Solenoid electron lenses

Fundamentals and design

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Structure

Motivation

▶ Magnetic lenses - important components of particle accelerators, microscopes
▶ Electromagnet solenoids - physical foundation of more advanced magnetic lenses
▶ Design constraints:
  ▶ Power consumption
  ▶ Physical size, material usage
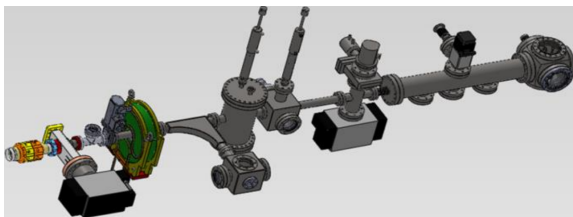 ▶ Characteristic parameters for interaction with other machine components

Figure: Schematic[1] of AREAL, an electron bunch-research oriented linac[2]

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[1]Grigoryan et al., "Status of AREAL RF Photogun Test Facility".
[2]Grigoryan et al., "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun".

Structure

Electron optics

Definition

Classic optics

- Light ray - photons

- Can pass through optically transparent solids

- Bends due to refractive index difference between media

Electron optics

- Electron beam - electrons

- Gets absorbed/loses energy due to interactions with atoms in media

- Bends due to Coulomb and Lorentz forces in the presense of external EM-fields
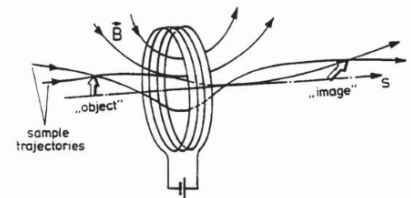
Magnet lenses
Requirements

A lens must have the following properties:

- ▶ deflection increases with increasing deviation of the beam from the optic axis;

- ▶ electron energy should not change, or change negligebly;

- ▶ symmetry of deflection on all sides of the optical axis;

- ▶ methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses

Solenoids

(a) A solenoid with S as optical axis[1]
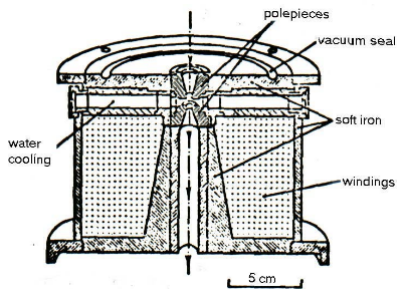
(b) Cross-section of a magnetic lens[2]

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[1]Rossbach and Schmüser, *Basic course on accelerator optics*.

[2]Egerton et al., *Physical principles of electron microscopy*.
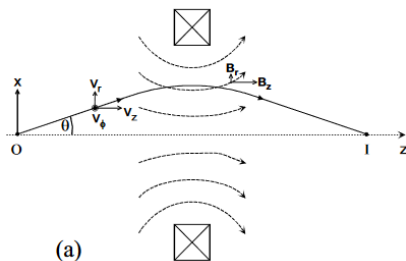
Magnet lenses
Solenoids

Figure: A solenoid cross-section along the optic axis[1]

$$F_{\varphi} = e\left(B_z v_r - v_z B_r\right) \qquad (1)$$

$$F_r = -e\left(v_z B_z\right) \qquad (2)$$

$$F_z = e\left(v_{\varphi} B_r\right) \qquad (3)$$

$$\gamma m\left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \qquad (4)$$

$$\gamma m\frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \qquad (5)$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \qquad (6)$$

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[1]Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses
Electron path equations

• From (5) follows:

$$\dot{\varphi} = \frac{e}{2\gamma m} B_z \qquad (9)$$

$$B_z(z, r) = \sum_n \frac{(-1)^n}{n! n!} \left(\frac{r}{2} \right)^{2n} \frac{\partial^{2n} B_{z,\,axis}}{\partial z^{2n}} \qquad (7)$$

• From (4) and (6) follows:

$$B_r(z, r) = \sum_n \frac{(-1)^n}{n! \, (n-1)!} \left(\frac{r}{2} \right)^{2n-1} \frac{\partial^{2n-1} B_{z,\,axis}}{\partial z^{2n-1}} \qquad (8)$$

