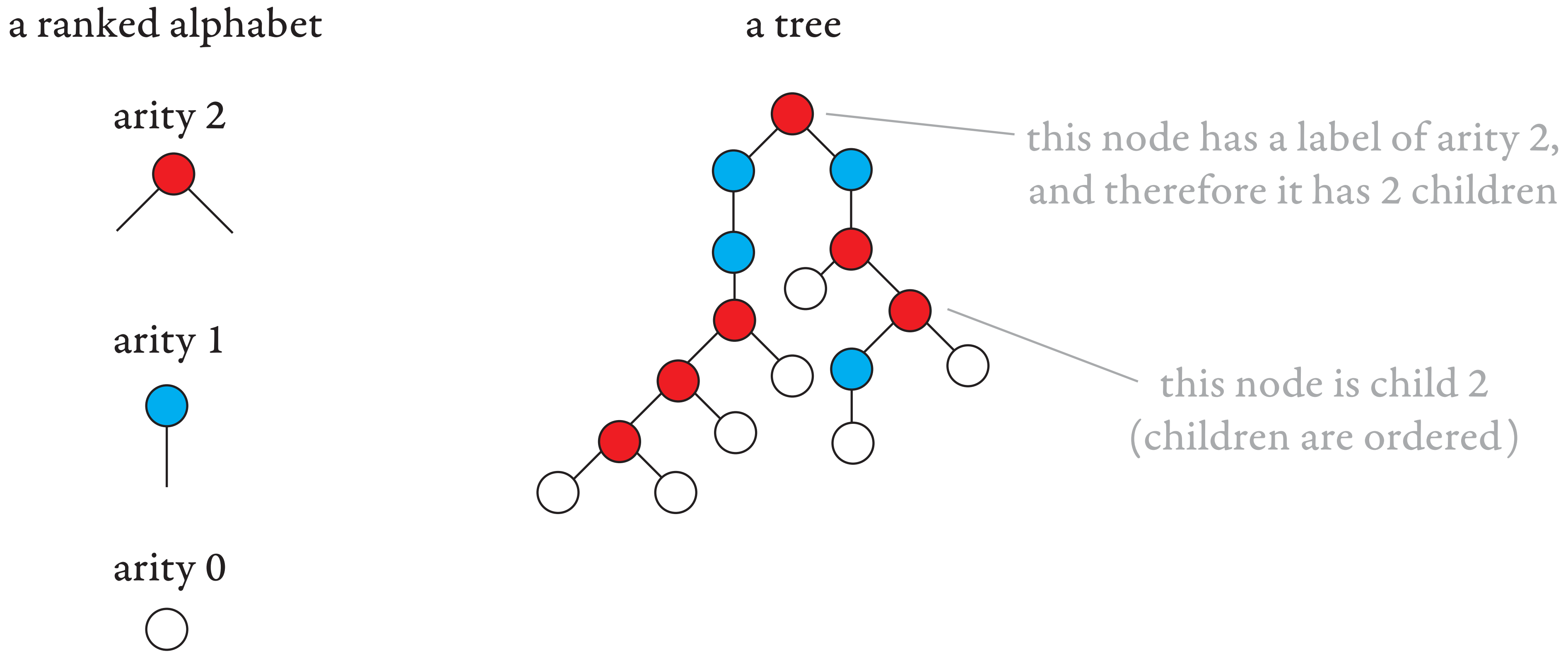
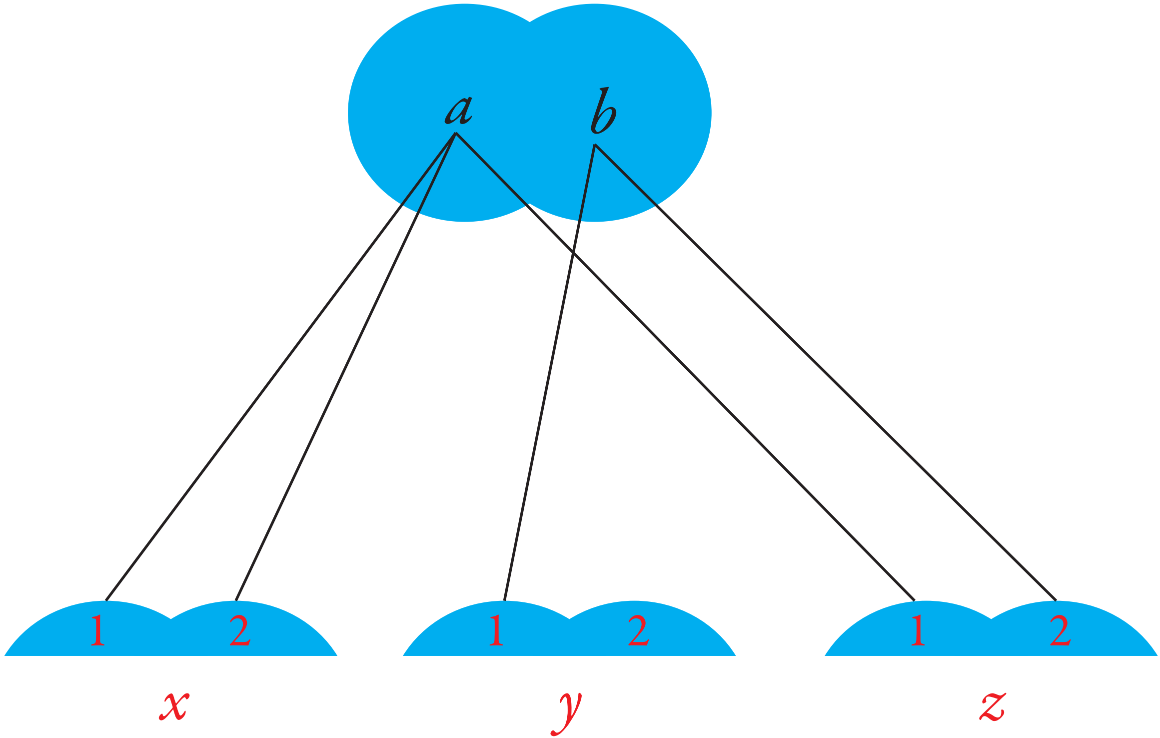
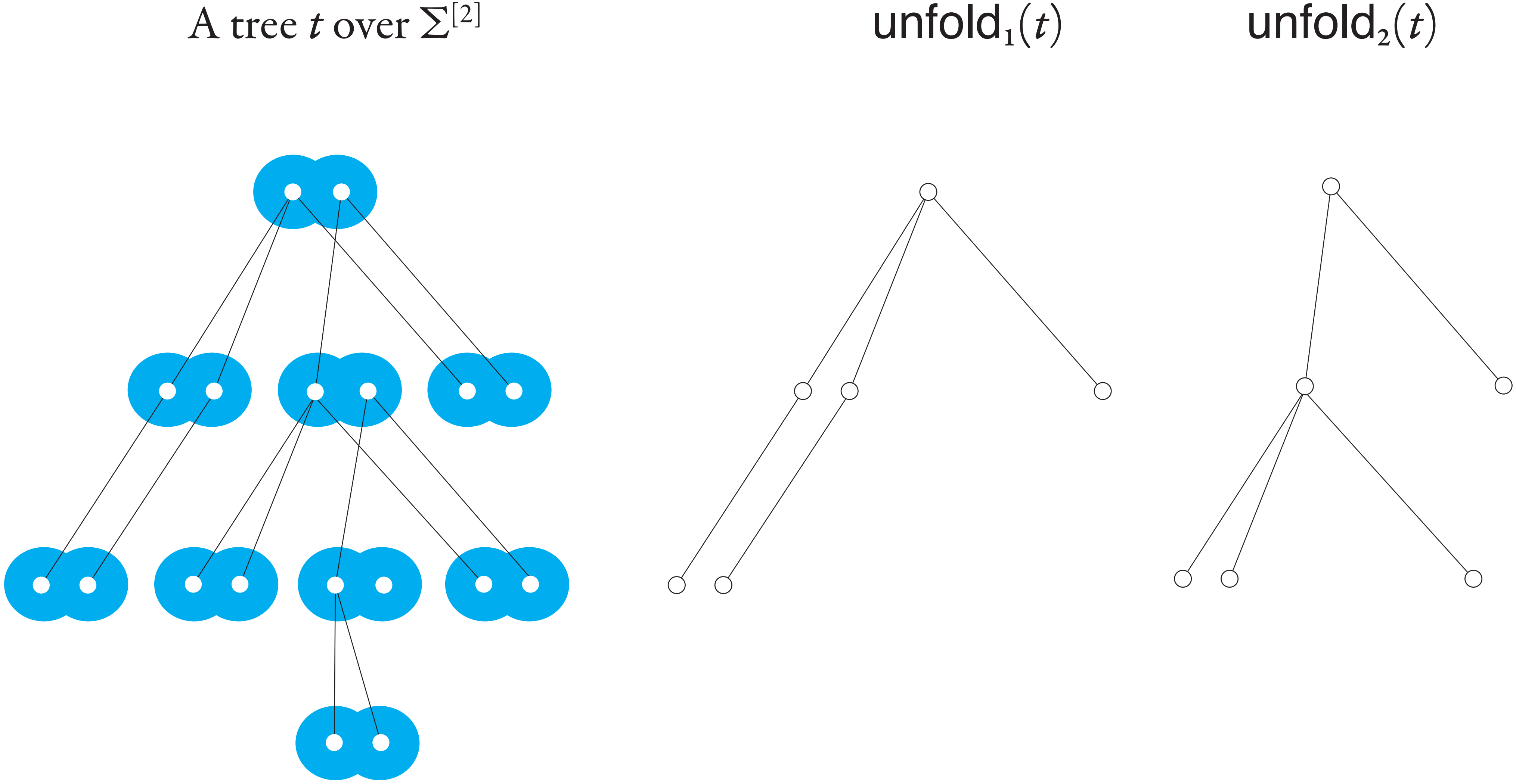
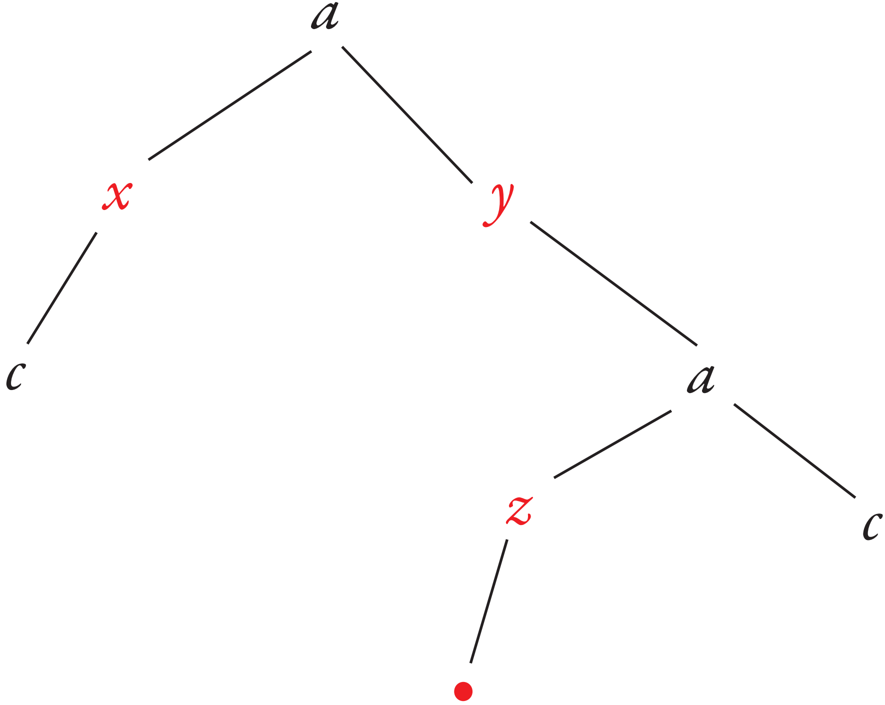
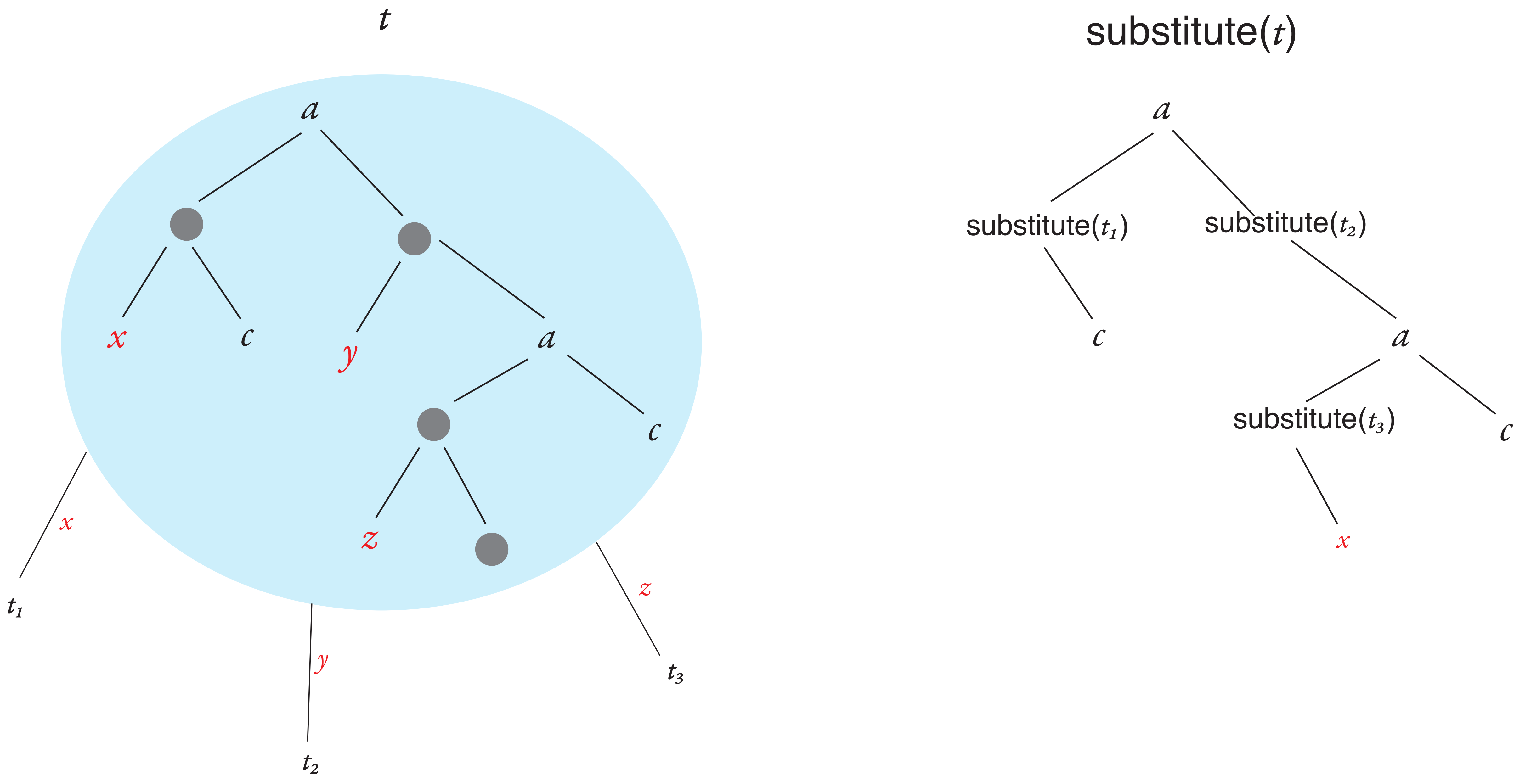

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)

