

# Solenoid electron lenses

Fundamentals and design

Anton Douginets    Andrii Yanovets

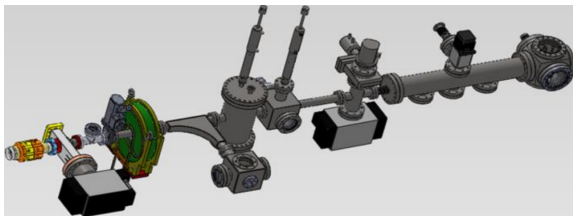
22.06.2020

# Structure

- 1 Motivation
- 2 Magnetic lens overview
  - General notions
  - Lens imperfections
- 3 Project methodology
  - Aim
  - Model
  - Optimization
- 4 Software demo
- 5 Summary, perspective

# Motivation

- ▶ Magnetic lenses - important components of particle accelerators, microscopes
- ▶ Electromagnet solenoids - physical foundation of more advanced magnetic lenses
- ▶ Design constraints:
  - ▶ Power consumption
  - ▶ Physical size, material usage
  - ▶ Characteristic parameters for interaction with other machine components



**Figure:** Schematic<sup>1</sup> of AREAL, an electron bunch-research oriented linac<sup>2</sup>

<sup>1</sup>Grigoryan et al., "Status of AREAL RF Photogun Test Facility".

<sup>2</sup>Grigoryan et al., "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun".

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# Electron optics

## Definition

### Classic optics

- Light ray - photons
- Can pass through optically transparent solids
- Bends due to refractive index difference between media

### Electron optics

- Electron beam - electrons
- Gets absorbed/loses energy due to interactions with atoms in media
- Bends due to Coulomb and Lorentz forces in the presence of external EM-fields

# Magnet lenses

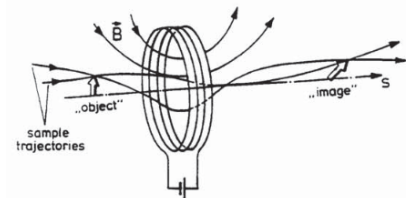
## Requirements

A lens must have the following properties:

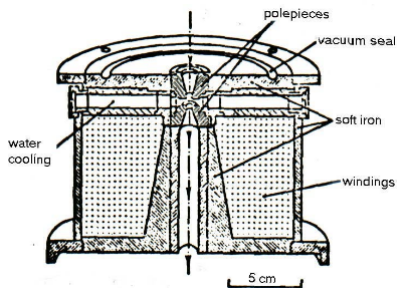
- ▶ deflection increases with increasing deviation of the beam from the optic axis;
- ▶ electron energy should not change, or change negligibly;
- ▶ symmetry of deflection on all sides of the optical axis;
- ▶ methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

# Magnet lenses

## Solenoids



(a) A solenoid with  $S$  as optical axis<sup>1</sup>



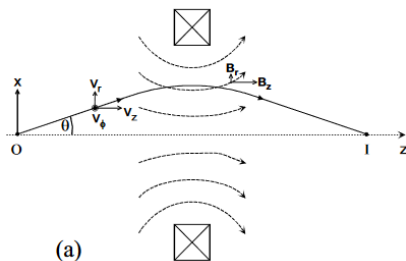
(b) Cross-section of a magnetic lens<sup>2</sup>

<sup>1</sup>Rossbach and Schmüser, *Basic course on accelerator optics*.

<sup>2</sup>Egerton et al., *Physical principles of electron microscopy*.

# Magnet lenses

## Solenoids



**Figure:** A solenoid cross-section along the optic axis<sup>1</sup>

$$F_{\varphi} = e (B_z v_r - v_z B_r) \quad (1)$$

$$F_r = -e (v_z B_z) \quad (2)$$

$$F_z = e (v_{\varphi} B_r) \quad (3)$$

$$\gamma m (\ddot{r} - r \dot{\varphi}^2) = -e r \dot{\varphi} B_z \quad (4)$$

$$\gamma m \frac{d}{dt} (r^2 \dot{\varphi}) = -e r (\dot{r} B_z + B_r \dot{z}) \quad (5)$$

$$\gamma m \ddot{z} = e r \dot{\varphi} B_r \quad (6)$$

<sup>1</sup>Egerton et al., *Physical principles of electron microscopy*.



