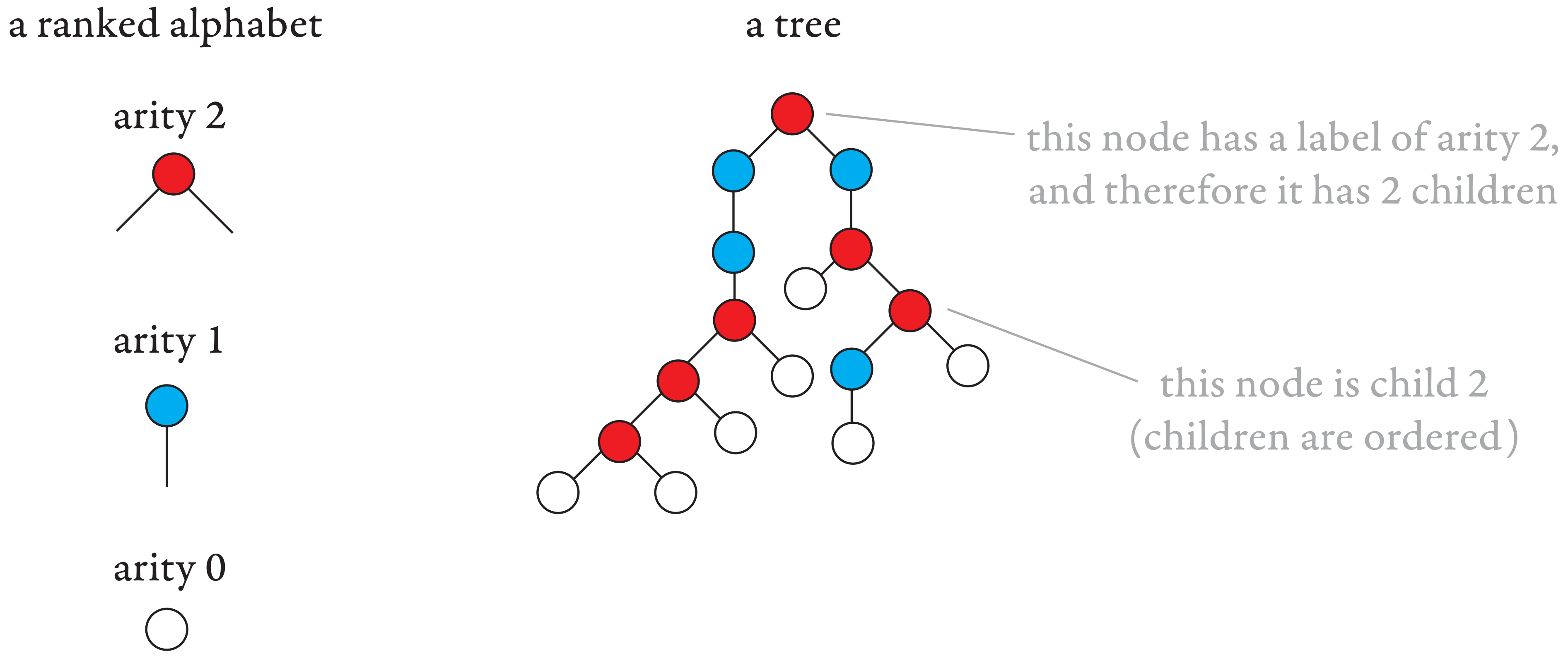
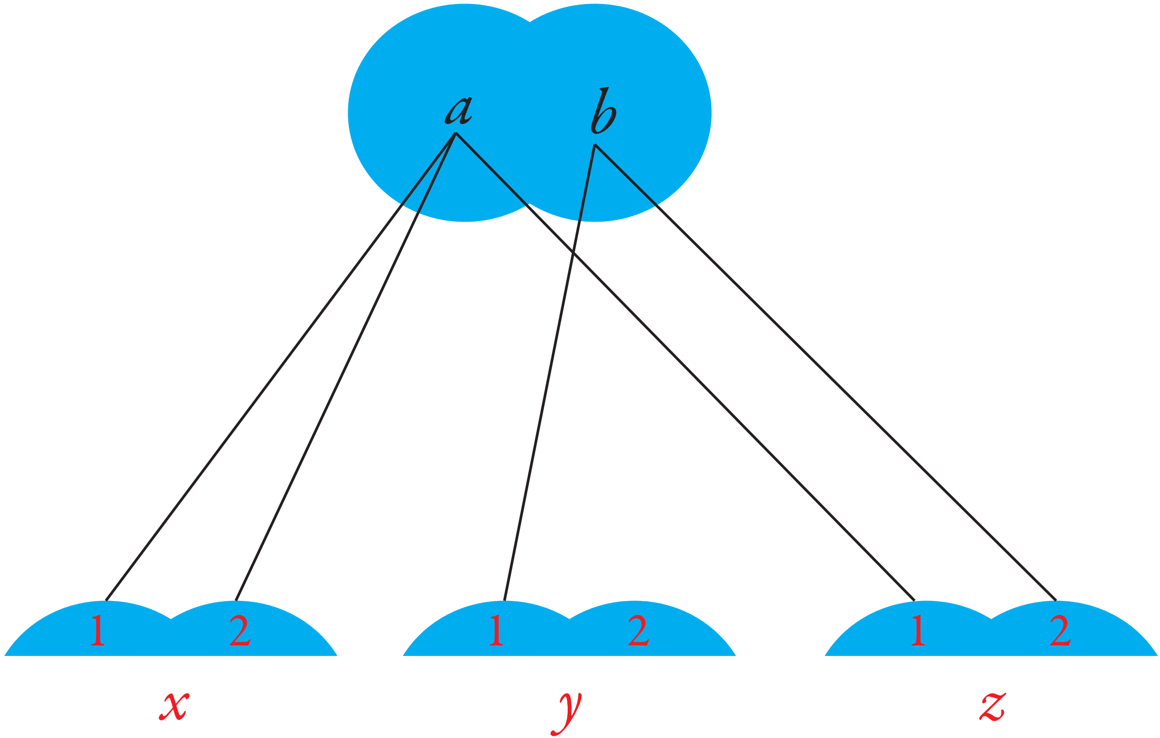
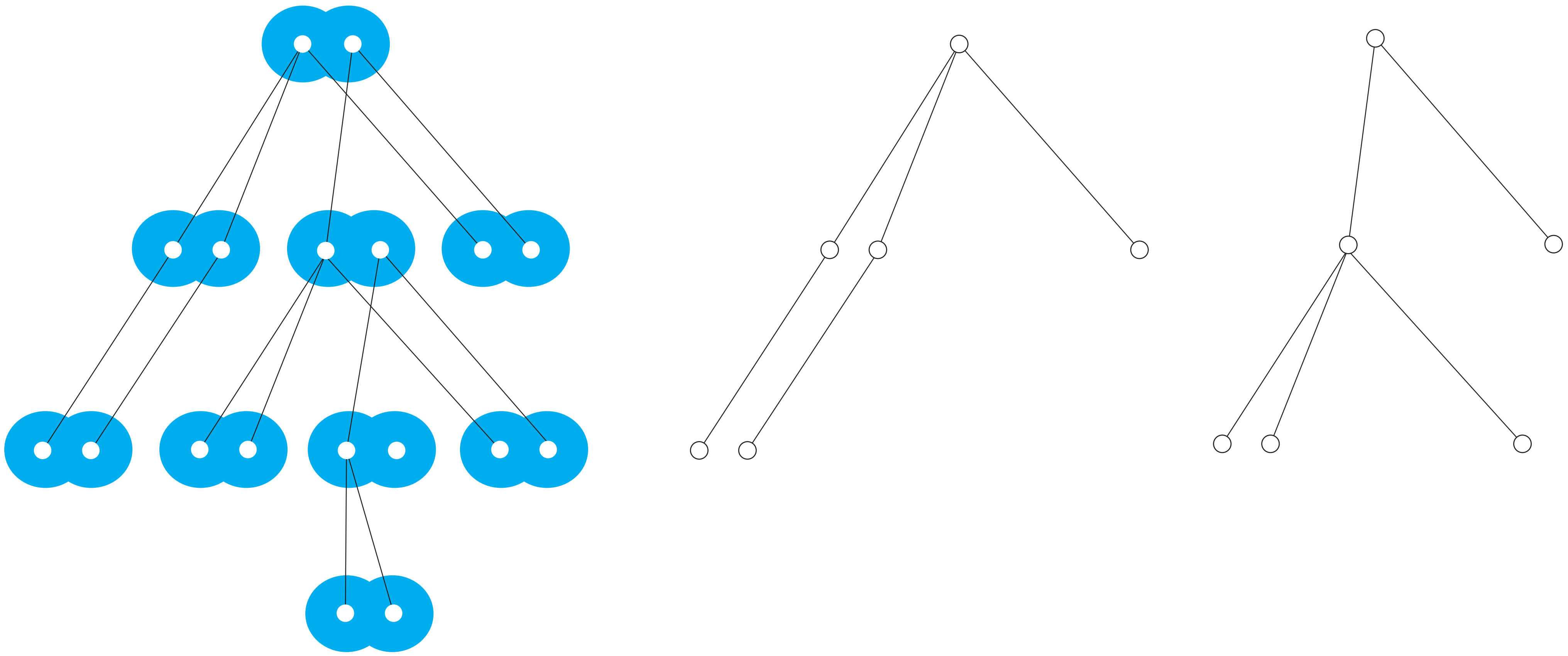
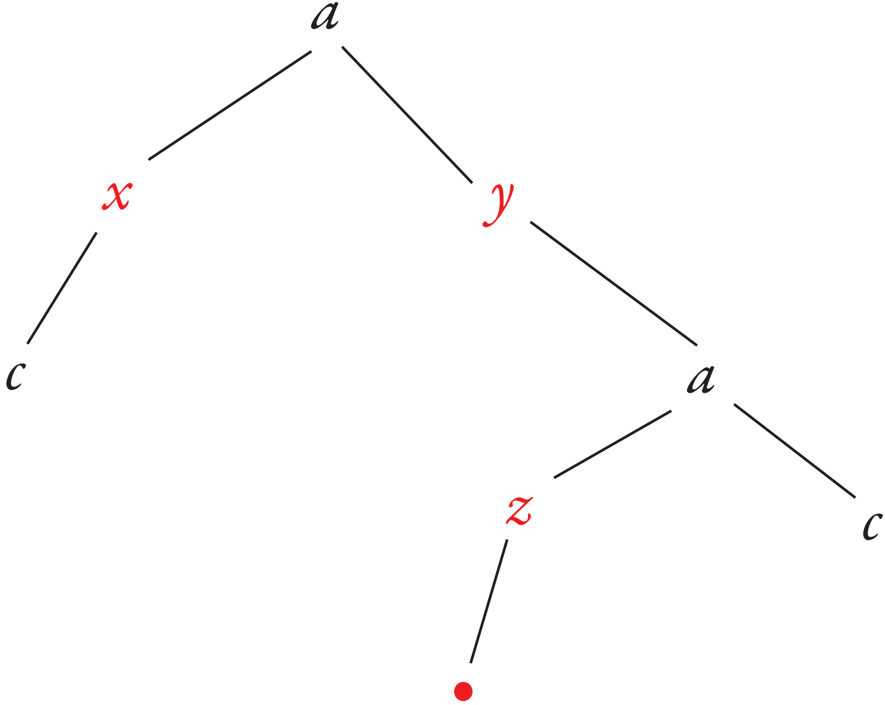
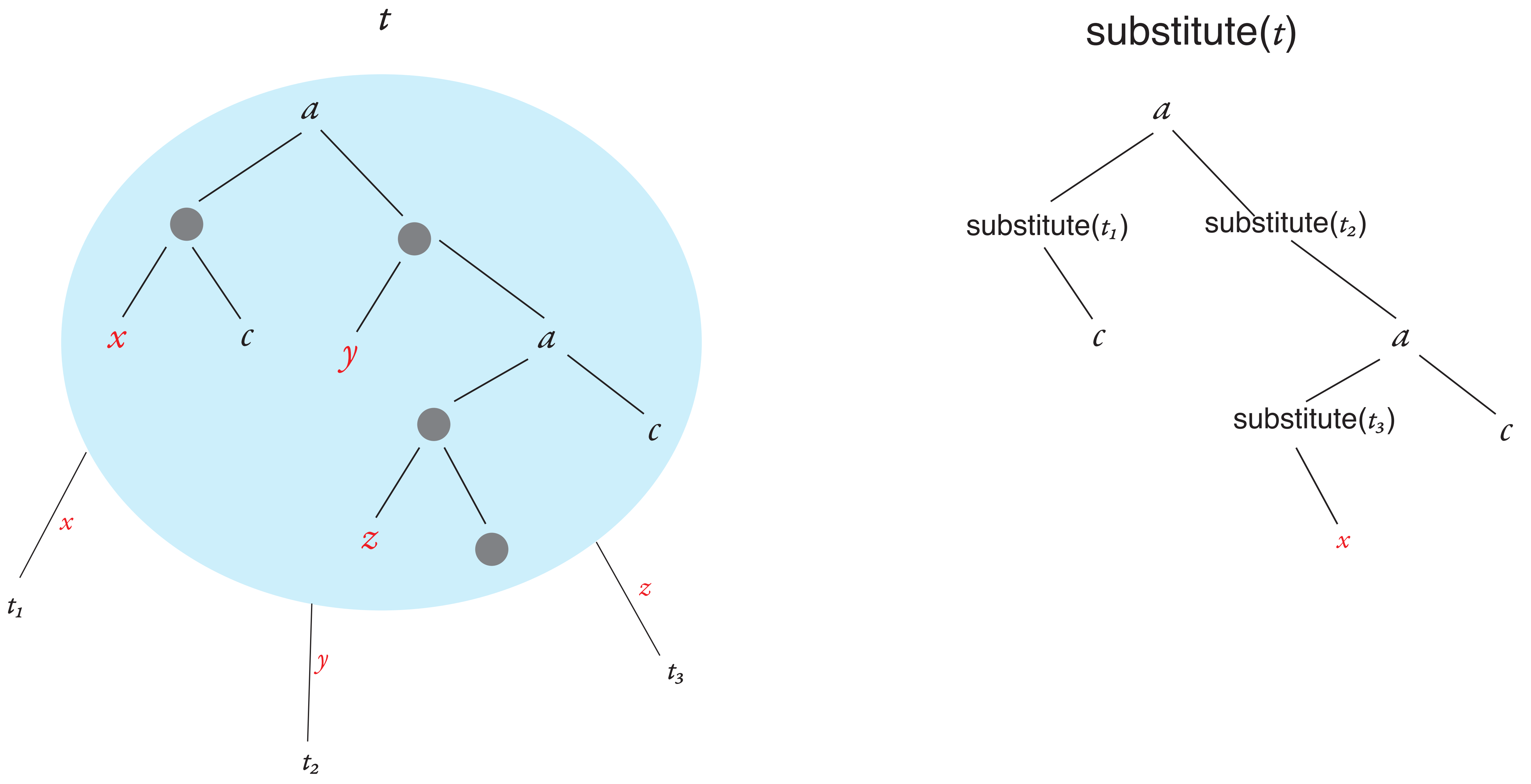

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)


a
b
1
2
1
2
1
2
x
y
z

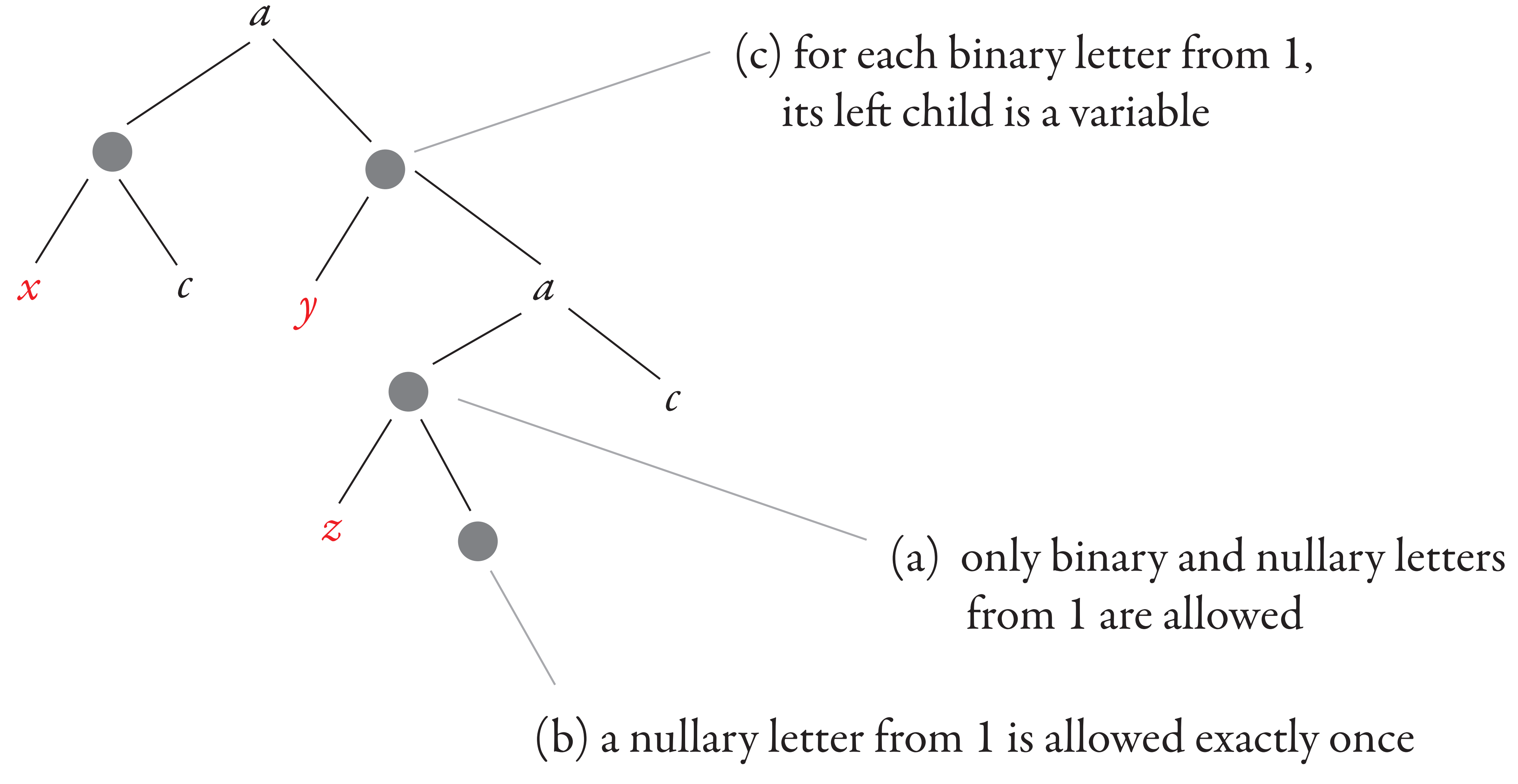
A tree t over $\Sigma^{[2]}$ $\qquad$ $\mathsf{unfold}_1(t)$ $\qquad$ $\mathsf{unfold}_2(t)$

