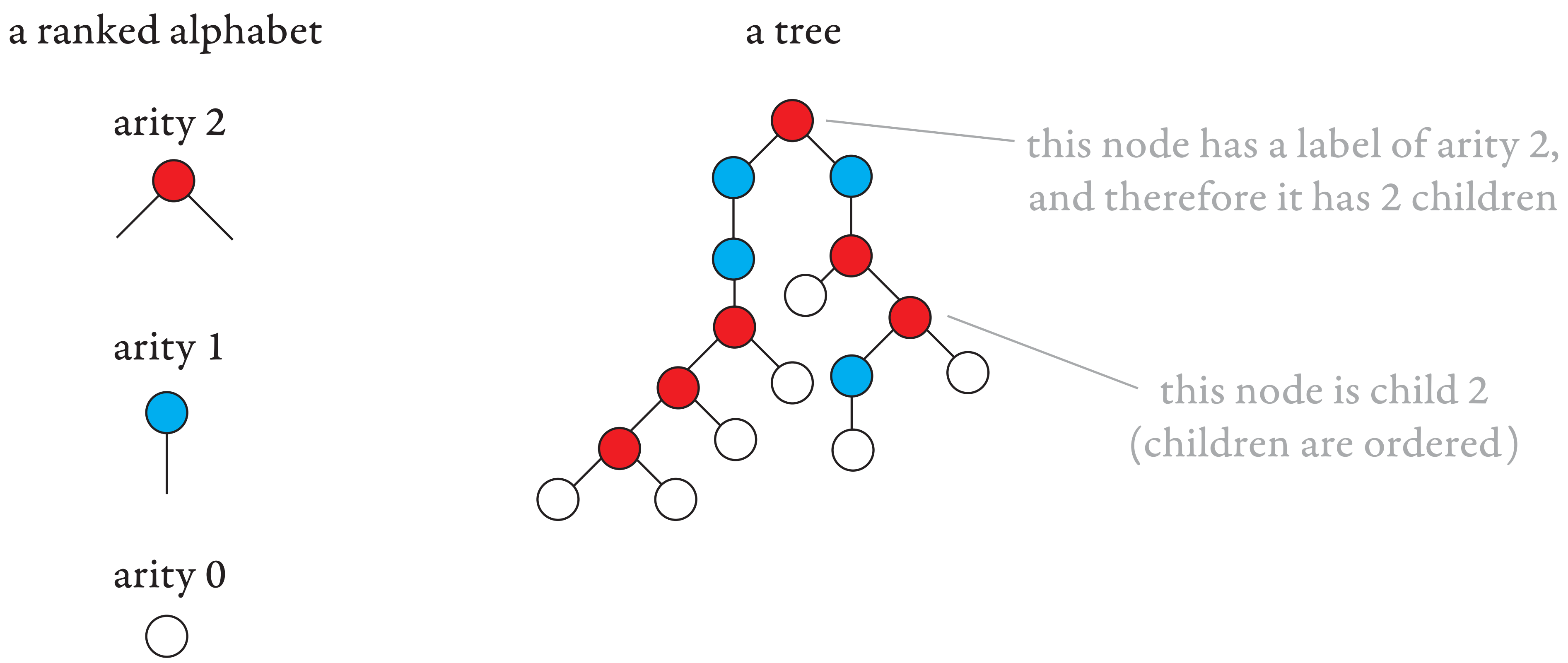
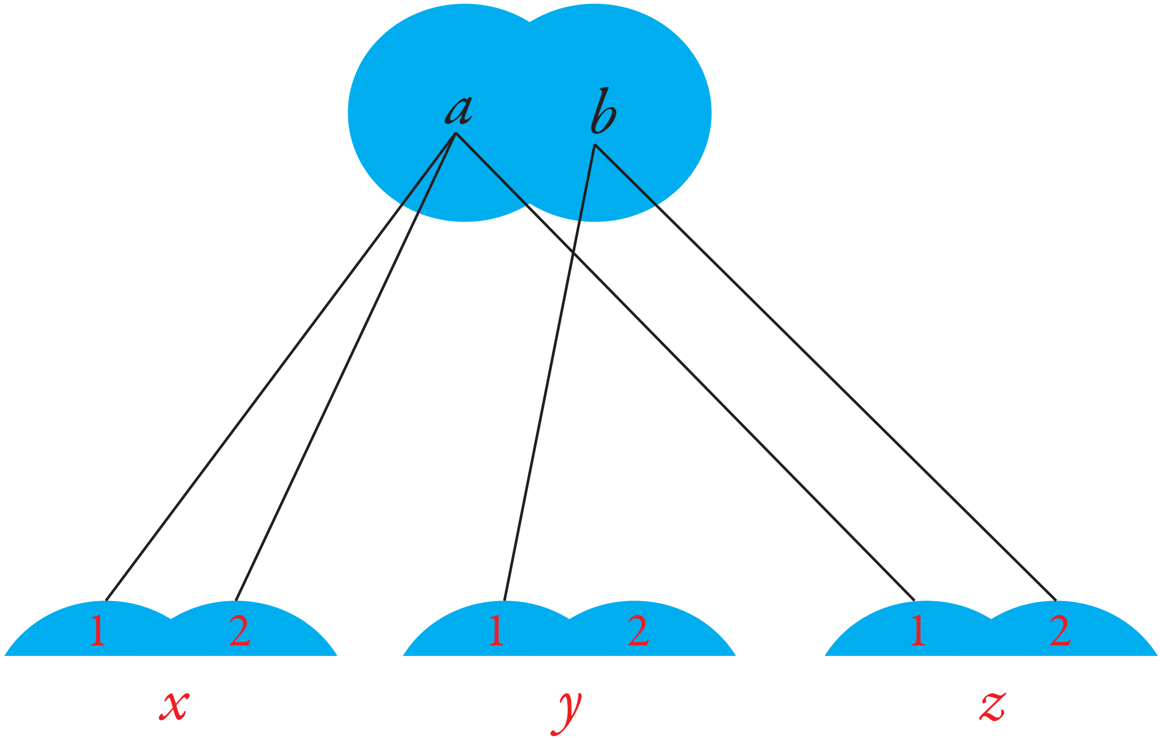
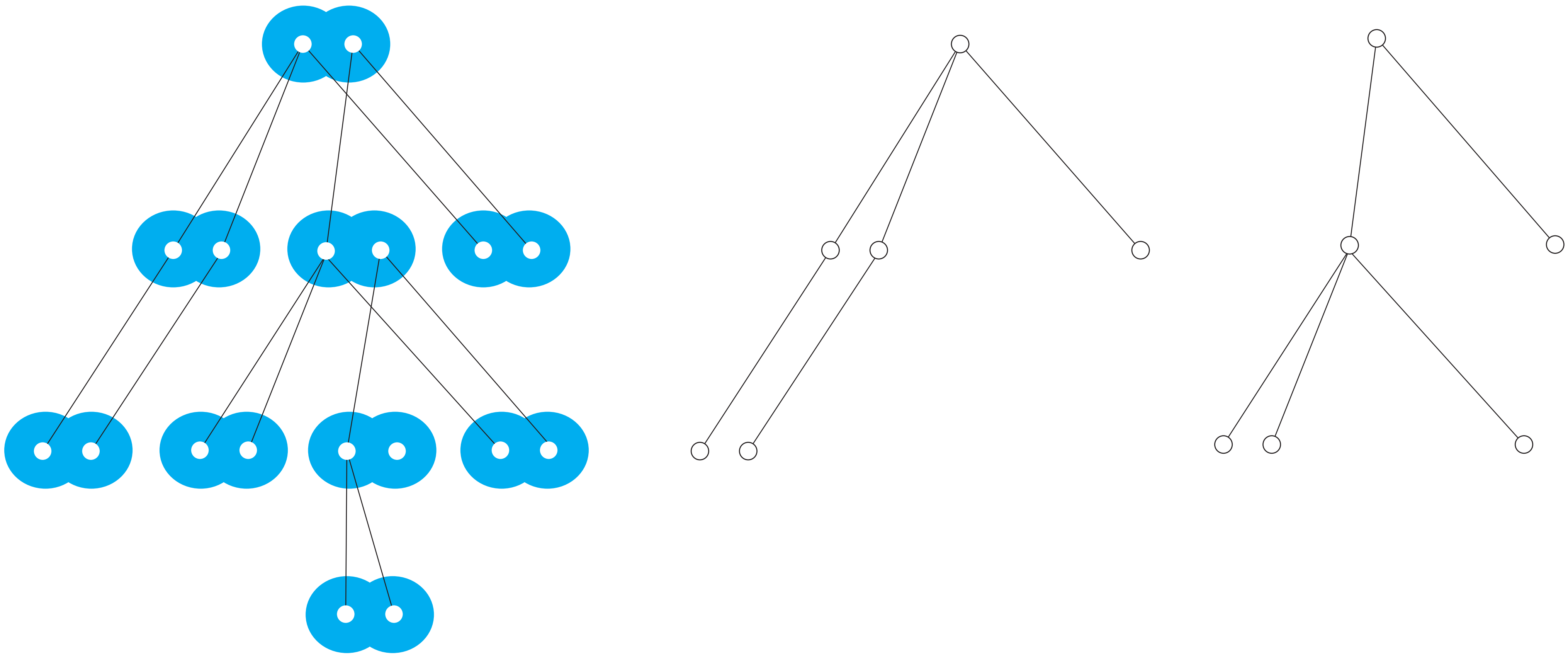
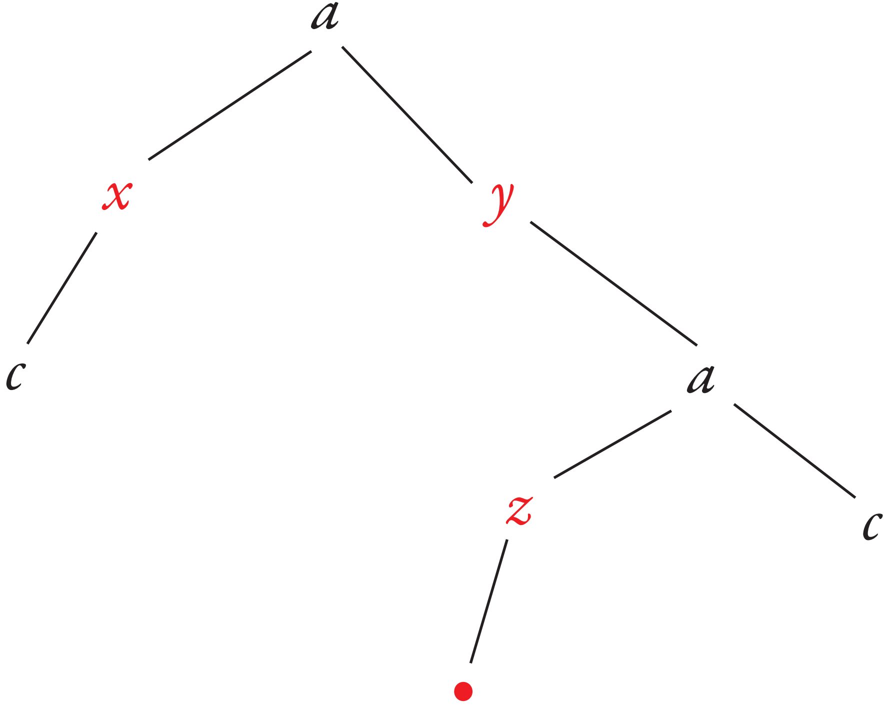

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)