# Magnet lenses

## Electron path equations

• From (5) follows:

$$\dot{\varphi} = \frac{e}{2\gamma m} B_z \quad (9)$$

$$B_z(z, r) = \sum_n \frac{(-1)^n}{n!n!} \left(\frac{r}{2}\right)^{2n} \frac{\partial^{2n} B_{z, axis}}{\partial z^{2n}} \quad (7)$$

• From (4) and (6) follows:

$$B_r(z, r) = \sum_n \frac{(-1)^n}{n!(n-1)!} \left(\frac{r}{2}\right)^{2n-1} \frac{\partial^{2n-1} B_{z, axis}}{\partial z^{2n-1}} \quad (8)$$

$$\ddot{r} = - \left(\frac{e}{2\gamma m}\right)^2 r B_z^2 \quad (10)$$

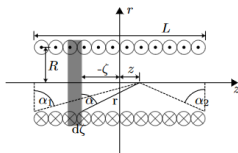
$$\ddot{z} = - \left(\frac{e}{2\gamma m}\right)^2 r^2 B_z B'_z \quad (11)$$

$$\ddot{r} = r'' \dot{z}^2 \approx r'' (\beta c)^2 \Rightarrow r'' = \left(\frac{e}{2p_z}\right)^2 r B_z^2 \quad (12)$$

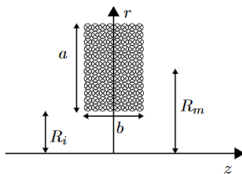
$$-\frac{r'}{r} = \frac{1}{f} \xrightarrow{\text{integrate (12)}} \frac{1}{f} = \left(\frac{e}{2p_z}\right)^2 \int_{-\infty}^{\infty} B_z^2 dz := \left(\frac{e}{2p_z}\right)^2 F_2 \quad (13)$$

# Magnet lenses

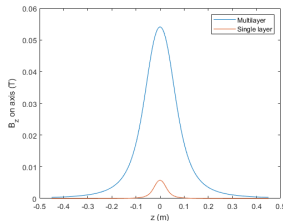
## Magnetic field of solenoid



(a) Single-wind solenoid<sup>1</sup>



(b) Solenoid with multilayered windings<sup>1</sup>



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

Approximate field of the solenoid (b)<sup>1</sup>:

$$B_z(z) = \frac{\mu_0 n I}{2} \left( \frac{\Delta z}{\sqrt{R^2 + \Delta z^2}} - \frac{\Delta z^*}{\sqrt{R^2 + \Delta z^{*2}}} \right) \quad (14)$$

$$\Delta z = z - L/2 \quad (15)$$

$$B_z(z) \approx \frac{\mu_0 n I}{4} \left( \frac{Rc^2}{(z^2 + Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2 + Rc^{*2})^{3/2}} \right) \quad (16)$$

$$Rc = R_{sq} + c, \quad R_{sq} = R_m \left( 1 + \frac{a^2}{24R_m^2} \right), \quad c^2 = \frac{b^2 - a^2}{12} \quad (17)$$

<sup>1</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

# Lens imperfections

There are 3 main limitations to consider when designing a solenoid lens:

## Source:

- Chromatic aberration

Spread of electron energies

- RMS Emittance growth

Spread of electron coordinates in position-and-momentum phase space

- Spherical aberration

Real lens  $\rightarrow$  different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc} \sqrt{\langle x^2 \rangle \langle \tilde{p}_x^2 \rangle - \langle x \tilde{p}_x \rangle^2} = \frac{1}{mc} \left( \frac{e^2 \sigma^4}{3\sqrt{2} p_{z,0}} F_3 + \frac{e^4 \sigma^4}{24\sqrt{2} p_{z,0}^3} F_4 \right) \quad (18)$$

