Solenoid electron lenses

Fundamentals and design

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Structure

Motivation

Demands on solenoid lenses:

- Low power, materials use
- Specific component size, focal length
- Minimal aberrations

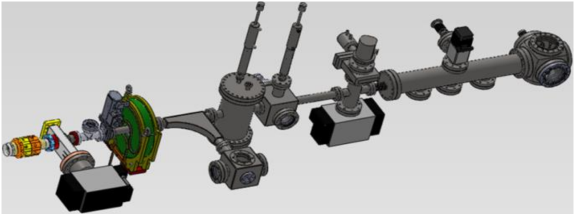


Figure: Schematic[1] of AREAL, an electron bunch-research oriented linac[2]

[1]Grigoryan et al., "Status of AREAL RF Photogun Test Facility".

[2]Grigoryan et al., "Advanced Research Electron Accelerator Laboratory Based on Photocathode RF Gun".

Structure

Electron optics

Definition

Classic optics	Electron optics
•Light ray - photons	•Electron beam - electrons
•Can pass through optically transparent solids	•Gets absorbed/loses energy due to interactions with atoms in media
•Bends due to refractive index difference between media	•Bends due to Coulomb and Lorentz forces in the presense of external EM-fields

Magnet lenses

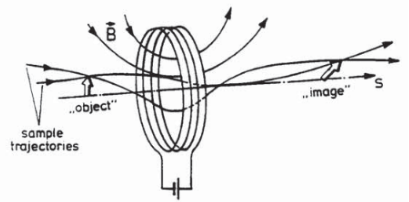
Requirements

A lens must have the following properties:

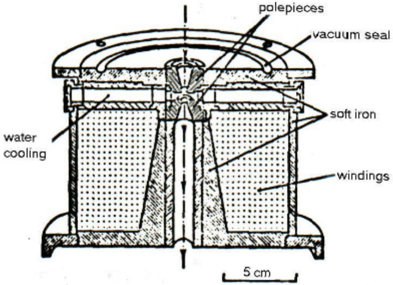
- deflection increases with increasing deviation of the beam from the optic axis;
- electron energy should not change, or change negligebly;
- symmetry of deflection on all sides of the optical axis;
- methods and laws of classical optics (such as thin lens formula and approximation, matrix formalism) are assumed to be applicable.

Magnet lenses

Solenoids



(a) A solenoid with S as optical axis[1]



(b) Cross-section of a magnetic lens[2]

[1] Rossbach and Schmüser, *Basic course on accelerator optics*.
[2] Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses

Solenoids

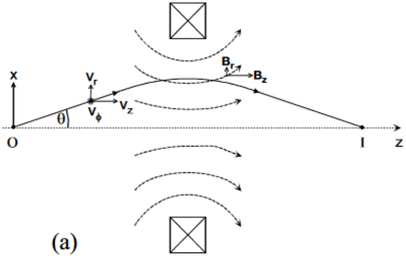


Figure: A solenoid cross-section along the optic axis[1]

$$F_\varphi = e\left(B_z v_r - v_z B_r\right) \quad (1)$$

$$F_r = -e\left(v_z B_z\right) \quad (2)$$

$$F_z = e\left(v_\varphi B_r\right) \quad (3)$$

$$\gamma m\left(\ddot{r} - r\dot{\varphi}^2\right) = -er\dot{\varphi}B_z \quad (4)$$

$$\gamma m\frac{d}{dt}\left(r^2\dot{\varphi}\right) = -er(\dot{r}B_z + B_r\dot{z}) \quad (5)$$

$$\gamma m\ddot{z} = er\dot{\varphi}B_r \quad (6)$$

[1] Egerton et al., *Physical principles of electron microscopy*.