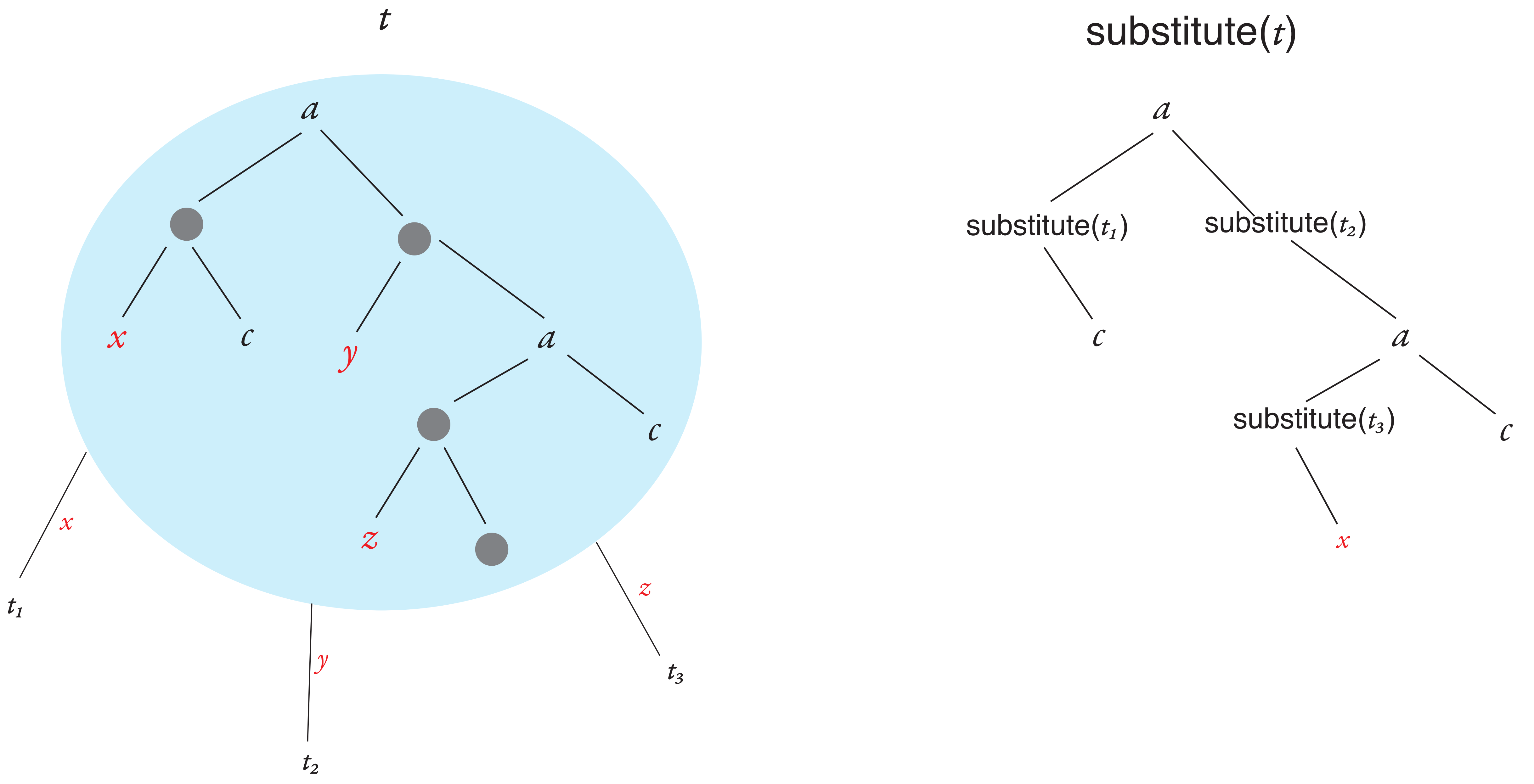


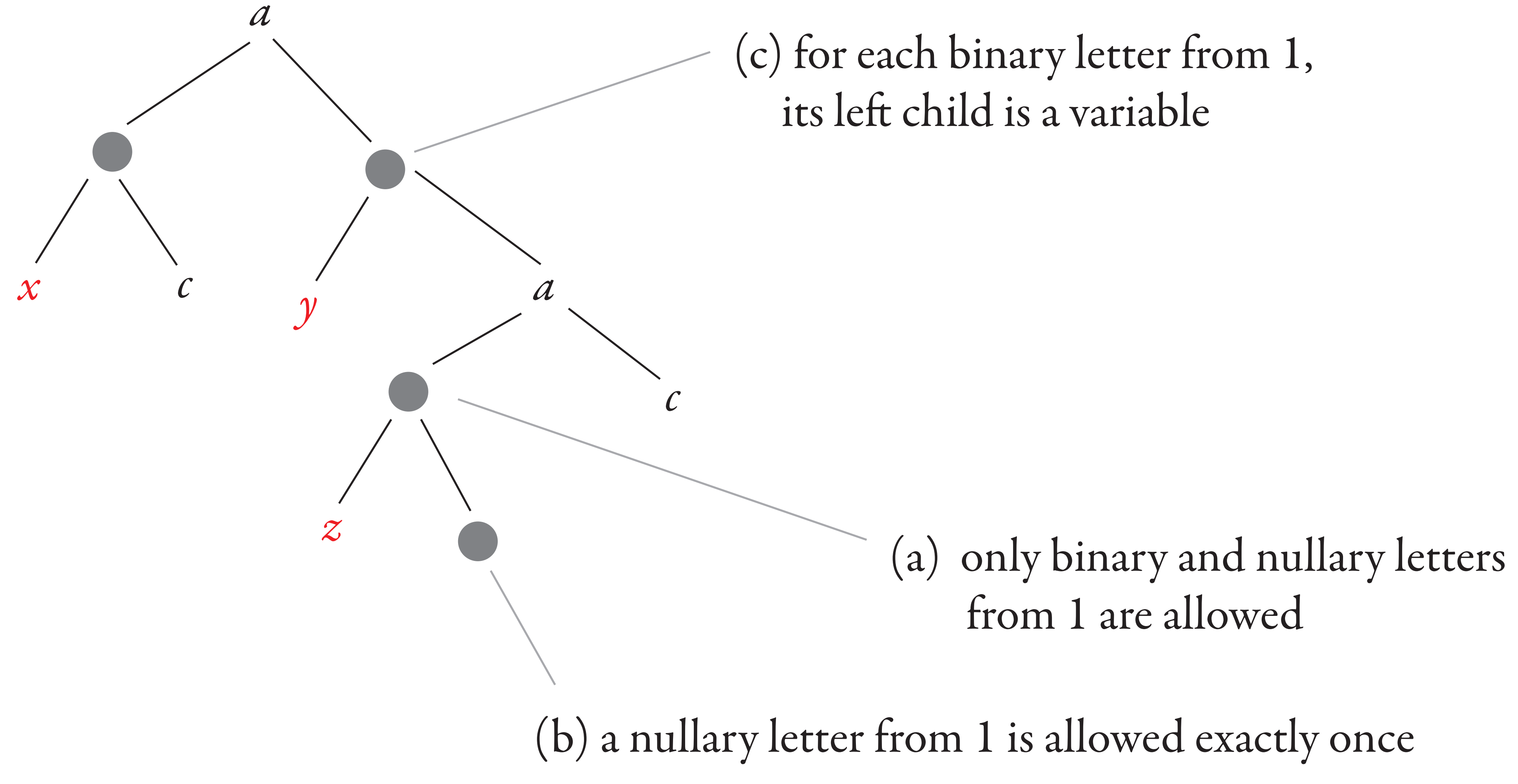
a
b
1
2
1
2
1
2
x
y
z

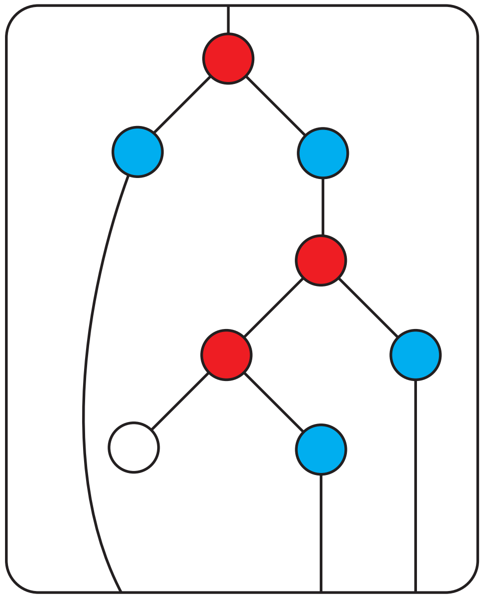
A tree t over $\Sigma^{[2]}$ $\quad$ $\mathsf{unfold}_1(t)$ $\quad$ $\mathsf{unfold}_2(t)$



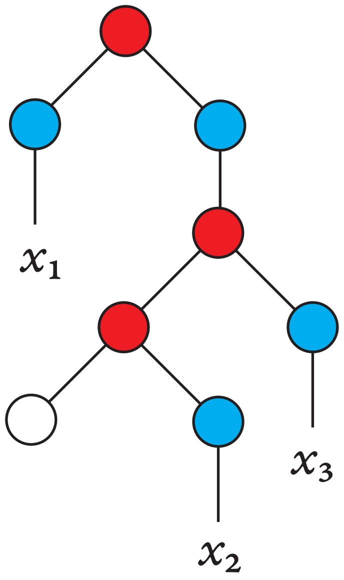

a
x
y
c
a
z
c


t
a
x
c
y
a
c
z
x
z
y
t_1
t_2
t_3
substitute(t)
a
substitute(t_1)
substitute(t_2)
c
a
substitute(t_3)
c
x

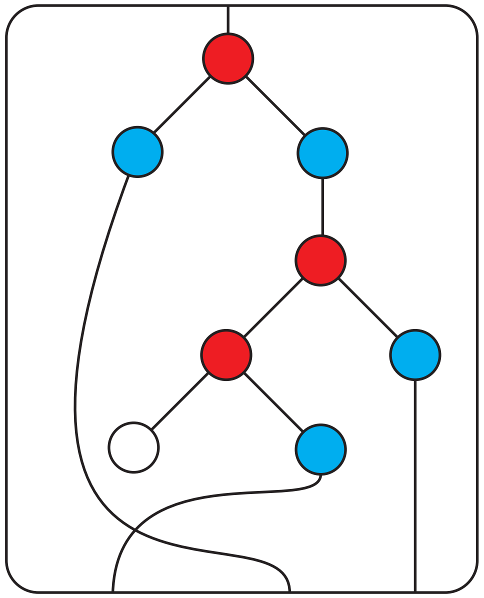

a
(c) for each binary letter from 1,
its left child is a variable
x
c
y
a
c
z
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once