Chromatic aberration:

$$r_c \approx \alpha f (\Delta E_0 / E_0) \approx \alpha C_c (\Delta E_0 / E_0) \quad (19)$$

# Lens impefections

## Spherical aberration<sup>1</sup>

$$\Delta f = c \cdot x^2 \quad (20)$$

$$x \approx f \cdot \tan(\alpha) \quad (21)$$

$$r_s = \Delta f \cdot \tan(\alpha) \approx c \cdot f^2 \tan^3(\alpha) \quad (22)$$

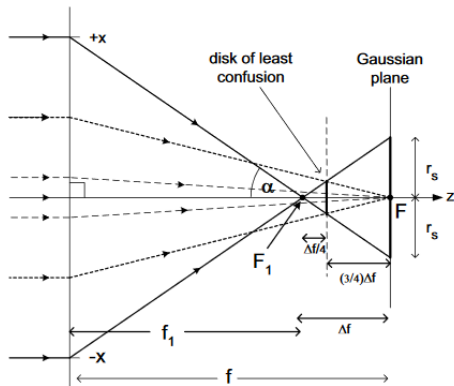
$$= C_s \cdot \left( \frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \quad (23)$$

$$F_3 = - \int \frac{B_z B_z'''}{2} dz \quad (24)$$

$$F_4 = \int B_z^4 dz \quad (25)$$

$$C_s = \frac{e}{96m\tilde{U}} \int \left( \frac{2e}{m\tilde{U}} B_z^4 + 5 (B_z')^2 - B_z B_z''' \right) R^4 dz \quad (26)$$

$$= \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12p_{z,0}^4} F_4 \quad (27)$$



**Figure:** Illustration of beam radius change due to spherical aberration

<sup>1</sup>Egerton et al., *Physical principles of electron microscopy*.

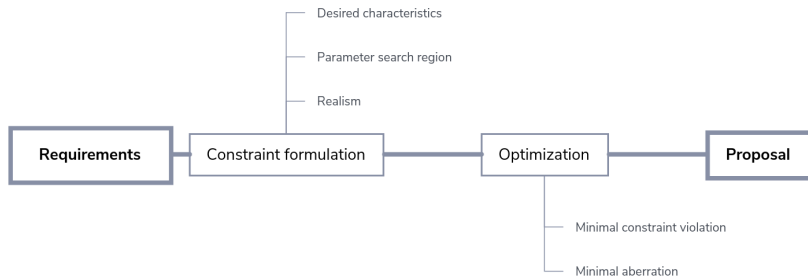
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# Project task

Simple solenoid lens design:

- ▶ Monochromatic e beam, fixed beam radius  $R$
- ▶ Target FWHM, peak  $B_z$ ,  $f$
- ▶ Optimize geometry, current for minimal spheric aberration



**Figure:** Generalized design process

# Solenoid model

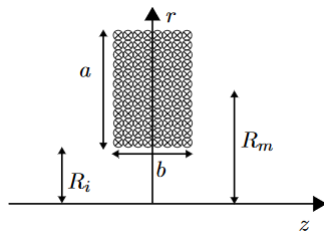
## ► Rectangular cross-section solenoid<sup>1</sup>

Two-loop field approximation:

$$B_z(z) \approx \frac{\mu_0 N I}{4} \left( \frac{Rc^2}{(z^2 + Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2 + Rc^{*2})^{3/2}} \right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2 - a^2}{12},$$

$$R_{sq} = R_m \left( 1 + \frac{a^2}{24R_m^2} \right).$$



**Figure:** Solenoid geometry:

$R_m$  - mean radius

$a$  - transverse width

$b$  - axial length

Parameters: geometry, scaling factor  $N \cdot I$  [Ampere-Turns]

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<sup>1</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

# Field integrals

For an axial beam, only the on-axis  $B_z$  is of significance<sup>1</sup>. The field's optical properties are described in terms of:

$$F_1 = \int B_z dz$$

$$F_3 = \int -\frac{B_z'' \cdot B_z}{2} dz$$

$$F_2 = \int B_z^2 dz$$

$$F_4 = \int B_z^4 dz$$

whereas the integration domain is  $(-\infty, \infty)$ .