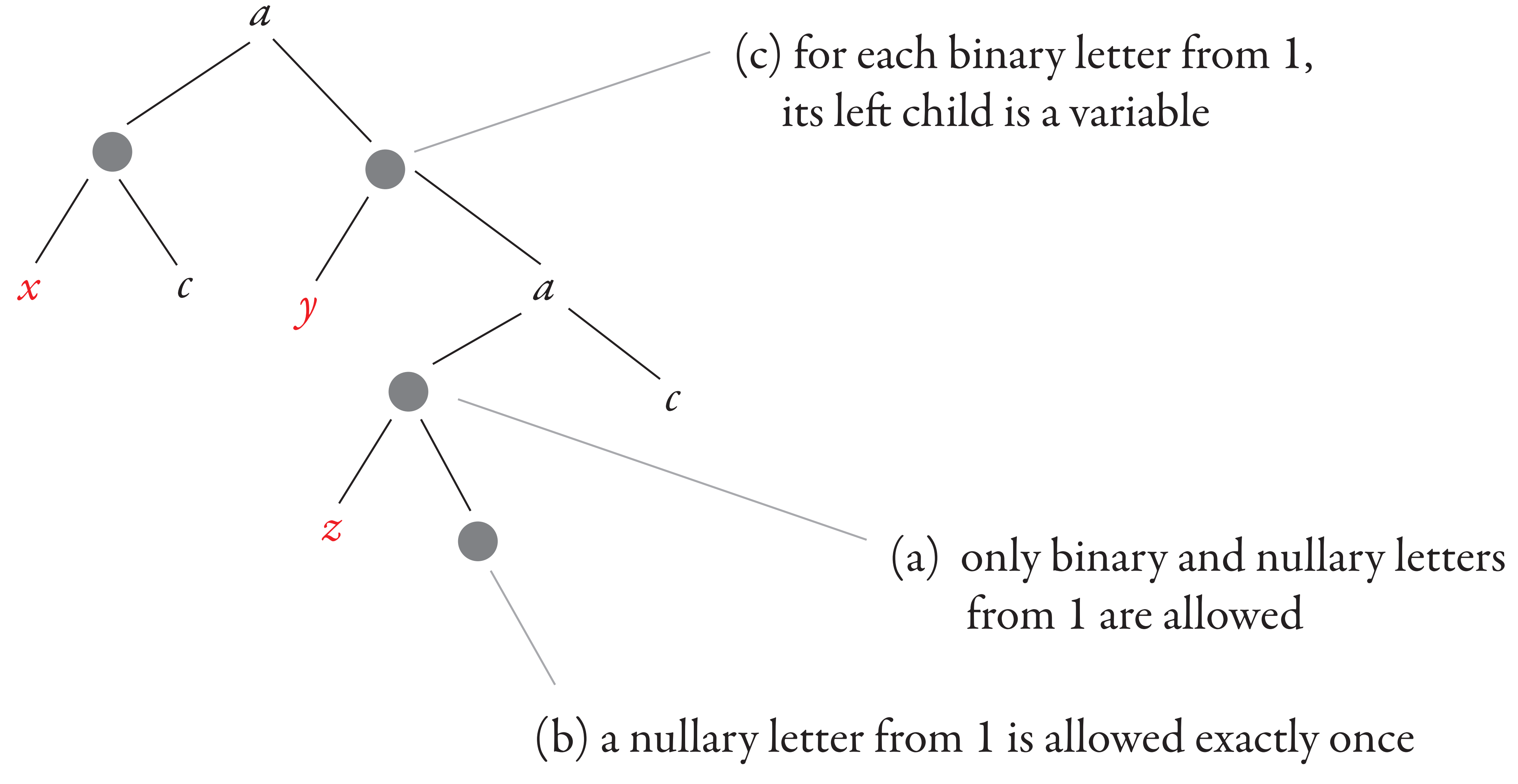

a
b
1
2
1
2
1
2
x
y
z

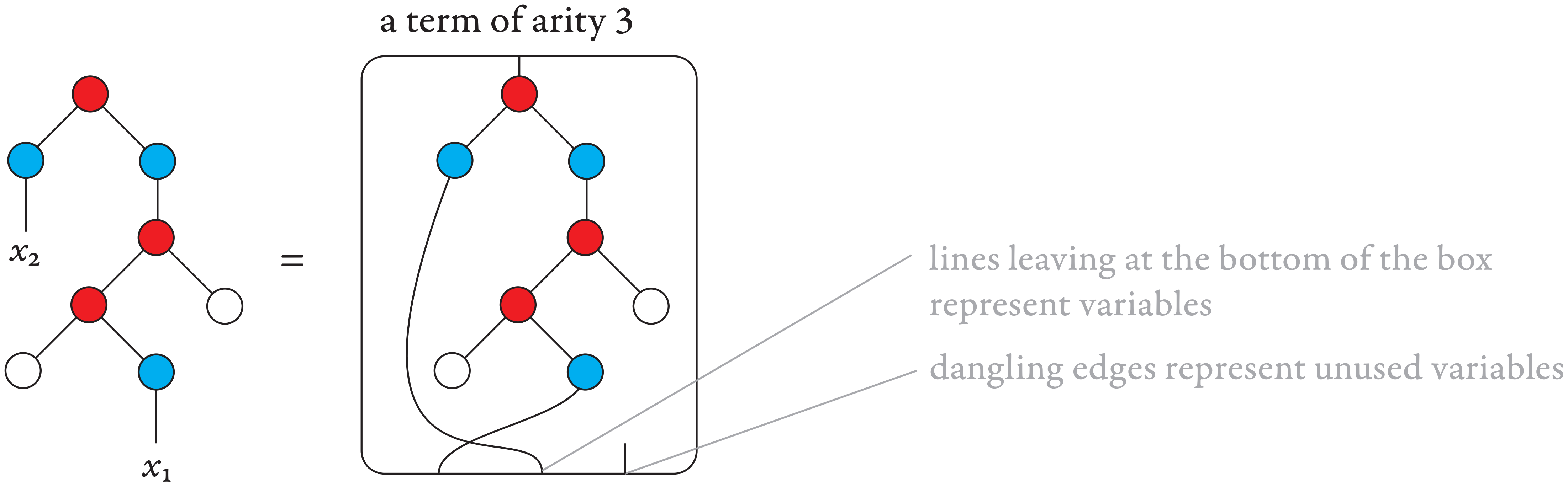

A tree t over $\Sigma^{[2]}$
$\mathsf{unfold}_1(t)$
$\mathsf{unfold}_2(t)$

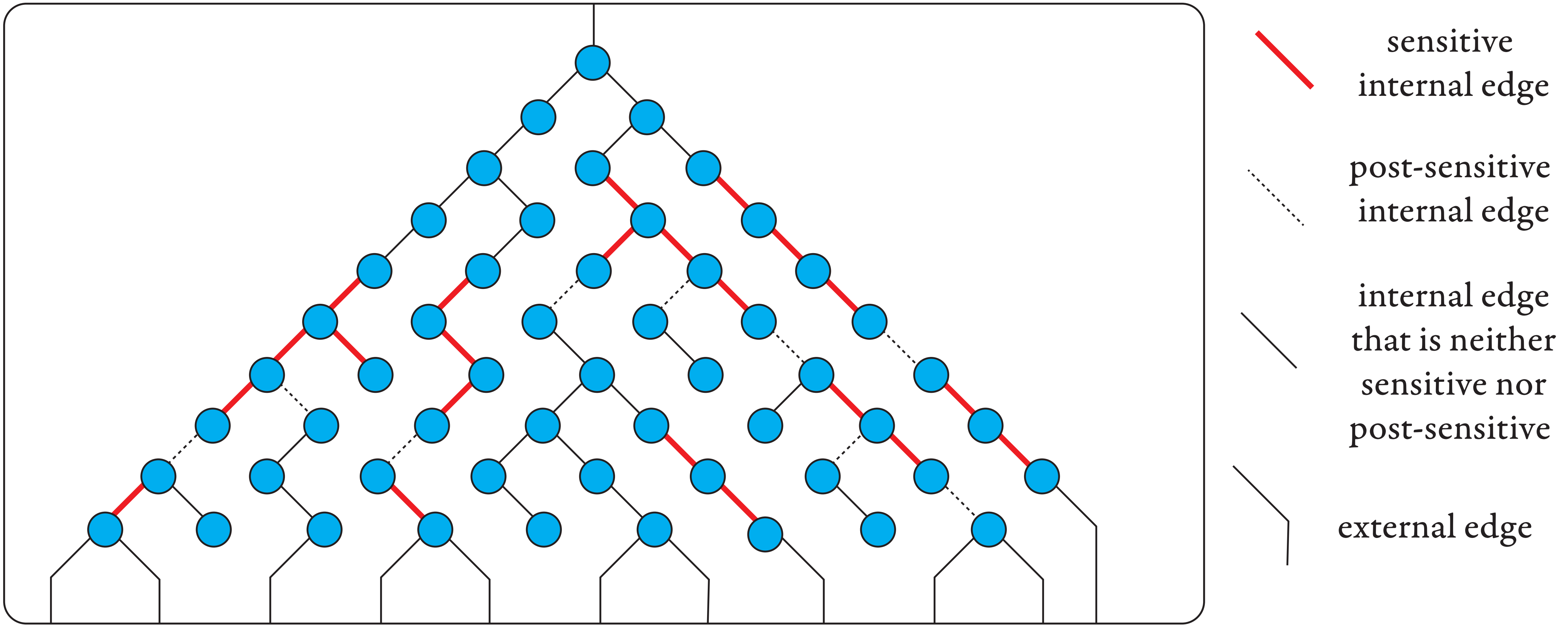

a
x
y
c
a
z
c

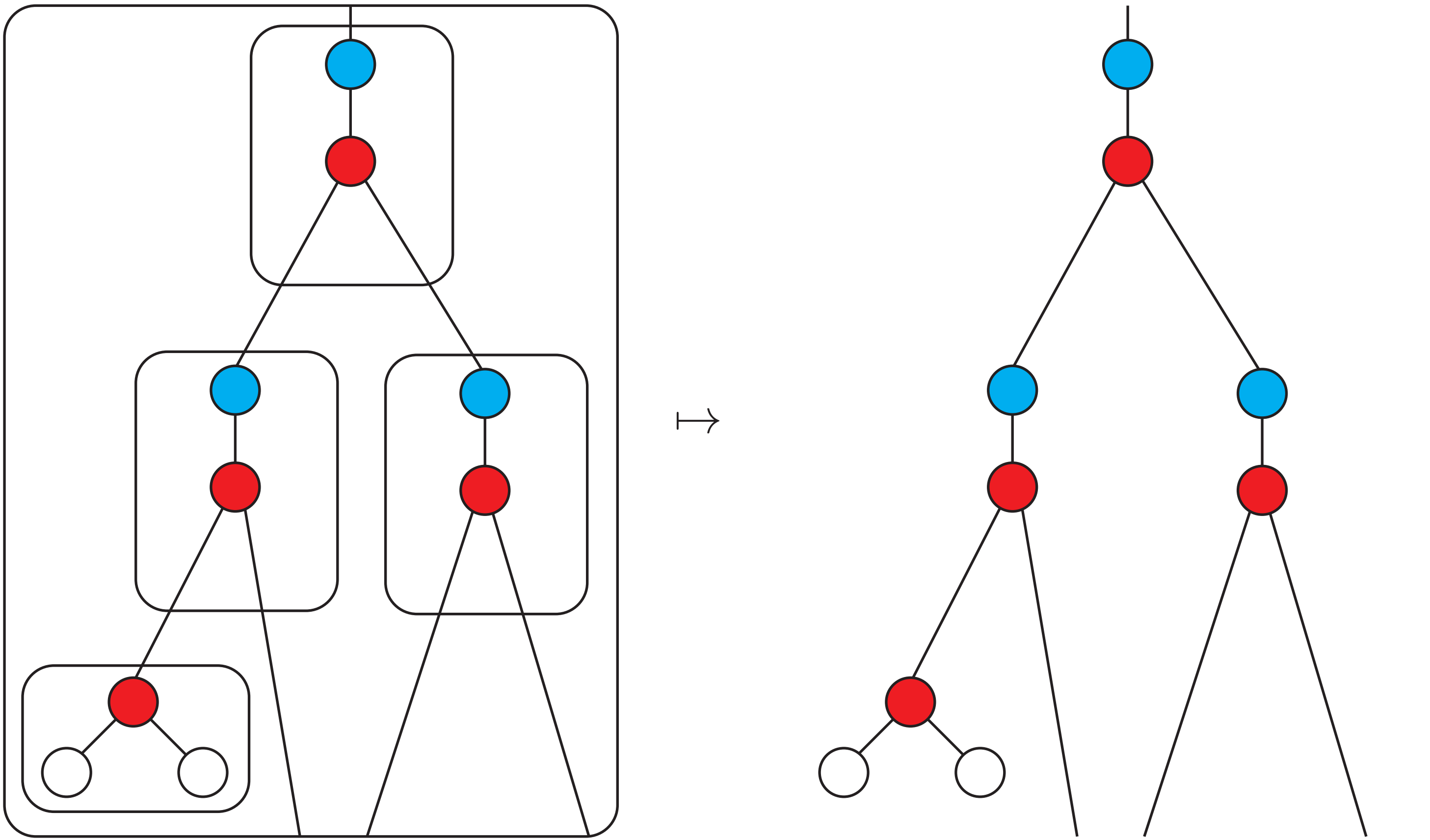


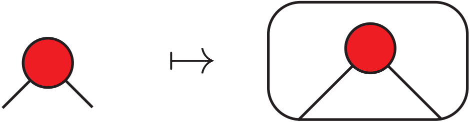
t
a
x
c
y
a
c
z
x
t1
y
t2
z
t3
substitute(t)
a
substitute(t1)
substitute(t2)
c
a
substitute(t3)
c
x


a
(c) for each binary letter from 1,
its left child is a variable
x
c
y
a
c
z
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

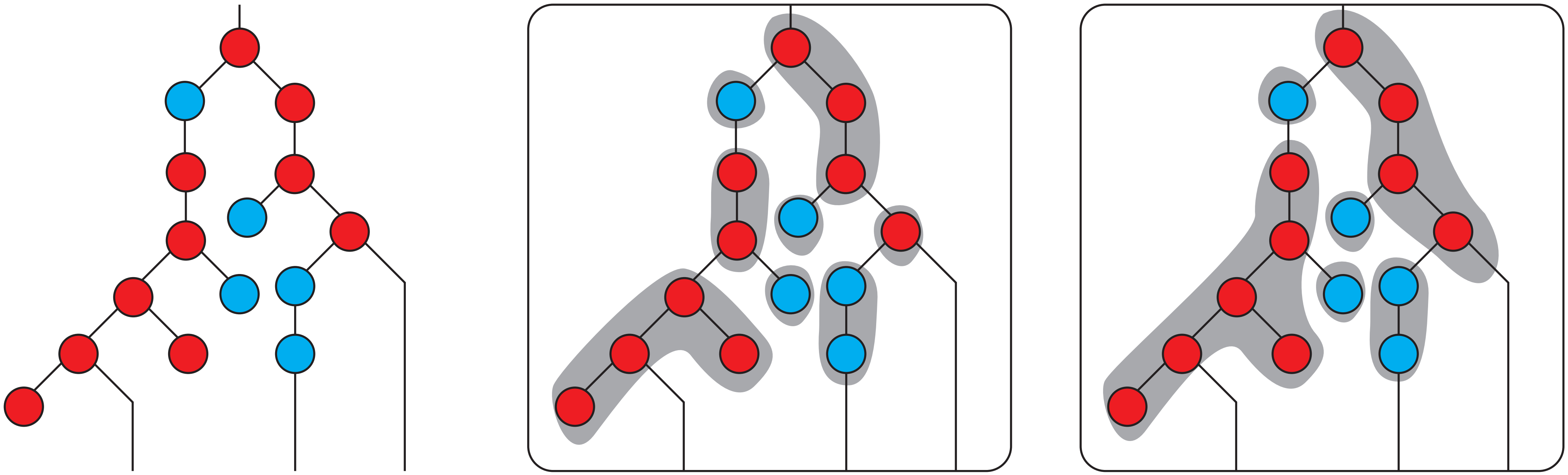

a term of arity 3
x_2
x_1
=
lines leaving at the bottom of the box represent variables
dangling edges represent unused variables

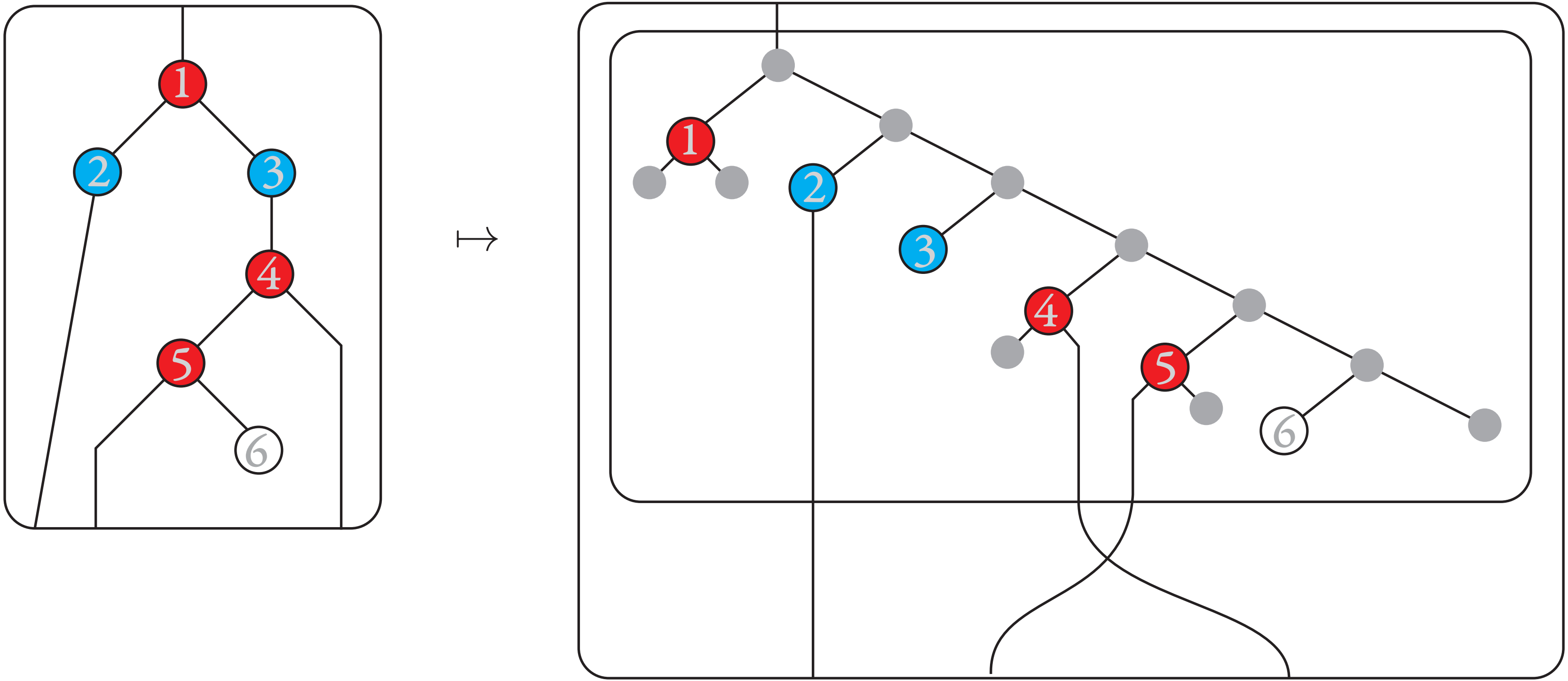

sensitive
internal edge
post-sensitive
internal edge
internal edge
that is neither
sensitive nor
post-sensitive
external edge



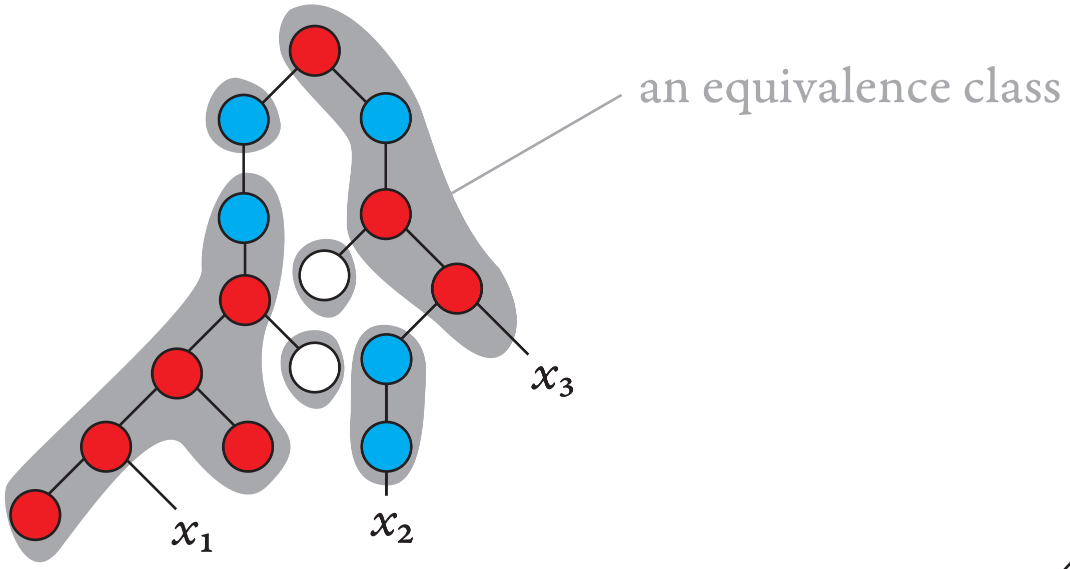


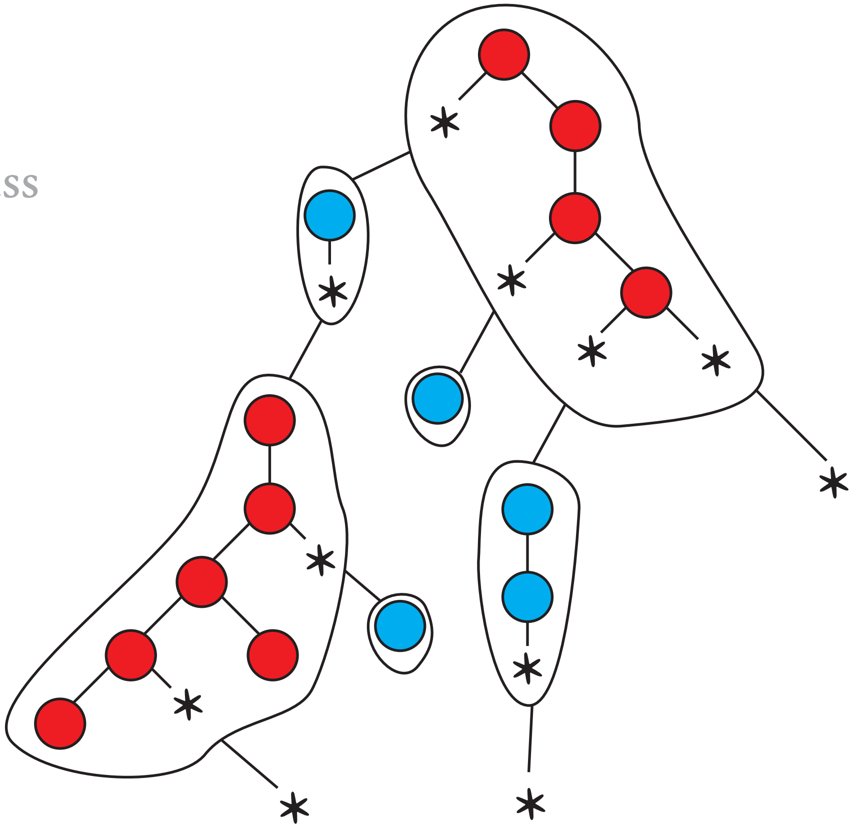
a term ancestor equivalence descendant equivalence