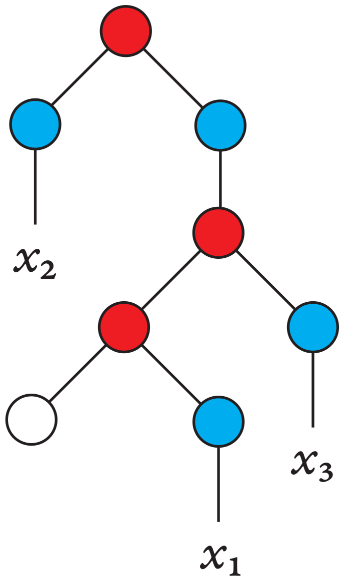
=



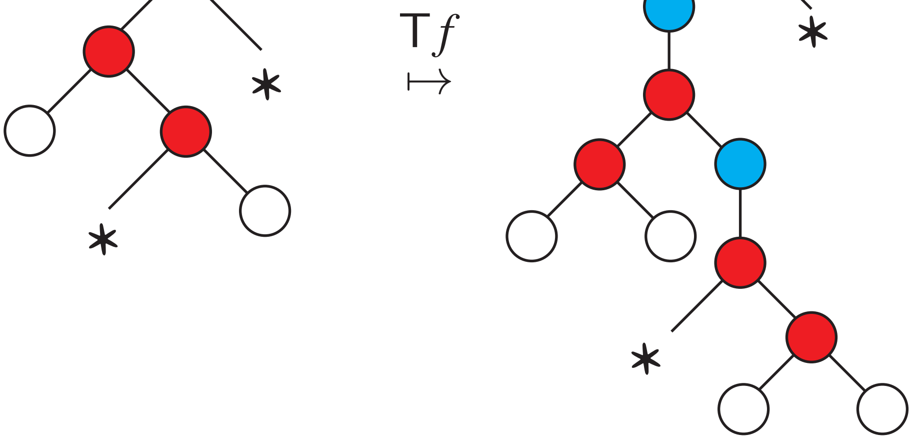
x_1
x_3
x_2



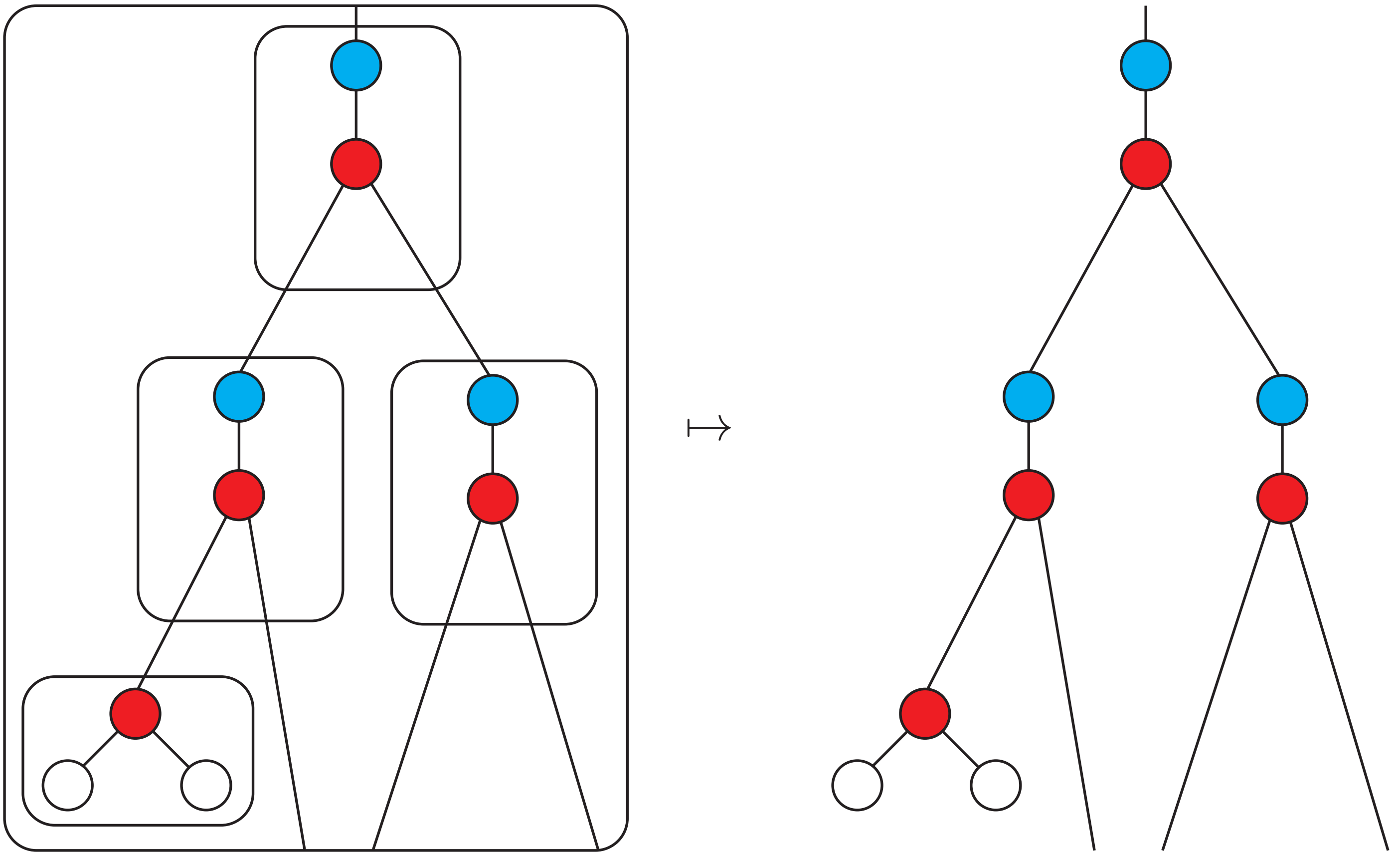
=

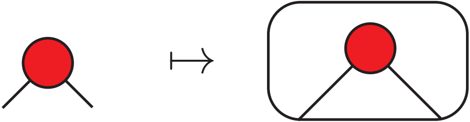


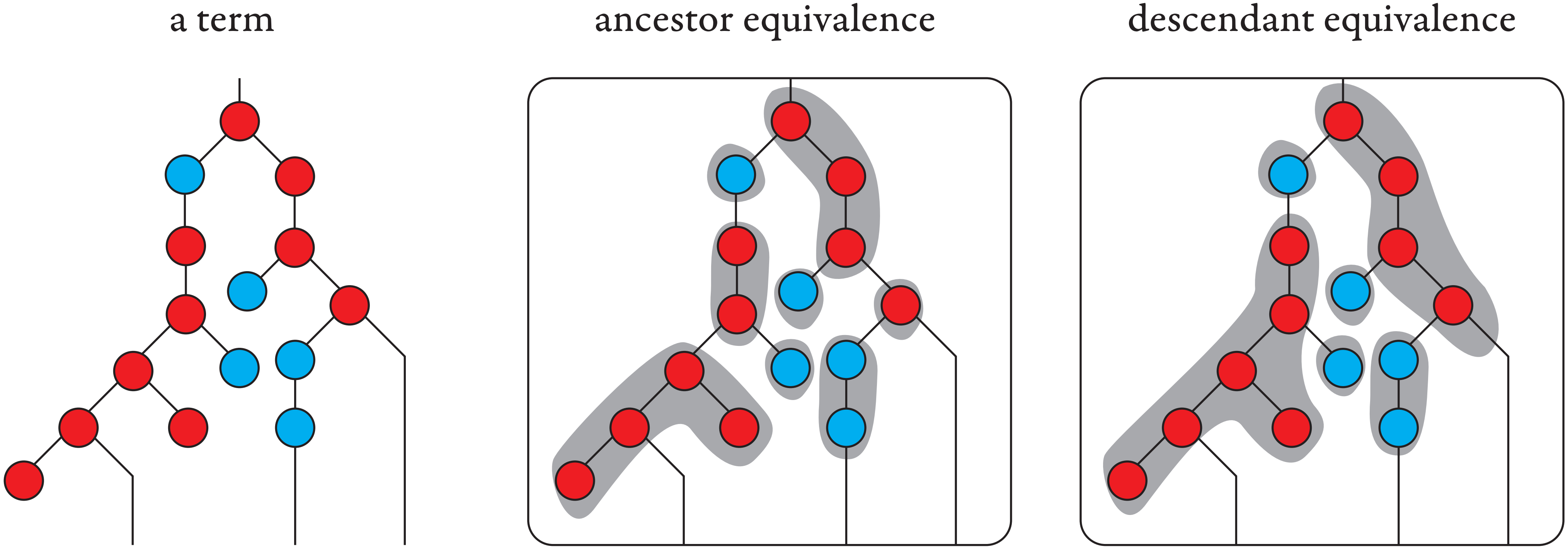
x_2
x_3
x_1

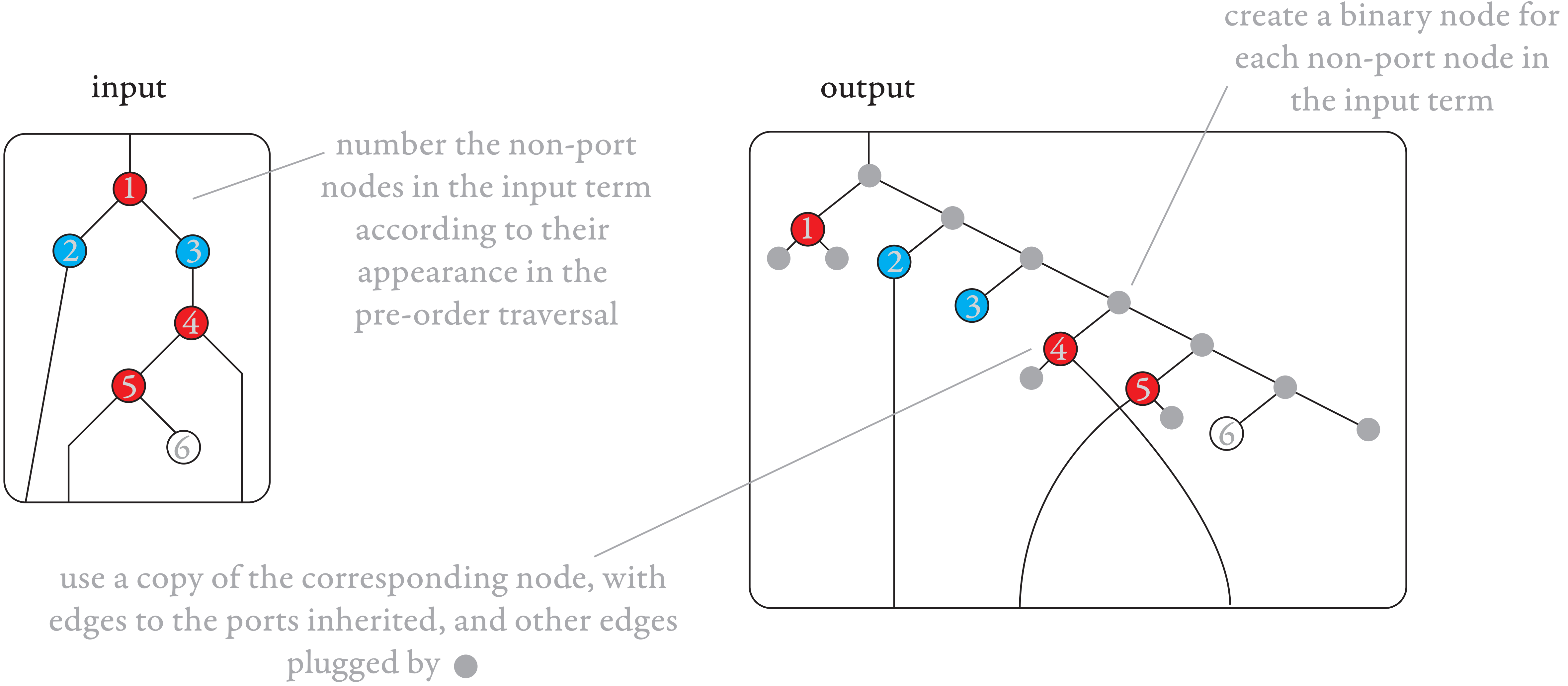


Tf

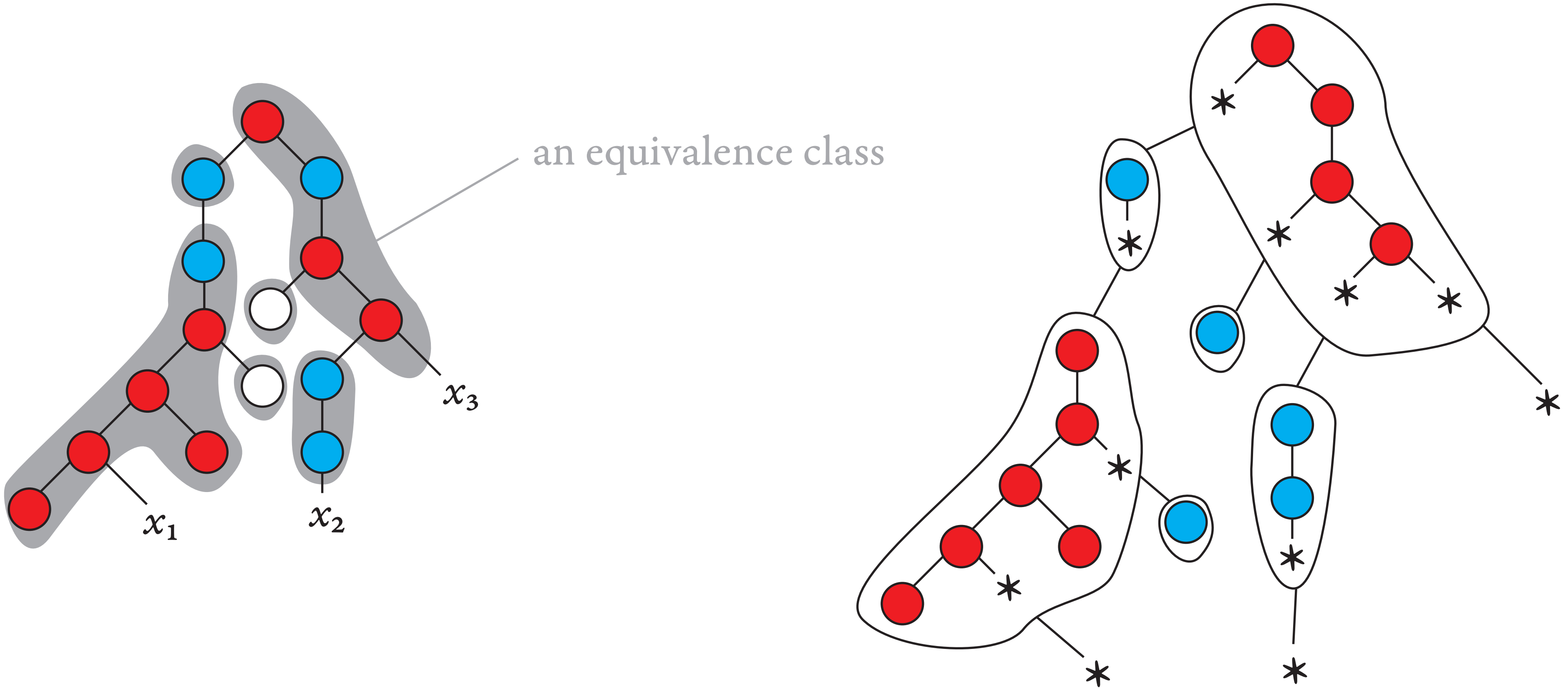


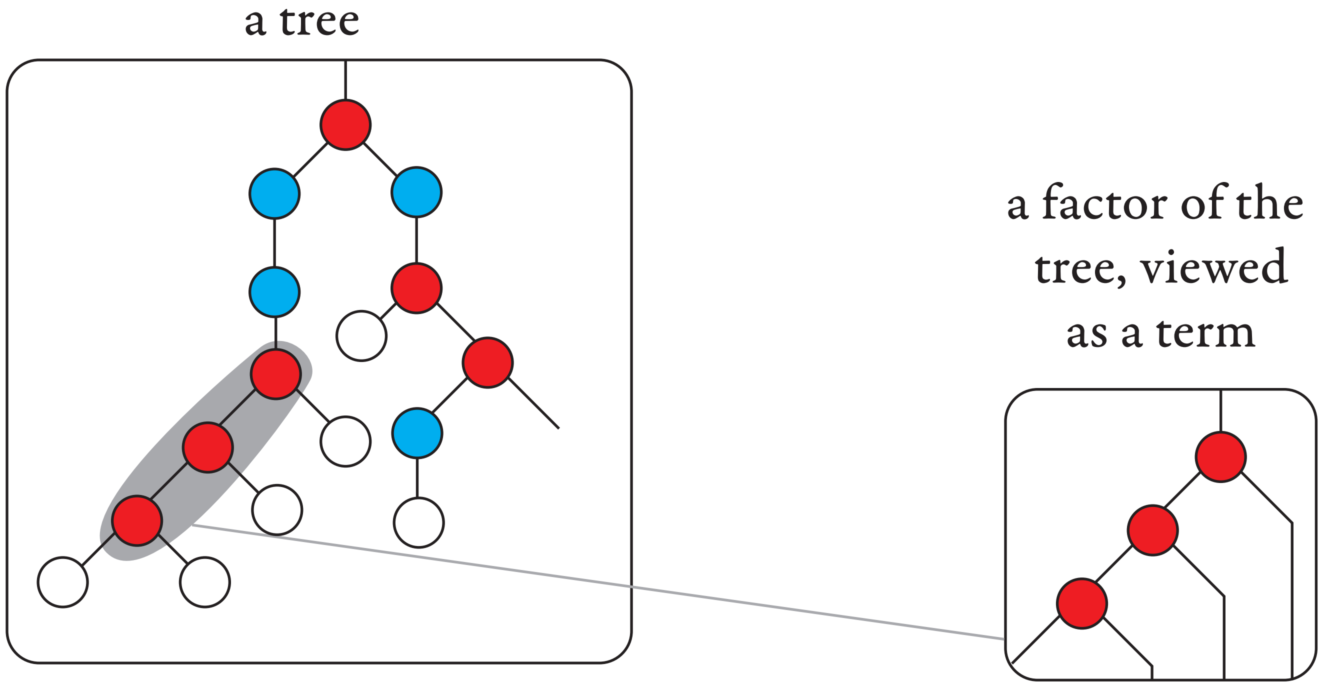


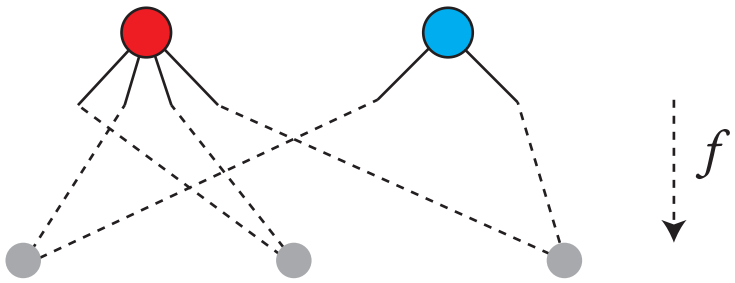

a term
ancestor equivalence
descendant equivalence

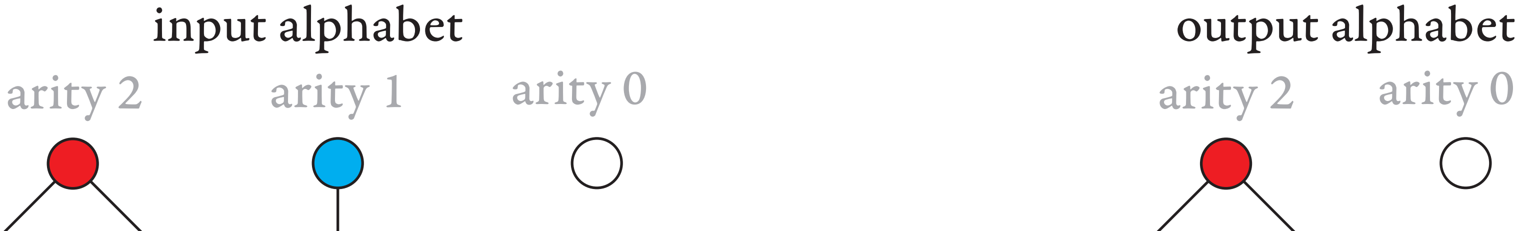

input
output
number the non-port nodes in the input term according to their appearance in the pre-order traversal
create a binary node for each non-port node in the input term
use a copy of the corresponding node, with edges to the ports inherited, and other edges plugged by
1
2
3
4
5
6
1
2
3
4
5
6

