TITLE

Subtitle

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Structure

Definition

Classic optics

- Light ray- photons
- Can pass through optically transparent solids
- Bends due to refractive index difference between mediums

Electron optics

- Electron beam- electrons
- Get's absorbed/loses energy due to interactions with atoms in solids
- Bends due to Coulomb and Lorentz forces in the presense of outer EM field

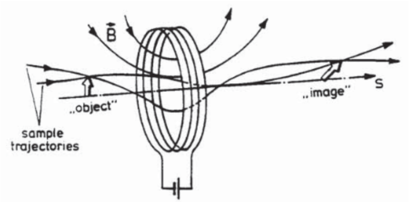
Magnet lenses
Requirements

The lense must have following properties:

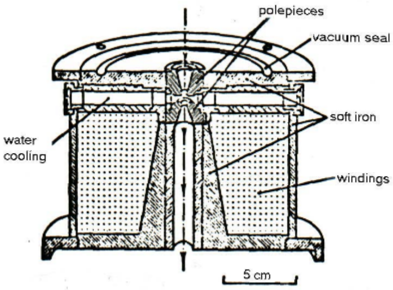
- deflection must increase with increasing deviation of the electron ray from the optic axis;
- single electron energy in the beam should not be altered by passing through the lense or it's change must be neglegible;
- for a constant deviation from the optic axis deflection must be symmetric with respect to rotation around optical axis;
- methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable;

Magnet lenses

Solenoids



(a) Solenoid with S, as optical axis[1]



(b) Cross section through a magnetic lens[2]

[1] "**Basic course on accelerator optics**", **Rossbach and P. Schmser**.
[2] "**Physical Principles of Electron Microscopy**", **Egerton, R.F.**.

Magnet lenses

Solenoids

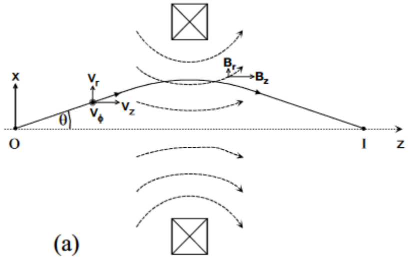


Figure: Solenoid cross-section along optic axis[3]

$$F_\varphi = e\left(B_z v_r - v_z B_r\right) \quad (1)$$

$$F_r = -e\left(v_z B_z\right) \quad (2)$$

$$F_z = e\left(v_\varphi B_r\right) \quad (3)$$

$$\gamma m\left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \quad (4)$$

$$\gamma m \frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \quad (5)$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \quad (6)$$

[3] 'Physical Principles of Electron Microscopy' , Egerton, R.F..

Magnet lenses

Electron path equations

$$B_z\left(z,r\right)=\sum_n \frac{(-1)^n}{n!n!}\left(\frac{r}{2}\right)^{2n}\frac{\partial^{2n}B_{z,\,axis}}{\partial z^{2n}} \tag{7}$$

$$B_r\left(z,r\right)=\sum_n \frac{(-1)^n}{n!\left(n-1\right)!}\left(\frac{r}{2}\right)^{2n-1}\frac{\partial^{2n-1}B_{z,\,axis}}{\partial z^{2n-1}} \tag{8}$$

- From (5) follows:

$$\dot{\varphi}=\frac{e}{2\gamma m}B_z \tag{9}$$

- From (4) and (6) follows:

$$\ddot{r}=-\left(\frac{e}{2\gamma m}\right)^2 rB_z^2 \tag{10}$$

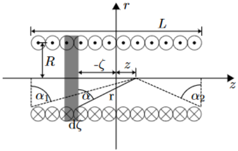
$$\ddot{z}=-\left(\frac{e}{2\gamma m}\right)^2 r^2B_zB_z' \tag{11}$$

$$\ddot{r}=r''\dot{z}^2\approx r''\left(\beta c\right)^2\Rightarrow r''=\left(\frac{e}{2p_z}\right)^2 rB_z^2 \tag{12}$$

