Solenoid electron lenses

Fundamentals and design

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Structure

Motivation

- Magnetic lenses - important components of particle accelerators, microscopes
- Electromagnet solenoids - physical foundation of more advanced magnetic lenses
- Design constraints:
  - Power consumption
  - Physical size, material usage
  - Characteristic parameters for interaction with other machine components

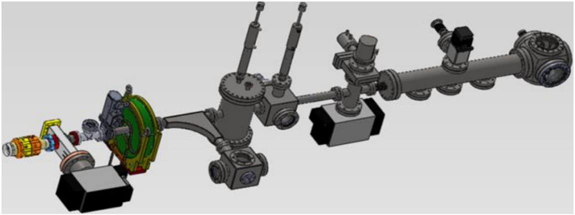


Figure: Schematic[1] of AREAL, an electron bunch-research oriented linac[2]

[1] **Grigoryan2014**.
[2] **Grigoryan2011**.

Structure

Electron optics

Definition

Classic optics

- Light ray - photons
- Can pass through optically transparent solids
- Bends due to refractive index difference between media

Electron optics

- Electron beam - electrons
- Gets absorbed/loses energy due to interactions with atoms in media
- Bends due to Coulomb and Lorentz forces in the presense of external EM-fields

Magnet lenses

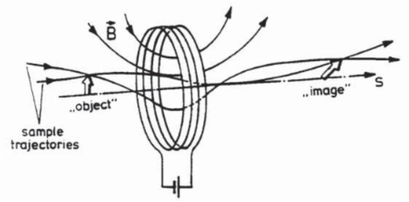
Requirements

A lens must have the following properties:

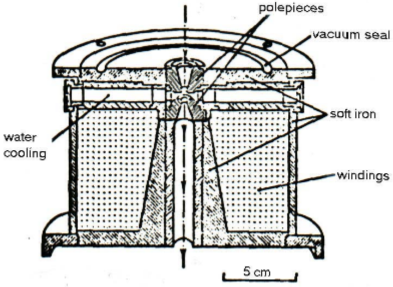
- deflection increases with increasing deviation of the beam from the optic axis;
- electron energy should not change, or change negligebly;
- symmetry of deflection on all sides of the optical axis;
- methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses

Solenoids



(a) A solenoid with S as optical axis[1]



(b) Cross-section of a magnetic lens[2]

[1] **a˙optics**.
[2] **Egerton**.

Magnet lenses

Solenoids

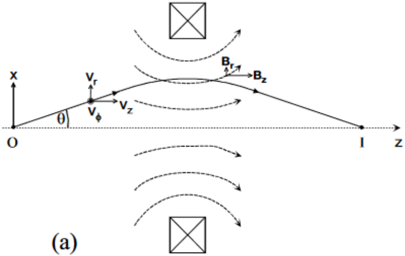


Figure: A solenoid cross-section along the optic axis[1]

$$F_\varphi = e\left(B_z v_r - v_z B_r\right) \quad (1)$$

$$F_r = -e\left(v_z B_z\right) \quad (2)$$

$$F_z = e\left(v_\varphi B_r\right) \quad (3)$$

$$\gamma m\left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \quad (4)$$

$$\gamma m\frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \quad (5)$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \quad (6)$$

[1] **Egerton**.

Magnet lenses

Electron path equations

$$B_z\,(z,r) = \sum_n \frac{(-1)^n}{n!n!}\left(\frac{r}{2}\right)^{2n} \frac{\partial^{2n} B_{z,\,axis}}{\partial z^{2n}} \tag{7}$$

$$B_r\,(z,r) = \sum_n \frac{(-1)^n}{n!\,(n-1)!}\left(\frac{r}{2}\right)^{2n-1} \frac{\partial^{2n-1} B_{z,\,axis}}{\partial z^{2n-1}} \tag{8}$$

- From (5) follows:

$$\dot{\varphi} = \frac{e}{2\gamma m} B_z \tag{9}$$

- From (4) and (6) follows:

$$\ddot{r} = -\left(\frac{e}{2\gamma m}\right)^2 r B_z^2 \tag{10}$$

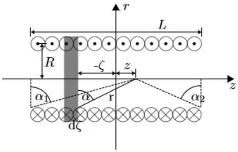
$$\ddot{z} = -\left(\frac{e}{2\gamma m}\right)^2 r^2 B_z B_z' \tag{11}$$

$$\ddot{r} = r''\dot{z}^2 \approx r''\,(\beta c)^2 \Rightarrow r'' = \left(\frac{e}{2p_z}\right)^2 r B_z^2 \tag{12}$$