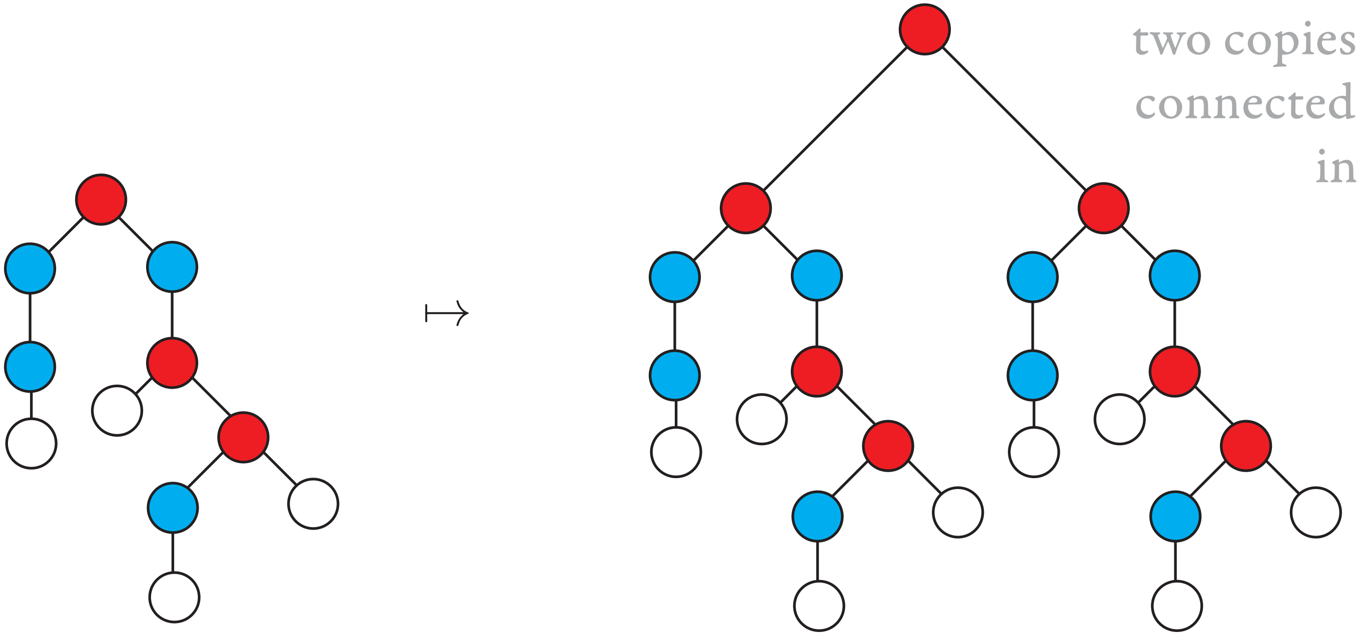
a factorisation equivalence



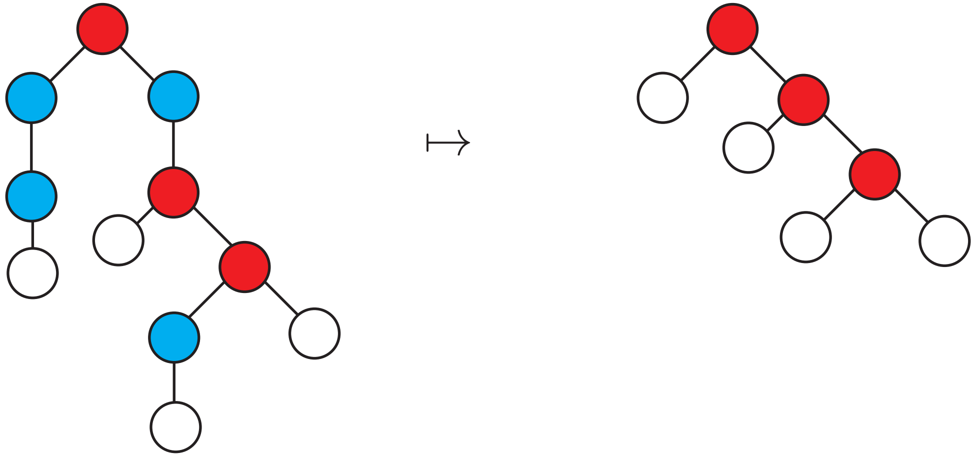

a tree
a factor of the tree, viewed as a term

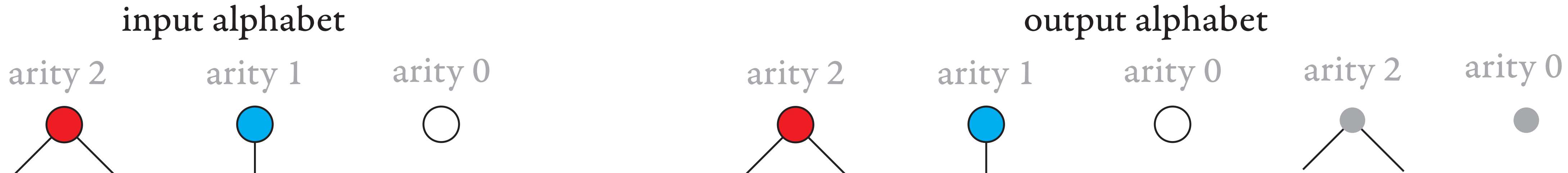

f

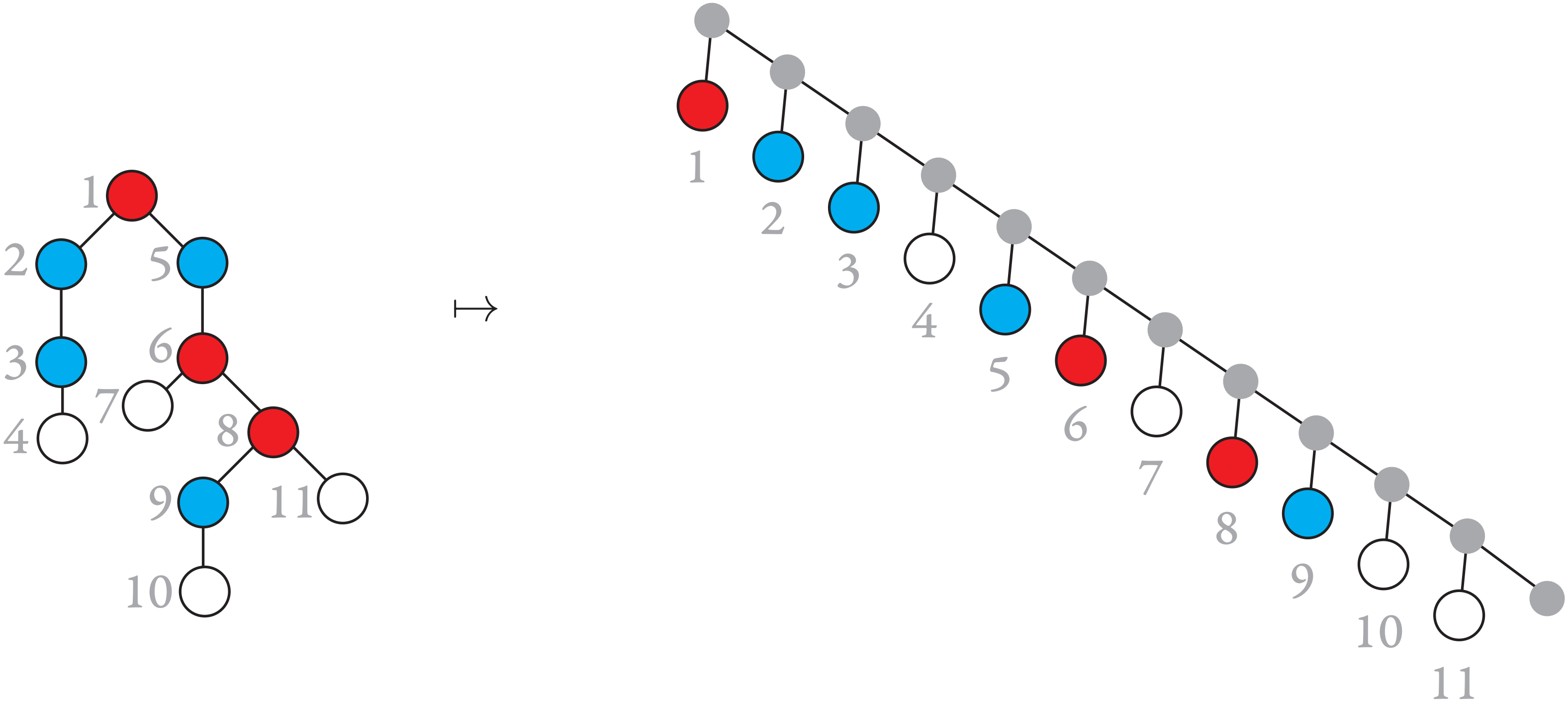

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0



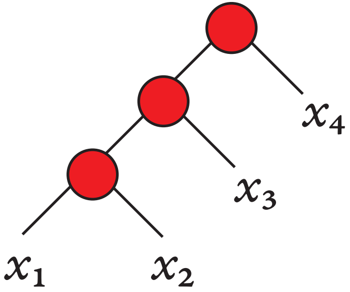
two copies of the input tree,
connected by a binary node
in the root



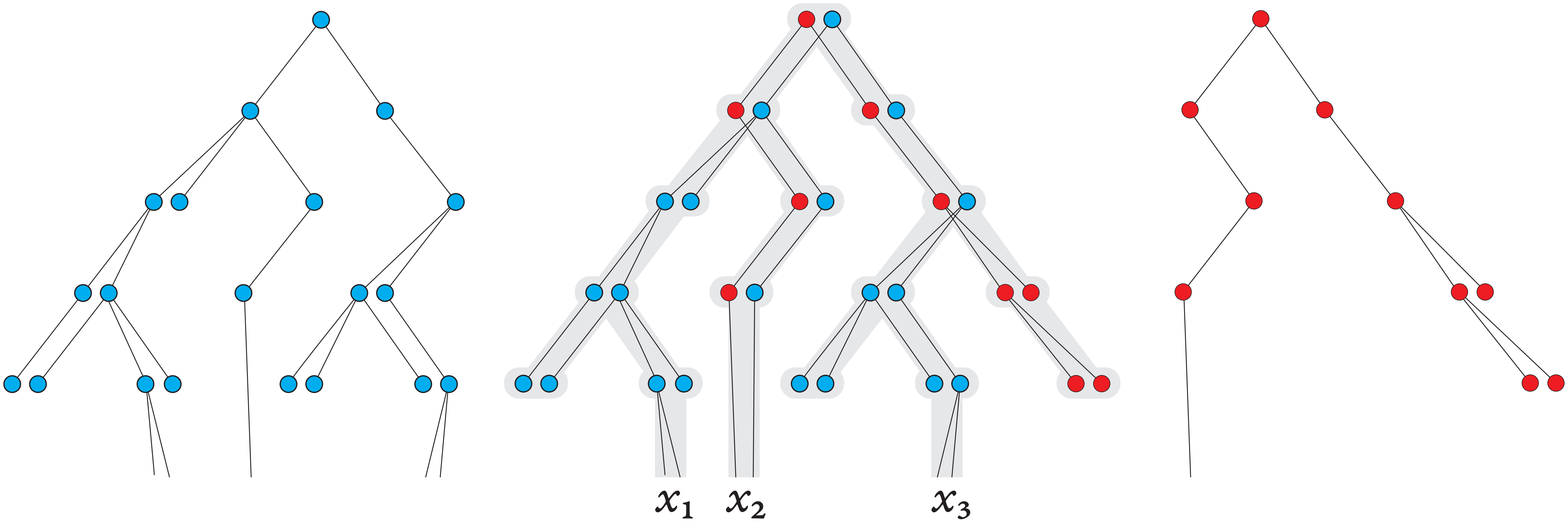

input alphabet
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 1
arity 0
arity 2
arity 0

