

a ranked alphabet

arity 2



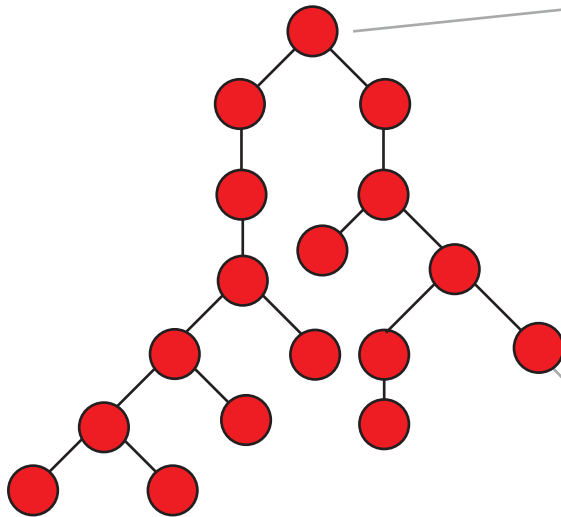
arity 1



arity 0



a tree



this node has a
label of arity 2,
and therefore it has
2 children

this node is
child 2
(children are
ordered)



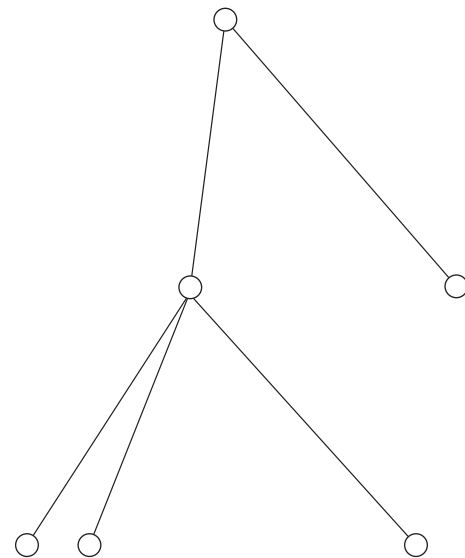
A tree t over $\Sigma^{[2]}$



$\text{unfold}_1(t)$



$\text{unfold}_2(t)$





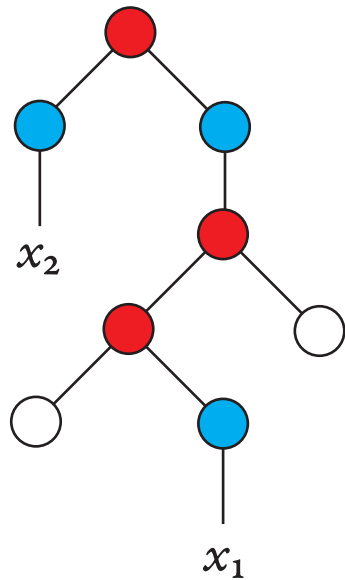
t



substitute(t)

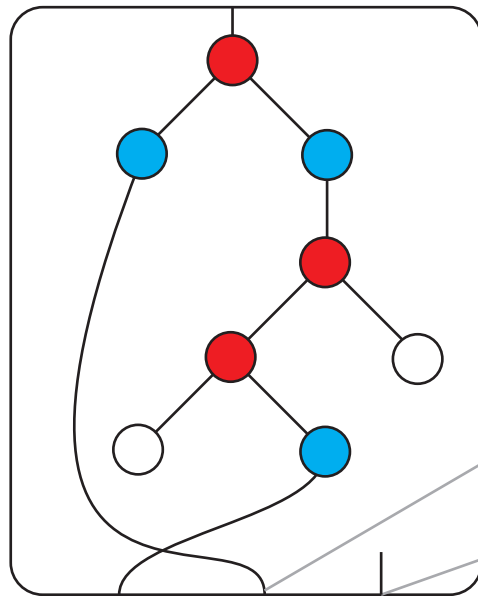






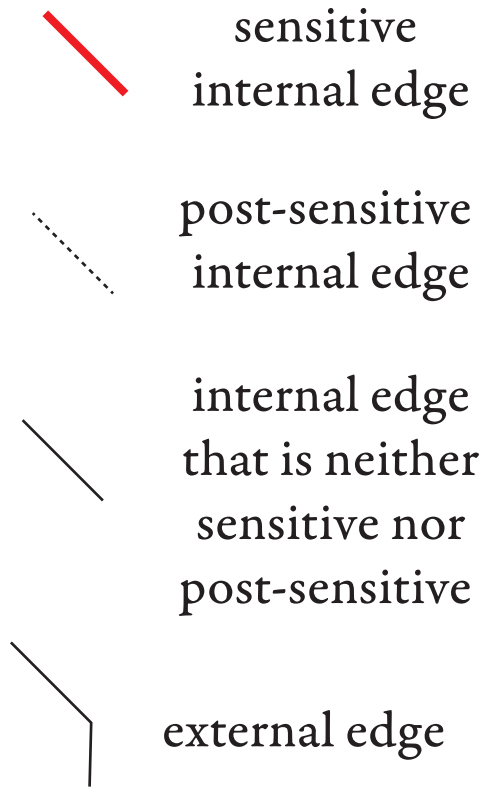
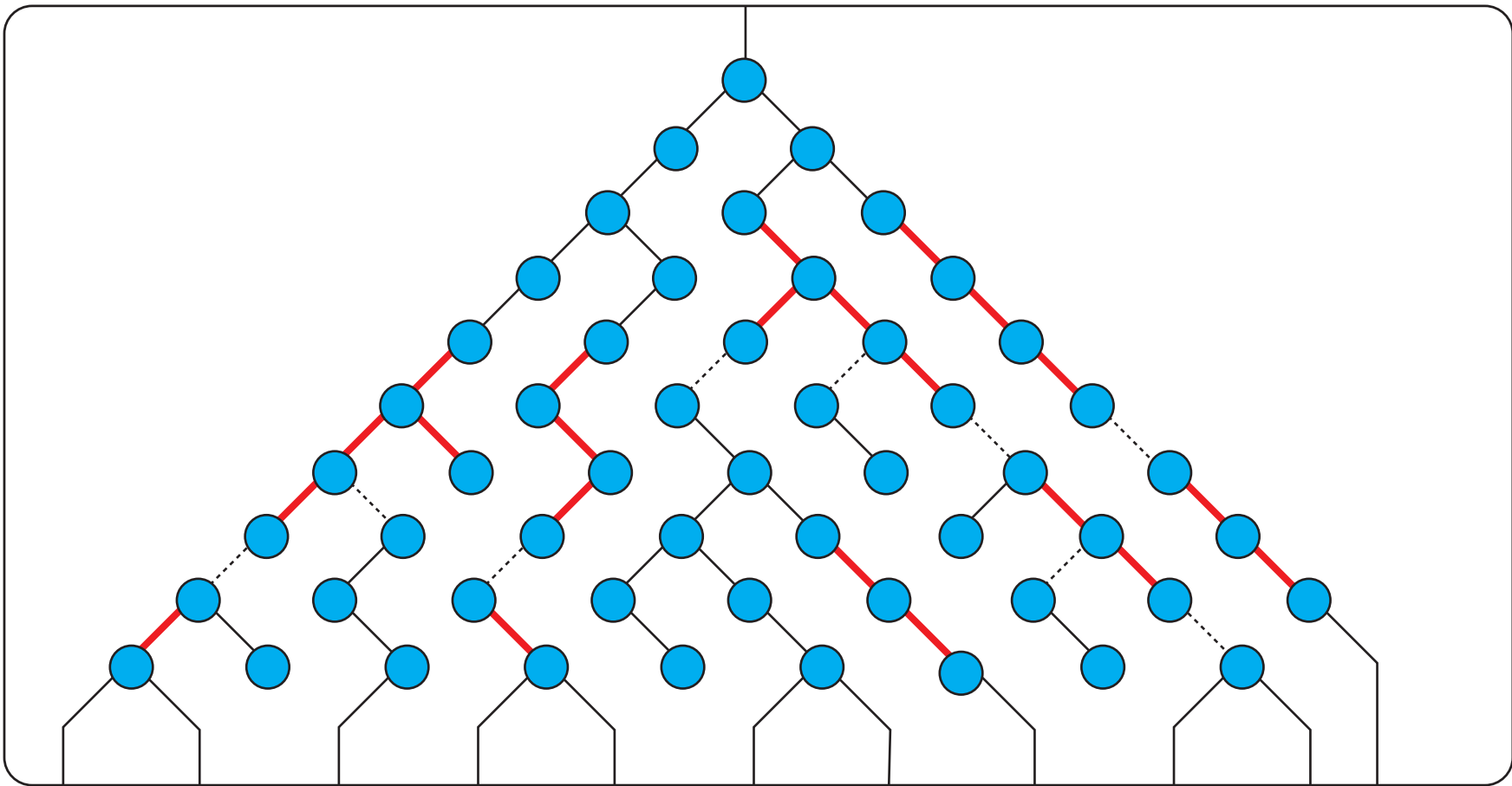
=

a term of arity 3



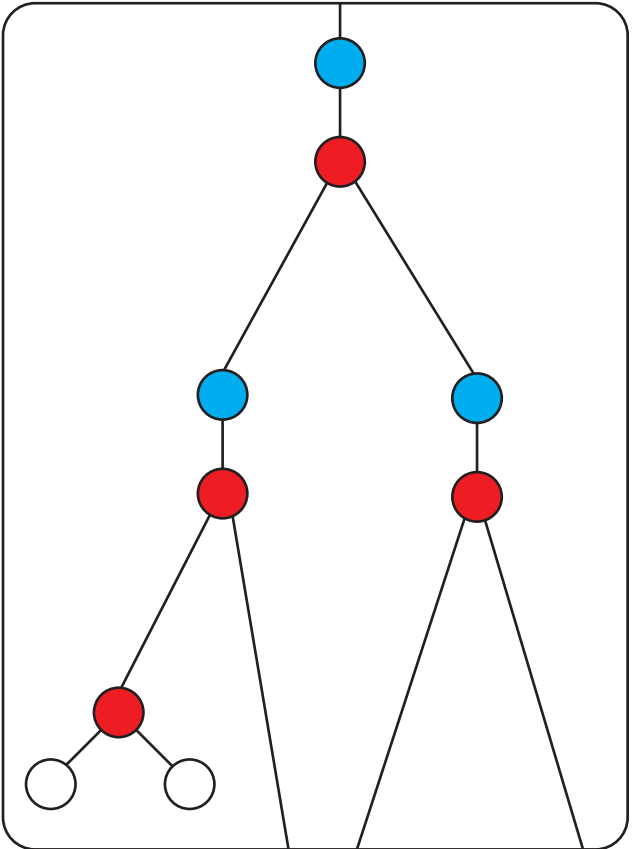
lines leaving at the bottom of the box
represent variables

dangling edges represent unused variables



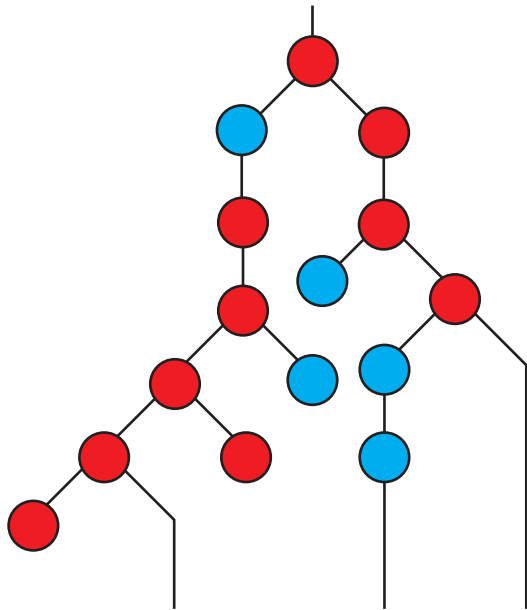


\mapsto





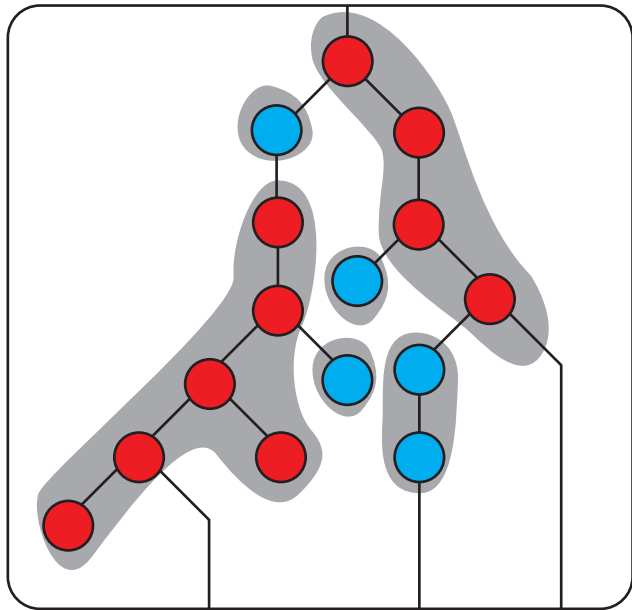
a term



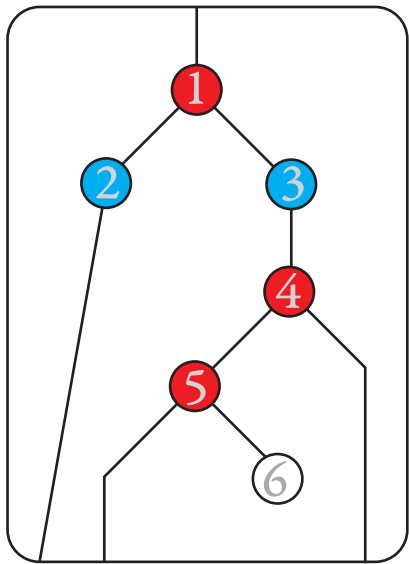
ancestor equivalence



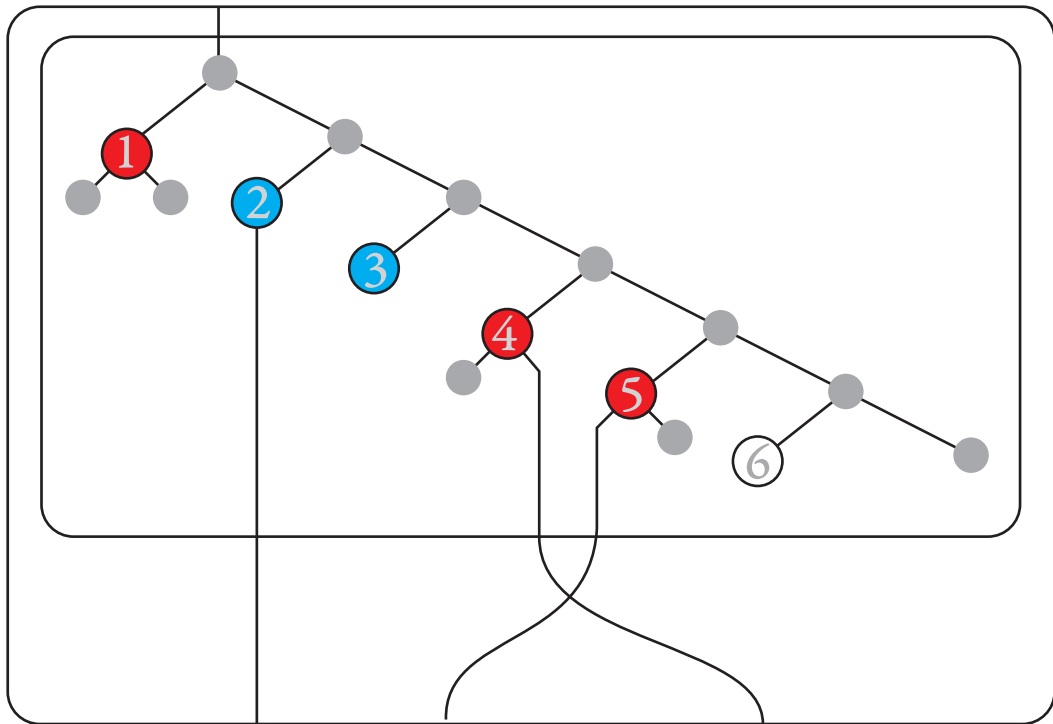
descendant equivalence





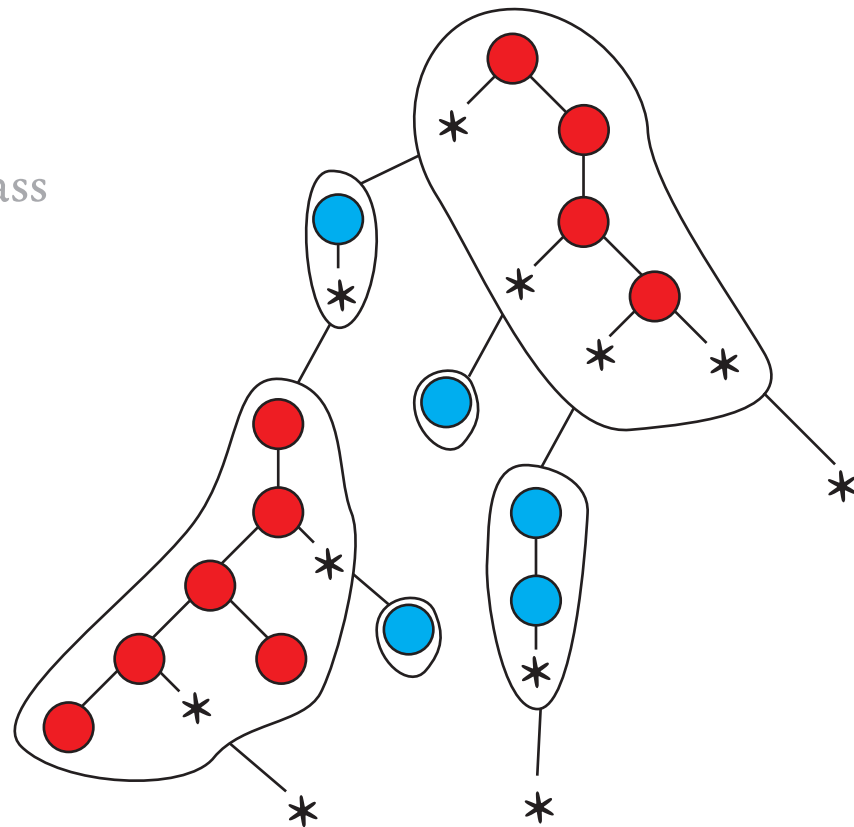


\mapsto

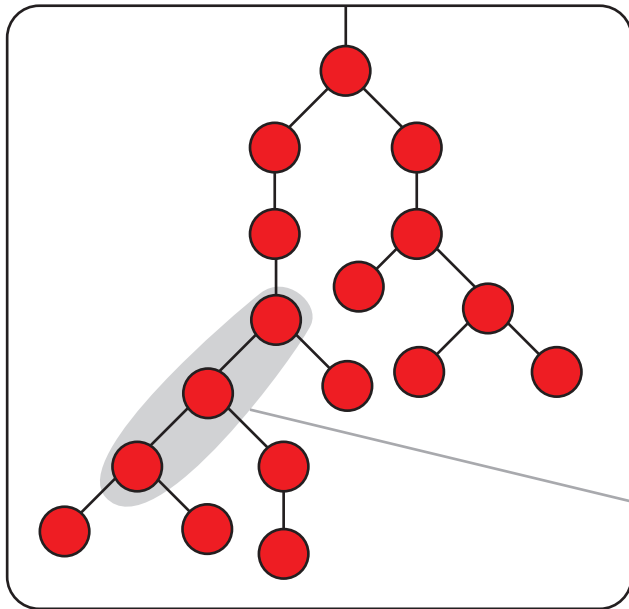


The diagram shows a graph with nodes and edges. The nodes are colored red, blue, or white. The graph is partitioned into three regions labeled x_1 , x_2 , and x_3 . The regions are shaded gray. The nodes are connected by edges. The nodes in x_1 are red. The nodes in x_2 are blue. The nodes in x_3 are red. There are two white nodes in the center of the graph.