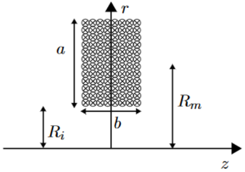
$$-\frac{r'}{r} = \frac{1}{f} \underset{integrate\,(12)}{\Rightarrow} \frac{1}{f} = \left(\frac{e}{2p_z}\right)^2 \int_{-\infty}^{\infty} B_z^2 dz := \left(\frac{e}{2p_z}\right)^2 F_2 \tag{13}$$

Magnet lenses

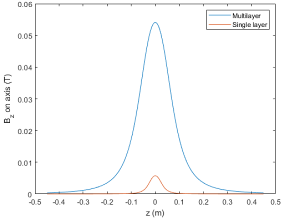
Magnetic field of solenoid



(a) Single-wind solenoid[1]



(b) Solenoid with multilayred windings[1]



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 nI}{2}\left(\frac{\triangle z}{\sqrt{R^2 + \triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2 + \triangle z^{*2}}}\right) \tag{14}$$

$$\triangle z = z - L/2 \tag{15}$$

Approximate field of the solenoid (b)[1]:

$$B_z\,(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}}\right) \tag{16}$$

$$Rc = R_{sq} + c,\ R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right),\ c^2 = \frac{b^2 - a^2}{12} \tag{17}$$

[1] **Disser**.

Lens impefections

There are 3 main limitations to consider when designing a solenoid lens:

	Source:
•Chromatic aberration	Spread of electron energies
•RMS Emittance growth	Spread of eletron coordinates in position-and-momentum phase space
•Spherical aberration	Real lens $\rightarrow$ different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc}\sqrt{\left\langle x^2 \right\rangle \left\langle \tilde{p}_x^2 \right\rangle - \left\langle x\tilde{p}_x \right\rangle^2} = \frac{1}{mc}\left(\frac{e^2\sigma^4}{3\sqrt{2}p_{z,0}} F_3 + \frac{e^4\sigma^4}{24\sqrt{2}p_{z,0}^3} F_4 \right) \tag{18}$$

Chromatic aberration:

$$r_c \approx \alpha f \left(\triangle E_0 / E_0 \right) \approx \alpha C_c \left(\triangle E_0 / E_0 \right) \tag{19}$$

Lens impefections

Spherical aberration[1]

$$\triangle f = c \cdot x^2 \tag{20}$$

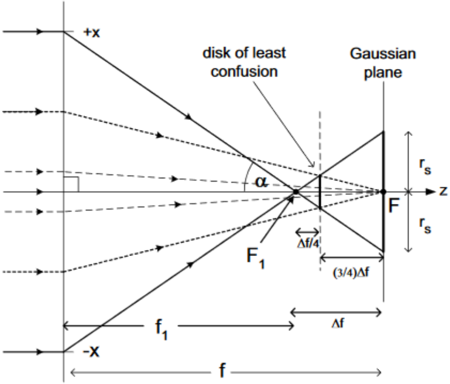
$$x \approx f \cdot \tan(\alpha) \tag{21}$$

$$r_s = \triangle f \cdot \tan(\alpha) \approx c \cdot f^2 \tan(\alpha)^3 \tag{22}$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \tag{23}$$

$$F_3 = - \int \frac{B_z B_z''}{2} dz \tag{24}$$

$$F_4 = \int B_z^4 dz \tag{25}$$



$$C_s = \frac{e}{96 m \tilde{U}} \int \left(\frac{2e}{m\tilde{U}} B_z^4 + 5 \left(B_z' \right)^2 - B_z B_z'' \right) R^4 dz \tag{26}$$

$$= \frac{e^2 R^4}{4 p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4 \tag{27}$$

Figure: Illustration of beam radius change due to spherical aberration

[1] **Egerton**.

Structure

Project task

Simple solenoid lens design:

- Monochromatic e beam, fixed beam radius R
- Target FWHM, peak B_z, f
- Optimize geometry, current for minimal spheric aberration

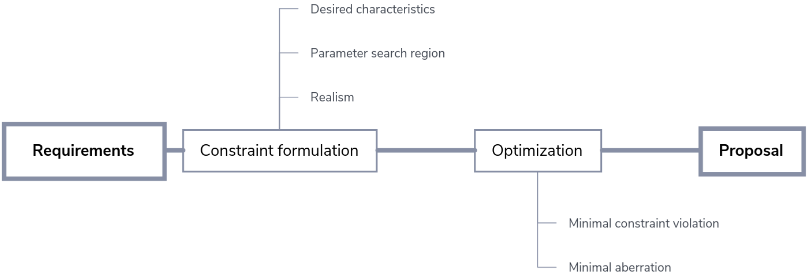


Figure: Generalized design process

Solenoid model

- Rectangular cross-section solenoid[1]

