Solenoid electron lenses

Fundamentals and design

Anton Douginets Andrii Yanovets

July 9, 2020

Structure

Motivation

Demands on solenoid lenses:

- Low power, materials use
- Specific component size, focal length
- Minimal aberrations

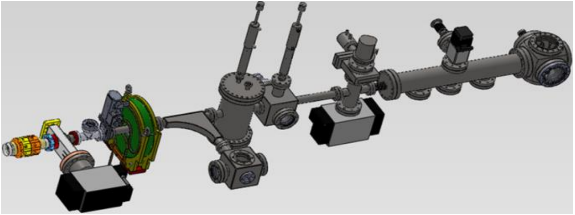


Figure: Schematic[1] of AREAL, an electron bunch-research oriented linac[2]

[1] Grigoryan et al., "Status of AREAL RF Photogun Test Facility".

[2] Grigoryan et al., "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun".

Structure

1 Motivation

2 Project methodology
- Aim
- Model
- Optimization

3 Software demo

4 Summary, perspective

Project task

Simple solenoid lens design:

- Monochromatic e beam, fixed beam radius R
- Target FWHM, peak B_z, f
- Optimize geometry, current for minimal aberrations

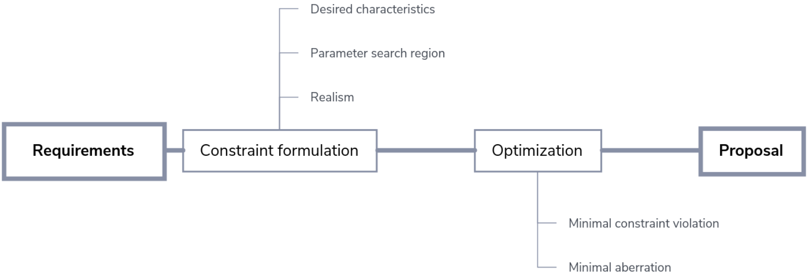


Figure: Generalized design process

Solenoid model

- Rectangular cross-section solenoid[1]

Two-loop field approximation:

$$B_z(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{(z^2+Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2+Rc^{*2})^{3/2}}\right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2-a^2}{12},$$

$$R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right).$$

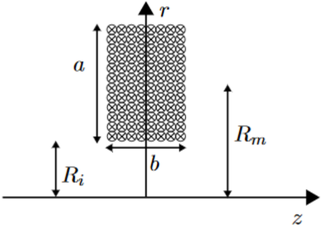


Figure: Solenoid geometry:
R_m - mean radius
a - transverse width
b - axial length

Parameters: geometry, scaling factor $N \cdot I$ [Ampere-Turns]

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Field integrals

For an axial beam, only the on-axis B_z is of significance[1]. The field's optical properties are described in terms of:

$$F_1 = \int B_z dz \qquad F_3 = \int -\frac{B_z'' \cdot B_z}{2} dz$$

$$F_2 = \int B_z^2 dz \qquad F_4 = \int B_z^4 dz$$

whereas the integration domain is $(-\infty, \infty)$.

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Solenoid characteristics

- Peak $B_z = B_z(0)$; effective field length $\rightleftarrows$ FWHM
- Focal length:

$$f = \left(\frac{2p_z}{e}\right)^2 \frac{1}{F_2}$$

- Spherical aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4$$

- Resulting focal spot size:

$$r_s = C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}}\right)^3$$

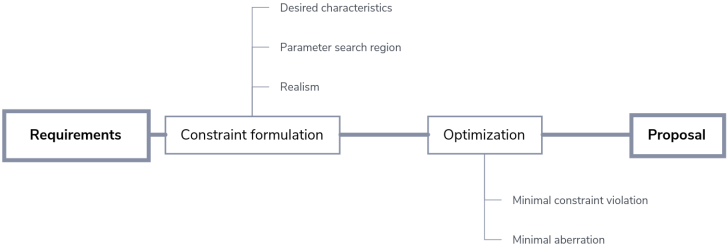
Considerations:

- - size, material usage
- Scaling factor, geometry - power consumption, material usage
- f, FWHM, c_s, r_s: interaction with other components, lens quality

Optimization

We used:

- Constrained Trust Region algorithm[1] - minimize c_s and constraint violation with dynamic "trust region" definition
- Interior Point algorithm:[2] - a more rigorous, less flexible implementation of the "trust region" concept



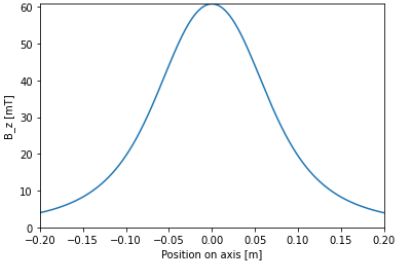
[1] *Documentation on SciPy's CTR implementation*.