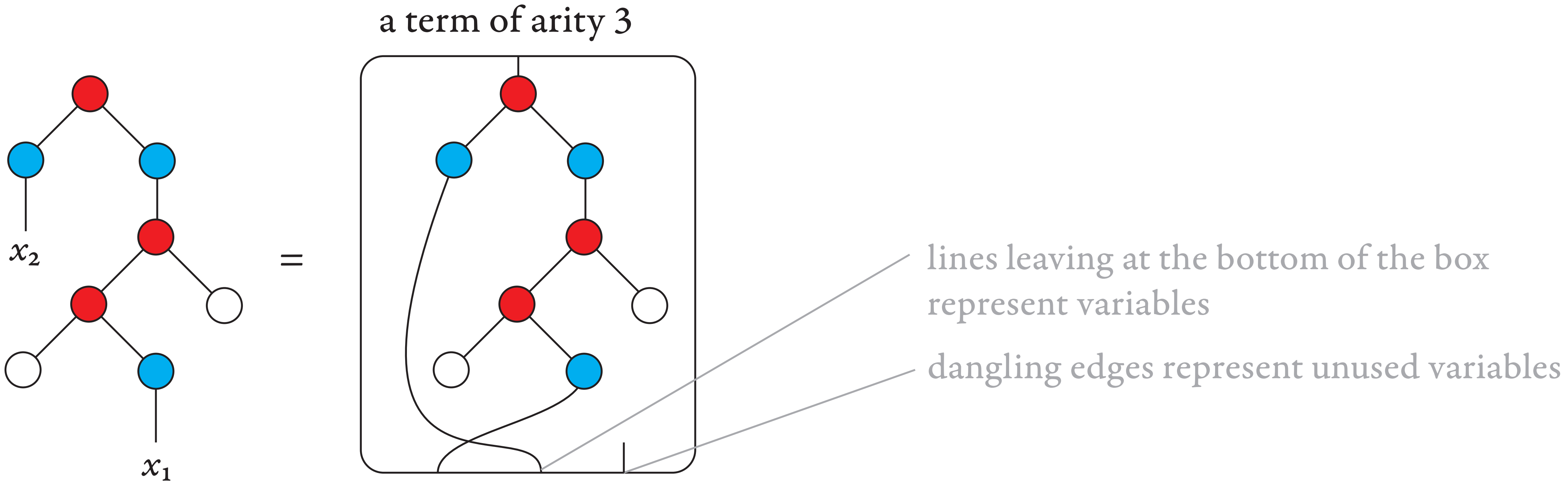


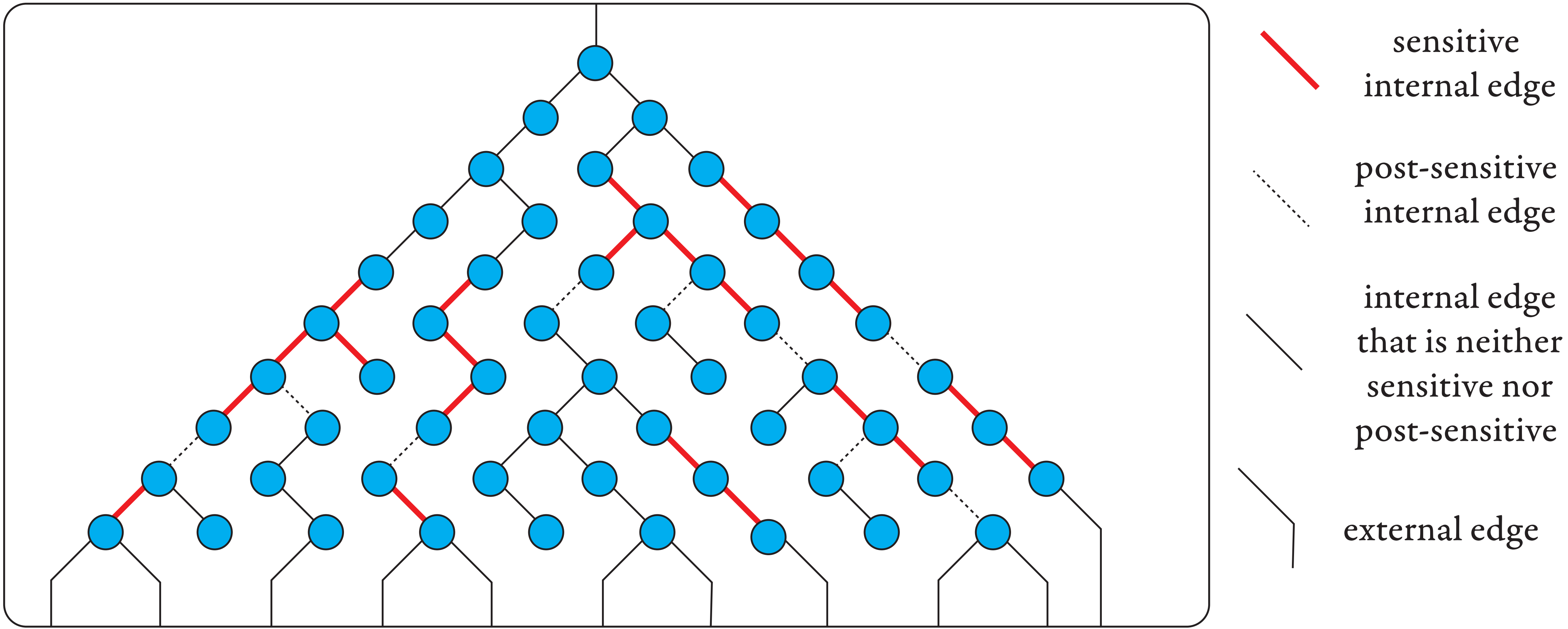

a
x
y
c
a
z
c

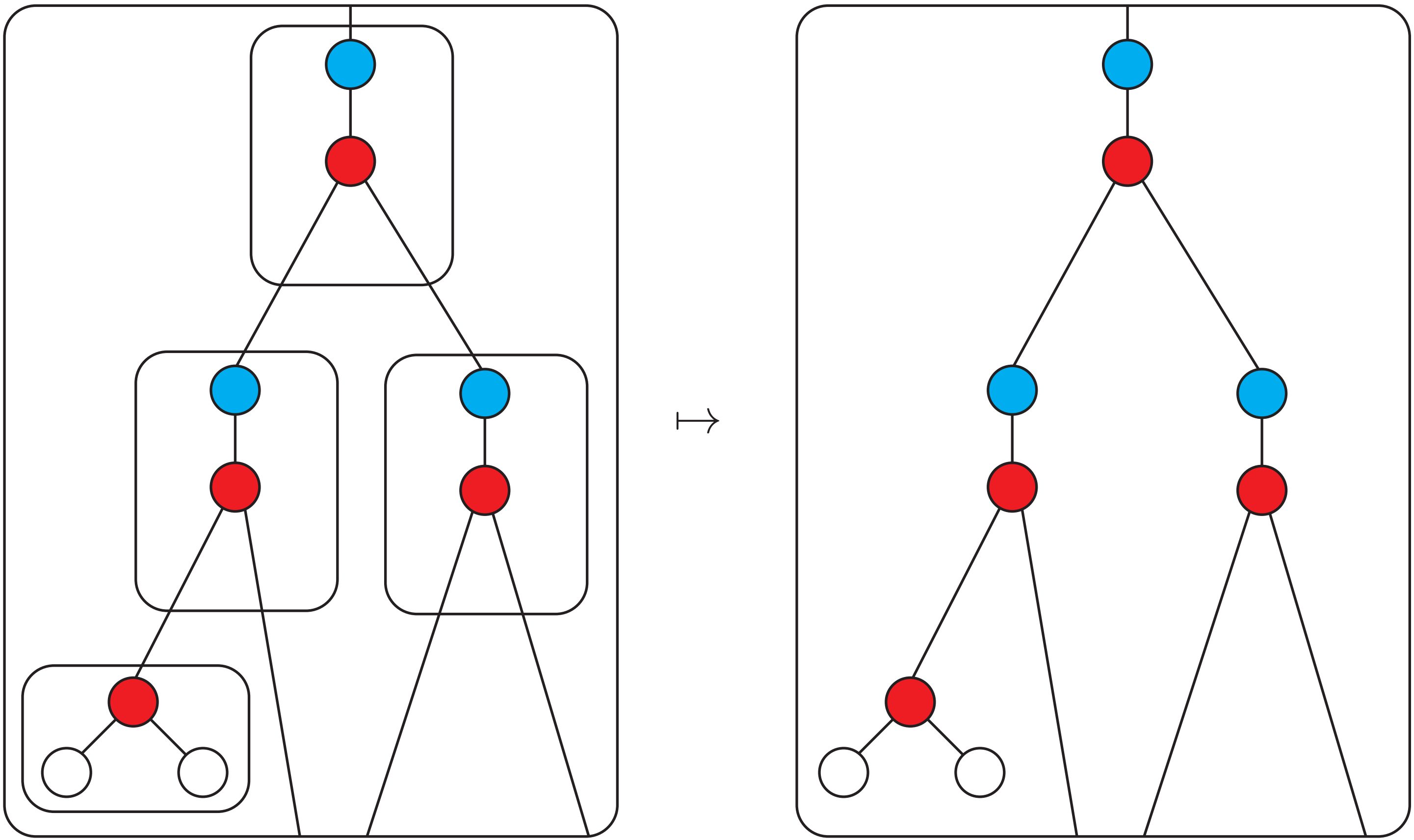


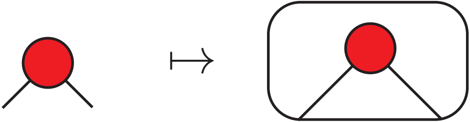
t
a
x
c
y
a
z
c
x
t1
y
t2
z
t3
substitute(t)
a
substitute(t1)
substitute(t2)
c
a
substitute(t3)
c
x


a
x
c
y
a
c
z
(c) for each binary letter from 1, its left child is a variable
(a) only binary and nullary letters from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

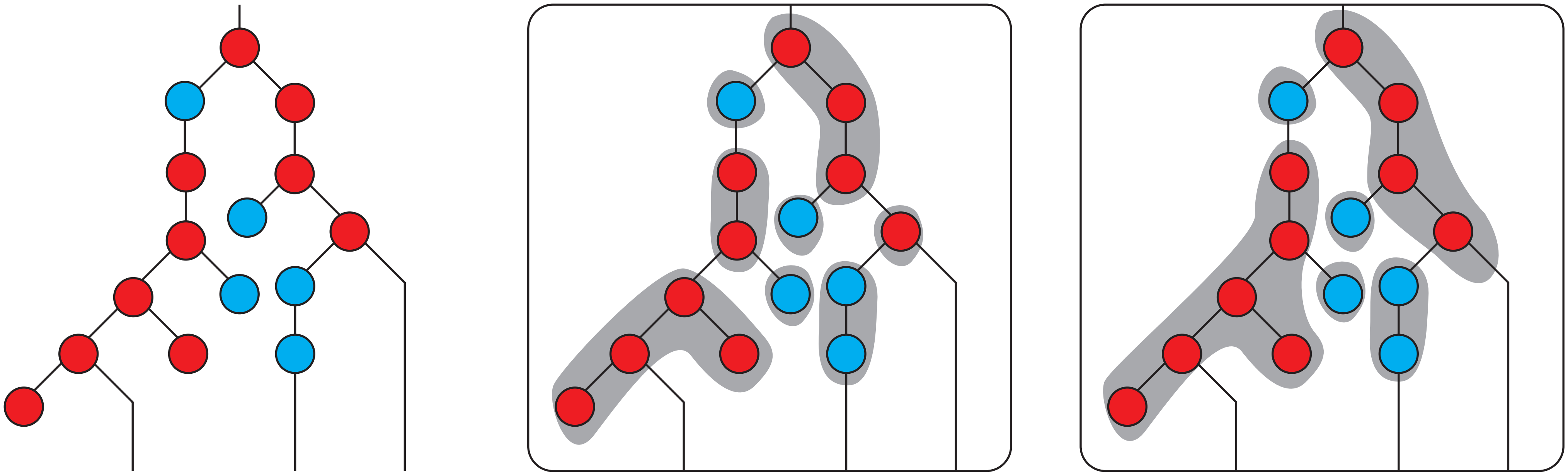

a term of arity 3
x_2
x_1
$=$
lines leaving at the bottom of the box represent variables
dangling edges represent unused variables

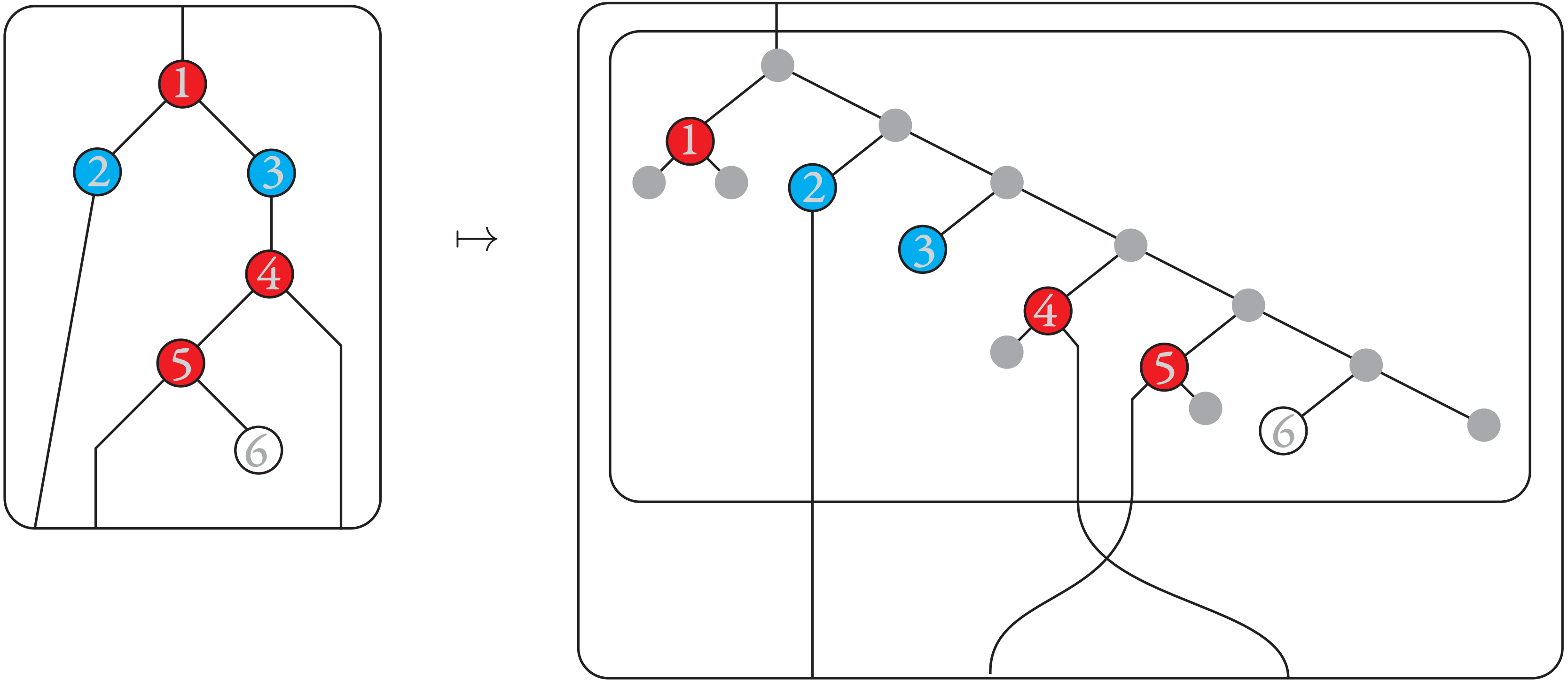

sensitive internal edge
post-sensitive internal edge
internal edge that is neither sensitive nor post-sensitive
external edge





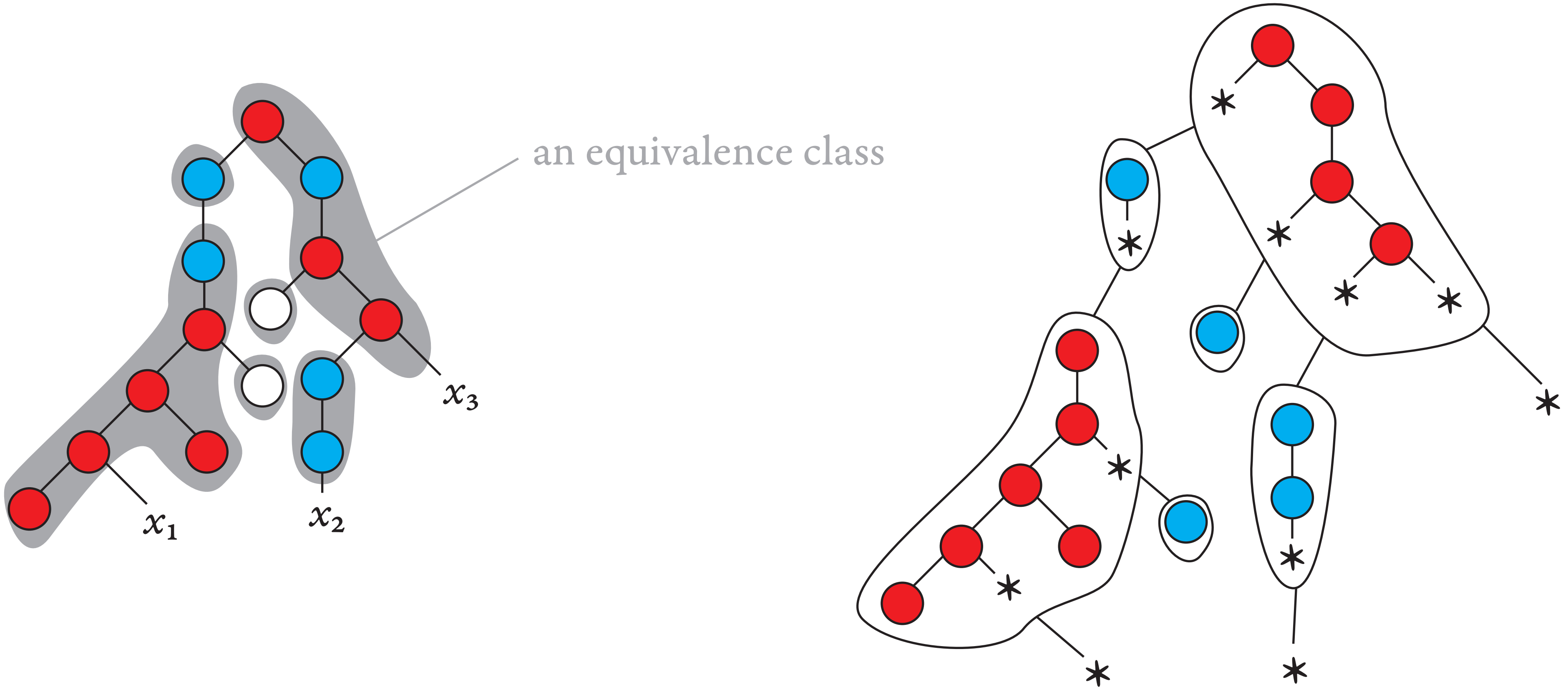
a term ancestor equivalence descendant equivalence