Two-loop field approximation:

$$B_z(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{(z^2+Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2+Rc^{*2})^{3/2}}\right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2 - a^2}{12},$$

$$R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right).$$

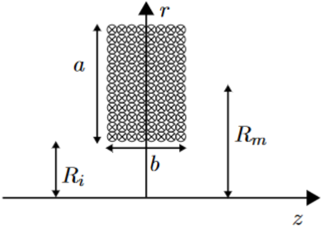


Figure: Solenoid geometry:
R_m - mean radius
a - transverse width
b - axial length

Parameters: geometry, scaling factor $N \cdot I$ [Ampere-Turns]

[1] **Disser**.

Field integrals

For an axial beam, only the on-axis B_z is of significance[1]. The field's optical properties are described in terms of:

$$F_1 = \int B_z dz \qquad F_3 = \int -\frac{B_z'' \cdot B_z}{2} dz$$

$$F_2 = \int B_z^2 dz \qquad F_4 = \int B_z^4 dz$$

whereas the integration domain is $(-\infty, \infty)$.

[1] **Disser**.

Solenoid characteristics

- Peak $B_z = B_z(0)$;
- Effective field length $\rightleftarrows$ FWHM
- Focal length:

$$f = \left(\frac{2p_z}{e}\right)^2 \frac{1}{F_2}$$

- Aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4$$

Considerations:

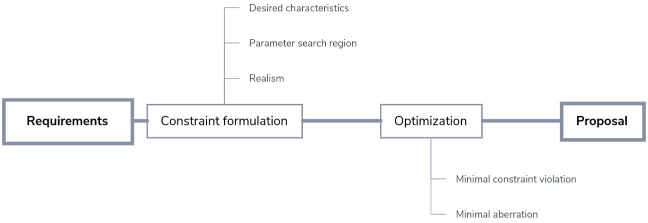
- Geometry - size, material usage
- Scaling factor - power, material usage
- f, FWHM, c_s: interaction with other components, fitness to purpose

Optimization

- Constrained Trust Region algorithm[1]:
  Iterative gradient search process; minimize merit function $m(p)$ of parameters $p = (s, R_m, a, b)$:

  $$m(p) = c_s(p) + k \cdot |C(p)|_2, \quad \Delta p \leq R_T;$$

  $C(p) = (C_1, C_2, ..., C_n)(p)$ - measure of constraint violation;
- Interior Point Algorithm[2]:
  Solves a sequence of approximate optimization problems, by transforming the original problem $\min_x f(x),\ h(x) = 0 \wedge g(x) \leq 0$ into $\min_x f(x) = \min_{x,s} f(x) - \mu \sum_i \ln(s_i),\ h(x) = 0 \wedge g(x) + s = 0.$



[1] **scipyctr**.
[2] **sqp**.

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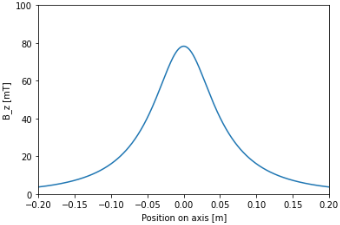
Software demonstration[1]

Evaluating the REGAE solenoid[2]

```
Parameters:
 - s: 9000.000
 - R_mean: 79.750 mm, a: 41.800 mm, b: 99.500 mm
Resulting characteristics:
 - Peak axial field: 78.279 mT
 - Effective field length: 102.000 mm
 - Focal distance for given E: 27.679 cm
 - Spherical aberration for given E: 1.407E-10 m
 - Focal spot radius: 6.636E-03 fm
```



(a) Testing the model

```
Settings:
 - g [mm]: [79.75, 41.8, 99.5]
 - s [A*N]: 9000
Targets:
 - peak B [mT]: 80
 - FWHM [mm]: 102
 - f [cm]: 27.7
 - g [mm]: (array([39.875, 20.9  , 49.75 ]),
array([119.625,  62.7  , 149.25 ]))
 - s [N*A]: [7000, 11000]
Result:
Parameters:
 - s: 8999.988
 - R_mean: 79.926 mm, a: 43.242 mm, b: 96.889 mm
Resulting characteristics:
 - Peak axial field: 77.297 mT
 - Effective field length: 104.000 mm
 - Focal distance for given E: 27.889 cm
 - Spherical aberration for given E: 1.367E-10 m
 - Focal spot radius: 6.300E-03 fm
```

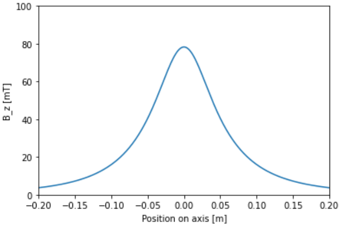
(b) Attempting to optimize within 10% margin

[1] **repo**.
[2] **Disser**.