[2] Byrd, Gilbert, and Nocedal, "A trust region method based on interior point techniques for nonlinear programming".

Software demonstration

Evaluating the REGAE solenoid[1]

```
Parameters:
 - s: 9000.000
 - R: 30.000 mm, a: 99.500 mm, b: 41.800 mm
Resulting characteristics:
 - Peak axial field: 60.855 mT
 - Effective field length: 148.000 mm
 - Focal distance for given E: 124.931 cm
 - Spherical aberration for given E, R:
5.861E-12 m
 - Focal spot radius: 3.006E-06 fm
```



(a) Testing the model

```
Settings:
 - g [mm]: [30, 99.5, 41.8]
 - s [A*N]: 9000
Targets:
 - peak B [mT]: 60.855
 - FWHM [mm]: 148
 - f [cm]: 124.931
 - g [mm]: [30, 99.5, 41.8]
 - s [N*A]: 9000
 - Margin [%]: 10
Result:
Parameters:
 - s: 8998.119
 - R: 30.759 mm, a: 103.938 mm, b: 41.821
mm
Resulting characteristics:
 - Peak axial field: 58.450 mT
 - Effective field length: 154.000 mm
 - Focal distance for given E: 129.898 cm
 - Spherical aberration for given E, R:
5.265E-12 m
 - Focal spot radius: 2.402E-06 fm
```

(b) Attempting to optimize within 10% margin

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Software demonstration

Results depend on initial conditions:

```
Settings:
 - g [mm]: [50, 50, 50]
 - s [A*N]: 8000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [45, 55]
 - f [cm]: [50, inf]
 - g [mm]: [[10, 10, 10], [200, 200, 200]]
 - s [N*A]: [4000, 12000]
 - Margin [%]: 10
Result:
Parameters:
 - s: 7292.988
 - R: 41.028 mm, a: 50.416 mm, b: 143.039
mm
Resulting characteristics:
 - Peak axial field: 100.000 mT
 - Effective field length: 54.000 mm
 - Focal distance for given E: 120.034 cm
 - Spherical aberration for given E, R:
1.328E-11 m
 - Focal spot radius: 7.680E-06 fm
```

(a) Optimizing for initial specifications

```
Settings:
 - g [mm]: [100, 100, 100]
 - s [A*N]: 5000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [45, 55]
 - f [cm]: [50, inf]
 - g [mm]: [[10, 10, 10], [200, 200, 200]]
 - s [N*A]: [4000, 12000]
 - Margin [%]: 10
Result:
Parameters:
 - s: 6547.926
 - R: 39.273 mm, a: 48.147 mm, b: 144.188
mm
Resulting characteristics:
 - Peak axial field: 100.000 mT
 - Effective field length: 46.000 mm
 - Focal distance for given E: 137.290 cm
 - Spherical aberration for given E, R:
1.155E-11 m
 - Focal spot radius: 4.463E-06 fm
```

(b) An approach from different start values

Software demonstration - observations

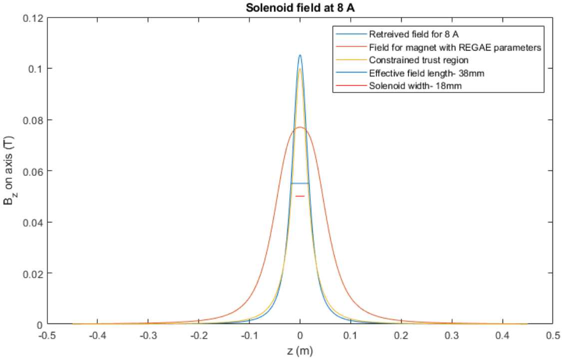
```
Settings:
 - g [mm]: [50, 50, 50]
 - s [A*N]: 8000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [50, 50]
 - f [cm]: [50, inf]
 - g [mm]: [[10, 10, 10], [200, 200, 200]]
 - s [N*A]: [4000, 12000]
 - Margin [%]: 10
Result:
Parameters:
 - s: 7985.177
 - R: 65.059 mm, a: 33.370 mm, b: 82.427 mm
Resulting characteristics:
 - Peak axial field: 65.533 mT
 - Effective field length: 112.000 mm
 - Focal distance for given E: 143.346 cm
 - Spherical aberration for given E, R:
5.481E-12 m
 - Focal spot radius: 1.861E-06 fm
```

