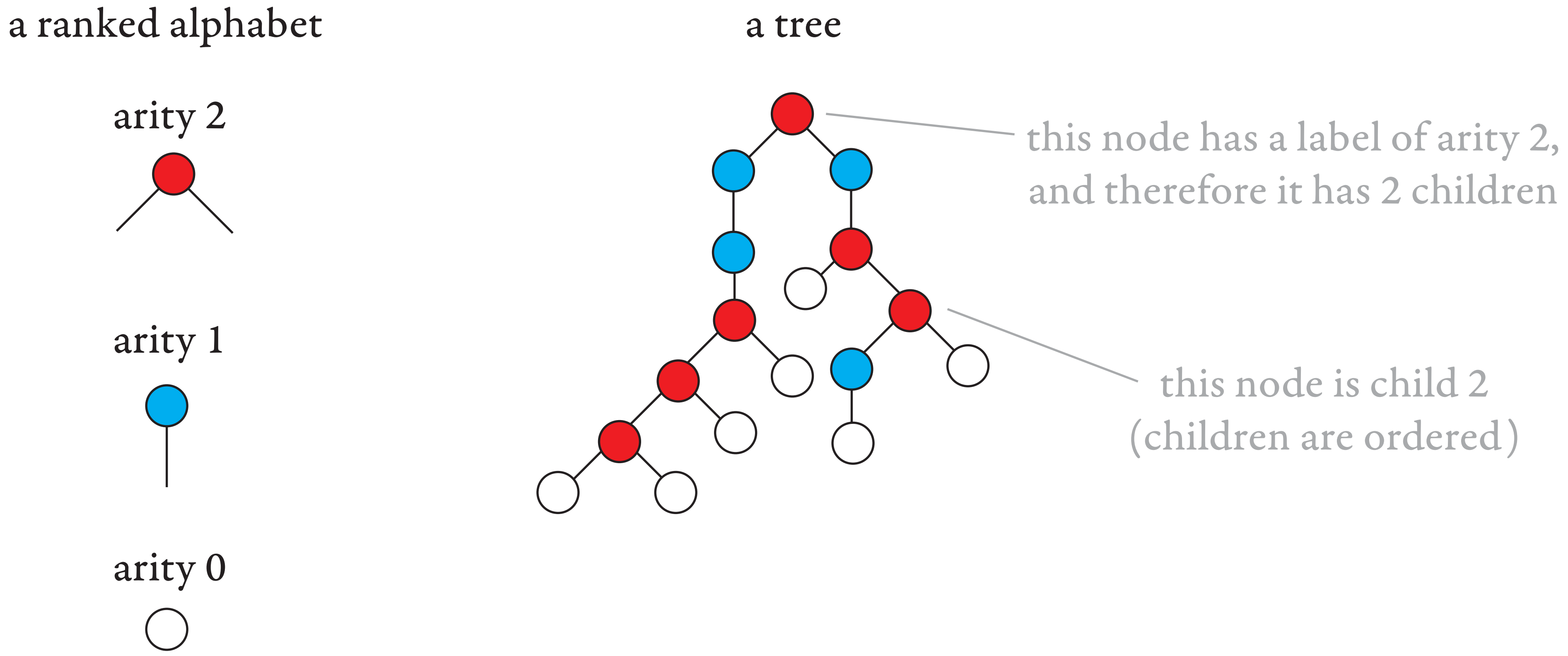
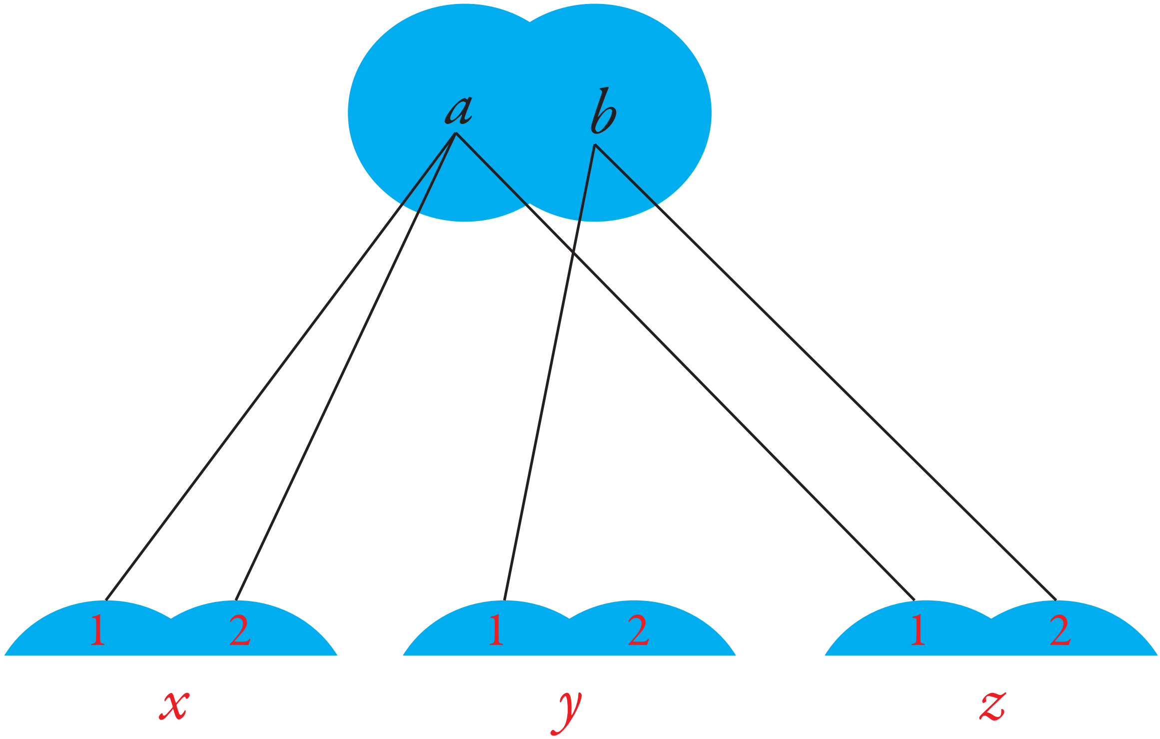
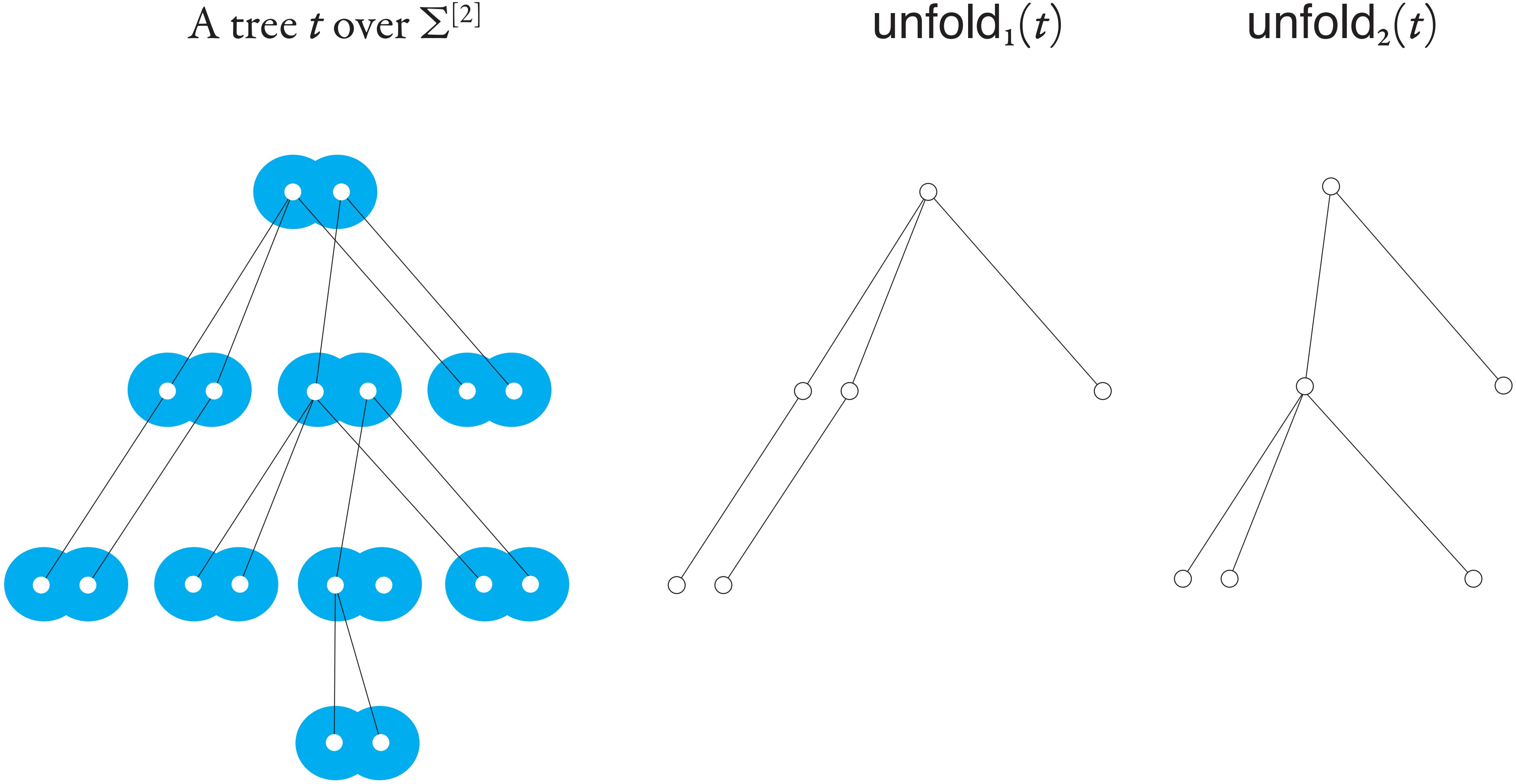
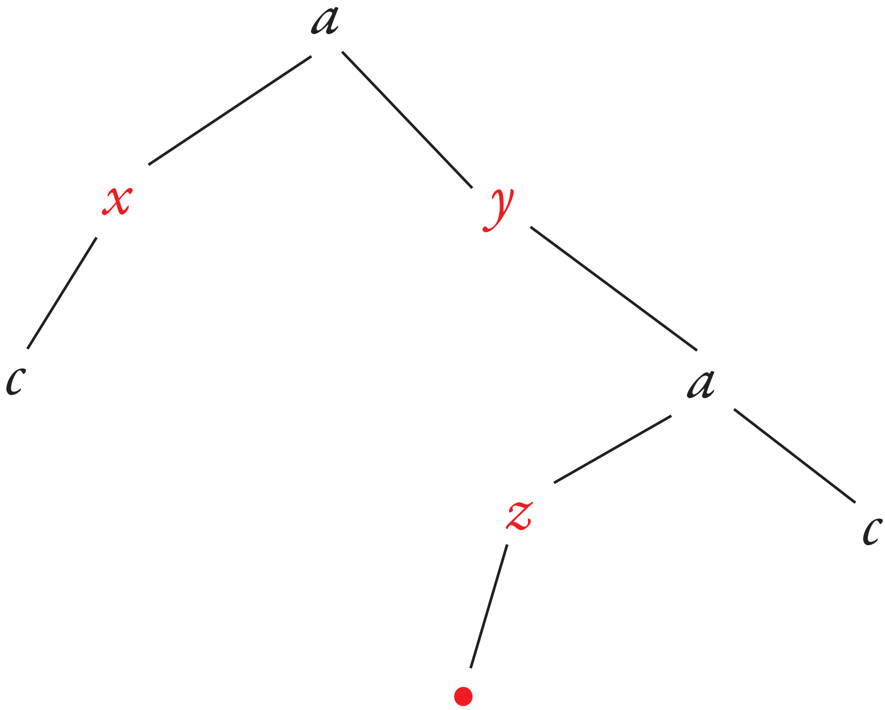
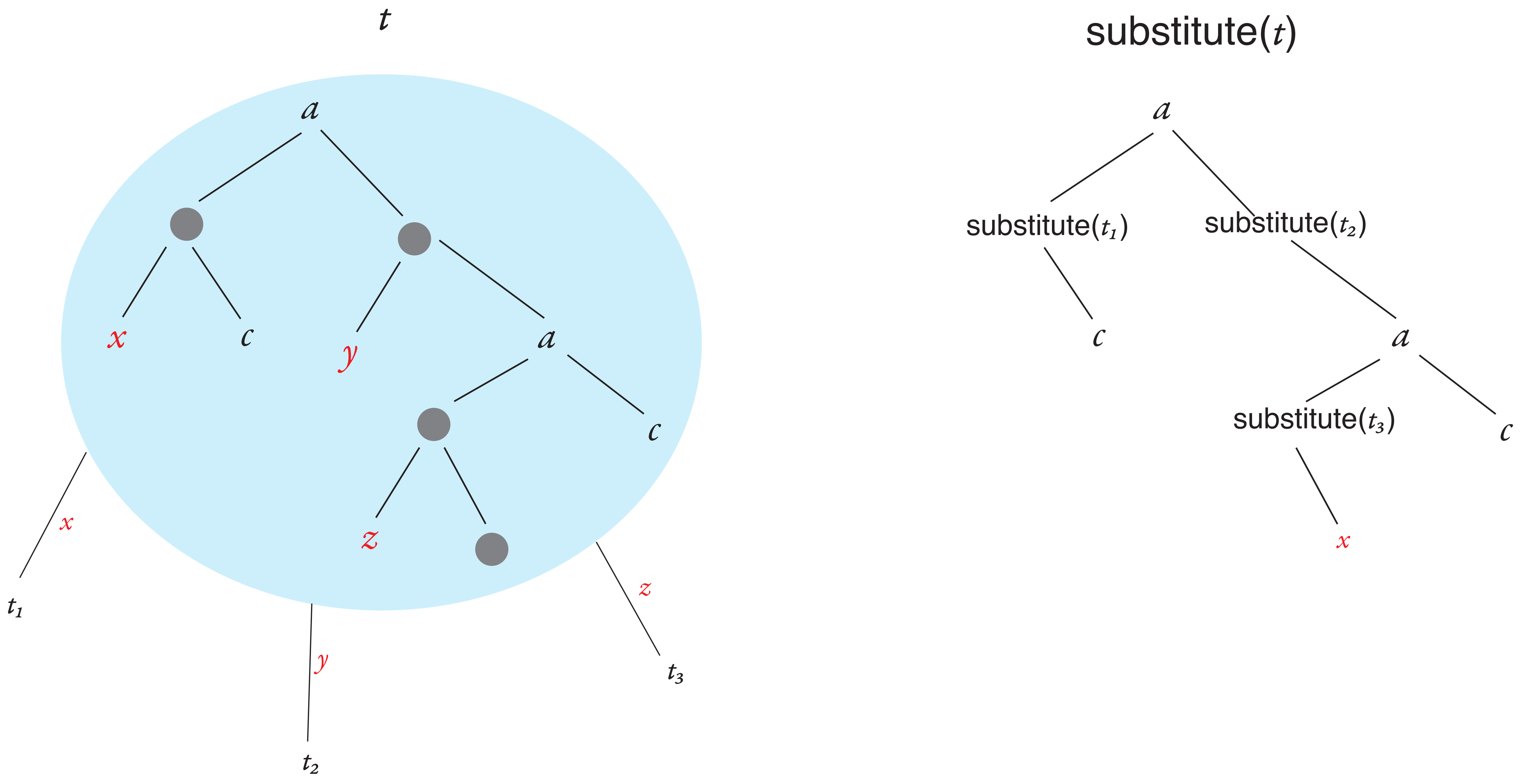