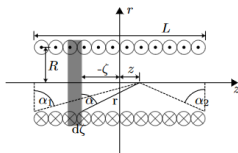$$\ddot{r} = - \left(\frac{e}{2\gamma m} \right)^2 r B_z^2 \qquad (10)$$

$$\ddot{z} = - \left(\frac{e}{2\gamma m} \right)^2 r^2 B_z B_z' \qquad (11)$$

$$\ddot{r} = r'' \dot{z}^2 \approx r'' (\beta c)^2 \Rightarrow r'' = \left(\frac{e}{2 p_z} \right)^2 r B_z^2 \qquad (12)$$
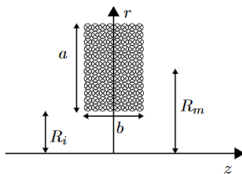
$$- \frac{r'}{r} = \frac{1}{f} \underset{integrate\ (12)}{\Rightarrow} \frac{1}{f} = \left(\frac{e}{2 p_z} \right)^2 \int_{-\infty}^{\infty} B_z^2 dz := \left(\frac{e}{2 p_z} \right)^2 F_2 \qquad (13)$$
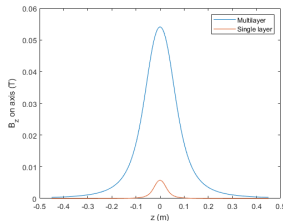
Magnet lenses

Magnetic field of solenoid

(a) Single-wind solenoid[1]

(b) Solenoid with multilayred windings[1]

(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 n I}{2} \left(\frac{\triangle z}{\sqrt{R^2 + \triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2 + \triangle z^{*2}}} \right) \quad (14)$$

$$\triangle z = z - L/2 \quad (15)$$

Approximate field of the solenoid (b)[1]:

$$B_z(z) \approx \frac{\mu_0 N I}{4} \left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}} \right) \quad (16)$$

$$Rc = R_{sq} + c, \ R_{sq} = R_m \left(1 + \frac{a^2}{24 R_m^2} \right), \ c^2 = \frac{b^2 - a^2}{12} \quad (17)$$

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Lens impefections

There are 3 main limitations to consider when designing a solenoid lens:

Source:

•Chromatic aberration → Spread of electron energies

•RMS Emittance growth → Spread of eletron coordinates in position-and-momentum phase space

•Spherical aberration → Real lens → different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc}\sqrt{\langle x^2\rangle\langle \tilde{p}_x^2\rangle - \langle x\tilde{p}_x\rangle^2} = \frac{1}{mc}\left(\frac{e^2\sigma^4}{3\sqrt{2}p_{z,0}}F_3 + \frac{e^4\sigma^4}{24\sqrt{2}p_{z,0}^3}F_4\right) \tag{18}$$

Chromatic aberration:

$$r_c \approx \alpha f\,(\triangle E_0/E_0) \approx \alpha C_c\,(\triangle E_0/E_0) \tag{19}$$

Lens impefections

Spherical aberration[1]

$$\triangle f = c \cdot x^2 \tag{20}$$

$$x \approx f \cdot \tan(\alpha) \tag{21}$$

$$r_s = \triangle f \cdot \tan(\alpha) \approx c \cdot f^2 \tan(\alpha)^3 \tag{22}$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \tag{23}$$

$$F_3 = - \int \frac{B_z B_z''}{2} dz \tag{24}$$

$$F_4 = \int B_z^4 dz \tag{25}$$

Figure: Illustration of beam radius change due to spherical aberration

$$C_s = \frac{e}{96m\bar{U}} \int \left(\frac{2e}{m\bar{U}} B_z^4 + 5 \left(B_z' \right)^2 - B_z B_z'' \right) R^4 dz \tag{26}$$

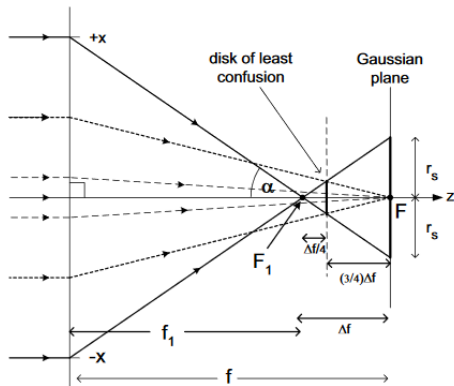$$= \frac{e^2 R^4}{4 p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4 \tag{27}$$

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[1]Egerton et al., *Physical principles of electron microscopy*.