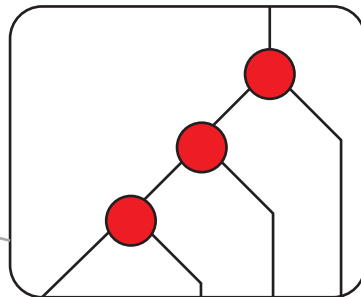
- an equivalence class



a tree



a term with 4 ports that represents part of the tree





input alphabet

arity 2



arity 1



arity 0



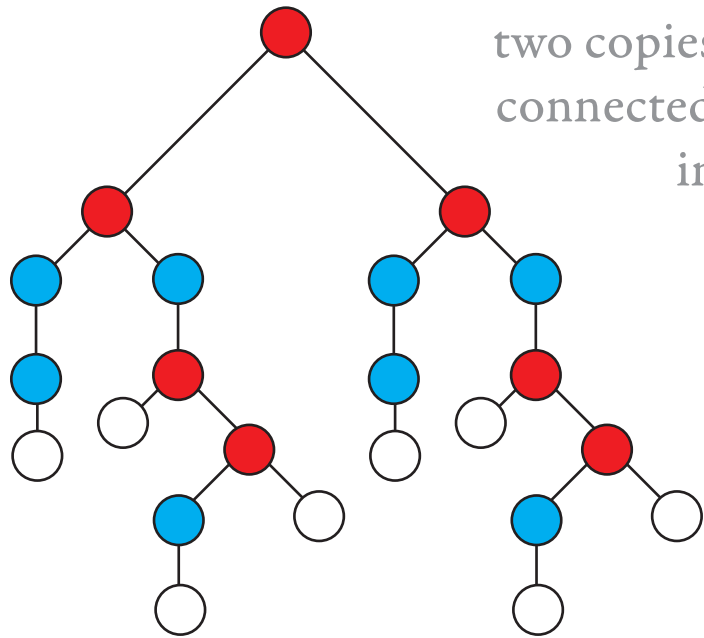
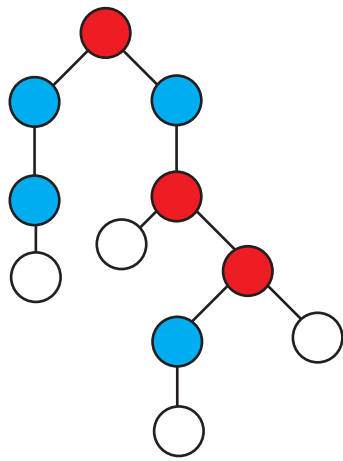
output alphabet

arity 2

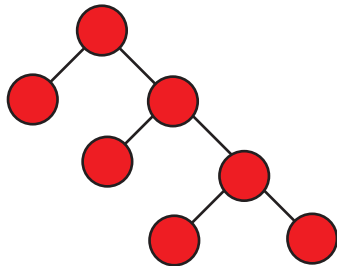
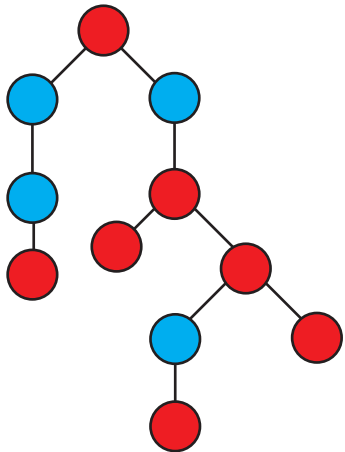


arity 0





two copies of the input tree,
connected by a binary node
in the root





input alphabet

arity 2



arity 1



arity 0



output alphabet

arity 2



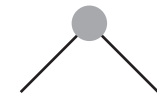
arity 1



arity 0

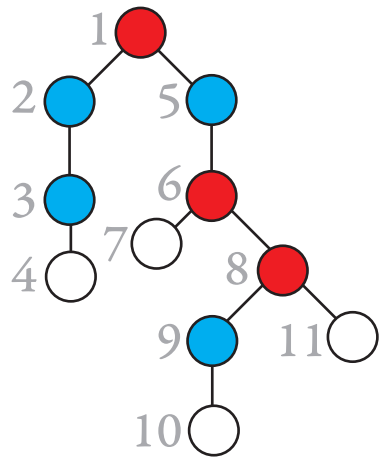


arity 2

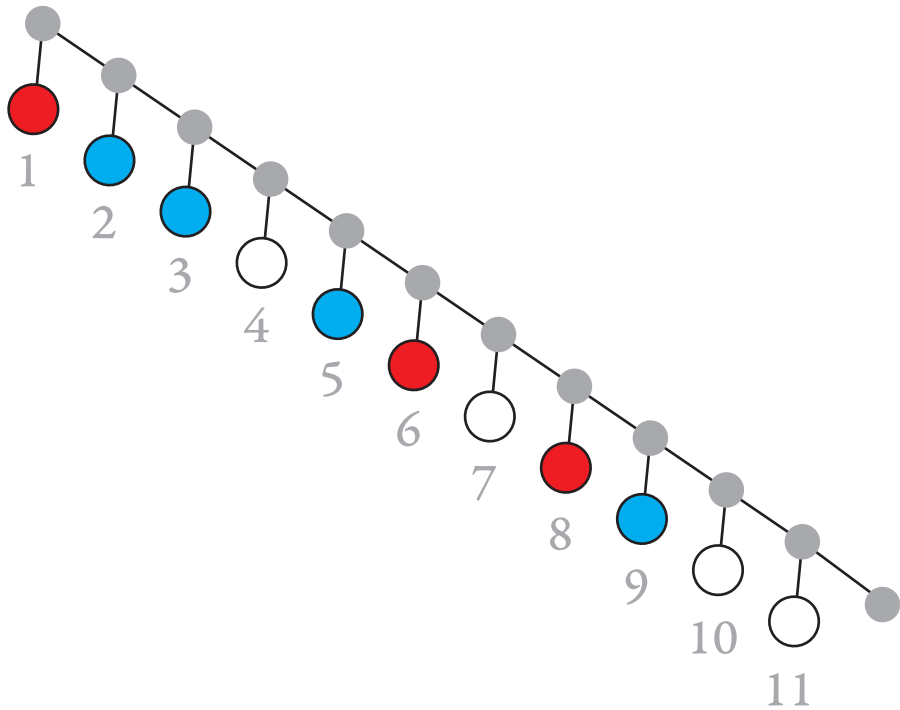


arity 0





\mapsto





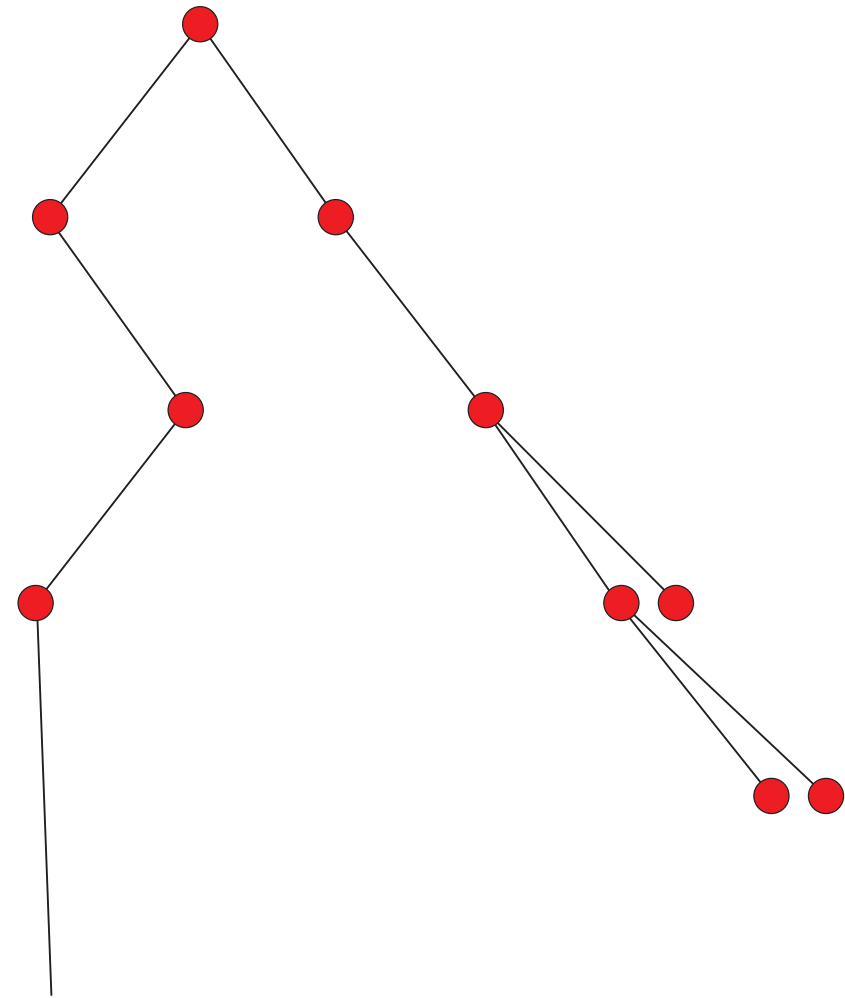
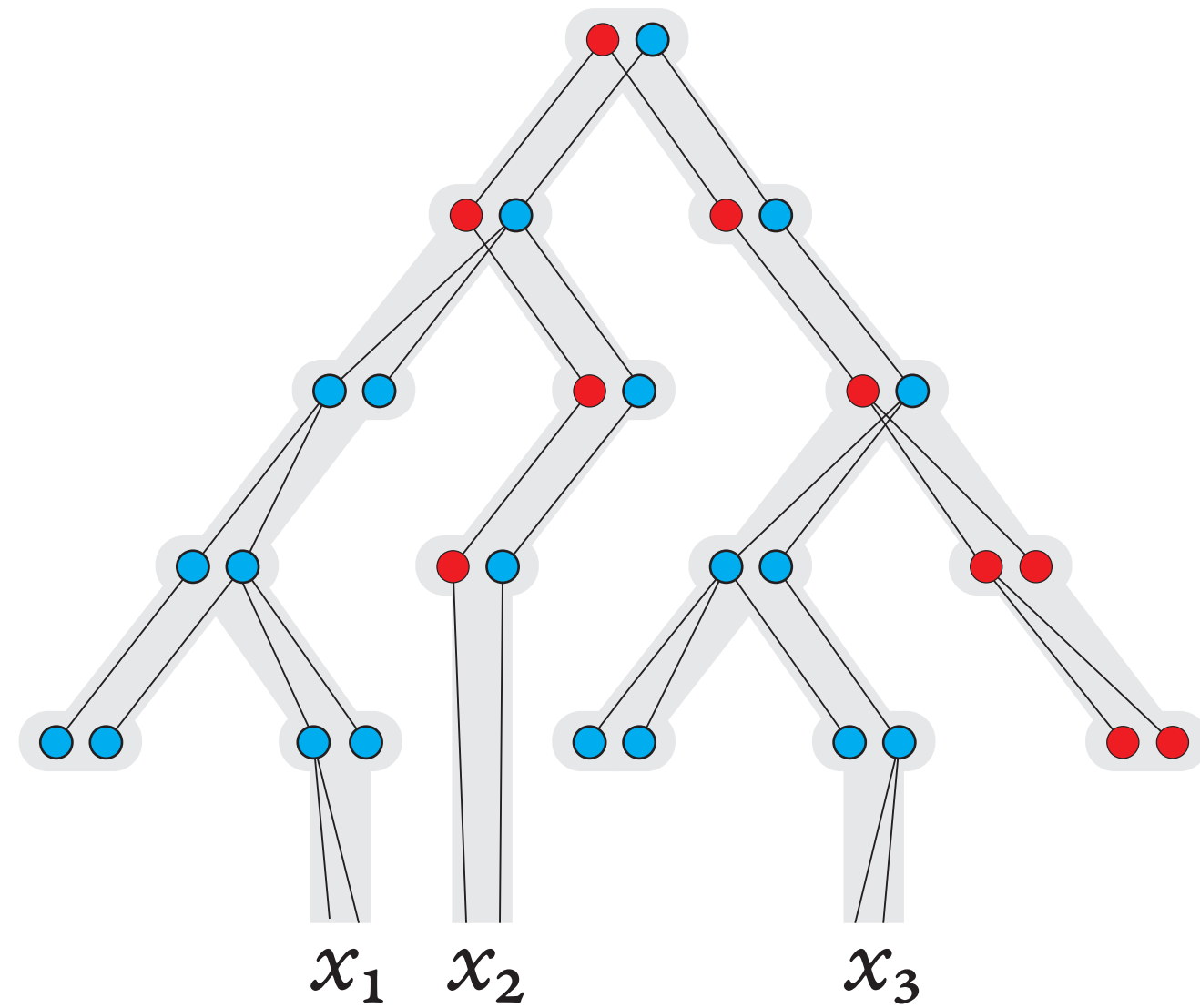


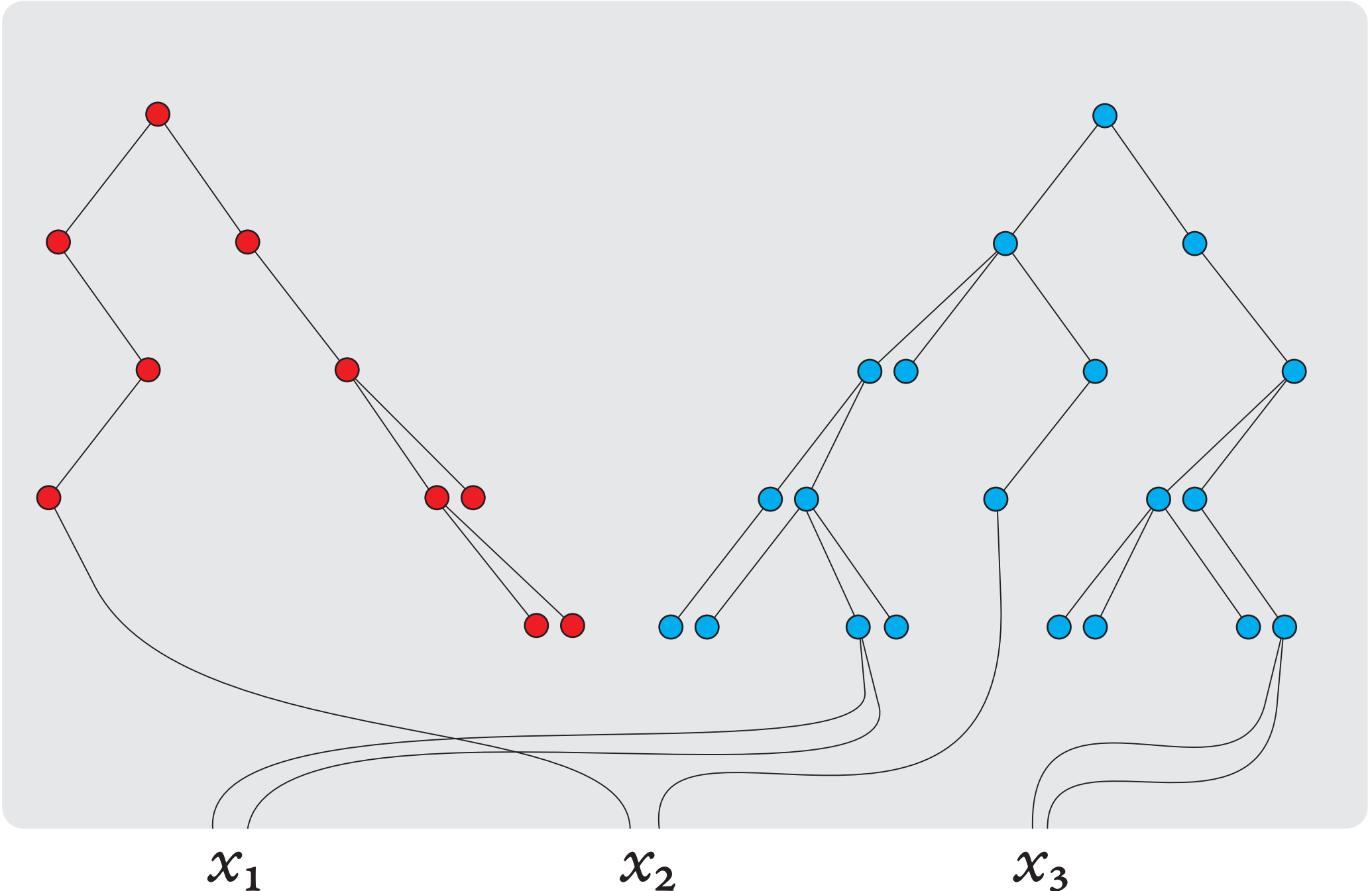
a term of arity 4



a term of arity 0





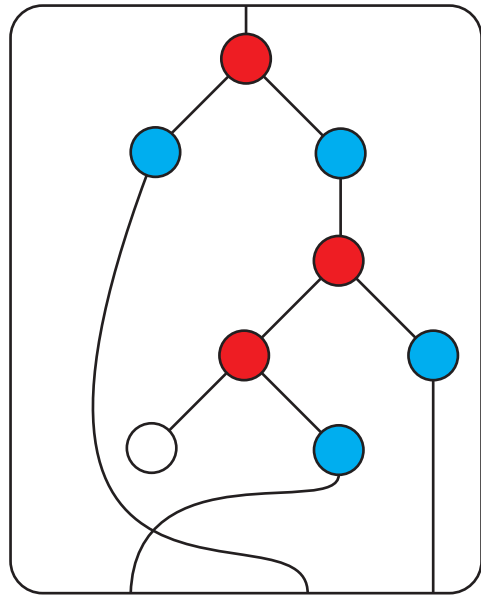




satisfies (*)

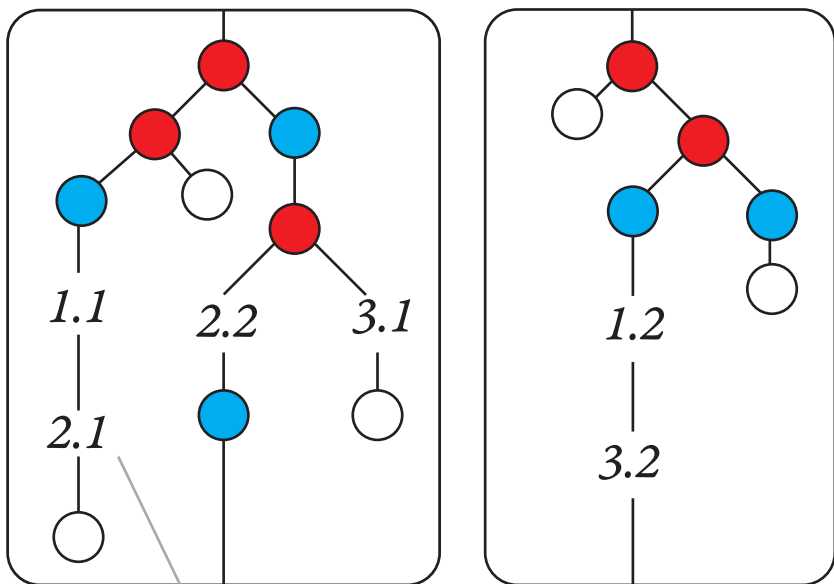
(*)

If the root has arity n ,
and $1 \leq i < j \leq n$, then
all ports of the j -th
subterm of the root are
after all ports of the
 i -th subterm of the root



violates (*)

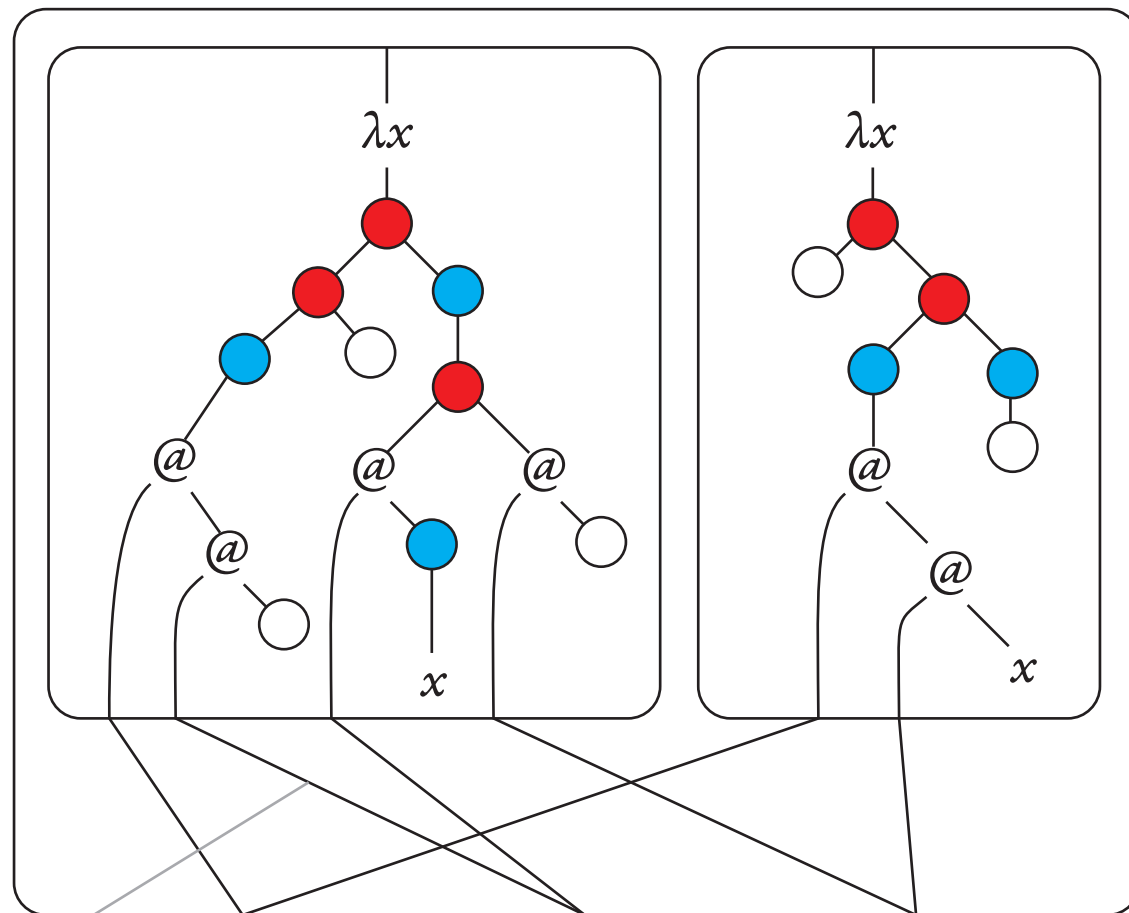
a register update



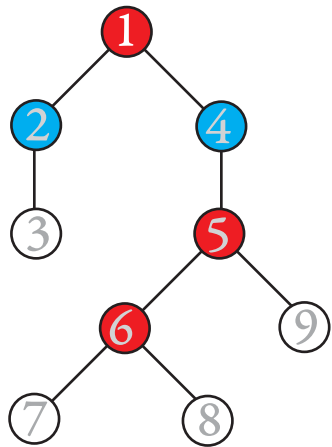
Variable $i.j$ represents register i in the j -th argument of the register update.

In the dual, this variable is mapped to the i -th edge which enters the j -th port of the reducer.