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)



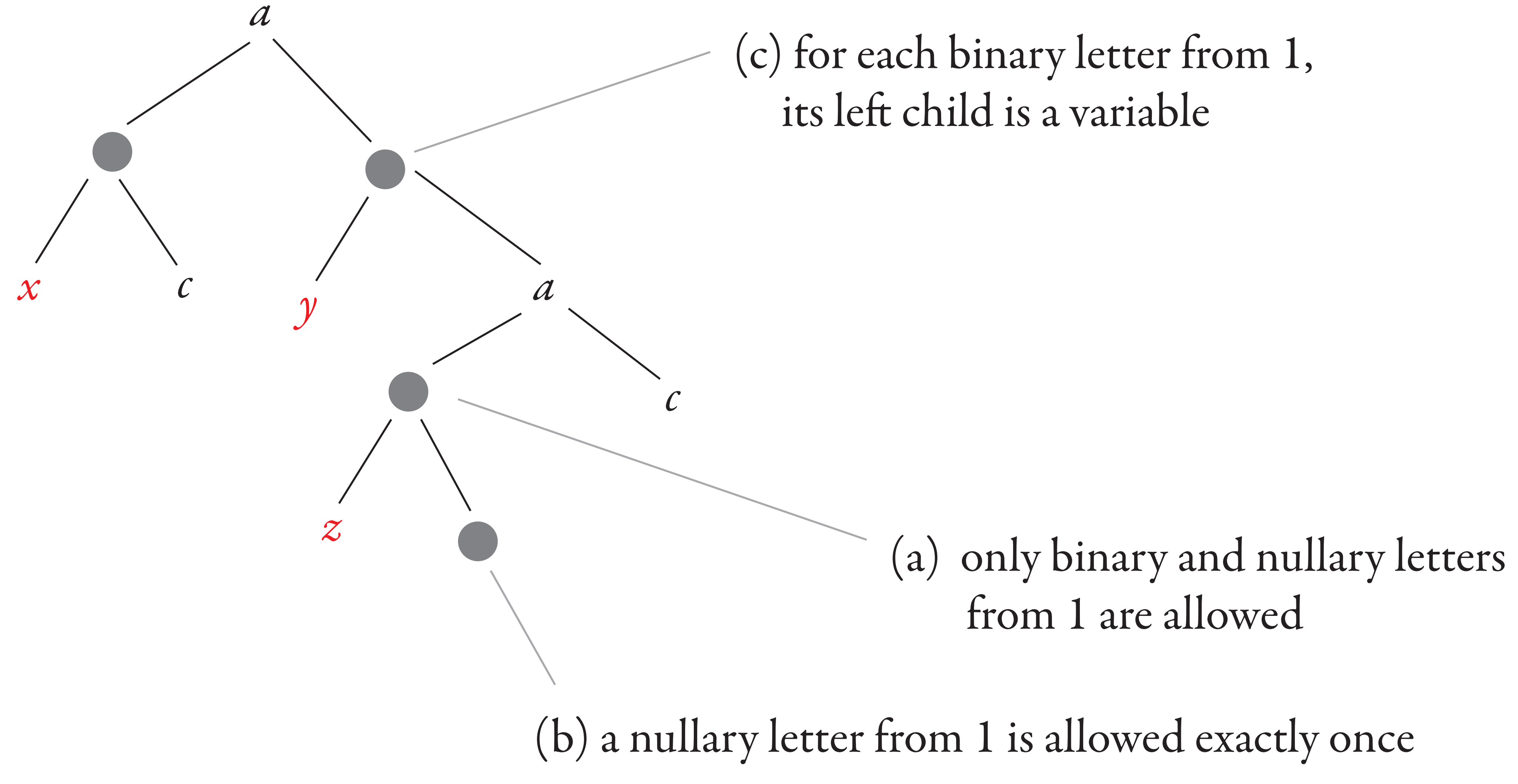
a
b
1
2
1
2
1
2
x
y
z


A tree t over $\Sigma^{[2]}$
$\mathsf{unfold}_1(t)$
$\mathsf{unfold}_2(t)$

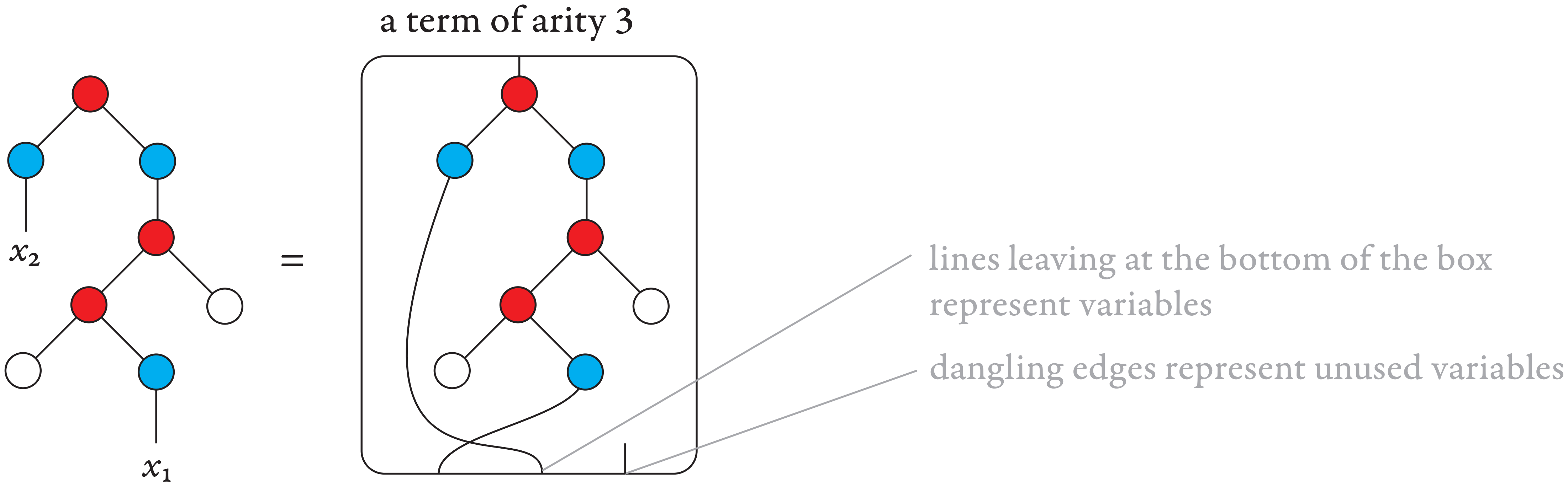

a
x
y
c
a
z
c

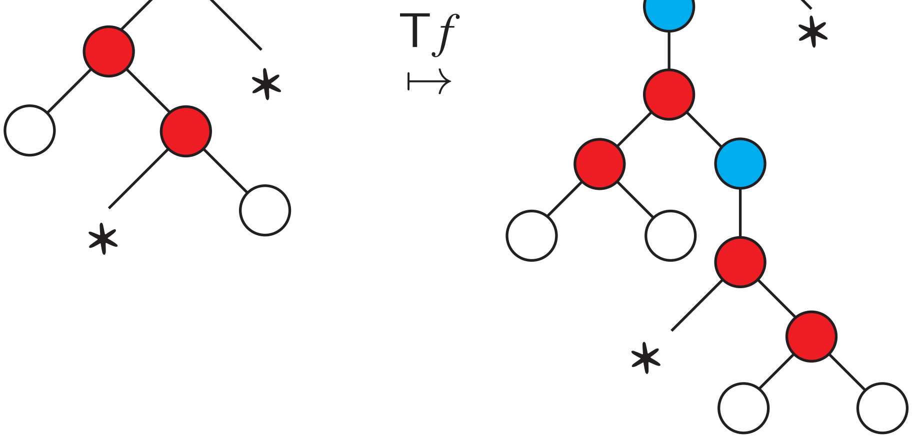


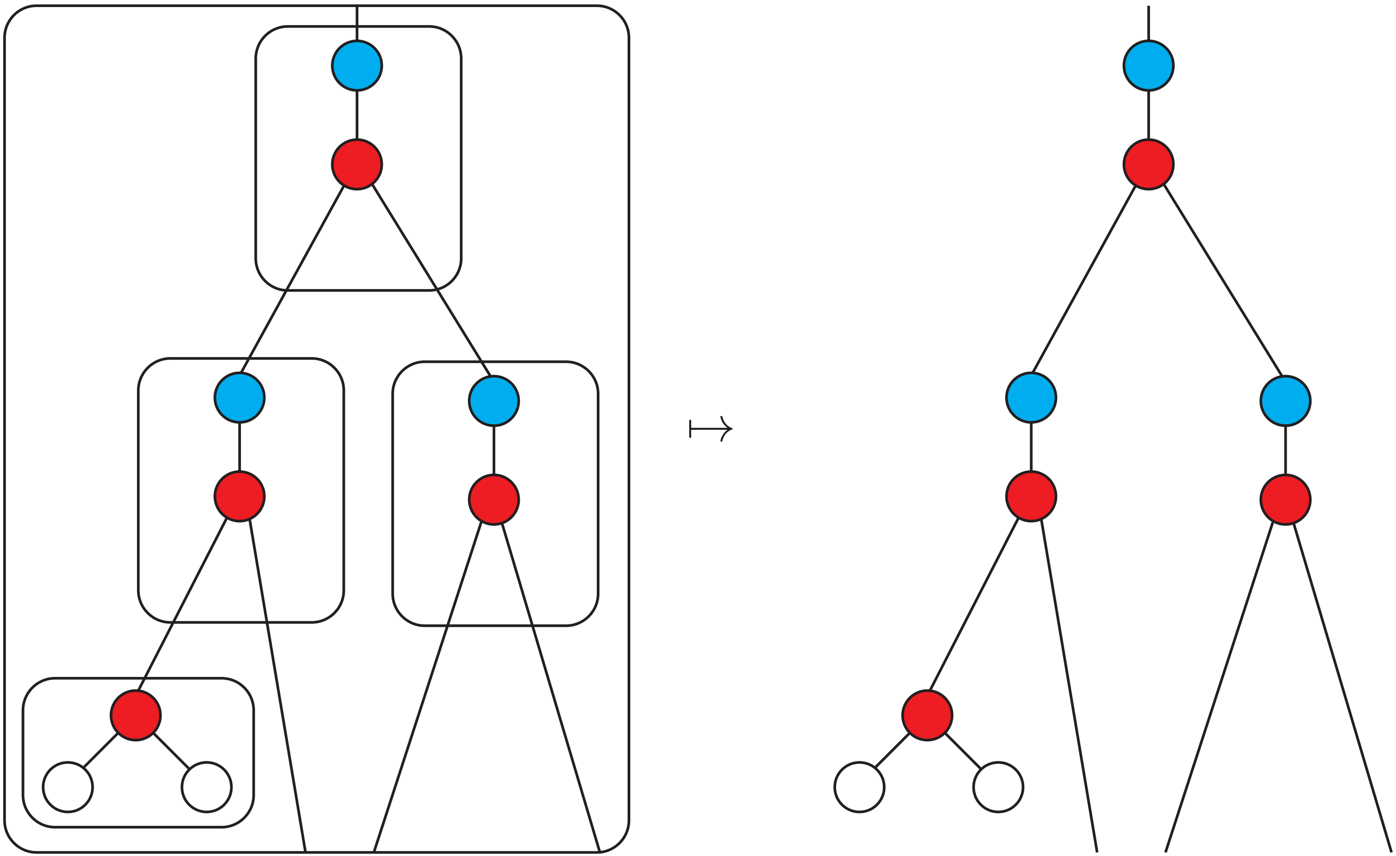
t
a
x
c
y
a
c
z
t_1
x
y
t_2
z
t_3
substitute(t)
a
substitute(t_1)
substitute(t_2)
c
a
substitute(t_3)
c
x