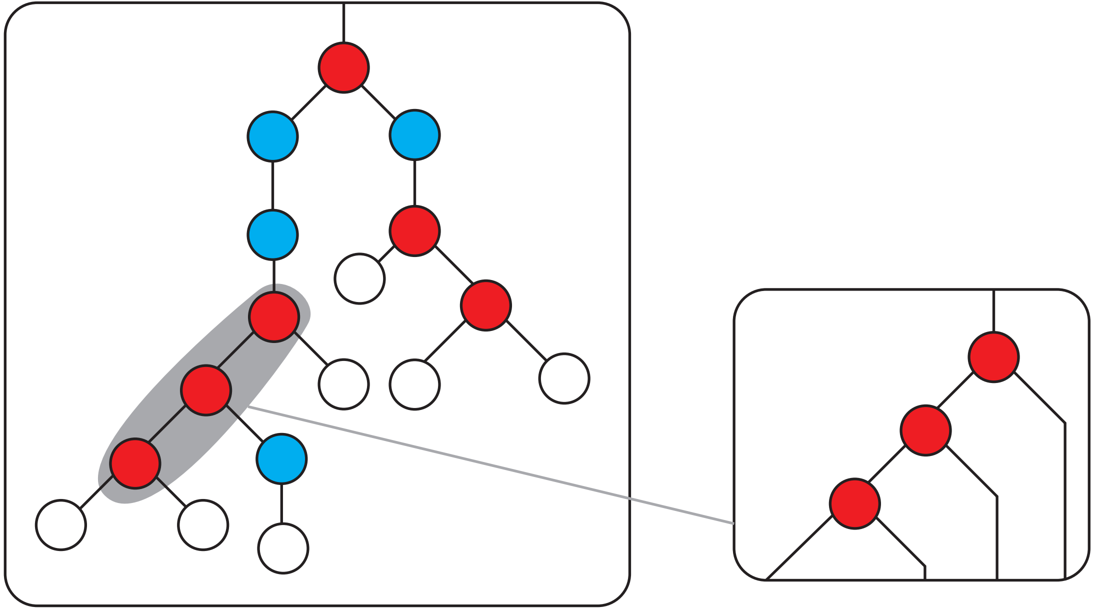

1
2
3
4
5
6
↦
1
2
3
4
5
6

a factorisation equivalence

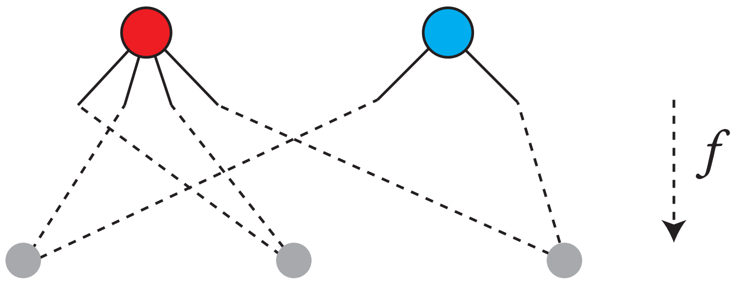




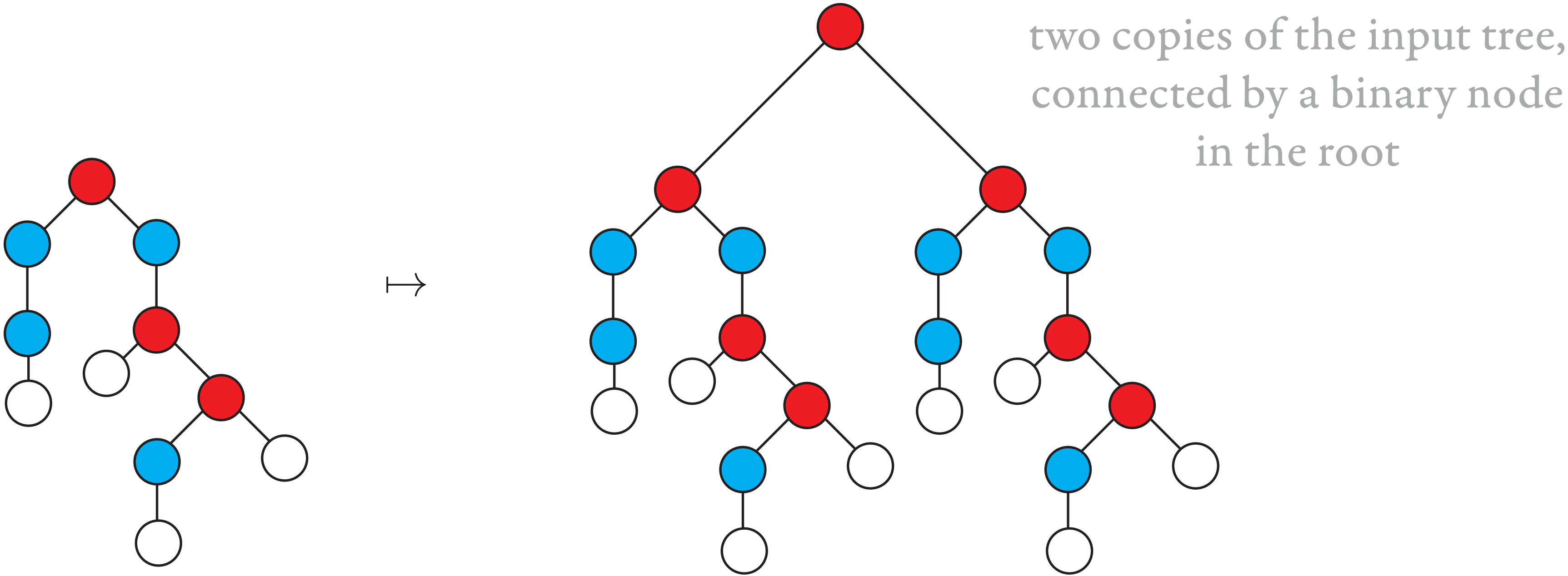
a tree

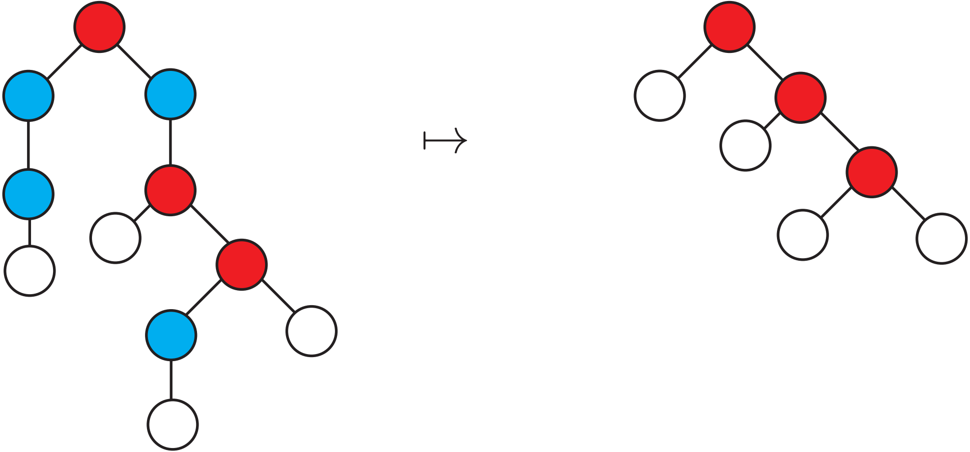


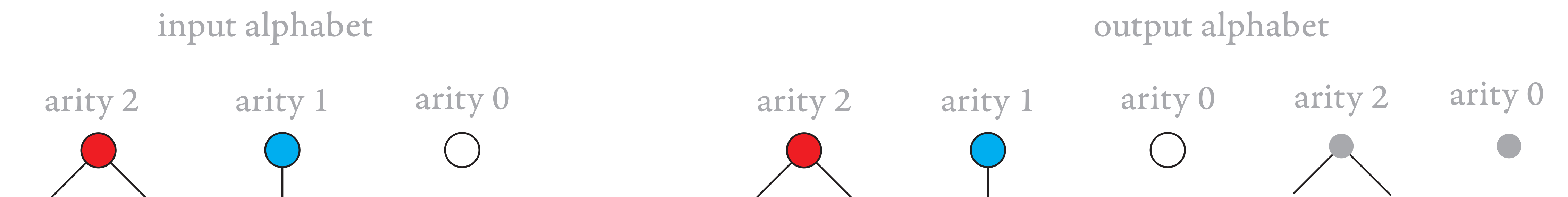
a term with 4 ports that represents part of the tree

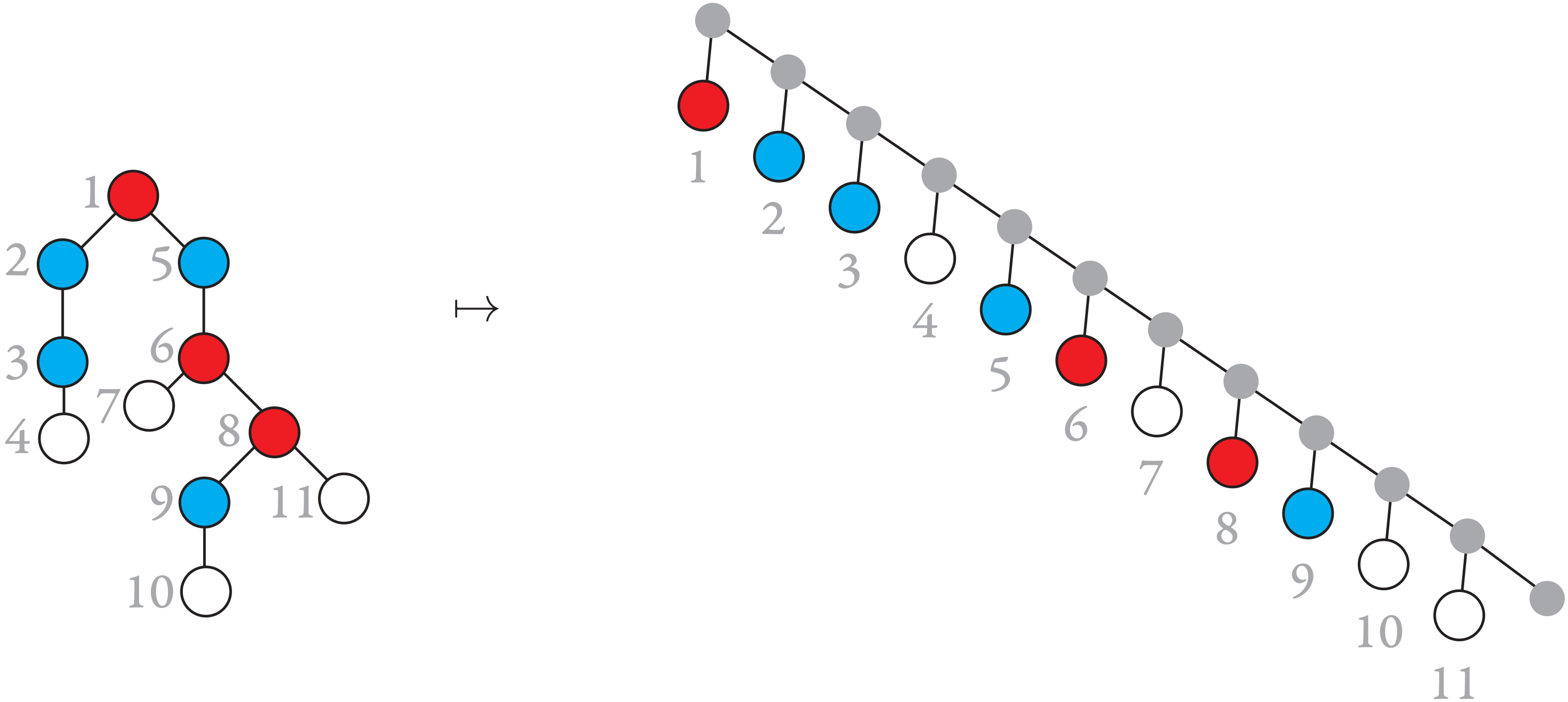

f

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0

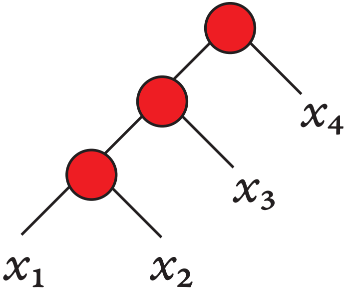

↦
two copies of the input tree,
connected by a binary node
in the root



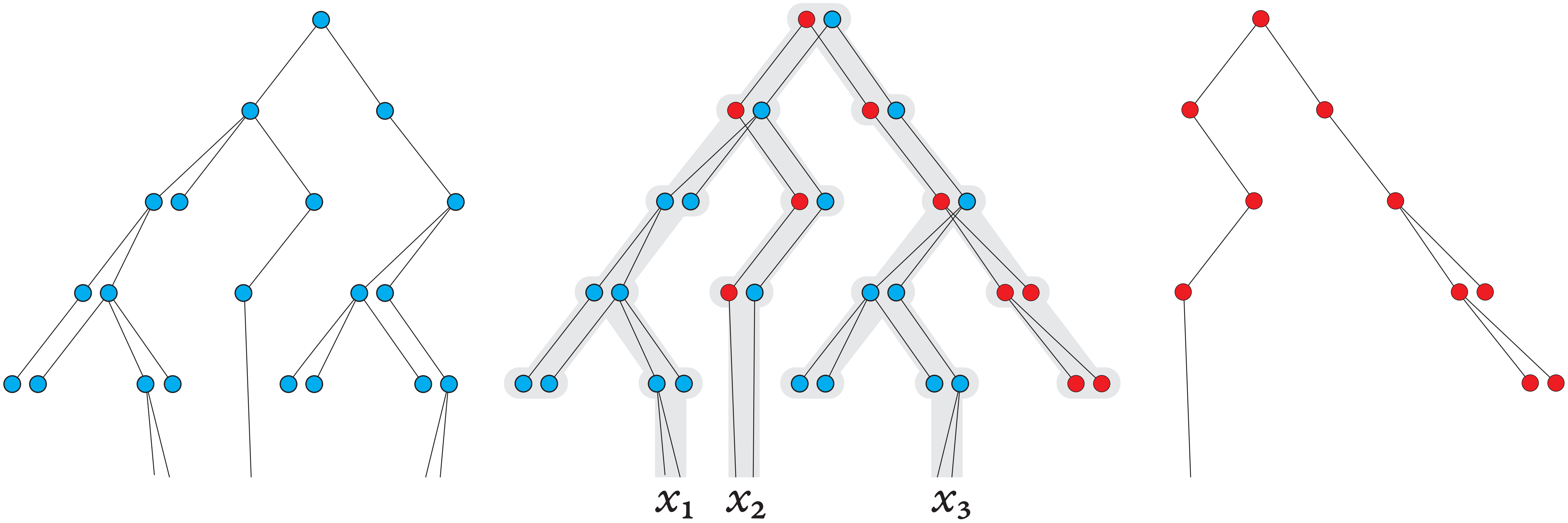

input alphabet
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 1
arity 0
arity 2
arity 0


1
2
3
4
5
6
7
8
9
10
11

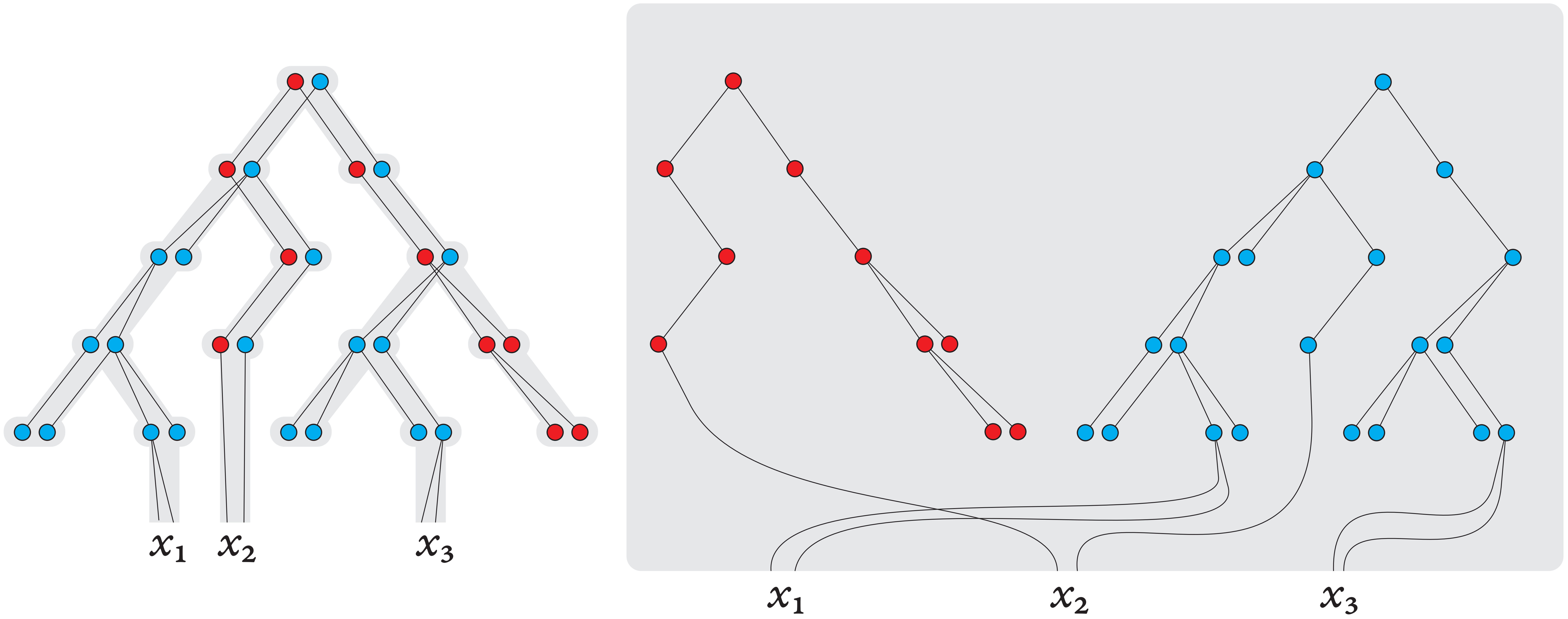
a term of arity 4

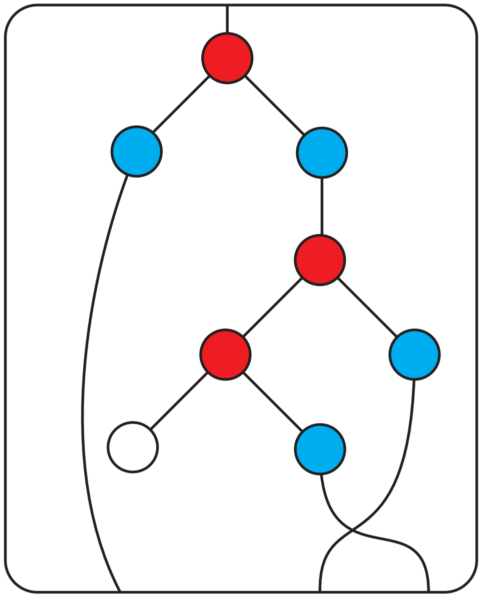


a term of arity 0



x_1
x_2
x_3

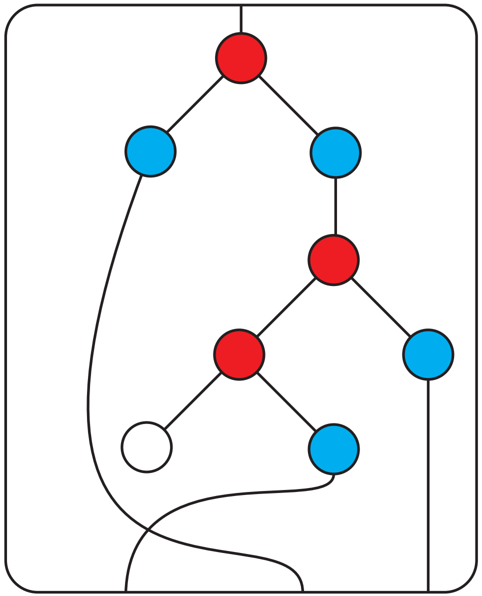

x_1
x_2
x_3
x_1
x_2
x_3



satisfies (*)

(*)

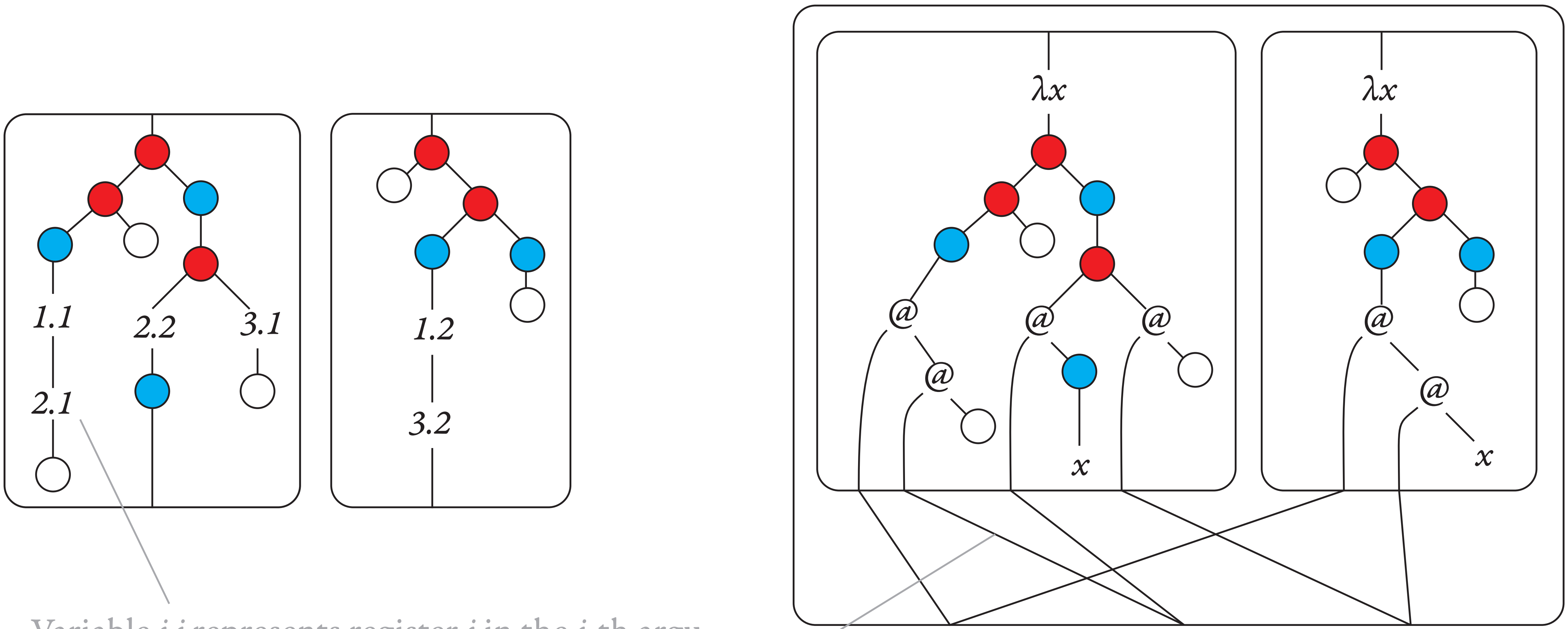
If the root has arity n, and $1 \leq i < j \leq n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root