Figure: Example of overconstraining

- Overconstraining throws the algorithm off
- Multiple configurations in (s, r, a, b) correspond to same minima
  $\rightarrow$ convergence depends on initial guess and search region
- Parameters, constraints seem to be weighted differently - the optimization space is not normalized
- Lower c_s does not always yield better spot size

Interior point Algorithm results

Retrieved parameters

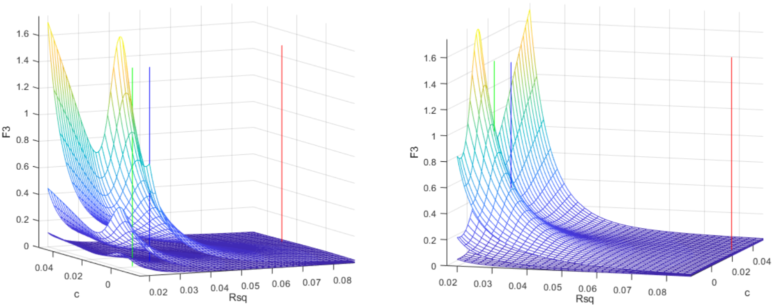


	Height a mm	Width b mm	Max field B_z mT	F. length f cm	Spherical ab. C_s	RMS emi. $\epsilon_{n.rms}$
Opt. parameters	17.6	17.6	105	50	$1.7e-9m$	$3.4e-10m$
REGAE[1]	99.5	41.8	79	30.5	$6.3e-11m$	$7.9e-11m$

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Interior point Algorithm results

F_3 Integral



(a) **(b)**

Figure: F_3 integral

Perspective

Potential for further development

Algorithm:

- Normalize parameters, constraints for better weighing
- Rework structure to allow for flexible model, optimization parameter choice

Models:

- Consider solenoids in ferromagnetic yokes
- Consider chromatic aberrations

General:

- Develop means for sweep-like study of characteristic response to parameters, constraints, initial values

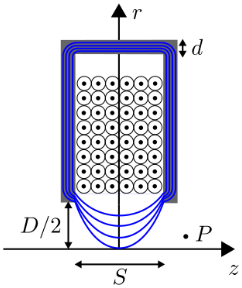


Figure: Solenoid in a yoke [1]

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Summary

- Nonlinear, multivariate optimization is highly sensitive to initial guess values
- Over-constraining in characteristic values throws the optimization off-course
- Minimal aberration can be achieved with different configurations

$\rightarrow$ Design is to be oriented on required characteristics and optimized in that region - no search for a “perfect ”lens.

- Further improvements to the software framework could provide simple tools to comprehensively study the design process, potential trade-offs and nuances.

Thank you for your attention!

References I

- Richard H Byrd, Jean Charles Gilbert, and Jorge Nocedal. "A trust region method based on interior point techniques for nonlinear programming". In: *Mathematical programming* 89.1 (2000), pp. 149–185.
- *Documentation on SciPy's CTR implementation*. URL: https://docs.scipy.org/doc/scipy/reference/optimize.minimize-trustconstr.html.
- T. Gehrke. "Design of Permanent Magnetic Solenoids for REGAE". MA thesis. Hamburg: Universität Hamburg, 2013.
- B. Grigoryan et al. "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun". In: *Proceedings of IPAC2011, San Sebastián, Spain*. 2011.

References II

B. Grigoryan et al. "Status of AREAL RF Photogun Test Facility". In: *Proceedings of IPAC2014, Dresden, Germany* (Dresden, Germany). International Particle Accelerator Conference 5. https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017. Geneva, Switzerland: JACoW, July 2014, pp. 620–623. ISBN: 978-3-95450-132-8. DOI: https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017. URL: http://jacow.org/ipac2014/papers/mopri017.pdf.