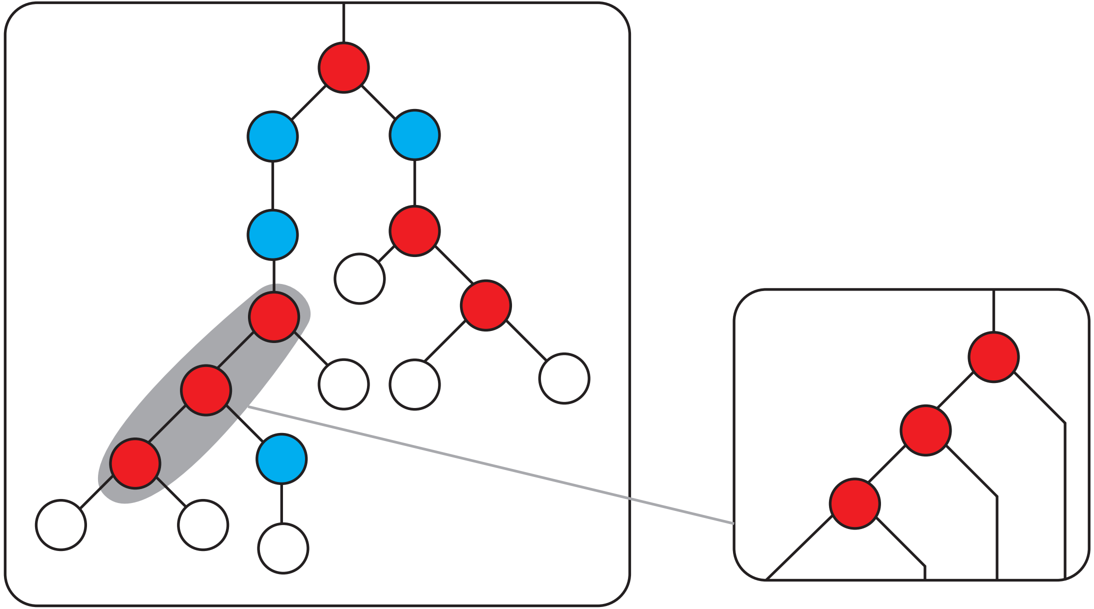


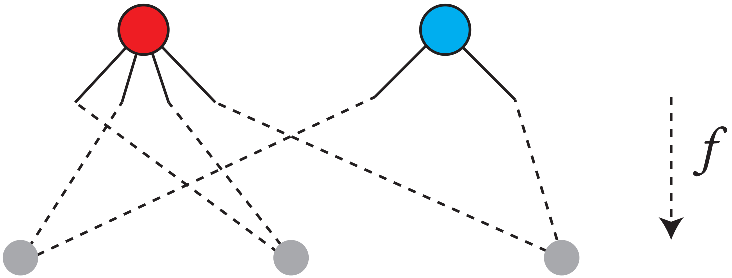


1
2
3
4
5
6
1
2
3
4
5
6

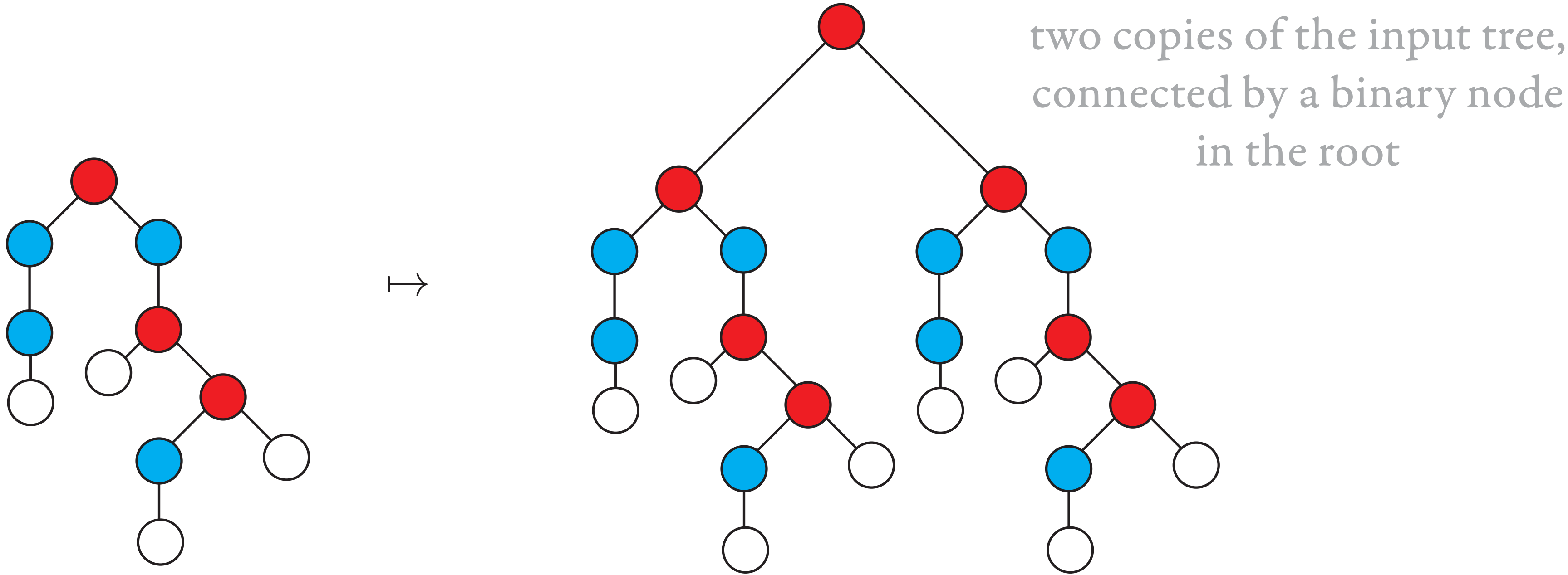
a factorisation equivalence

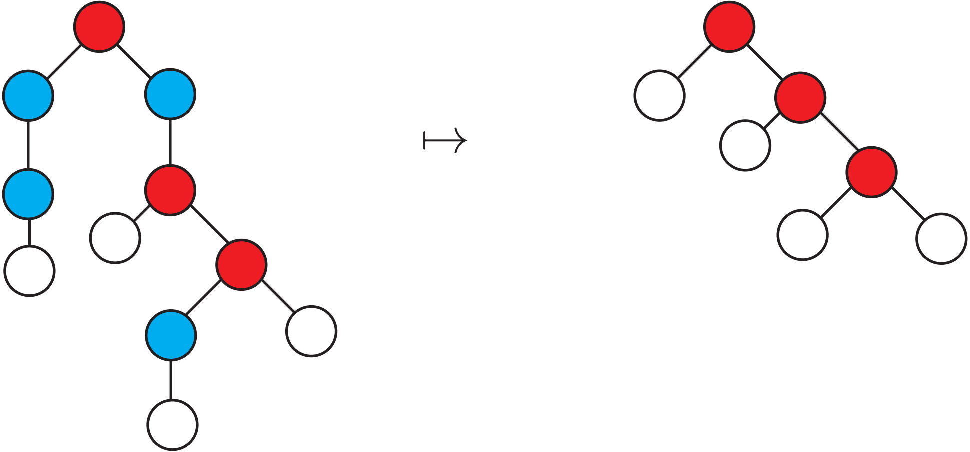


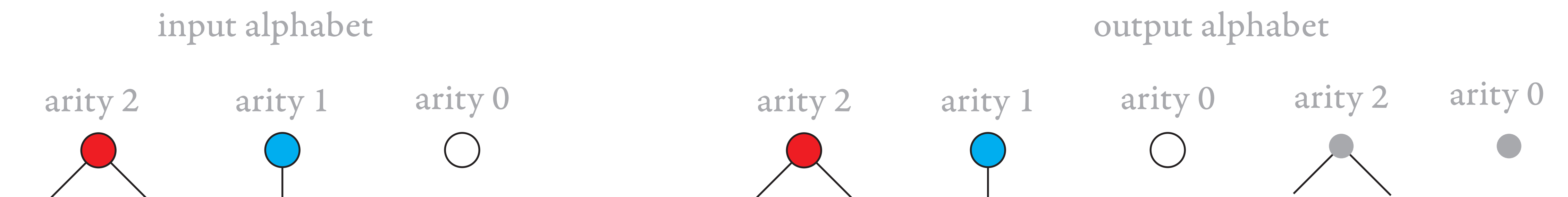

a tree
a term with 4 ports that represents part of the tree

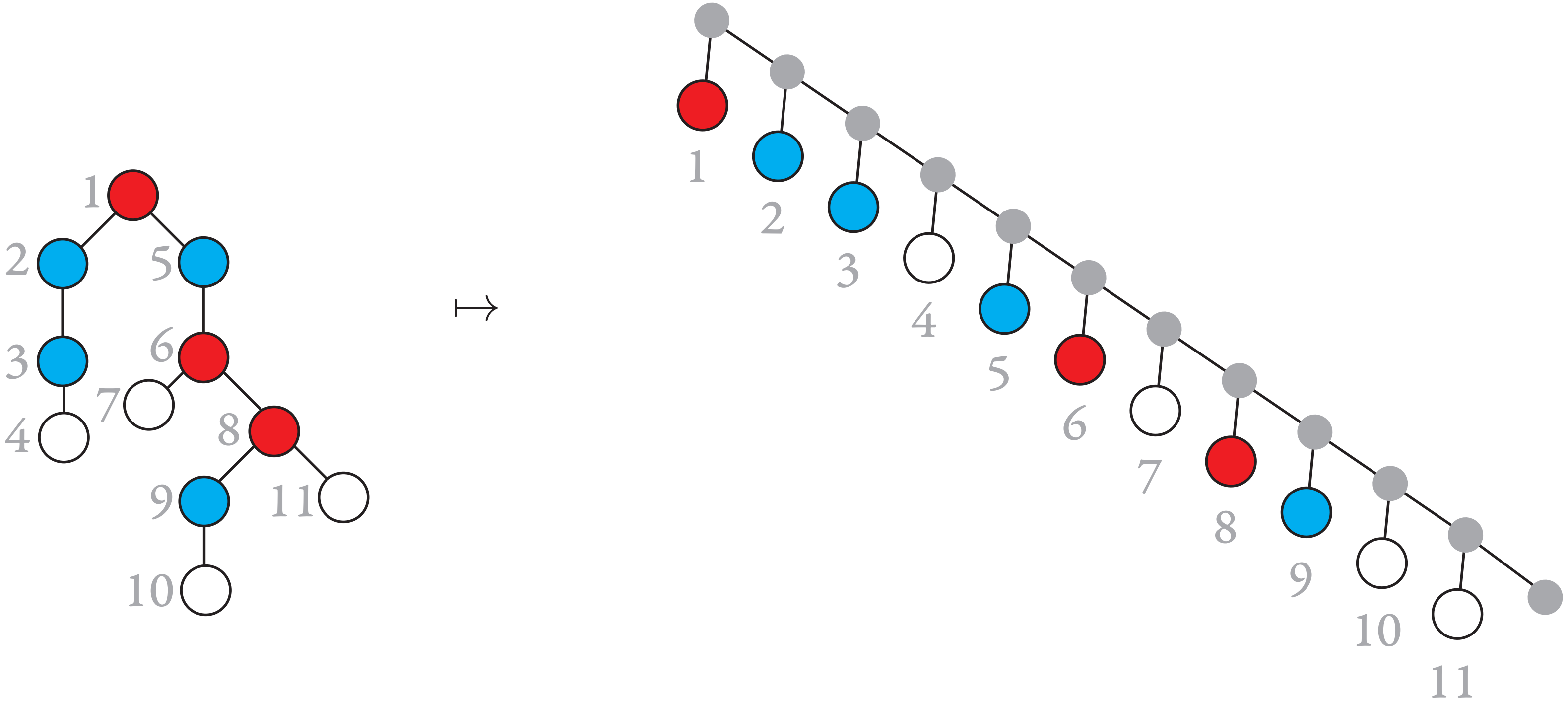

f


input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0

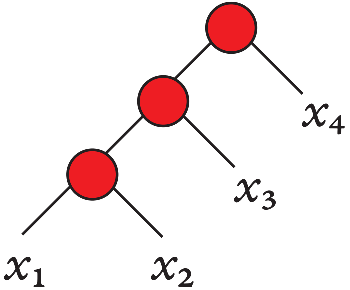

$\mapsto$
two copies of the input tree,
connected by a binary node
in the root



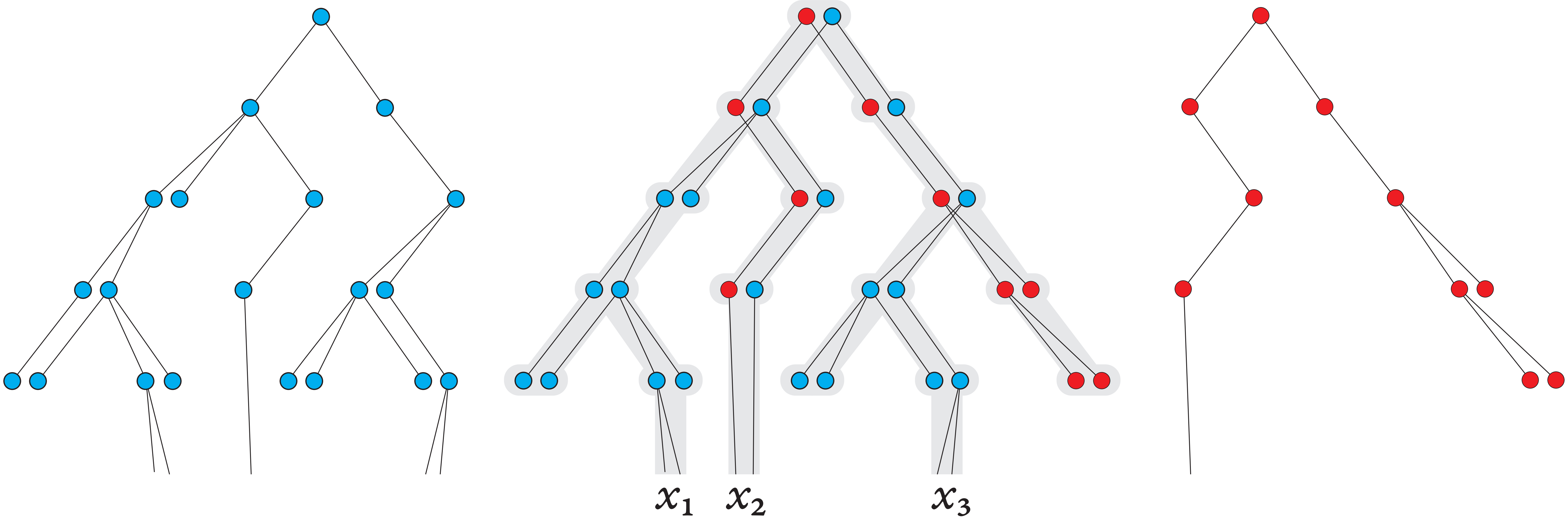

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 0

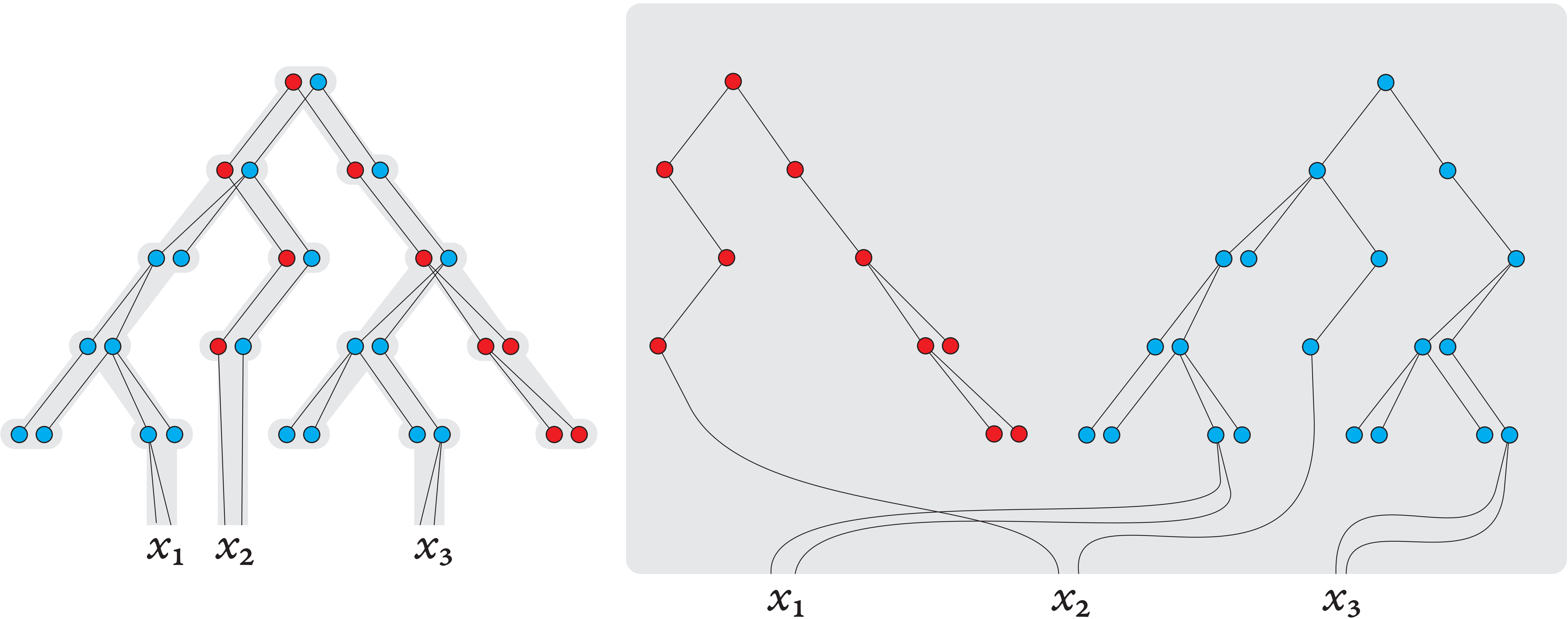

1
2
5
3
6
7
4
8
9
11
10
↦
1
2
3
4
5
6
7
8
9
10
11

a term of arity 4

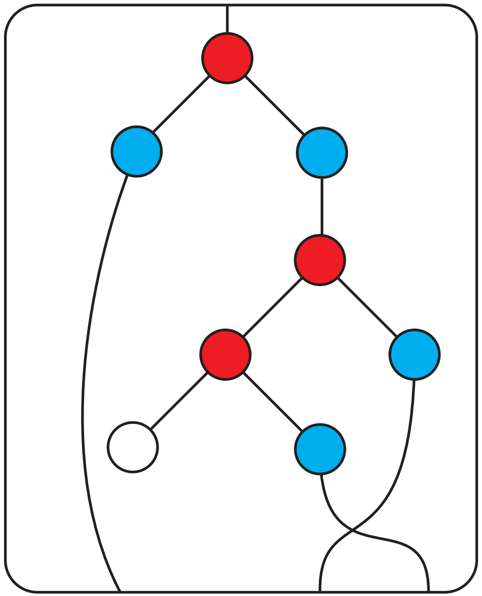


a term of arity 0


x_1
x_2
x_3



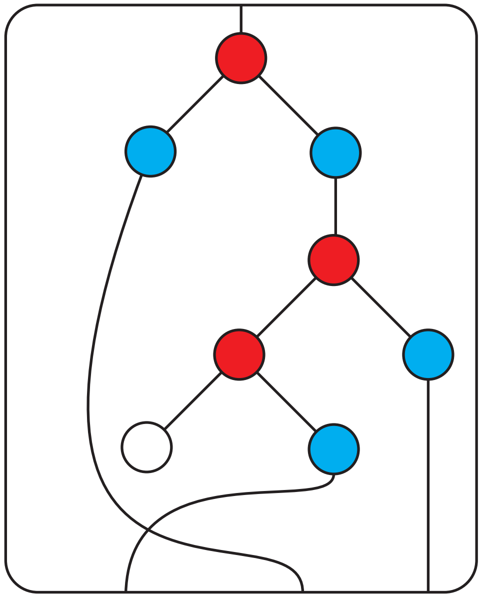
x_1 x_2 x_3
x_1 x_2 x_3



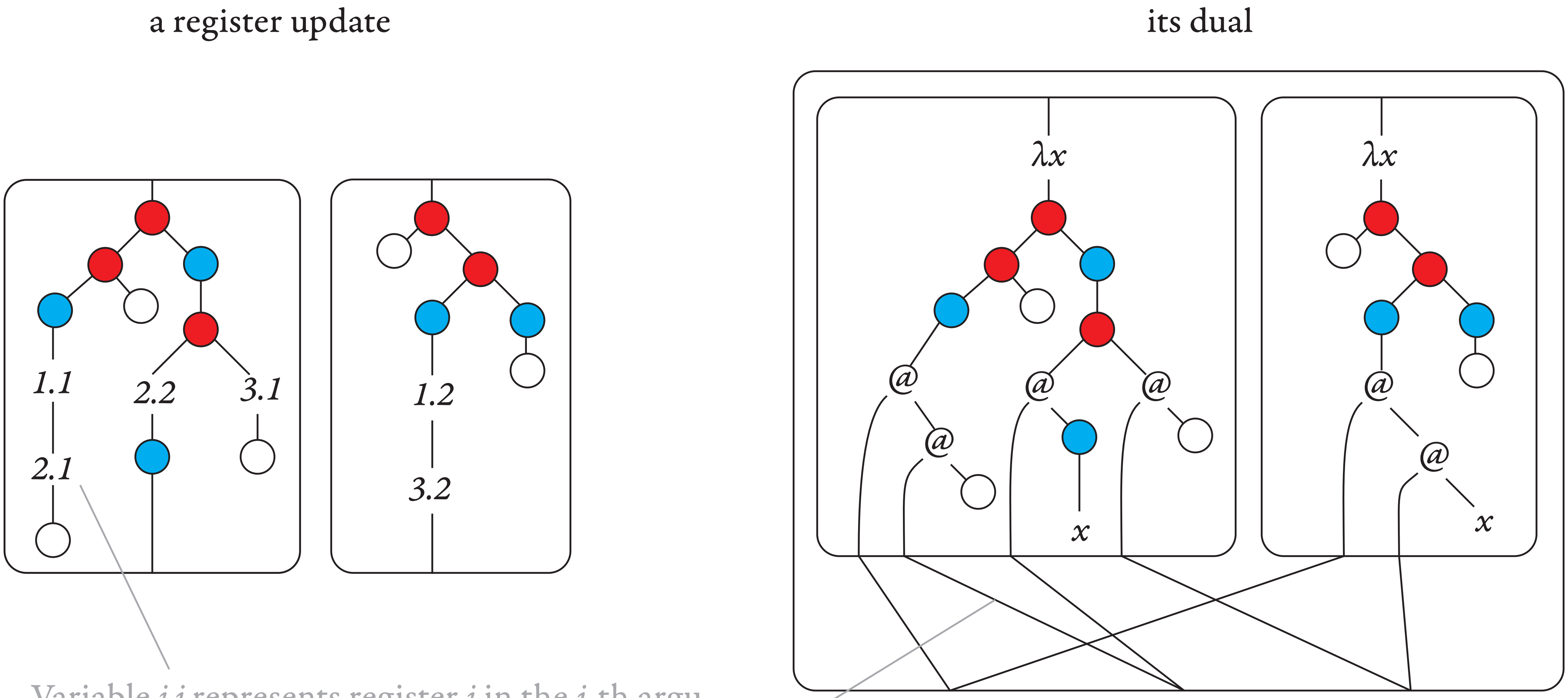
satisfies (*)

(*)

If the root has arity n, and $1 \leq i < j \leq n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root



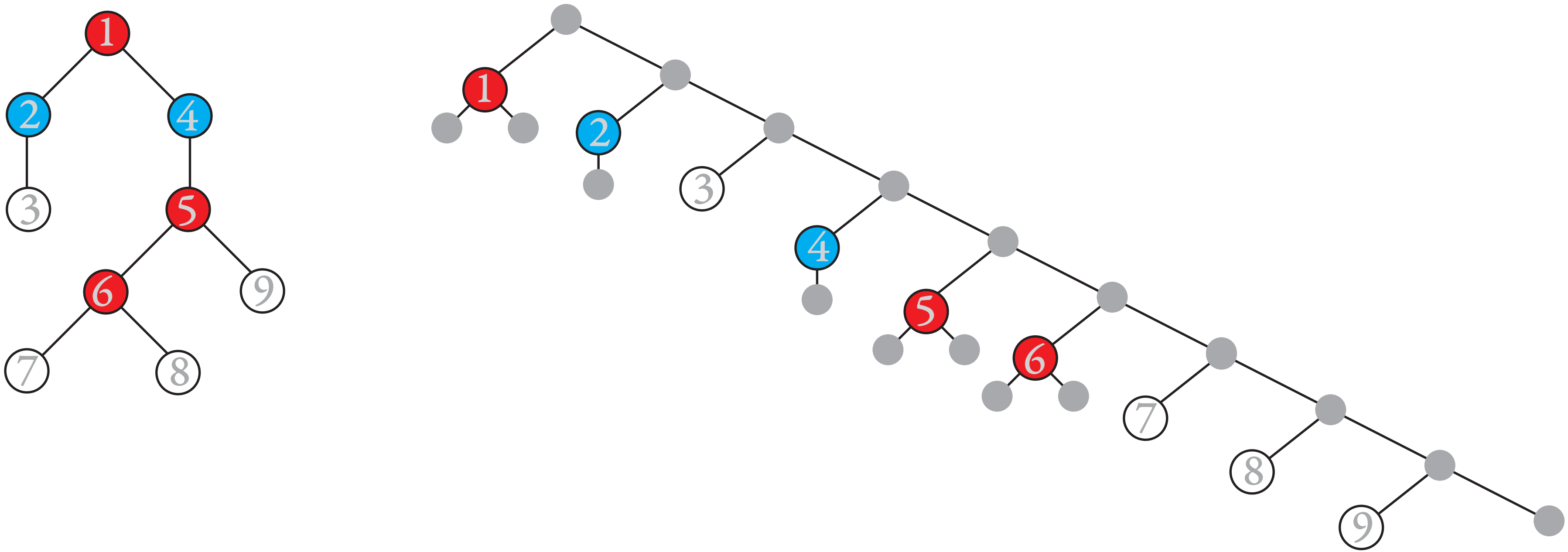
violates (*)

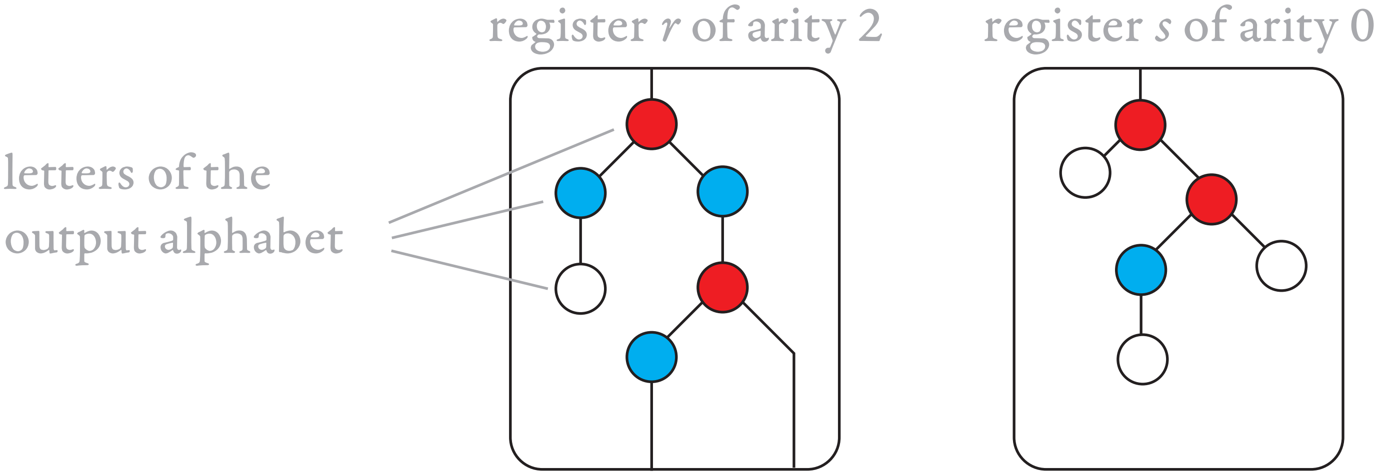


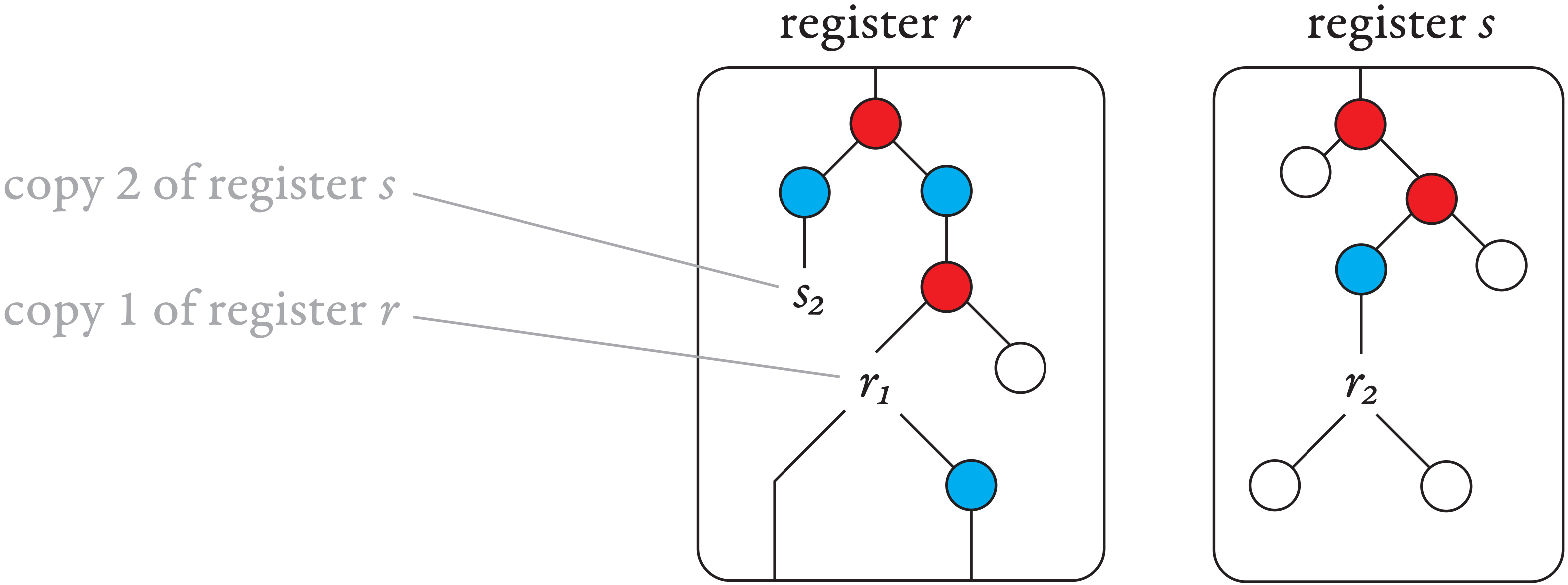
Variable *i.j* represents register *i* in the *j*-th argument of the reigster update.
In the dual, this variable is mapped to the *i*-th edge which enters the *j*-th port of the reducer.

