Solenoid electron lenses

Fundamentals and design

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Structure

Electron optics

Definition

Classic optics

- Light ray - photons
- Can pass through optically transparent solids
- Bends due to refractive index difference between media

Electron optics

- Electron beam - electrons
- Gets absorbed/loses energy due to interactions with atoms in media
- Bends due to Coulomb and Lorentz forces in the presense of external EM-fields

Magnet lenses

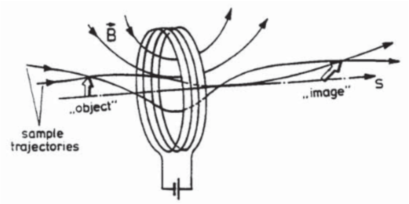
Requirements

A lens must have the following properties:

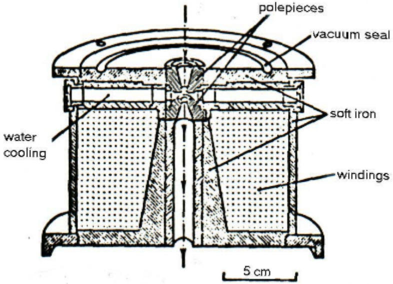
- deflection increases with increasing deviation of the beam from the optic axis;
- electron energy should not change, or change negligebly;
- symmetry of deflection on all sides of the optical axis;
- methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses

Solenoids



(a) A solenoid with S as optical axis[1]



(b) Cross-section of a magnetic lens[2]

[1] Rossbach and Schmüser, *Basic course on accelerator optics*.
[2] Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses

Solenoids

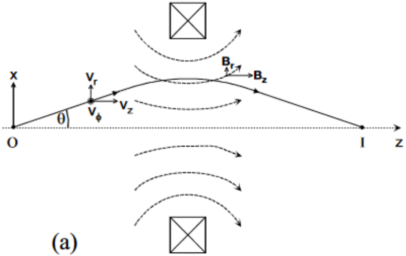


Figure: A solenoid cross-section along the optic axis[3]

$$F_\varphi = e\left(B_z v_r - v_z B_r\right) \tag{1}$$

$$F_r = -e\left(v_z B_z\right) \tag{2}$$

$$F_z = e\left(v_\varphi B_r\right) \tag{3}$$

$$\gamma m\left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \tag{4}$$

$$\gamma m\frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \tag{5}$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \tag{6}$$

[3]Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses

Electron path equations

$$B_z\left(z,r\right) = \sum_n \frac{(-1)^n}{n!n!}\left(\frac{r}{2}\right)^{2n}\frac{\partial^{2n}B_{z,\,axis}}{\partial z^{2n}} \tag{7}$$

$$B_r\left(z,r\right) = \sum_n \frac{(-1)^n}{n!\left(n-1\right)!}\left(\frac{r}{2}\right)^{2n-1}\frac{\partial^{2n-1}B_{z,\,axis}}{\partial z^{2n-1}} \tag{8}$$

- From (5) follows:

$$\dot{\varphi} = \frac{e}{2\gamma m}B_z \tag{9}$$

- From (4) and (6) follows:

$$\ddot{r} = -\left(\frac{e}{2\gamma m}\right)^2 rB_z^2 \tag{10}$$

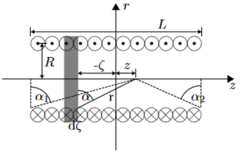
$$\ddot{z} = -\left(\frac{e}{2\gamma m}\right)^2 r^2 B_z B_z' \tag{11}$$

$$\ddot{r} = r''\dot{z}^2 \approx r''\left(\beta c\right)^2 \Rightarrow r'' = \left(\frac{e}{2p_z}\right)^2 rB_z^2 \tag{12}$$

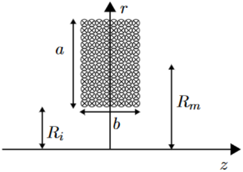
$$-\frac{r'}{r} = \frac{1}{f} \underset{integrate\,(12)}{\Rightarrow} \frac{1}{f} = \left(\frac{e}{2p_z}\right)^2 \int_{-\infty}^{\infty} B_z^2 dz := \left(\frac{e}{2p_z}\right)^2 F_2 \tag{13}$$