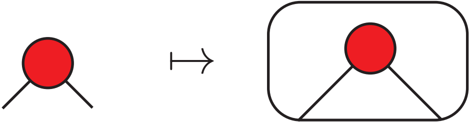


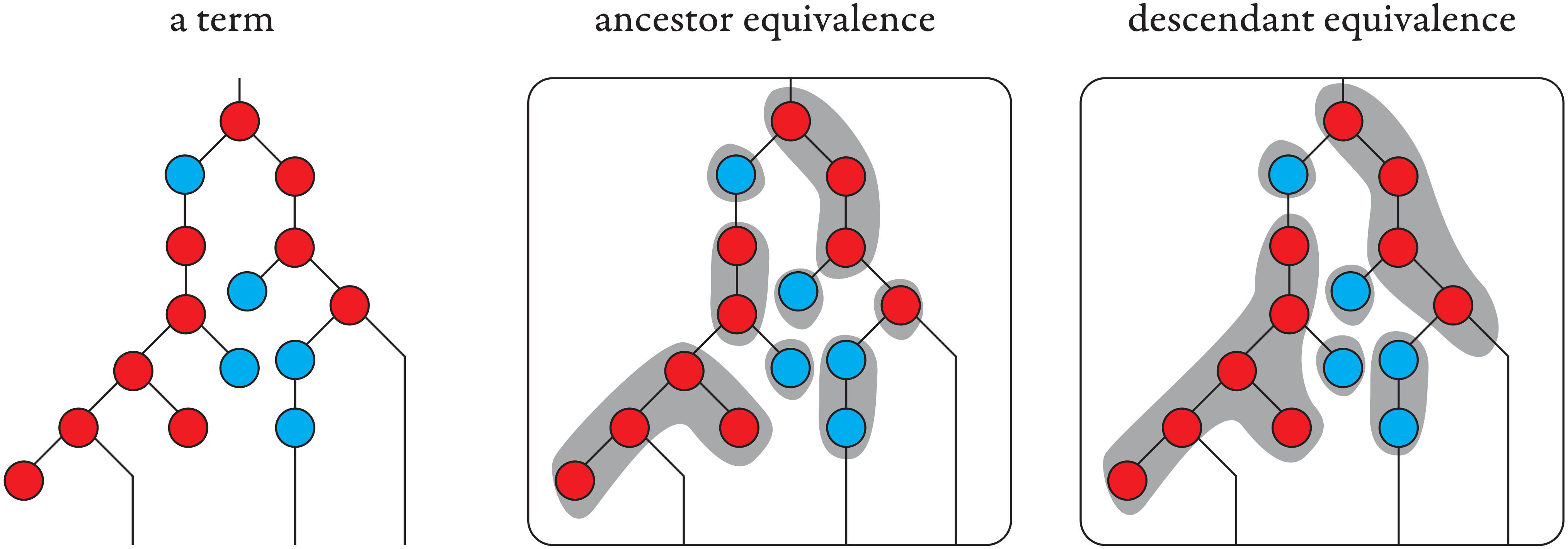
a
x
c
y
a
z
c
(c) for each binary letter from 1,
its left child is a variable
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

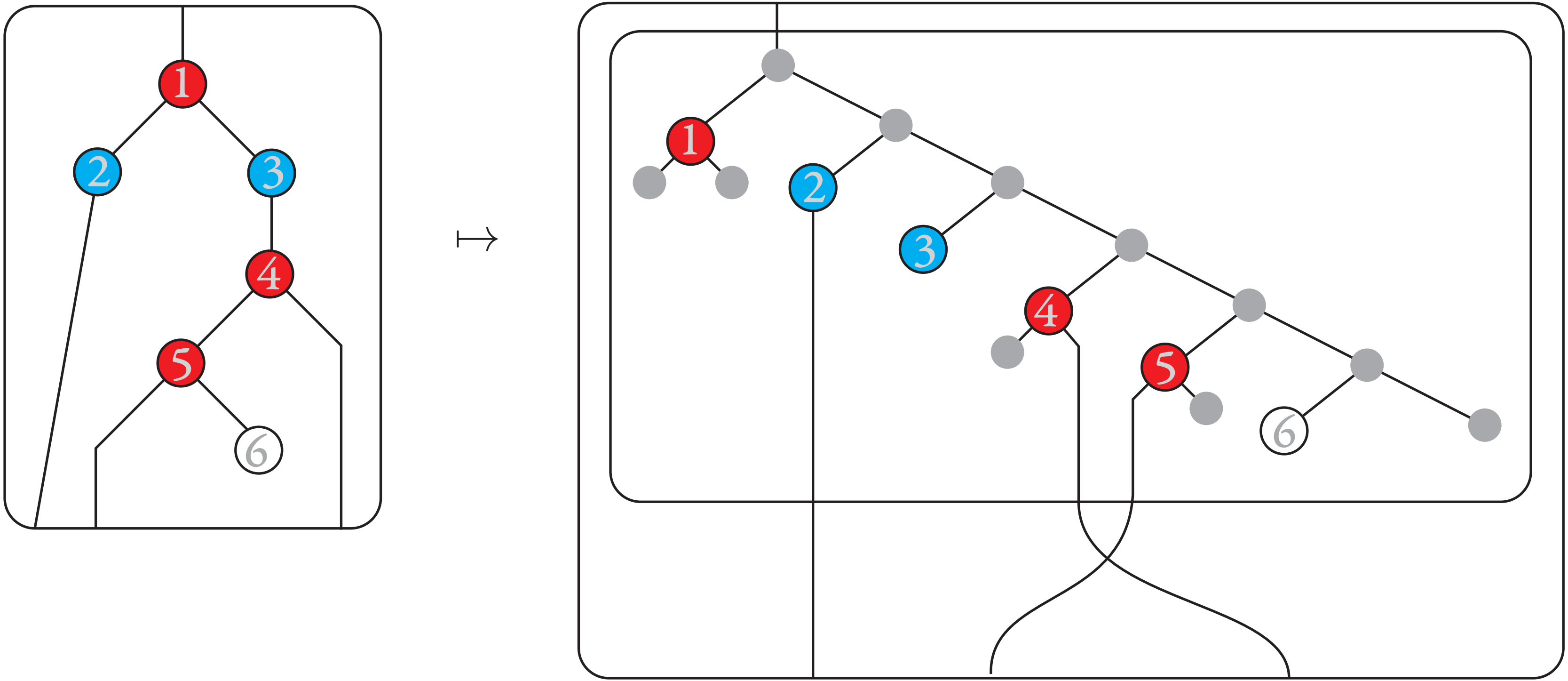

a term of arity 3
x_2
x_1
=
lines leaving at the bottom of the box represent variables
dangling edges represent unused variables