input

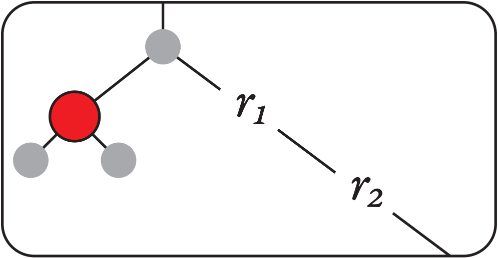
output

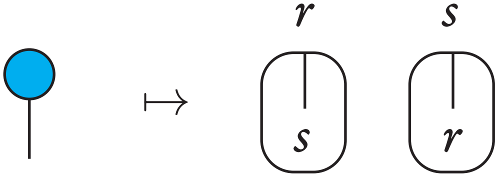


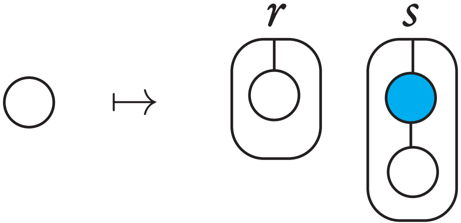

register r of arity 2
register s of arity 0
letters of the output alphabet

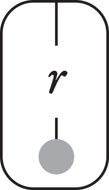

register r
register s
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2

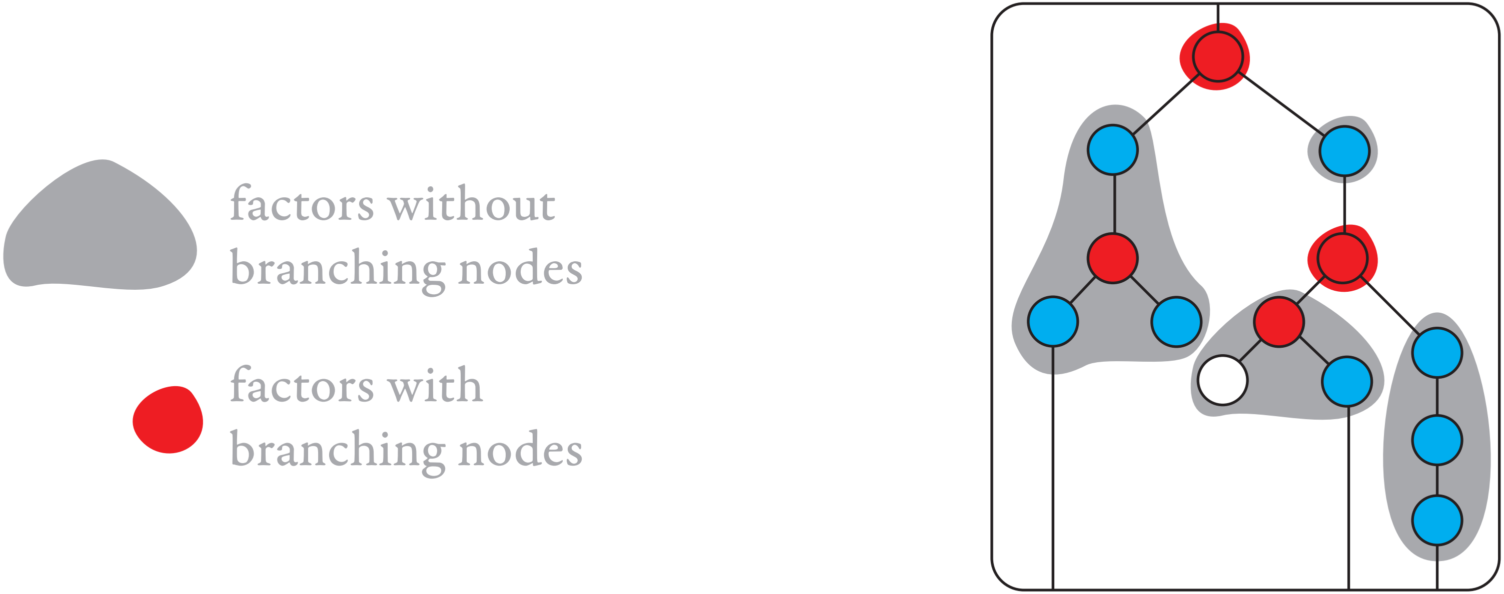

r
r
r
s

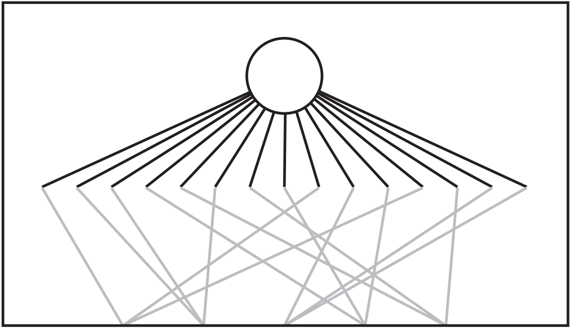

r_1
r_2


r
s
s
r

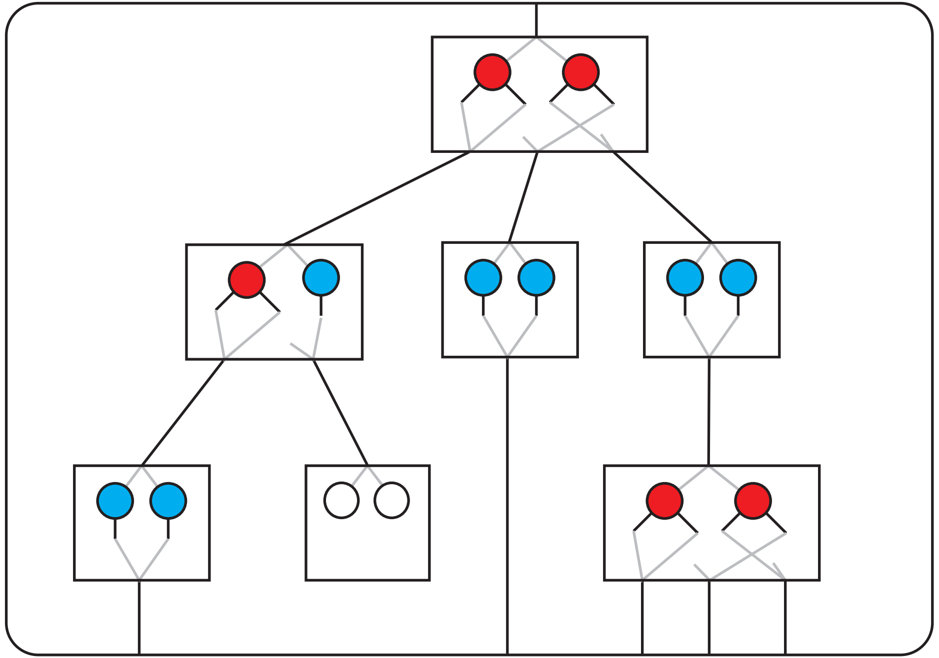

r
s


r

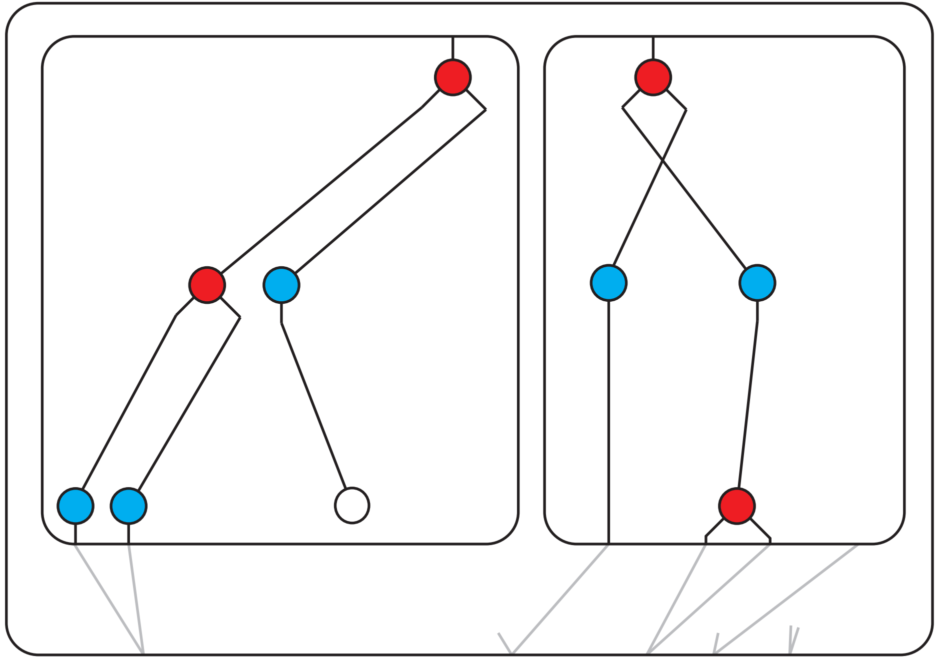

factors without branching nodes
factors with branching nodes

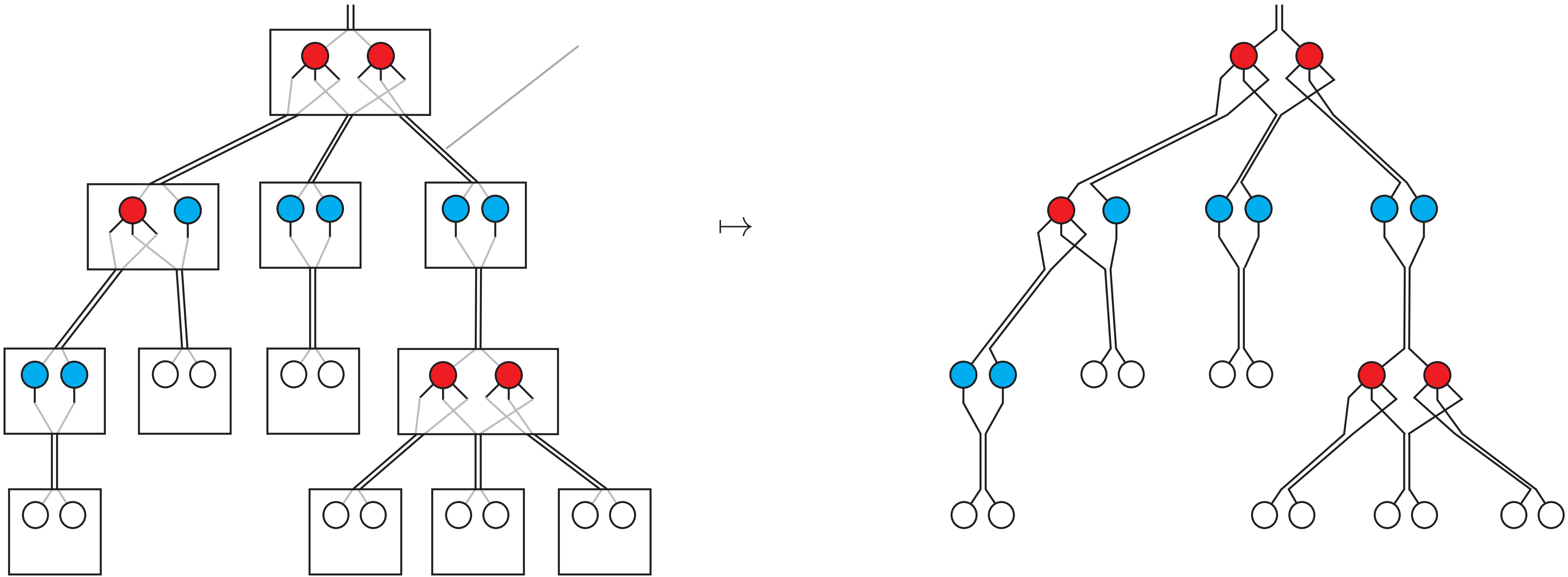


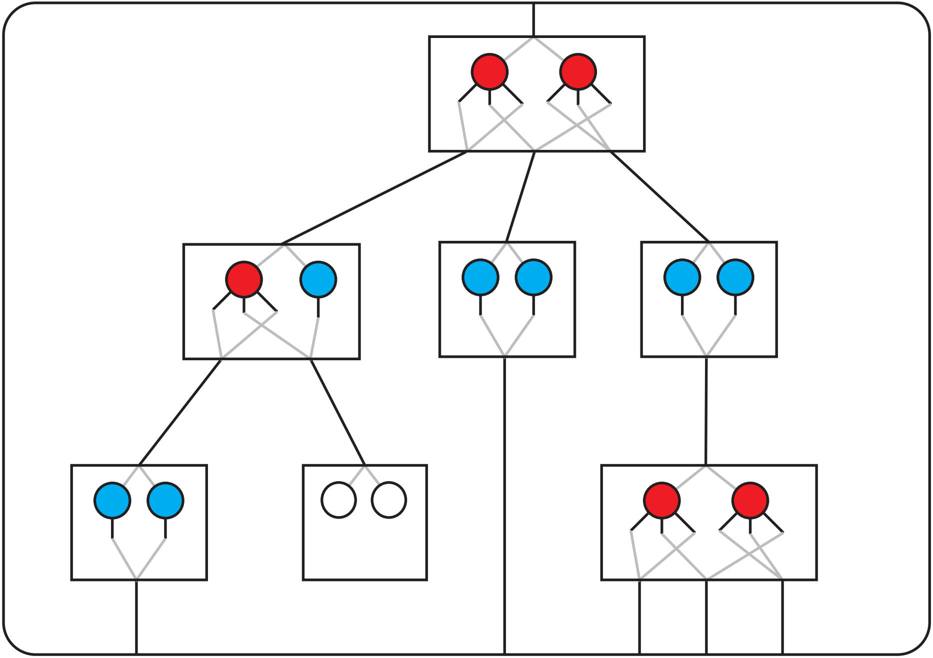
a term of matrix powers

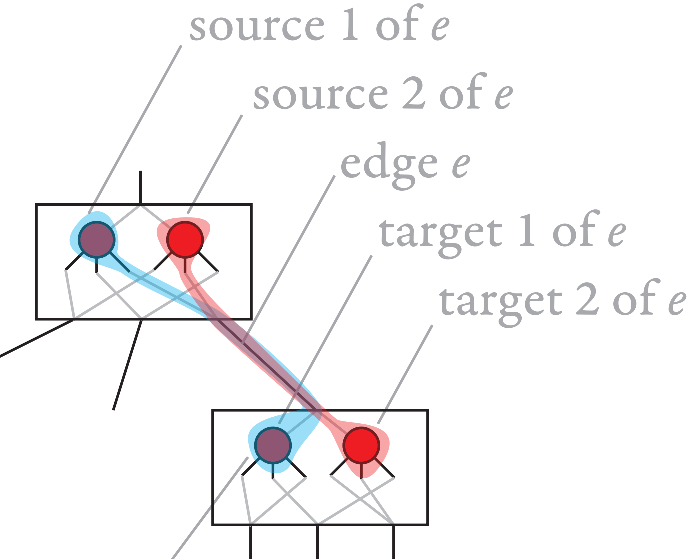


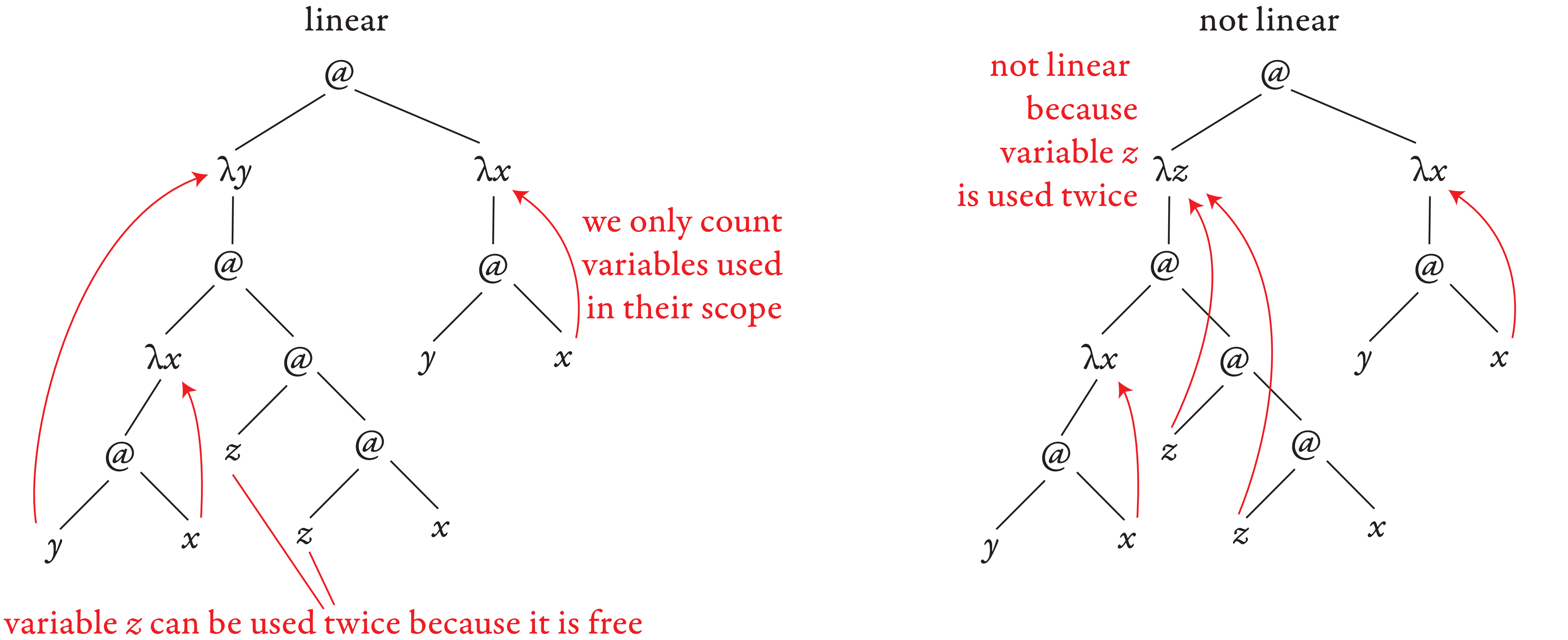
its term unfolding



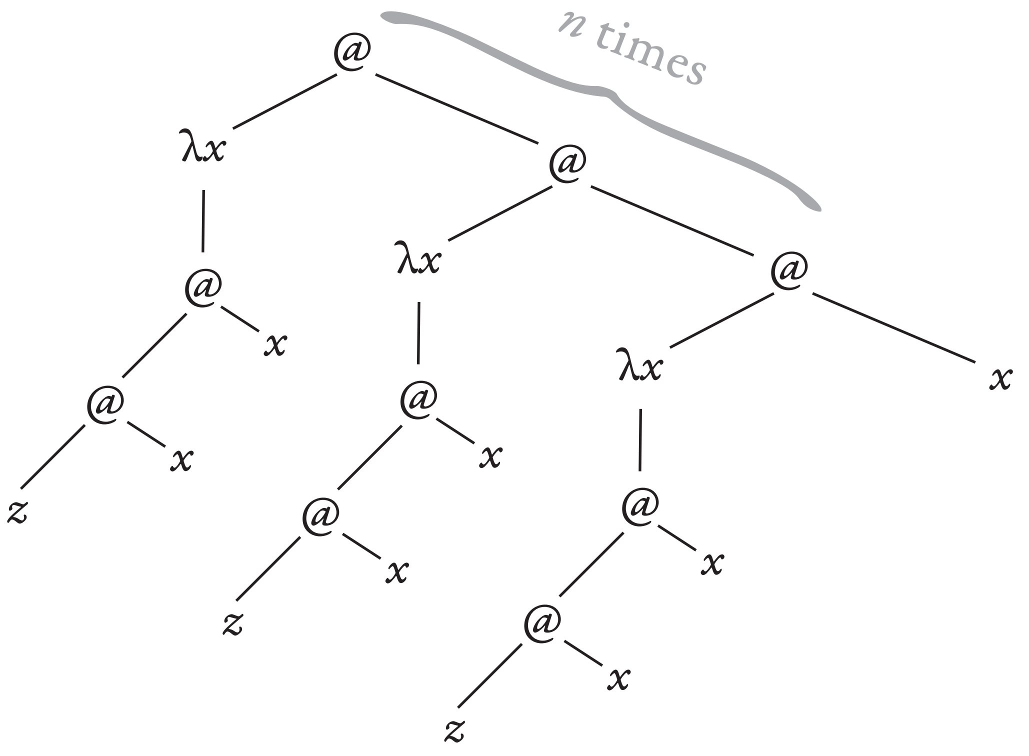




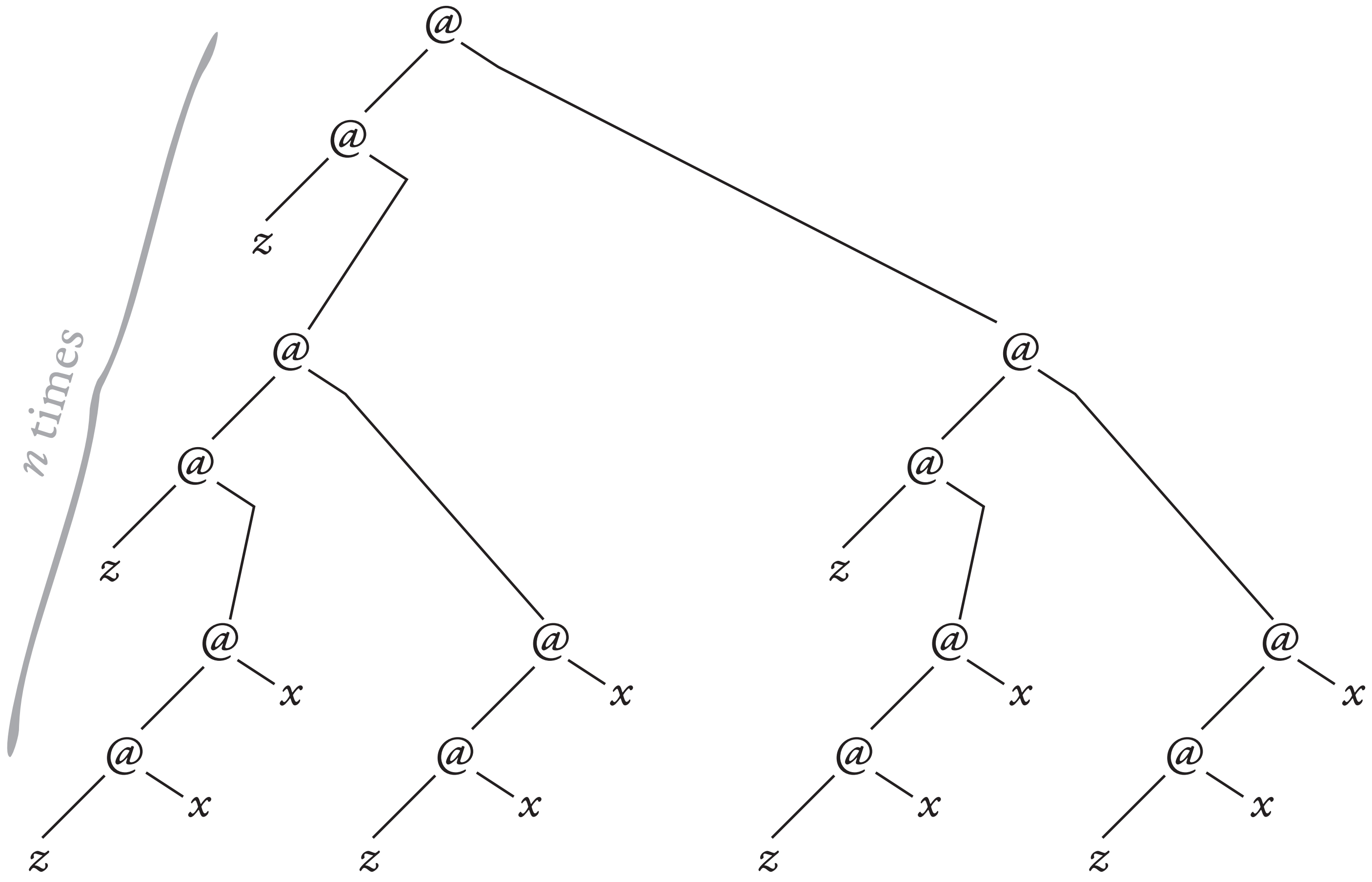

source 1 of *e*
source 2 of *e*
edge *e*
target 1 of *e*
target 2 of *e*


linear
not linear
we only count
variables used
in their scope
variable z can be used twice because it is free
not linear
because
variable z
is used twice