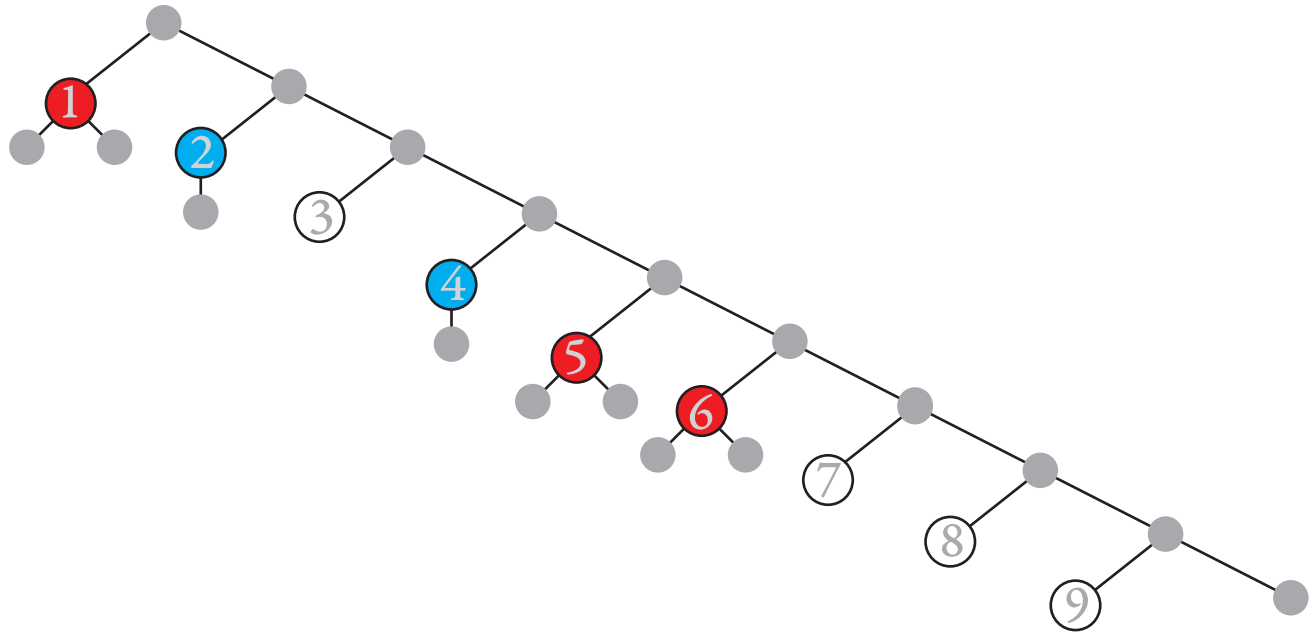
its dual



input

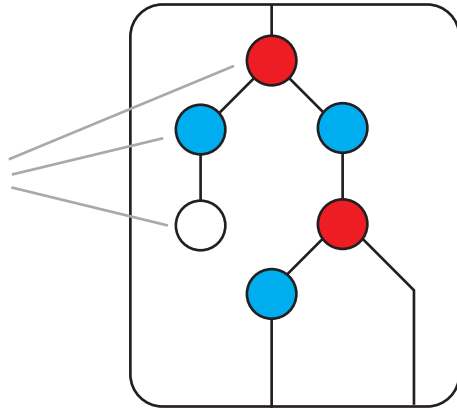


output

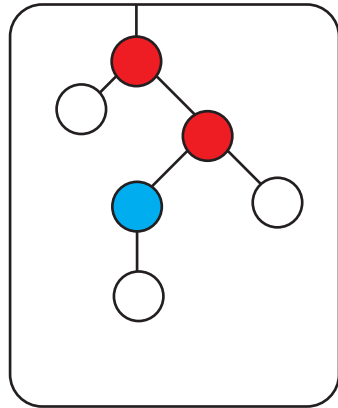


register r of arity 2

letters of the
output alphabet



register s of arity 0

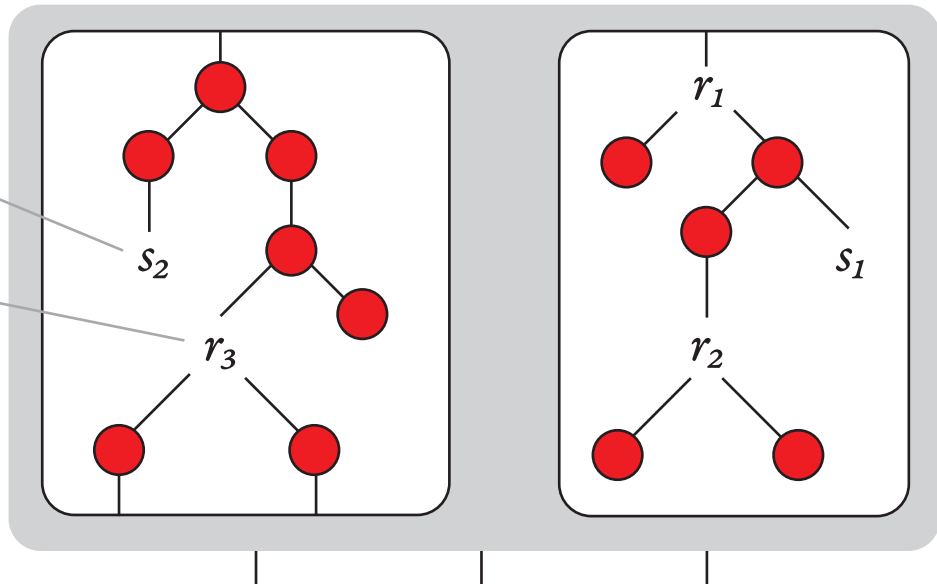


register r of arity 2

register s of arity 0

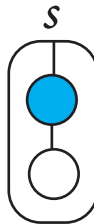
register s from argument 2

register r from argument 3









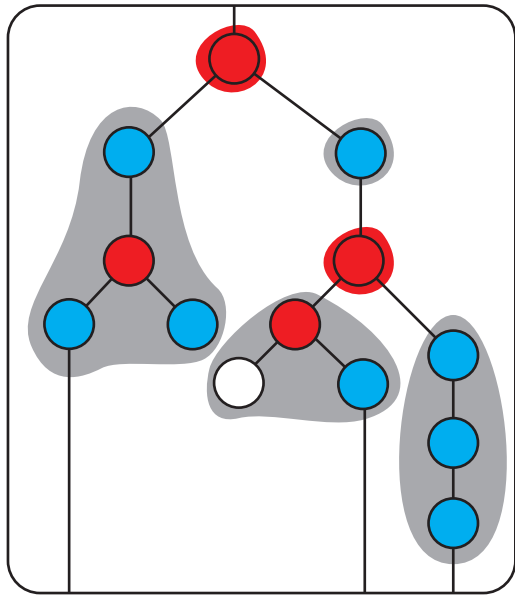




factors without
branching nodes

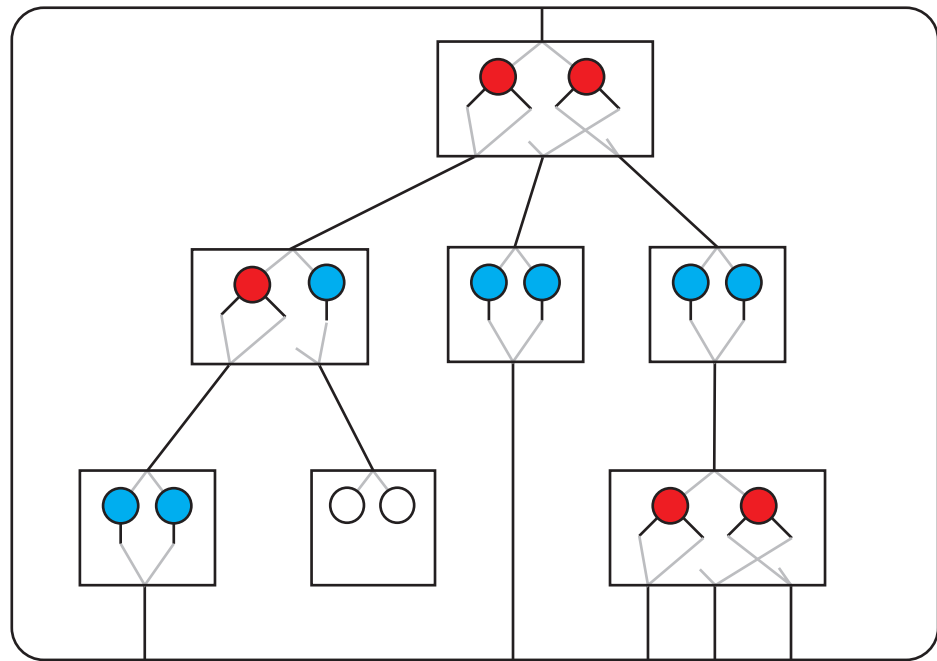


factors with
branching nodes

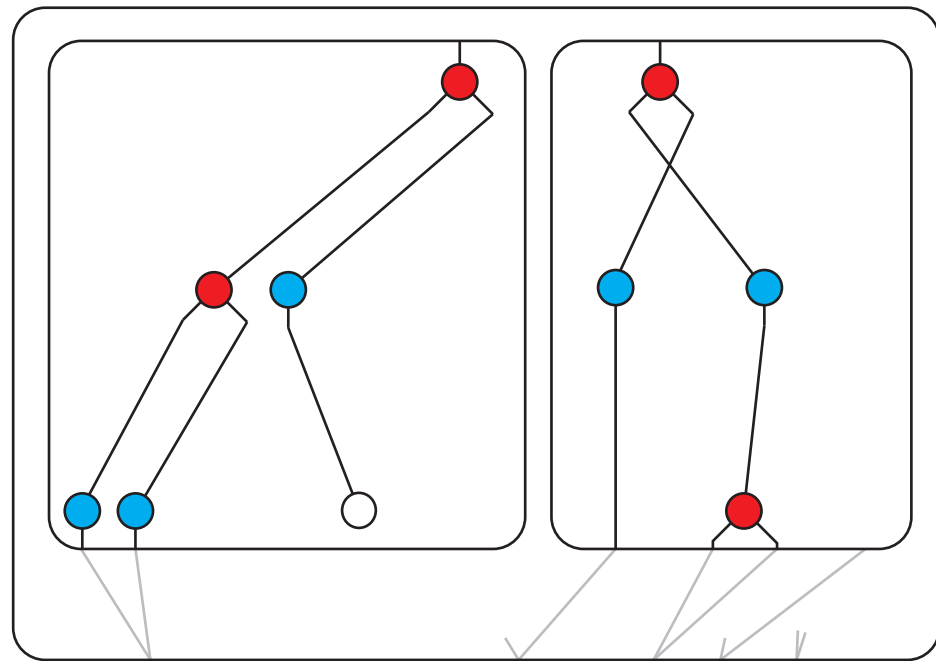


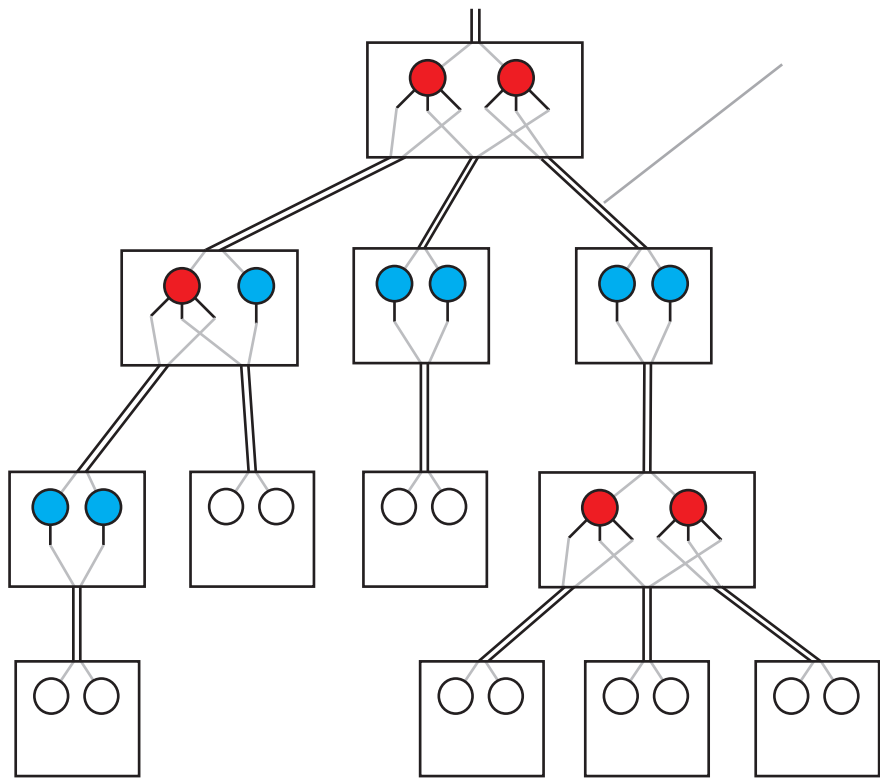


a term of matrix powers

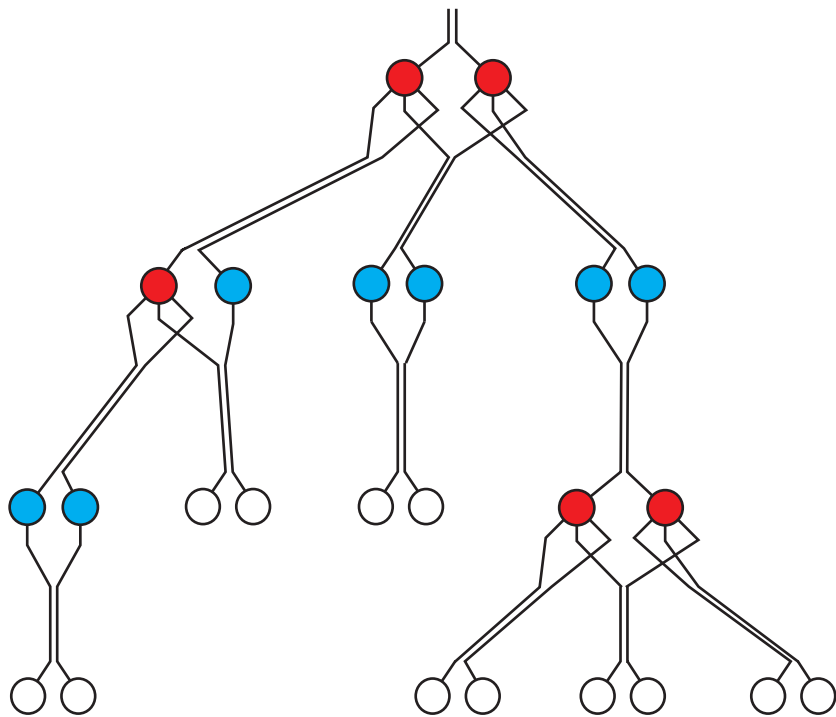


its term unfolding





\mapsto





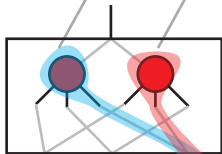
source 1 of e

source 2 of e

edge e

target 1 of e

target 2 of e



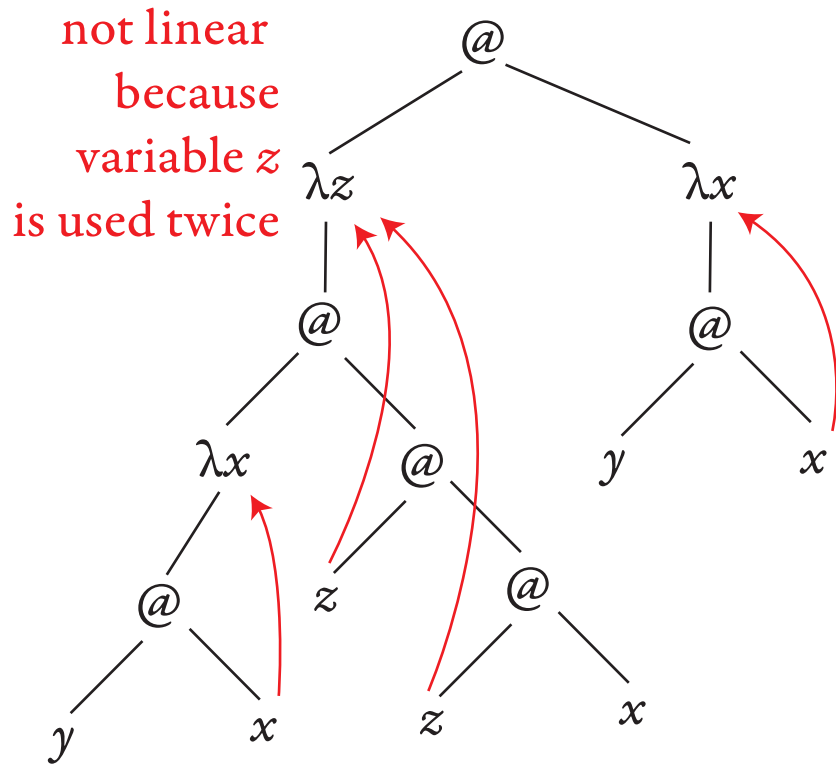
linear



we only count
variables used
in their scope

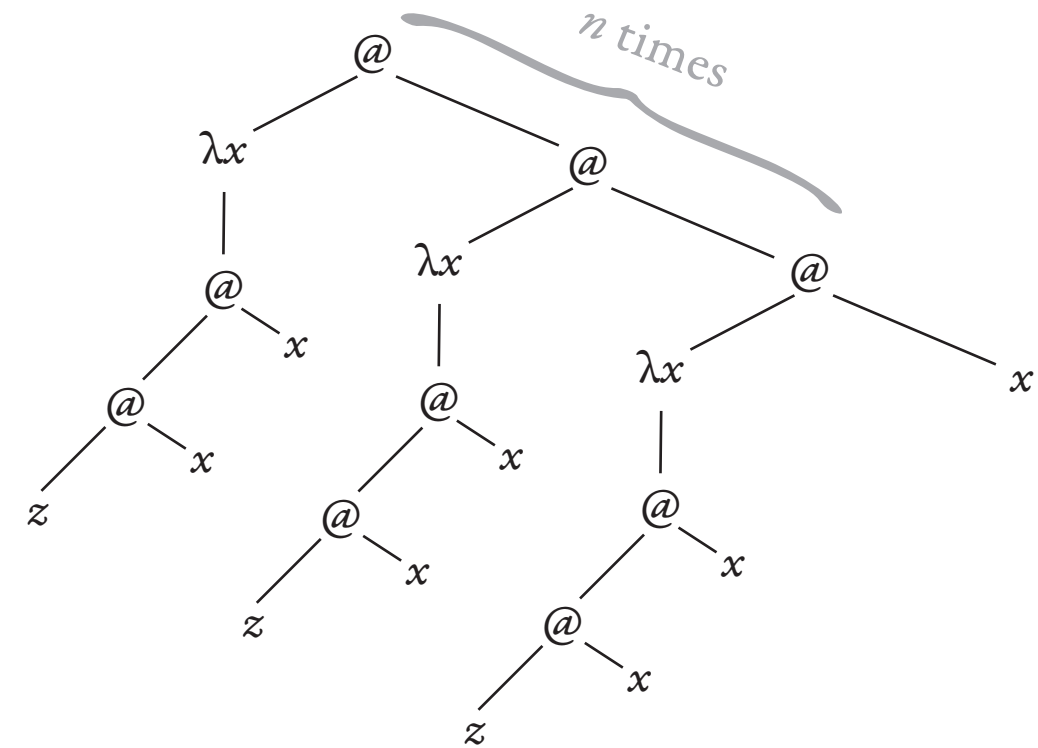
variable z can be used twice because it is free