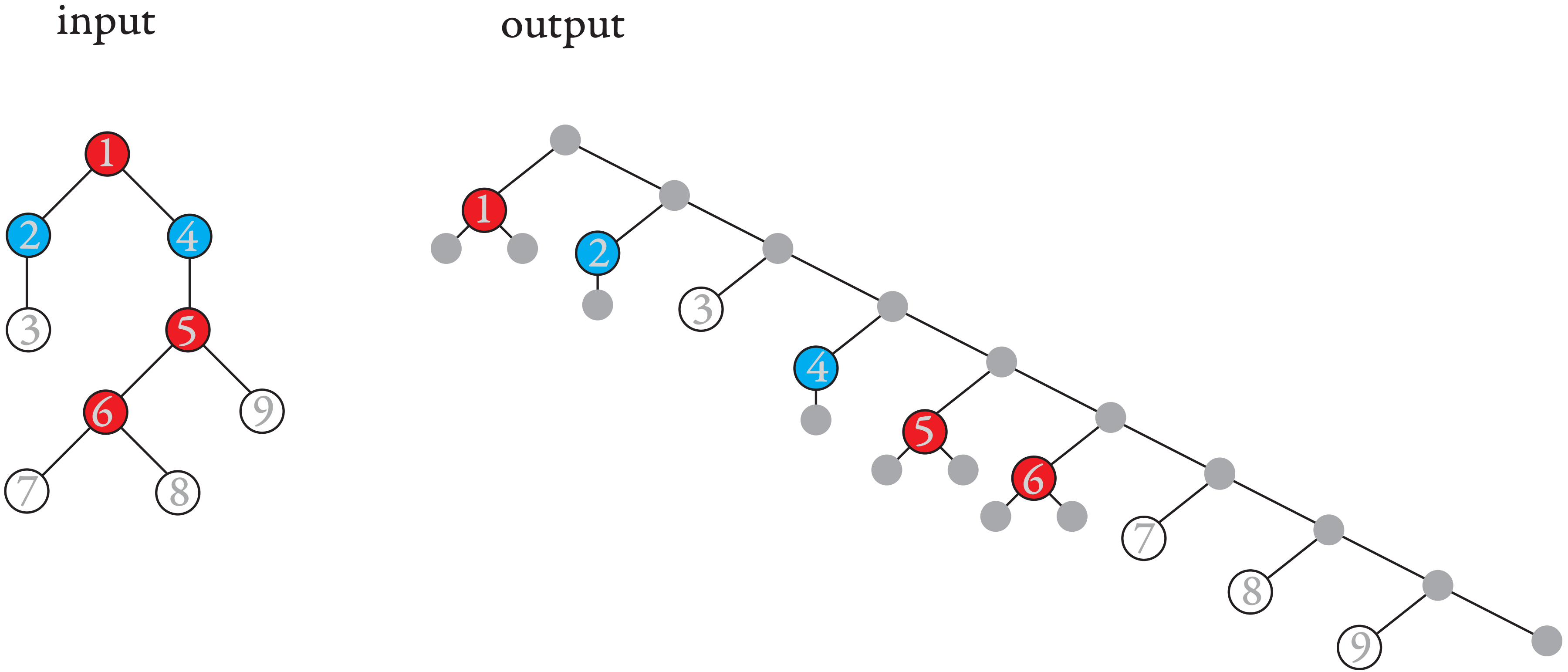
violates (*)

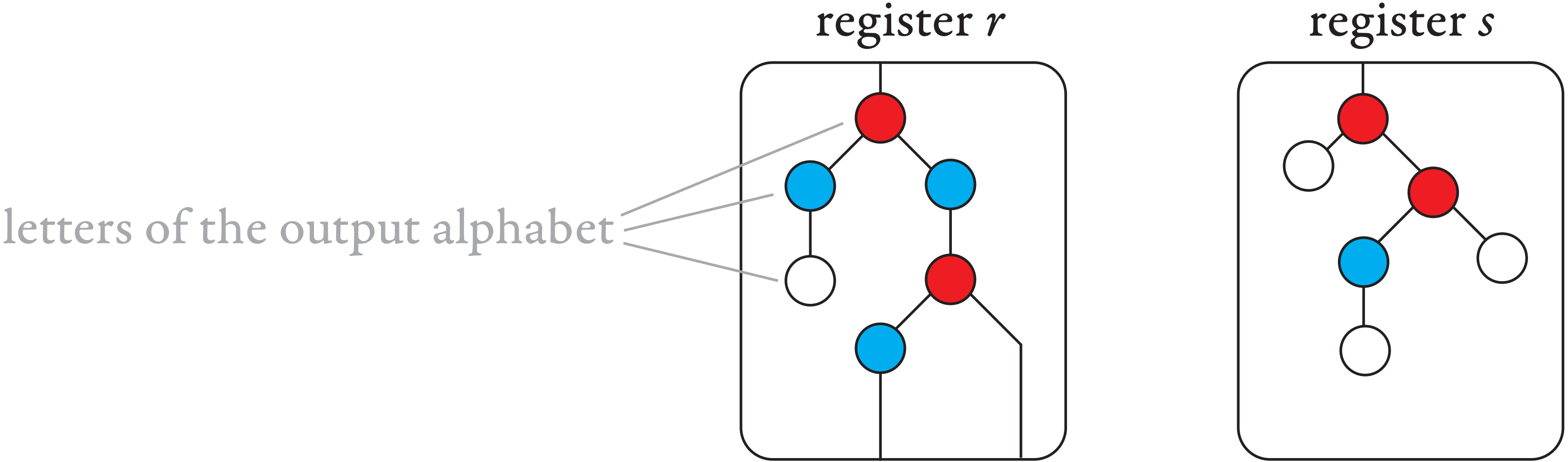
a register update

its dual

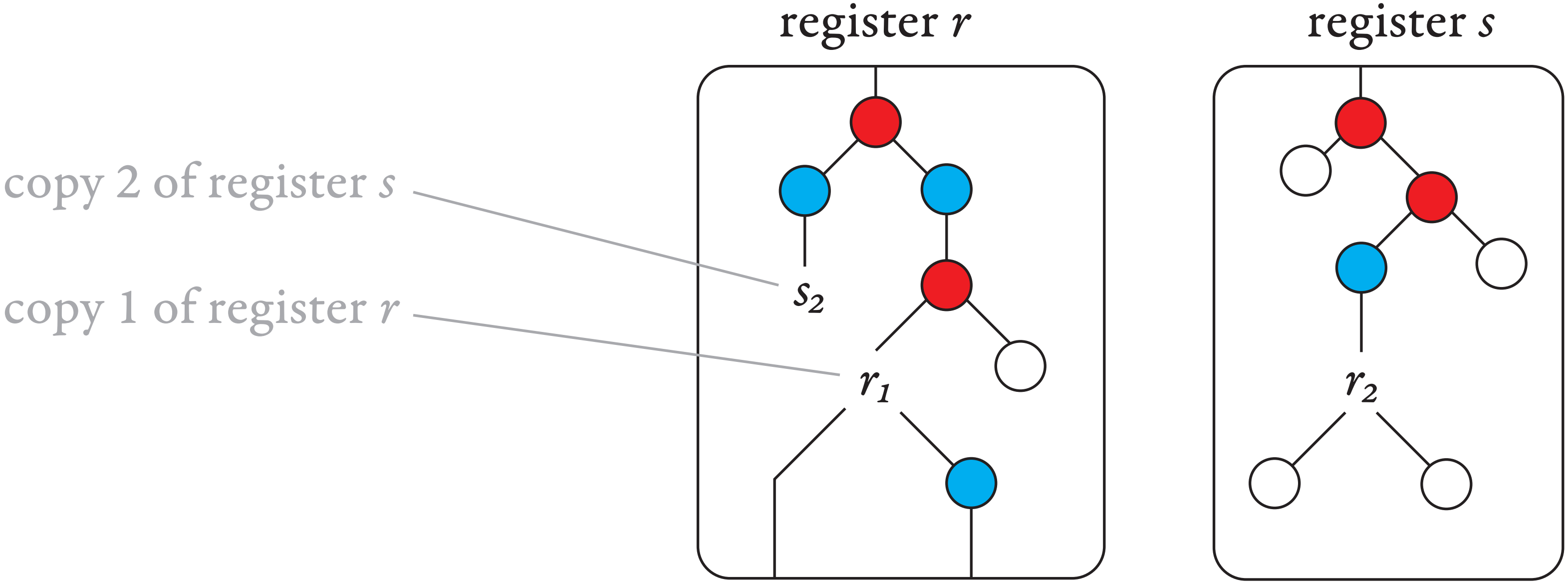


Variable *i.j* represents register *i* in the *j*-th argument of the reigster update.
In the dual, this variable is mapped to the *i*-th edge which enters the *j*-th port of the reducer.

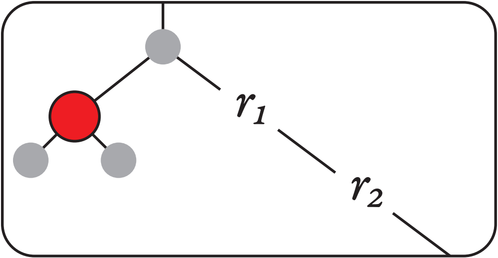

input
output
1
2
3
4
5
6
7
8
9
1
2
3
4
5
6
7
8
9

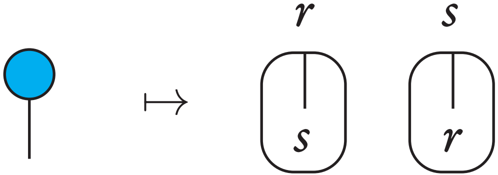


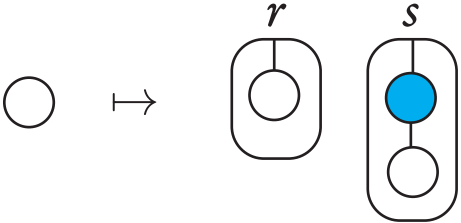
register *r*
register *s*
letters of the output alphabet