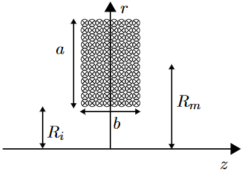
$$-\frac{r'}{r}=\frac{1}{f}\underset{integrate\,(12)}{\Rightarrow}\frac{1}{f}=\left(\frac{e}{2p_z}\right)^2\int_{-\infty}^{\infty}B_z^2dz:=\left(\frac{e}{2p_z}\right)^2F_2 \tag{13}$$

Magnet lenses

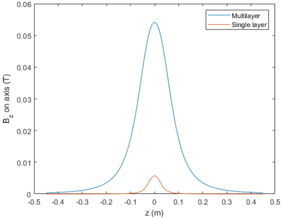
Magnetic field of solenoid



(a) Solenoid with a single layer of windings[4]



(b) Solenoid with multilayred windings[5]



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 nI}{2}\left(\frac{\triangle z}{\sqrt{R^2+\triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2+\triangle z^{*2}}}\right) \quad (14)$$

$$\triangle z = z - L/2 \quad (15)$$

Approximate field of the solenoid (b):

$$B_z(z) \approx \frac{\mu_0 nI}{2}\left(\frac{Rc^2}{(z^2+Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2+Rc^{*2})^{3/2}}\right) \quad (16)$$

$$Rc = R_{sq} + c,\ R_{sq} = R_m\left(1+\frac{a^2}{24R_m^2}\right),\ c^2 = \frac{b^2-a^2}{12} \quad (17)$$

[4] 'Design of Permanent MagneticSolenoids for REGAE' , Gehrke T..

[5] 'Design of Permanent MagneticSolenoids for REGAE' , Gehrke T..

Lense impefections

Defects

There are 3 main defects to be considering when designing a solenoid lense:

	Source
• Chromatic aberration	Spread of electron energies
• RMS Emittance	Spread of eletron coordinates in position-and-momentum phase space
• Spherical aberration	Dependence of f from distance from optical axis, due to lense specific coefficient C_s

$$\epsilon_{n,rms} = \frac{1}{mc}\sqrt{\langle x^2\rangle \langle \tilde{p}_x^2\rangle - \langle x\tilde{p}_x\rangle^2} = \frac{1}{mc}\left(\frac{e^2\sigma^4}{3\sqrt{2}p_{z,0}}F_3 + \frac{e^4\sigma^4}{24\sqrt{2}p_{z,0}^3}F_4\right) \quad (18)$$

$$r_c \approx \alpha f\left(\triangle E_0/E_0\right) \approx \alpha C_c\left(\triangle E_0/E_0\right) \quad (19)$$

Lense impefections

Spherical aberration

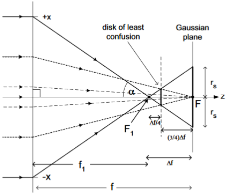


Figure: Def. of the beam radius change due to spherical aberration[6]

$$\triangle f = c \cdot x^2 \quad (20)$$

$$x \approx f \cdot tan\,(\alpha) \quad (21)$$

$$r_s = \triangle f \cdot tan\,(\alpha) \approx c \cdot f^2 tan\,(\alpha)^3 \quad (22)$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \quad (23)$$

$$F_3 = -\int \frac{B_z B_z''}{2} dz \quad (24)$$

$$F_4 = \int B_z^4 dz \quad (25)$$

$$C_s = \frac{e}{96 m\tilde{U}} \int \left(\frac{2e}{m\tilde{U}} B_z^4 + 5 \left(B_z'\right)^2 - B_z B_z'' \right) R^4 dz \quad (26)$$

$$= \frac{e^2 R^4}{4 p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4 \quad (27)$$

[6] **˙Physical Principles of Electron Microscopy˙, Egerton, R.F.**

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Summary

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References I