not linear

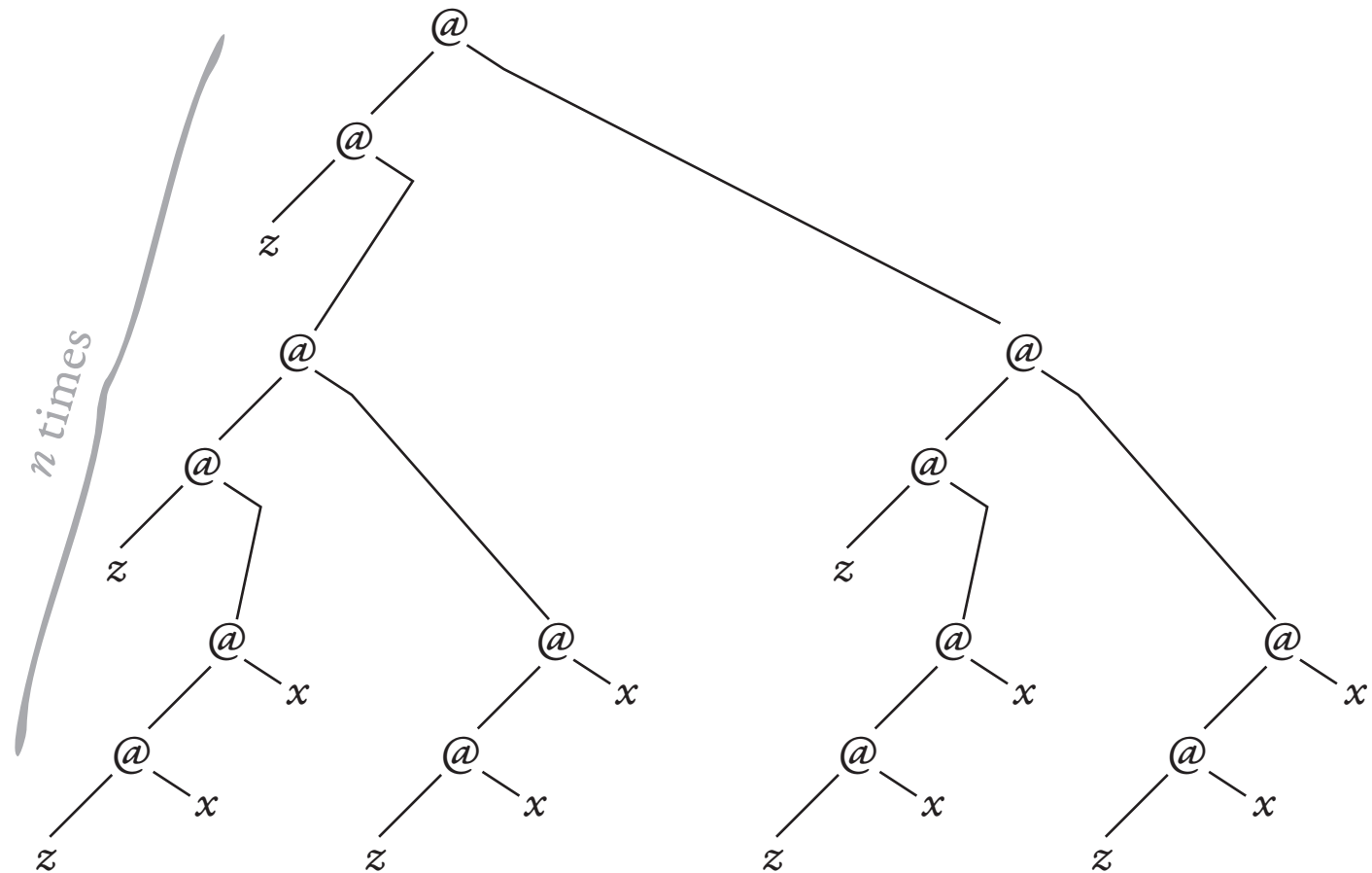


not linear
because
variable z
is used twice

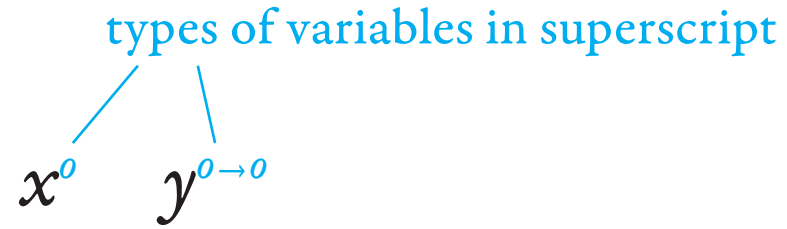
a λ -term of size $O(n)$



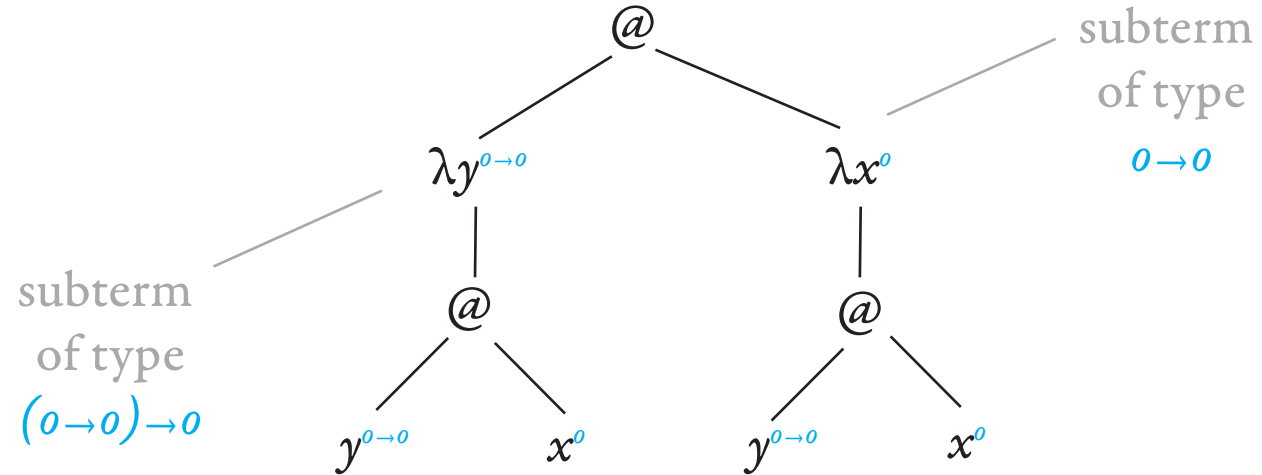
its normal form of size $O(2^n)$



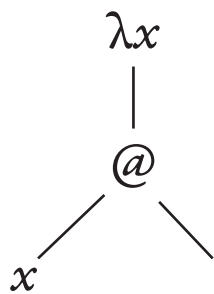
variables



λ -term of type o



@



$\lambda x.$

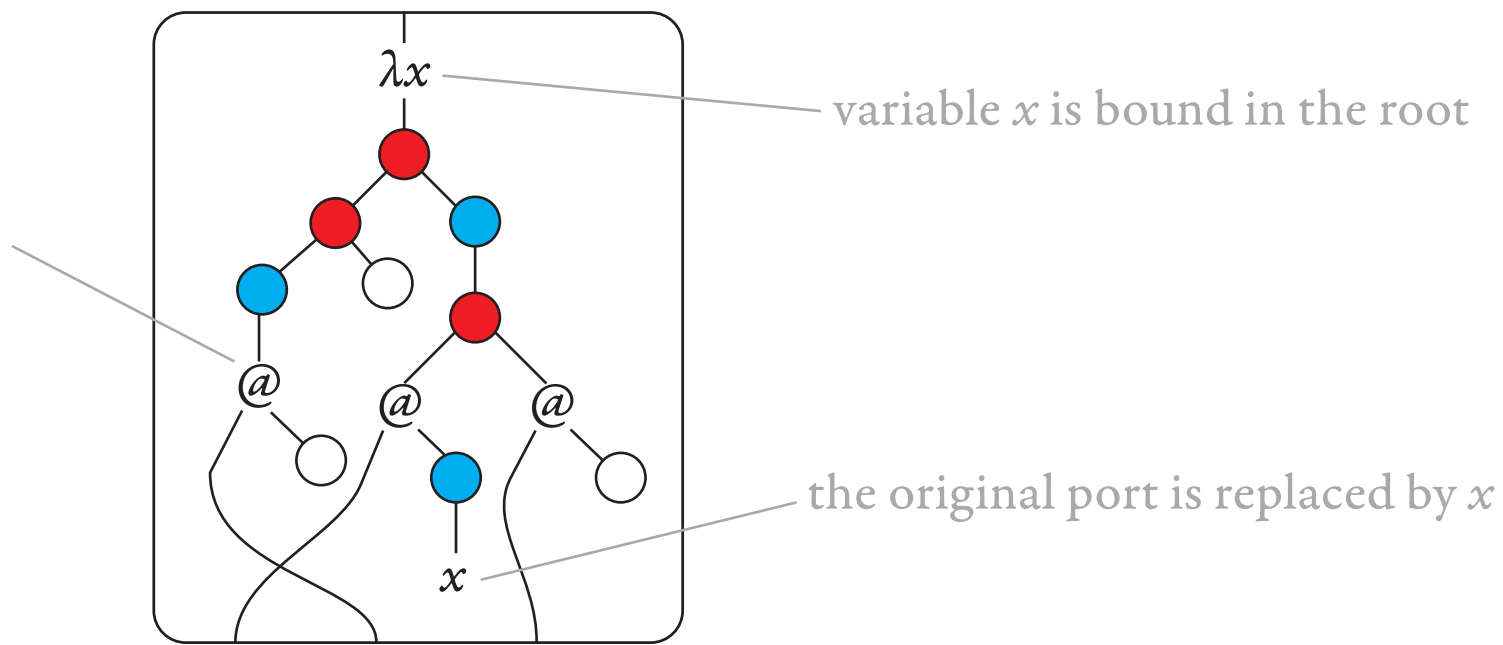


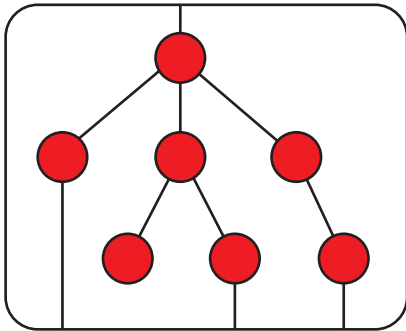
r



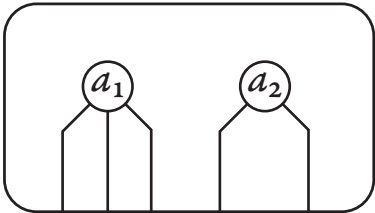
placeholder for the term
stored in the unique register
of the 2nd child

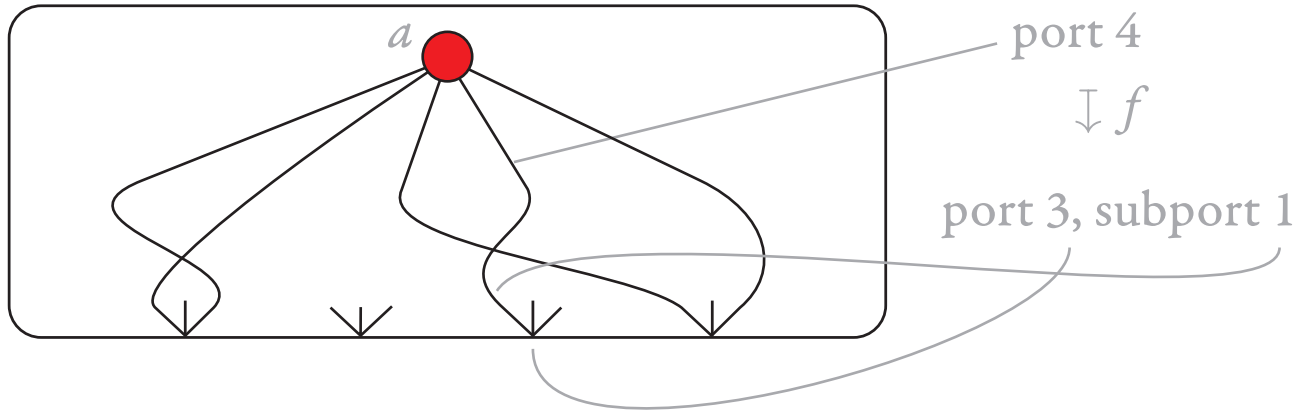






dangling edges
represent ports



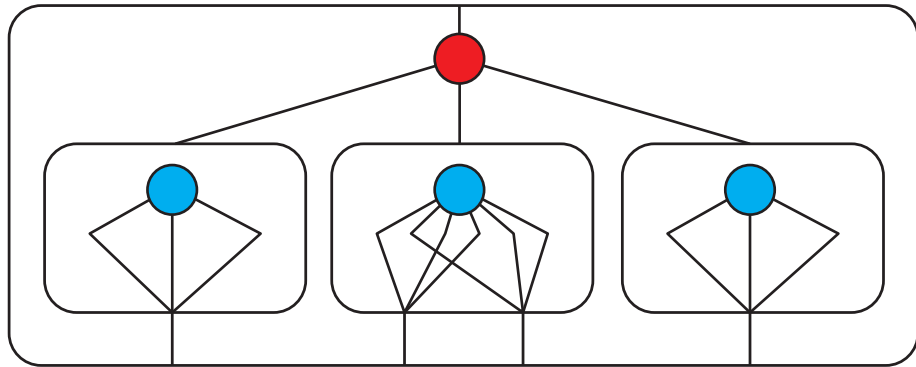


the root is from Σ

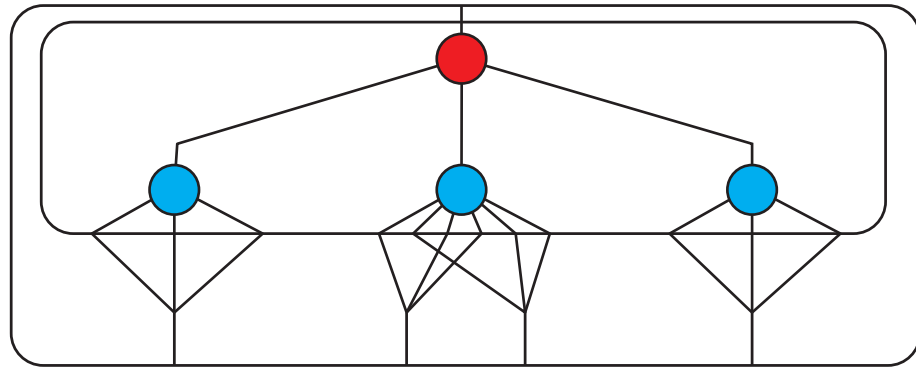
all children are from Γ

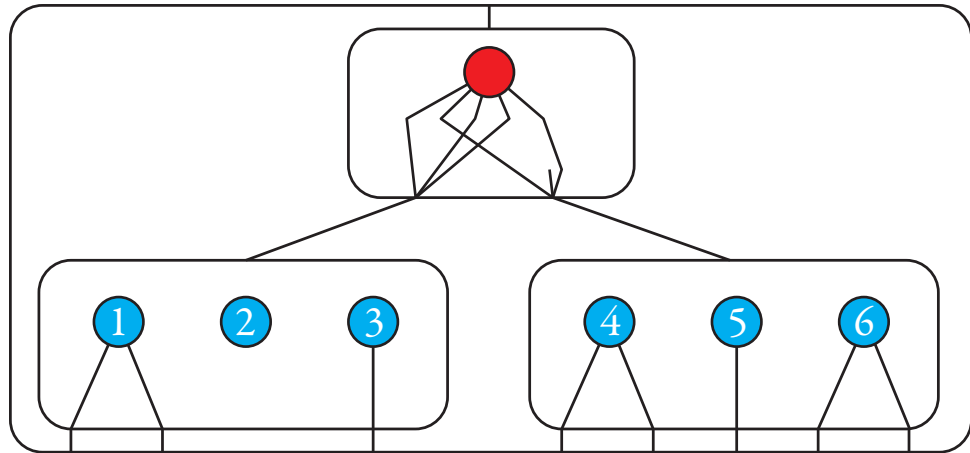


input

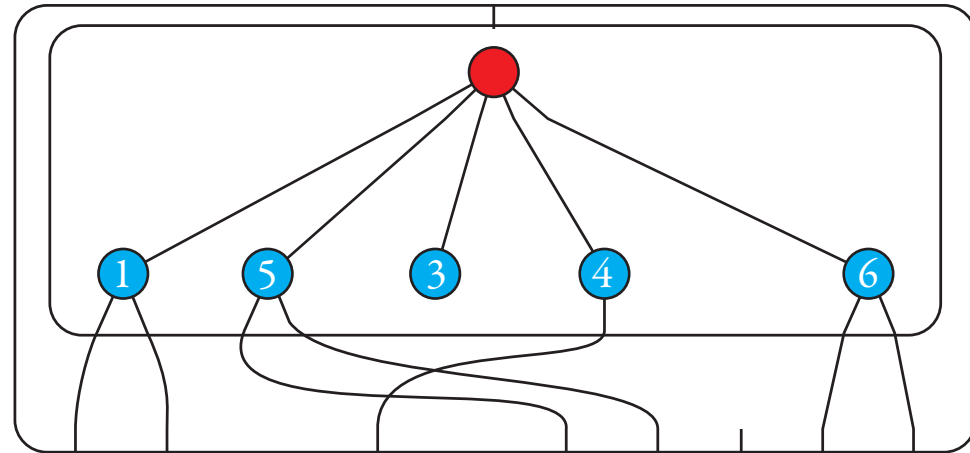


output

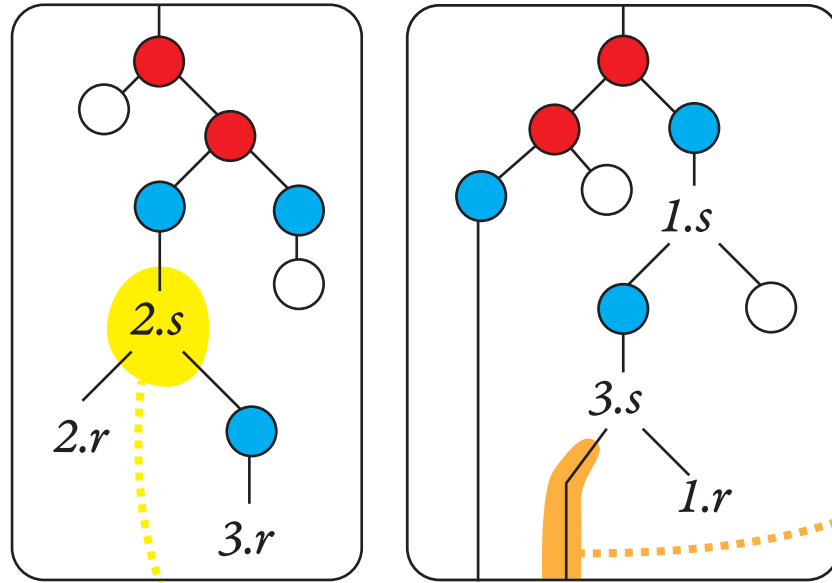




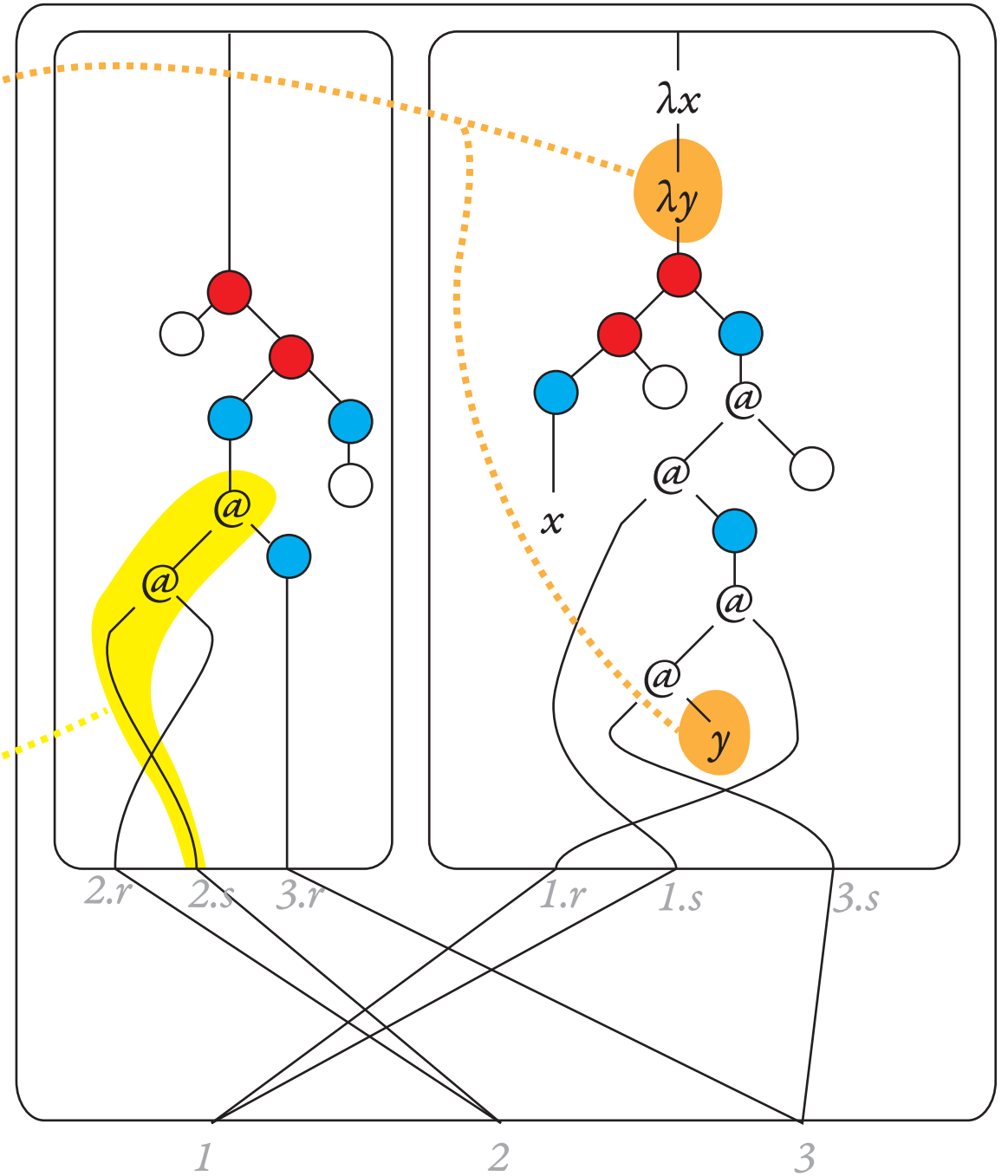
\mapsto



a register update



its dual



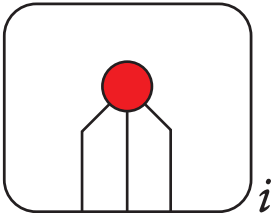
The diagram shows a binary tree structure. The root node is red. Its left child is red, and its right child is blue. The red node's left child is blue, and its right child is white. The blue node's left child is blue, and its right child is white. A yellow circle labeled r_1 highlights the blue node that is the right child of the root. An orange shape labeled r_2 highlights the subtree rooted at the blue node that is the left child of the root. A dashed orange line labeled r_3 indicates a path from the root to the right child of the root, and then to the right child of that node.