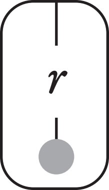

register r
register s
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2

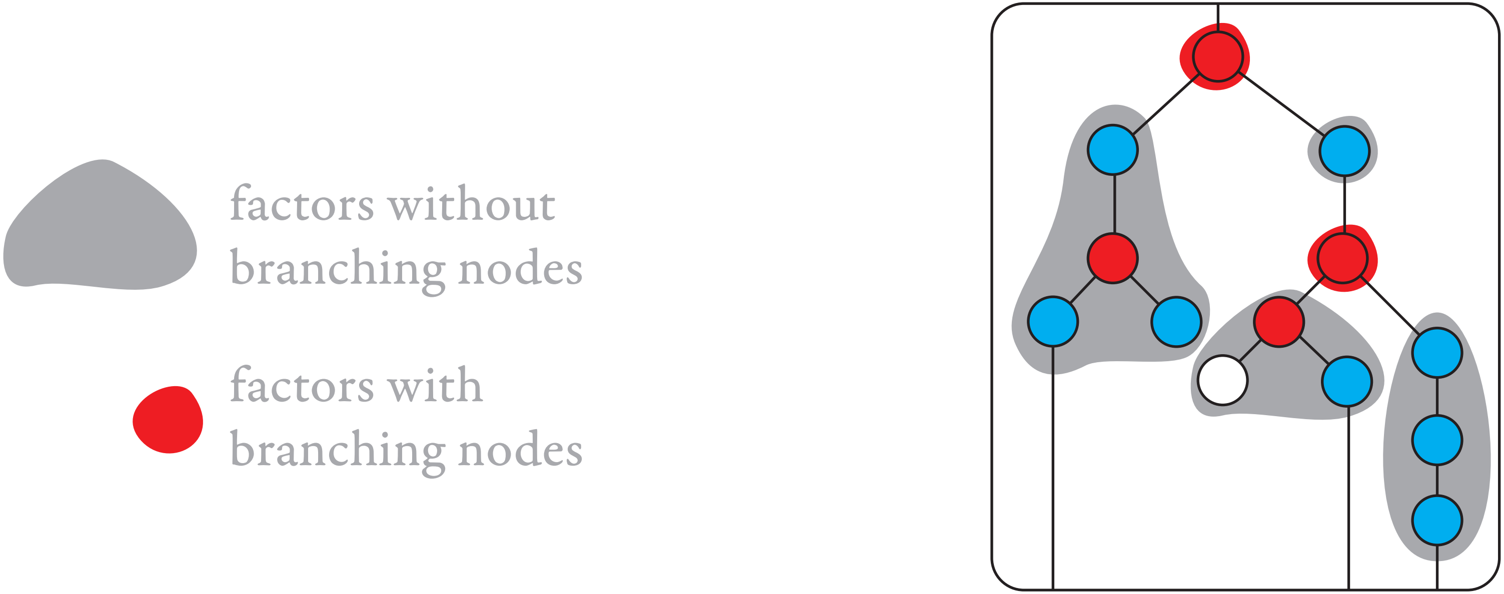

r
r
r
s

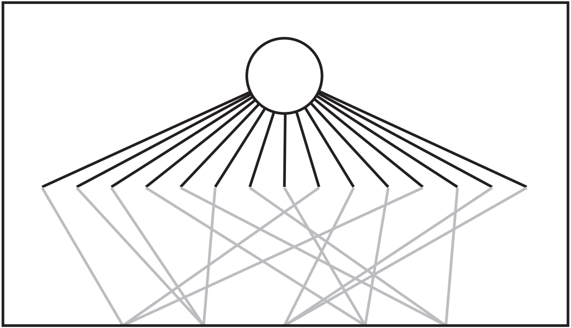

r_1
r_2


r
s
s
r

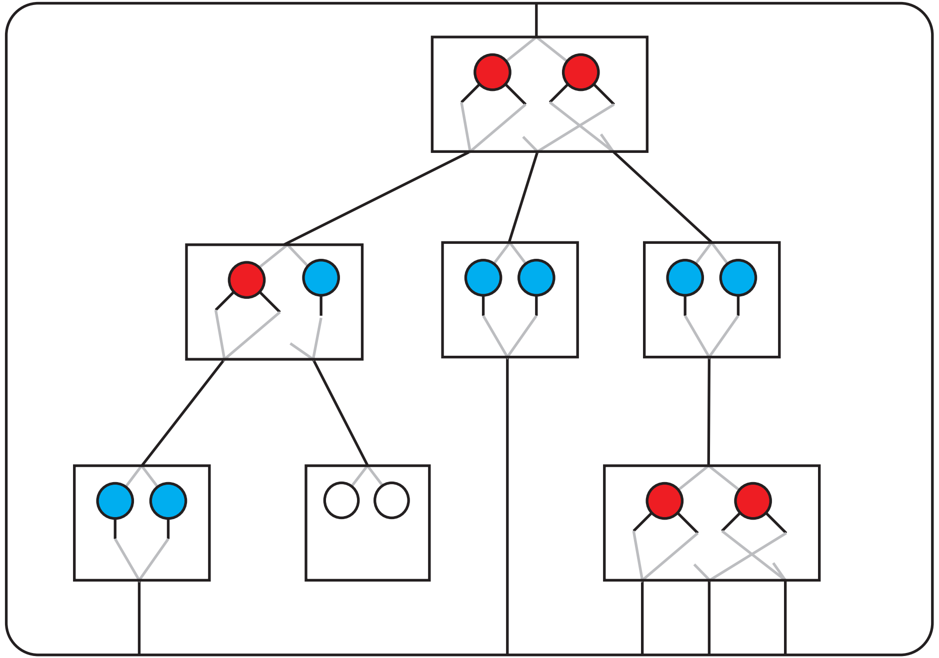

r
s


r

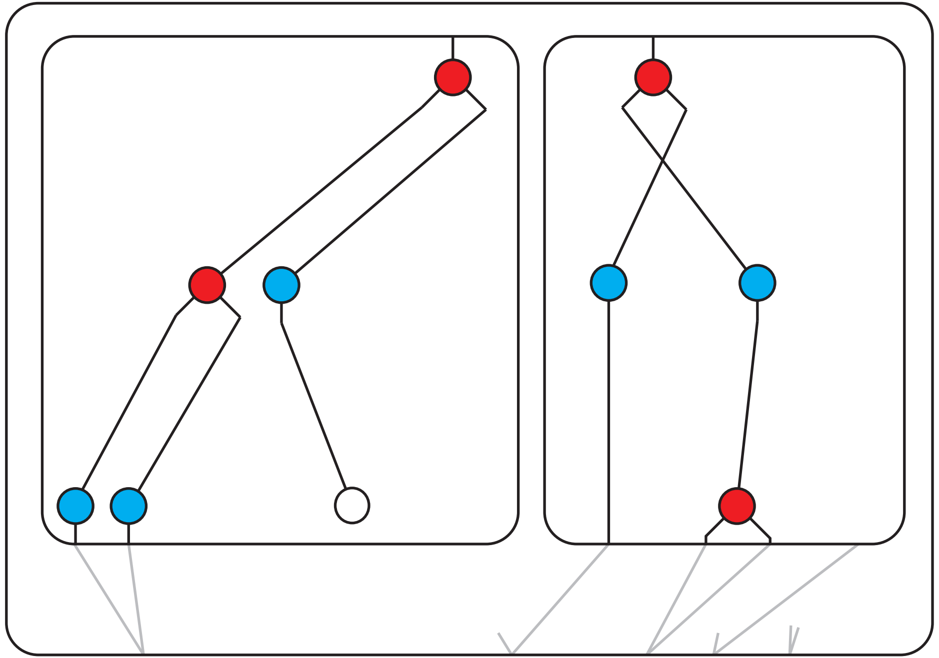

factors without branching nodes
factors with branching nodes

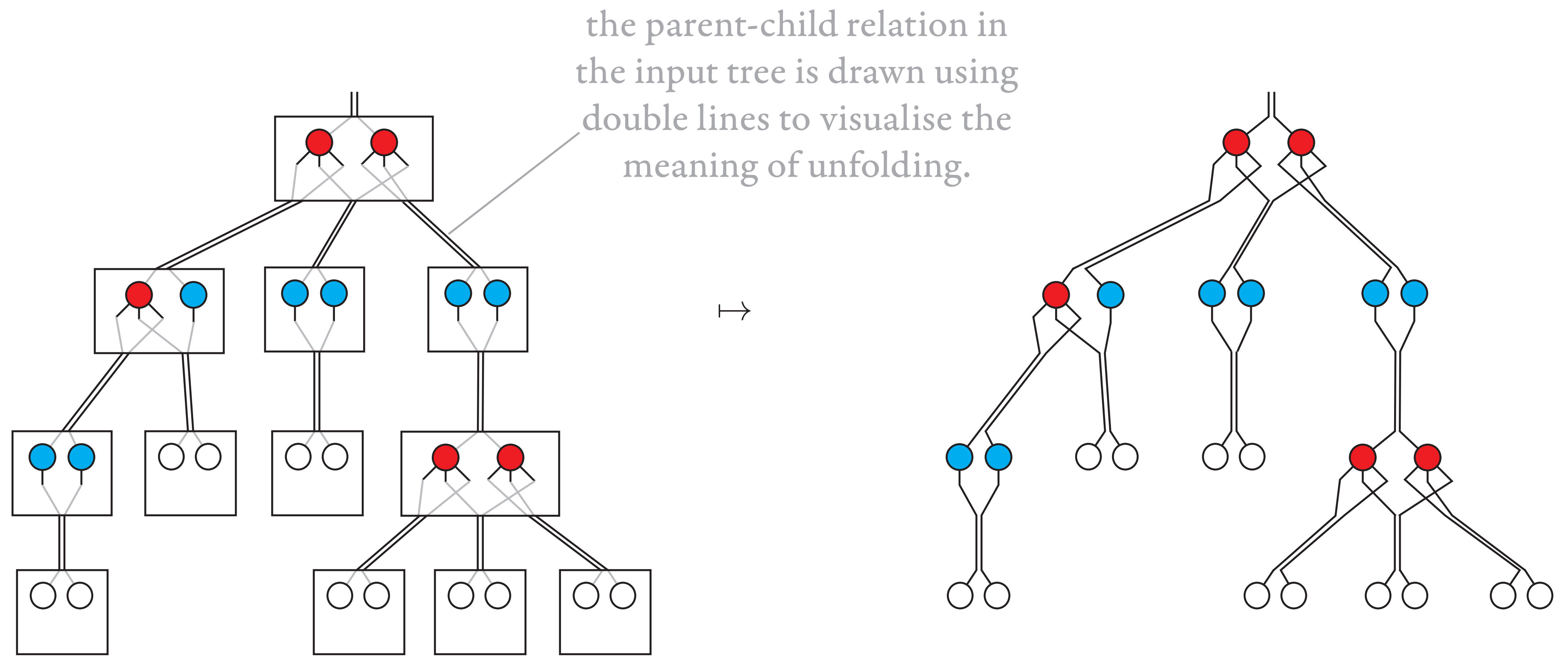


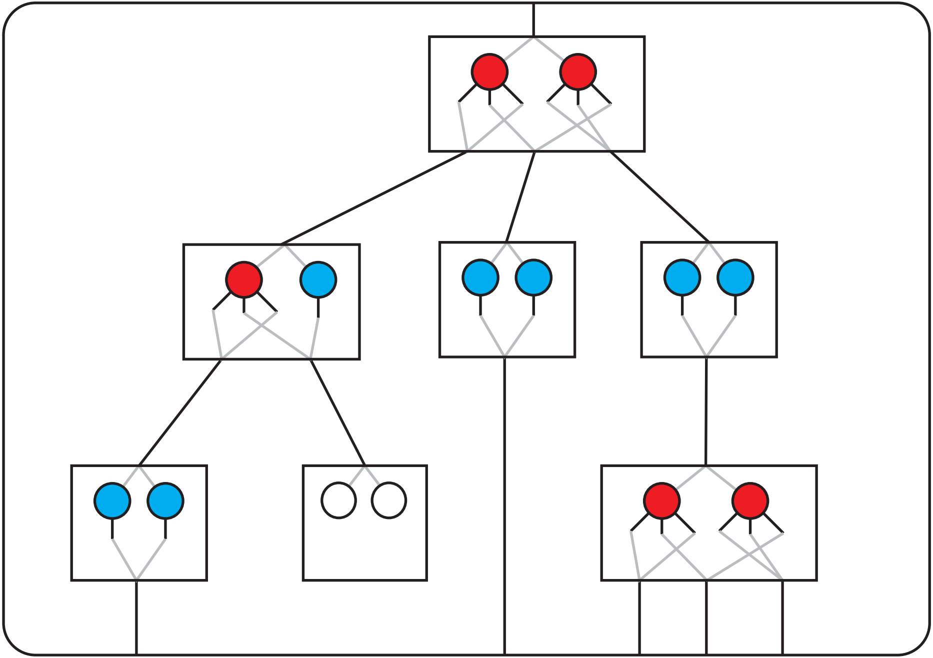
a term of matrix powers

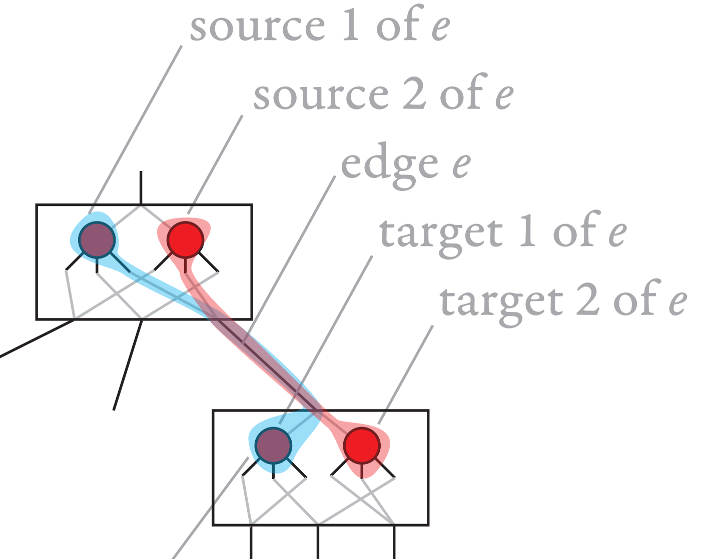


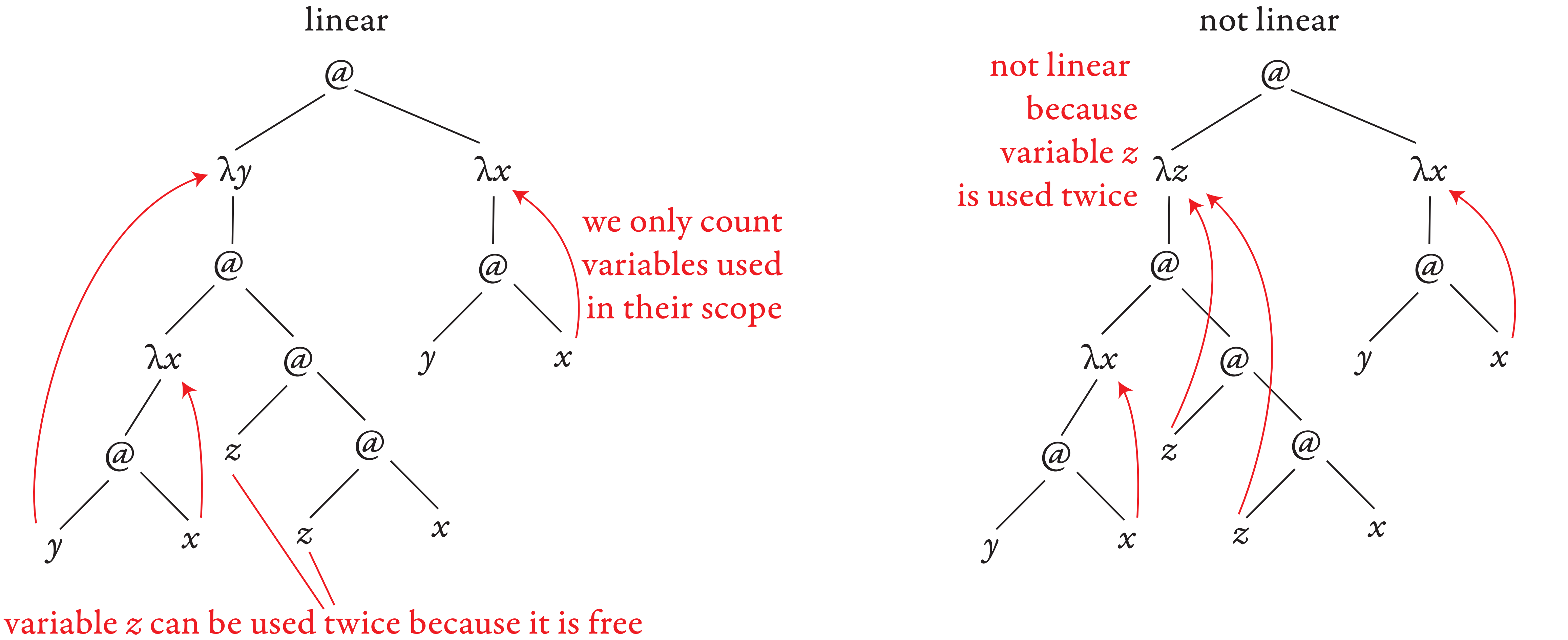
its term unfolding



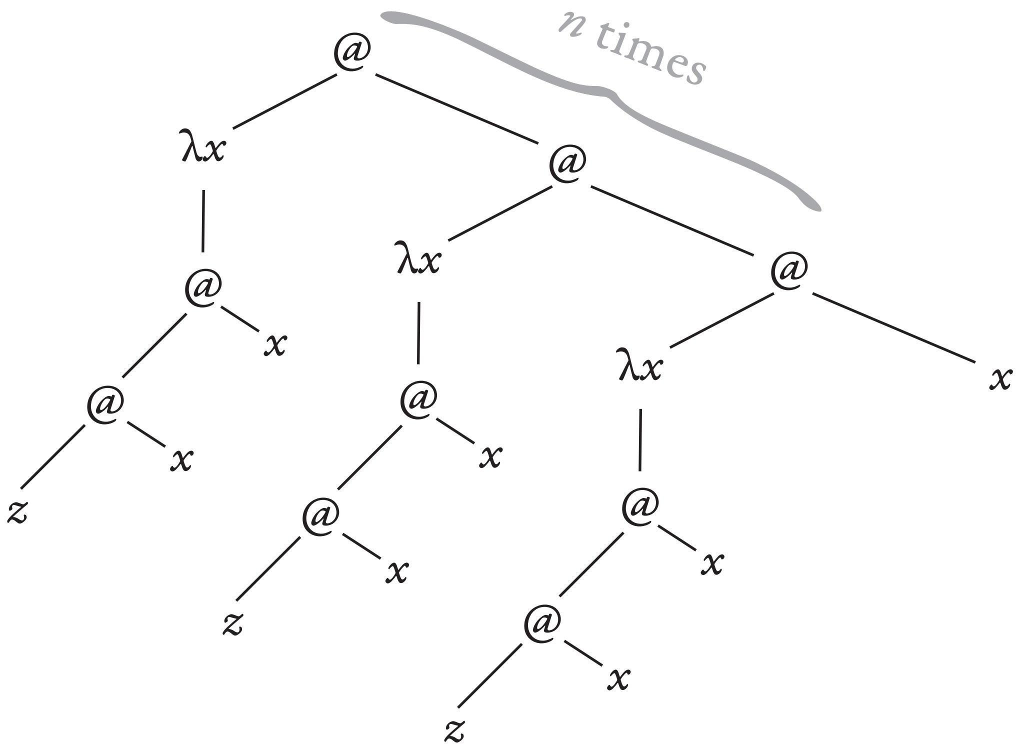

the parent-child relation in the input tree is drawn using double lines to visualise the meaning of unfolding.
↦



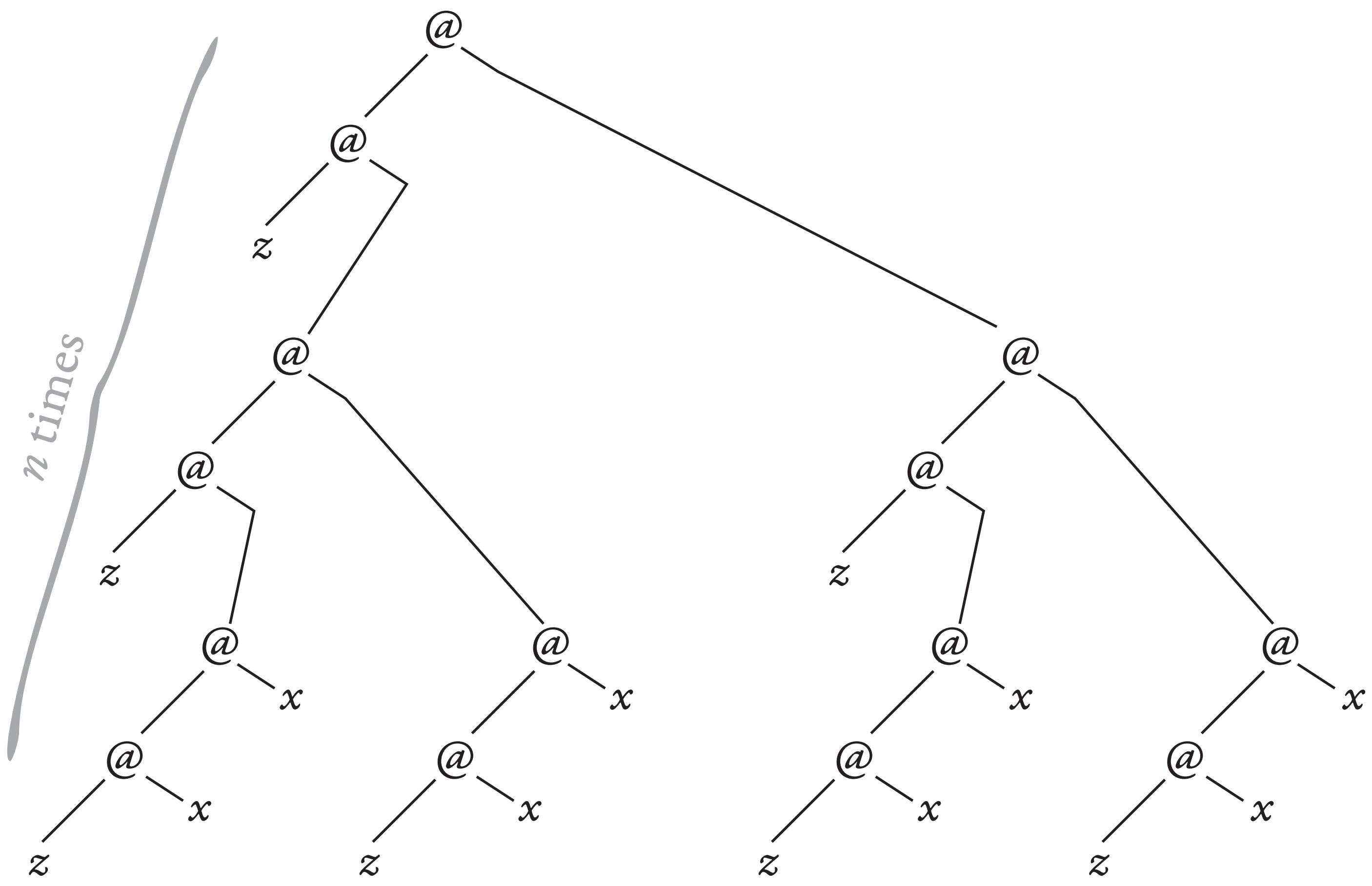

source 1 of e
source 2 of e
edge e
target 1 of e
target 2 of e


linear
not linear
we only count
variables used
in their scope
variable z can be used twice because it is free
not linear
because
variable z
is used twice