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<sup>1</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".



# Solenoid characteristics

- ▶ Peak  $B_z = B_z(0)$ ;
- ▶ Effective field length  $\rightleftharpoons$  FWHM
- ▶ Focal length:

$$f = \left( \frac{2p_z}{e} \right)^2 \frac{1}{F_2}$$

- ▶ Aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12p_{z,0}^4} F_4$$

## Considerations:

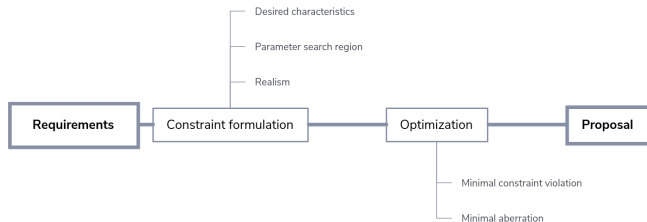
- ▶ Geometry - size, material usage
- ▶ Scaling factor - power, material usage
- ▶  $f$ , FWHM,  $c_s$ : interaction with other components, fitness to purpose

# Optimization

- ▶ Constrained Trust Region algorithm<sup>1</sup>:  
Iterative gradient search process; minimize merit function  $m(p)$  of parameters  $p = (s, R_m, a, b)$ :

$$m(p) = c_s(p) + k \cdot |C(p)|_2, \quad \Delta p \leq R_T;$$

$C(p) = (C_1, C_2, \dots, C_n)(p)$  - measure of constraint violation;



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<sup>1</sup>Documentation on SciPy's CTR implementation.

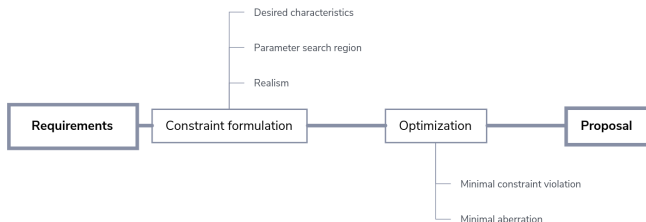
# Optimization

- Interior Point Algorithm<sup>2</sup>:  
Solves a sequence of approximate optimization problems, by transforming the original problem

$$\min_p f(p), \quad h(p) = 0 \wedge g(p) \leq 0$$

into

$$\min_p f(p) = \min_{p,s} f(p) - \mu \sum_i \ln(s_i), \quad h(p) = 0 \wedge g(p) + s = 0.$$



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<sup>2</sup>Byrd, Gilbert, and Nocedal, "A trust region method based on interior point techniques for nonlinear programming".

# Software demonstration<sup>1</sup>

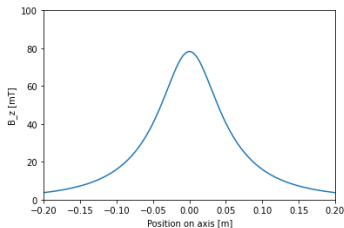
## Evaluating the REGAE solenoid<sup>2</sup>

### Parameters:

- s: 9000.000
- R<sub>mean</sub>: 79.750 mm, a: 41.800 mm, b: 99.500 mm

### Resulting characteristics:

- Peak axial field: 78.279 mT
- Effective field length: 102.000 mm
- Focal distance for given E: 27.679 cm
- Spherical aberration for given E:  $1.407 \times 10^{-10}$  m
- Focal spot radius:  $6.636 \times 10^{-3}$  fm



