

Solenoid electron lenses

Fundamentals and design

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Structure

1 Motivation

2 Magnetic lens overview

3 Summary

Motivation

- ▶ Magnetic lenses - important components of particle accelerators, microscopes
- ▶ Electromagnet solenoids - physical foundation of more advanced magnetic lenses
- ▶ Design constraints:
 - ▶ Power consumption
 - ▶ Physical size, material usage
 - ▶ Characteristic parameters for interaction with other machine components

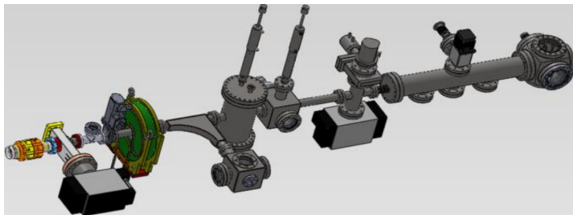


Figure: Schematic¹ of AREAL, an electron bunch-research oriented linac²

¹Grigoryan et al., "Status of AREAL RF Photogun Test Facility".

²Grigoryan et al., "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun".

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Electron optics

Definition

Classic optics

- Light ray - photons
- Can pass through optically transparent solids
- Bends due to refractive index difference between media

Electron optics

- Electron beam - electrons
- Gets absorbed/loses energy due to interactions with atoms in media
- Bends due to Coulomb and Lorentz forces in the presence of external EM-fields

Magnet lenses

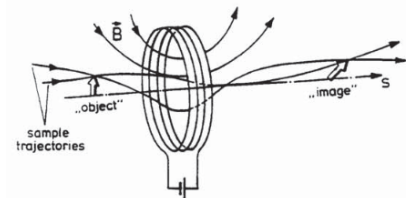
Requirements

A lens must have the following properties:

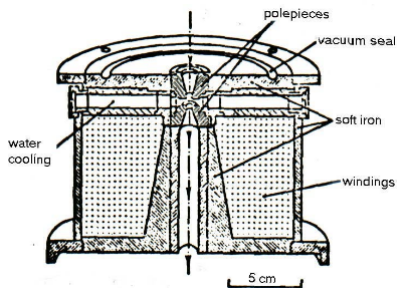
- ▶ deflection increases with increasing deviation of the beam from the optic axis;
- ▶ electron energy should not change, or change negligibly;
- ▶ symmetry of deflection on all sides of the optical axis;
- ▶ methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses

Solenoids



(a) A solenoid with S as optical axis¹



(b) Cross-section of a magnetic lens²

¹Rossbach and Schmöser, *Basic course on accelerator optics*.

²Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses

Solenoids

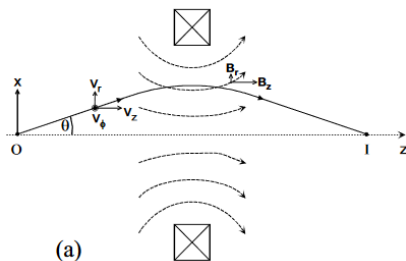


Figure: A solenoid cross-section along the optic axis¹

$$F_{\varphi} = e (B_z v_r - v_z B_r) \quad (1)$$

$$F_r = -e (v_z B_z) \quad (2)$$

$$F_z = e (v_{\varphi} B_r) \quad (3)$$

$$\gamma m (\ddot{r} - r \dot{\varphi}^2) = -e r \dot{\varphi} B_z \quad (4)$$

$$\gamma m \frac{d}{dt} (r^2 \dot{\varphi}) = -e r (\dot{r} B_z + B_r \dot{z}) \quad (5)$$

$$\gamma m \ddot{z} = e r \dot{\varphi} B_r \quad (6)$$

¹Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses

Electron path equations

• From (5) follows:

$$\dot{\varphi} = \frac{e}{2\gamma m} B_z \quad (9)$$

$$B_z(z, r) = \sum_n \frac{(-1)^n}{n!n!} \left(\frac{r}{2}\right)^{2n} \frac{\partial^{2n} B_{z, axis}}{\partial z^{2n}} \quad (7)$$

• From (4) and (6) follows:

$$B_r(z, r) = \sum_n \frac{(-1)^n}{n!(n-1)!} \left(\frac{r}{2}\right)^{2n-1} \frac{\partial^{2n-1} B_{z, axis}}{\partial z^{2n-1}} \quad (8)$$

$$\ddot{r} = - \left(\frac{e}{2\gamma m}\right)^2 r B_z^2 \quad (10)$$

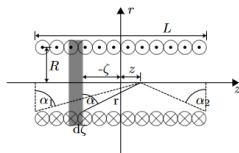
$$\ddot{z} = - \left(\frac{e}{2\gamma m}\right)^2 r^2 B_z B'_z \quad (11)$$

$$\ddot{r} = r'' \dot{z}^2 \approx r'' (\beta c)^2 \Rightarrow r'' = \left(\frac{e}{2p_z}\right)^2 r B_z^2 \quad (12)$$