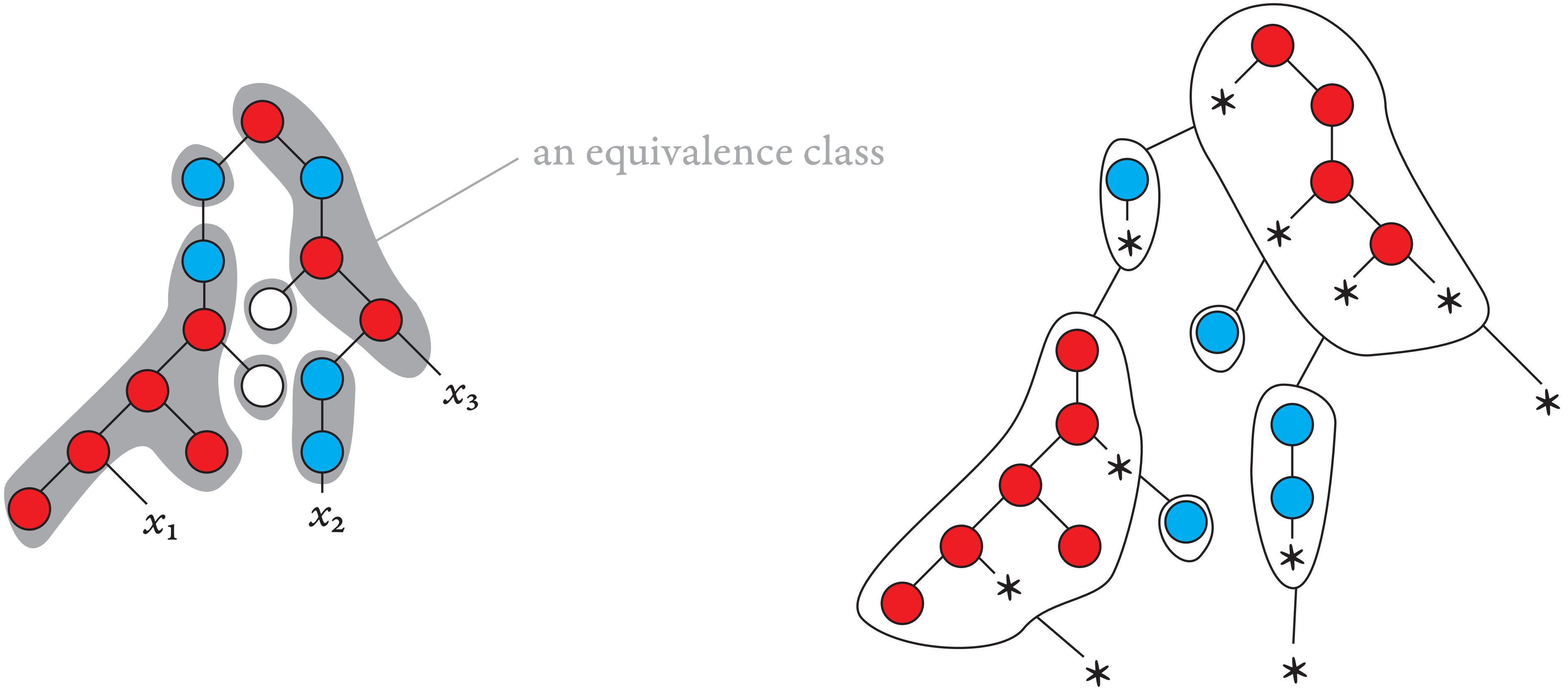


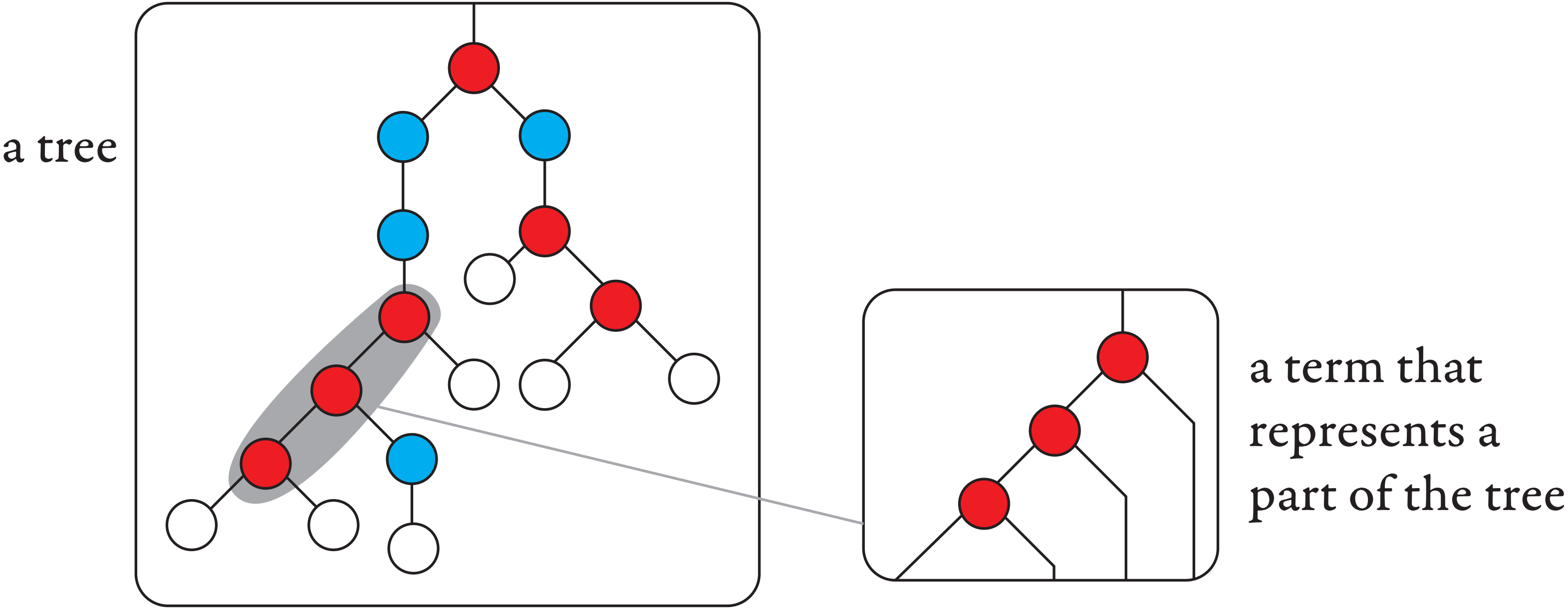

a term
ancestor equivalence
descendant equivalence

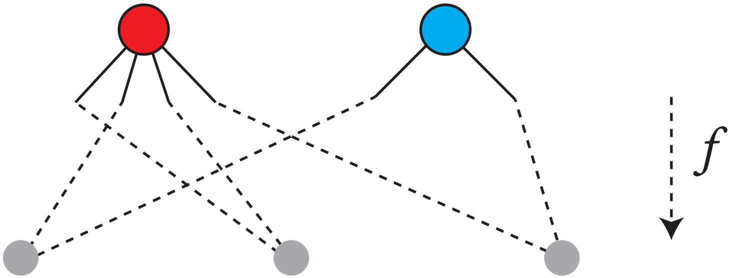


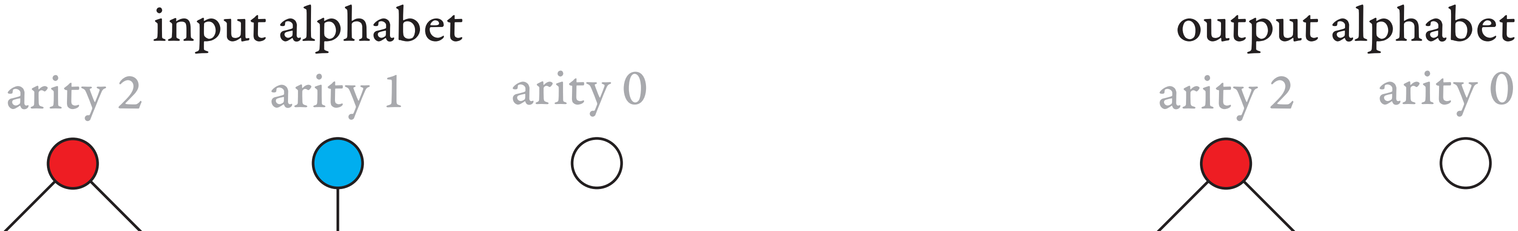
1
2
3
4
5
6
$\mapsto$
1
2
3
4
5
6

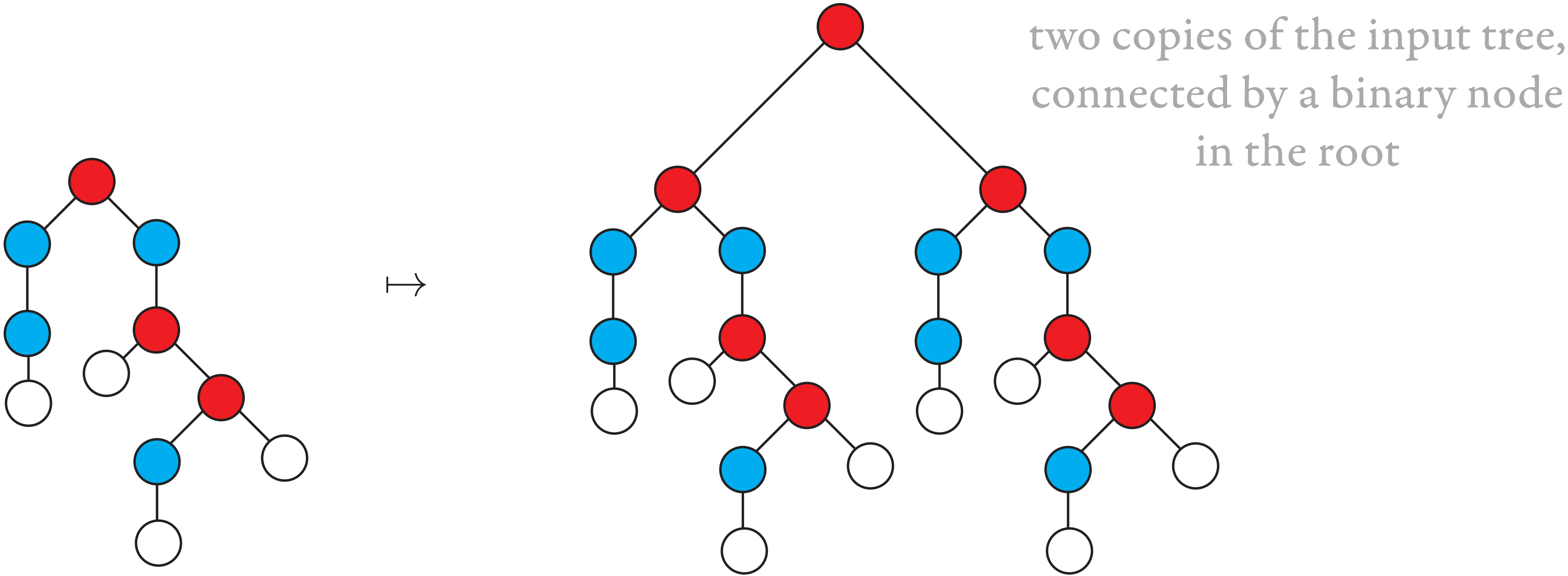
a factorisation equivalence

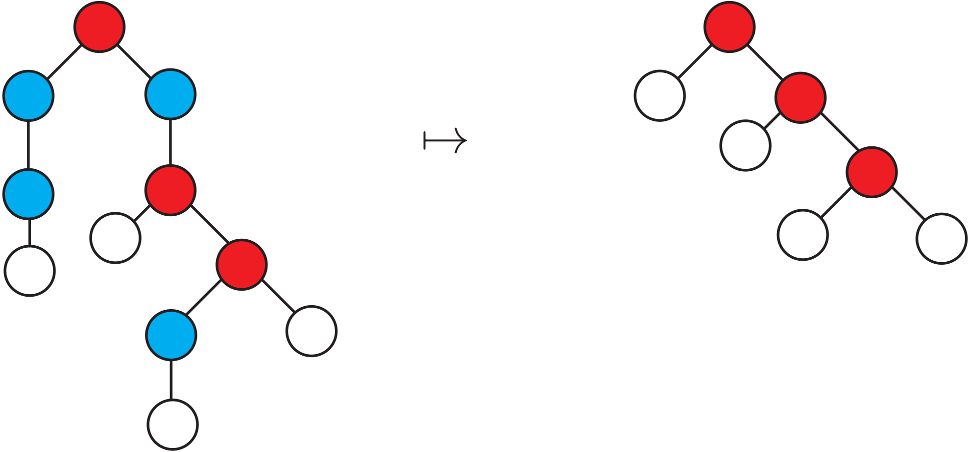


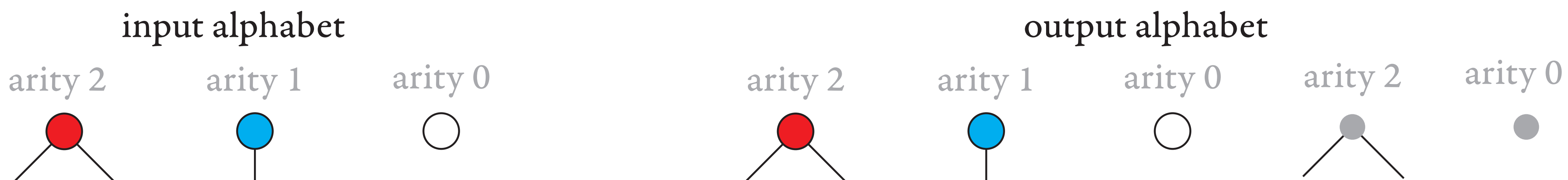

a tree
a term that
represents a
part of the tree

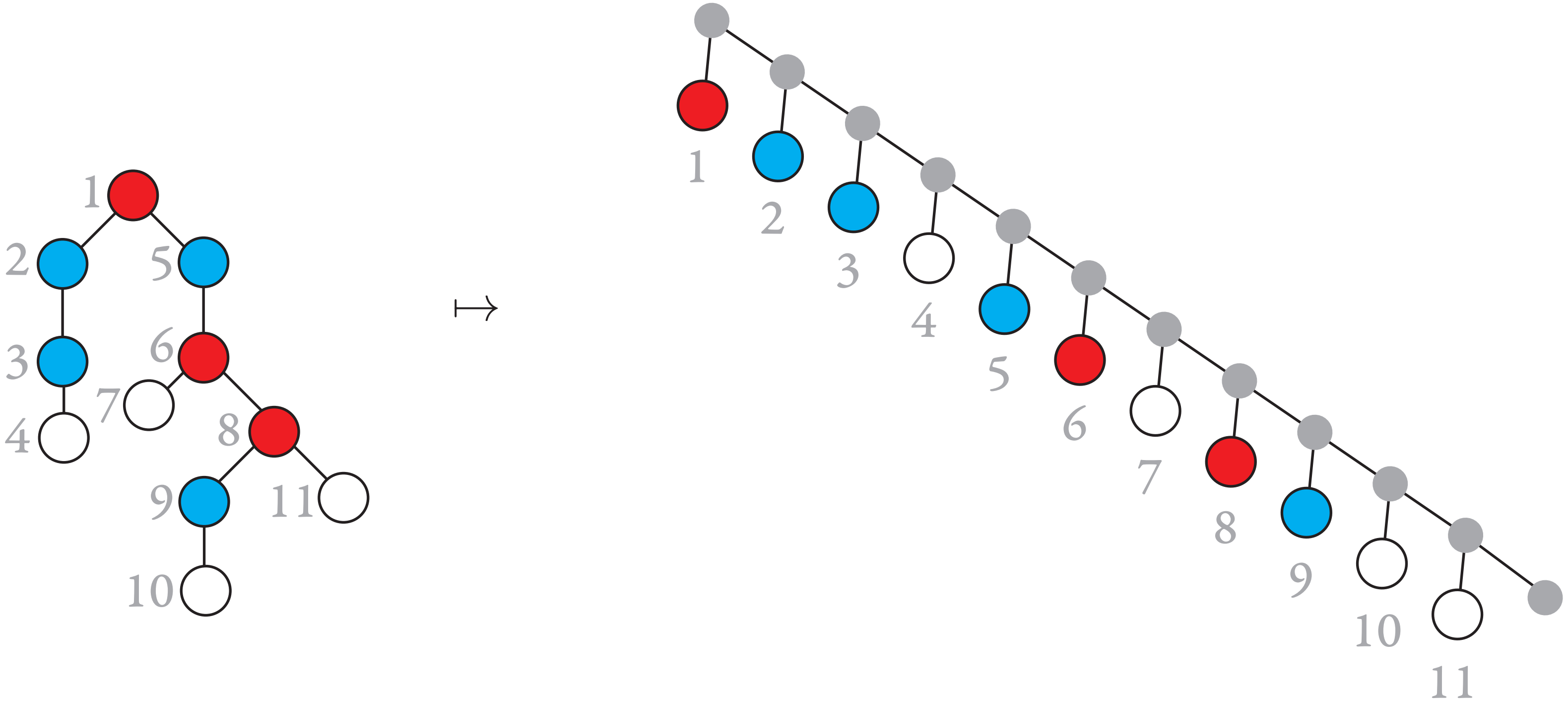

f


input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0