(a) Testing the model

### Settings:

- g [mm]: [79.75, 41.8, 99.5]
- s [A\*N]: 9000

### Targets:

- peak B [mT]: 80
- FWHM [mm]: 102
- f [cm]: 27.7
- g [mm]: (array([39.875, 20.9 , 49.75 ]), array([119.625, 62.7 , 149.25 ]))
- s [N\*A]: [7000, 11000]

### Result:

#### Parameters:

- s: 8999.988
- R<sub>mean</sub>: 79.926 mm, a: 43.242 mm, b: 96.889 mm

#### Resulting characteristics:

- Peak axial field: 77.297 mT
- Effective field length: 104.000 mm
- Focal distance for given E: 27.889 cm
- Spherical aberration for given E:  $1.367 \times 10^{-10}$  m
- Focal spot radius:  $6.300 \times 10^{-3}$  fm

(b) Attempting to optimize within 10% margin

<sup>1</sup>Douginets and Yanovets, *solensim project repository*.

<sup>2</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

# Software demonstration

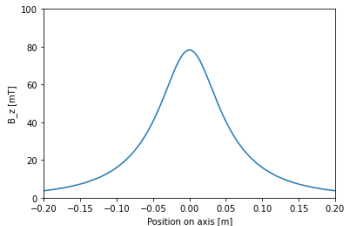
## General optimization

### Parameters:

- s: 9000.000
- R mean: 79.750 mm, a: 41.800 mm, b: 99.500 mm

### Resulting characteristics:

- Peak axial field: 78.279 mT
- Effective field length: 102.000 mm
- Focal distance for given E: 27.679 cm
- Spherical aberration for given E:  $1.407 \times 10^{-10}$  m
- Focal spot radius:  $6.636 \times 10^{-3}$  fm



(a) REGAE Solenoid<sup>1</sup>

### Settings:

- g [mm]: [30, 40, 50]
- s [A\*N]: 4000

### Targets:

- peak B [mT]: [100, 100]
- FWHM [mm]: [25, 75]
- f [cm]: [50, 50]
- g [mm]: (array([15., 20., 25.]), array([45., 60., 75.]))
- s [N\*A]: (2000, 6000)

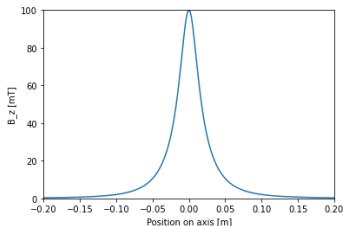
### Result:

#### Parameters:

- s: 4000.781
- R mean: 28.438 mm, a: 26.823 mm, b: 47.544 mm

#### Resulting characteristics:

- Peak axial field: 100.000 mT
- Effective field length: 34.000 mm
- Focal distance for given E: 50.000 cm
- Spherical aberration for given E:  $1.242 \times 10^{-10}$  m
- Focal spot radius:  $9.936 \times 10^{-4}$  fm