Structure

Project task

Simple solenoid lens design:

- Monochromatic e beam, fixed beam radius R
- Target FWHM, peak B_z, f
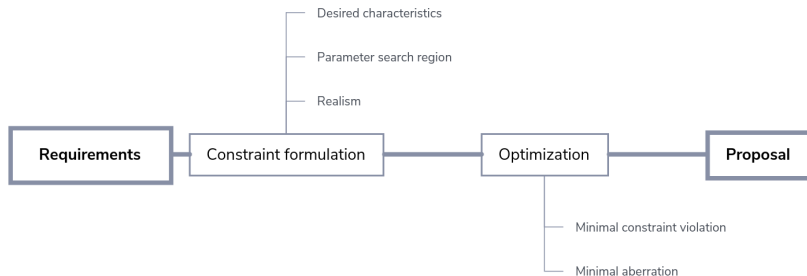- Optimize geometry, current for minimal spheric aberration

Figure: Generalized design process

Solenoid model

▶ Rectangular cross-section solenoid[1]

Two-loop field approximation:

$$B_z\left(z\right) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{\left(z^2+Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2+Rc^{*2}\right)^{3/2}}\right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2-a^2}{12},$$

$$R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right).$$
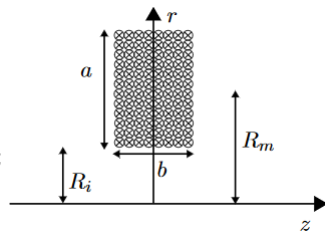

Figure: Solenoid geometry:
R_m - mean radius
a - transverse width
b - axial length

Parameters: geometry, scaling factor $N \cdot I$ [Ampere-Turns]

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Field integrals

For an axial beam, only the on-axis B_z is of significance[1]. The field's optical properties are described in terms of:

$$F_1 = \int B_z \, dz \qquad\qquad F_3 = \int -\frac{B_z'' \cdot B_z}{2} \, dz$$

$$F_2 = \int B_z^2 \, dz \qquad\qquad F_4 = \int B_z^4 \, dz$$

whereas the integration domain is $(-\infty, \infty)$.

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Solenoid characteristics

- ▶ Peak $B_z = B_z(0)$;
- ▶ Effective field length $\rightleftarrows$ FWHM
- ▶ Focal length:

$$f = \left(\frac{2p_z}{e}\right)^2 \frac{1}{F_2}$$

- ▶ Aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12p_{z,0}^4} F_4$$

Considerations:

- ▶ Geometry - size, material usage
- ▶ Scaling factor - power, material usage
- ▶ f, FWHM, c_s: interaction with other components, fitness to purpose

Optimization

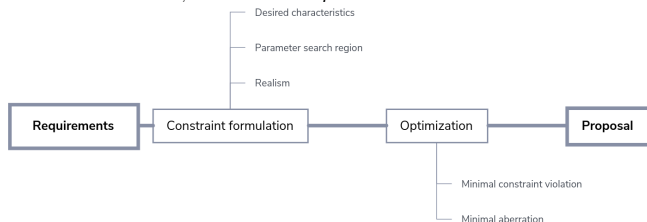▶ Constrained Trust Region algorithm[1]:
  Iterative gradient search process; minimize merit function $m(p)$ of parameters $p = (s, R_m, a, b)$:

  $$m(p) = c_s(p) + k \cdot |C(p)|_2, \quad \Delta p \leq R_T;$$

  $C(p) = (C_1, C_2, ..., C_n)(p)$ - measure of constraint violation;

▶ Interior Point Algorithm[2]:
  Solves a sequence of approximate optimization problems, by transforming the original problem $\min_x f(x)$, $h(x) = 0 \wedge g(x) \leq 0$ into $\min_x f(x) = \min_{x,s} f(x) - \mu \sum_i \ln(s_i)$, $h(x) = 0 \wedge g(x) + s = 0$.