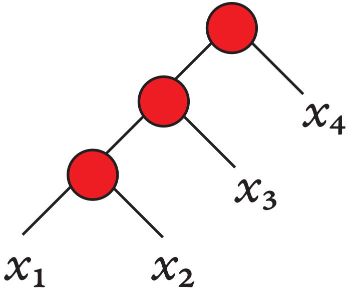

↦
two copies of the input tree,
connected by a binary node
in the root



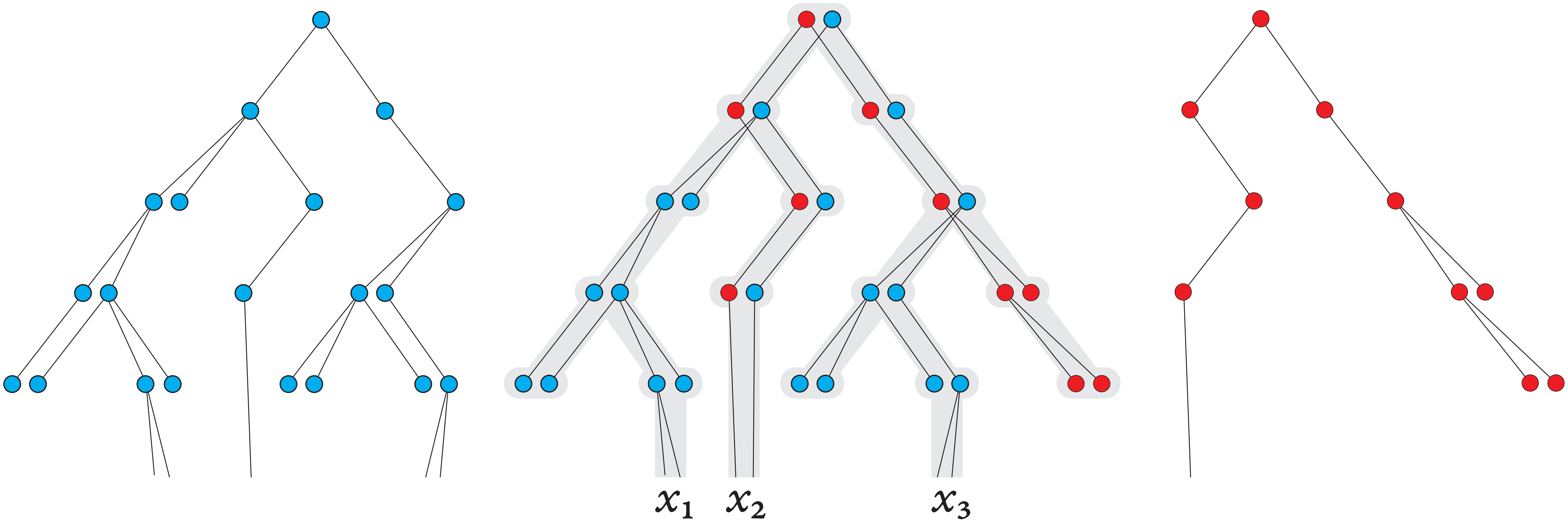

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 0

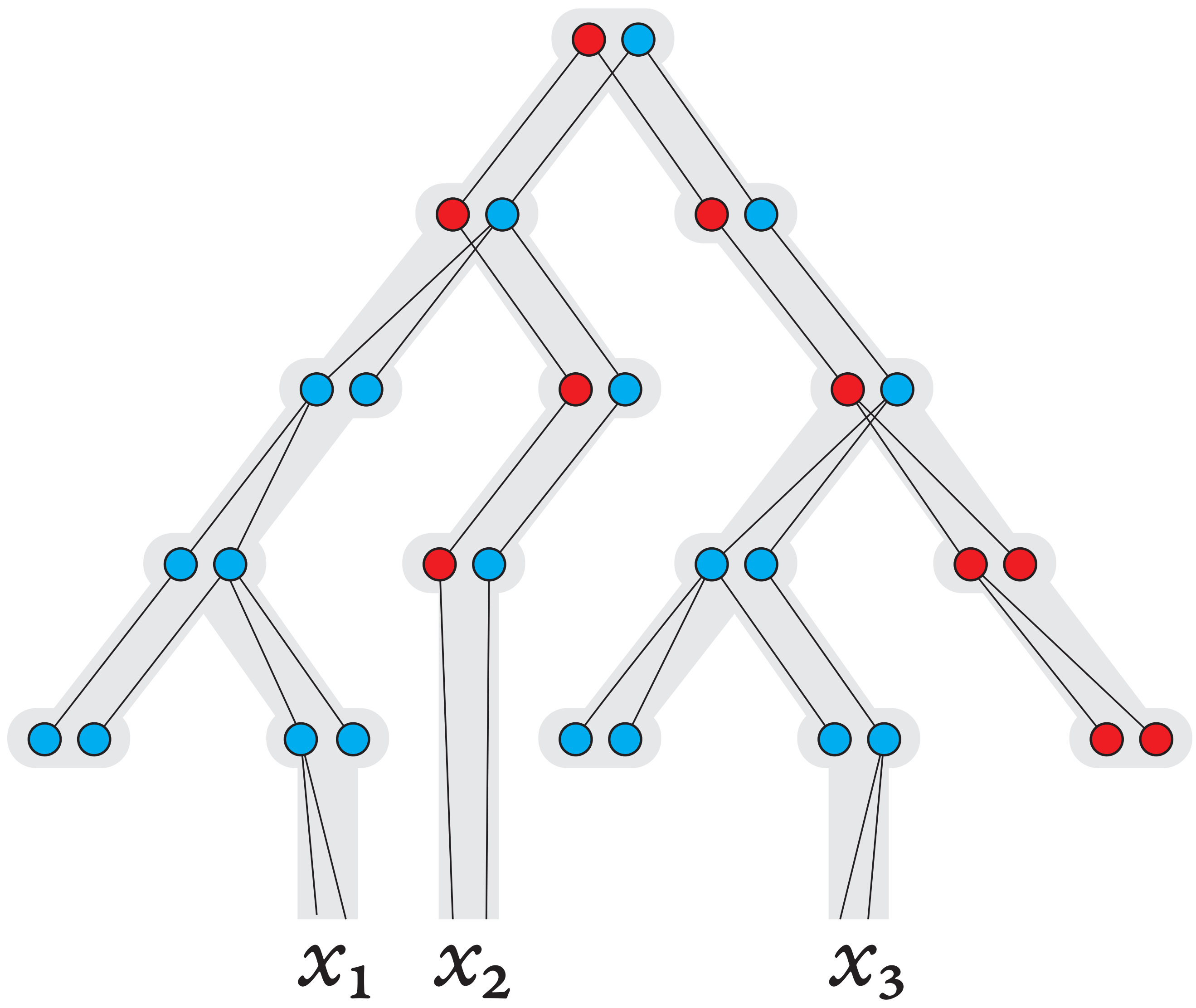

1
2
3
4
5
6
7
8
9
10
11
↦
1
2
3
4
5
6
7
8
9
10
11

a term of arity 4

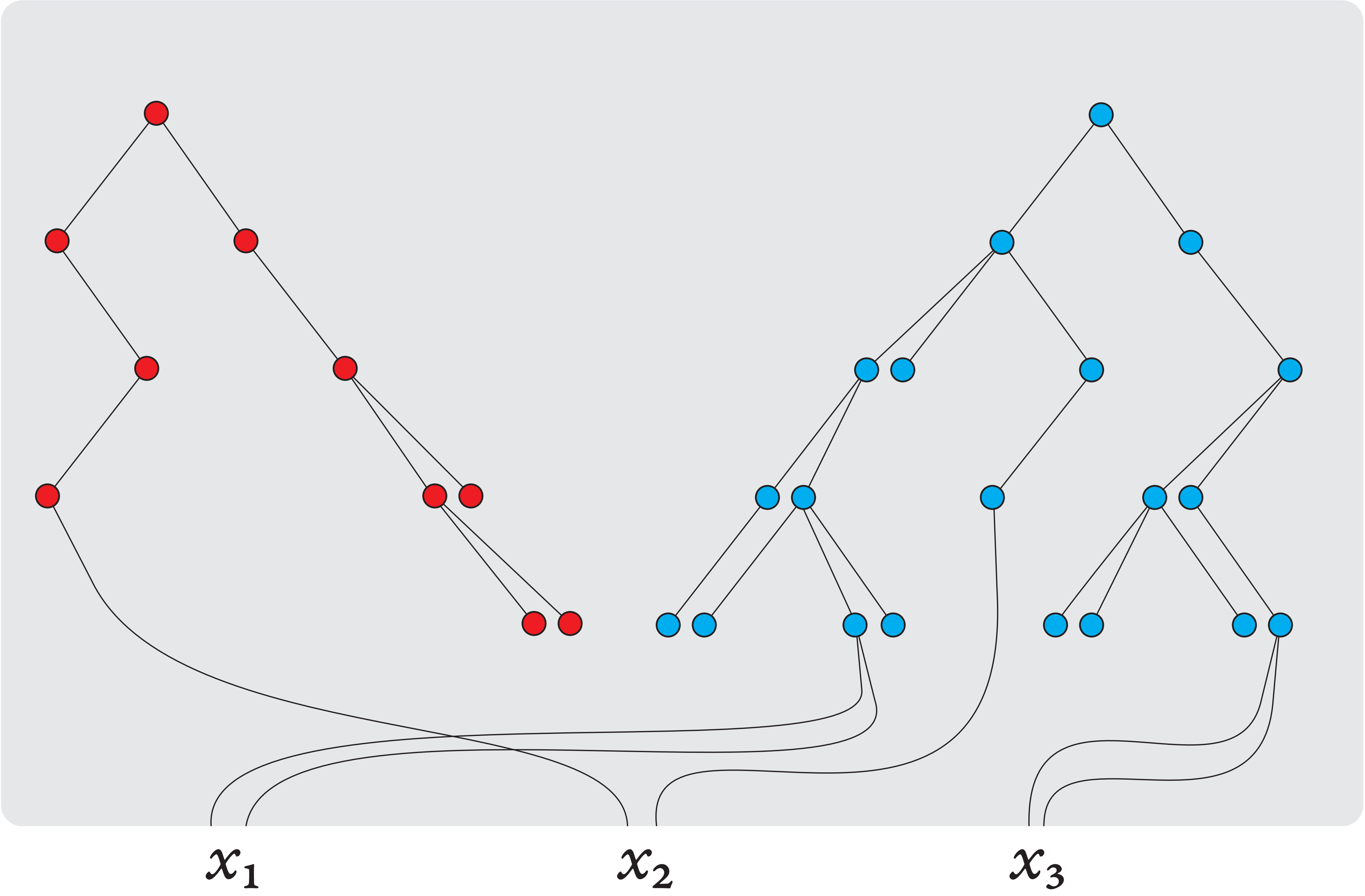


a term of arity 0

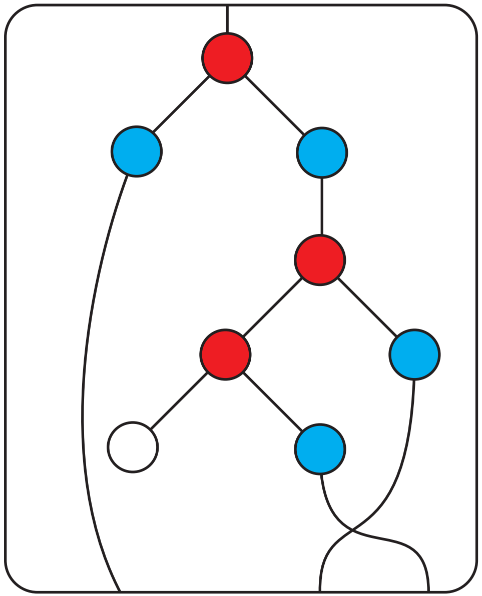

x_1
x_2
x_3



x_1
x_2
x_3



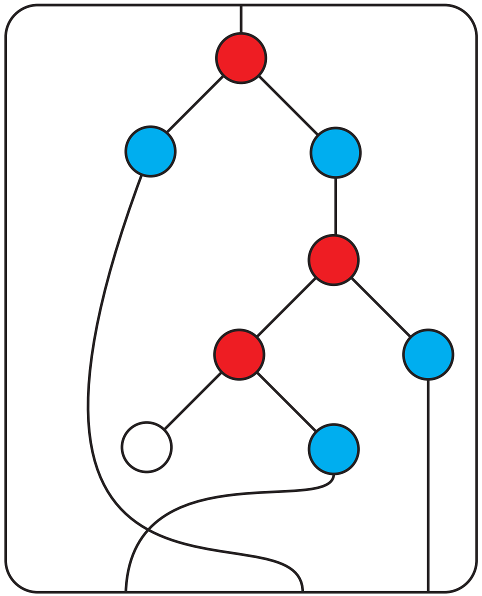
x_1
x_2
x_3



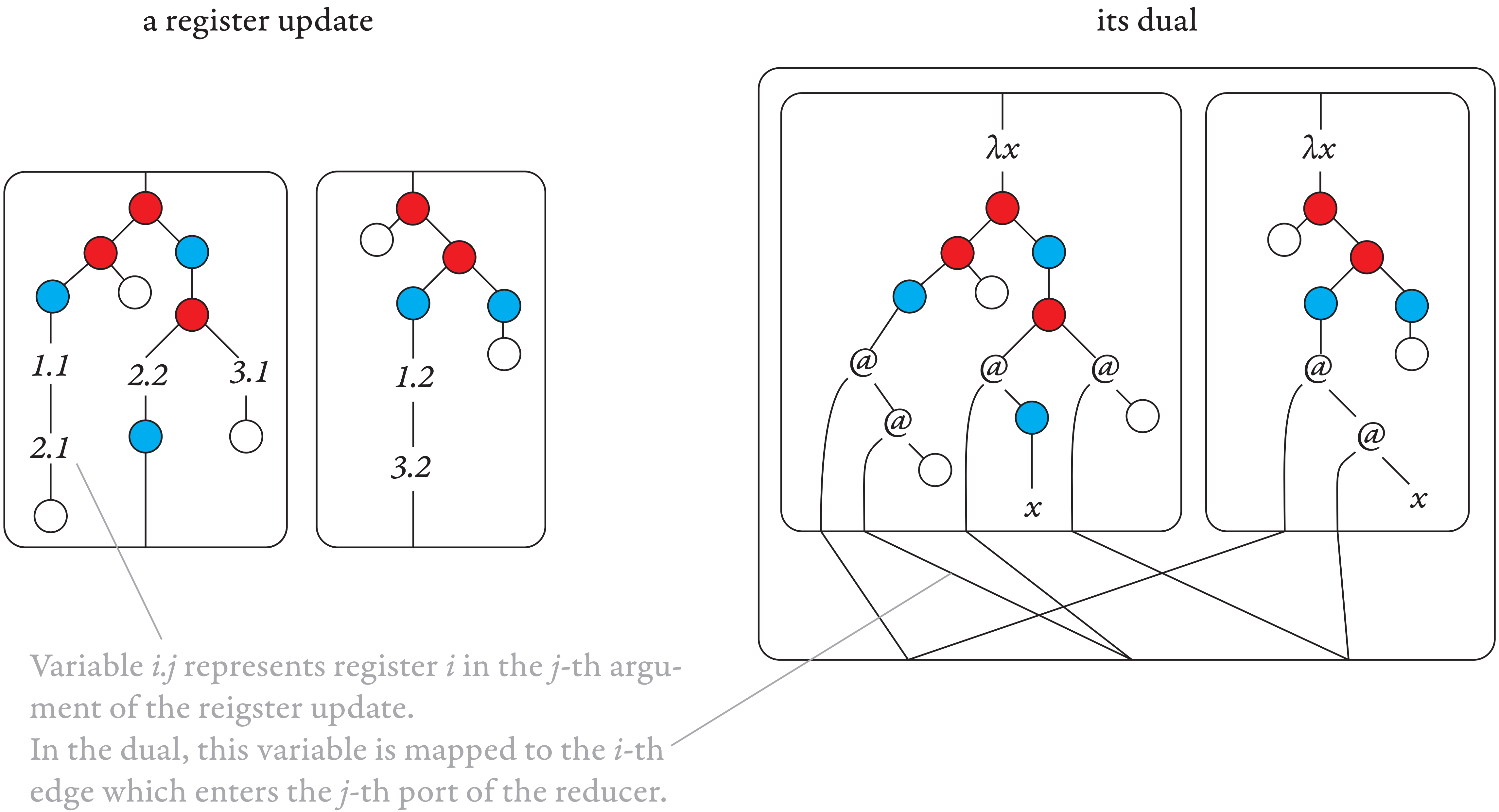
satisfies (*)

