

Regular and first-order tree-to-tree functions

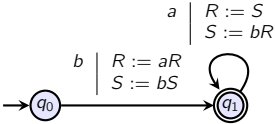
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Regular and first-order word transductions

$$f : A^* \rightarrow B^*$$

Streaming string transducers (SST)		Monotone SST
MSO transductions	$x \geq y \mapsto y \geq x$	FO transductions
List functions	Basic functions (eg. $abaa \mapsto aaba$) + composition	FO list functions

Regular and first-order tree-to-tree transductions

$$f : \mathcal{T}(A) \rightarrow \mathcal{T}(B)$$

Streaming tree transducers (STT)

MSO transductions

?

Monotone STT

FO transductions

?

Regular and first-order tree-to-tree transductions

$$f : \mathcal{T}(A) \rightarrow \mathcal{T}(B)$$

Streaming tree transducers (STT)

MSO transductions

Tree-to-tree functions

Monotone STT

FO transductions

FO tree-to-tree functions

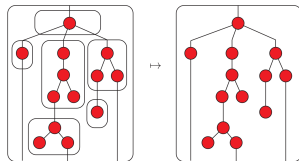
Regular and first-order tree-to-tree functions

$$f : \mathcal{T}(A) \rightarrow \mathcal{T}(B)$$

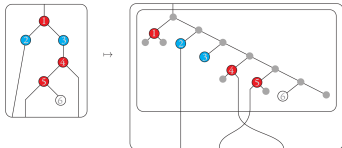
$$\text{unit} : A \rightarrow \mathcal{T}(A)$$



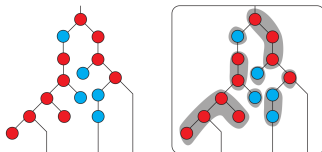
$$\text{flat} : \mathcal{T}(\mathcal{T}(A)) \rightarrow \mathcal{T}(A)$$



$$\text{Depth-first-search} : \mathcal{T}(A) \rightarrow \mathcal{T}(A)$$



$$\text{block} : \mathcal{T}(A + B) \rightarrow \mathcal{T}(\mathcal{T}(A) + \mathcal{T}(B))$$



Regular and first-order tree-to-tree functions

Theorem

Term functions = MSO transductions

Theorem

FO term functions = FO transductions

By product 2

Temporal logic for trees
corresponding to FO.

By product 2

Evaluation of simply typed linear
 λ -terms is a FO transduction.

What's next?

- ▶ Implementations
- ▶ Graph-to-graph functions
- ▶ Poly-regular functions

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- ▶ Implementations
- ▶ Graph-to-graph functions
- ▶ Poly-regular functions

Thank you for your attention!