Software demonstration

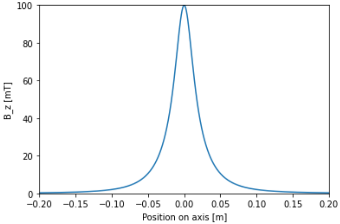
General optimization

```
Parameters:
 - s: 9000.000
 - R_mean: 79.750 mm, a: 41.800 mm, b: 99.500 mm
Resulting characteristics:
 - Peak axial field: 78.279 mT
 - Effective field length: 102.000 mm
 - Focal distance for given E: 27.679 cm
 - Spherical aberration for given E: 1.407E-10 m
 - Focal spot radius: 6.636E-03 fm
```



(a) REGAE Solenoid[1]

```
Settings:
 - g [mm]: [30, 40, 50]
 - s [A*N]: 4000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [25, 75]
 - f [cm]: [50, 50]
 - g [mm]: (array([15., 20., 25.]), array([45.,
60., 75.]))
 - s [N*A]: (2000, 6000)
Result:
Parameters:
 - s: 4000.781
 - R_mean: 28.438 mm, a: 26.823 mm, b: 47.544 mm
Resulting characteristics:
 - Peak axial field: 100.000 mT
 - Effective field length: 34.000 mm
 - Focal distance for given E: 50.000 cm
 - Spherical aberration for given E: 1.242E-10 m
 - Focal spot radius: 9.936E-04 fm
```

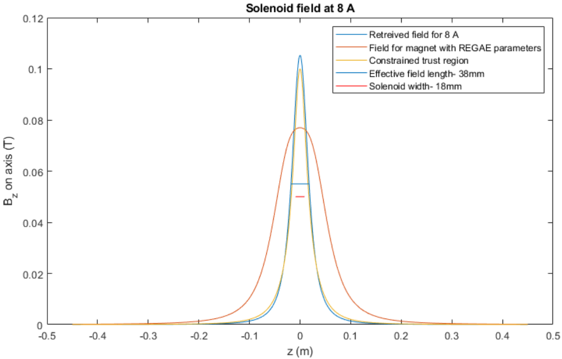


(b) Optimizing for provided constraints

[1] **Disser**.

Interior point Algorithm results

Retrieved parameters

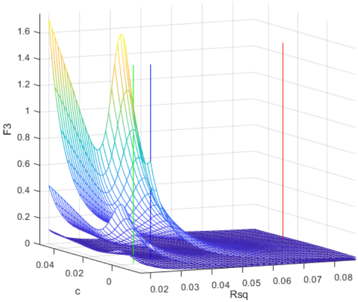


	Height a mm	Width b mm	Max field B_z mT	F. length f cm	Spherical ab. C_s	RMS emi. $\epsilon_{n.rms}$
Opt. parameters	17.6	17.6	105	50	$1.7e-9m$	$3.4e-10m$
REGAE[1]	99.5	41.8	79	30.5	$6.3e-11m$	$7.9e-11m$

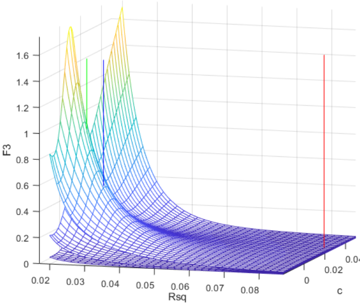
[1] **Disser**.

Interior point Algorithm results

F_3 Integral



(a)



(b)

Figure: F_3 integral

Observations

- **Correlated requirements pose over-constraining problems**
- Multiple configurations in (s, r, a, b) correspond to same minima
  $\rightarrow$ convergence depends on initial guess and search region
- P-space scaled differently along various axes $\rightarrow$ weak s-adjustment
- Interval constraints are weighted differently

Perspective

Potential for further development

Algorithm:

- Normalize P-space for better parameter and constraint weighing

Model:

- Consider solenoids in ferromagnetic yokes
- Study tradeoffs in desired characteristics

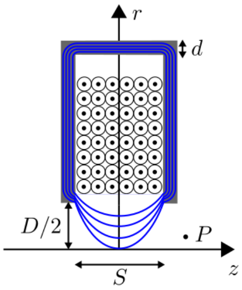


Figure: Solenoid in yoke [1]

[1] **Disser**.

Summary

- Electromagnet solenoids are a flexible electron lens implementation

- Minimal aberration can be achieved with different configurations
  $\rightarrow$ Design is to be oriented on required characteristics

Thank you for your attention!

References I