Magnet lenses

Electron path equations

$$B_z\left(z,r\right)=\sum_n\frac{\left(-1\right)^n}{n!n!}\left(\frac{r}{2}\right)^{2n}\frac{\partial^{2n}B_{z,\text{ axis}}}{\partial z^{2n}} \tag{7}$$

$$B_r\left(z,r\right)=\sum_n\frac{\left(-1\right)^n}{n!\left(n-1\right)!}\left(\frac{r}{2}\right)^{2n-1}\frac{\partial^{2n-1}B_{z,\text{ axis}}}{\partial z^{2n-1}} \tag{8}$$

- From (5) follows:

$$\dot{\varphi}=\frac{e}{2\gamma m}B_z \tag{9}$$

- From (4) and (6) follows:

$$\ddot{r}=-\left(\frac{e}{2\gamma m}\right)^2 rB_z^2 \tag{10}$$

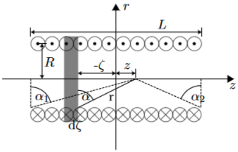
$$\ddot{z}=-\left(\frac{e}{2\gamma m}\right)^2 r^2 B_z B_z' \tag{11}$$

$$\ddot{r}=r''\dot{z}^2\approx r''\left(\beta c\right)^2\Rightarrow r''=\left(\frac{e}{2p_z}\right)^2 rB_z^2 \tag{12}$$

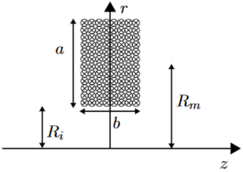
$$-\frac{r'}{r}=\frac{1}{f}\underset{integrate\,(12)}{\Rightarrow}\frac{1}{f}=\left(\frac{e}{2p_z}\right)^2\int_{-\infty}^{\infty}B_z^2dz:=\left(\frac{e}{2p_z}\right)^2F_2 \tag{13}$$

Magnet lenses

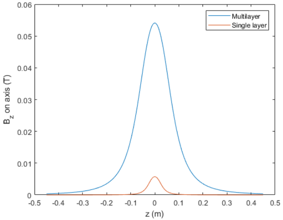
Magnetic field of solenoid



(a) Single-wind solenoid[1]



(b) Solenoid with multilayred windings[1]



(c) Fields produced by two coils with same N and L=b

Field of the solenoid (a):

$$B_z(z) = \frac{\mu_0 nI}{2}\left(\frac{\triangle z}{\sqrt{R^2 + \triangle z^2}} - \frac{\triangle z^*}{\sqrt{R^2 + \triangle z^{*2}}}\right) \tag{14}$$

$$\triangle z = z - L/2 \tag{15}$$

Approximate field of the solenoid (b)[1]:

$$B_z(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{\left(z^2 + Rc^2\right)^{3/2}} + \frac{Rc^{*2}}{\left(z^2 + Rc^{*2}\right)^{3/2}}\right) \tag{16}$$

$$Rc = R_{sq} + c,\; R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right),\; c^2 = \frac{b^2 - a^2}{12} \tag{17}$$

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Lens impefections

There are 3 main limitations to consider when designing a solenoid lens:

	Source:
•Chromatic aberration	Spread of electron energies
•RMS Emittance growth	Spread of eletron coordinates in position-and-momentum phase space
•Spherical aberration	Real lens $\rightarrow$ different refraction based on distance from axis

Emittance:

$$\epsilon_{n,rms} = \frac{1}{mc}\sqrt{\left\langle x^2 \right\rangle \left\langle \tilde{p}_x^2 \right\rangle - \left\langle x\tilde{p}_x \right\rangle^2} = \frac{1}{mc}\left(\frac{e^2\sigma^4}{3\sqrt{2}p_{z,0}}F_3 + \frac{e^4\sigma^4}{24\sqrt{2}p_{z,0}^3}F_4\right) \tag{18}$$

Chromatic aberration:

$$r_c \approx \alpha f\left(\triangle E_0 / E_0\right) \approx \alpha C_c\left(\triangle E_0 / E_0\right) \tag{19}$$

