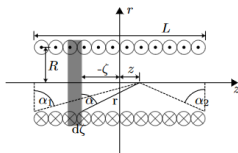$$\ddot{r} = -\left(\frac{e}{2\gamma m}\right)^2 r B_z^2 \qquad (10)$$

$$\ddot{z} = -\left(\frac{e}{2\gamma m}\right)^2 r^2 B_z B_z' \qquad (11)$$

$$\ddot{r} = r'' \dot{z}^2 \approx r''\left(\beta c\right)^2 \Rightarrow r'' = \left(\frac{e}{2p_z}\right)^2 r B_z^2 \qquad (12)$$
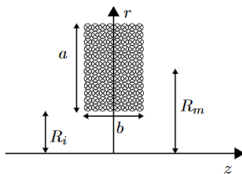
$$-\frac{r'}{r} = \frac{1}{f} \underset{integrate\ (12)}{\Rightarrow} \frac{1}{f} = \left(\frac{e}{2p_z}\right)^2 \int\limits_{-\infty}^{\infty} B_z^2\,dz \coloneqq \left(\frac{e}{2p_z}\right)^2 F_2 \qquad (13)$$
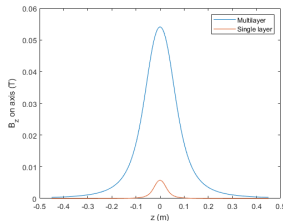
Magnet lenses

Magnetic field of solenoid

(a) Single-wind solenoid[1]

(b) Solenoid with multilayred windings[1]

(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 nI}{2}\left(\frac{\triangle z}{\sqrt{R^2 + \triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2 + \triangle z^{*2}}}\right) \qquad (14)$$

$$\triangle z = z - L/2 \qquad (15)$$

Approximate field of the solenoid (b)[1]:

$$B_z(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}}\right) \qquad (16)$$

$$Rc = R_{sq} + c, \; R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right), \; c^2 = \frac{b^2 - a^2}{12} \qquad (17)$$

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Lens impefections

There are 3 main limitations to consider when designing a solenoid lens:

	Source:
•Chromatic aberration	Spread of electron energies
•RMS Emittance growth	Spread of eletron coordinates in position-and-momentum phase space
•Spherical aberration	Real lens $\rightarrow$ different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc} \sqrt{\left\langle x^2 \right\rangle \left\langle \tilde{p}_x^2 \right\rangle - \left\langle x\tilde{p}_x \right\rangle^2} = \frac{1}{mc} \left(\frac{e^2 \sigma^4}{3\sqrt{2} p_{z,0}} F_3 + \frac{e^4 \sigma^4}{24\sqrt{2} p_{z,0}^3} F_4 \right) \tag{18}$$

Chromatic aberration:

$$r_c \approx \alpha f \left(\triangle E_0 / E_0 \right) \approx \alpha C_c \left(\triangle E_0 / E_0 \right) \tag{19}$$

Lens impefections

Spherical aberration[1]

$$\triangle f = c \cdot x^2 \qquad (20)$$

$$x \approx f \cdot \tan(\alpha) \qquad (21)$$

$$r_s = \triangle f \cdot \tan(\alpha) \approx c \cdot f^2 \tan(\alpha)^3 \qquad (22)$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \qquad (23)$$

$$F_3 = -\int \frac{B_z B_z''}{2} dz \qquad (24)$$

$$F_4 = \int B_z^4 dz \qquad (25)$$

Figure: Illustration of beam radius change due to spherical aberration

$$C_s = \frac{e}{96m\bar{U}} \int \left(\frac{2e}{m\bar{U}} B_z^4 + 5\left(B_z'\right)^2 - B_z B_z'' \right) R^4 dz \qquad (26)$$

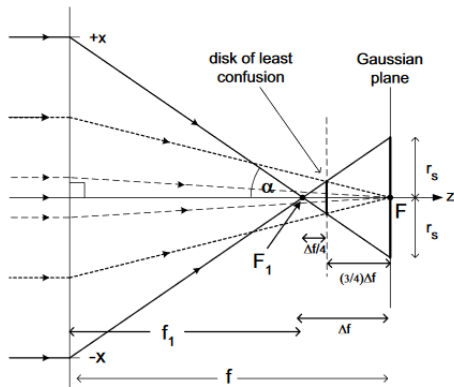$$= \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12p_{z,0}^4} F_4 \qquad (27)$$

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[1]Egerton et al., *Physical principles of electron microscopy*.