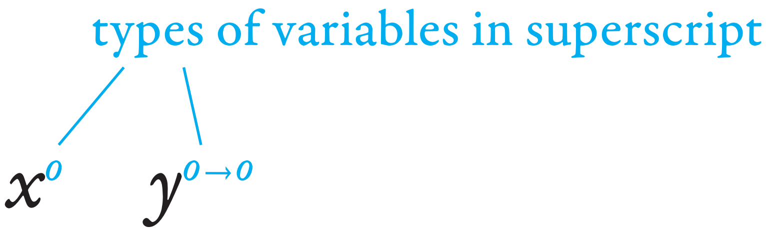
a λ-term of size $O(n)$



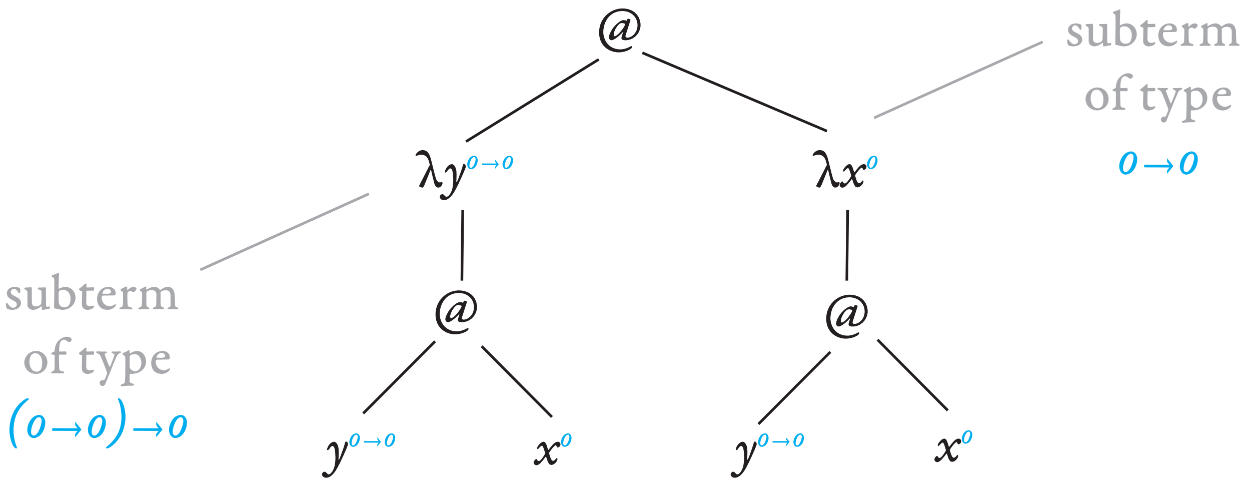
its normal form of size $O(2^n)$



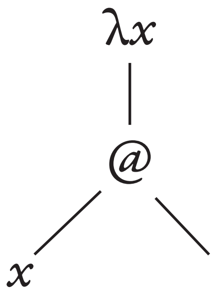
variables



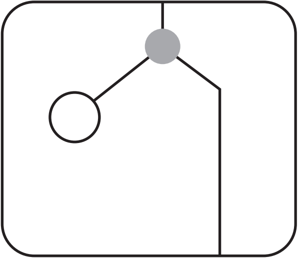
λ-term of type o

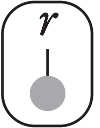


@

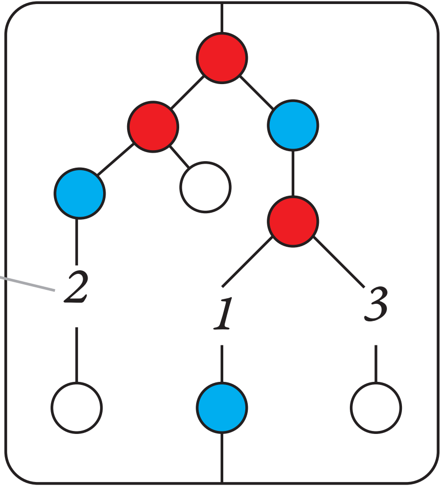


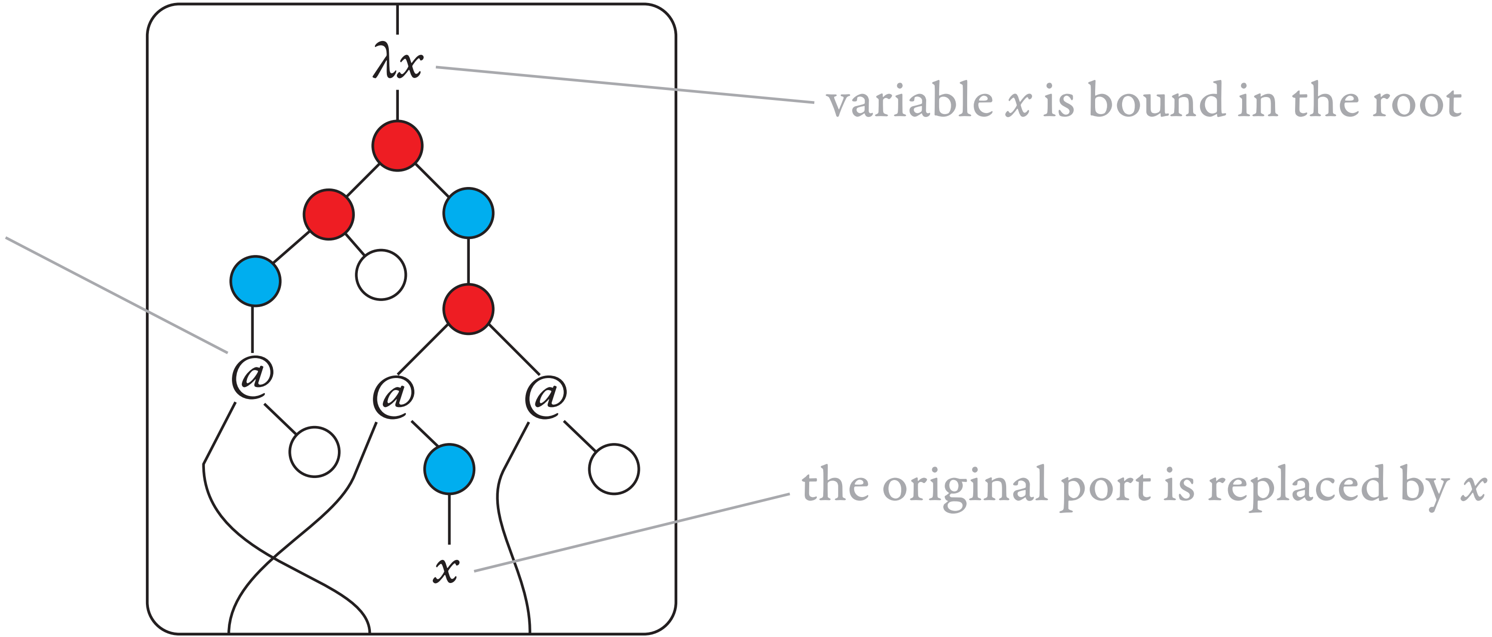
λx.

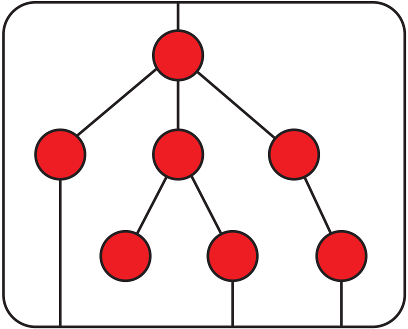



r

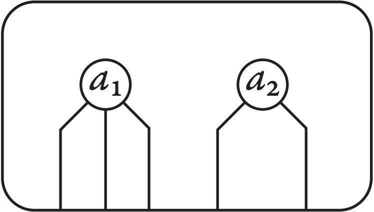
placeholder for the term
stored in the unique register
of the 2nd child



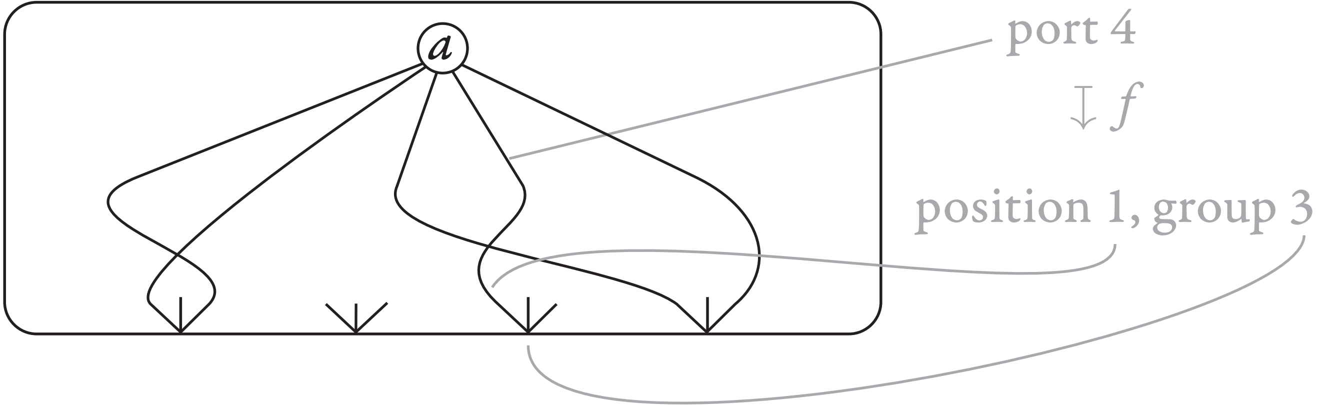

λx
@
@
@
x
variable x is bound in the root
the original port is replaced by x



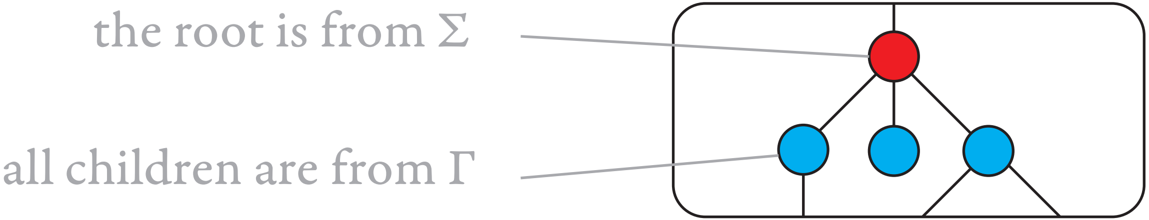
dangling edges
represent ports

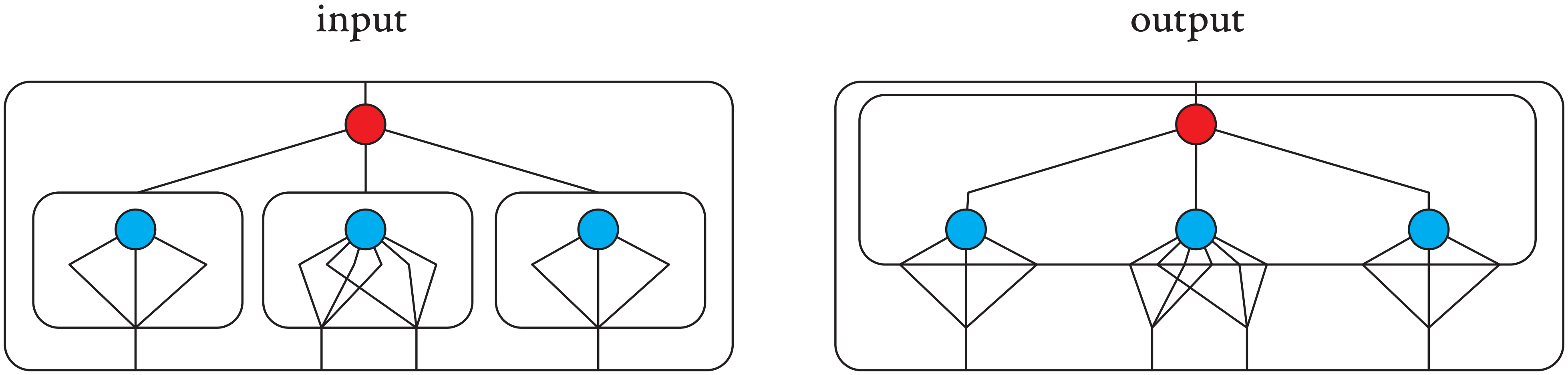


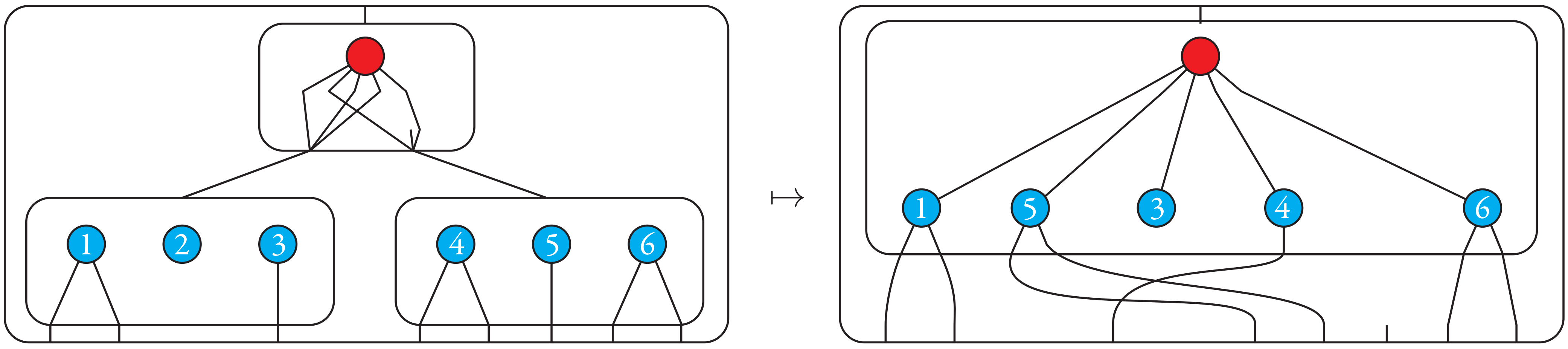
a_1
a_2



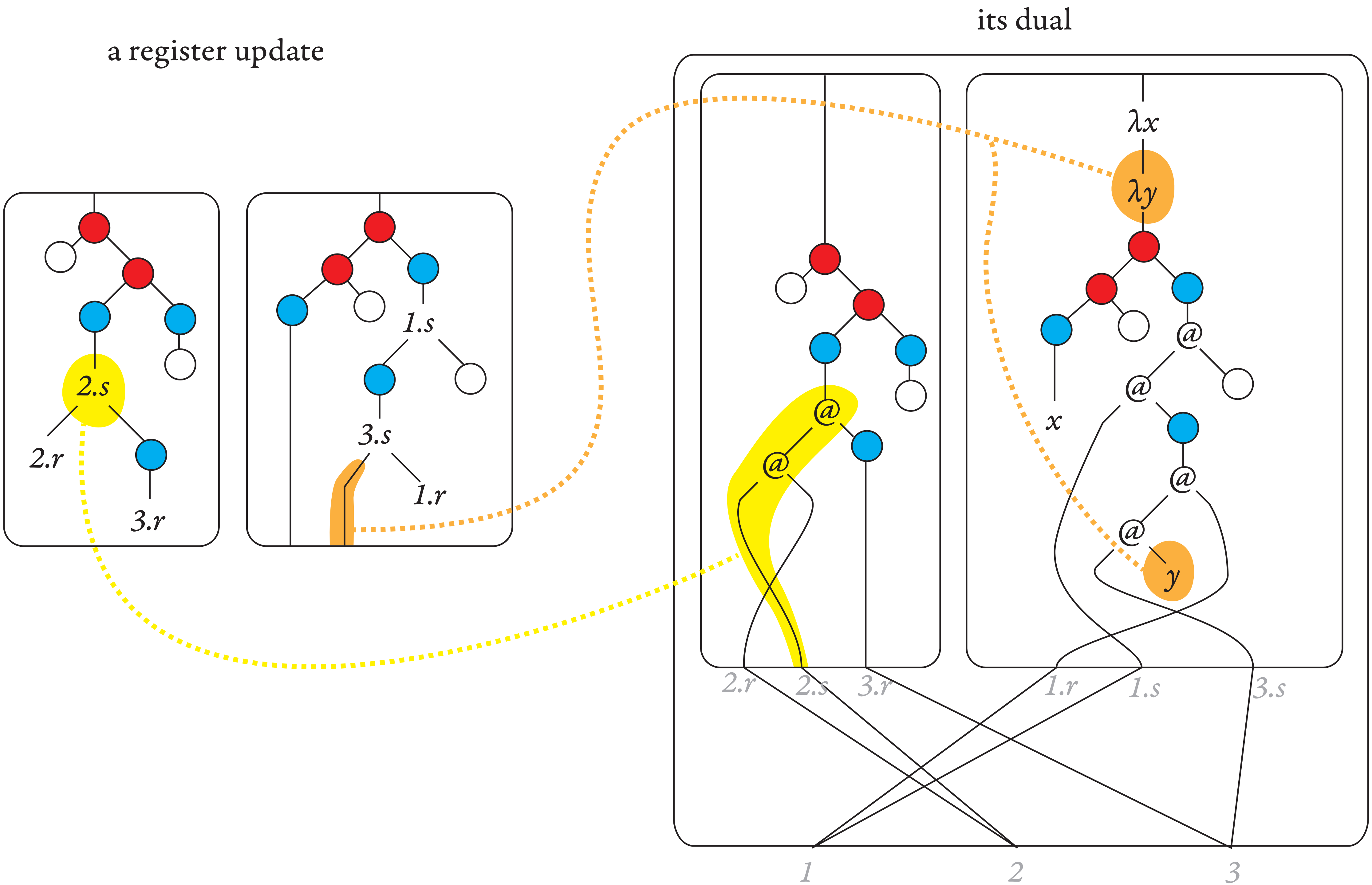
a
port 4
$\downarrow f$
position 1, group 3

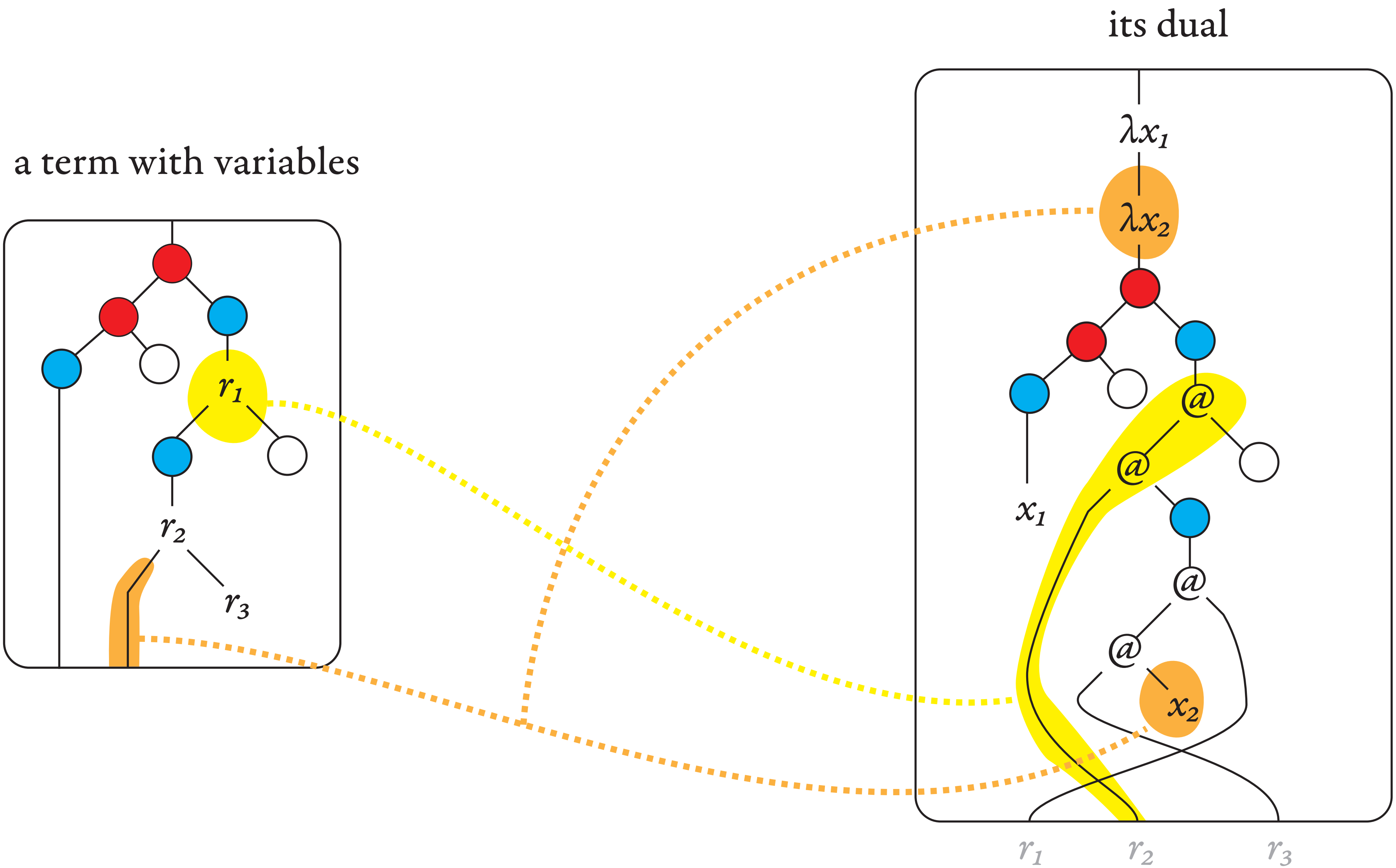

the root is from Σ
all children are from Γ

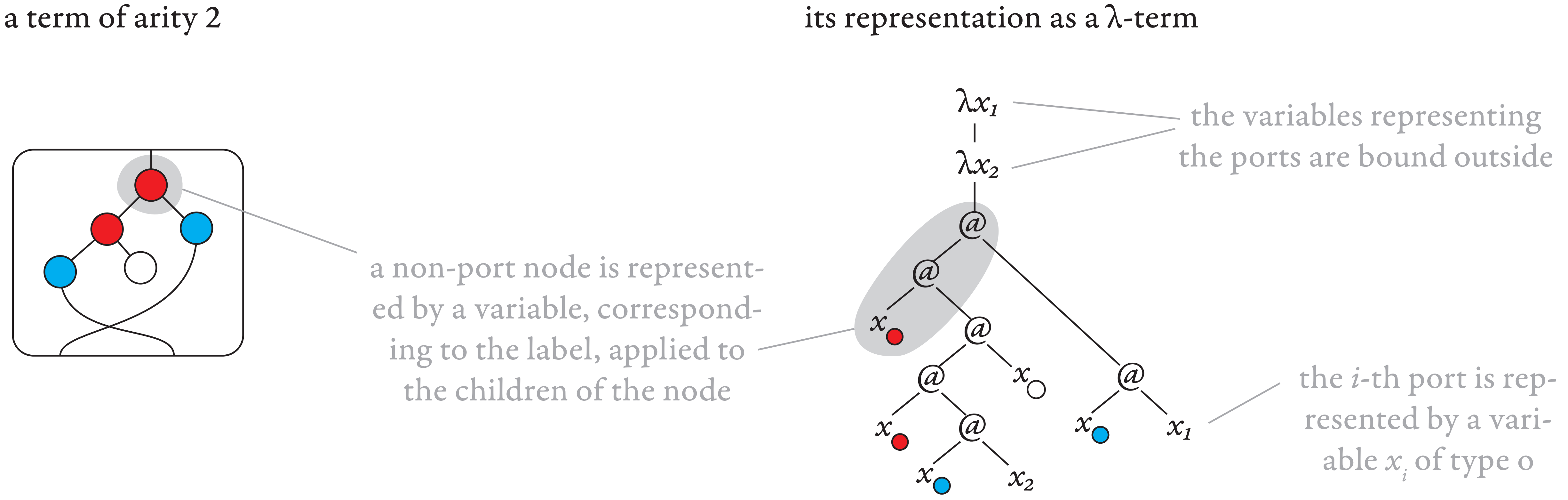

input
output

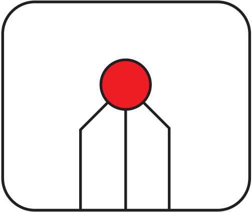


1
2
3
4
5
6
↦
1
5
3
4
6

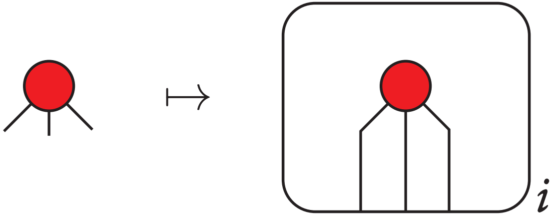

a register update
its dual
2.s
2.r
3.r
1.s
3.s
1.r
λx
λy
x
y
@
2.r 2.s 3.r 1.r 1.s 3.s
1 2 3