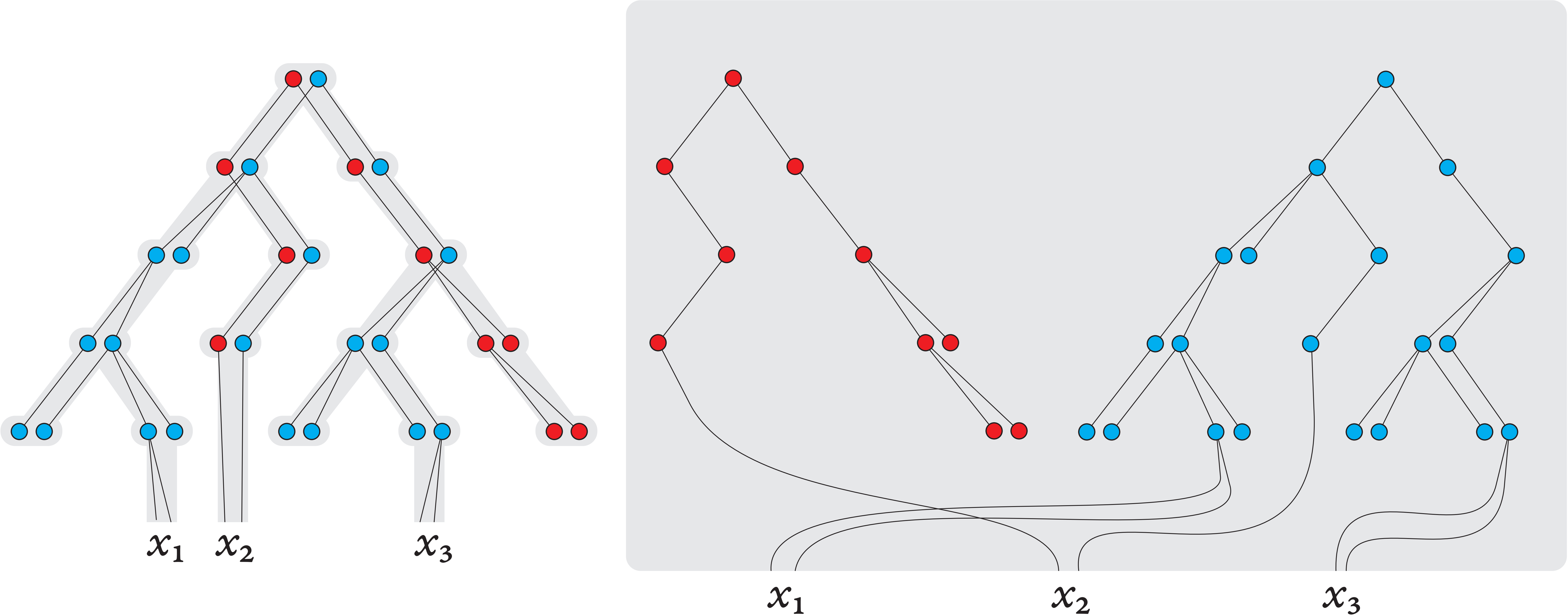

1
2
3
4
5
6
7
8
9
10
11
1
2
3
4
5
6
7
8
9
10
11

a term of arity 4

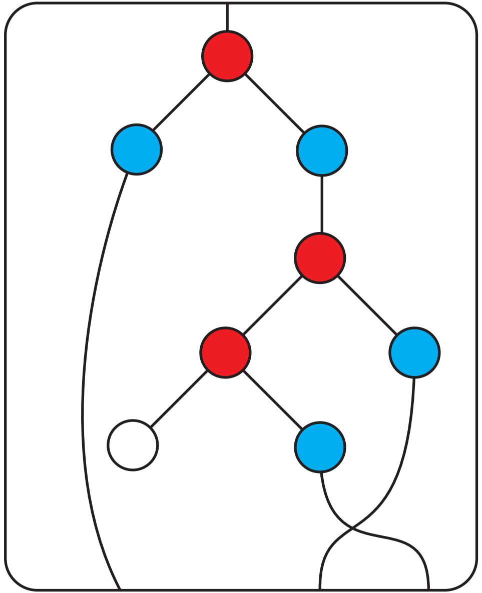


a term of arity 0


x_1
x_2
x_3



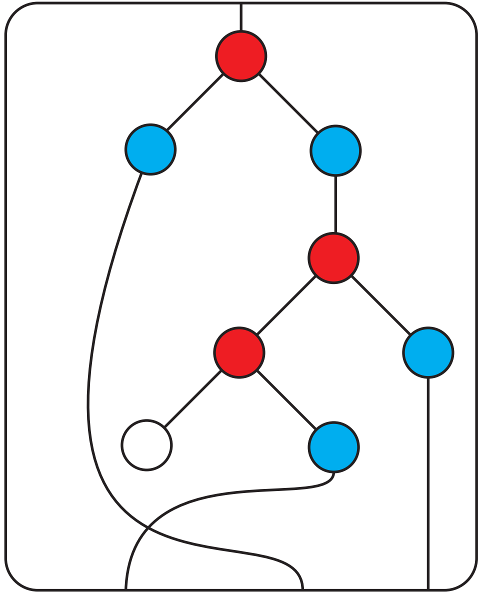
x_1 x_2 x_3
x_1 x_2 x_3



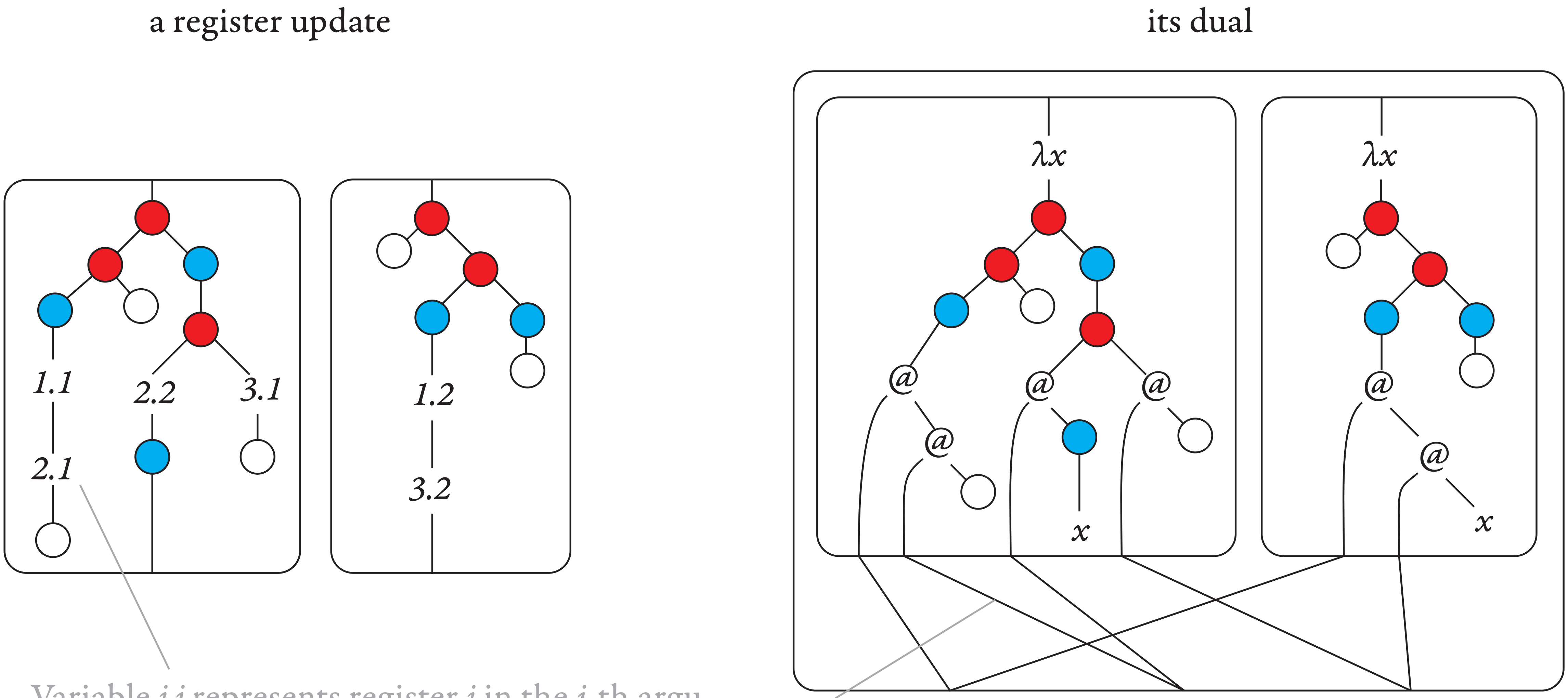
satisfies (*)

(*)

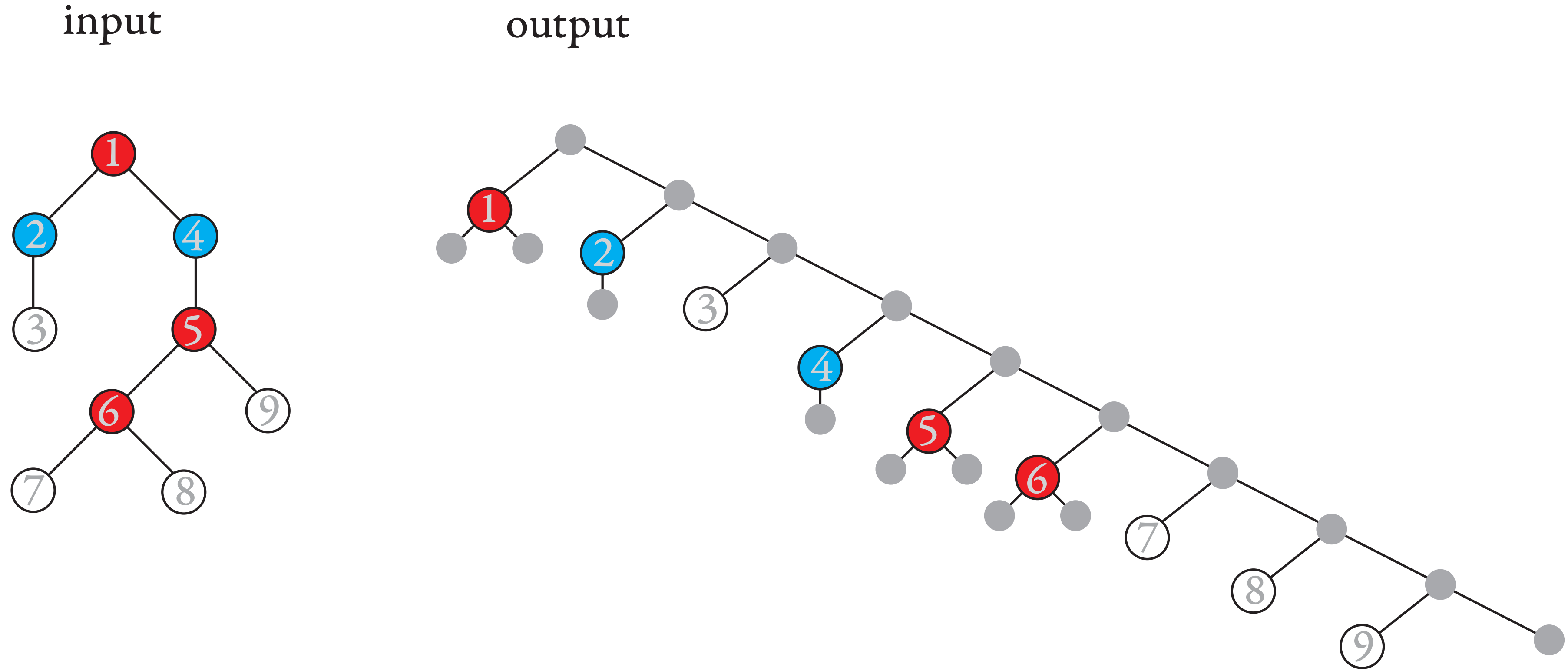
If the root has arity n, and $1 \le i < j \le n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root

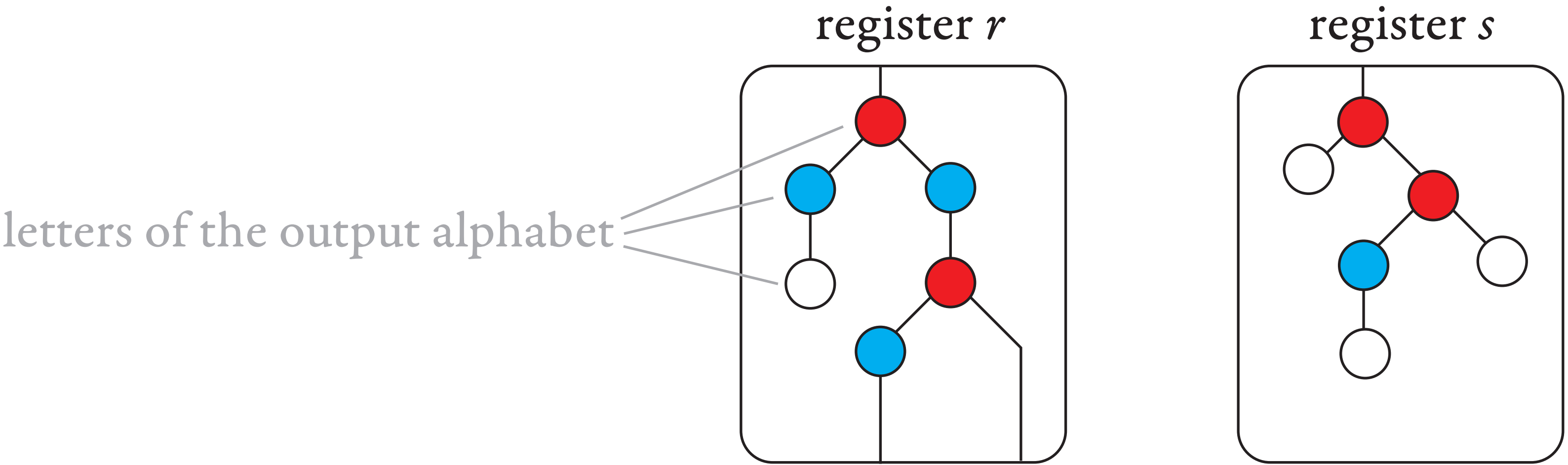


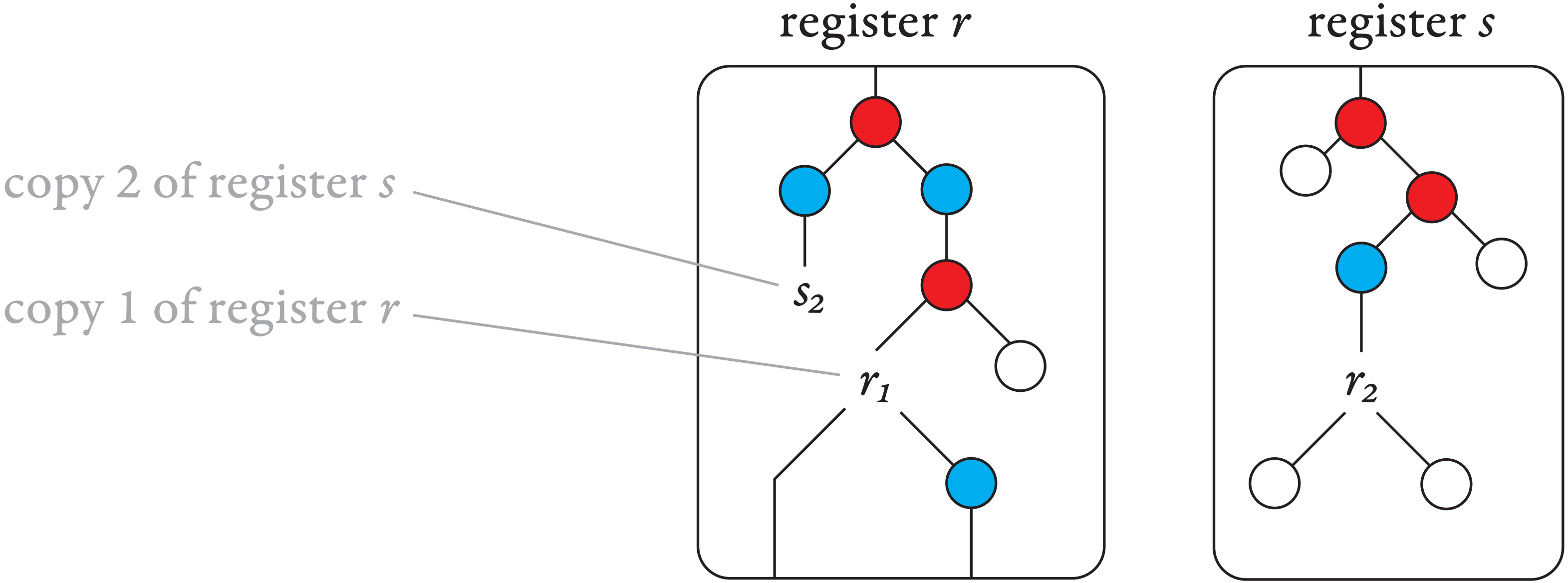
violates (*)



Variable $i.j$ represents register i in the j-th argument of the reigster update.
In the dual, this variable is mapped to the i-th edge which enters the j-th port of the reducer.

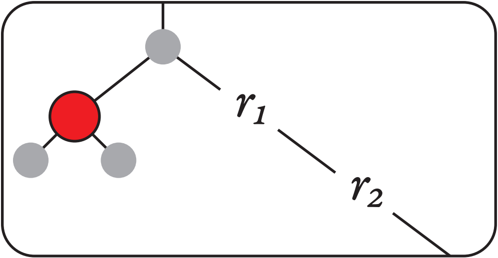

input
output
1
2
4
3
5
6
9
7
8
1
2
3
4
5
6
7
8
9


register r
register s
letters of the output alphabet

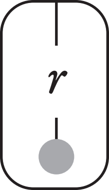


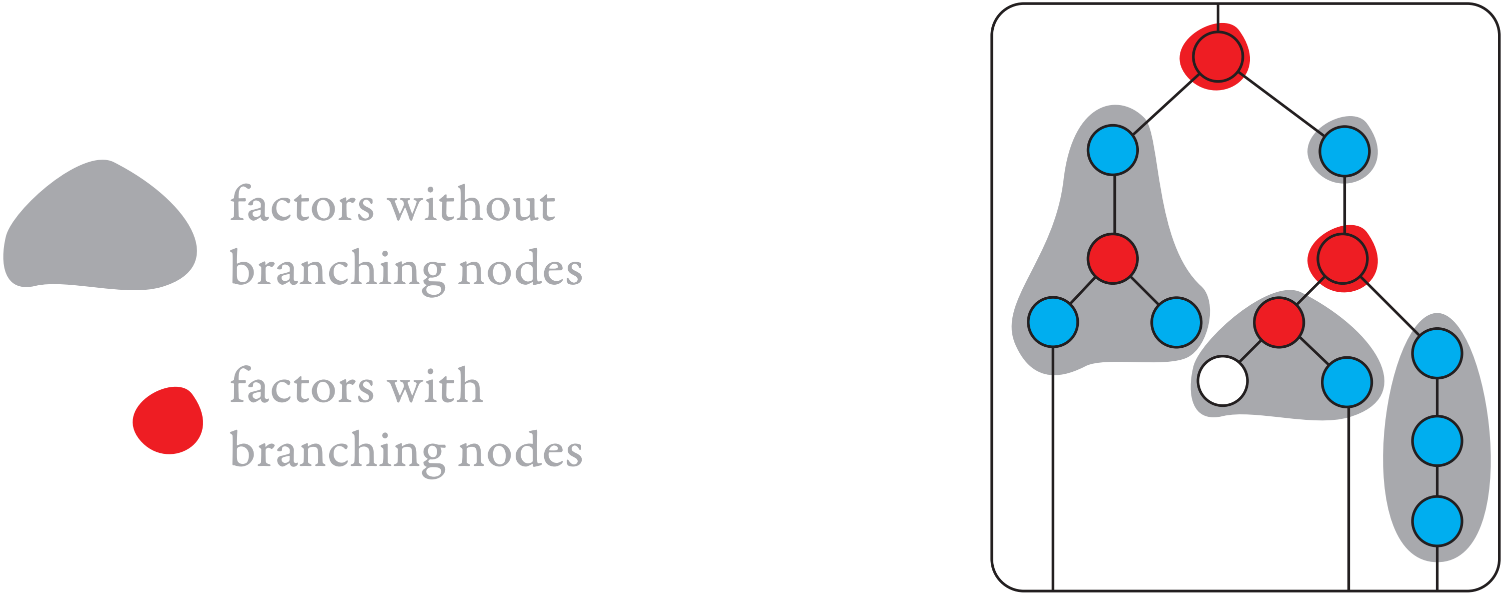
register r
register s
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2