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[1] *Documentation on SciPy's CTR implementation.*
[2] Wikipedia, *Sequential Quadratic Programming.*
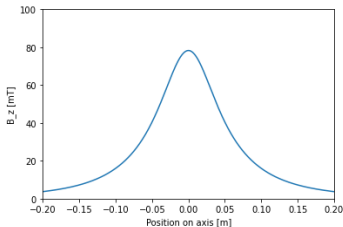
Software demonstration[1]

Evaluating the REGAE solenoid[2]

```
Parameters:
 - s: 9000.000
 - R_mean: 79.750 mm, a: 41.800 mm, b: 99.500 mm
Resulting characteristics:
 - Peak axial field: 78.279 mT
 - Effective field length: 102.000 mm
 - Focal distance for given E: 27.679 cm
 - Spherical aberration for given E: 1.407E-10 m
 - Focal spot radius: 6.636E-03 fm
```


(a) Testing the model

```
Settings:
 - g [mm]: [79.75, 41.8, 99.5]
 - s [A*N]: 9000
Targets:
 - peak B [mT]: 80
 - FWHM [mm]: 102
 - f [cm]: 27.7
 - g [mm]: (array([39.875, 20.9  , 49.75 ]),
array([119.625,  62.7  , 149.25 ]))
 - s [N*A]: [7000, 11000]
Result:
Parameters:
 - s: 8999.988
 - R_mean: 79.926 mm, a: 43.242 mm, b: 96.889 mm
Resulting characteristics:
 - Peak axial field: 77.297 mT
 - Effective field length: 104.000 mm
 - Focal distance for given E: 27.889 cm
 - Spherical aberration for given E: 1.367E-10 m
 - Focal spot radius: 6.300E-03 fm
```

(b) Attempting to optimize within 10% margin

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[1] Douginets and Yanovets, *solensim project repository*.
[2] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".
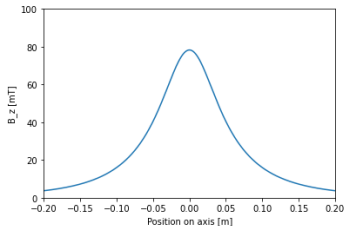
Software demonstration

General optimization

Parameters:
- s: 9000.000
- R_mean: 79.750 mm, a: 41.800 mm, b: 99.500 mm
Resulting characteristics:
- Peak axial field: 78.279 mT
- Effective field length: 102.000 mm
- Focal distance for given E: 27.679 cm
- Spherical aberration for given E: 1.407E-10 m
- Focal spot radius: 6.636E-03 fm

Settings:
- g [mm]: [30, 40, 50]
- s [A*N]: 4000
Targets:
- peak B [mT]: [100, 100]
- FWHM [mm]: [25, 75]
- f [cm]: [50, 50]
- g [mm]: (array([15., 20., 25.]), array([45., 60., 75.]))
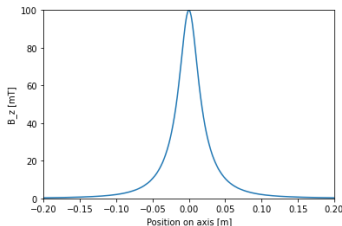- s [N*A]: (2000, 6000)
Result:
Parameters:
- s: 4000.781
- R_mean: 28.438 mm, a: 26.823 mm, b: 47.544 mm
Resulting characteristics:
- Peak axial field: 100.000 mT
- Effective field length: 34.000 mm
- Focal distance for given E: 50.000 cm
- Spherical aberration for given E: 1.242E-10 m
- Focal spot radius: 9.936E-04 fm

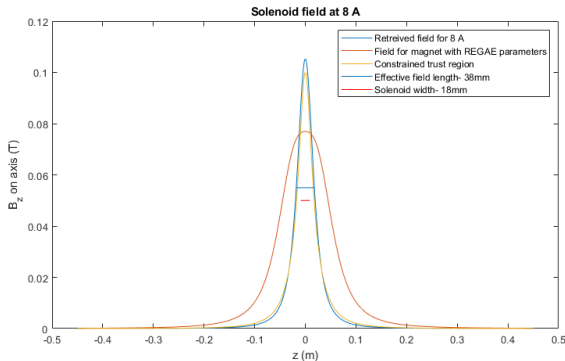**(a)** REGAE Solenoid[1]