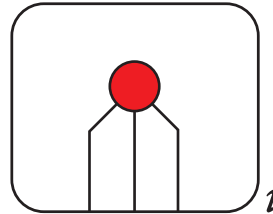
[illegible]

a non-port node is represented by a variable, corresponding to the label, applied to the children of the node

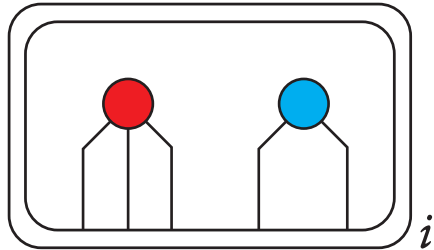
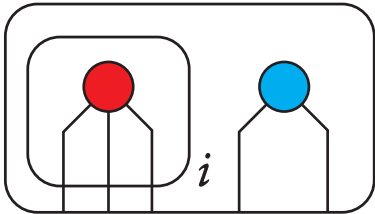
- the variables representing the ports are bound outside

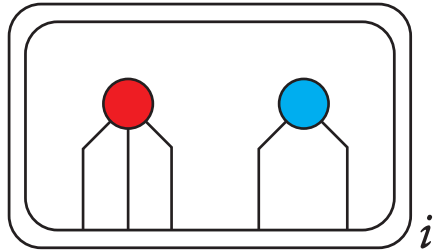
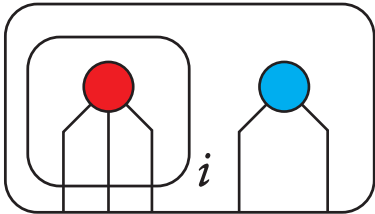
the i -th port is represented by a variable x_i of type \mathbf{o}

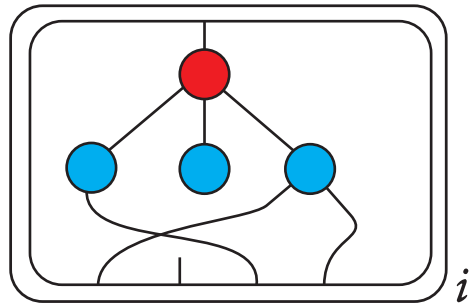
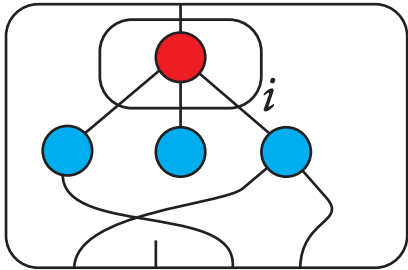


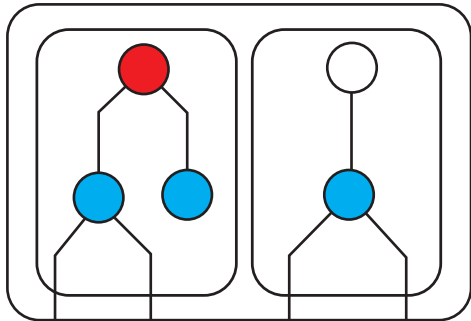
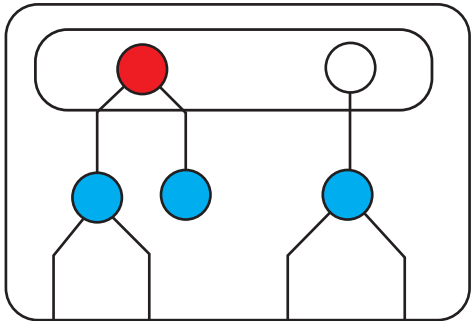


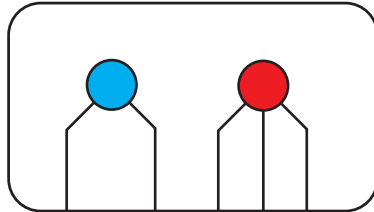




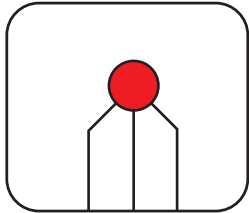


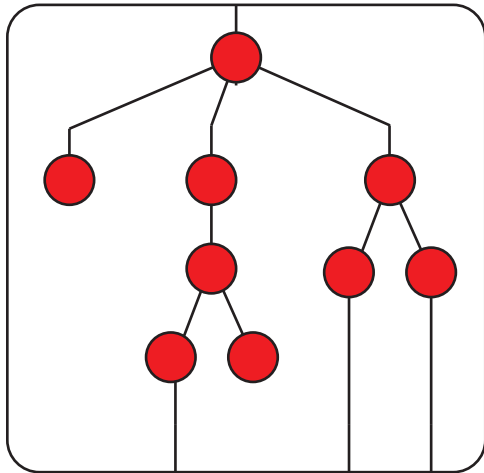
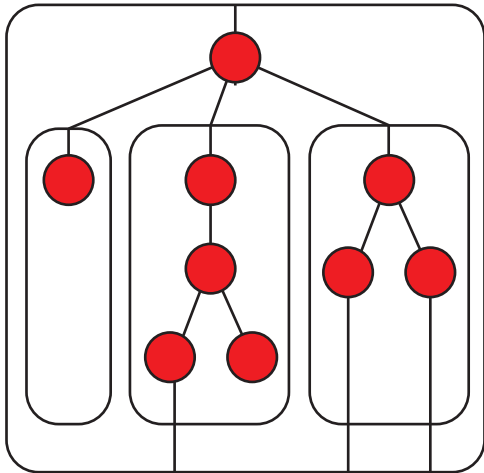


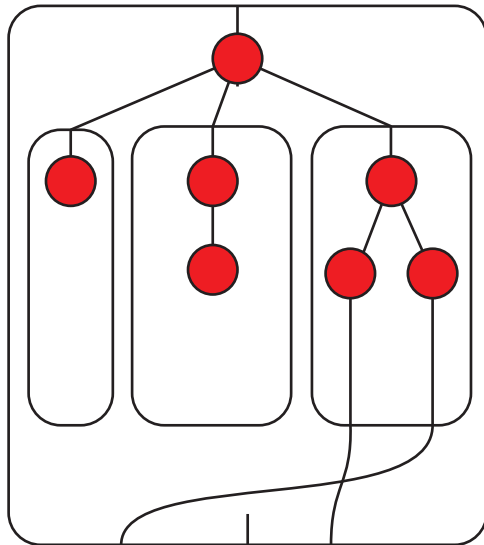
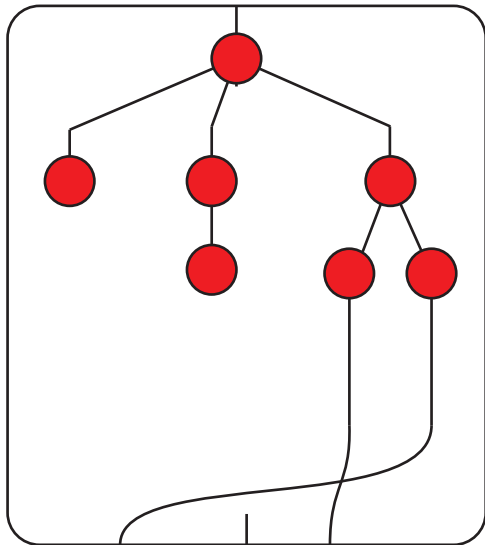


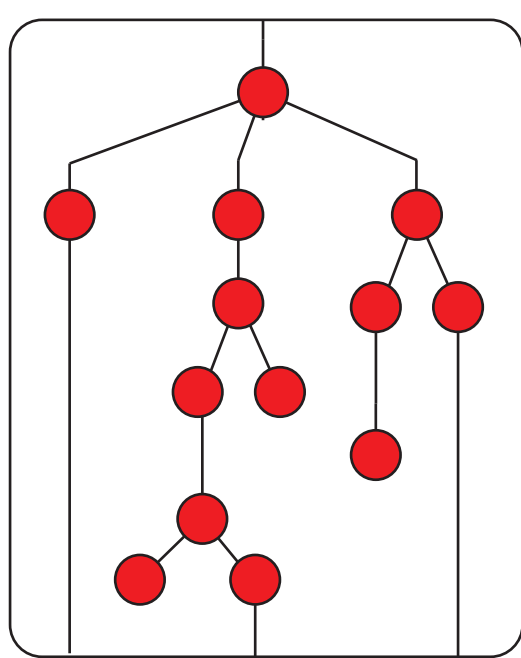
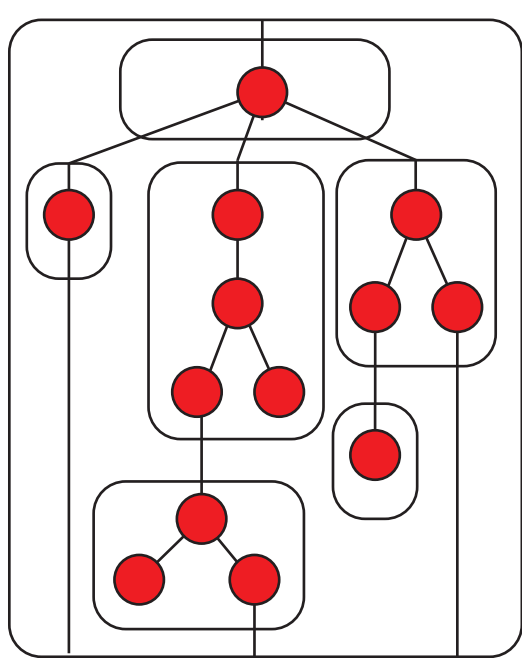


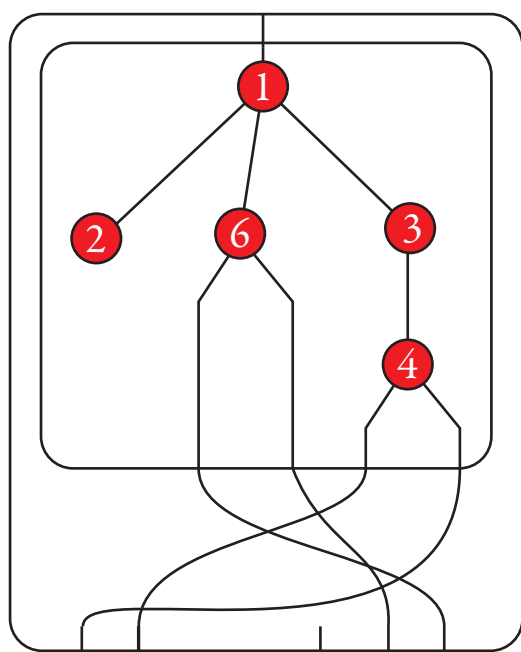


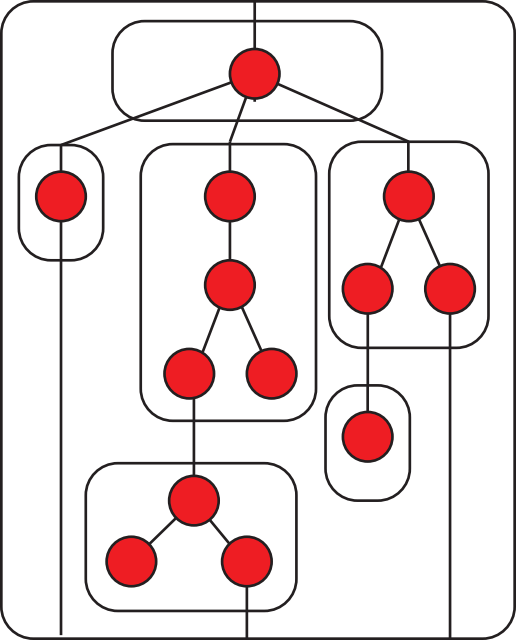




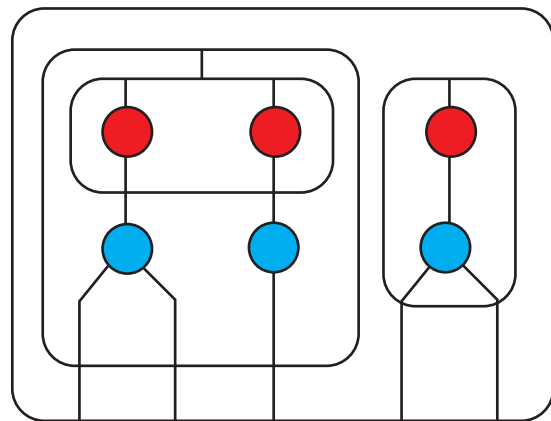
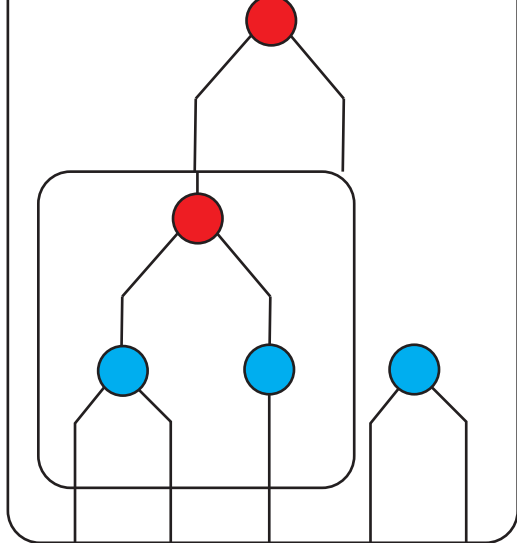
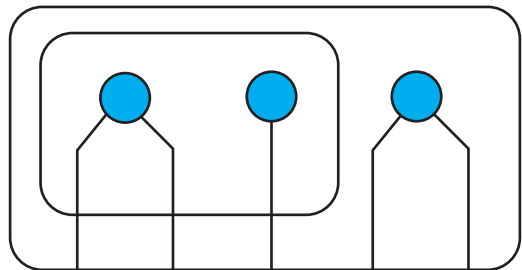


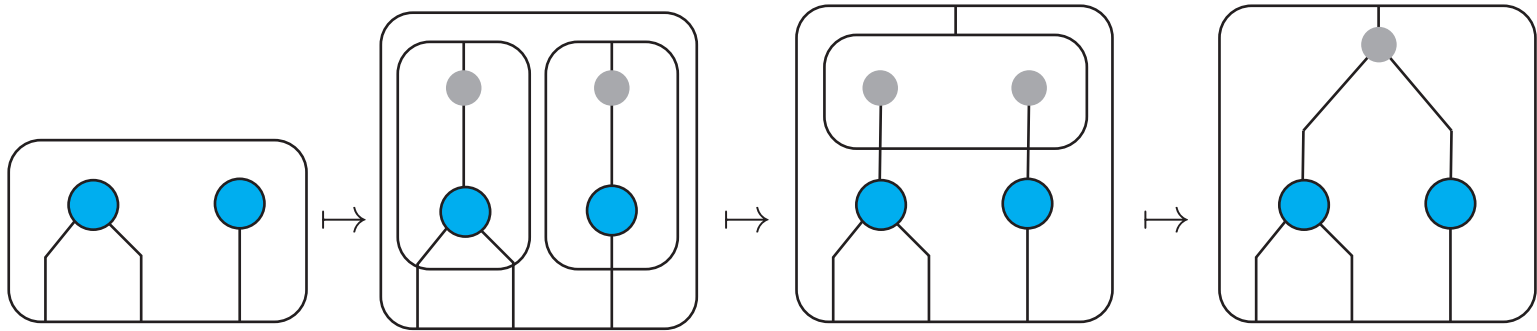






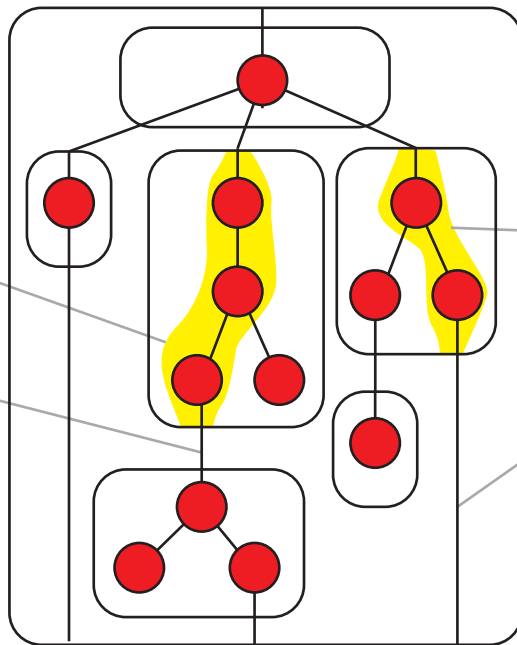




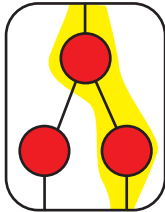




the subbranch
corresponding to
an internal edge

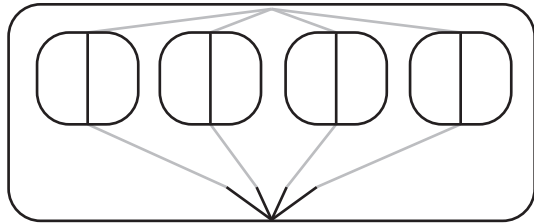


the subbranch
corresponding to
an external edge



a branch can be visualised as
a term with a distinguished
root-to-port path

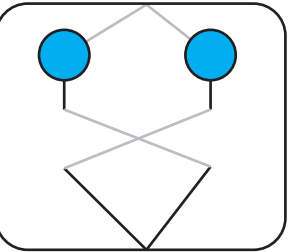




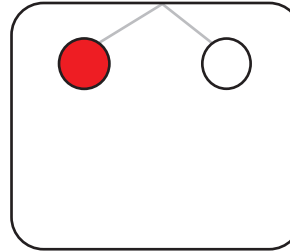
a tuple of k identity terms
with all their ports folded
into one

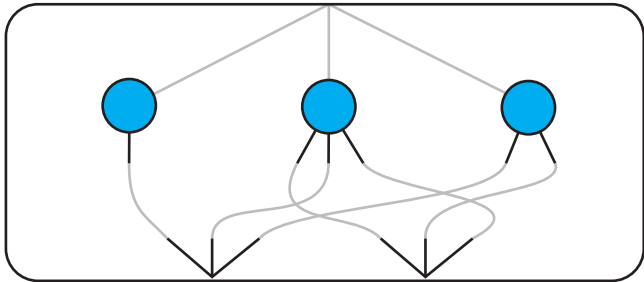
$$\Sigma = \{ \text{blue circle with stem}, \text{red circle}, \text{white circle} \}$$