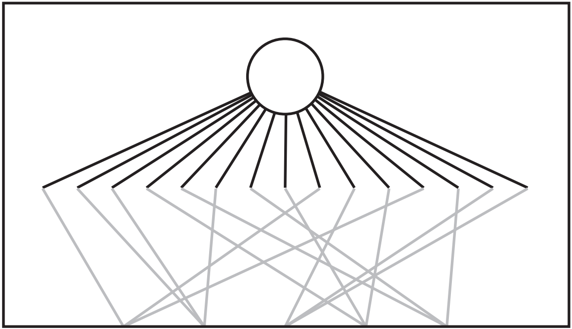

r
r
r
s

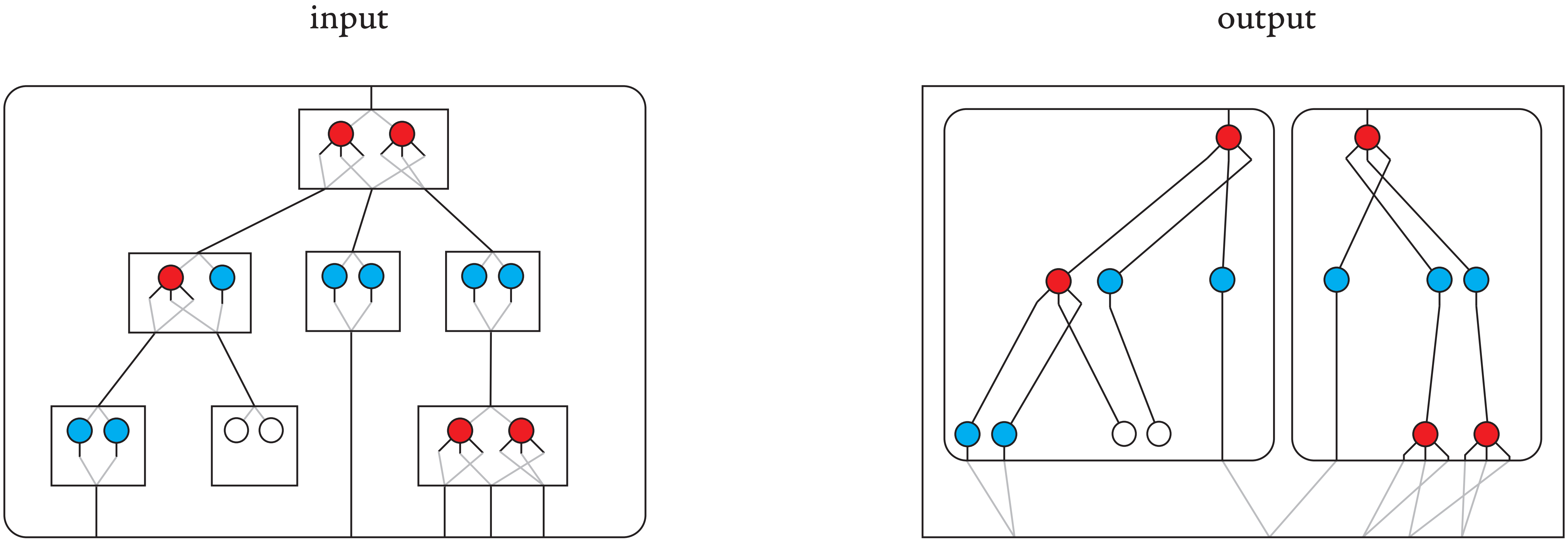

r_1
r_2

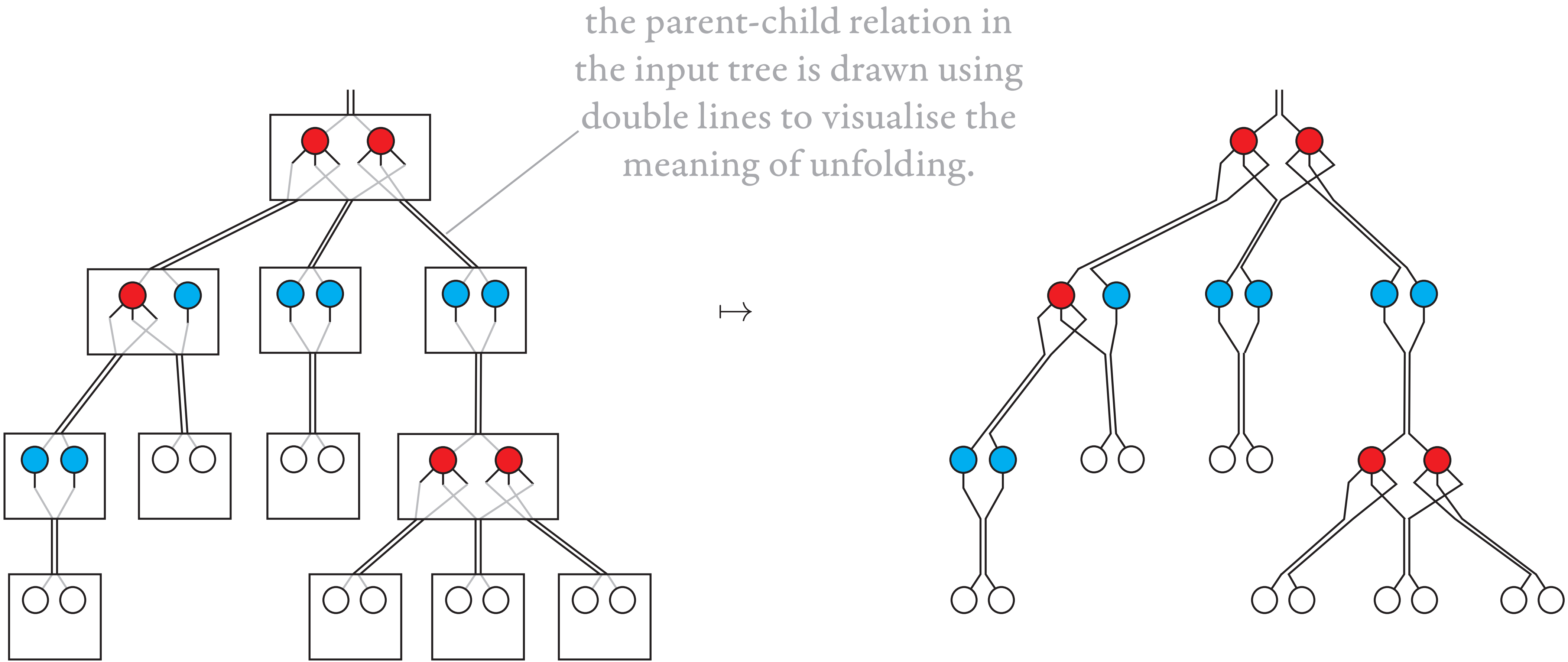

r
s
s
r
r
s

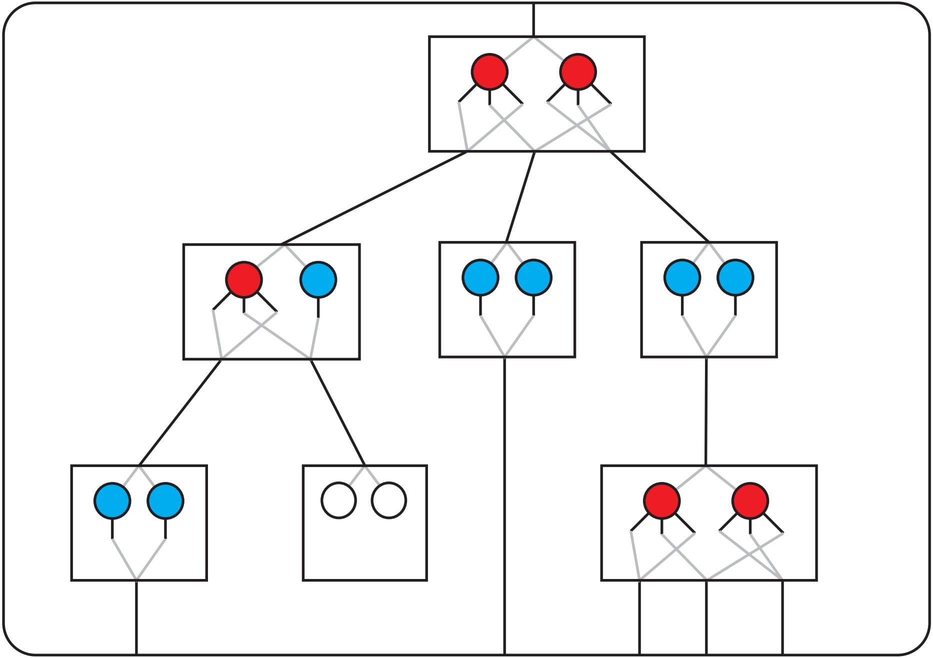

r

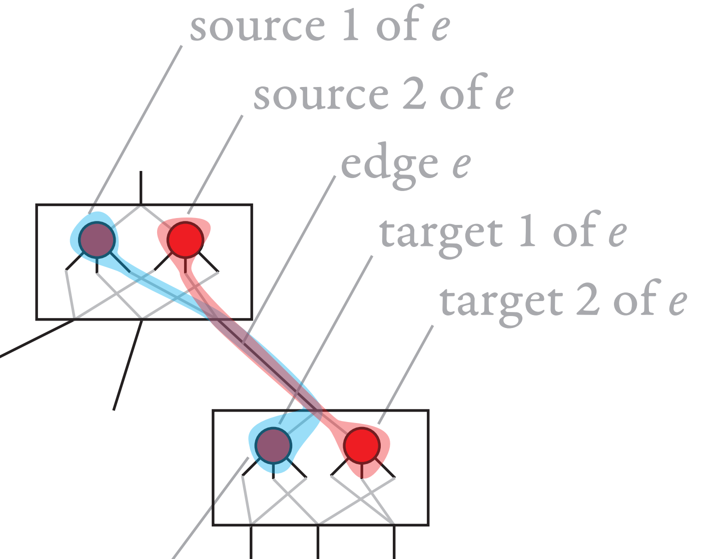

factors without
branching nodes
factors with
branching nodes



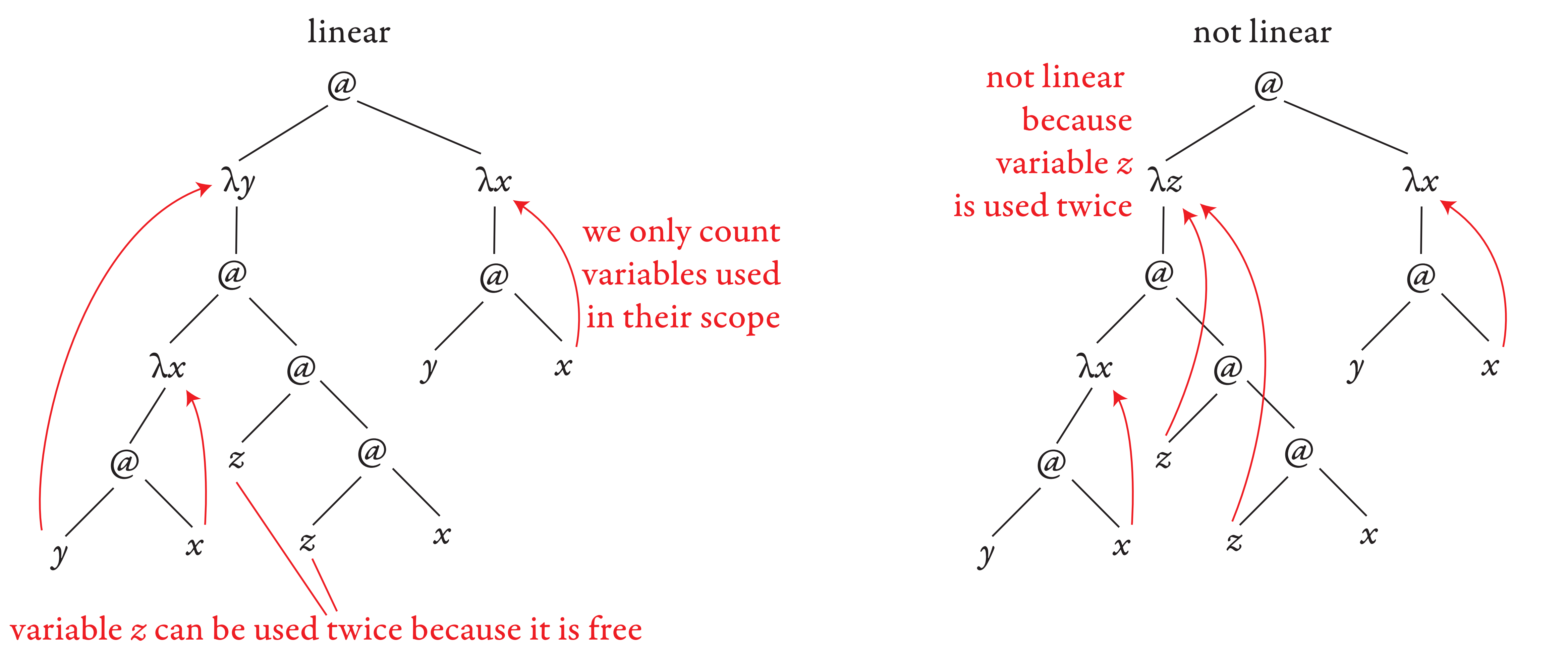

input
output


the parent-child relation in the input tree is drawn using double lines to visualise the meaning of unfolding.
↦