(b) Optimizing for provided constraints

[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Interior point Algorithm results
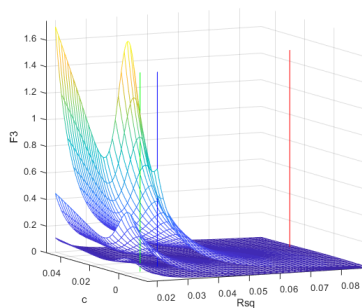
Retrieved parameters

Solenoid field at 8 A

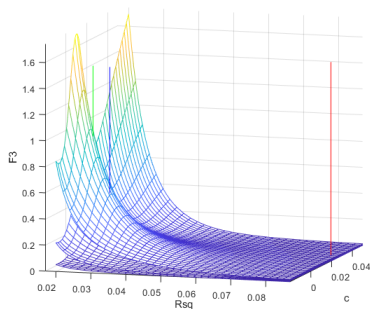|	Height a mm	Width b mm	Max field B_z mT	F. length f cm	Spherical ab. C_s	RMS emi. $\epsilon_{n.rms}$
Opt. parameters	17.6	17.6	105	50	$1.7e-9m$	$3.4e-10m$
REGAE[1]	99.5	41.8	79	30.5	$6.3e-11m$	$7.9e-11m$

[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Interior point Algorithm results

F_3 Integral

(a)

(b)

Figure: F_3 integral

Observations

▶ **Correlated requirements pose over-constraining problems**

▶ Multiple configurations in (s, r, a, b) correspond to same minima
  $\rightarrow$ convergence depends on initial guess and search region

▶ P-space scaled differently along various axes $\rightarrow$ weak s-adjustment

▶ Interval constraints are weighted differently

Perspective

Potential for further development

Algorithm:

▶ Normalize P-space for better parameter and constraint weighing

Model:

▶ Consider solenoids in ferromagnetic yokes
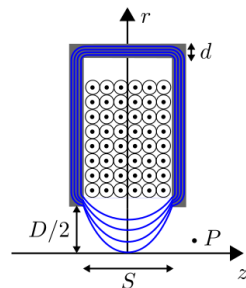
▶ Study tradeoffs in desired characteristics

Figure: Solenoid in yoke [1]

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[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Summary

▶ Electromagnet solenoids are a flexible electron lens implementation

▶ Minimal aberration can be achieved with different configurations
  $\rightarrow$ Design is to be oriented on required characteristics

Thank you for your attention!

References I

📄 *Documentation on SciPy's CTR implementation*. URL: https://docs.scipy.org/doc/scipy/reference/optimize.minimize-trustconstr.html.

📄 A. Douginets and A. Yanovets. *solensim project repository*. URL: https://github.com/adouge/solensim.

📄 R.F. Egerton et al. *Physical principles of electron microscopy*. Vol. 56. Springer, 2005.

📄 T. Gehrke. "Design of Permanent Magnetic Solenoids for REGAE". MA thesis. Hamburg: Universität Hamburg, 2013.

📄 B. Grigoryan et al. "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun". In: *Proceedings of IPAC2011, San Sebastián, Spain*. 2011.

References II

B. Grigoryan et al. "Status of AREAL RF Photogun Test Facility". In: *Proceedings of IPAC2014, Dresden, Germany* (Dresden, Germany). International Particle Accelerator Conference 5. https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017. Geneva, Switzerland: JACoW, July 2014, pp. 620–623. ISBN: 978-3-95450-132-8. DOI: https://doi.org/10.18429/JACoW-IPAC2014-MOPRI017. URL: http://jacow.org/ipac2014/papers/mopri017.pdf.

J Rossbach and P Schmüser. *Basic course on accelerator optics*. Tech. rep. P00011673, 1993.

Wikipedia. *Sequential Quadratic Programming*. URL: https://en.wikipedia.org/wiki/Sequential_quadratic_programming.