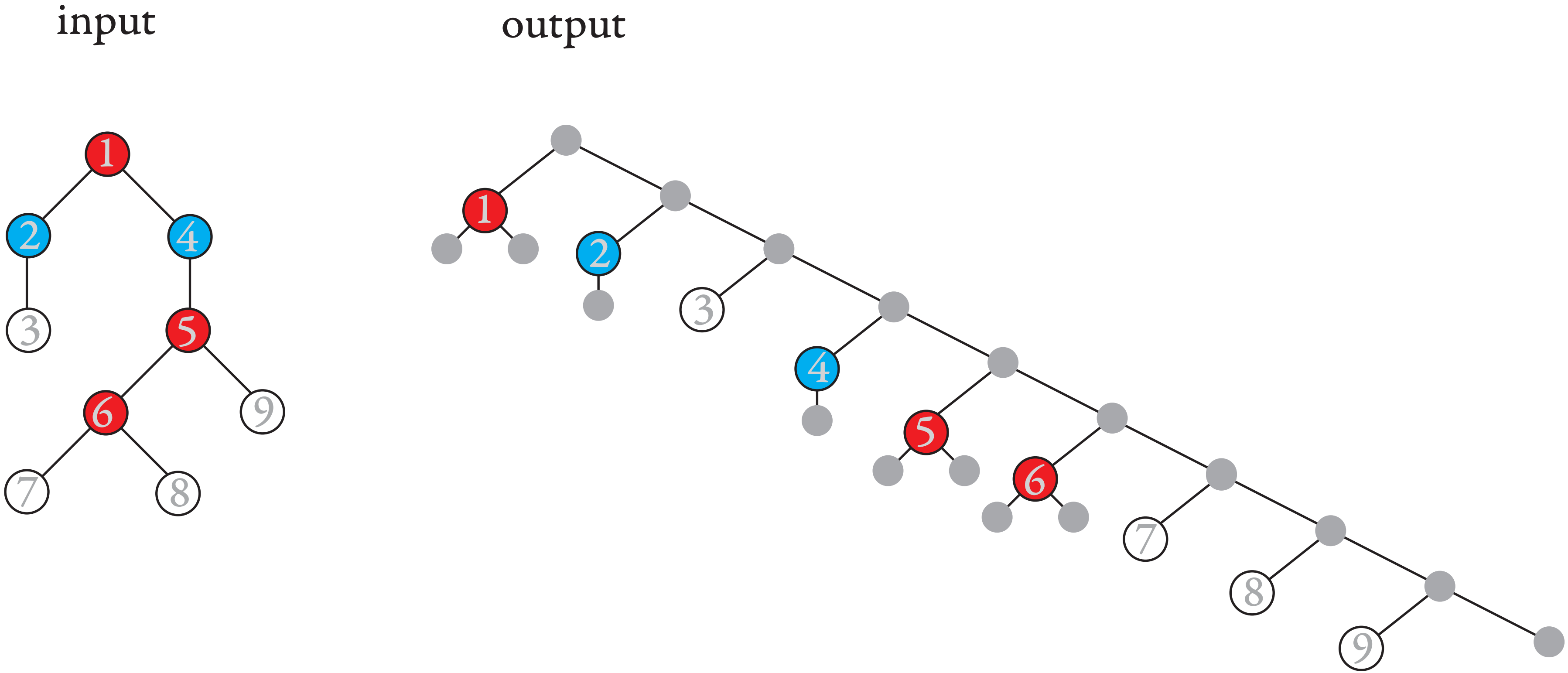
(*)

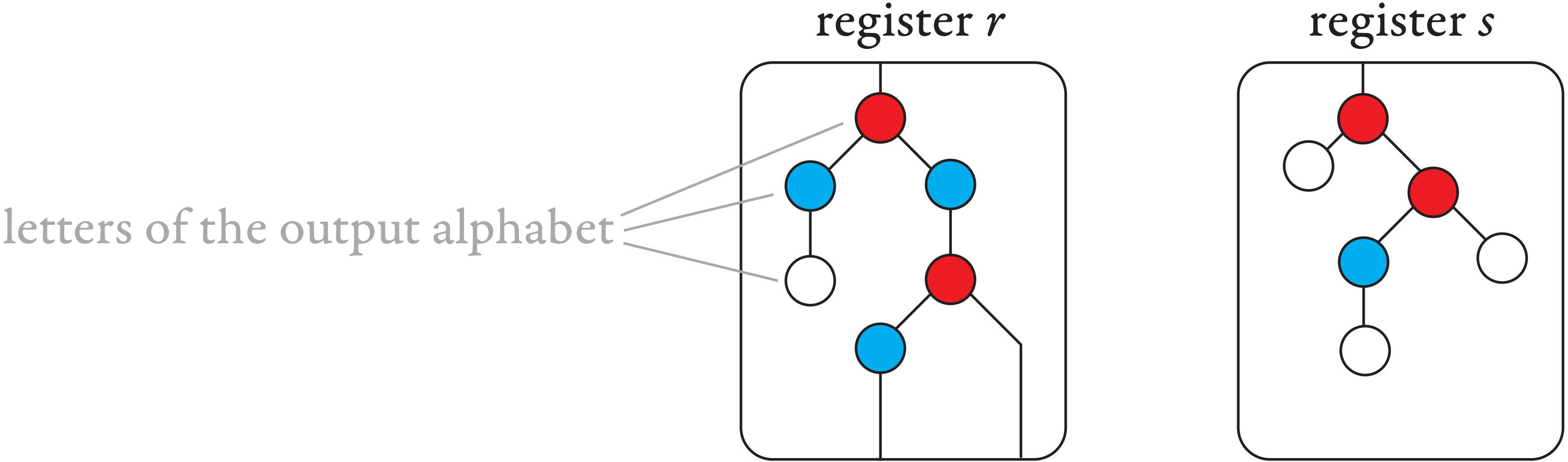
If the root has arity n, and $1 \leq i < j \leq n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root



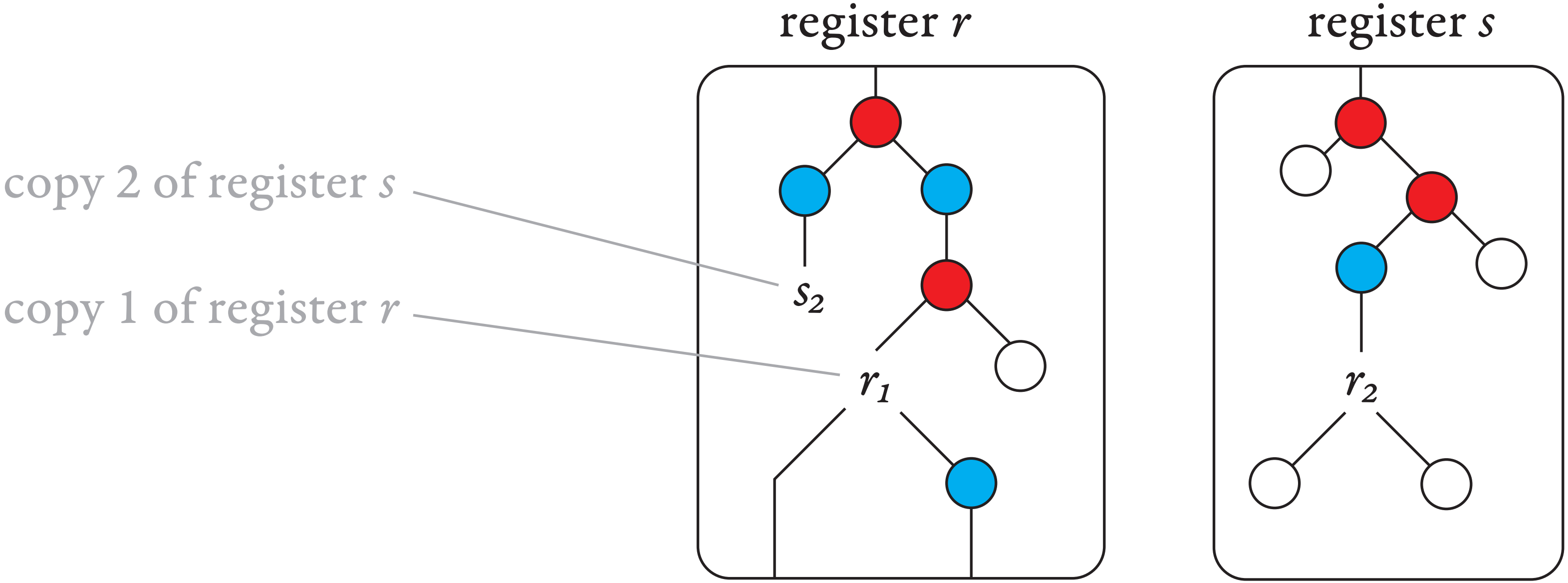
violates (*)


a register update
its dual
1.1
2.1
2.2
3.1
1.2
3.2
λx
λx
@
@
@
@
@
@
x
x
Variable i.j represents register i in the j-th argument of the reigster update.
In the dual, this variable is mapped to the i-th edge which enters the j-th port of the reducer.

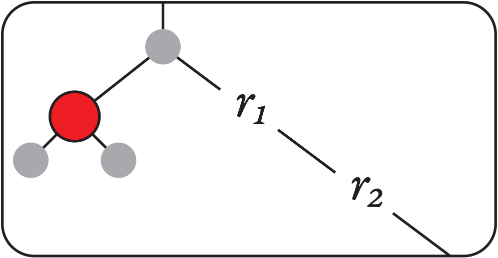

input
output
1
2
3
4
5
6
7
8
9
1
2
3
4
5
6
7
8
9

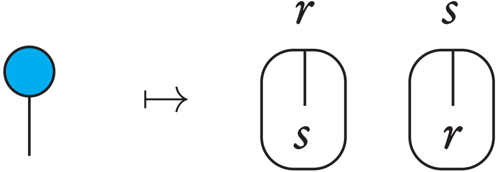


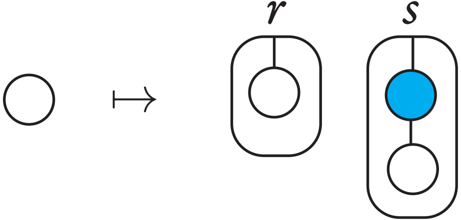
register r
register s
letters of the output alphabet