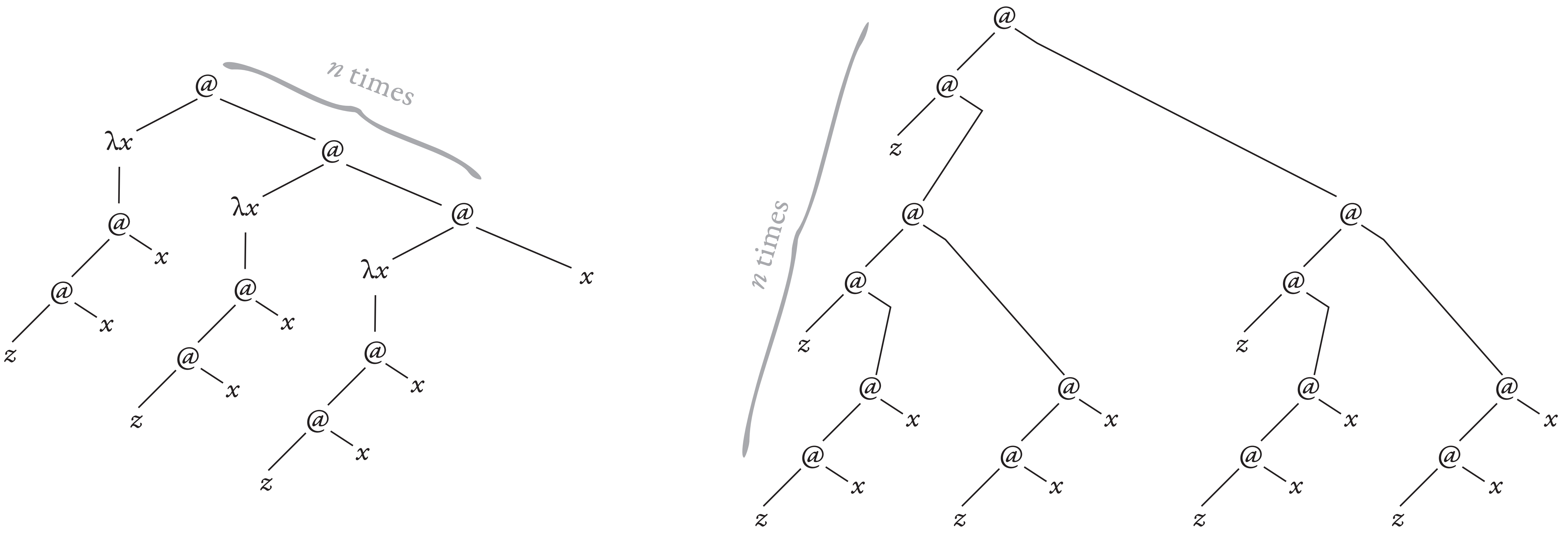


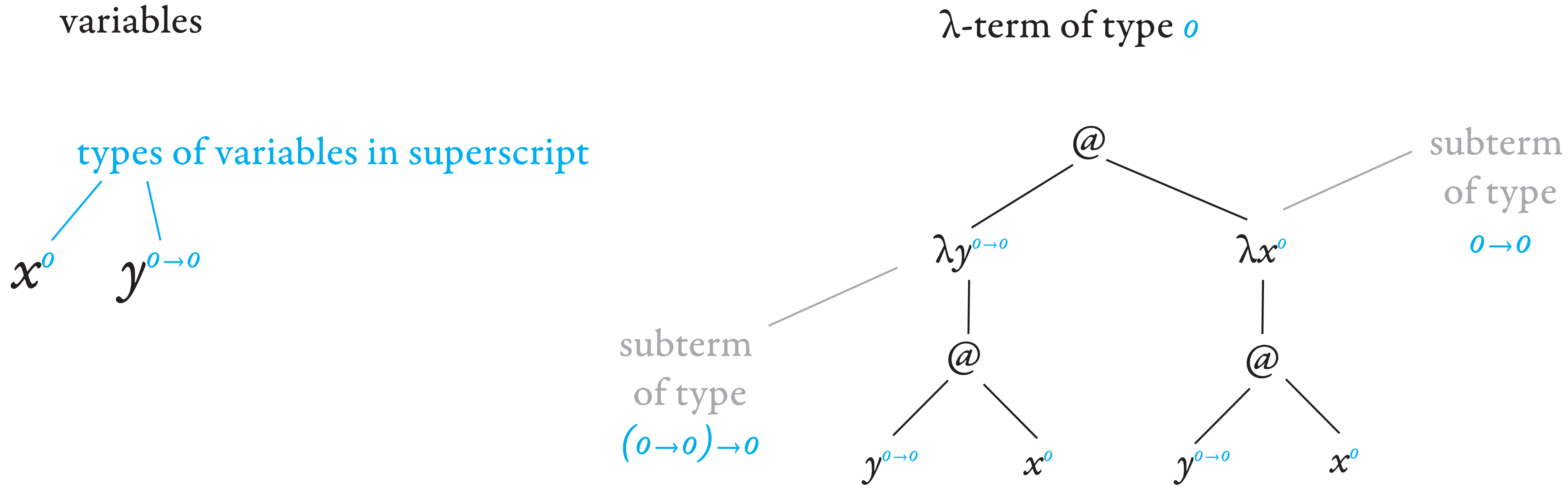


source 1 of e
source 2 of e
edge e
target 1 of e
target 2 of e


linear
not linear
we only count variables used in their scope
variable z can be used twice because it is free
not linear because variable z is used twice

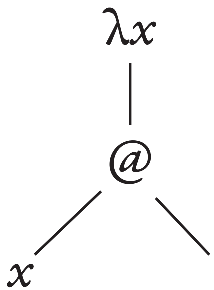
a λ-term of size $O(n)$ its normal form of size $O(2^n)$



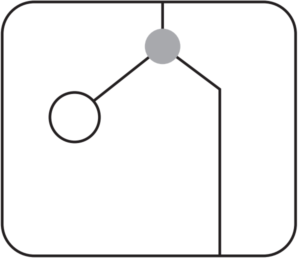


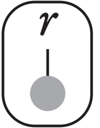
variables
types of variables in superscript
x^{o}
$y^{o\to o}$
λ-term of type o
@
$\lambda y^{o\to o}$
λx^{o}
@
@
$y^{o\to o}$
x^{o}
$y^{o\to o}$
x^{o}
subterm of type $(o\to o)\to o$
subterm of type $o\to o$

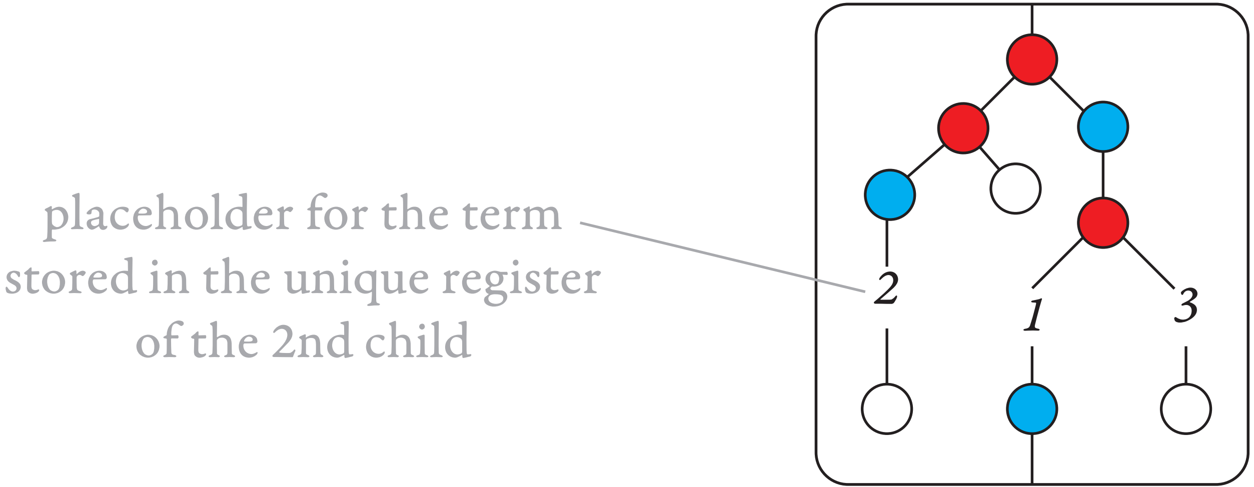
@

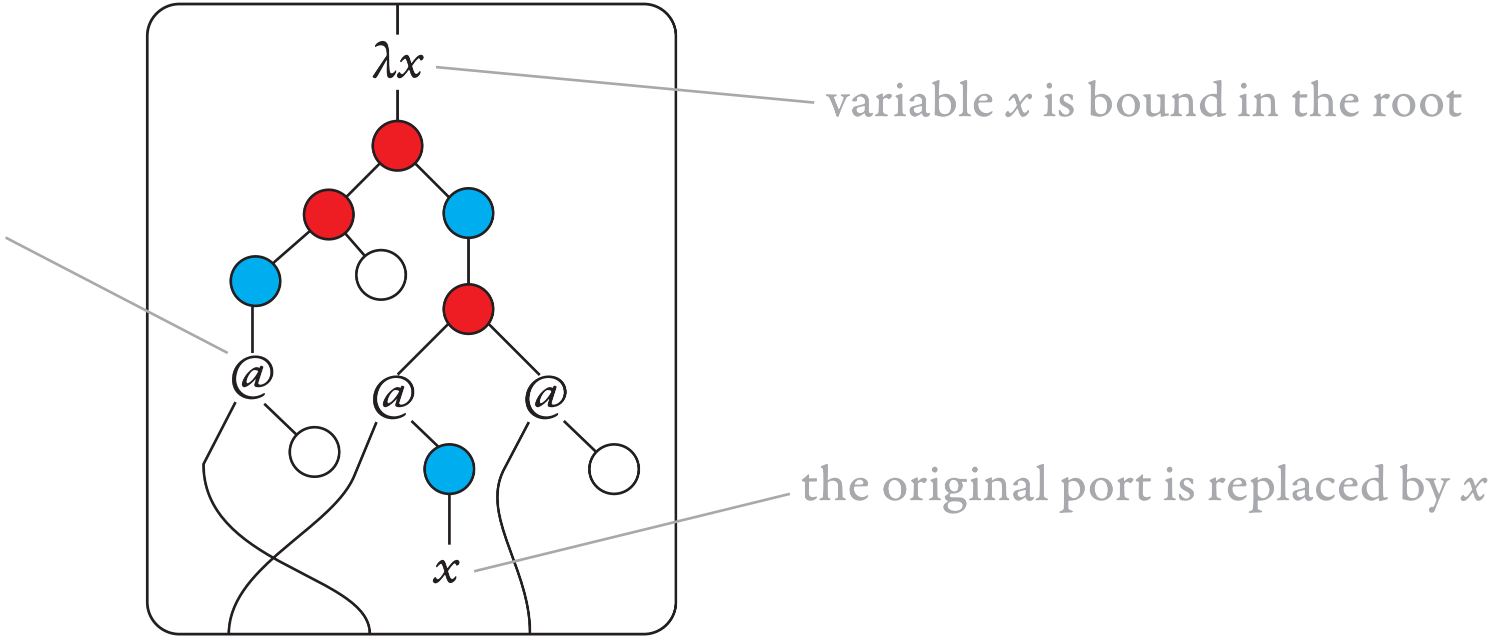


$\lambda x.$

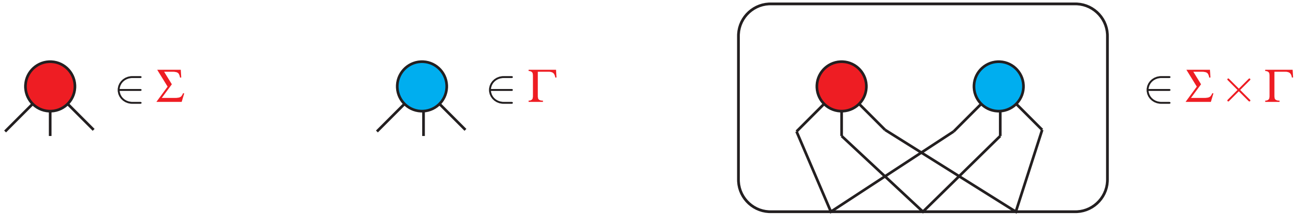


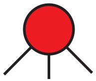

r

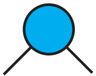

placeholder for the term
stored in the unique register
of the 2nd child
2
1
3

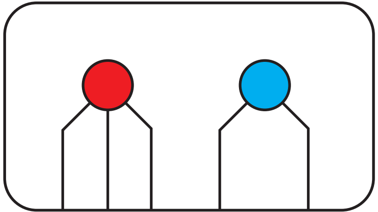


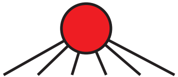
λx
variable x is bound in the root
@
@
@
the original port is replaced by x
x