$$a \in \Sigma^{[2]}$$

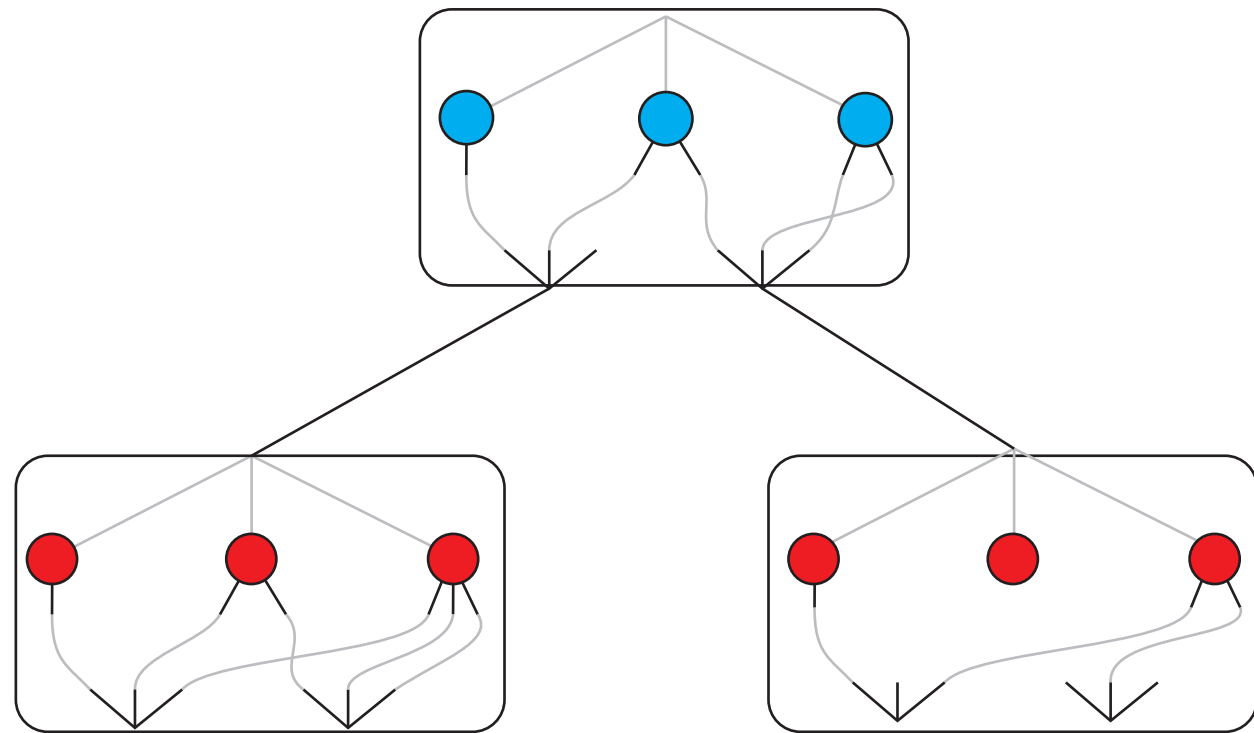


$$b \in \Sigma^{[2]}$$

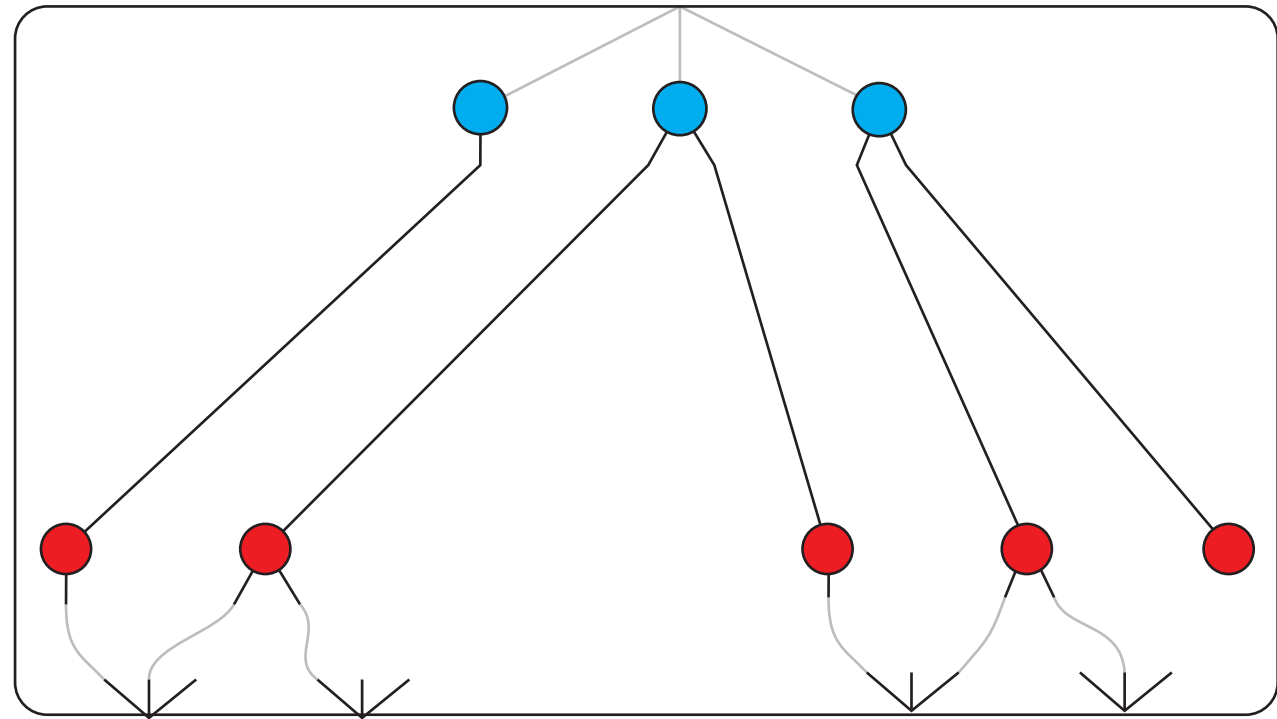


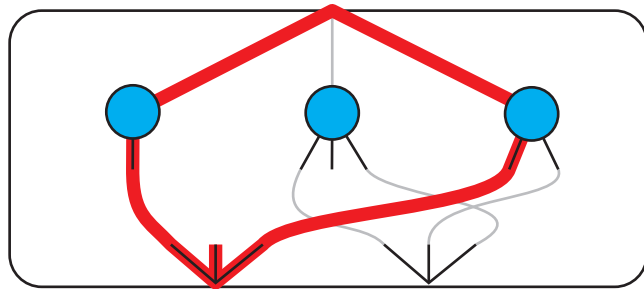


a shallow term of matrix powers



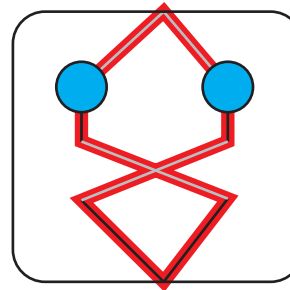
its shallow unfolding





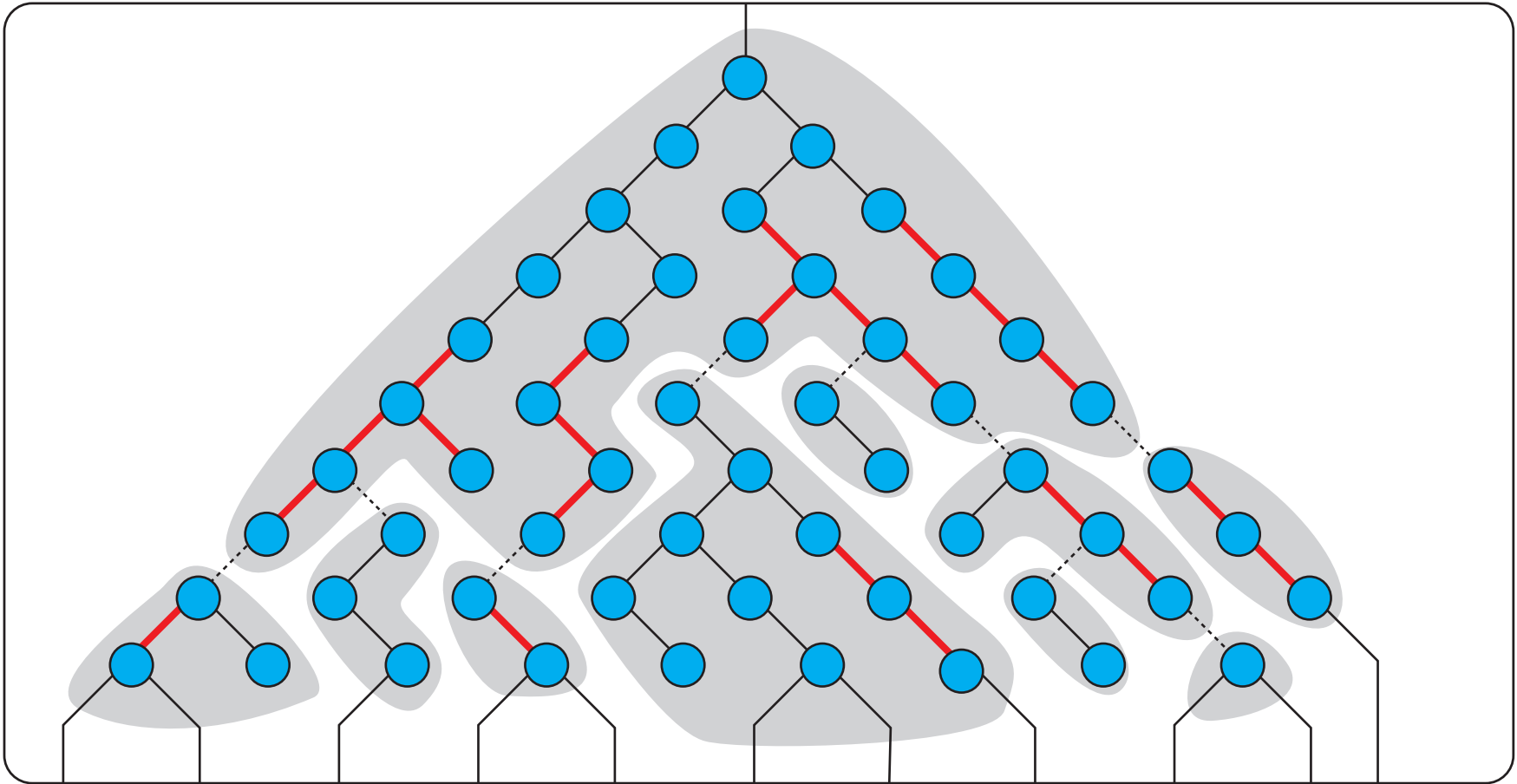
twist of port 1

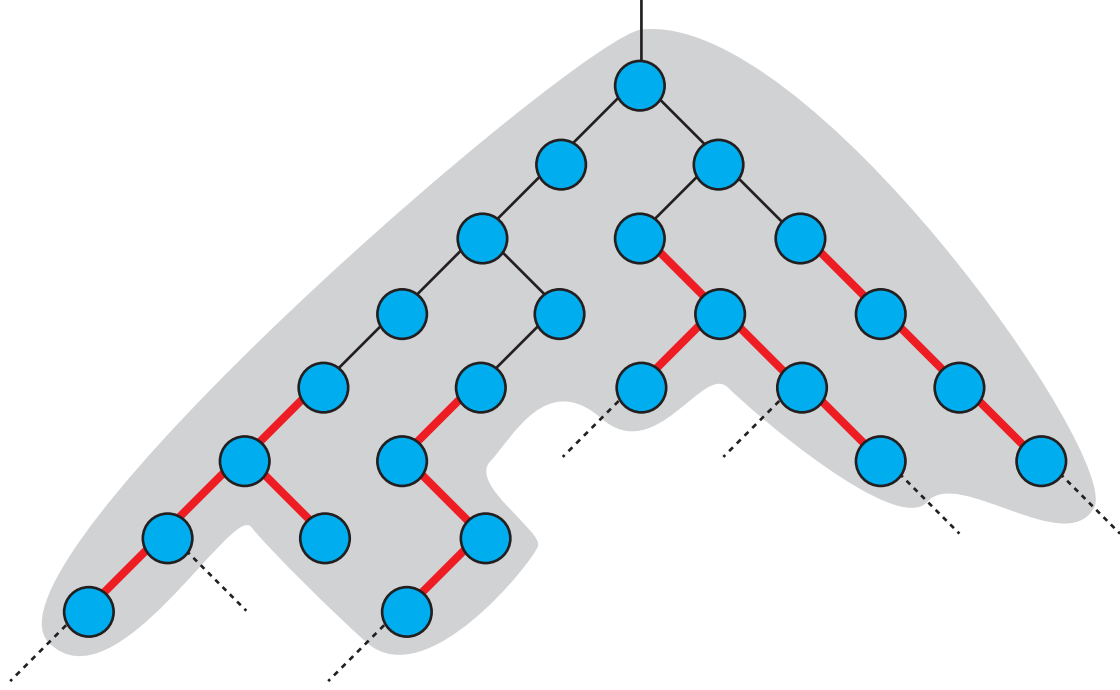
1	2	3
↑		↑
1	2	3

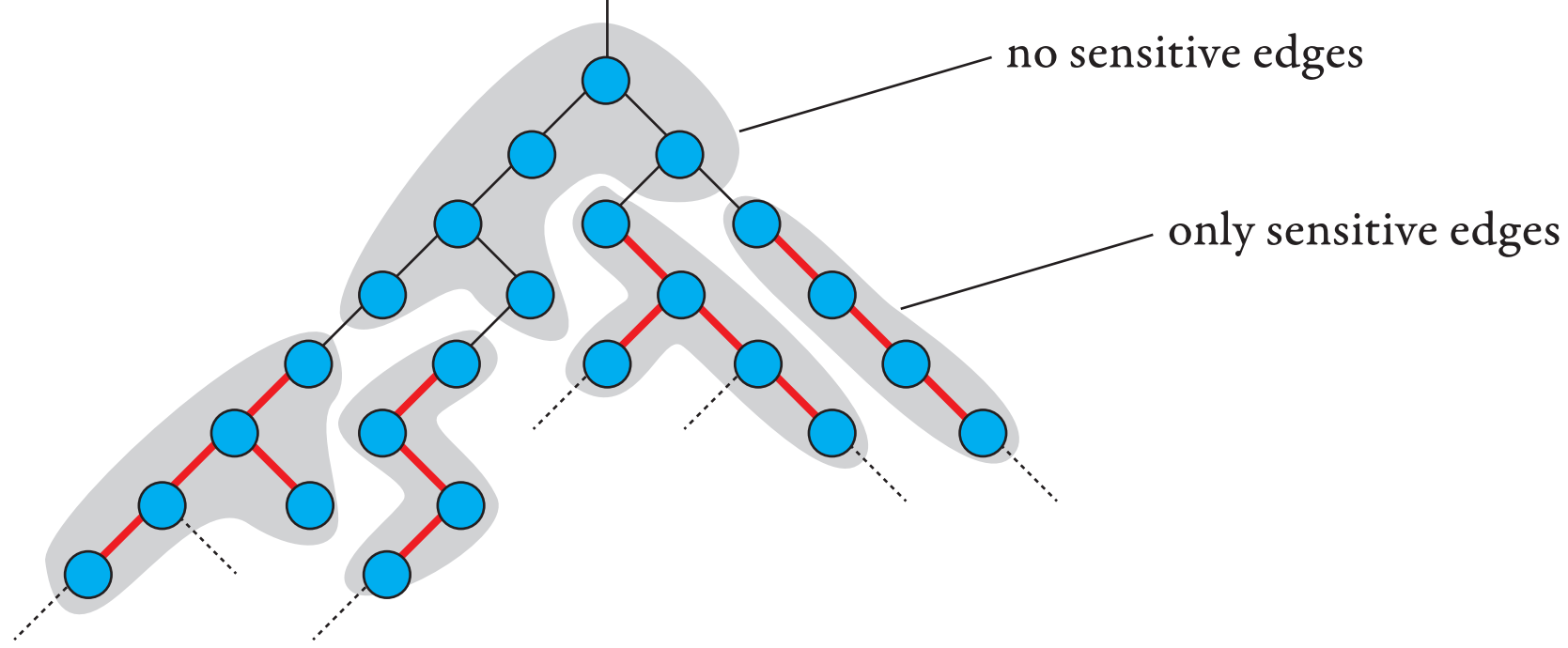


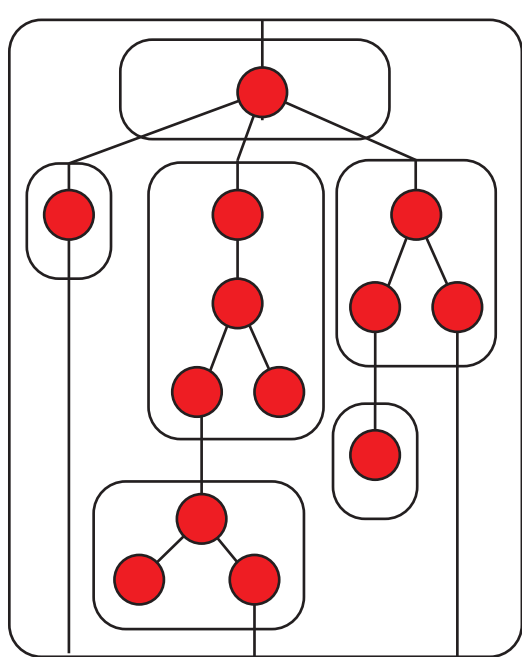
twist of port 1

1	2
↗	↖
1	2

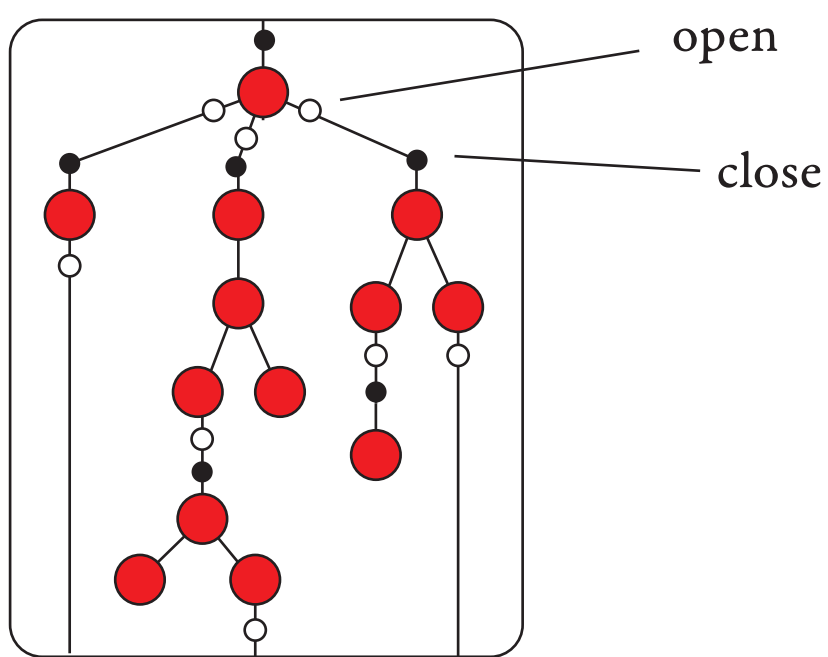






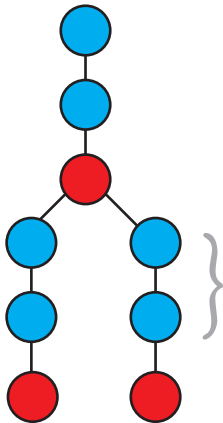
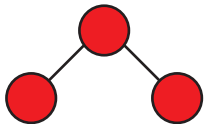


\mapsto



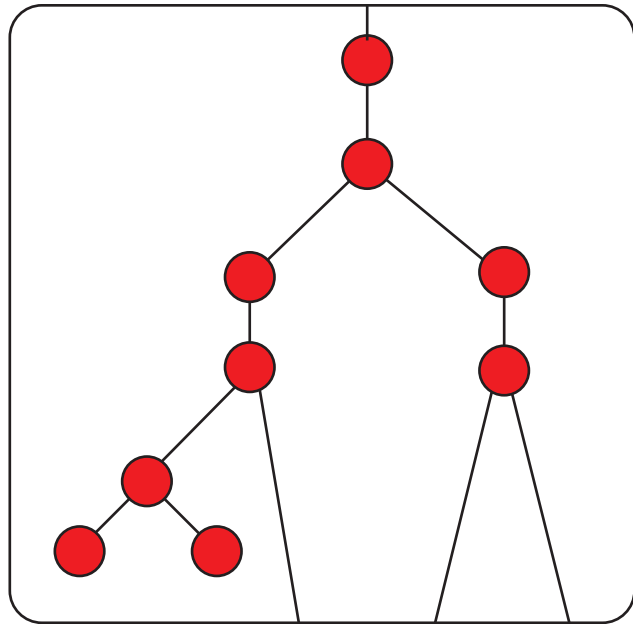
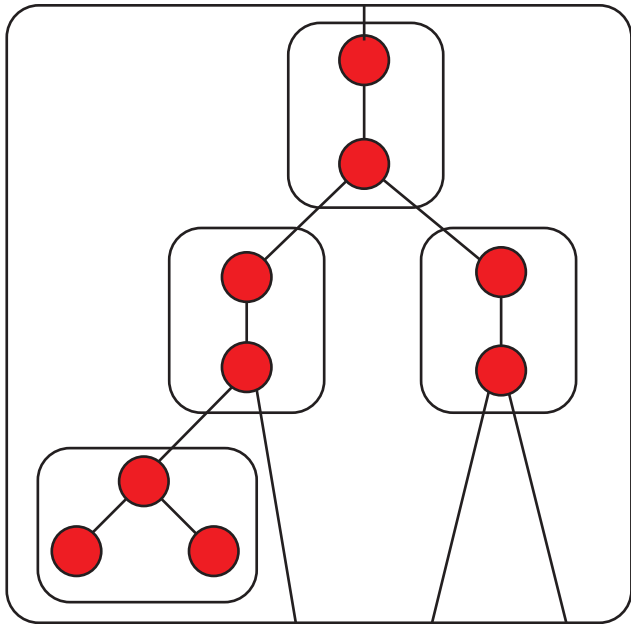






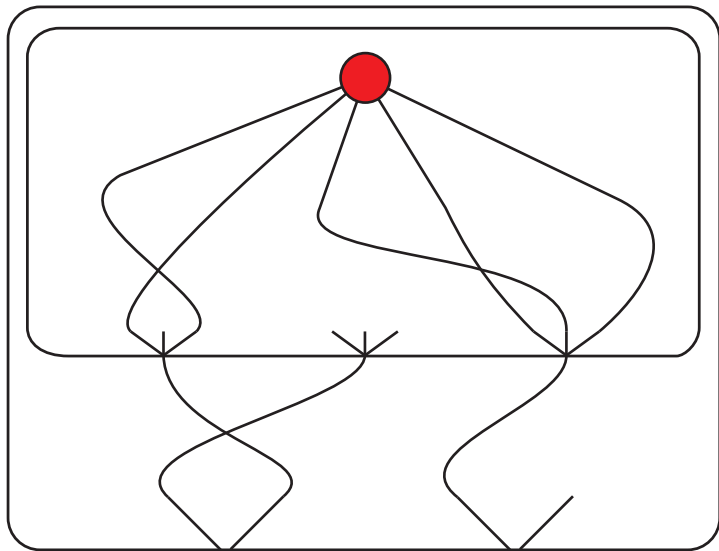
$$k - 1 = 2$$



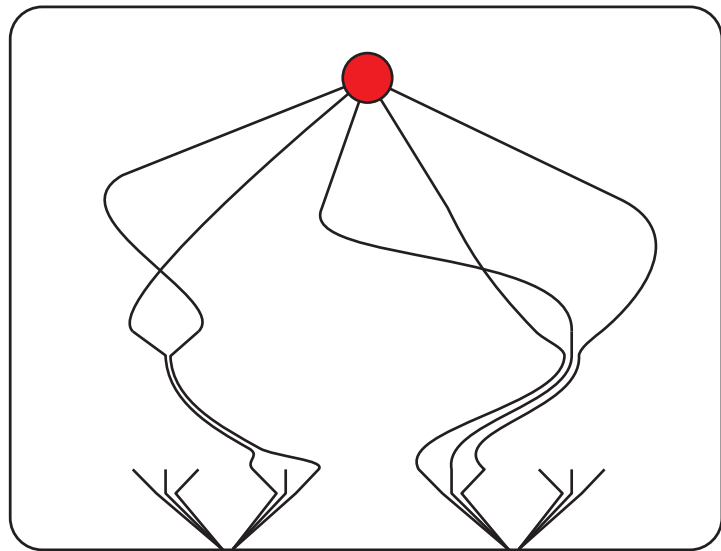


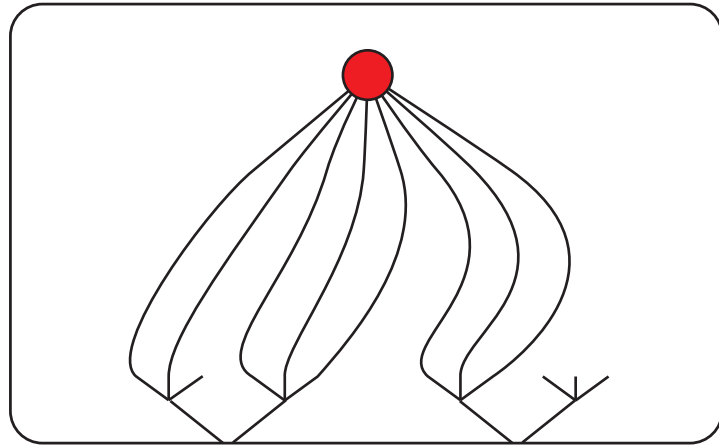
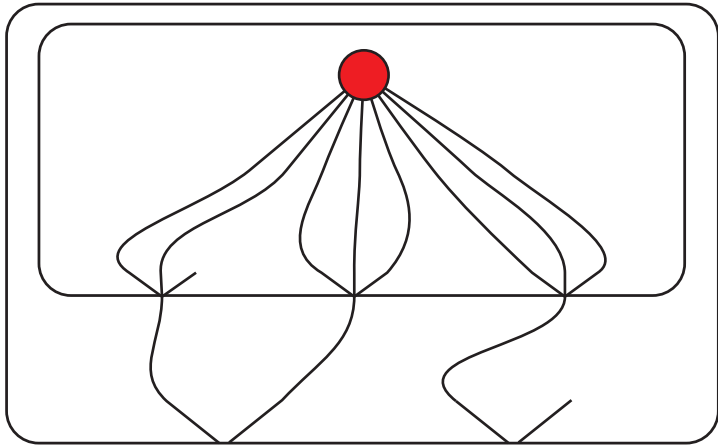