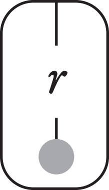

register r
register s
copy 2 of register s
copy 1 of register r
s2
r1
r2

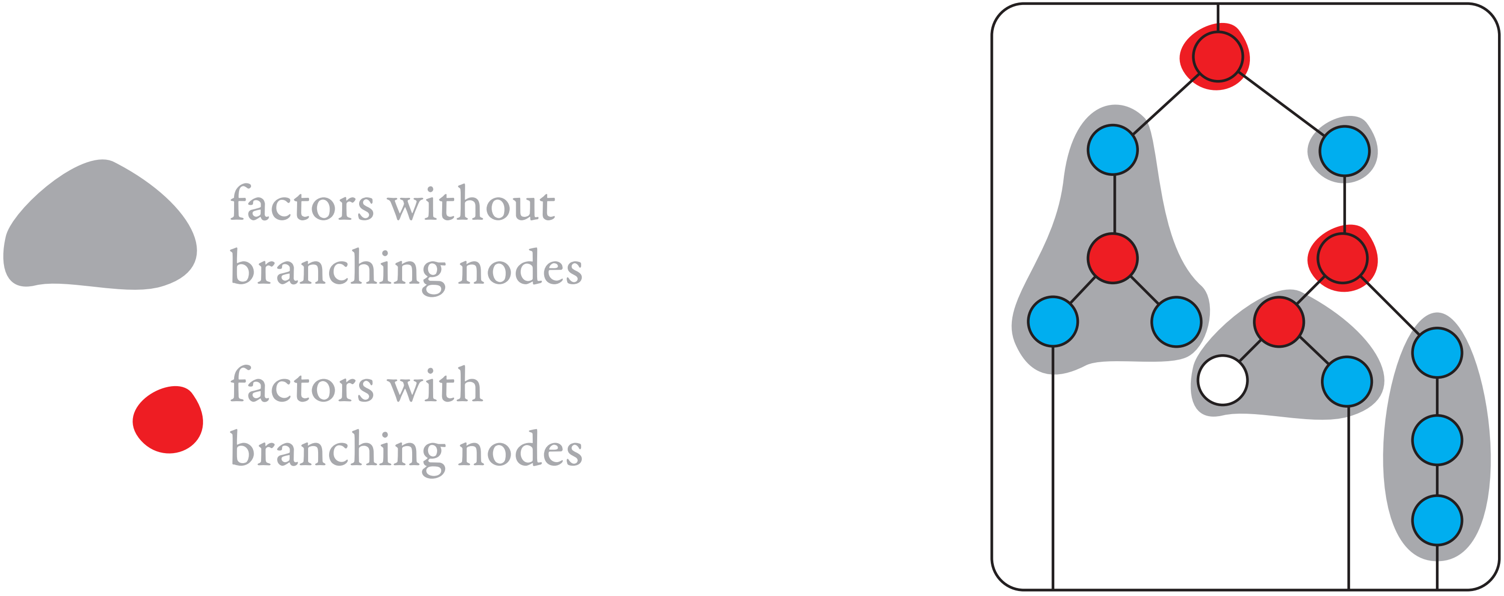

$\mapsto$
r
r
r
$\mapsto$
s

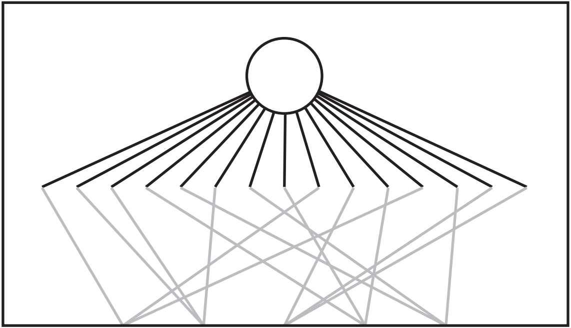

r_1
r_2


r
s
s
r

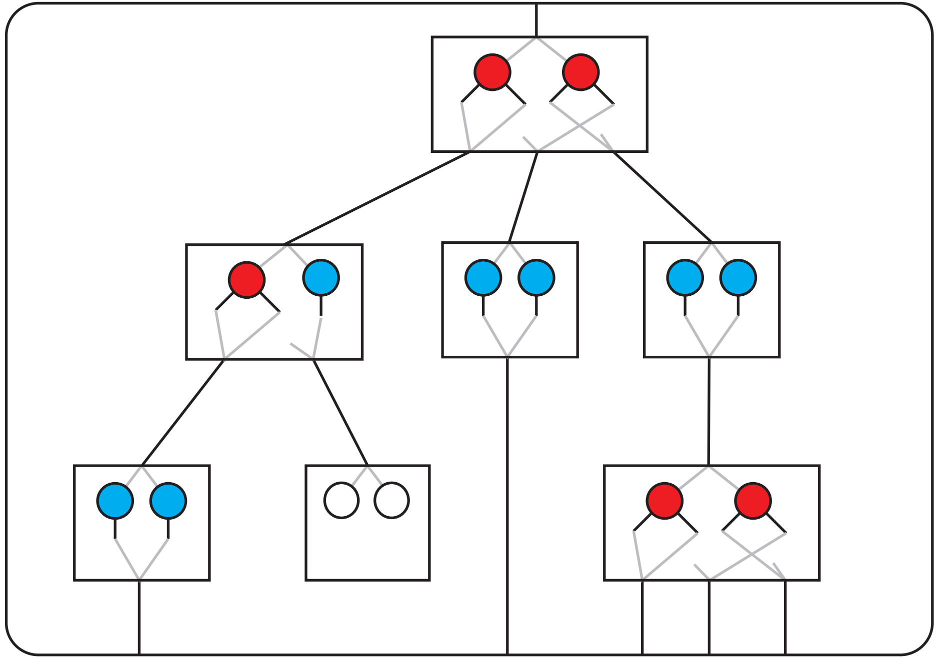

r
s


r

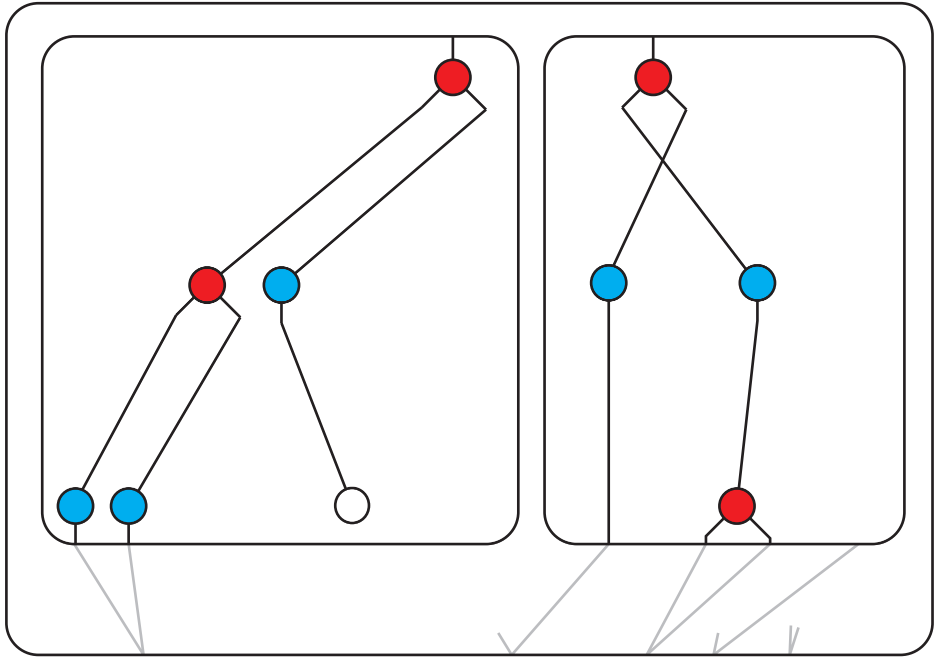

factors without
branching nodes
factors with
branching nodes

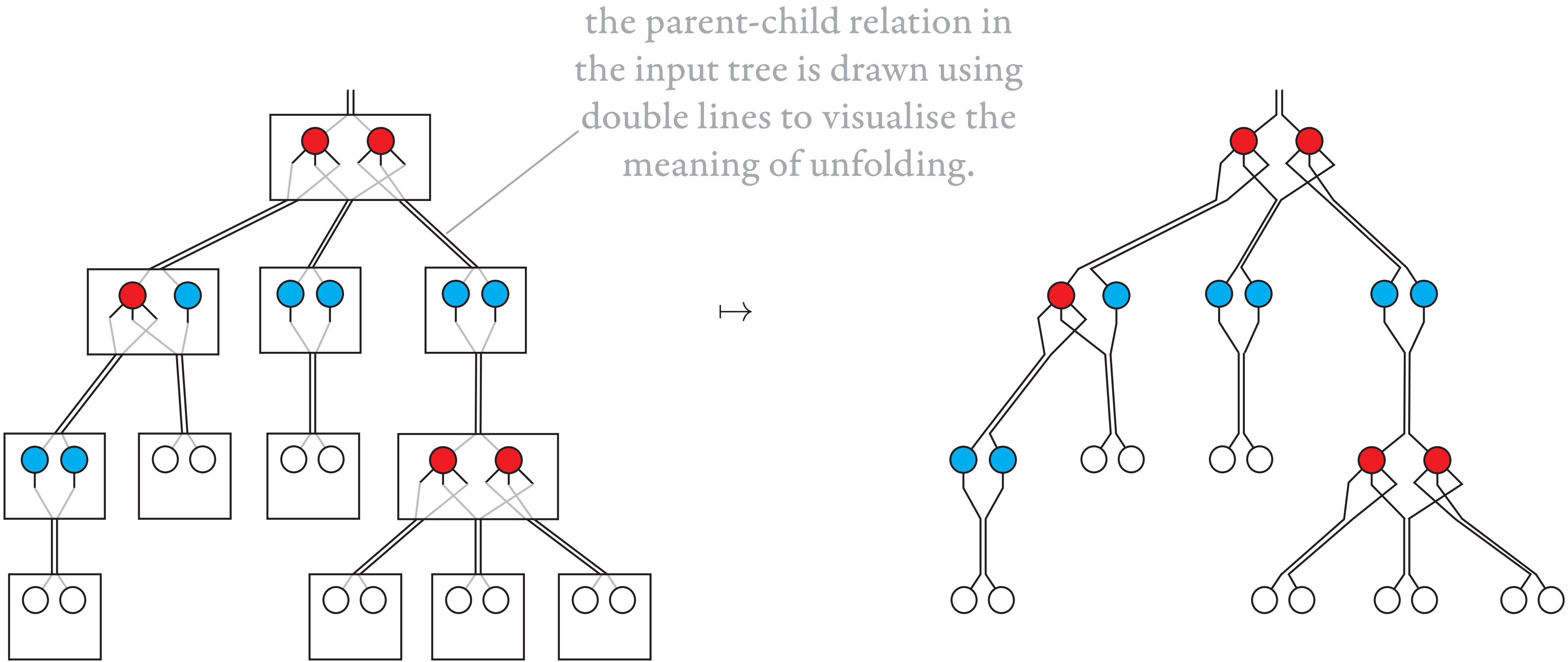


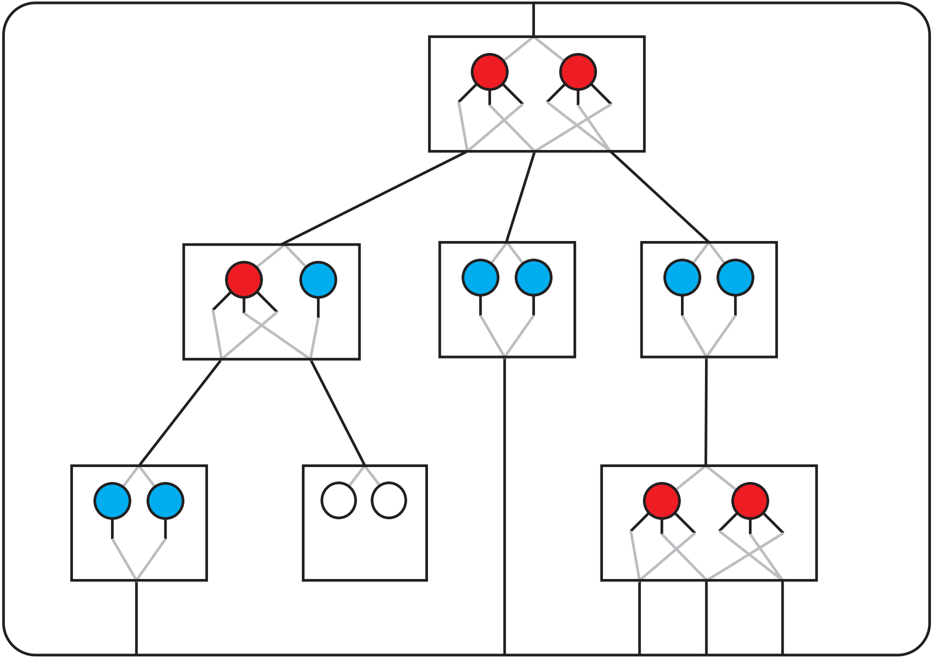
a term of matrix powers

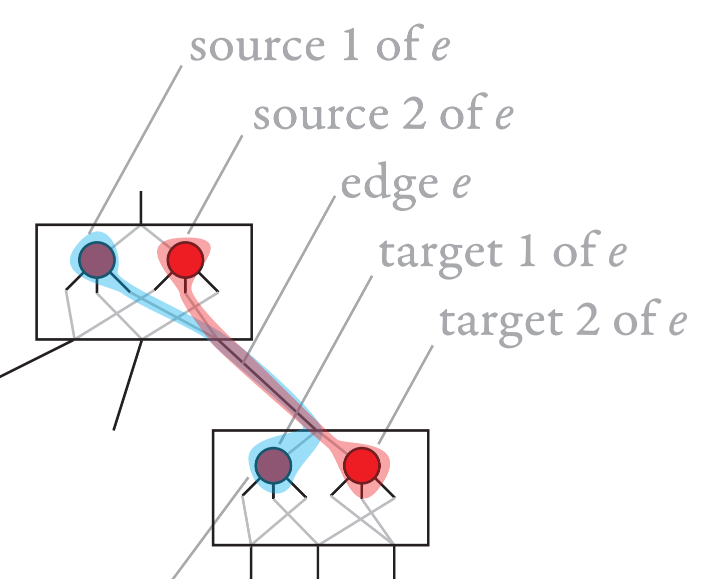


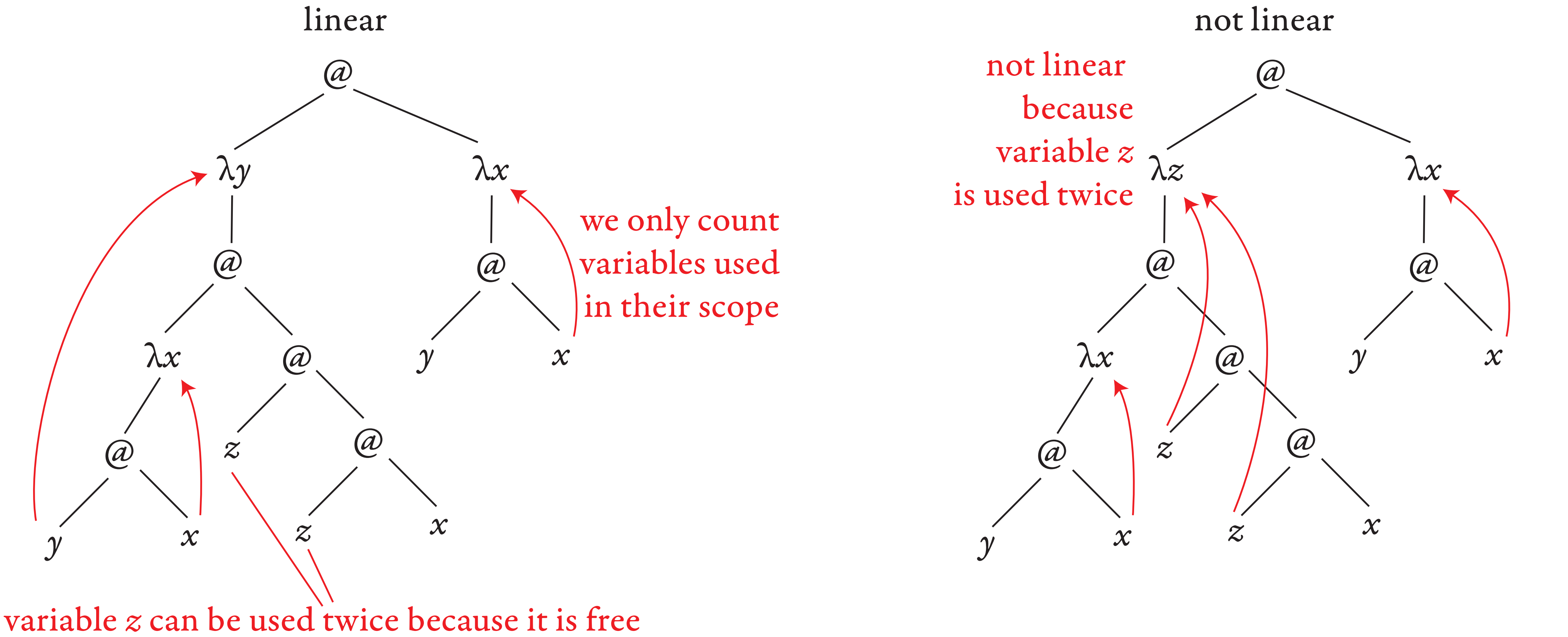
its term unfolding



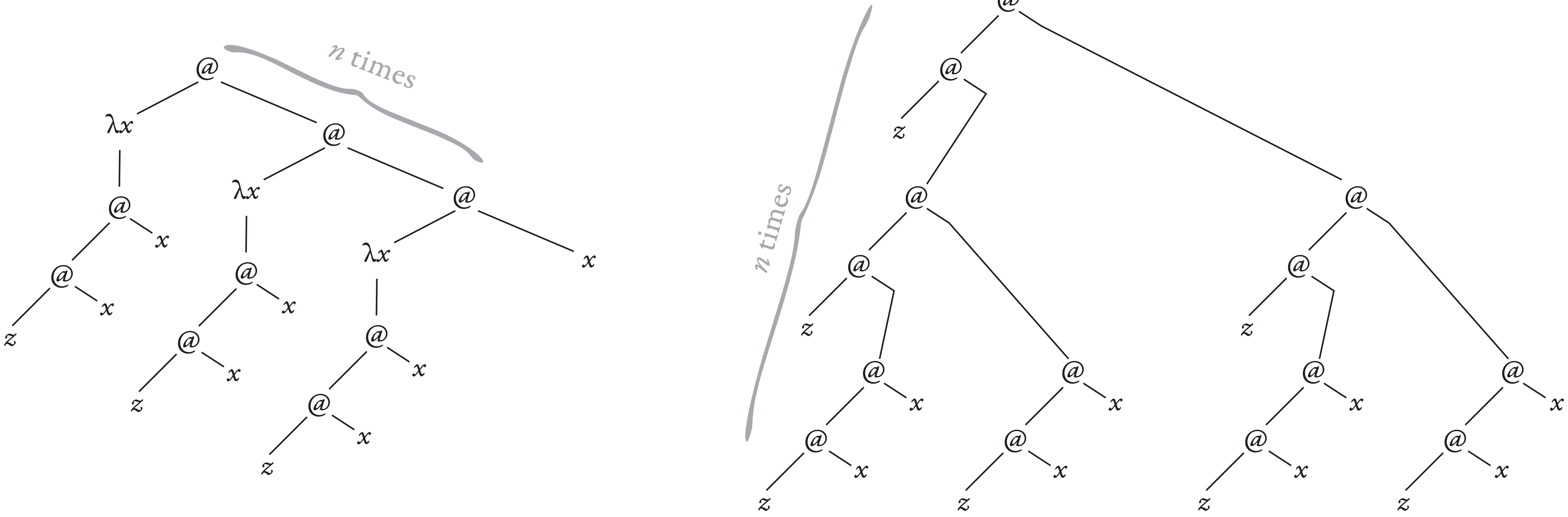

the parent-child relation in the input tree is drawn using double lines to visualise the meaning of unfolding.