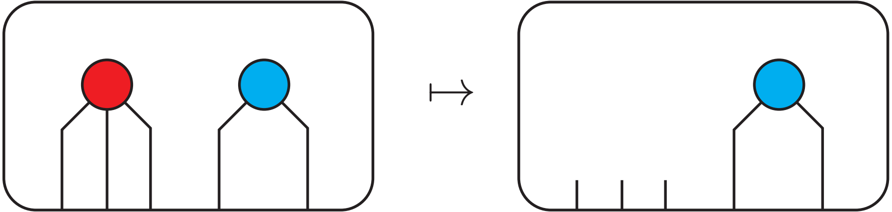

a term with variables
its dual
r_1
r_2
r_3
λx_1
λx_2
@
@
@
@
x_1
x_2
r_1
r_2
r_3

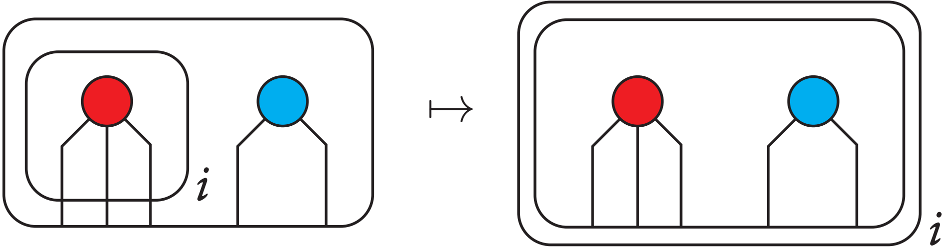

a term of arity 2
its representation as a λ-term
λx1
λx2
the variables representing the ports are bound outside
@
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
x
x1
x2
the i-th port is represented by a variable xi of type o

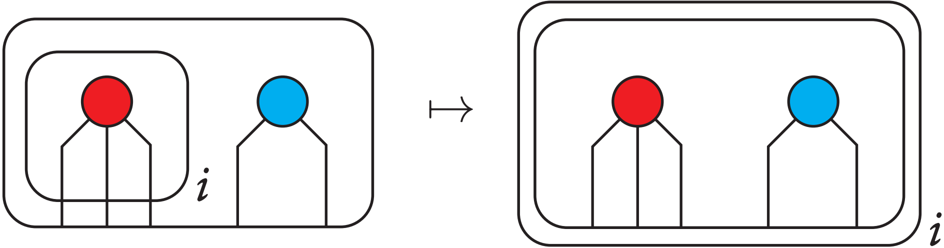


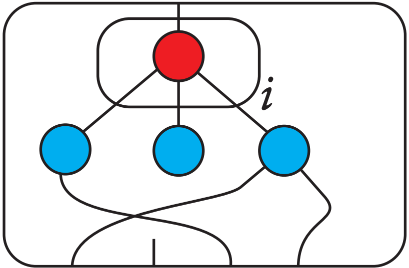
i


i

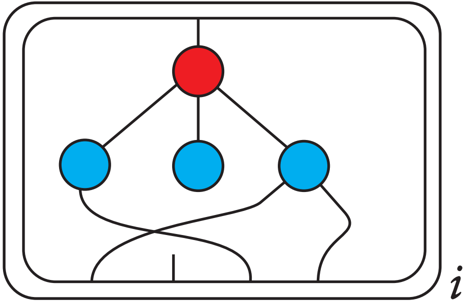


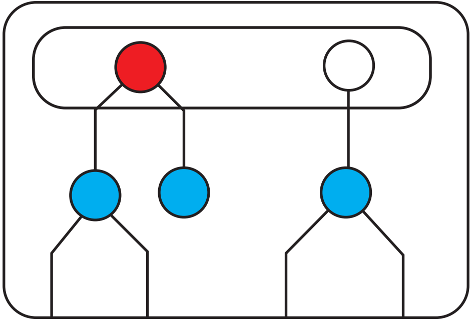

i
i


i
i



$\mapsto$





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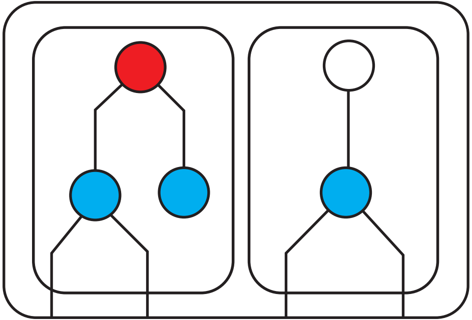

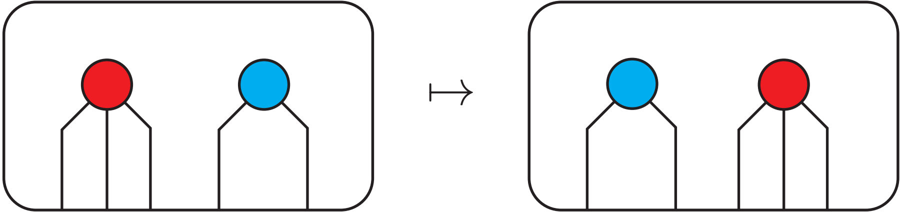

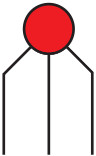

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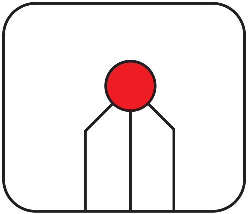

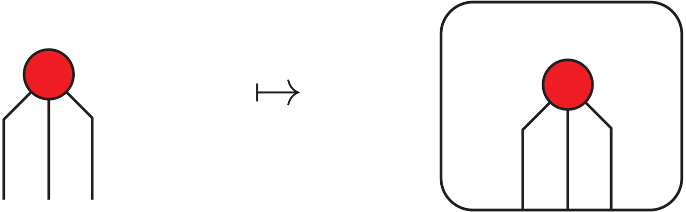

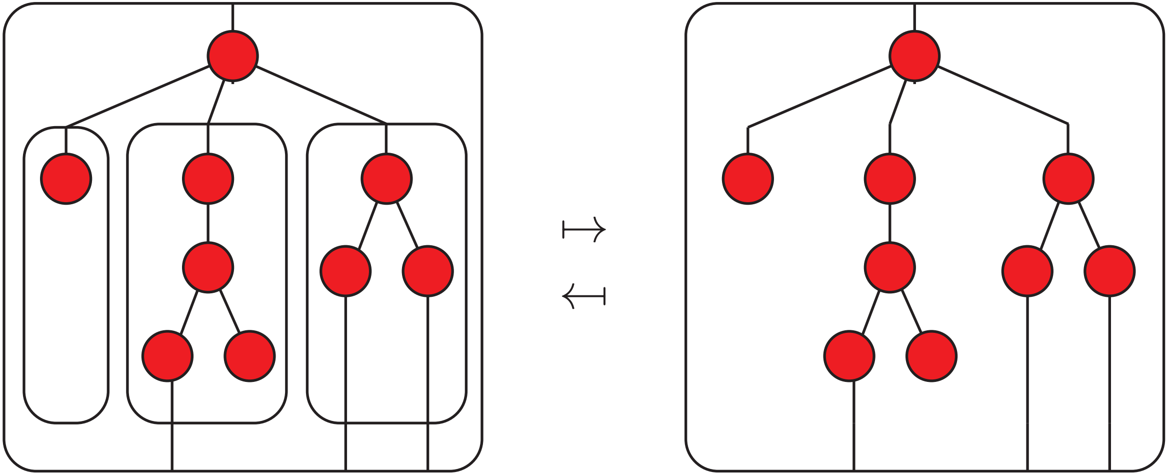

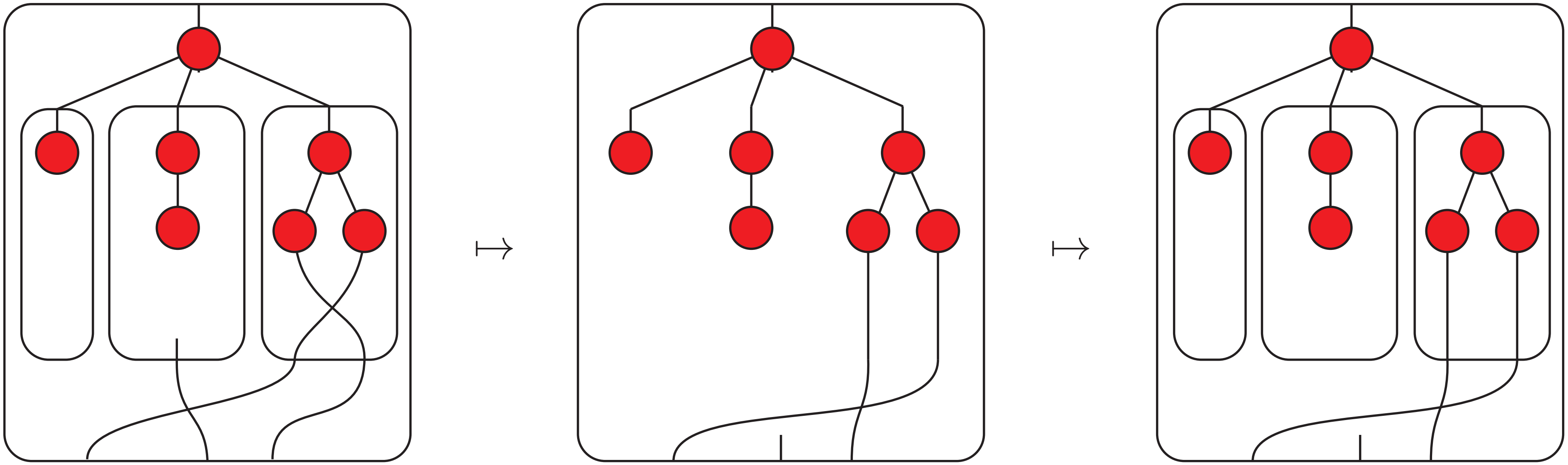

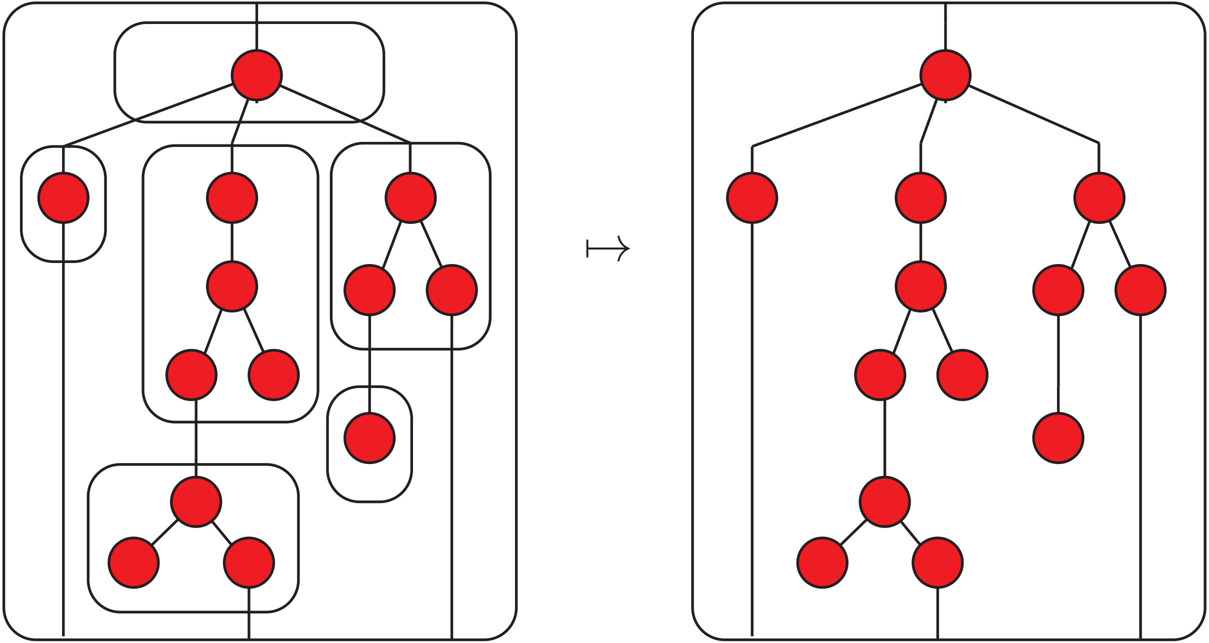

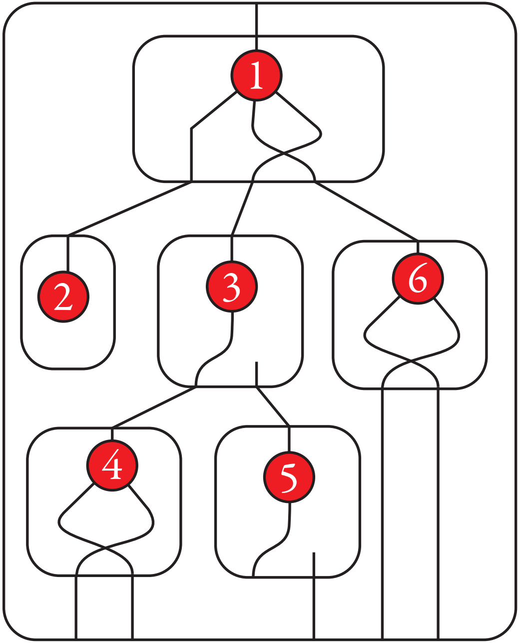

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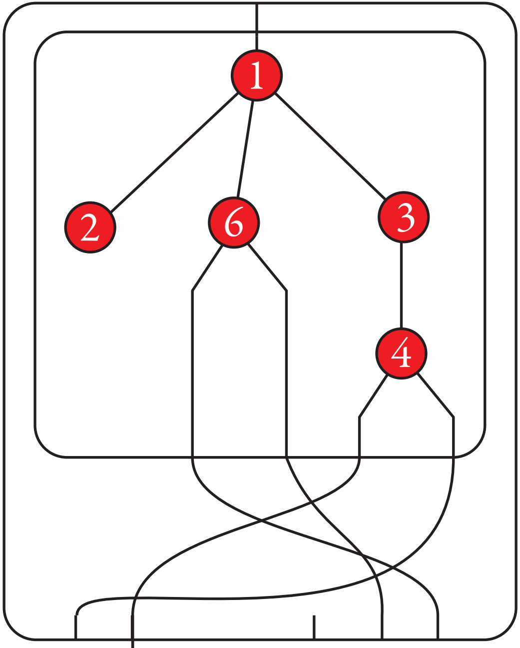
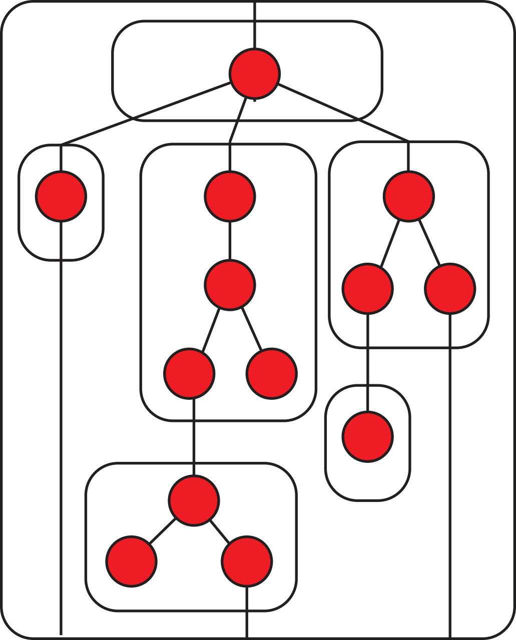
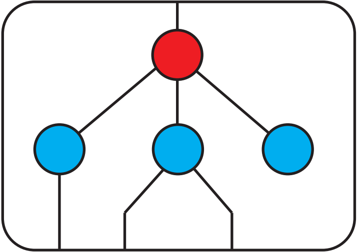
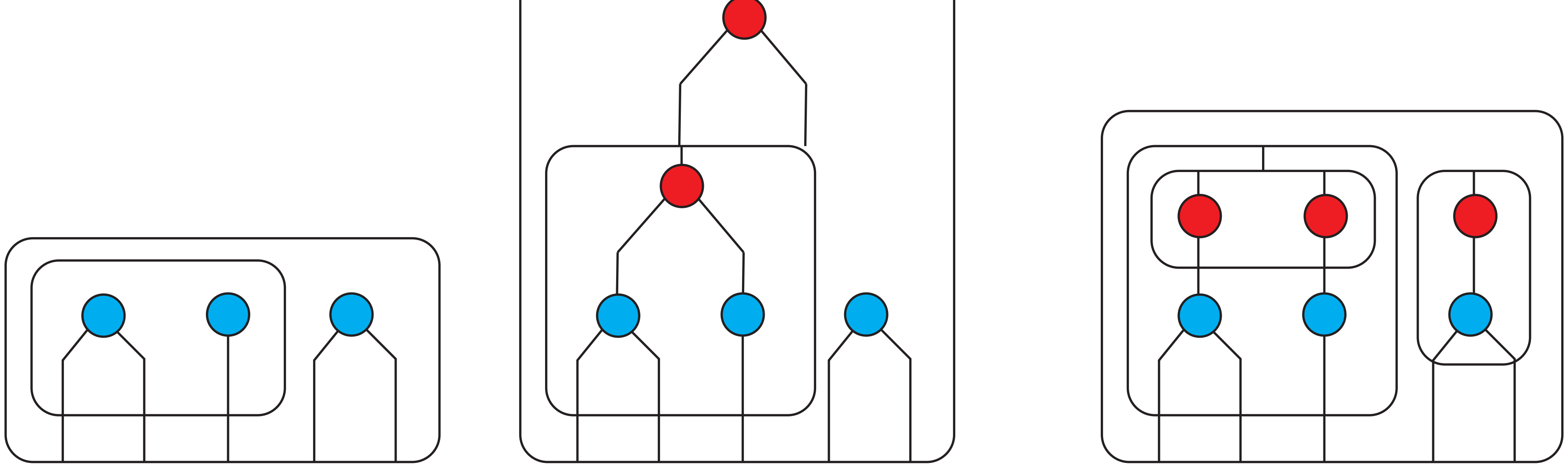
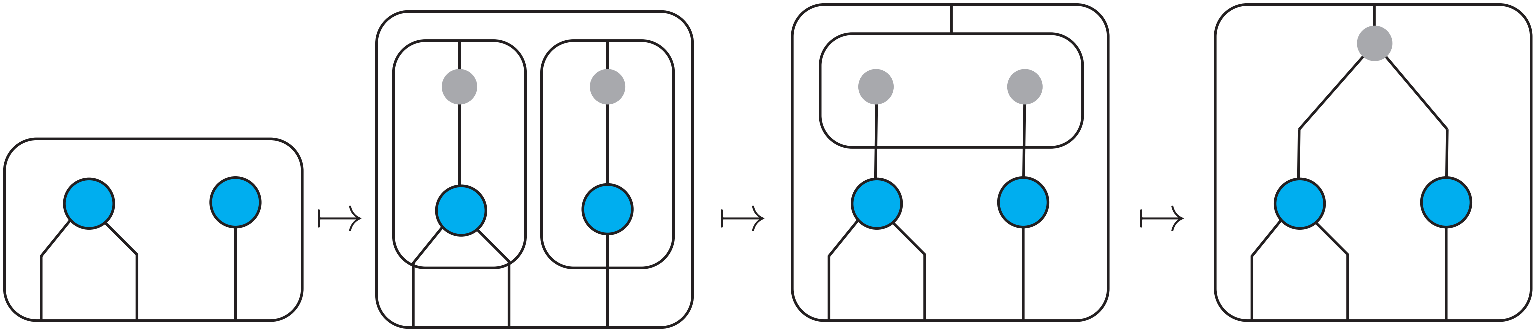
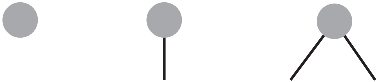


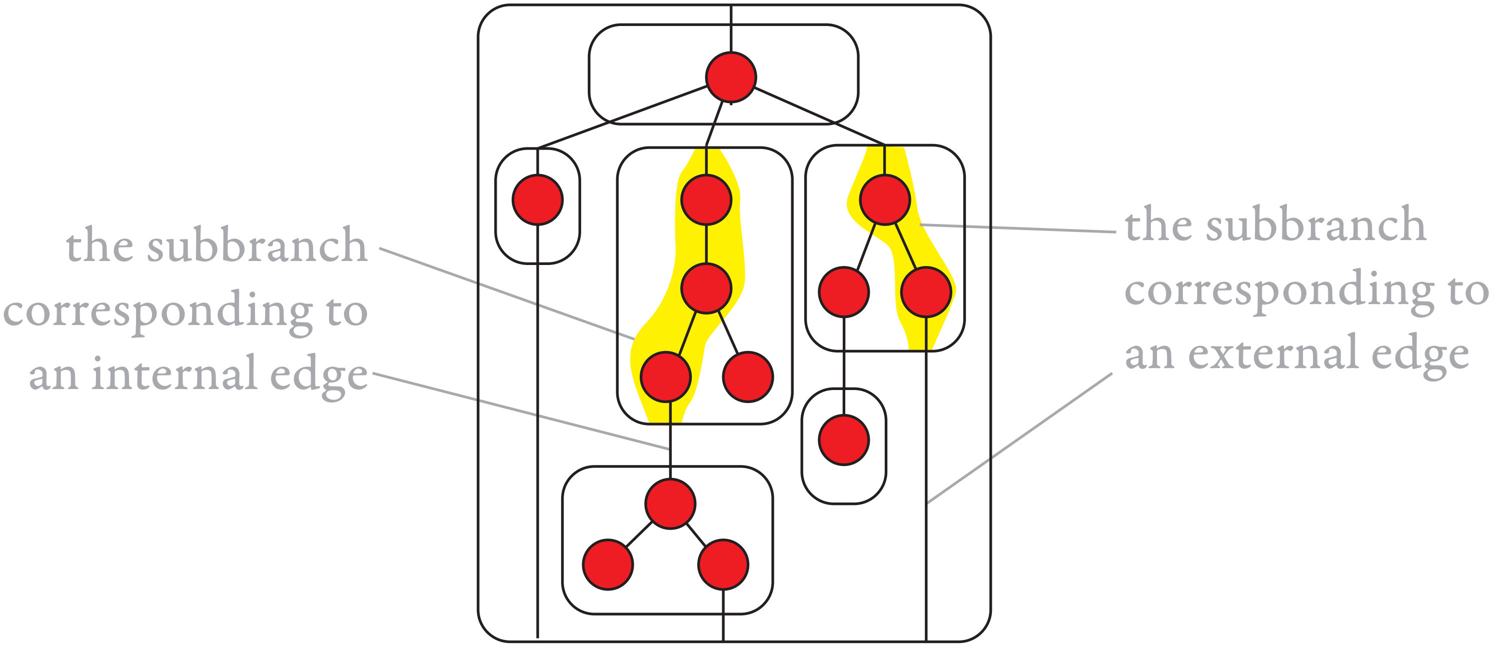


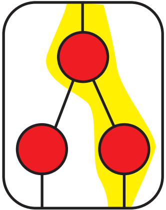




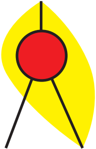


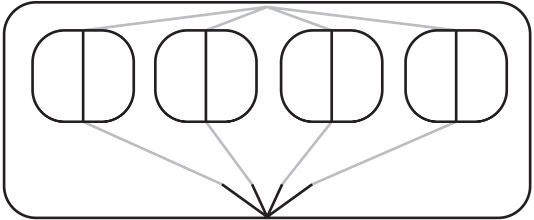



the subbranch corresponding to an internal edge
the subbranch corresponding to an external edge