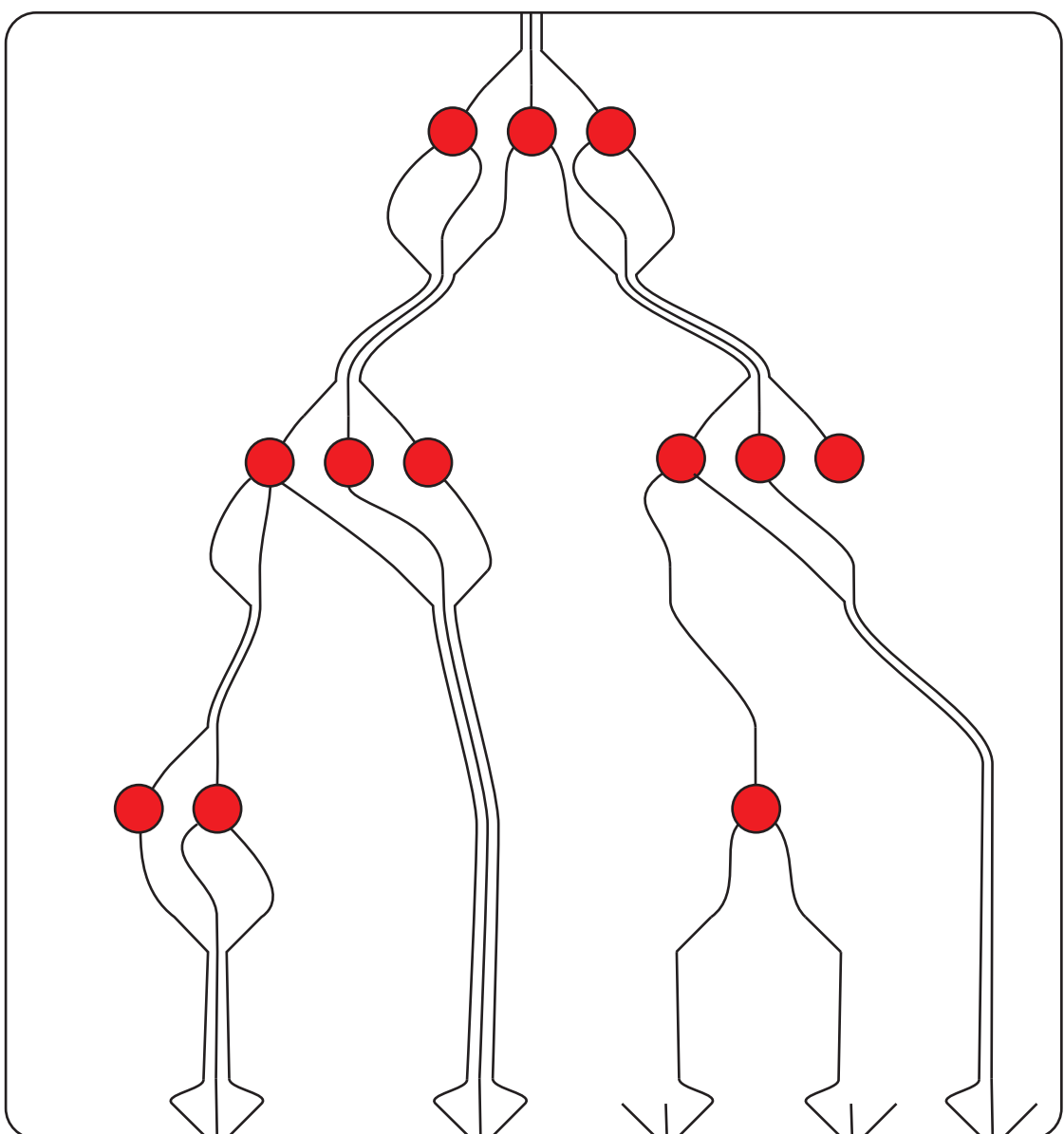
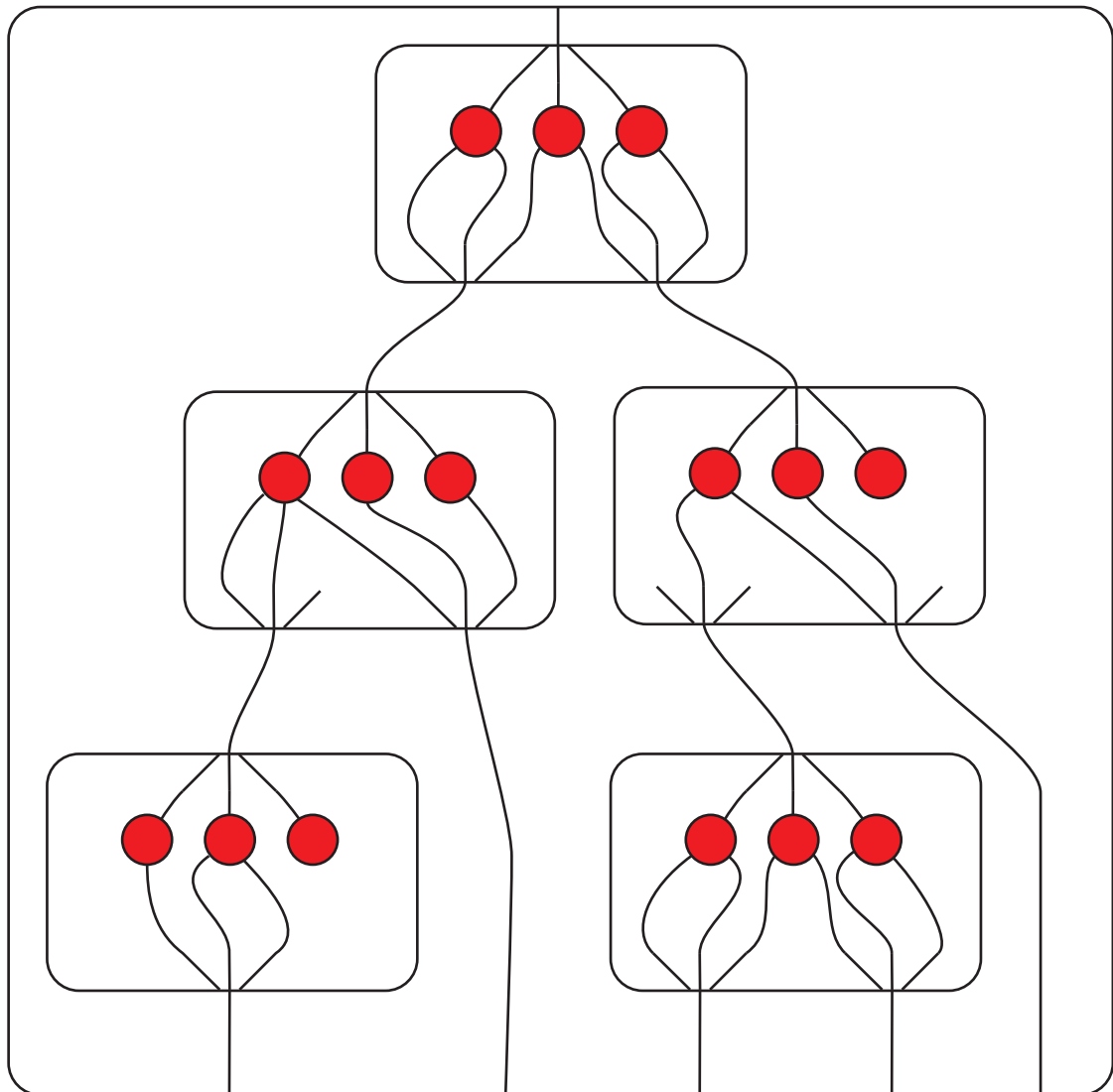
$F_2 F_3 \Sigma$



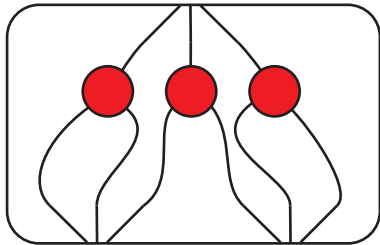
$F_6 \Sigma$



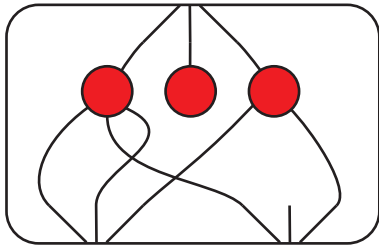




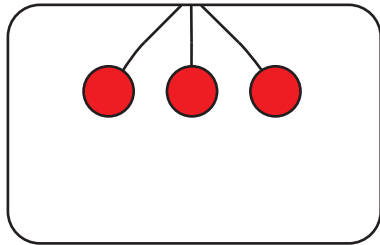
arity 2



arity 2



arity 0

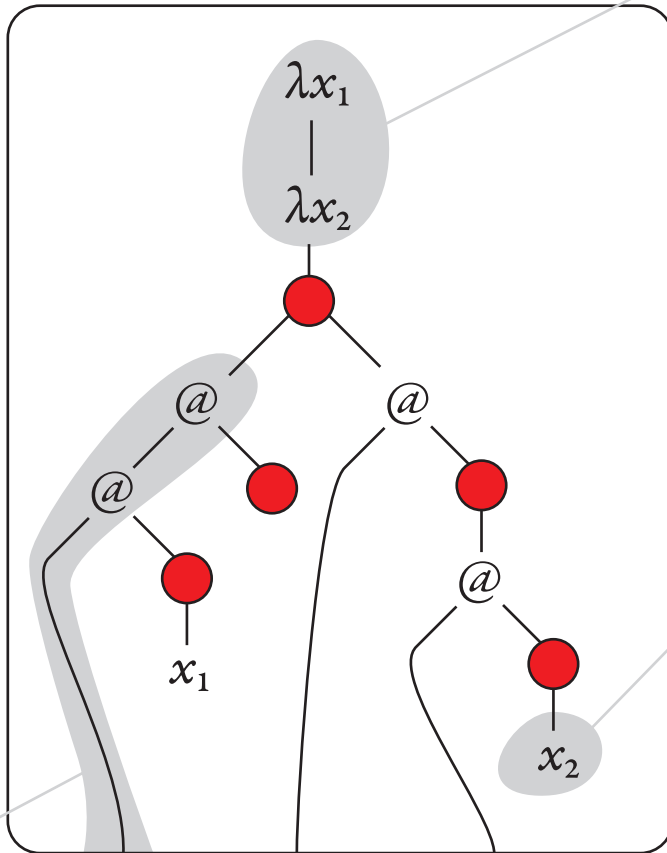
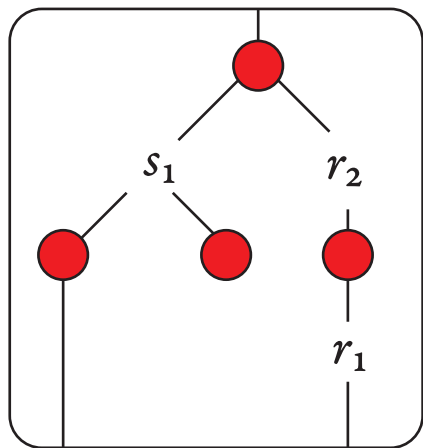


a term t with
placeholders

its λ -representation

one bound
variable
for each port
of t

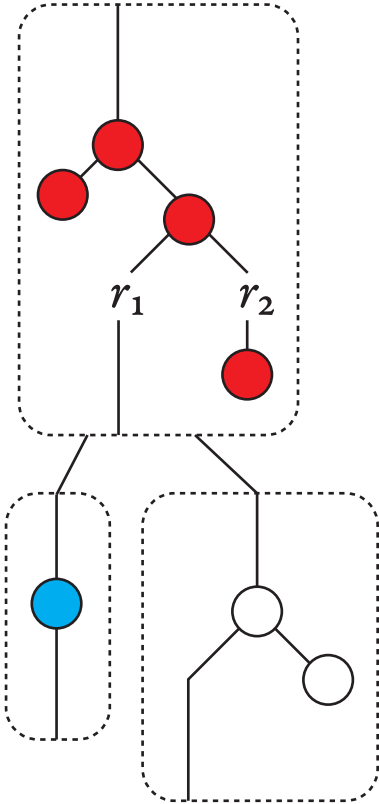
each port of t
is replaced by
a corresponding
variable



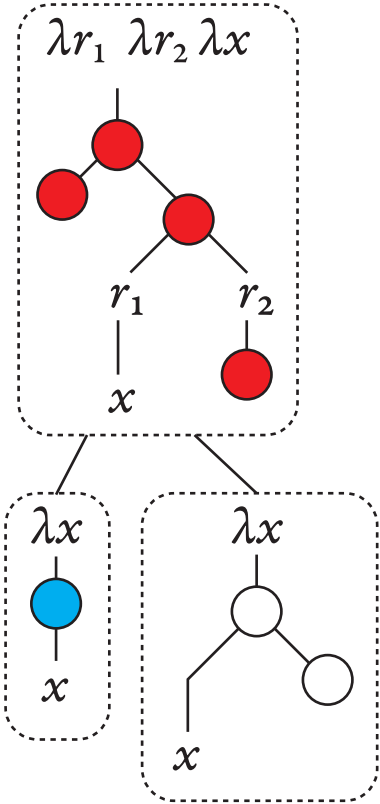
each placeholder of t is replaced by
a port applied to its children using $@$

tree of register updates

λ -term

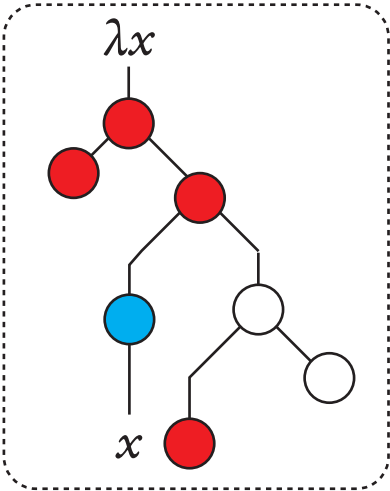


represent
as a λ -term
 \mapsto

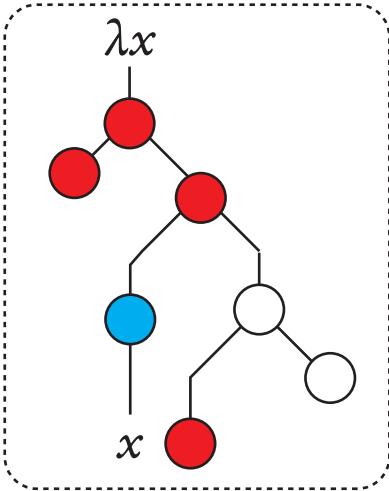


evaluate \Downarrow

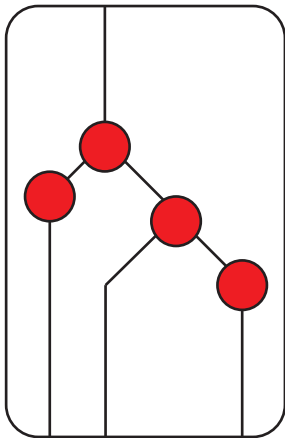
\Downarrow evaluate



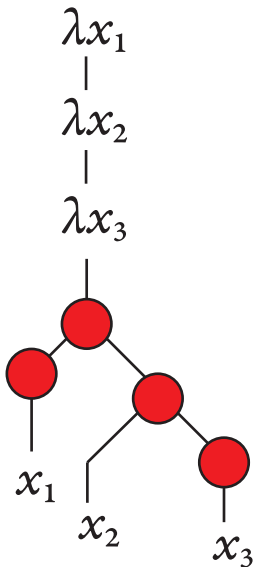
represent
as a λ -term
 \mapsto

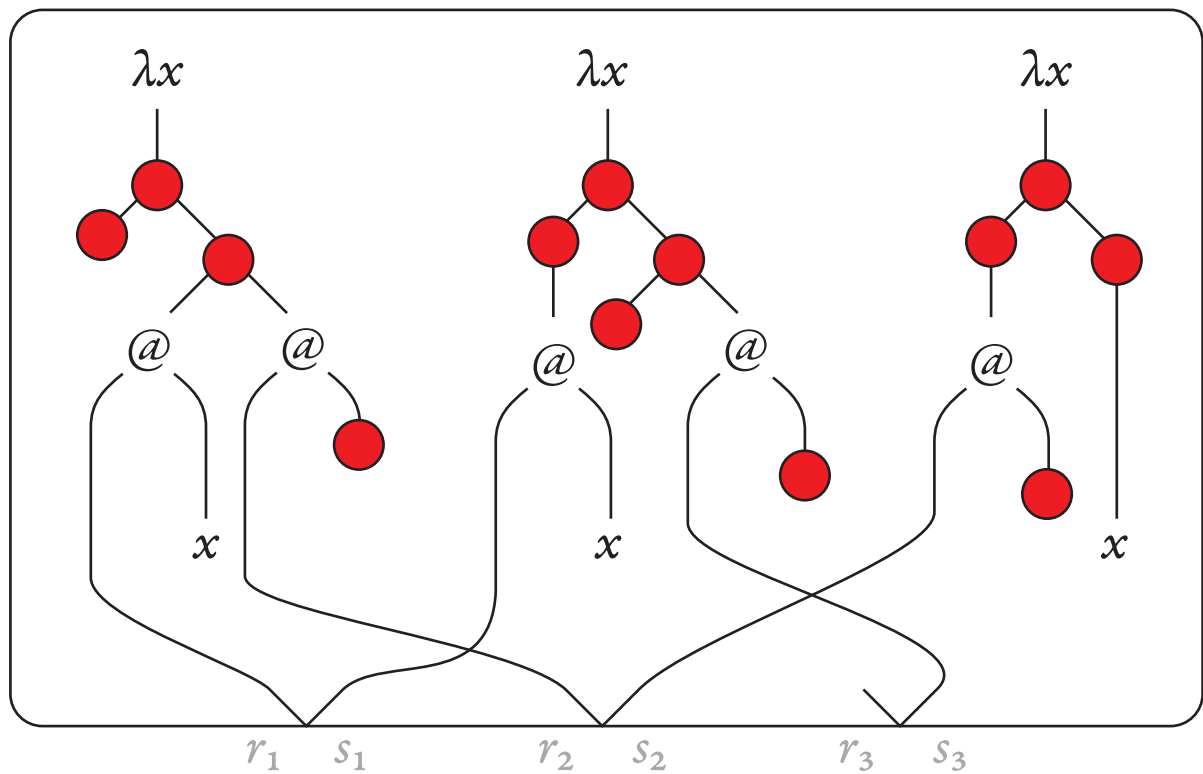
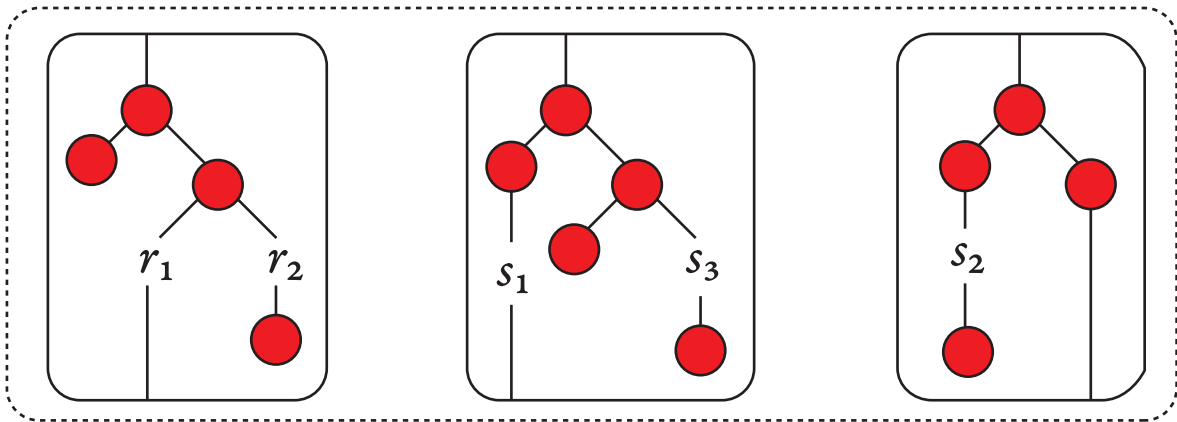