Lens impefections

Spherical aberration[1]

$$\triangle f = c \cdot x^2 \tag{20}$$

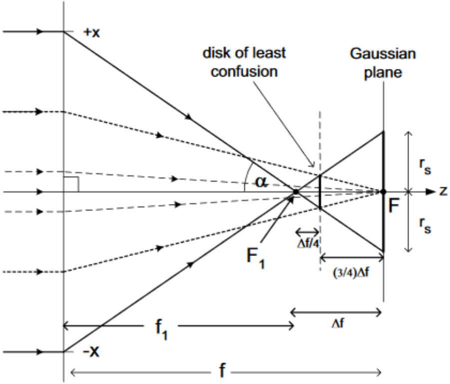
$$x \approx f \cdot tan(\alpha) \tag{21}$$

$$r_s = \triangle f \cdot tan(\alpha) \approx c \cdot f^2 tan(\alpha)^3 \tag{22}$$

$$= C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}} \right)^3 \tag{23}$$

$$F_3 = - \int \frac{B_z B_z''}{2} dz \tag{24}$$

$$F_4 = \int B_z^4 dz \tag{25}$$



$$C_s = \frac{e}{96 m \tilde{U}} \int \left(\frac{2e}{m\tilde{U}} B_z^4 + 5 \left(B_z' \right)^2 - B_z B_z'' \right) R^4 dz \tag{26}$$

$$= \frac{e^2 R^4}{4 p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4 \tag{27}$$

Figure: Illustration of beam radius change due to spherical aberration

[1] Egerton et al., *Physical principles of electron microscopy*.

Structure

Project task

Simple solenoid lens design:

- Monochromatic e beam, fixed beam radius R
- Target FWHM, peak B_z, f
- Optimize geometry, current for minimal aberrations

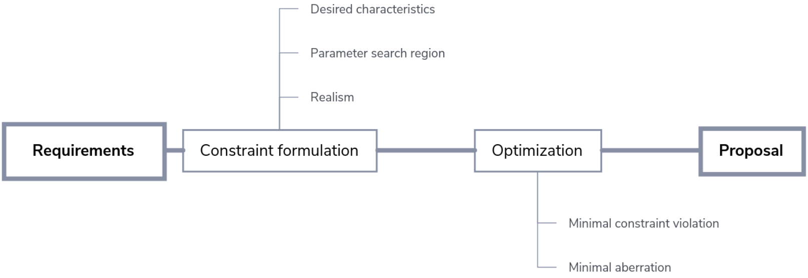
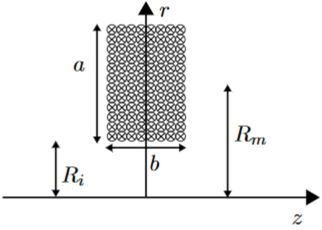


Figure: Generalized design process

Solenoid model

- Rectangular cross-section solenoid[1]

Two-loop field approximation:



$$B_z(z) \approx \frac{\mu_0 NI}{4}\left(\frac{Rc^2}{(z^2+Rc^2)^{3/2}} + \frac{Rc^{*2}}{(z^2+Rc^{*2})^{3/2}}\right);$$

$$Rc = R_{sq} + c, \text{ where } c^2 = \frac{b^2 - a^2}{12},$$

$$R_{sq} = R_m\left(1 + \frac{a^2}{24R_m^2}\right).$$

Figure: Solenoid geometry:
R_m - mean radius
a - transverse width
b - axial length

Parameters: geometry, scaling factor $N \cdot I$ [Ampere-Turns]

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Field integrals

For an axial beam, only the on-axis B_z is of significance[1]. The field's optical properties are described in terms of:

$$F_1 = \int B_z dz \qquad F_3 = \int -\frac{B_z'' \cdot B_z}{2} dz$$

$$F_2 = \int B_z^2 dz \qquad F_4 = \int B_z^4 dz$$

whereas the integration domain is $(-\infty, \infty)$.

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Solenoid characteristics

- Peak $B_z = B_z(0)$; effective field length $\rightleftarrows$ FWHM
- Focal length:

$$f = \left(\frac{2p_z}{e}\right)^2 \frac{1}{F_2}$$

- Spherical aberration coefficient:

$$c_s = \frac{e^2 R^4}{4p_{z,0}^2} F_3 + \frac{e^4 R^4}{12 p_{z,0}^4} F_4$$

- Resulting focal spot size:

$$r_s = C_s \cdot \left(\frac{r_{in}}{f - \frac{C_s r_{in}^2}{f^2}}\right)^3$$

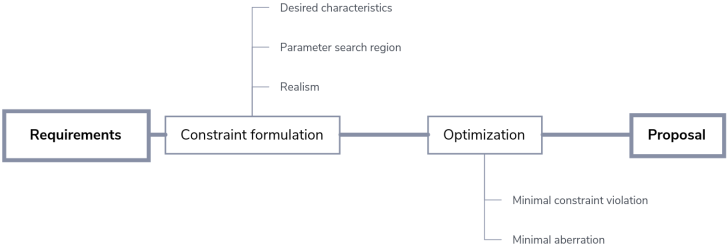
Considerations:

- - size, material usage
- Scaling factor, geometry - power consumption, material usage
- f, FWHM, c_s, r_s: interaction with other components, lens quality

Optimization

We used:

- Constrained Trust Region algorithm[1] - minimize c_s and constraint violation with dynamic "trust region" definition
- Interior Point algorithm:[2] - a more rigorous, less flexible implementation of the "trust region" concept



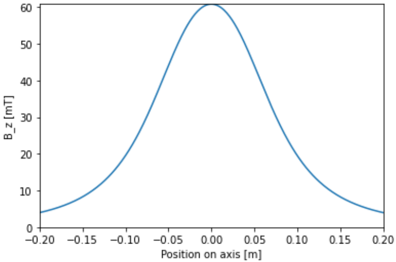
[1] *Documentation on SciPy's CTR implementation*.

[2] Byrd, Gilbert, and Nocedal, "A trust region method based on interior point techniques for nonlinear programming".

Software demonstration

Evaluating the REGAE solenoid[1]

```
Parameters:
 - s: 9000.000
 - R: 30.000 mm, a: 99.500 mm, b: 41.800 mm
Resulting characteristics:
 - Peak axial field: 60.855 mT
 - Effective field length: 148.000 mm
 - Focal distance for given E: 124.931 cm
 - Spherical aberration for given E, R:
5.861E-12 m
 - Focal spot radius: 3.006E-06 fm
```



(a) Testing the model

```
Settings:
 - g [mm]: [30, 99.5, 41.8]
 - s [A*N]: 9000
Targets:
 - peak B [mT]: 60.855
 - FWHM [mm]: 148
 - f [cm]: 124.931
 - g [mm]: [30, 99.5, 41.8]
 - s [N*A]: 9000
 - Margin [%]: 10
Result:
Parameters:
 - s: 8998.119
 - R: 30.759 mm, a: 103.938 mm, b: 41.821
mm
Resulting characteristics:
 - Peak axial field: 58.450 mT
 - Effective field length: 154.000 mm
 - Focal distance for given E: 129.898 cm
 - Spherical aberration for given E, R:
5.265E-12 m
 - Focal spot radius: 2.402E-06 fm
```

(b) Attempting to optimize within 10% margin

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Software demonstration

Results depend on initial conditions:

```
Settings:
 - g [mm]: [50, 50, 50]
 - s [A*N]: 8000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [45, 55]
 - f [cm]: [50, inf]
 - g [mm]: [[10, 10, 10], [200, 200, 200]]
 - s [N*A]: [4000, 12000]
 - Margin [%]: 10
Result:
Parameters:
 - s: 7292.988
 - R: 41.028 mm, a: 50.416 mm, b: 143.039
mm
Resulting characteristics:
 - Peak axial field: 100.000 mT
 - Effective field length: 54.000 mm
 - Focal distance for given E: 120.034 cm
 - Spherical aberration for given E, R:
1.328E-11 m
 - Focal spot radius: 7.680E-06 fm
```

(a) Optimizing for initial specifications

```
Settings:
 - g [mm]: [100, 100, 100]
 - s [A*N]: 5000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [45, 55]
 - f [cm]: [50, inf]
 - g [mm]: [[10, 10, 10], [200, 200, 200]]
 - s [N*A]: [4000, 12000]
 - Margin [%]: 10
Result:
Parameters:
 - s: 6547.926
 - R: 39.273 mm, a: 48.147 mm, b: 144.188
mm
Resulting characteristics:
 - Peak axial field: 100.000 mT
 - Effective field length: 46.000 mm
 - Focal distance for given E: 137.290 cm
 - Spherical aberration for given E, R:
1.155E-11 m
 - Focal spot radius: 4.463E-06 fm
```