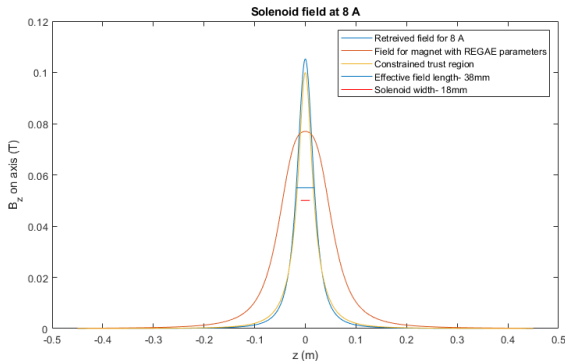


(b) Optimizing for provided constraints

<sup>1</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

# Interior point Algorithm results

## Retrieved parameters

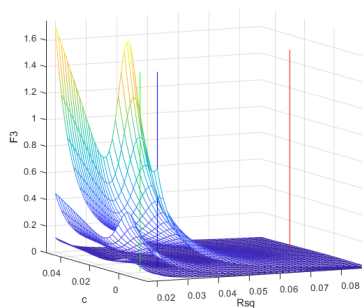


Opt. parameters	<u>Height <math>a</math> mm</u>	<u>Width <math>b</math> mm</u>	<u>Max field <math>B_z</math> mT</u>	<u>F. length <math>f</math> cm</u>	<u>Spherical ab. <math>C_S</math></u>	<u>RMS emi. <math>\epsilon_{n,rms}</math></u>
REGAE <sup>1</sup>	17.6	17.6	105	50	$1.7e - 9m$	$3.4e - 10m$
	99.5	41.8	79	30.5	$6.3e - 11m$	$7.9e - 11m$

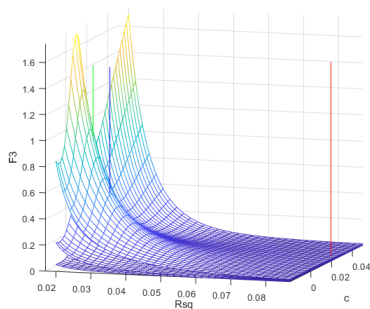
<sup>1</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

# Interior point Algorithm results

$F_3$  Integral



(a)



(b)

Figure:  $F_3$  integral

# Observations

- ▶ **Correlated requirements pose over-constraining problems**
- ▶ Multiple configurations in  $(s, r, a, b)$  correspond to same minima  
→ convergence depends on initial guess and search region
- ▶ P-space scaled differently along various axes → weak  $s$ -adjustment
- ▶ Interval constraints are weighted differently



# Perspective

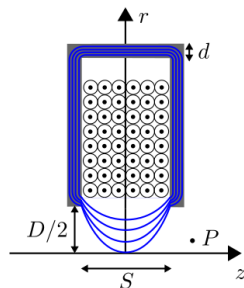
Potential for further development

Algorithm:

- ▶ Normalize P-space for better parameter and constraint weighing

Model:

- ▶ Consider solenoids in ferromagnetic yokes
- ▶ Study tradeoffs in desired characteristics



**Figure:** Solenoid in yoke <sup>1</sup>

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<sup>1</sup>Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

# Summary

- ▶ Electromagnet solenoids are a flexible electron lens implementation
- ▶ Minimal aberration can be achieved with different configurations  
→ Design is to be oriented on required characteristics

**Thank you for your attention!**

# References I

-  Richard H Byrd, Jean Charles Gilbert, and Jorge Nocedal. “A trust region method based on interior point techniques for nonlinear programming”. In: *Mathematical programming* 89.1 (2000), pp. 149–185.
-  Documentation on SciPy’s CTR implementation. URL: <https://docs.scipy.org/doc/scipy/reference/optimize.minimize-trustconstr.html>.
-  A. Douginets and A. Yanovets. *solensim project repository*. URL: <https://github.com/adouge/solensim>.
-  R.F. Egerton et al. *Physical principles of electron microscopy*. Vol. 56. Springer, 2005.
-  T. Gehrke. “Design of Permanent Magnetic Solenoids for REGAE”. MA thesis. Hamburg: Universität Hamburg, 2013.
-  B. Grigoryan et al. “Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun”. In: *Proceedings of IPAC2011, San Sebastián, Spain*. 2011.

# References II



B. Grigoryan et al. “Status of AREAL RF Photogun Test Facility”.  
In: *Proceedings of IPAC2014, Dresden, Germany* (Dresden, Germany). International Particle Accelerator Conference 5.  
<https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017>. Geneva, Switzerland: JACoW, July 2014, pp. 620–623. ISBN: 978-3-95450-132-8. DOI: <https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017>. URL: <http://jacow.org/ipac2014/papers/mopri017.pdf>.



J Rossbach and P Schmüser. *Basic course on accelerator optics*. Tech. rep. P00011673, 1993.