Structure

Project task

Simple solenoid lens design:

- ▶ Monochromatic e beam, fixed beam radius R
- ▶ Target FWHM, peak B_z, f
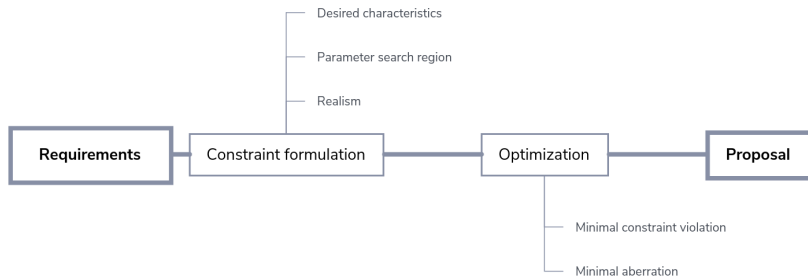- ▶ Optimize geometry, current for minimal spheric aberration

Figure: Generalized design process

Solenoid model

▶ Rectangular cross-section solenoid[1]

Two-loop field approximation:

Figure: Solenoid geometry:
R_m - mean radius
a - transverse width
b - axial length

$$B_z\left(z\right) \approx \frac{\mu_0 NI}{4} \left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}} \right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2 - a^2}{12},$$

$$R_{sq} = R_m \left(1 + \frac{a^2}{24 R_m^2} \right).$$

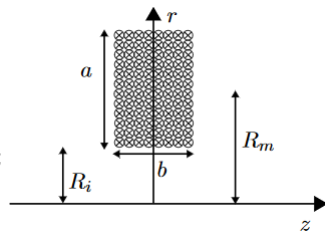Parameters: geometry, scaling factor $N \cdot I$ [Ampere-Turns]

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Field integrals

For an axial beam, only the on-axis B_z is of significance[1]. The field's optical properties are described in terms of:

$$F_1 = \int B_z \, dz$$

$$F_2 = \int B_z^2 \, dz$$

$$F_3 = \int -\frac{B_z'' \cdot B_z}{2} \, dz$$

$$F_4 = \int B_z^4 \, dz$$

whereas the integration domain is $(-\infty, \infty)$.

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Solenoid characteristics

- Peak $B_z = B_z(0)$;
- Effective field length $\rightleftarrows$ FWHM
- Focal length:

$$f = \left(\frac{2p_z}{e}\right)^2 \frac{1}{F_2}$$

- Aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12p_{z,0}^4} F_4$$

Considerations:

- Geometry - size, material usage
- Scaling factor - power, material usage
- f, FWHM, c_s: interaction with other components, fitness to purpose

Optimization

▶ Constrained Trust Region algorithm[1]:
  Iterative gradient search process; minimize merit function $m(p)$ of
  parameters $p = (s, R_m, a, b)$:

  $$m(p) = c_s(p) + k \cdot |C(p)|_2, \quad \Delta p \leq R_T;$$

  $C(p) = (C_1, C_2, ..., C_n)(p)$ - measure of constraint violation;

▶ Sequential Quadratic Programming[2]:
  A more rigid, constrained Gauss-Newton-type gradient search
  algorithm

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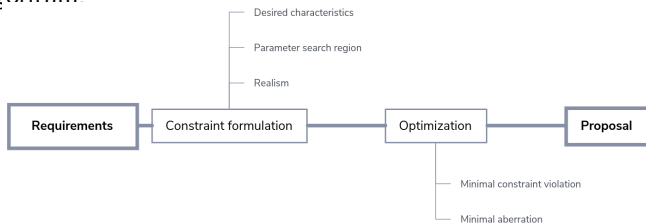
[1]*Documentation on SciPy's CTR implementation.*
[2]Wikipedia, *Sequential Quadratic Programming.*

Software demo[1]

Subtitle

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[1]Douginets and Yanovets, *solensim project repository*.

Perspective

Subtitle

Summary

Subtitle

Thank you for your attention!

References I

📄 *Documentation on SciPy's CTR implementation*. URL: https://docs.scipy.org/doc/scipy/reference/optimize.minimize-trustconstr.html.

📄 A. Douginets and A. Yanovets. *solensim project repository*. URL: https://github.com/adouge/solensim.

📄 R.F. Egerton et al. *Physical principles of electron microscopy*. Vol. 56. Springer, 2005.

📄 T. Gehrke. "Design of Permanent Magnetic Solenoids for REGAE". MA thesis. Hamburg: Universität Hamburg, 2013.

📄 B. Grigoryan et al. "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun". In: *Proceedings of IPAC2011, San Sebastián, Spain*. 2011.

References II

📄 B. Grigoryan et al. "Status of AREAL RF Photogun Test Facility". In: *Proceedings of IPAC2014, Dresden, Germany* (Dresden, Germany). International Particle Accelerator Conference 5. https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017. Geneva, Switzerland: JACoW, July 2014, pp. 620–623. ISBN: 978-3-95450-132-8. DOI: https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017. URL: http://jacow.org/ipac2014/papers/mopri017.pdf.

📄 J Rossbach and P Schmüser. *Basic course on accelerator optics*. Tech. rep. P00011673, 1993.

📄 Wikipedia. *Sequential Quadratic Programming*. URL: https://en.wikipedia.org/wiki/Sequential_quadratic_programming.