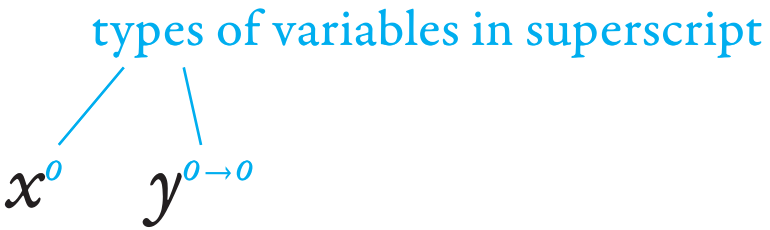
a λ-term of size $O(n)$



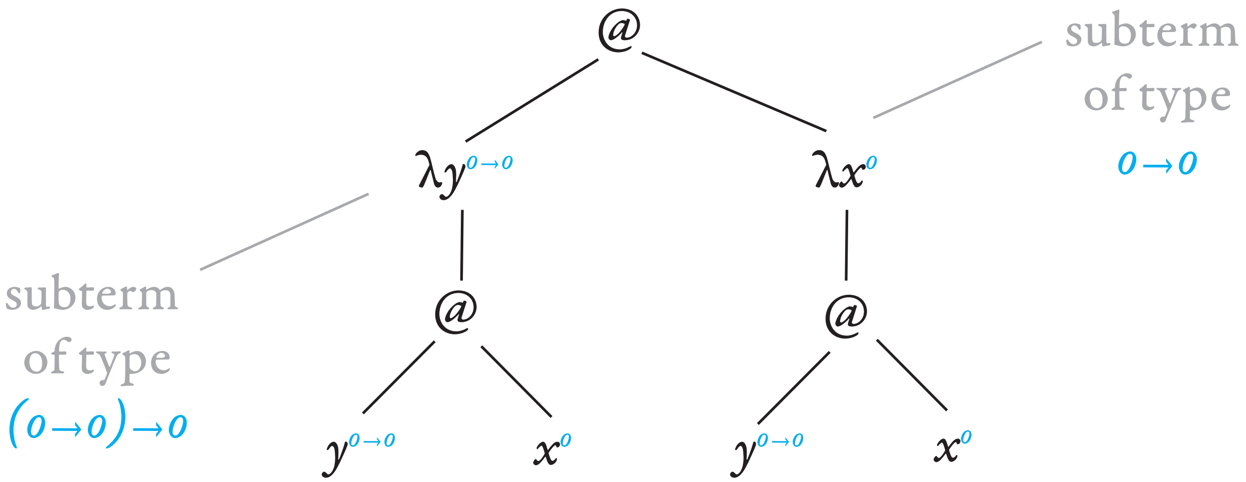
its normal form of size $O(2^n)$



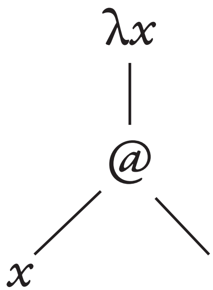
variables



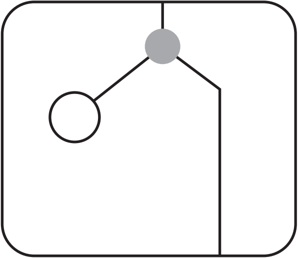
λ-term of type o

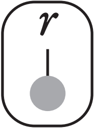


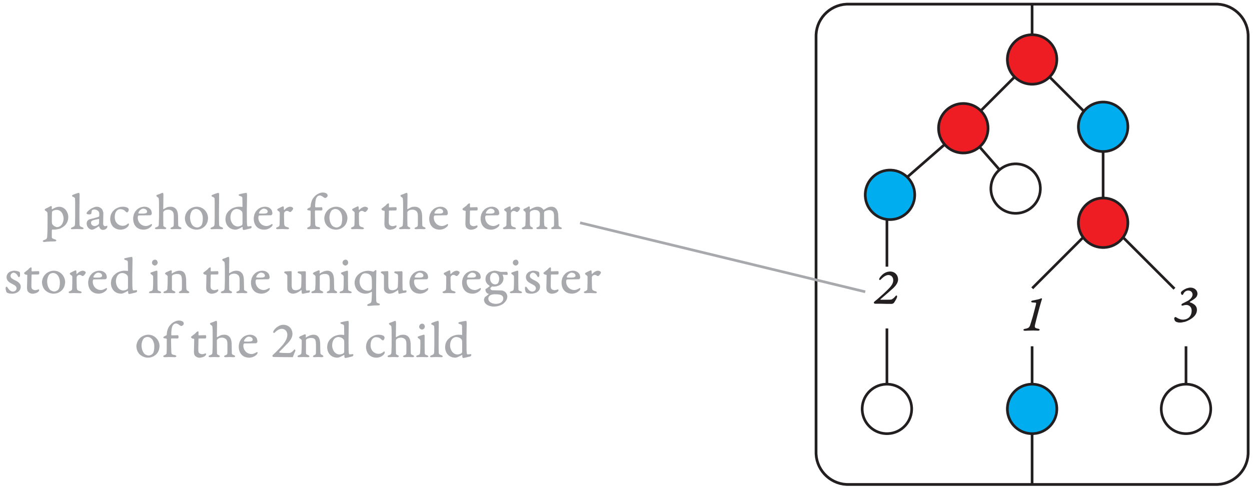
@

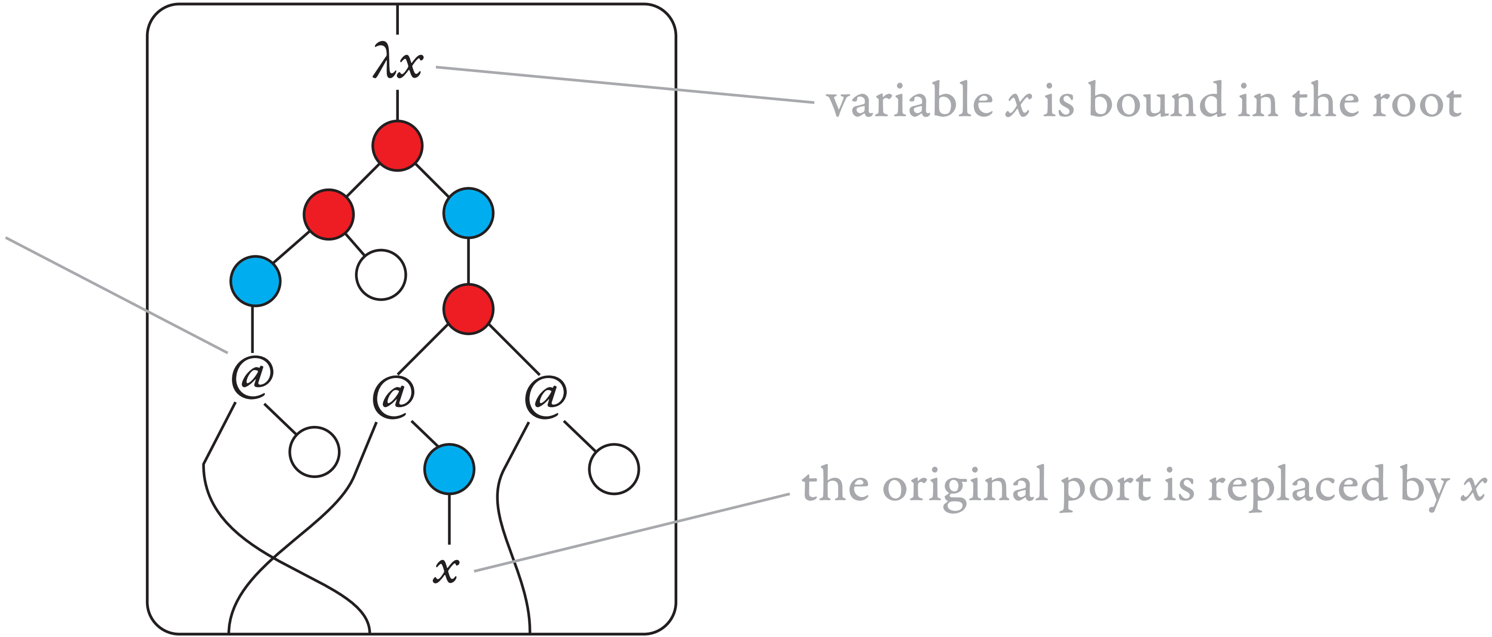


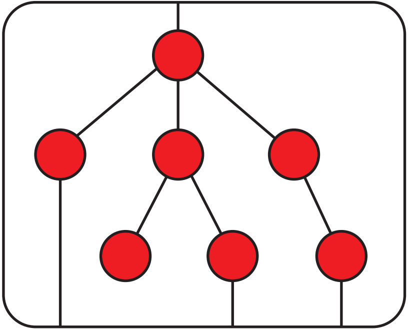
$\lambda x.$



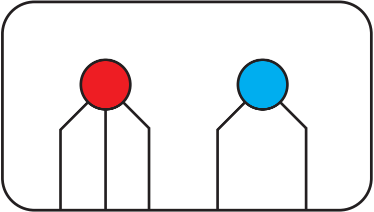

r

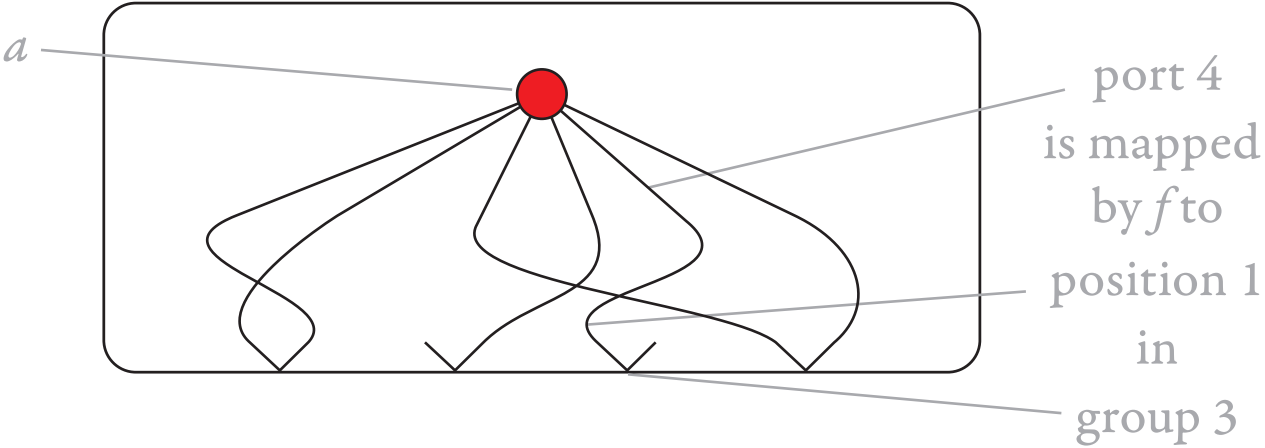

placeholder for the term
stored in the unique register
of the 2nd child
2
1
3


λx
variable x is bound in the root
@
@
@
x
the original port is replaced by x

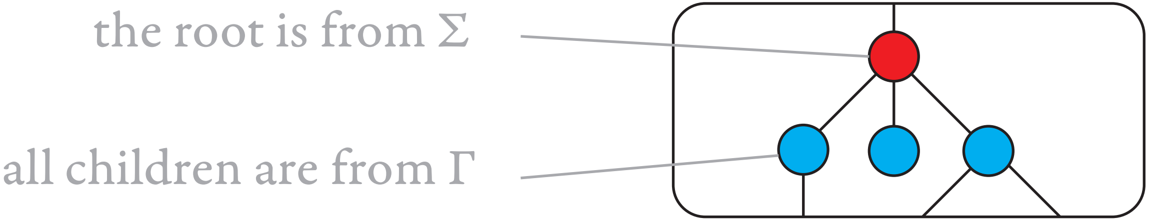


dangling edges
represent ports

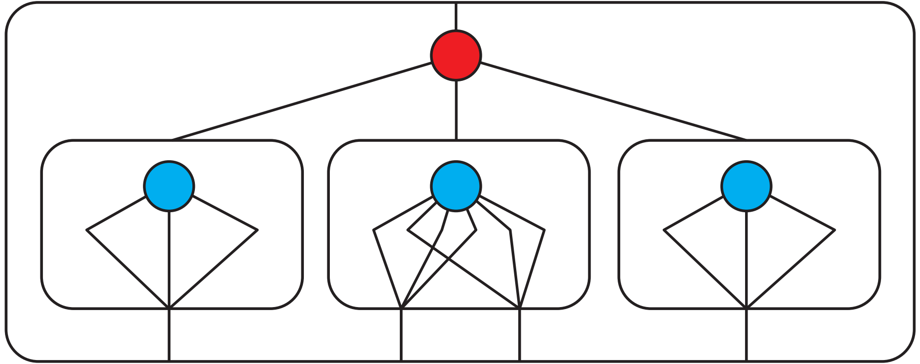




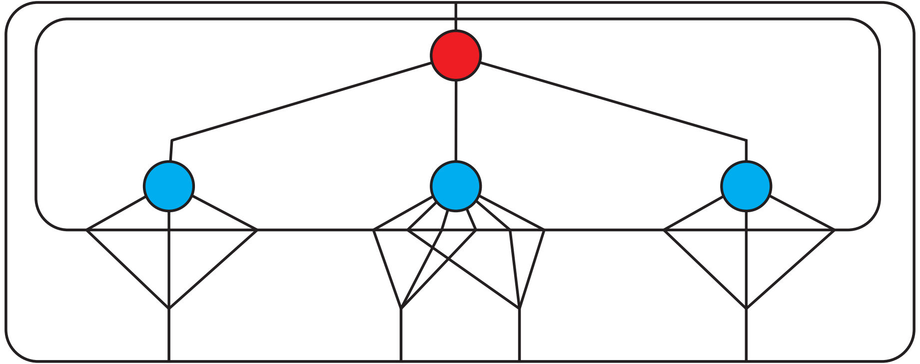
a
port 4
is mapped
by f to
position 1
in
group 3

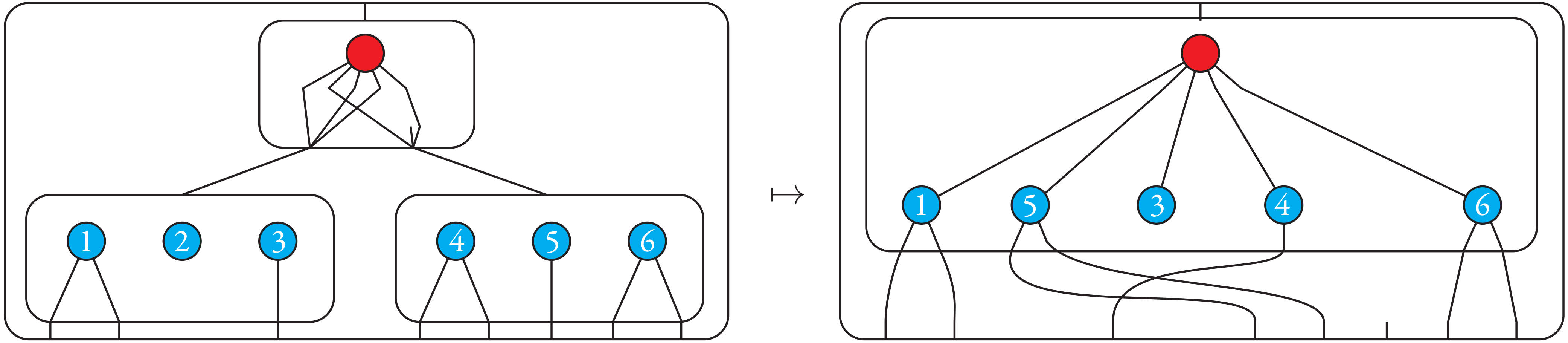

the root is from Σ
all children are from Γ

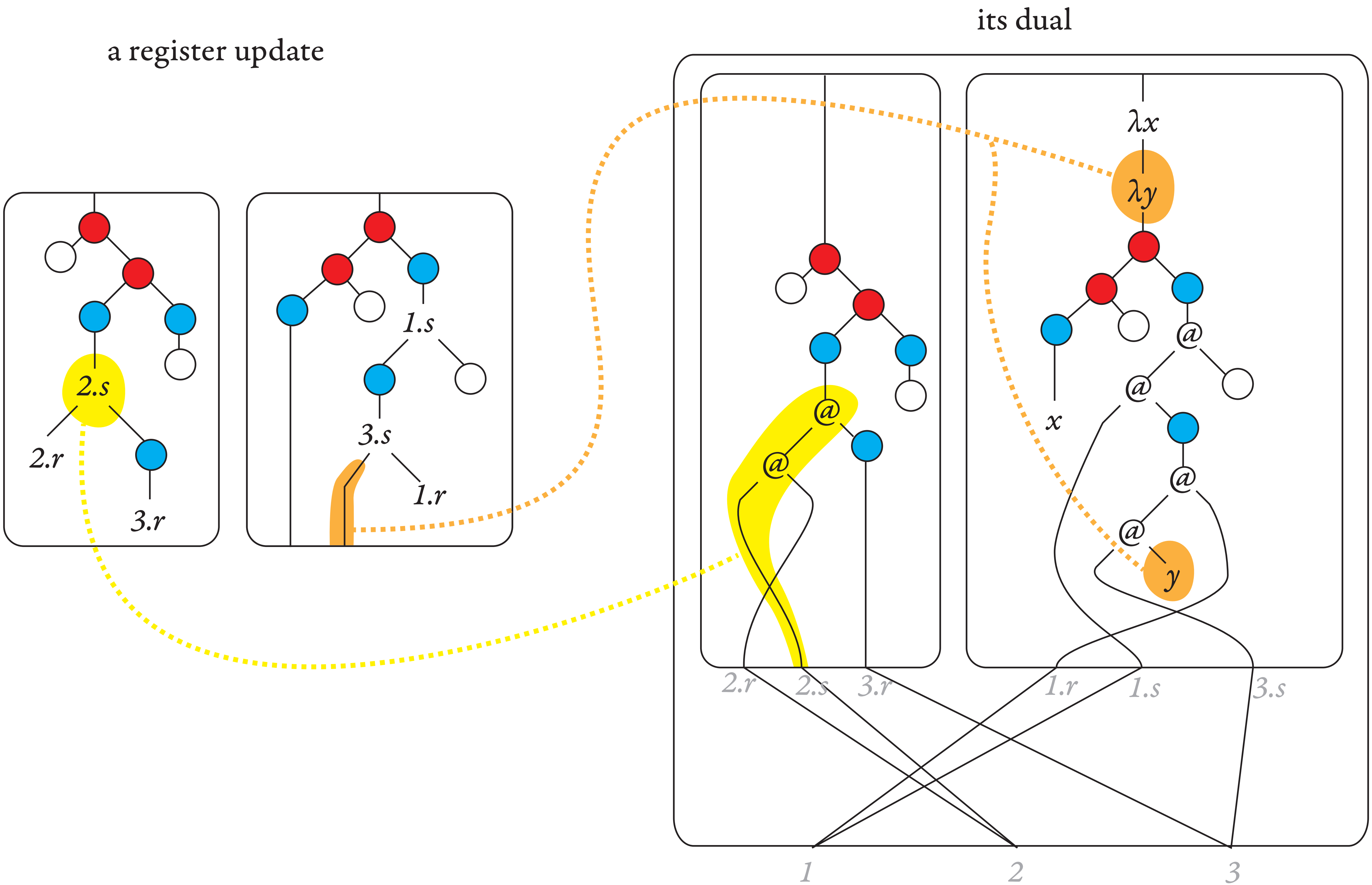
input



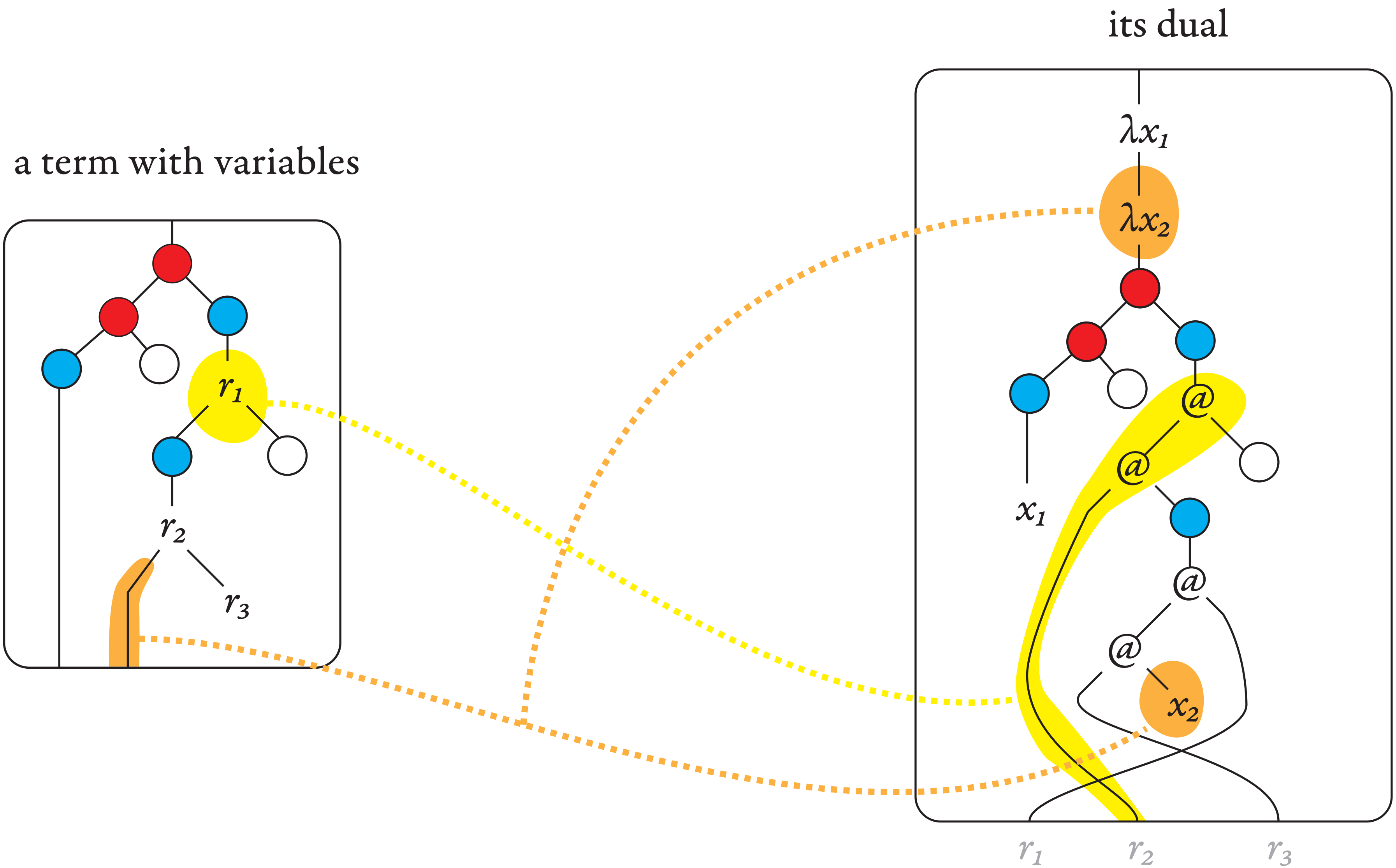
output



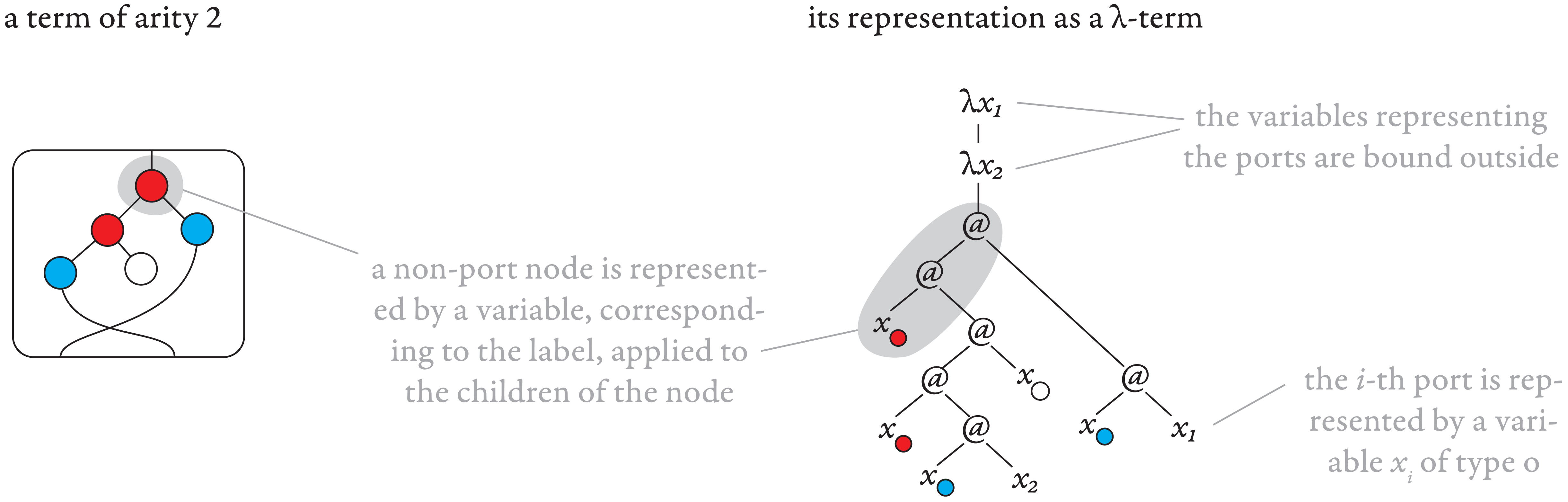

1
2
3
4
5
6
↦
1
5
3
4
6



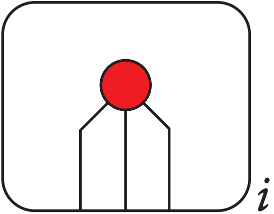
a register update
its dual
2.s
2.r
3.r
1.s
3.s
1.r
λx
λy
@
x
y
2.r
2.s
3.r
1.r
1.s
3.s
1
2
3

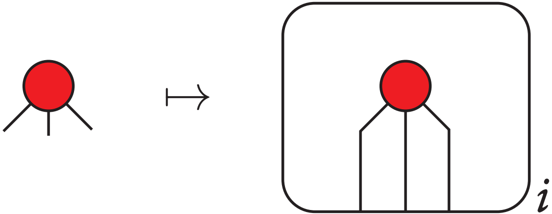


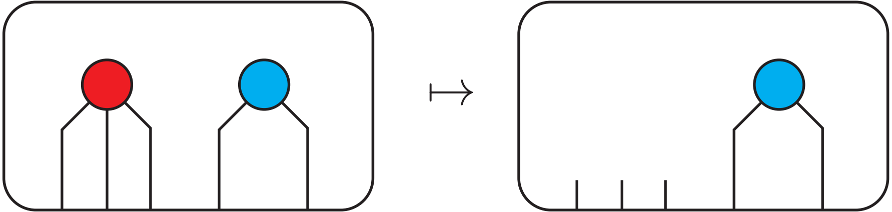
a term with variables
its dual
r_1
r_2
r_3
λx_1
λx_2
@
@
@
@
x_1
x_2
r_1
r_2
r_3

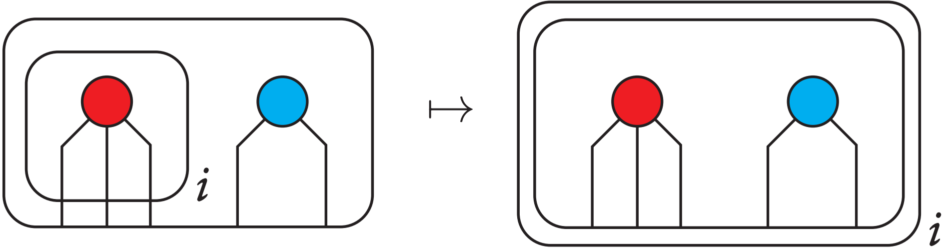


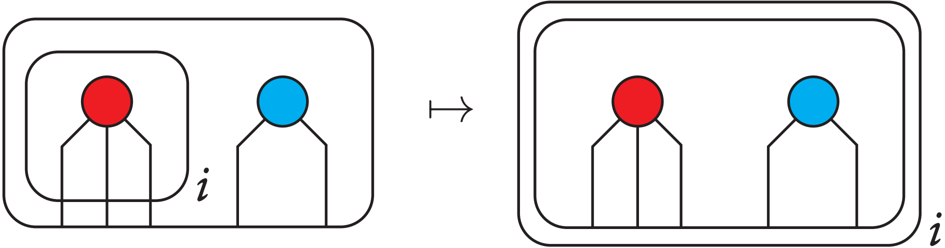
a term of arity 2
its representation as a λ-term
λx1
λx2
the variables representing the ports are bound outside
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
@
x1
x2
the i-th port is represented by a variable xi of type o


i

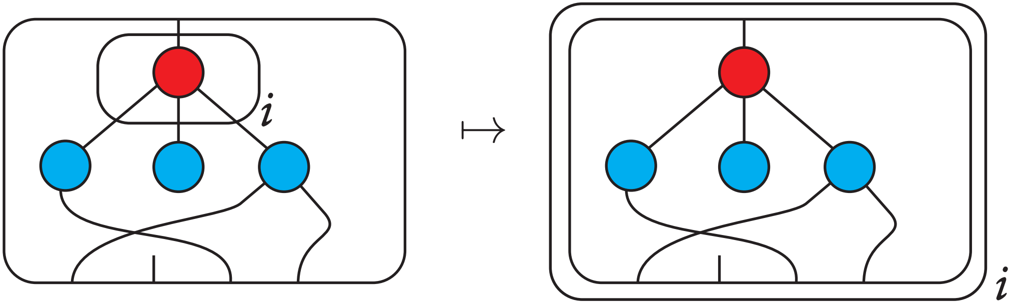

i

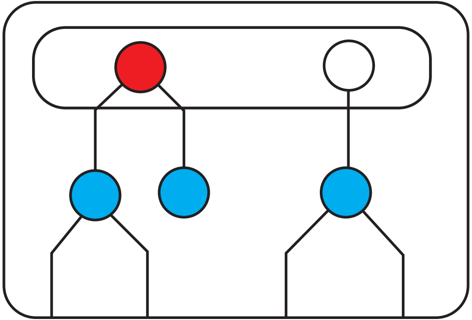



i
i



i
i


i
↦
i



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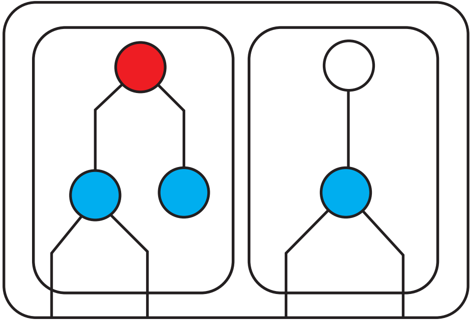