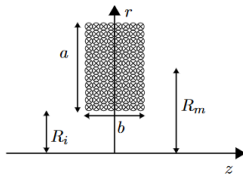
$$-\frac{r'}{r} = \frac{1}{f} \xrightarrow{\text{integrate (12)}} \frac{1}{f} = \left(\frac{e}{2p_z}\right)^2 \int_{-\infty}^{\infty} B_z^2 dz := \left(\frac{e}{2p_z}\right)^2 F_2 \quad (13)$$

Magnet lenses

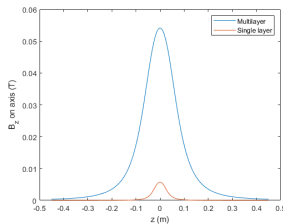
Magnetic field of solenoid



(a) Single-wind solenoid¹



(b) Solenoid with multilayered windings¹



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 n I}{2} \left(\frac{\Delta z}{\sqrt{R^2 + \Delta z^2}} - \frac{\Delta z^*}{\sqrt{R^2 + \Delta z^{*2}}} \right) \quad (14)$$

$$\Delta z = z - L/2 \quad (15)$$

Approximate field of the solenoid (b):

$$B_z(z) \approx \frac{\mu_0 n I}{4} \left(\frac{Rc^2}{(z^2 + Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2 + Rc^{*2})^{3/2}} \right) \quad (16)$$

$$Rc = R_{sq} + c, \quad R_{sq} = R_m \left(1 + \frac{a^2}{24R_m^2} \right), \quad c^2 = \frac{b^2 - a^2}{12} \quad (17)$$

¹Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Lens imperfections

There are 3 main limitations to consider when designing a solenoid lens:

Source:

- Chromatic aberration

Spread of electron energies

- RMS Emittance growth

Spread of electron coordinates in position-and-momentum phase space

- Spherical aberration

Real lens \rightarrow different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc} \sqrt{\langle x^2 \rangle \langle \tilde{p}_x^2 \rangle - \langle x \tilde{p}_x \rangle^2} = \frac{1}{mc} \left(\frac{e^2 \sigma^4}{3\sqrt{2} p_{z,0}} F_3 + \frac{e^4 \sigma^4}{24\sqrt{2} p_{z,0}^3} F_4 \right) \quad (18)$$

Chromatic aberration:

$$r_c \approx \alpha f (\Delta E_0 / E_0) \approx \alpha C_c (\Delta E_0 / E_0) \quad (19)$$

Lens impefections

Spherical aberration¹

$$\Delta f = c \cdot x^2 \quad (20)$$

$$x \approx f \cdot \tan(\alpha) \quad (21)$$

$$r_s = \Delta f \cdot \tan(\alpha) \approx c \cdot f^2 \tan^3(\alpha) \quad (22)$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \quad (23)$$

$$F_3 = - \int \frac{B_z B_z'''}{2} dz \quad (24)$$

$$F_4 = \int B_z^4 dz \quad (25)$$

$$C_s = \frac{e}{96m\tilde{U}} \int \left(\frac{2e}{m\tilde{U}} B_z^4 + 5 (B_z')^2 - B_z B_z''' \right) R^4 dz \quad (26)$$

$$= \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12p_{z,0}^4} F_4 \quad (27)$$

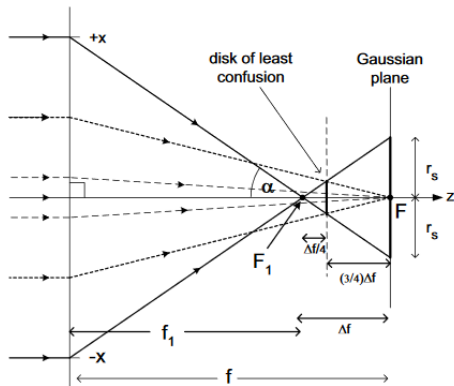







Figure: Illustration of beam radius change due to spherical aberration

¹Egerton et al., *Physical principles of electron microscopy*.

Summary

Subtitle

References I

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-  T. Gehrke. “Design of Permanent Magnetic Solenoids for REGAE”. MA thesis. Hamburg: Universität Hamburg, 2013.
-  B. Grigoryan et al. “Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun”. In: *Proceedings of IPAC2011, San Sebastián, Spain*. 2011.
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-  J Rossbach and P Schmüser. *Basic course on accelerator optics*. Tech. rep. P00011673, 1993.