a term

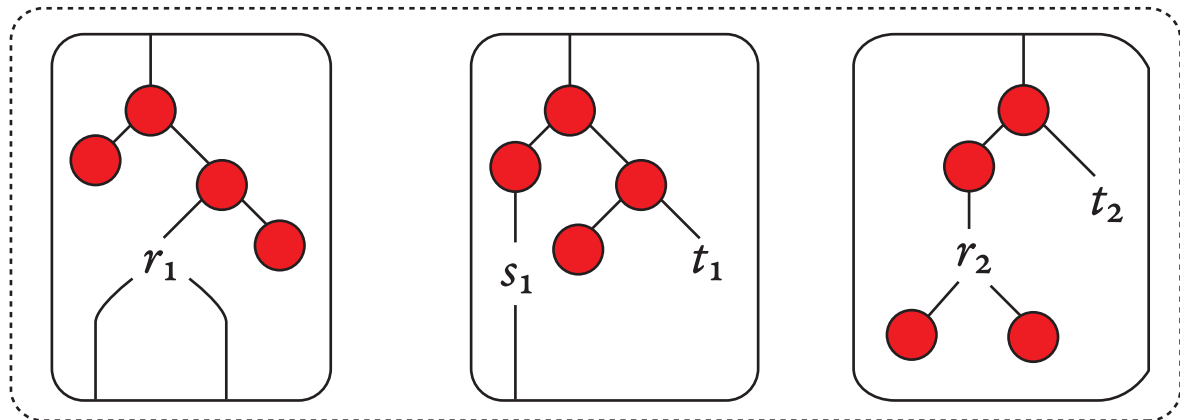


its λ -representation

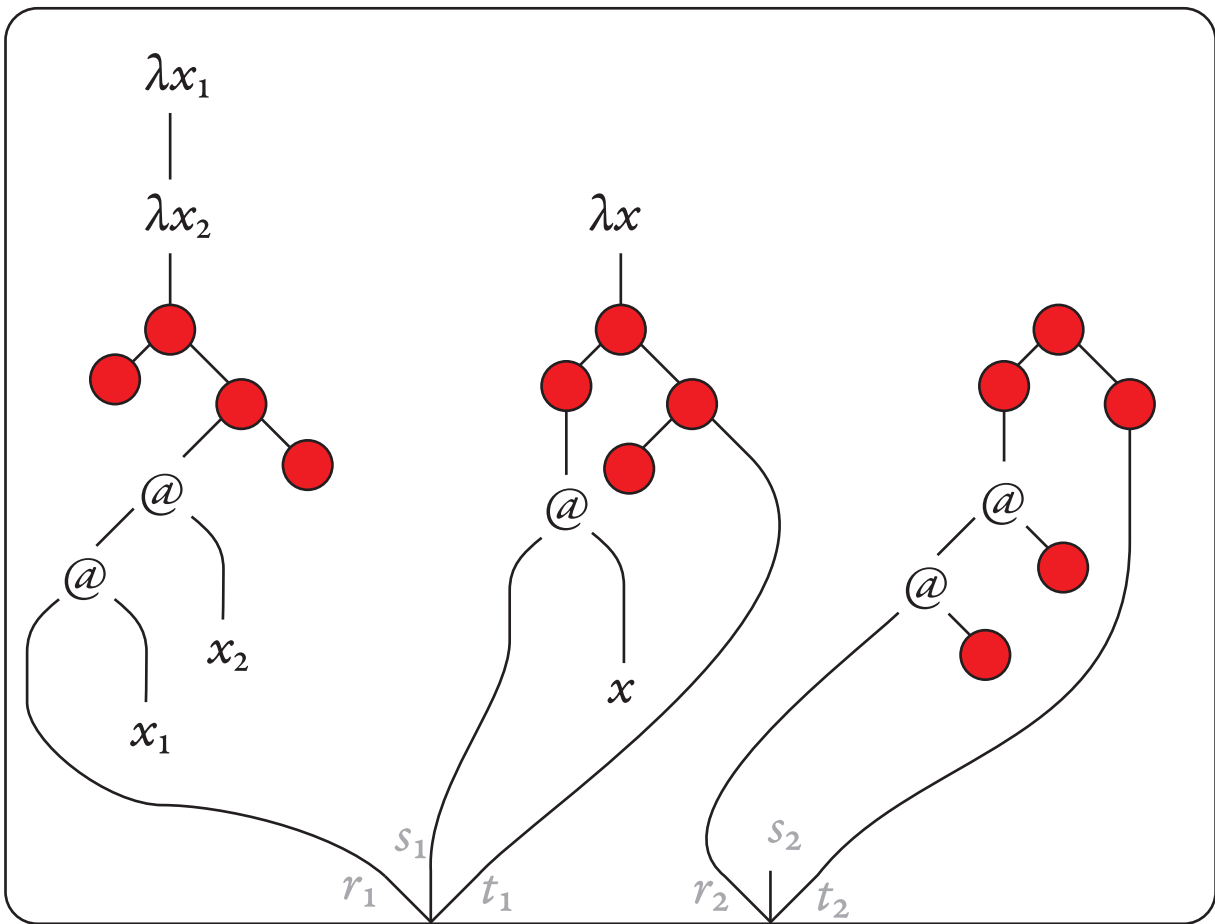


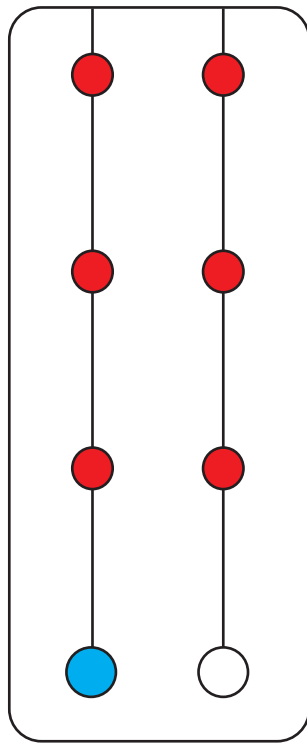
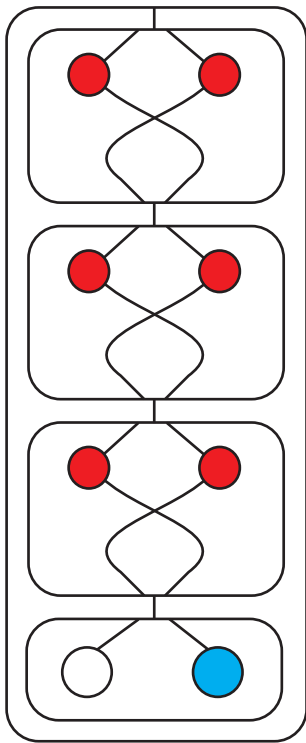


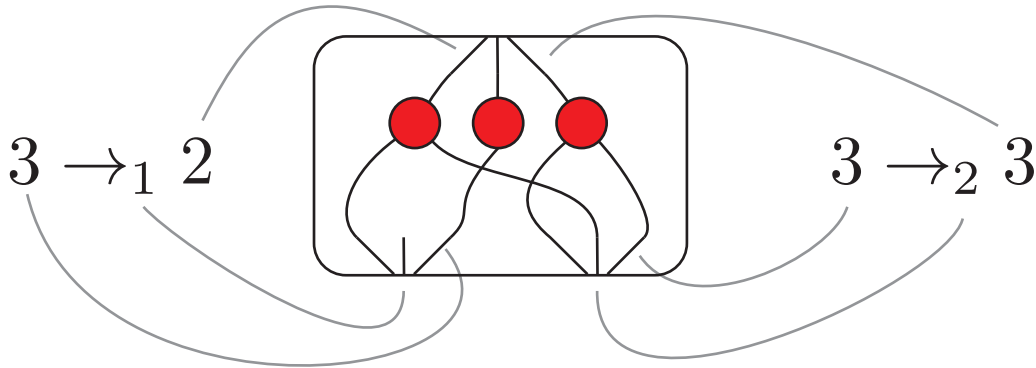
a register update

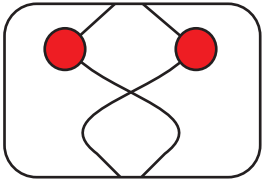


its λ -representation

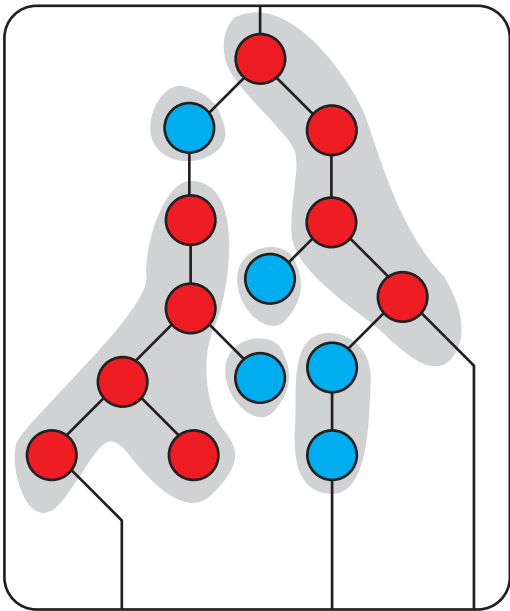




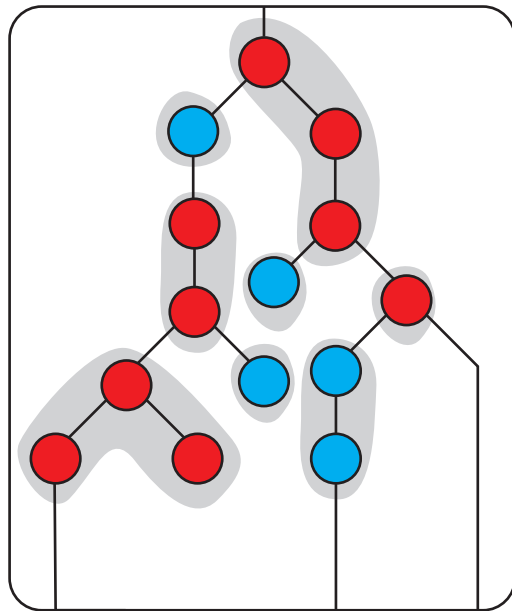


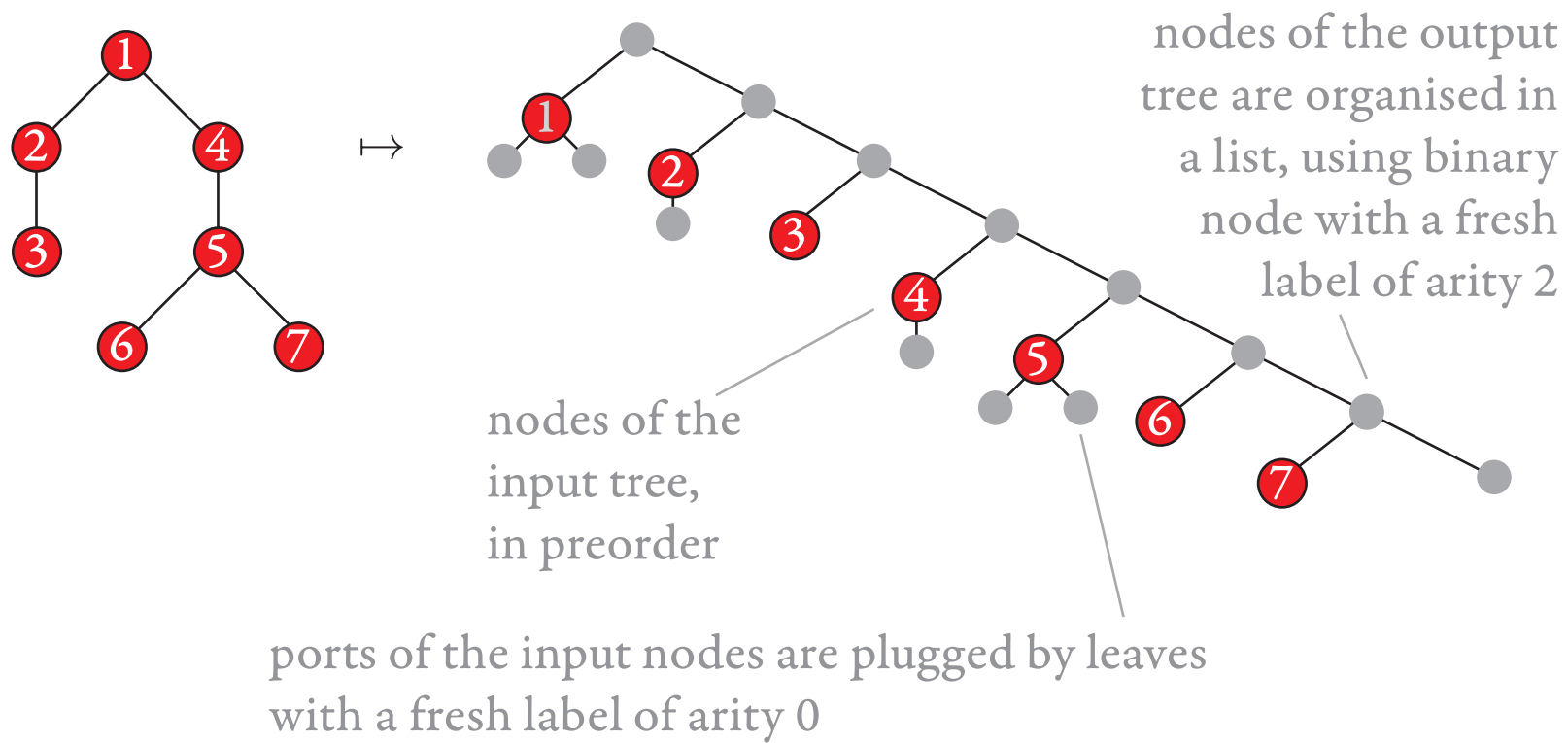


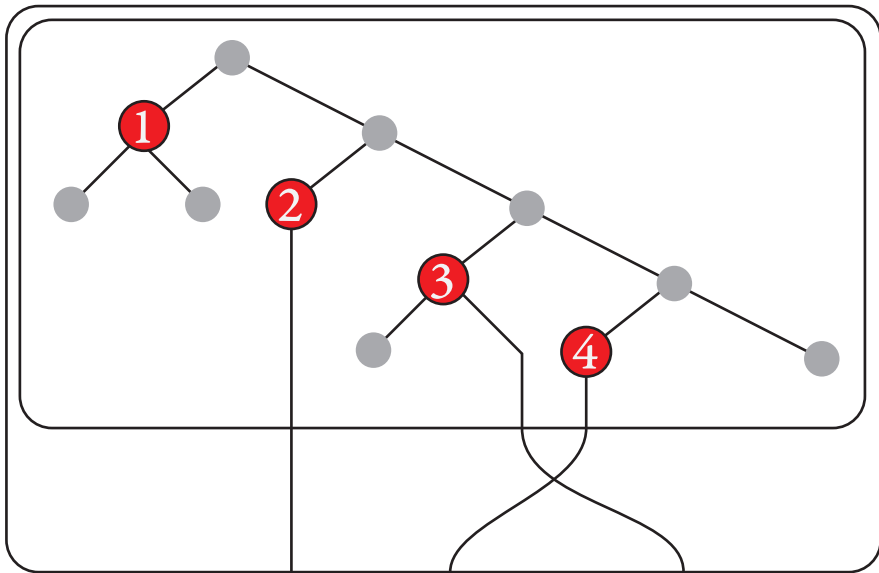
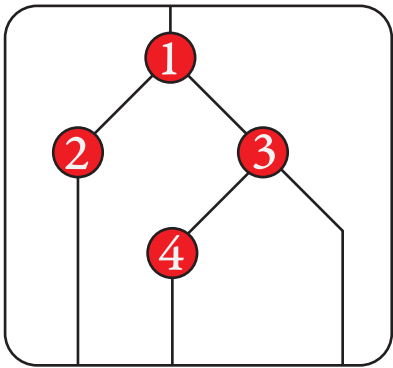
↑-equivalence



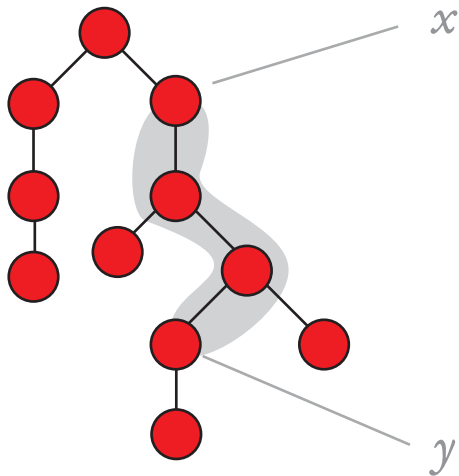
\downarrow -equivalence



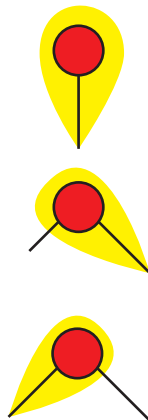




tree with a path
from x to y



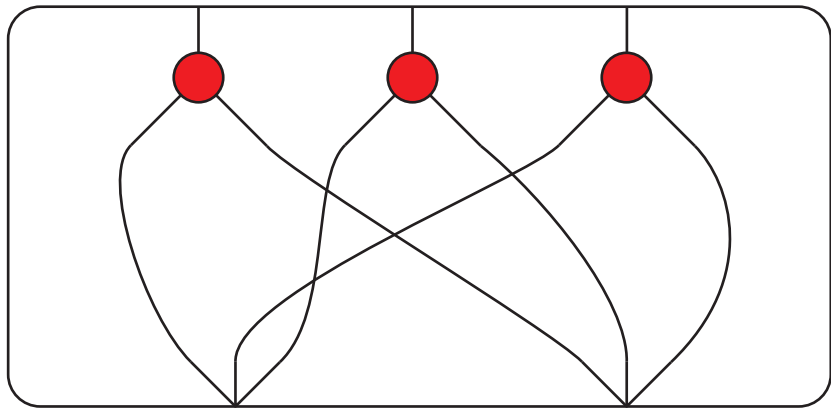
word corresponding
to the path

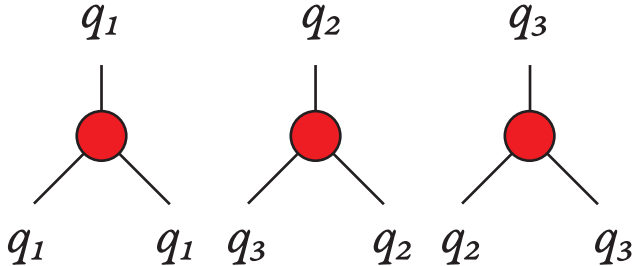


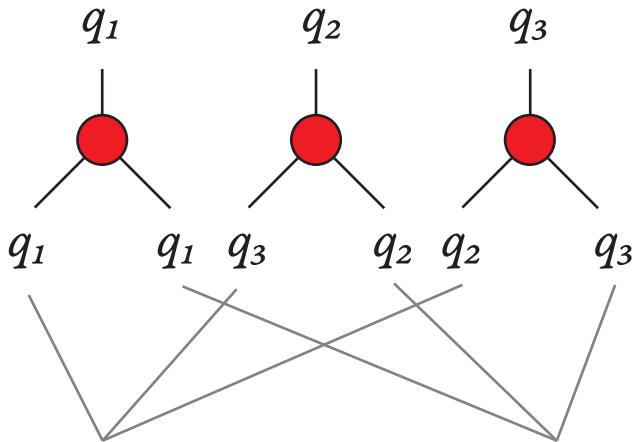
copy 1
of a

copy 2
of a

copy 3
of a

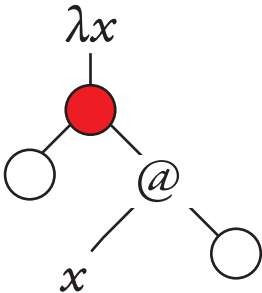




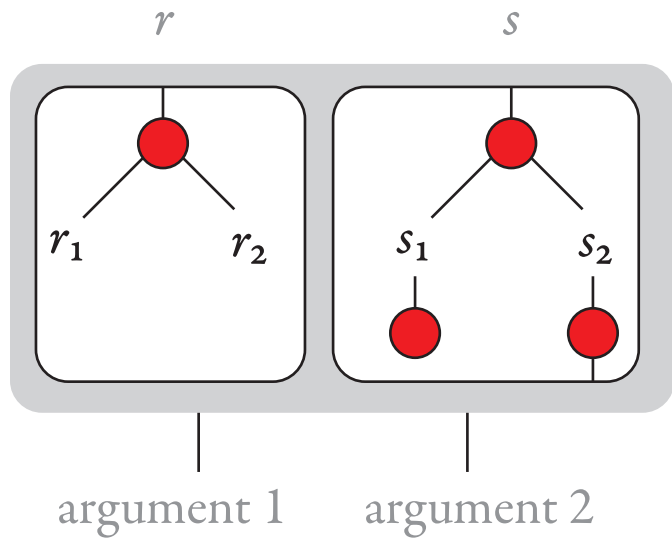


each state appears
exactly once as
a first child

each state appears
exactly once as
a second child



a register valuation



its λ -representation