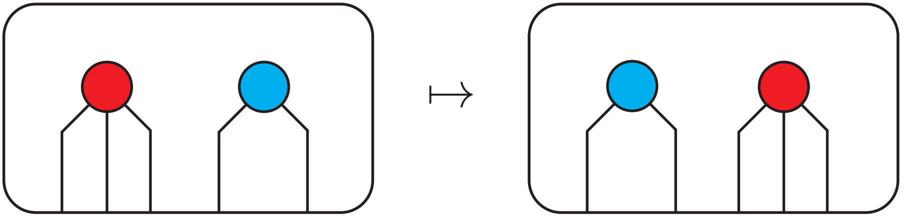

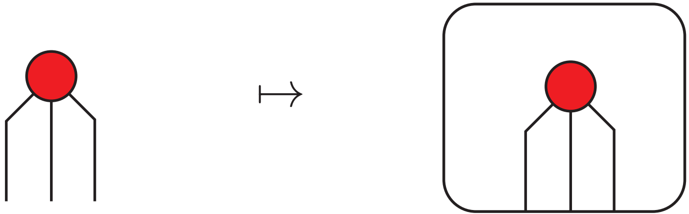

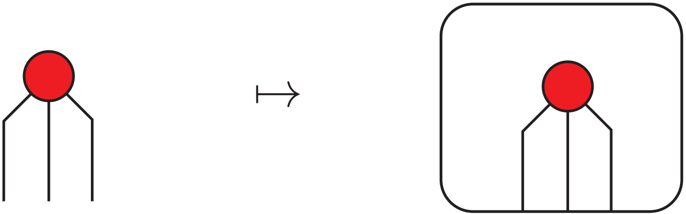

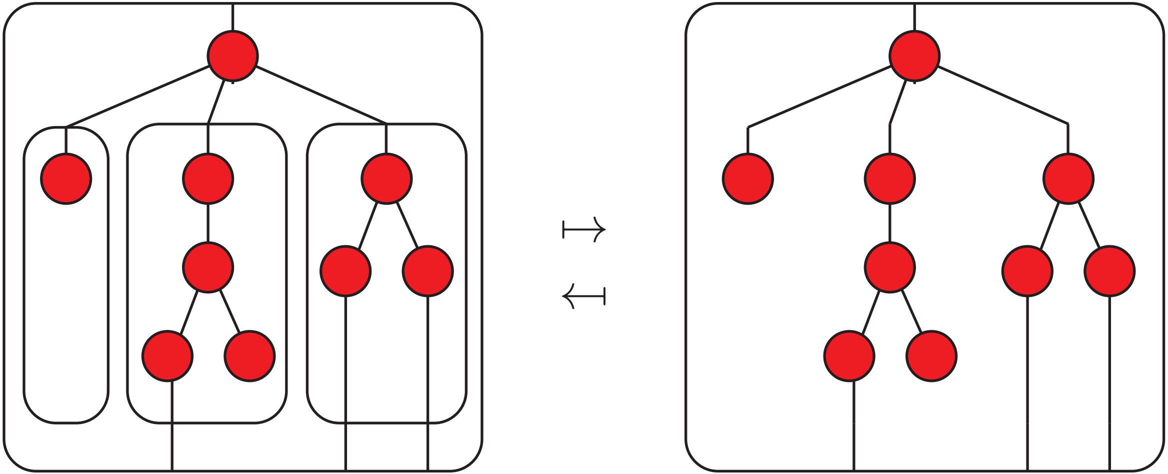

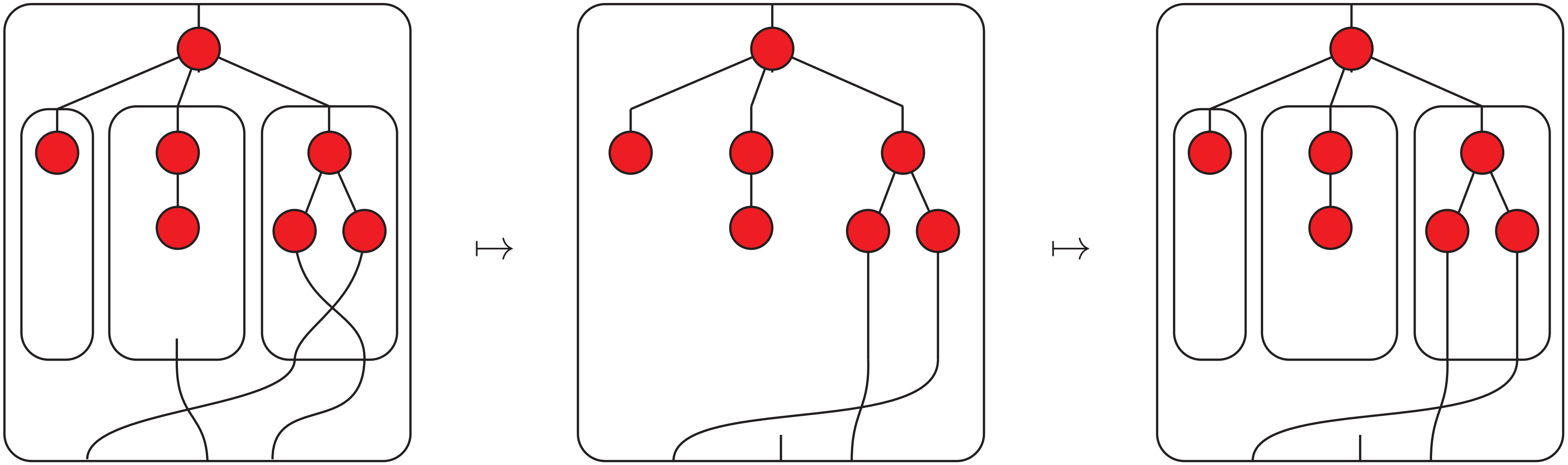

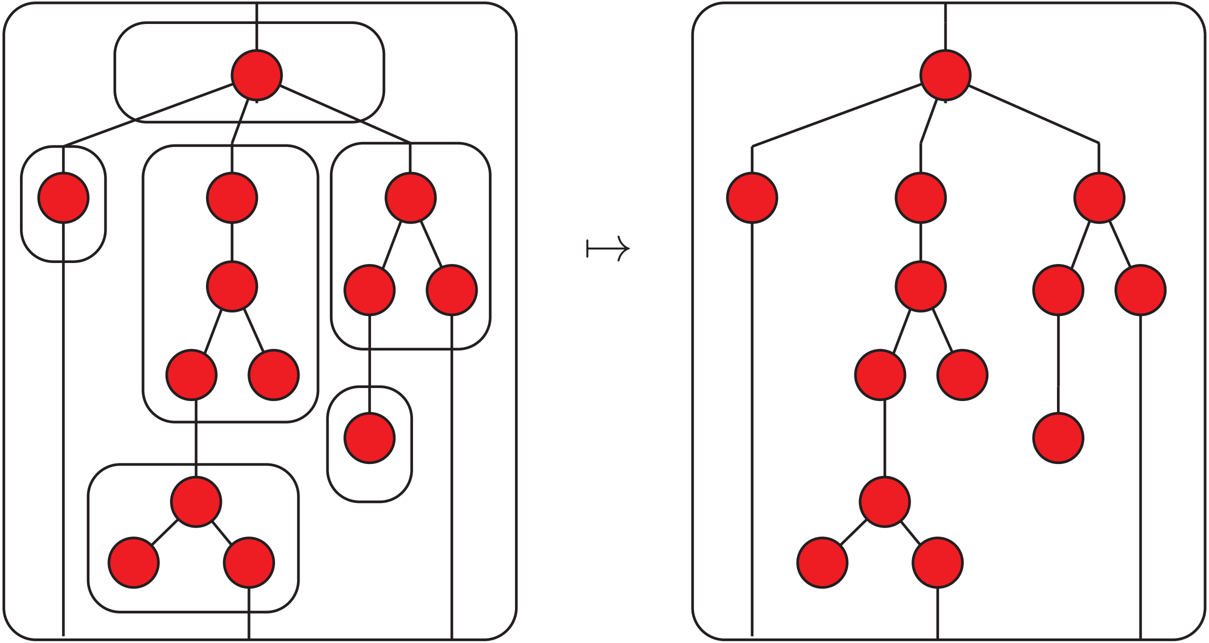

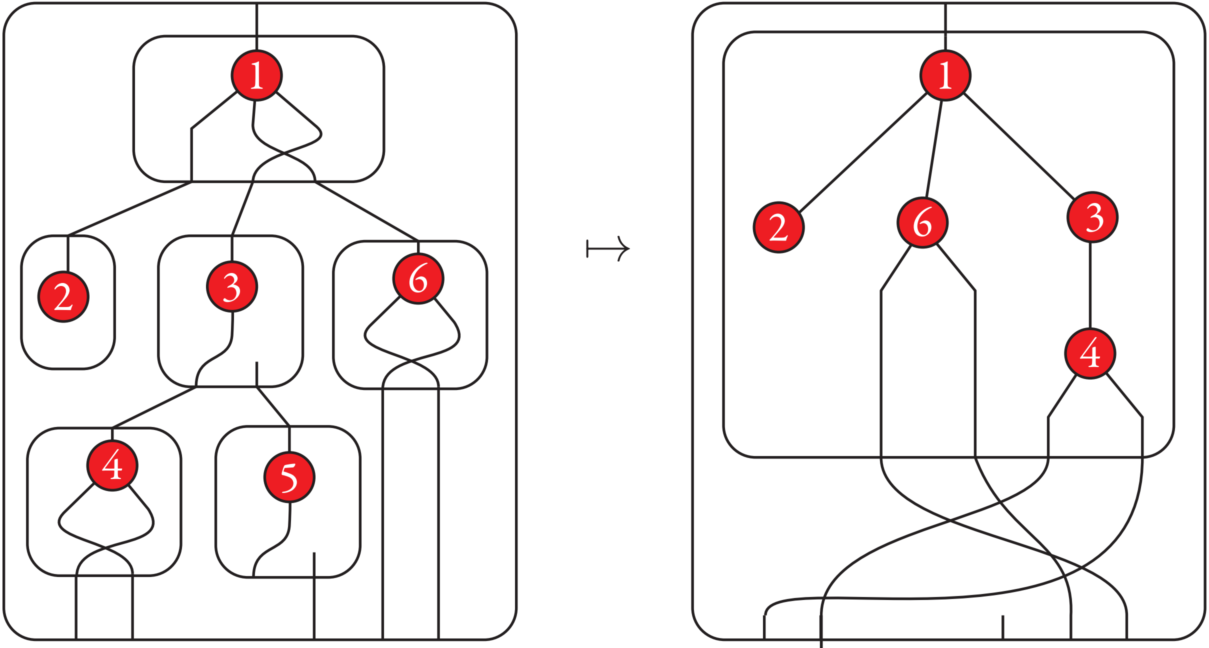
1
2
3
6
4
5
↦
1
2
6
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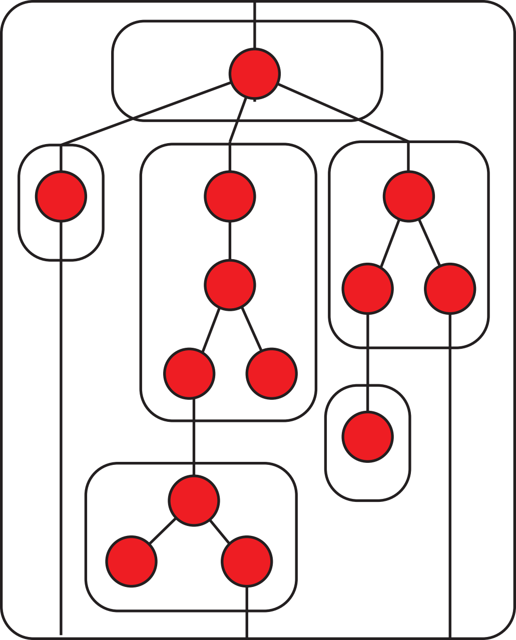
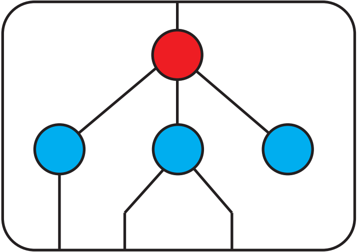
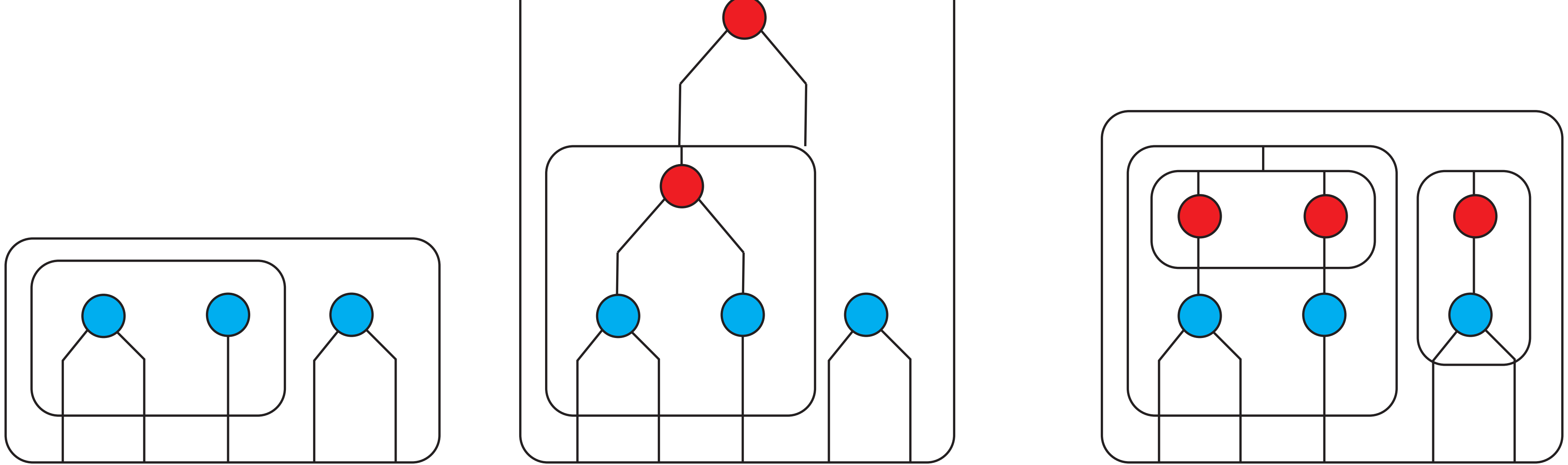
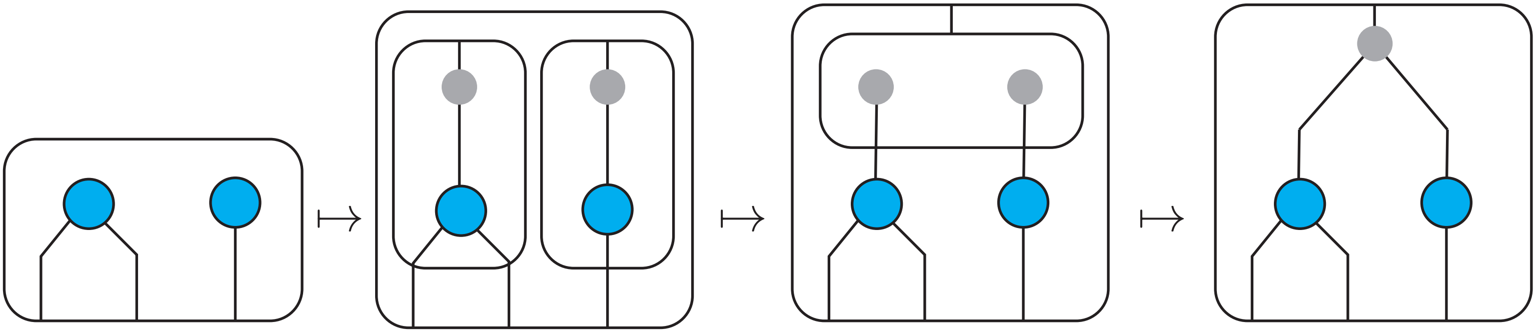
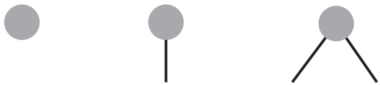
4

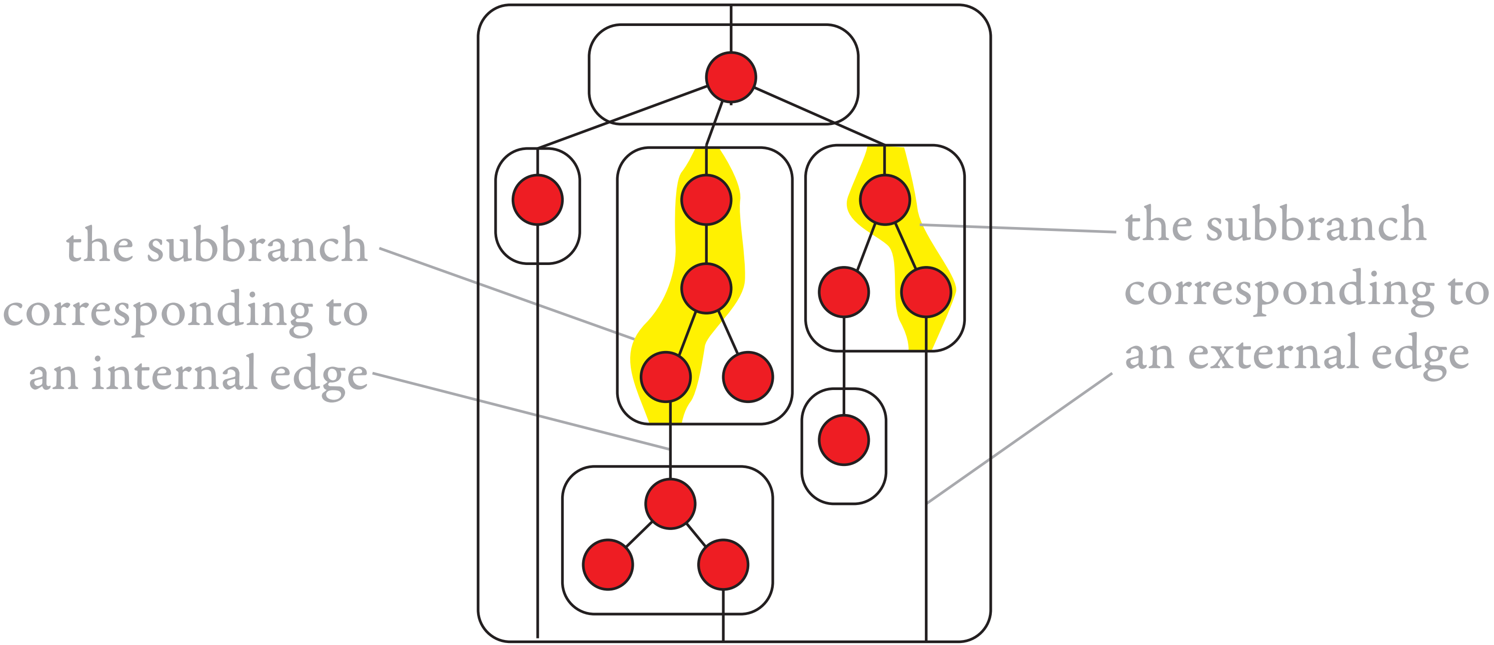


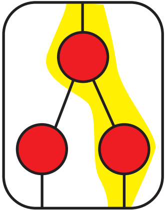




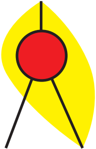


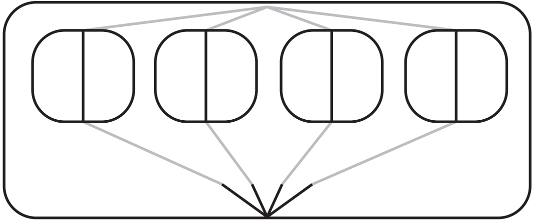



the subbranch
corresponding to
an internal edge
the subbranch
corresponding to
an external edge