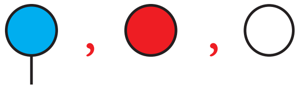


a branch can be visualised as
a term with a distinguished
root-to-port path

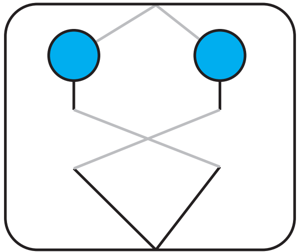




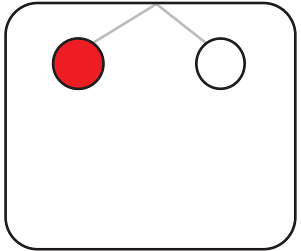
a tuple of k identity terms with all their ports folded into one

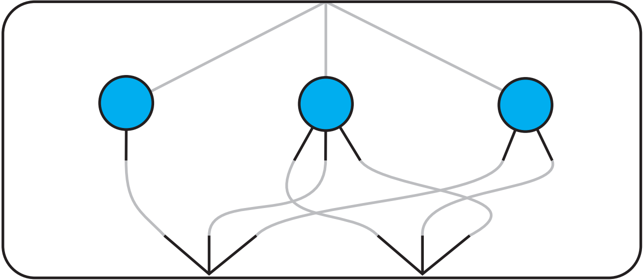
$\Sigma = \{$  $\}$

$a \in \Sigma^{[2]}$

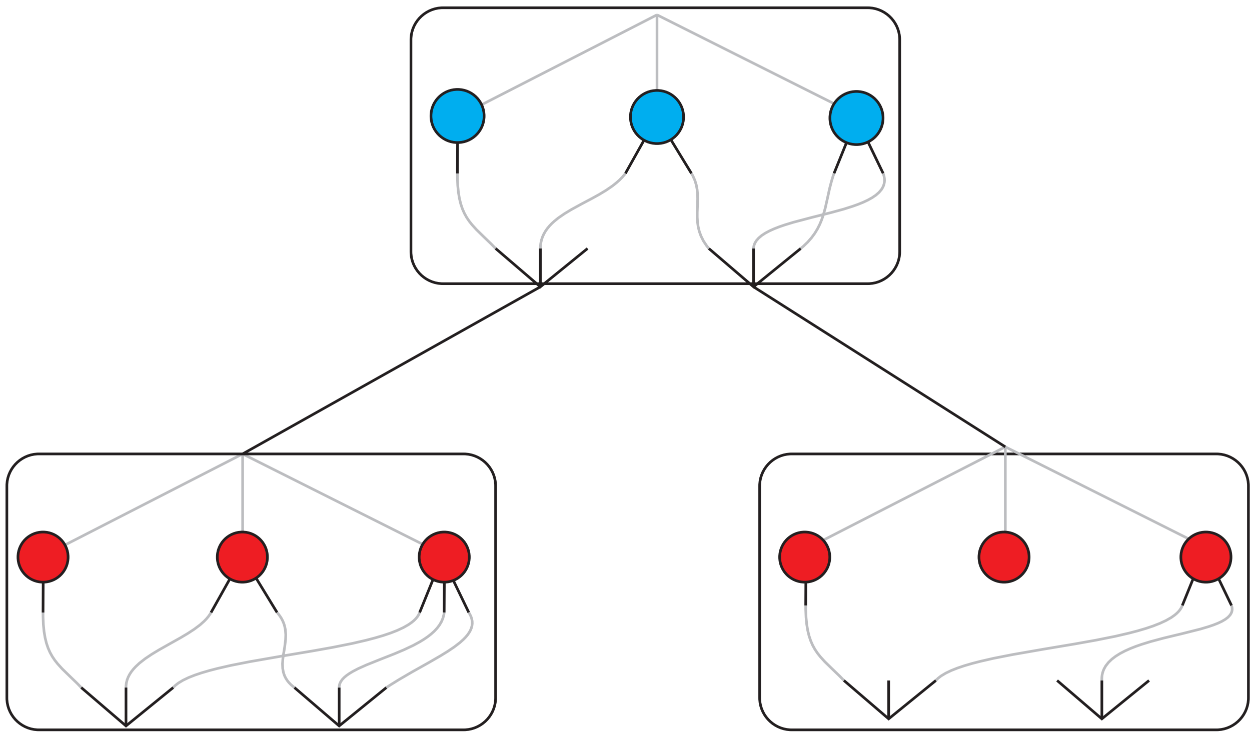


$b \in \Sigma^{[2]}$

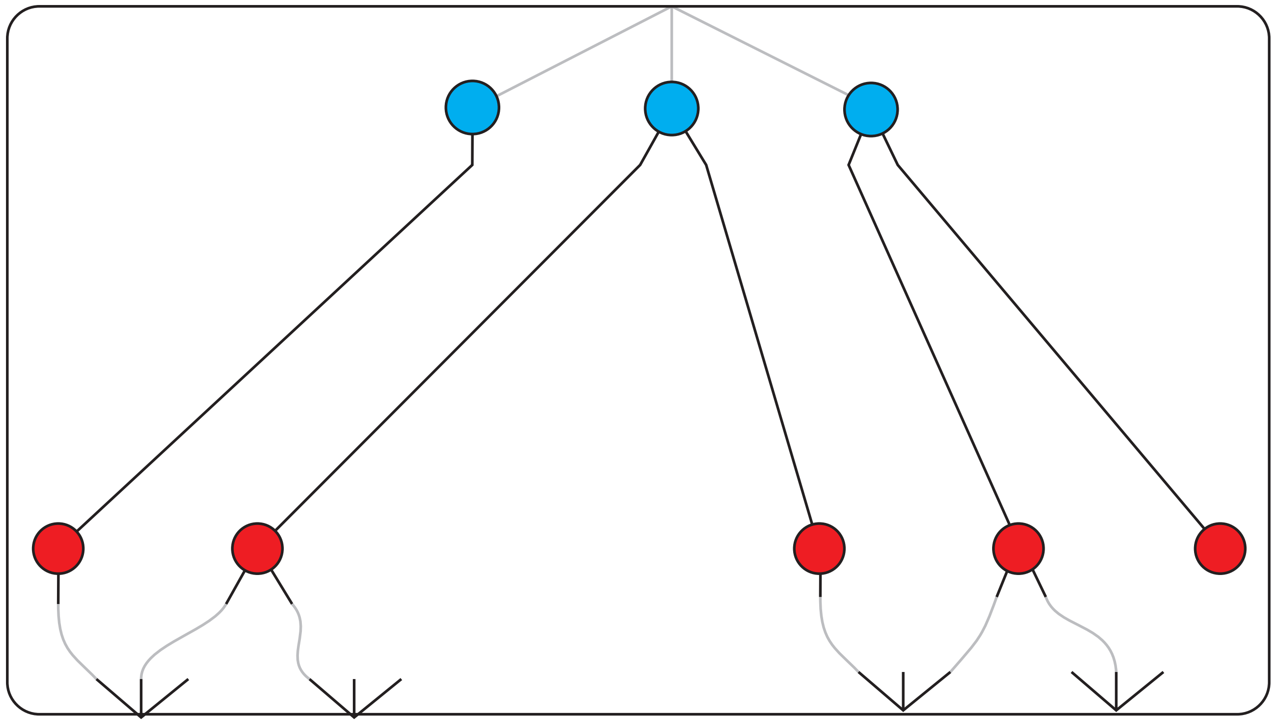


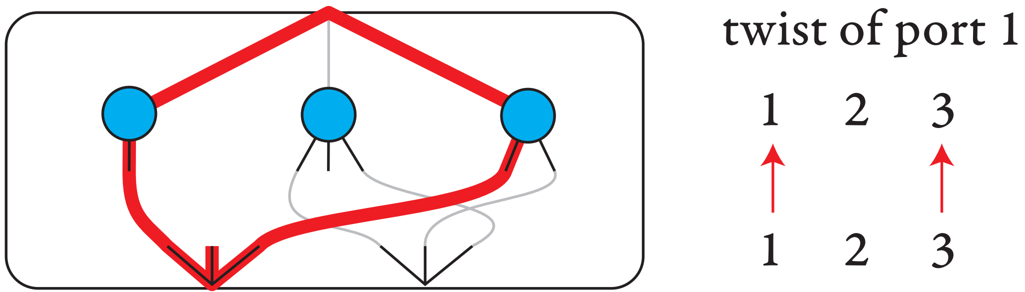


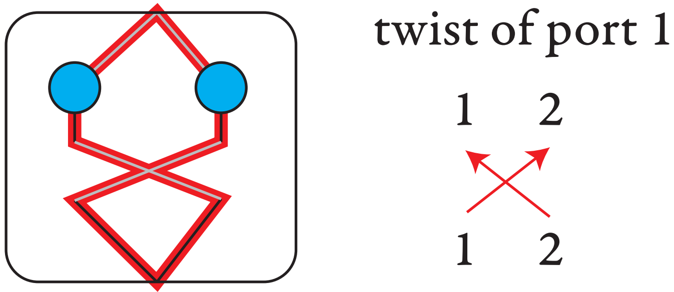
a shallow term of matrix powers

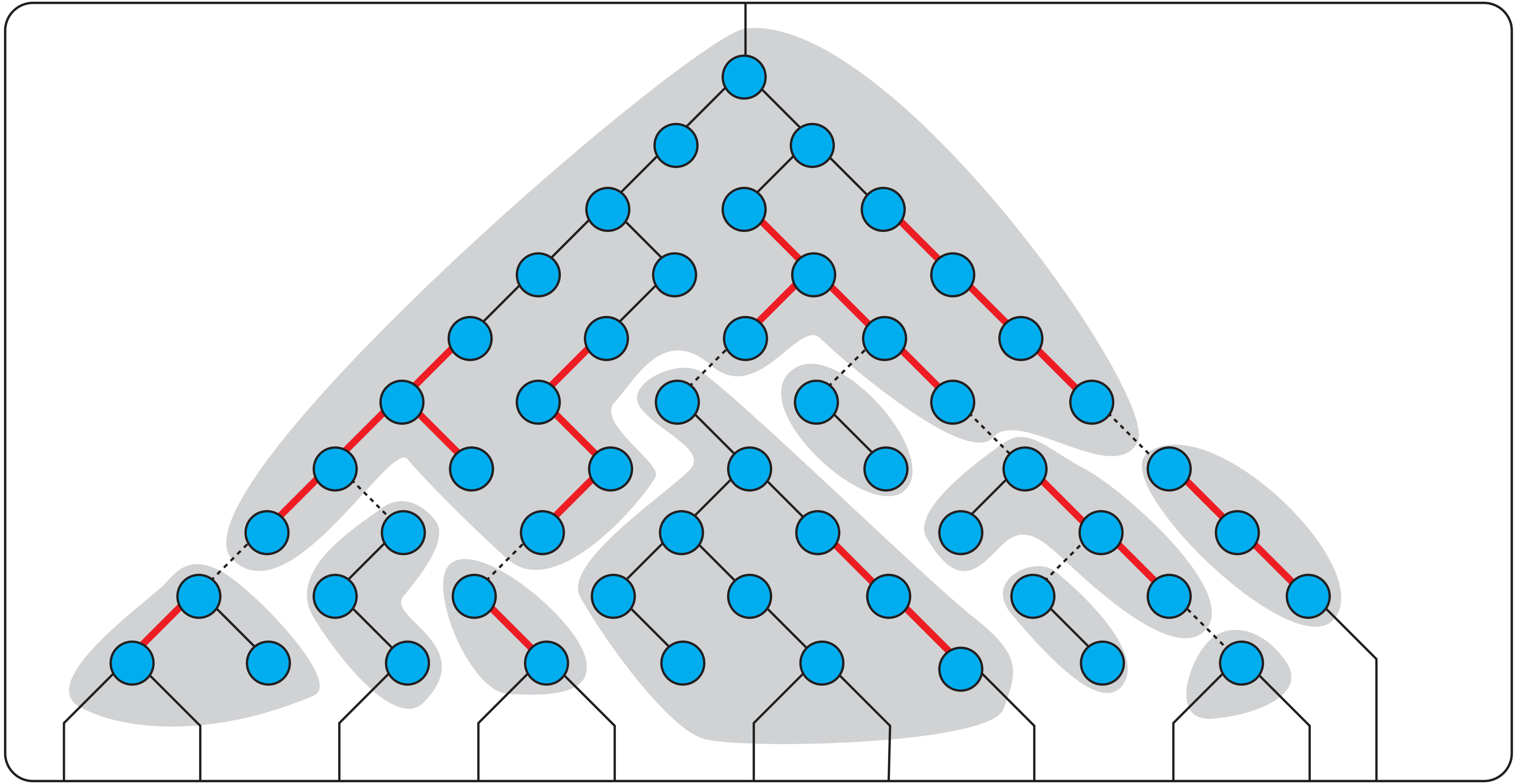


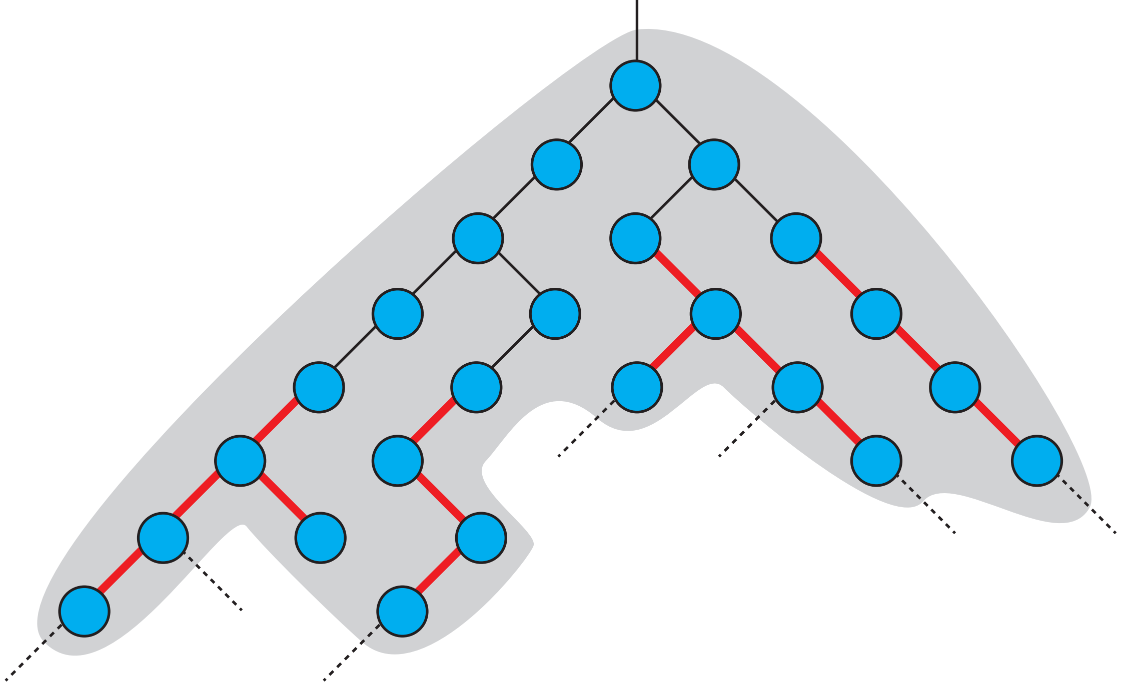
its shallow unfolding

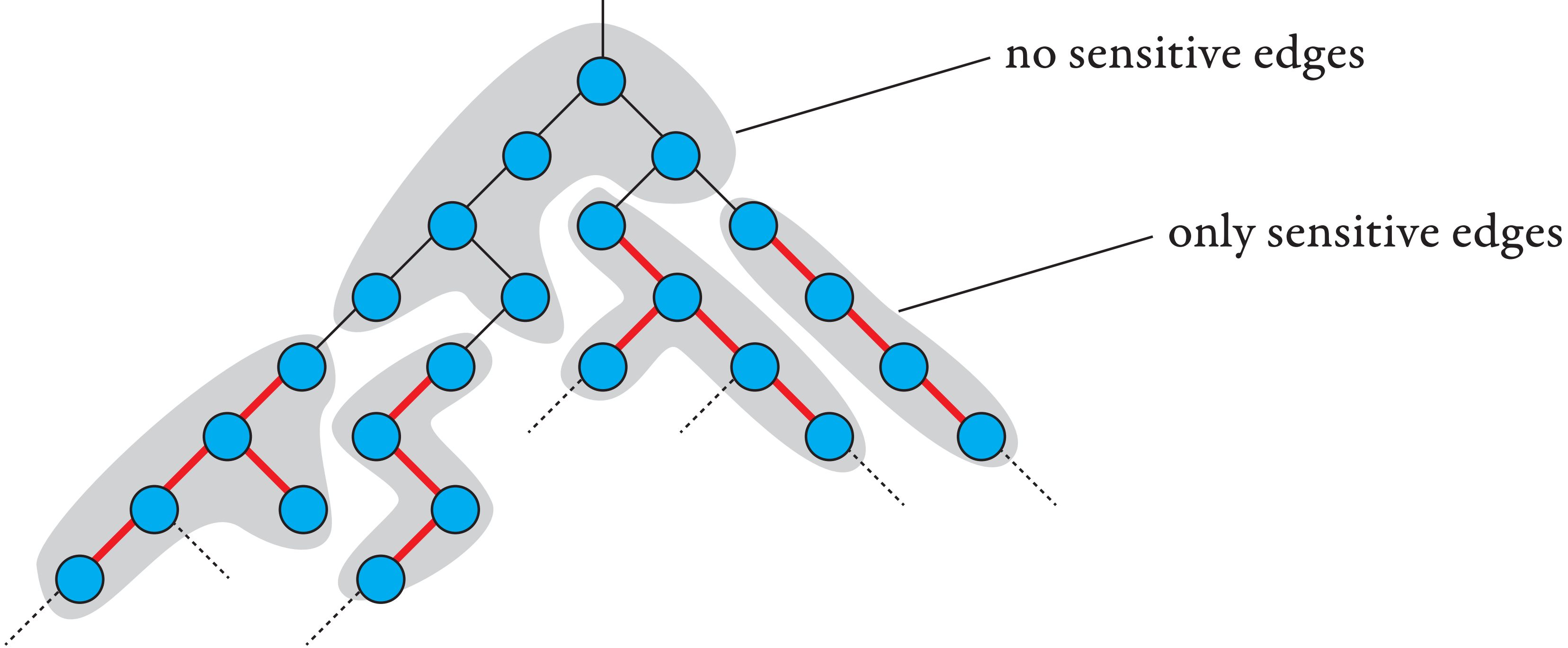


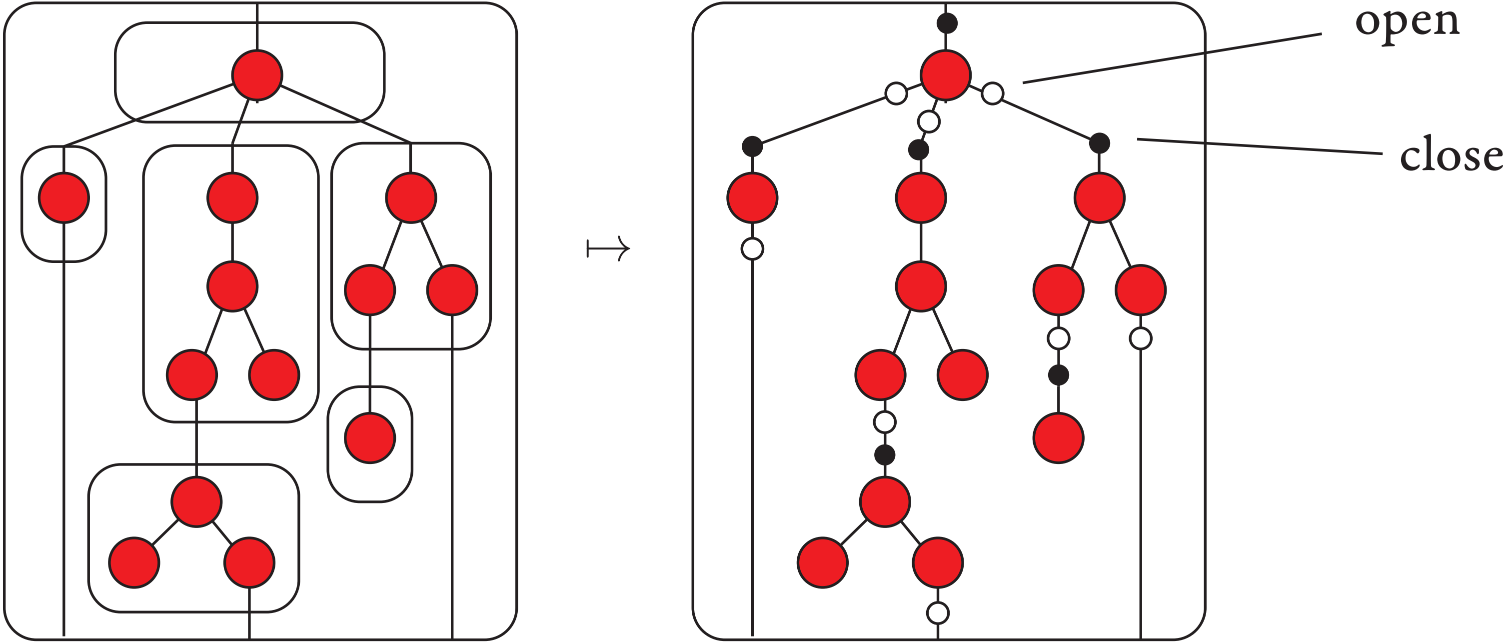

twist of port 1
1 2 3
1 2 3

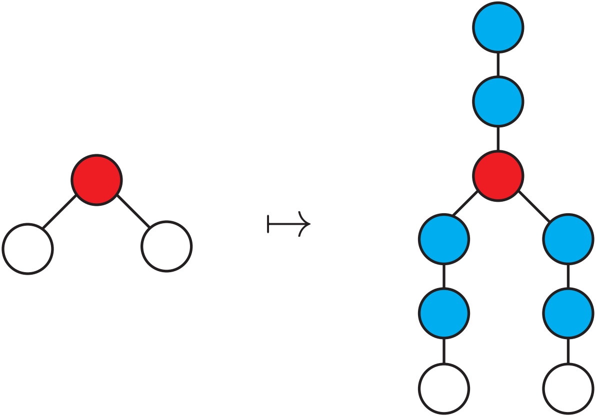

twist of port 1
1 2
1 2

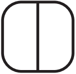