Magnet lenses

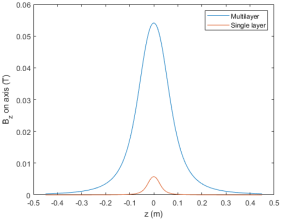
Magnetic field of solenoid



(a) Single-wind solenoid[4]



(b) Solenoid with multilayred windings[4]



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 nI}{2}\left(\frac{\triangle z}{\sqrt{R^2 + \triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2 + \triangle z^{*2}}}\right) \quad (14)$$

$$\triangle z = z - L/2 \quad (15)$$

Approximate field of the solenoid (b):

$$B_z\,(z) \approx \frac{\mu_0 nI}{2}\left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}}\right) \quad (16)$$

$$Rc = R_{sq} + c,\; R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right),\; c^2 = \frac{b^2 - a^2}{12} \quad (17)$$

[4] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Lens impefections

There are 3 main limitations to consider when designing a solenoid lens:

	Source:
• Chromatic aberration	Spread of electron energies
• RMS Emittance growth	Spread of eletron coordinates in position-and-momentum phase space
• Spherical aberration	Real lens $\rightarrow$ different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc}\sqrt{\left\langle x^2 \right\rangle \left\langle \tilde{p}_x^2 \right\rangle - \langle x\tilde{p}_x \rangle^2} = \frac{1}{mc}\left(\frac{e^2\sigma^4}{3\sqrt{2}p_{z,0}} F_3 + \frac{e^4\sigma^4}{24\sqrt{2}p_{z,0}^3} F_4 \right) \quad (18)$$

Chromatic aberration:

$$r_c \approx \alpha f \left(\triangle E_0 / E_0\right) \approx \alpha C_c \left(\triangle E_0 / E_0\right) \quad (19)$$

Lens impefections

Spherical aberration[5]

$$\triangle f = c \cdot x^2 \tag{20}$$

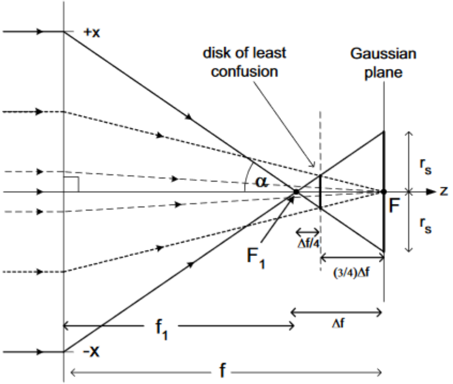
$$x \approx f \cdot \tan(\alpha) \tag{21}$$

$$r_s = \triangle f \cdot \tan(\alpha) \approx c \cdot f^2 \tan(\alpha)^3 \tag{22}$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \tag{23}$$

$$F_3 = -\int \frac{B_z B_z''}{2} dz \tag{24}$$

$$F_4 = \int B_z^4 dz \tag{25}$$



$$C_s = \frac{e}{96 m\tilde{U}} \int \left(\frac{2e}{m\tilde{U}} B_z^4 + 5 \left(B_z' \right)^2 - B_z B_z'' \right) R^4 dz \tag{26}$$

$$= \frac{e^2 R^4}{4 p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4 \tag{27}$$

Figure: Illustration of beam radius change due to spherical aberration

[5] Egerton et al., *Physical principles of electron microscopy*.

Summary

Subtitle

References I

[1] Ray F Egerton et al. *Physical principles of electron microscopy*. Vol. 56. Springer, 2005.

[2] T. Gehrke. “Design of Permanent Magnetic Solenoids for REGAE”. MA thesis. Hamburg: Universität Hamburg, 2013.

[3] J Rossbach and P Schmüser. *Basic course on accelerator optics*. Tech. rep. P00011673, 1993.