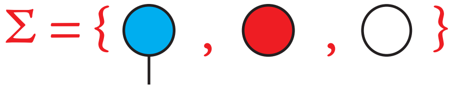


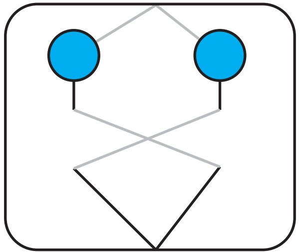
a branch can be visualised as
a term with a distinguished
root-to-port path

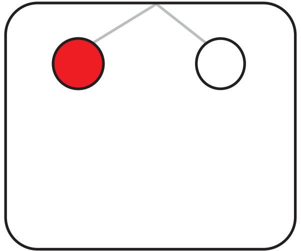


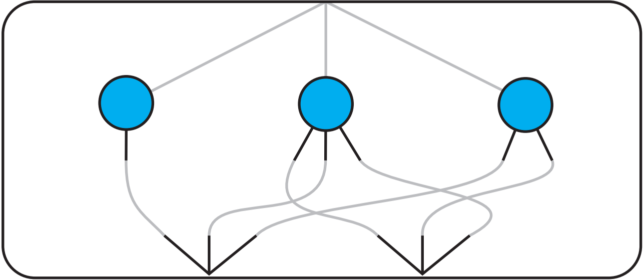


a tuple of *k* identity terms with all their ports folded into one

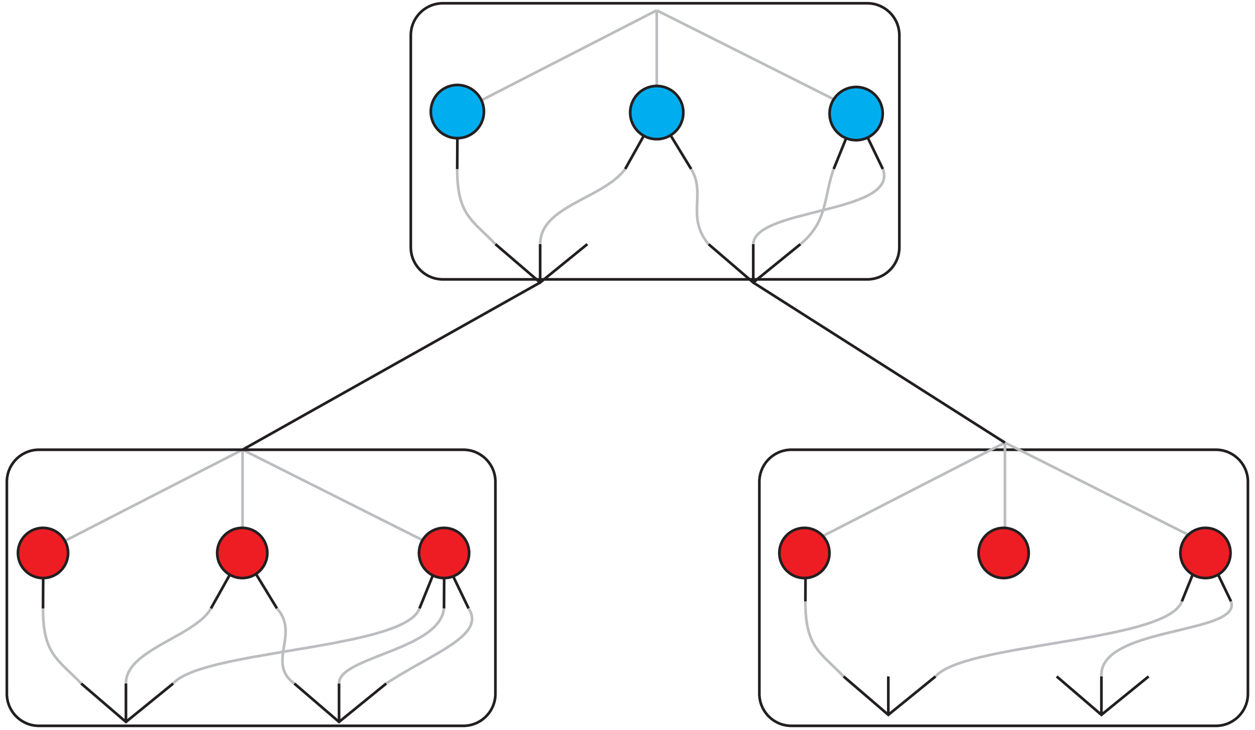


$a \in \Sigma^{[2]}$ 

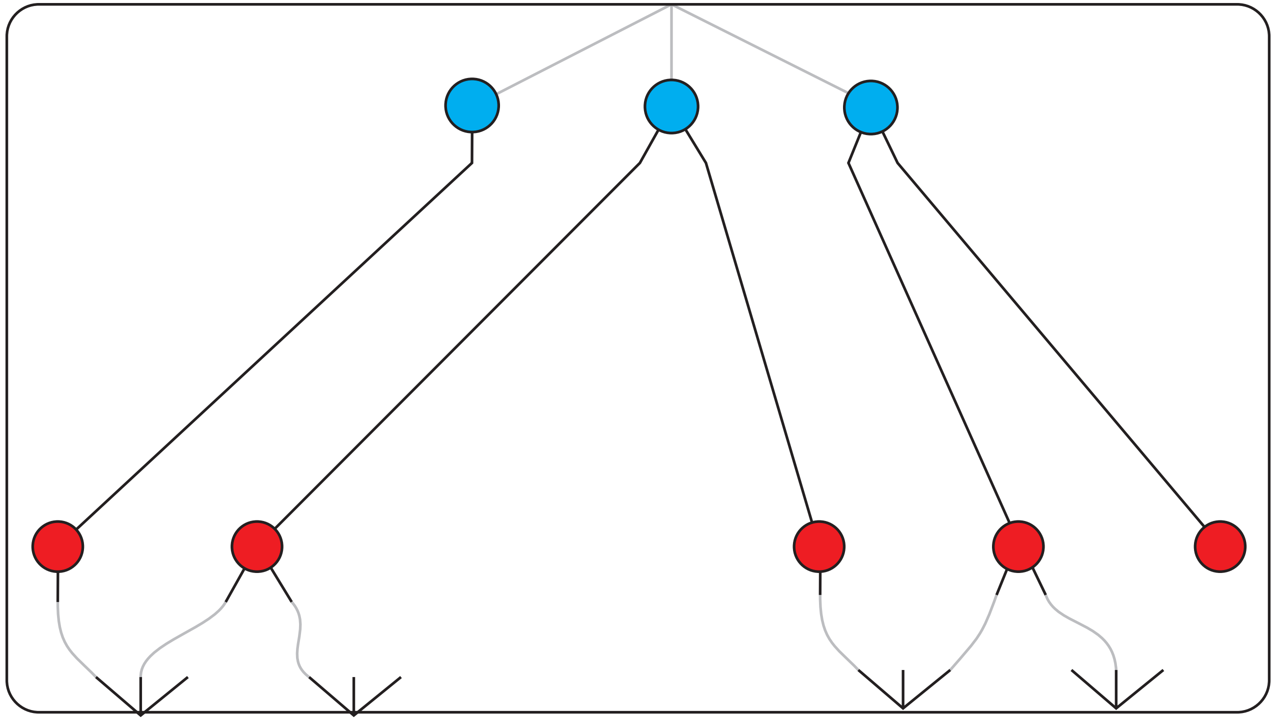
$b \in \Sigma^{[2]}$ 

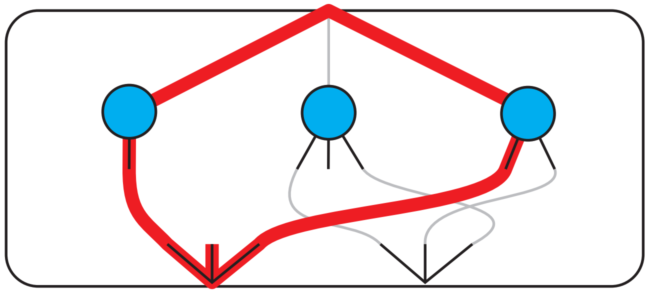


a shallow term of matrix powers



its shallow unfolding



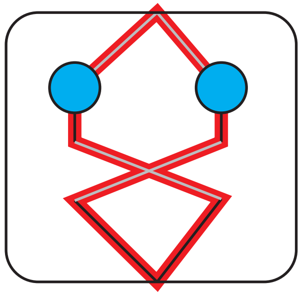


twist of port 1

1 2 3

↑ ↑

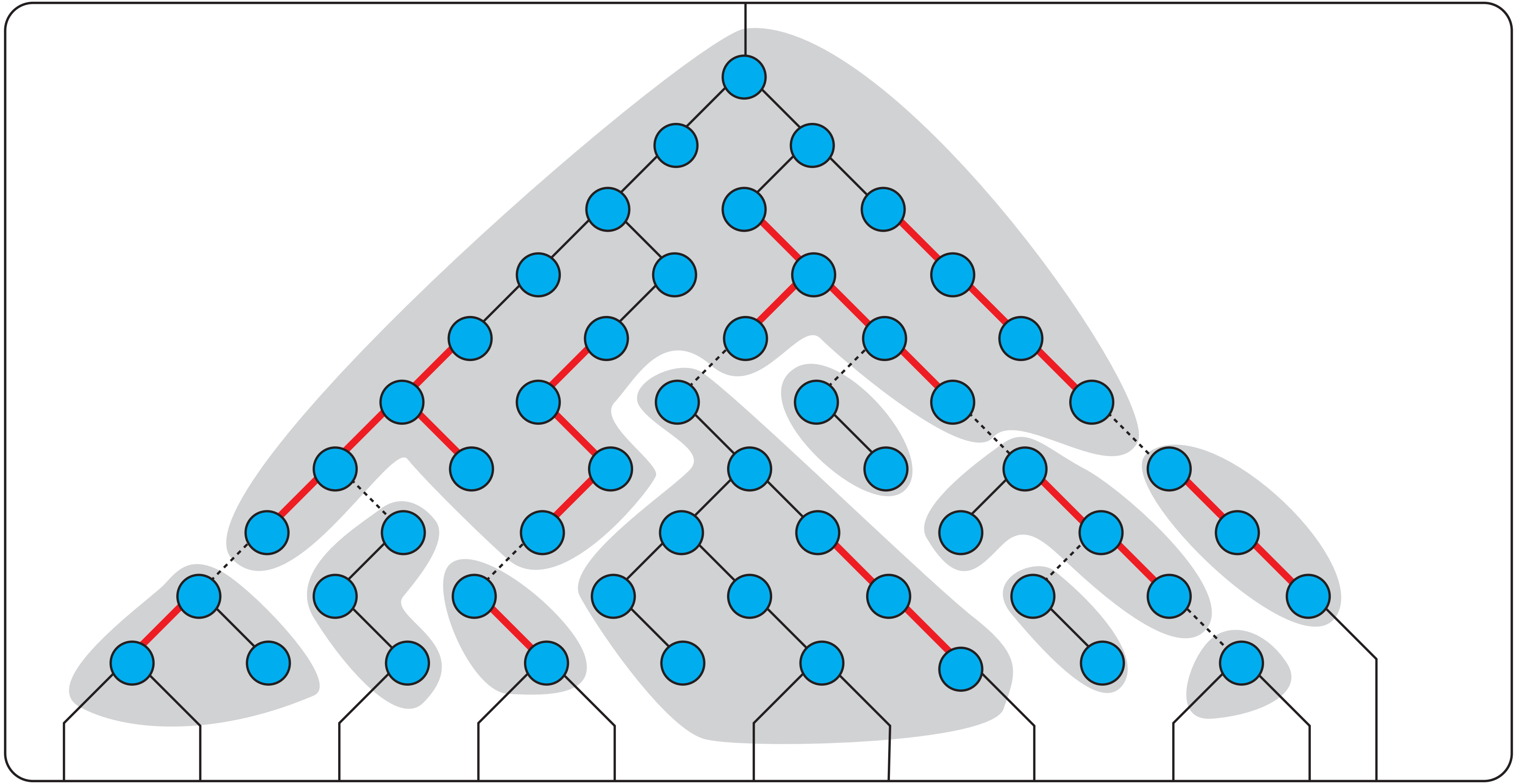
1 2 3

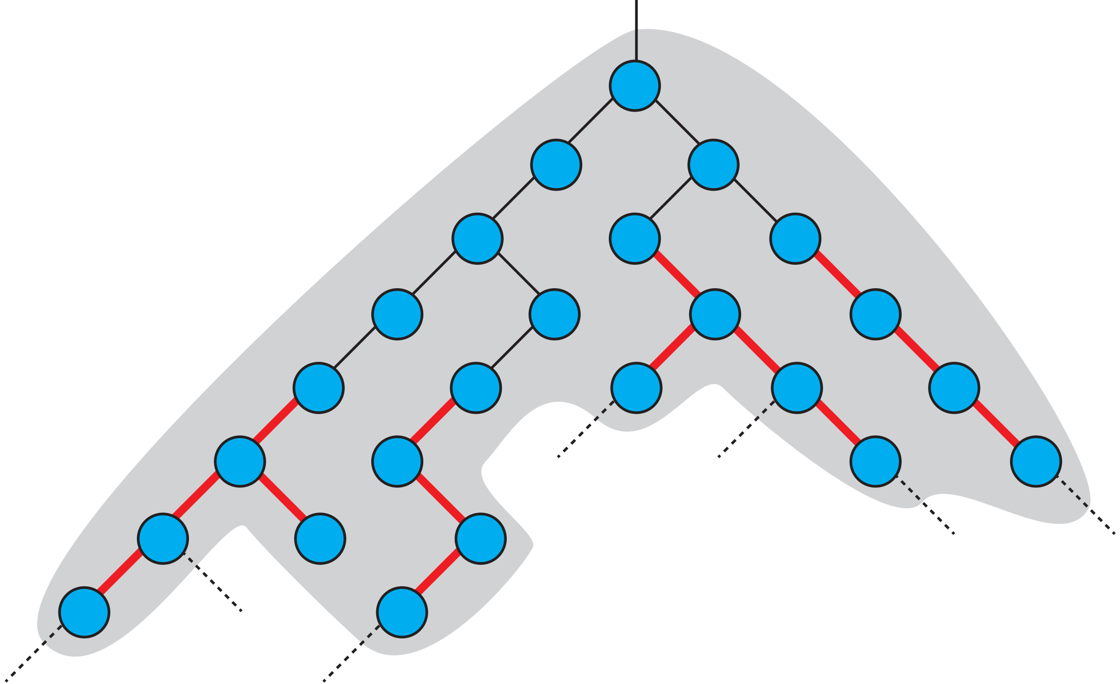


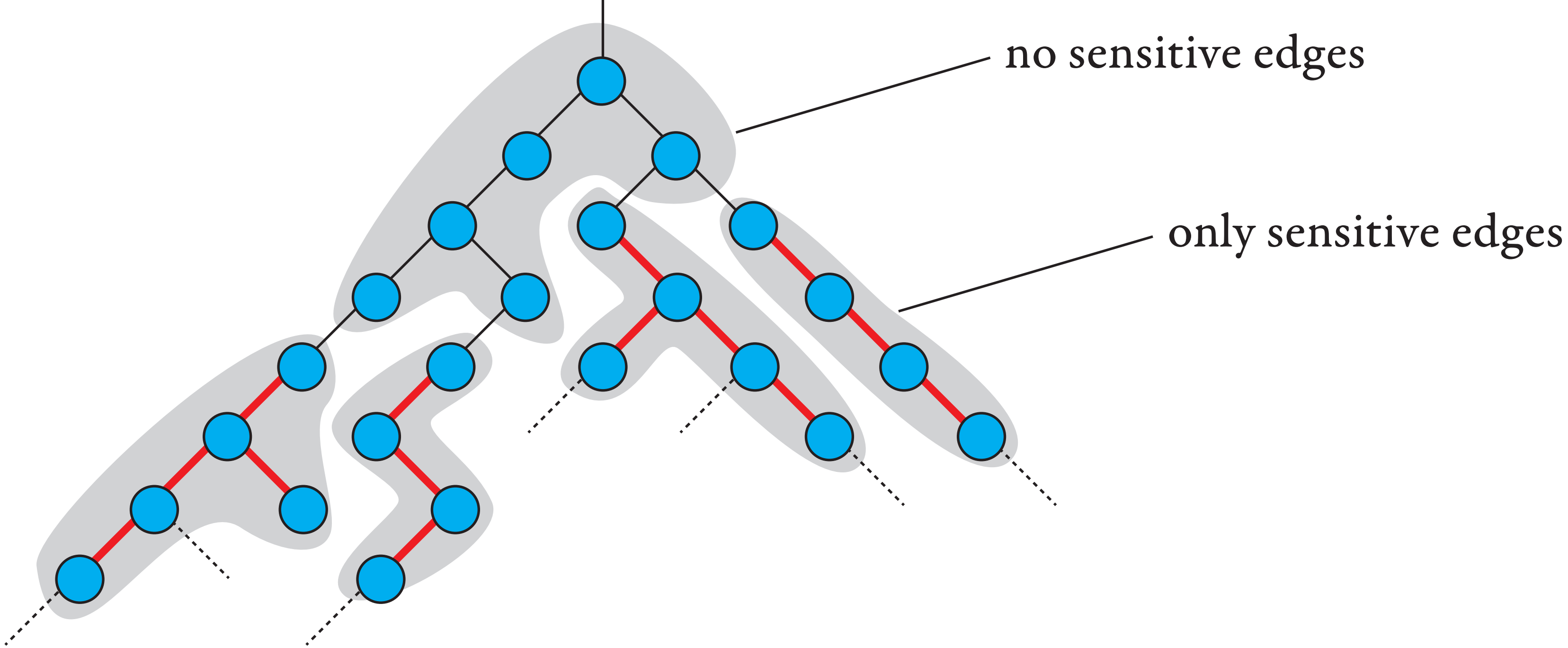
twist of port 1

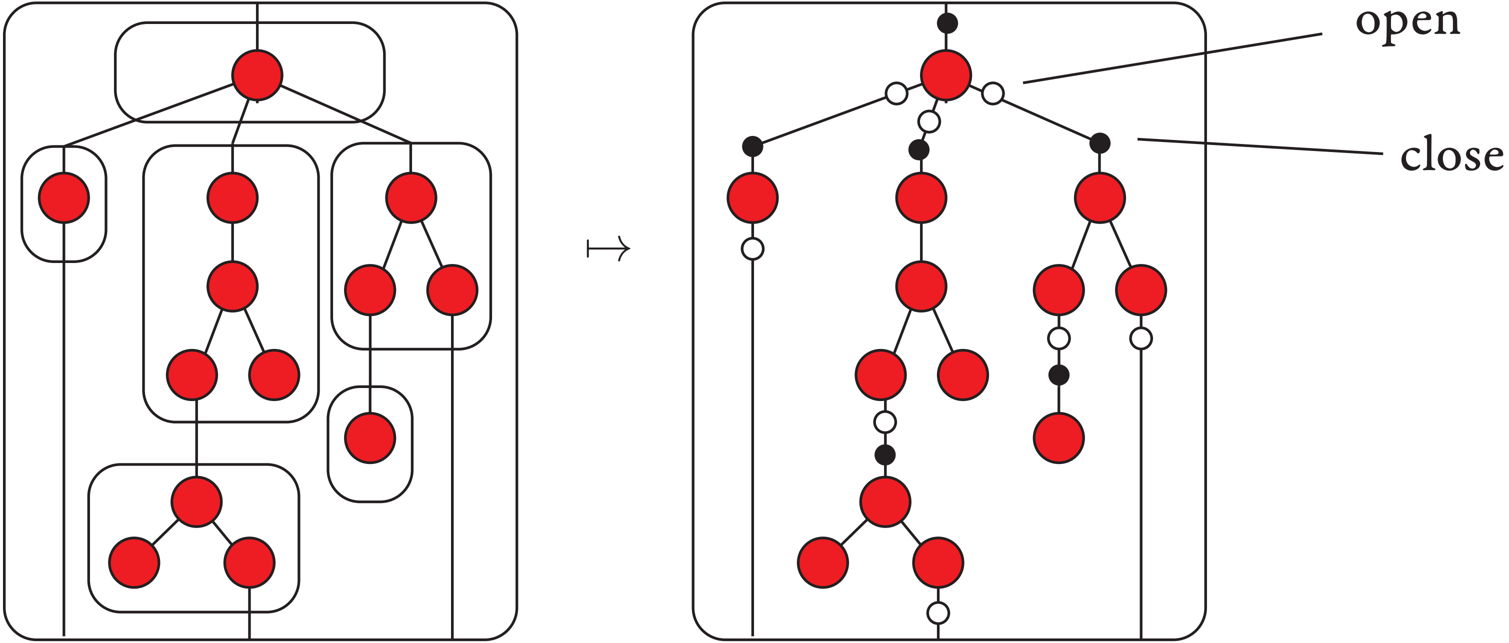
1 2

1 2






no sensitive edges
only sensitive edges


open
close