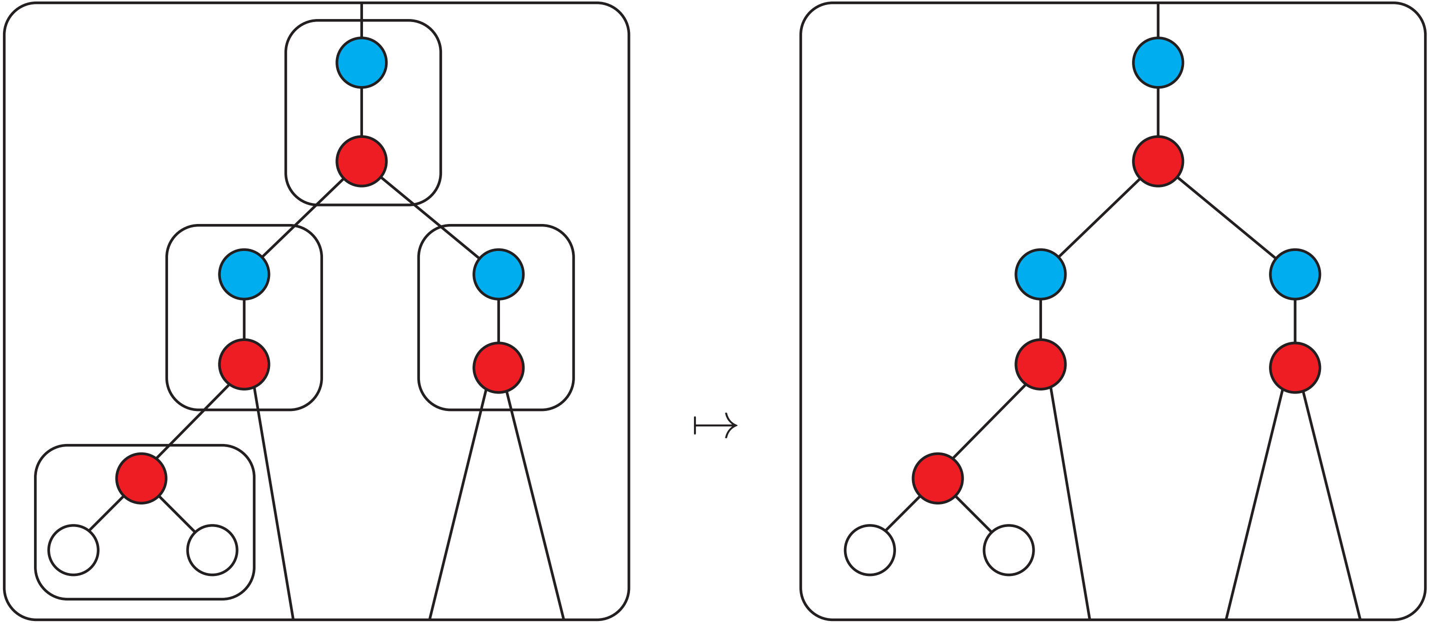


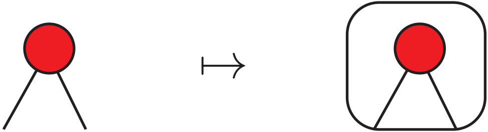

no sensitive edges
only sensitive edges

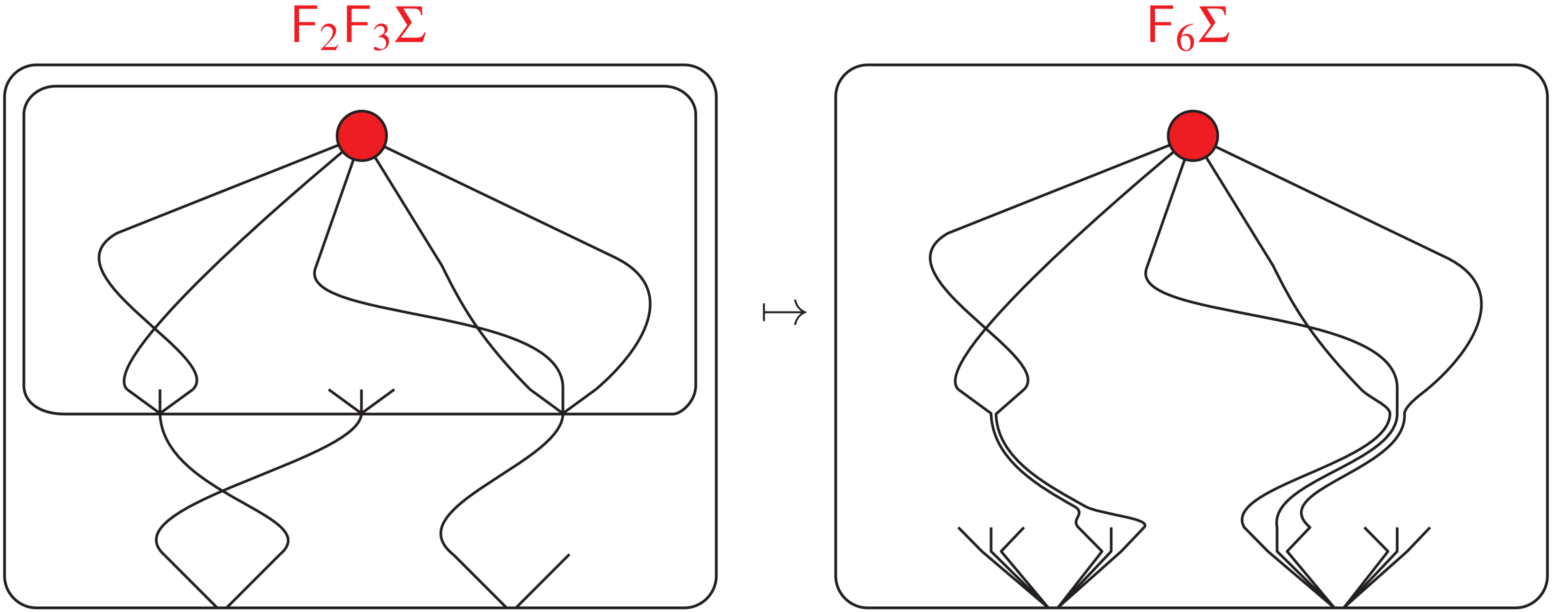

$\mapsto$
open
close



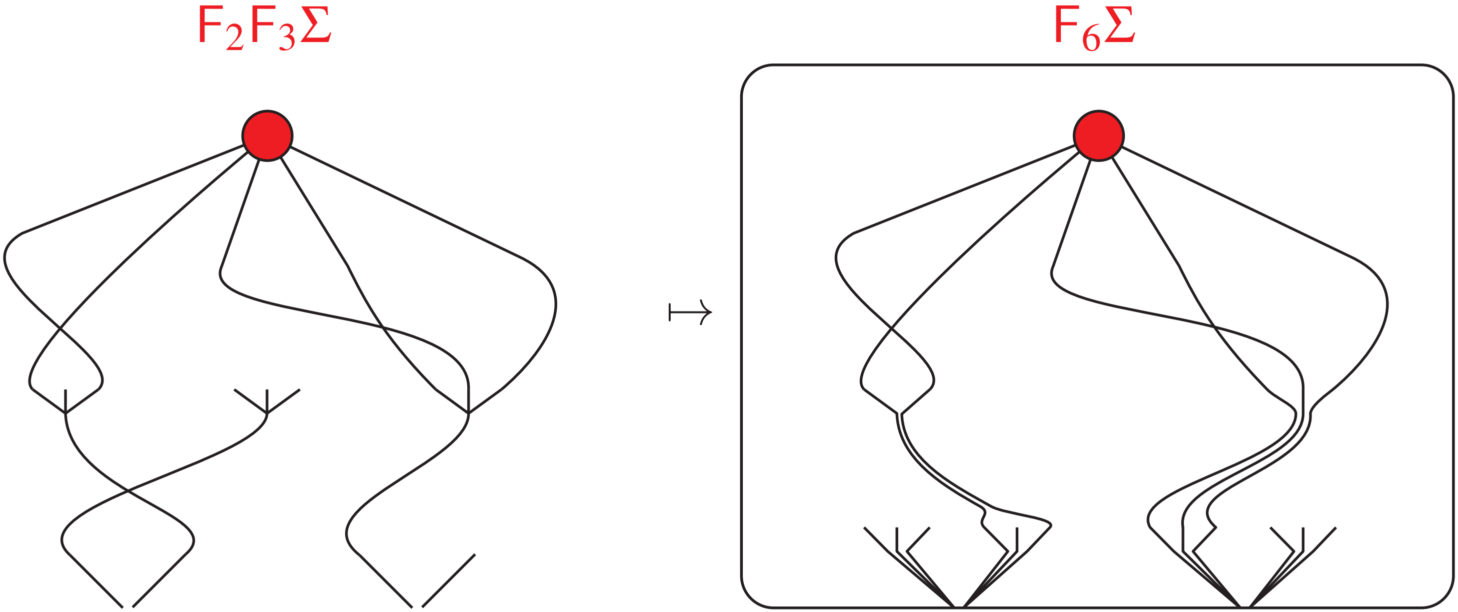


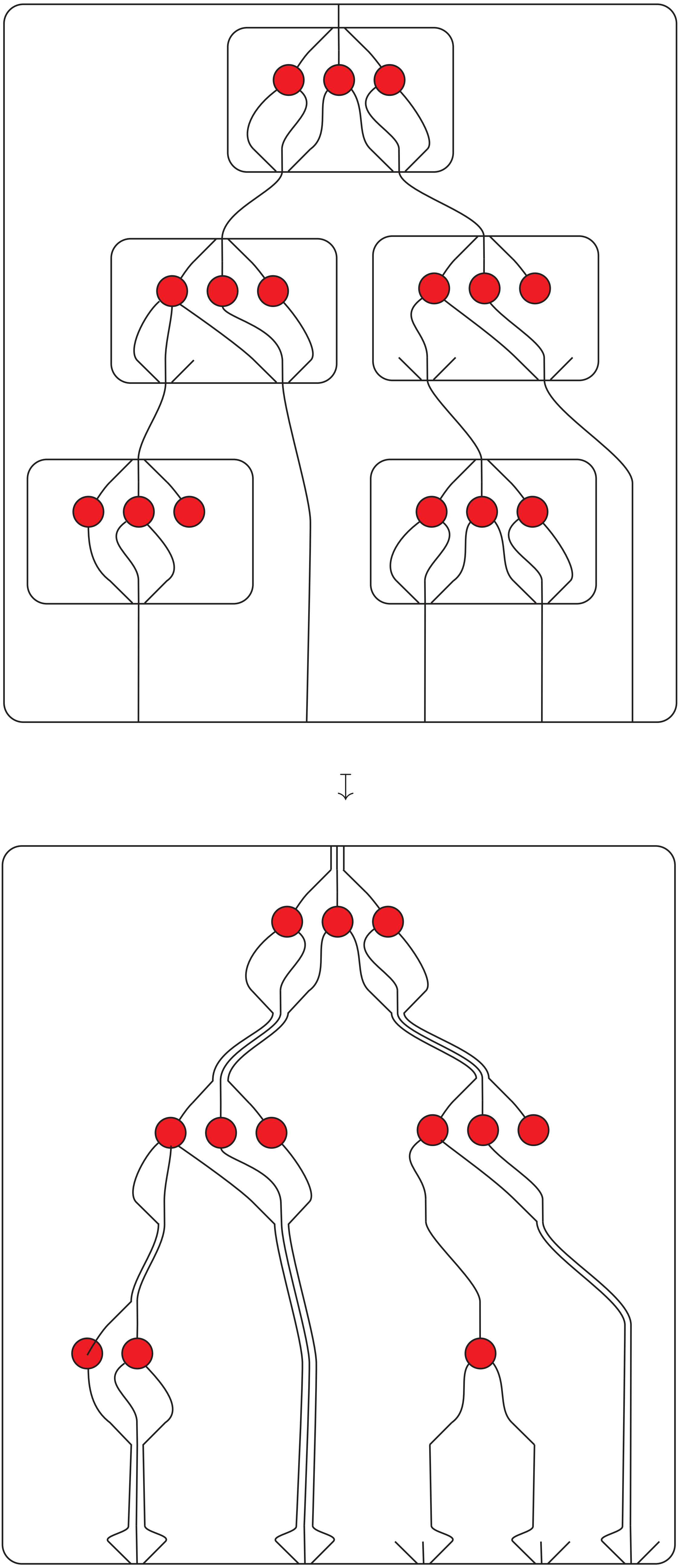


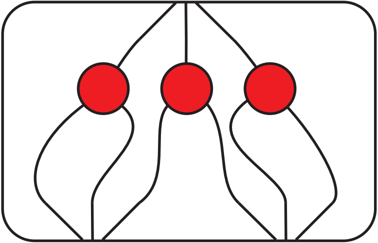


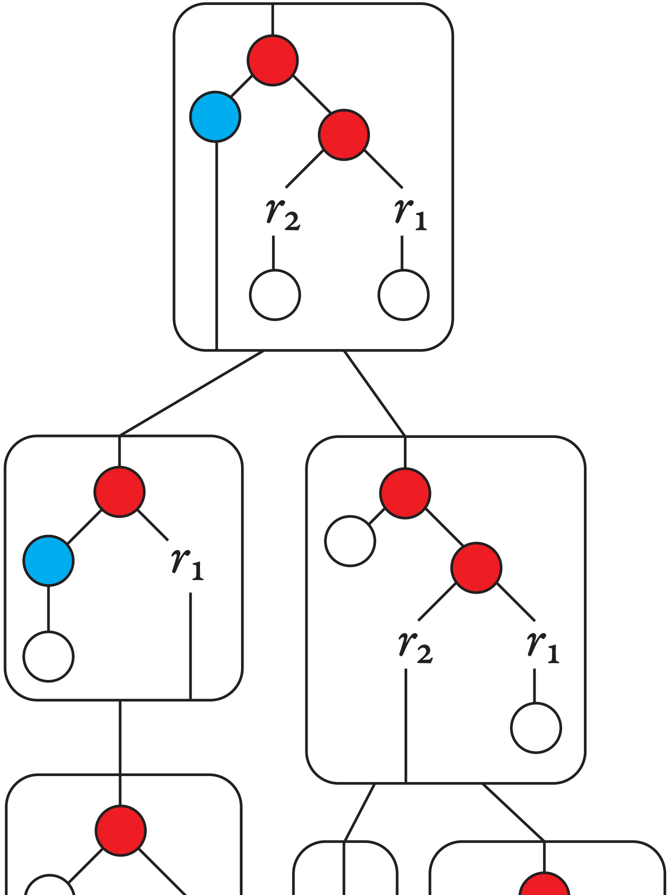


$F_2F_3\Sigma$
$\mapsto$
$F_6\Sigma$

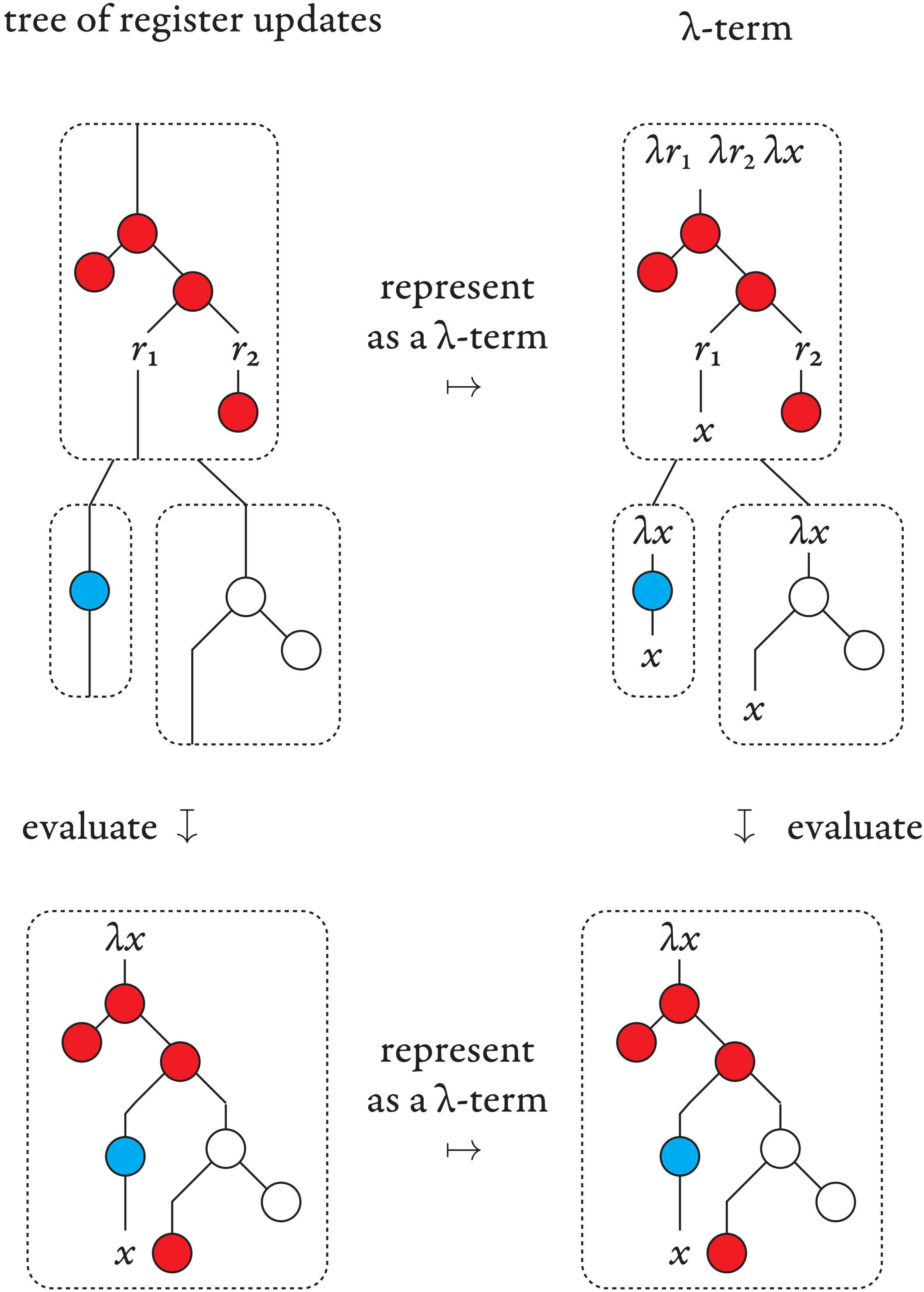

$F_2F_3\Sigma$
$\mapsto$
$F_6\Sigma$





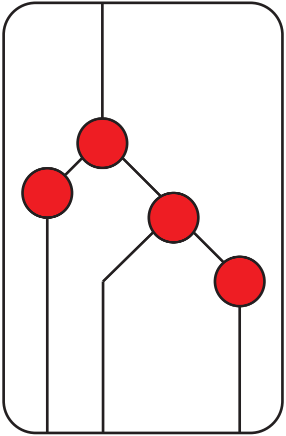


r_2
r_1
r_1
r_2
r_1

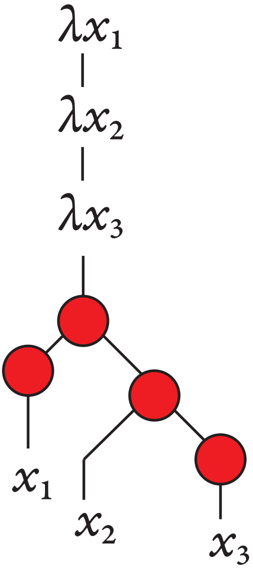


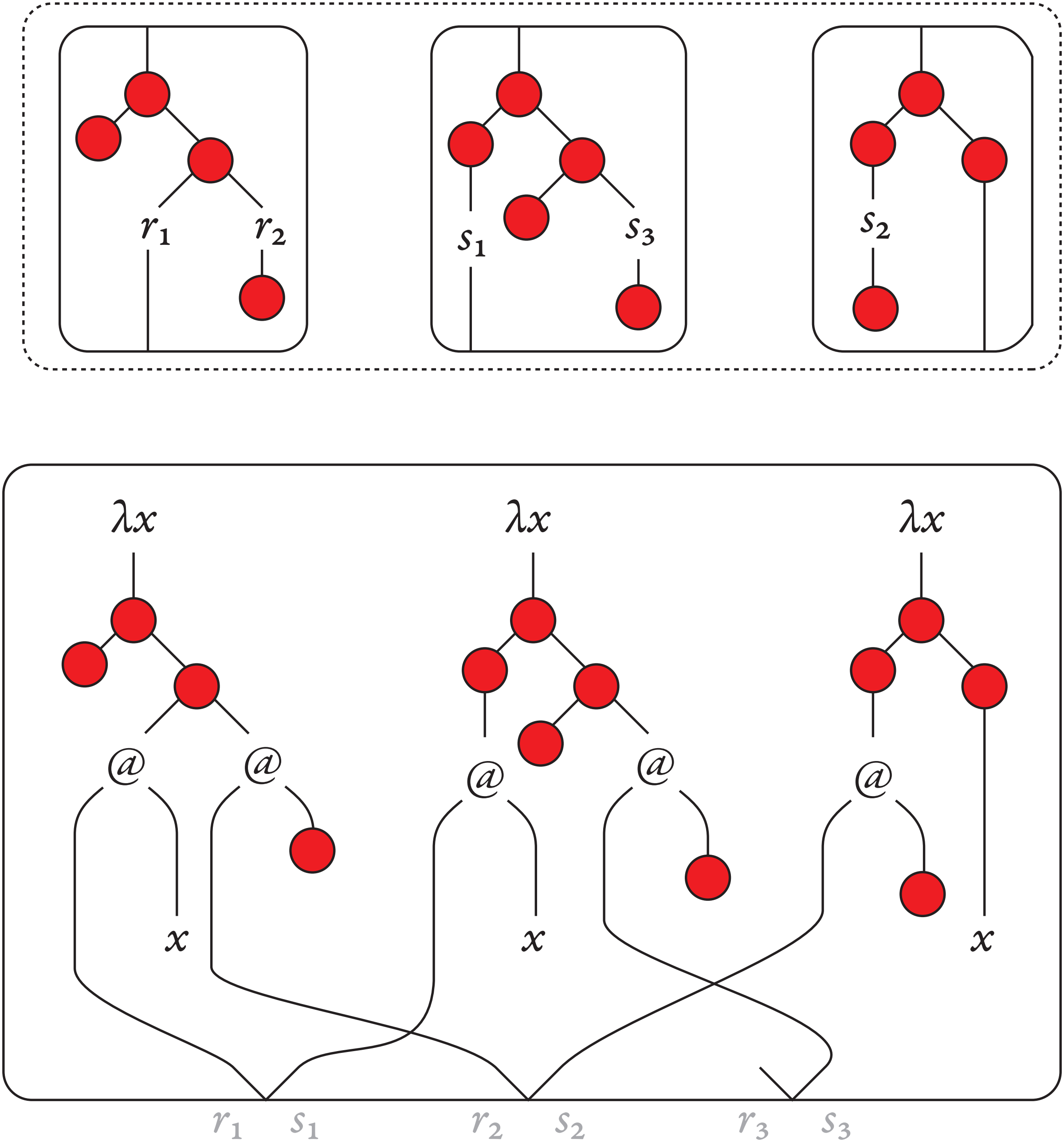
tree of register updates
λ-term
r_1
r_2
represent as a λ-term ↦
$\lambda r_1 \; \lambda r_2 \, \lambda x$
x
λx
evaluate ↧
↧ evaluate

a term

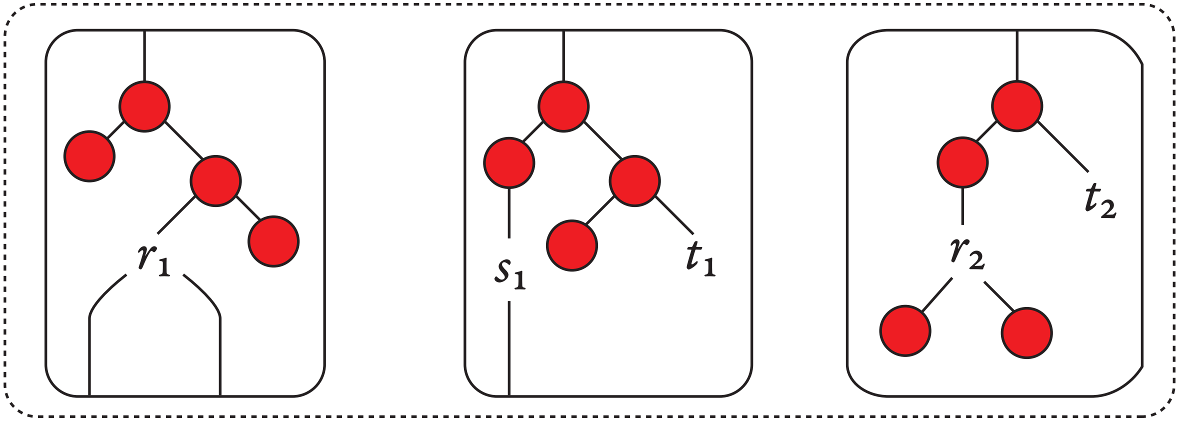


its λ-representation




r_1
r_2
s_1
s_3
s_2
λx
λx
λx
@
@
@
@
@
x
x
x
r_1 s_1
r_2 s_2
r_3 s_3

a register update



its λ-representation