$\in \Sigma$
$\in \Gamma$
$\in \Sigma \times \Gamma$

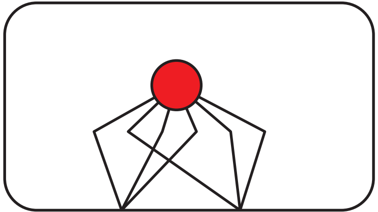
 $\in \Sigma$

 $\in \Gamma$

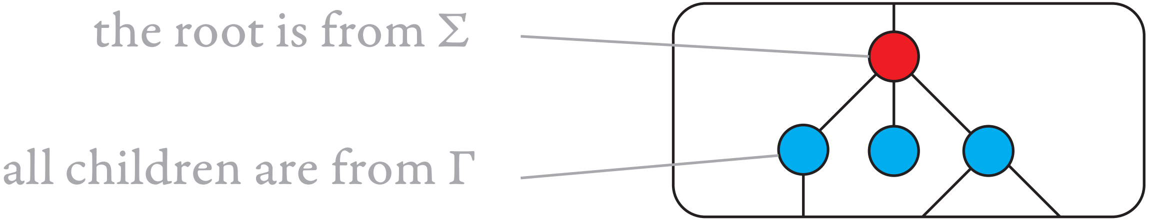
 $\in \Sigma \otimes \Gamma$



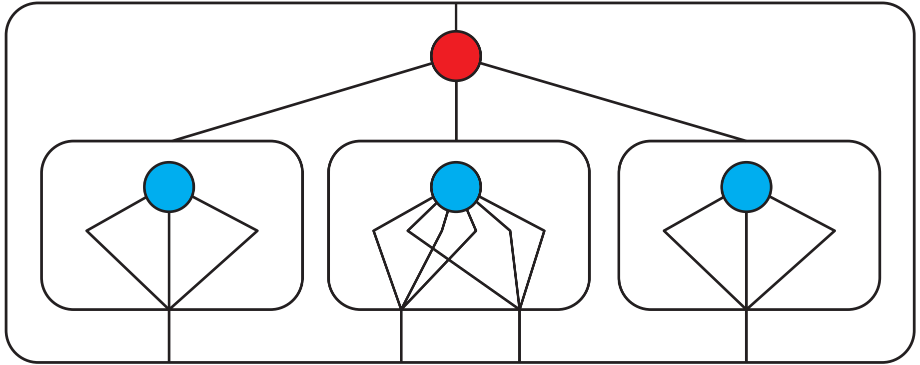
$\in \Sigma$



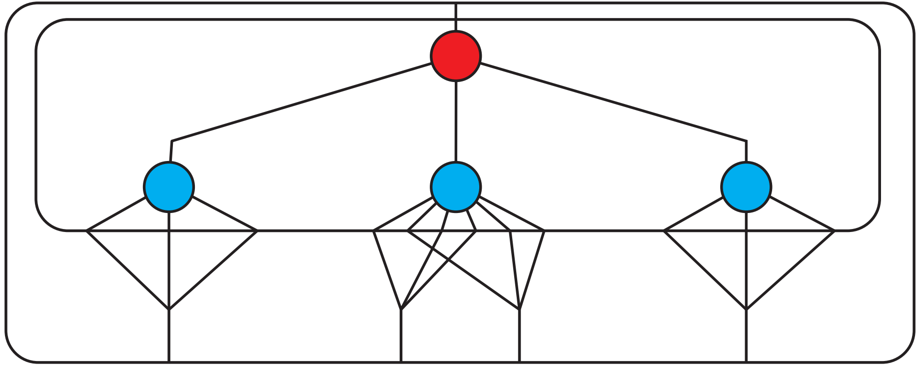
$\in F_3\Sigma$

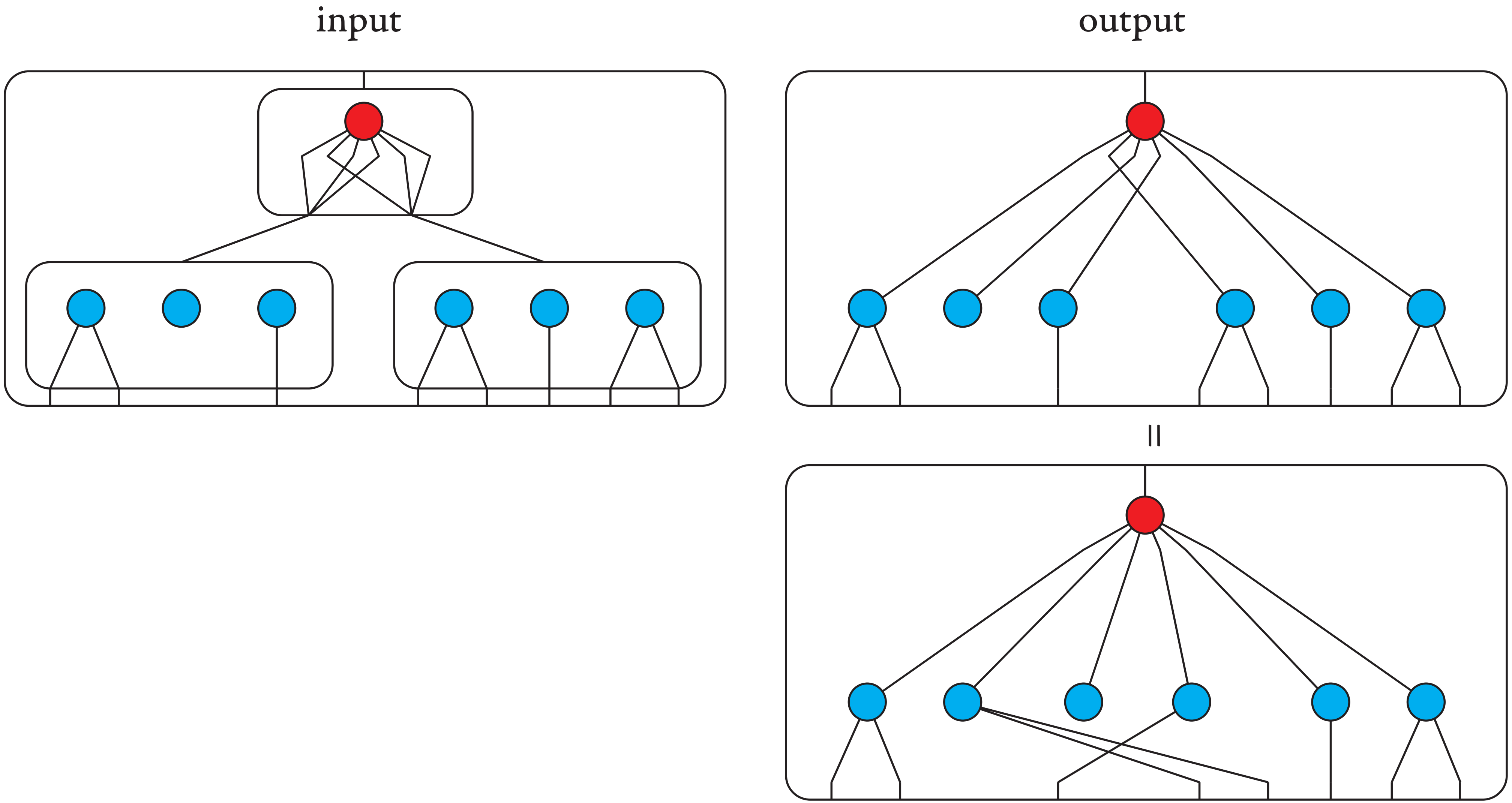

the root is from Σ
all children are from Γ

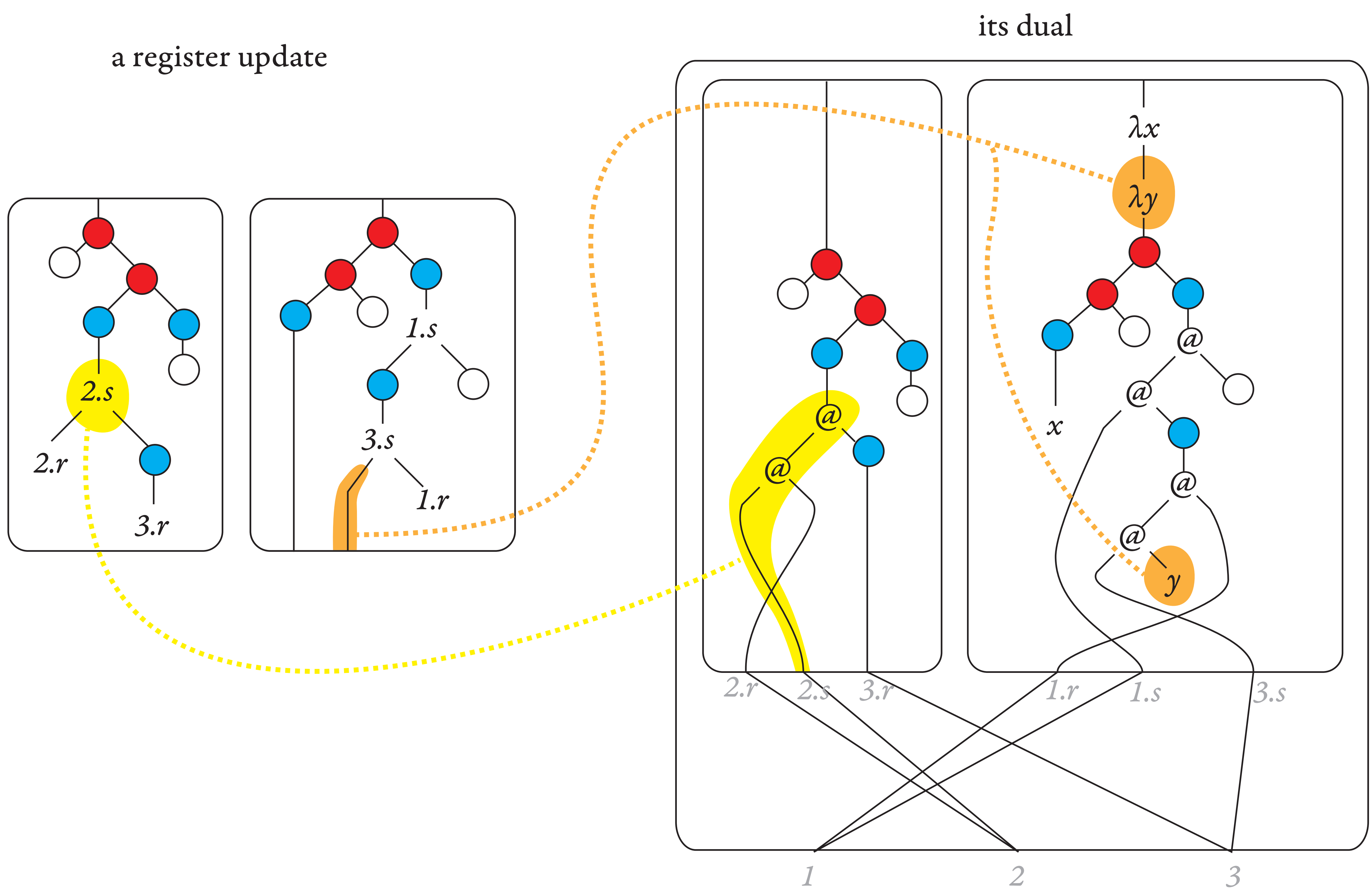
input



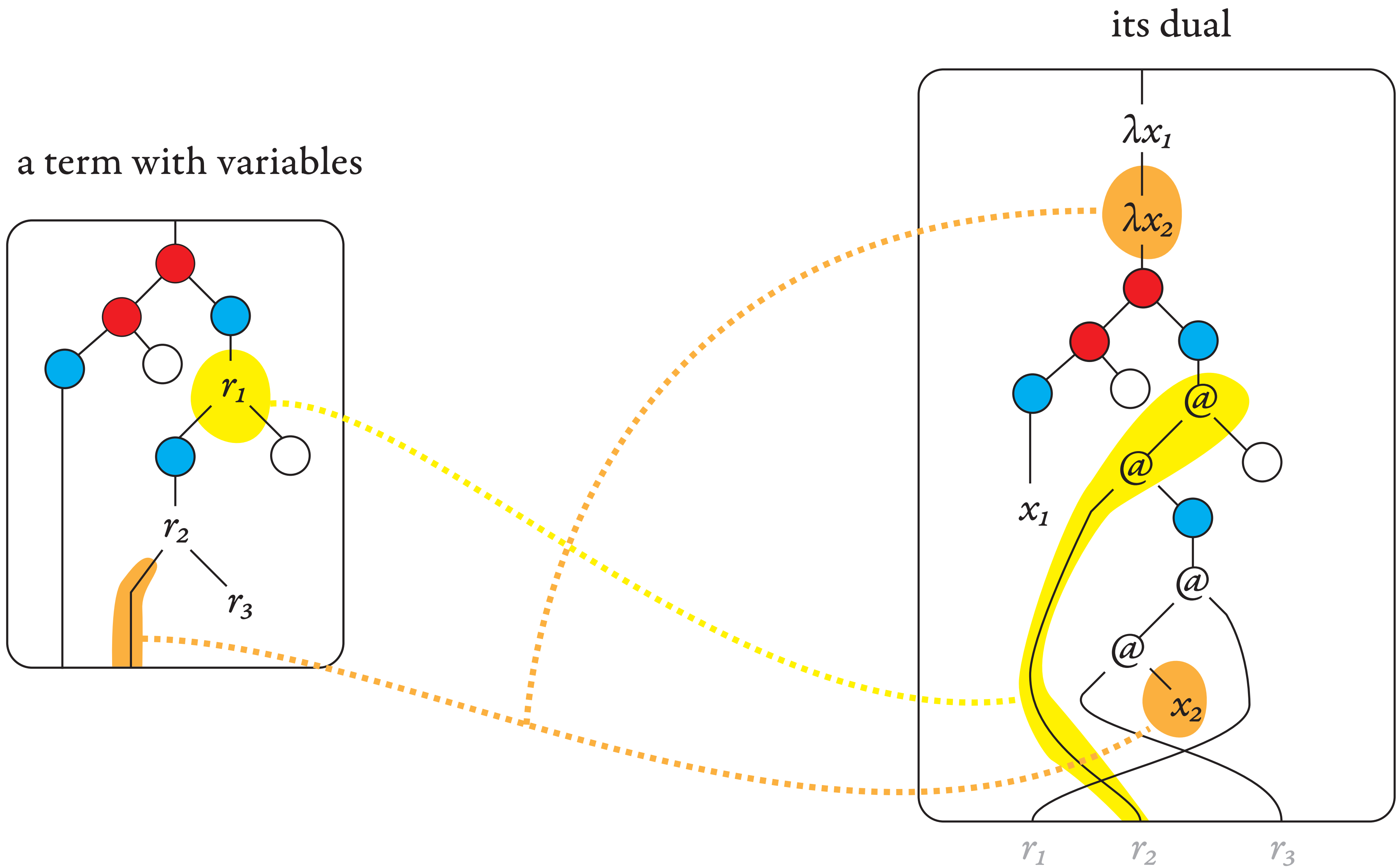
output



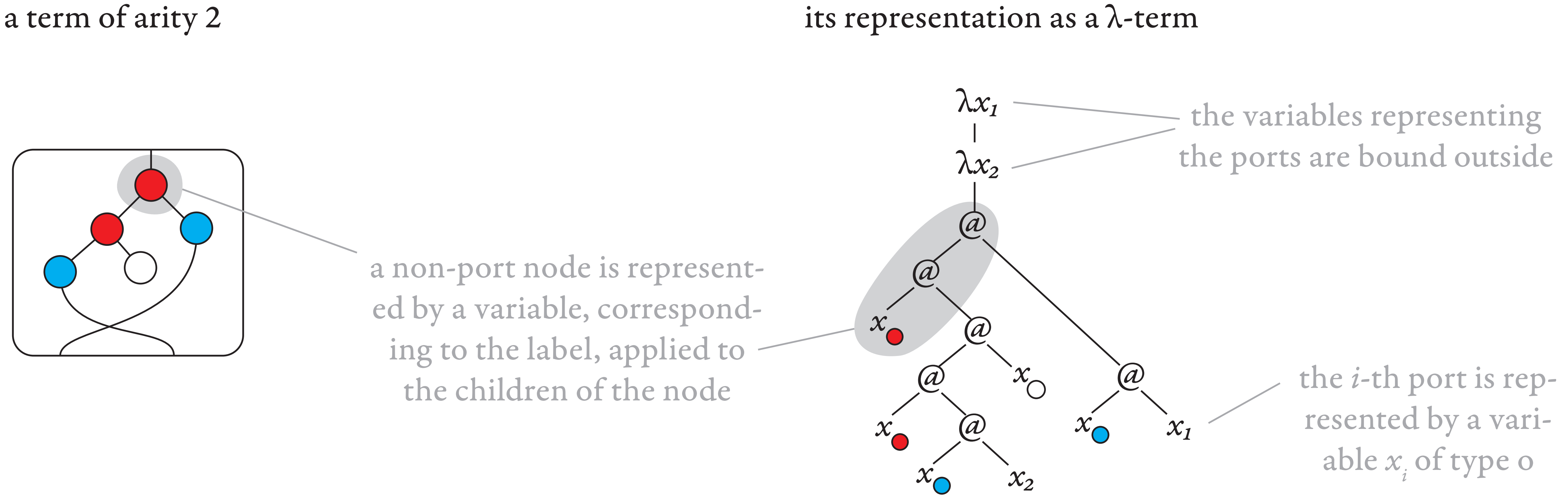

input
output
=



a register update
its dual
2.s
2.r
3.r
1.s
3.s
1.r
λx
λy
x
@
y
2.r
2.s
3.r
1.r
1.s
3.s
1
2
3



a term with variables
its dual
λx_1
λx_2
r_1
r_2
r_3
x_1
x_2
@
@
@
@
r_1
r_2
r_3


a term of arity 2
its representation as a λ-term
λx_1
λx_2
the variables representing the ports are bound outside
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
@
x
x_1
x_2
the i-th port is represented by a variable x_i of type o