(b) An approach from different start values

Software demonstration - observations

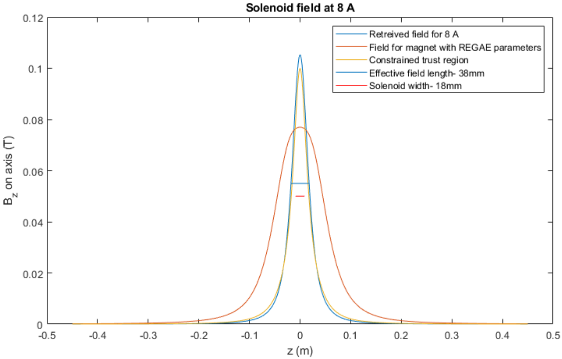
```
Settings:
 - g [mm]: [50, 50, 50]
 - s [A*N]: 8000
Targets:
 - peak B [mT]: [100, 100]
 - FWHM [mm]: [50, 50]
 - f [cm]: [50, inf]
 - g [mm]: [[10, 10, 10], [200, 200, 200]]
 - s [N*A]: [4000, 12000]
 - Margin [%]: 10
Result:
Parameters:
 - s: 7985.177
 - R: 65.059 mm, a: 33.370 mm, b: 82.427 mm
Resulting characteristics:
 - Peak axial field: 65.533 mT
 - Effective field length: 112.000 mm
 - Focal distance for given E: 143.346 cm
 - Spherical aberration for given E, R:
5.481E-12 m
 - Focal spot radius: 1.861E-06 fm
```

Figure: Example of overconstraining

- Overconstraining throws the algorithm off
- Multiple configurations in (s, r, a, b) correspond to same minima
  $\rightarrow$ convergence depends on initial guess and search region
- Parameters, constraints seem to be weighted differently - the optimization space is not normalized

Interior point Algorithm results

Retrieved parameters

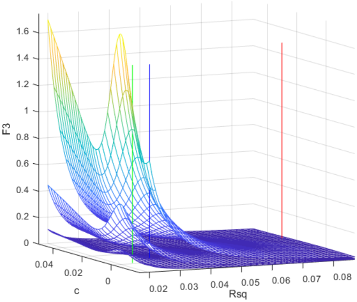


	Height a mm	Width b mm	Max field B_z mT	F. length f cm	Spherical ab. C_s	RMS emi. $\epsilon_{n.rms}$
Opt. parameters	17.6	17.6	105	50	$1.7e-9m$	$3.4e-10m$
REGAE[1]	99.5	41.8	79	30.5	$6.3e-11m$	$7.9e-11m$

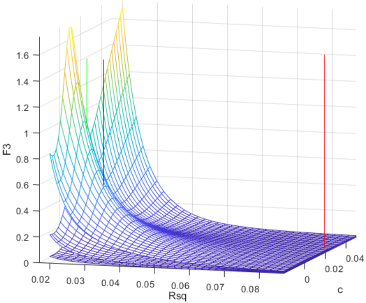
[1]Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Interior point Algorithm results

F_3 Integral



(a)



(b)

Figure: F_3 integral

Perspective

Potential for further development

Algorithm:

- Normalize parameters, constraints for better weighing
- Rework structure to allow for flexible model, optimization parameter choice

Models:

- Consider solenoids in ferromagnetic yokes
- Consider chromatic aberrations

General:

- Develop means for sweep-like study of characteristic response to parameters, constraints, initial values

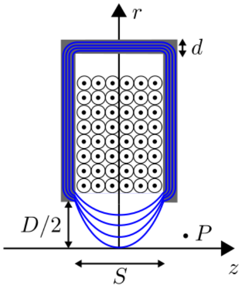


Figure: Solenoid in a yoke [1]

[1] Gehrke, "Design of Permanent Magnetic Solenoids for REGAE".

Summary

- Nonlinear, multivariate optimization is highly sensitive to initial guess values

- Over-constraining in characteristic values throws the optimization off-course

- Minimal aberration can be achieved with different configurations

$\rightarrow$ Design is to be oriented on required characteristics and optimized in that region - no search for a "perfect" lens.

- Further improvements to the software framework could provide simple tools to comprehensively study the design process, potential trade-offs and nuances.

Thank you for your attention!

References I

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