↦



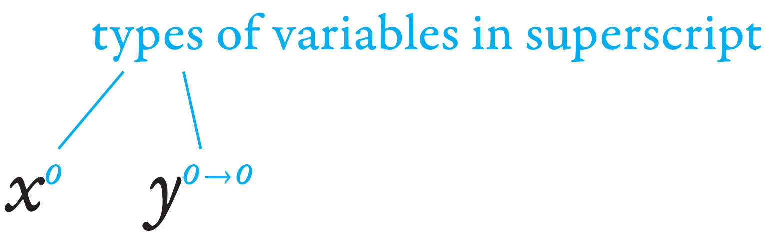

source 1 of *e*
source 2 of *e*
edge *e*
target 1 of *e*
target 2 of *e*


linear
not linear
we only count variables used in their scope
variable z can be used twice because it is free
not linear because variable z is used twice

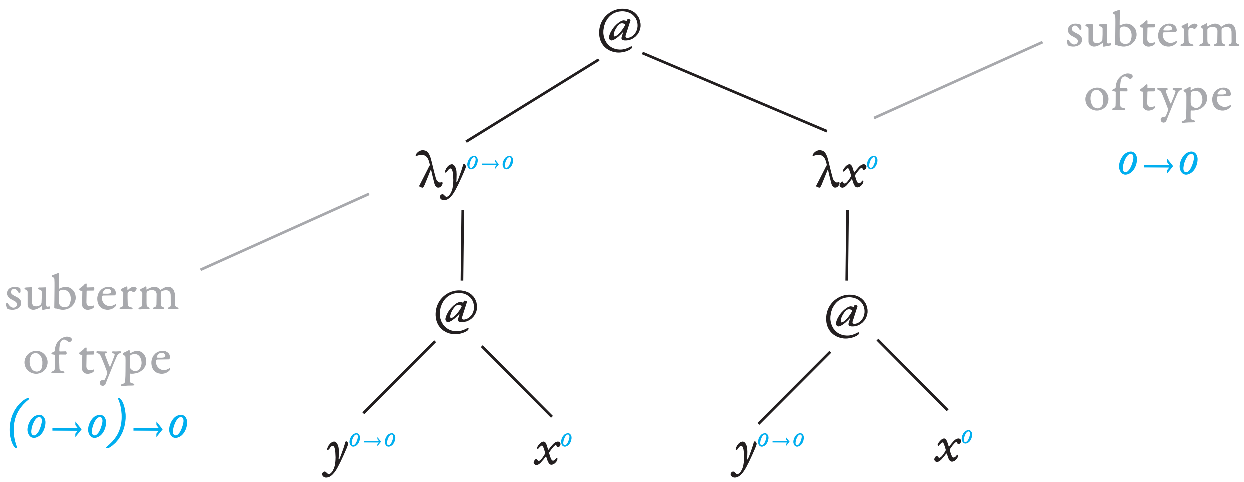
a λ-term of size $O(n)$ its normal form of size $O(2^n)$



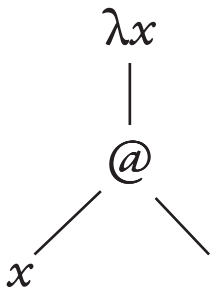
variables



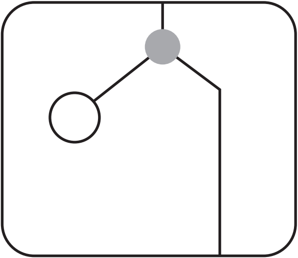
λ-term of type o

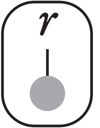


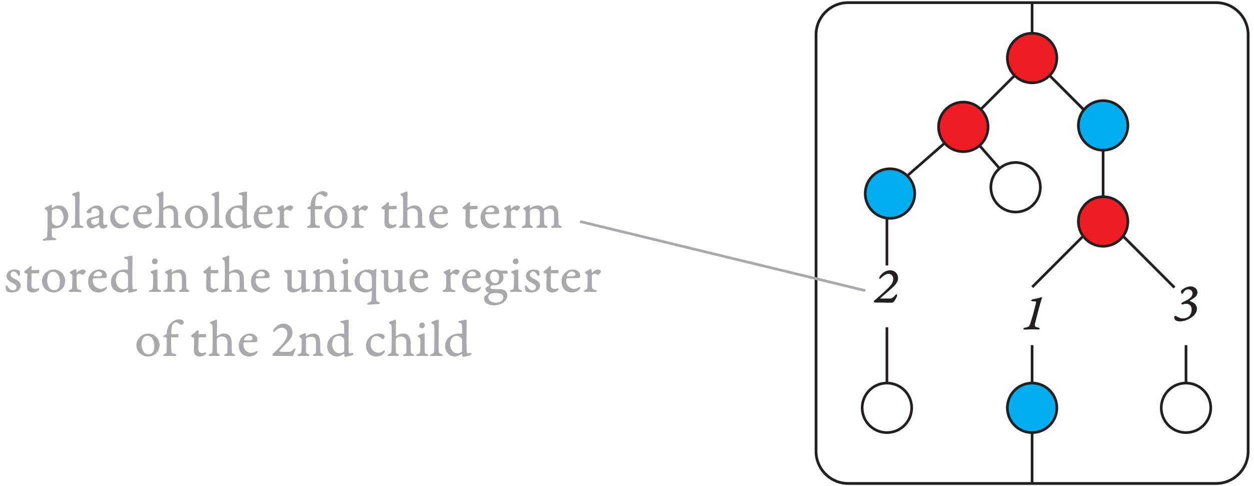
@

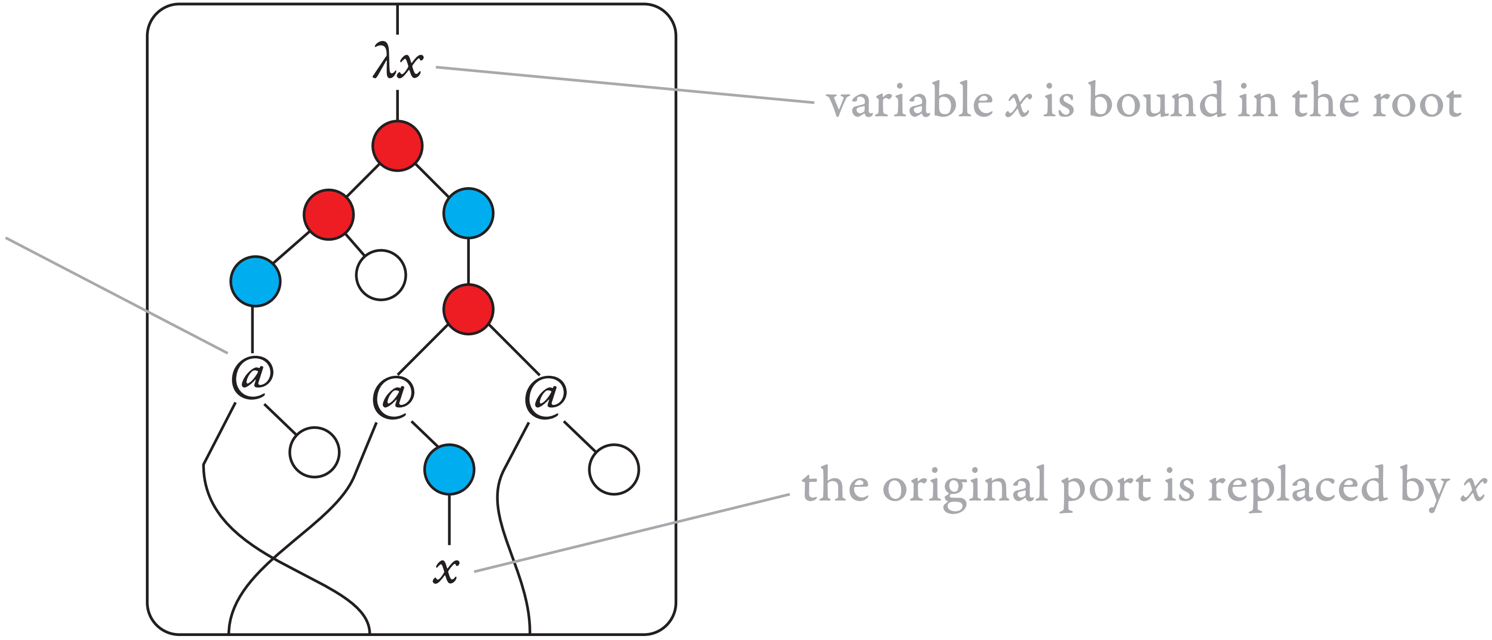


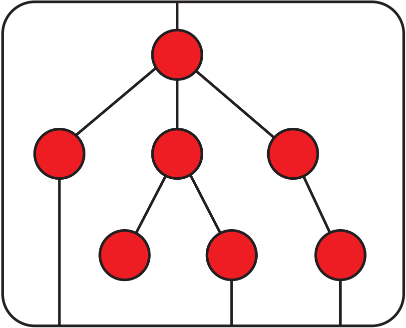
$\lambda x.$

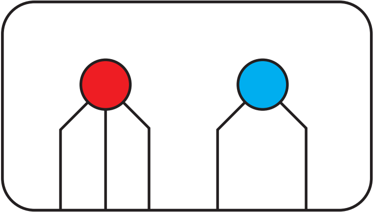


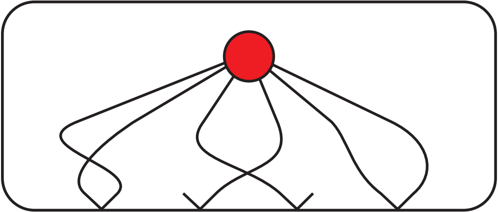

r

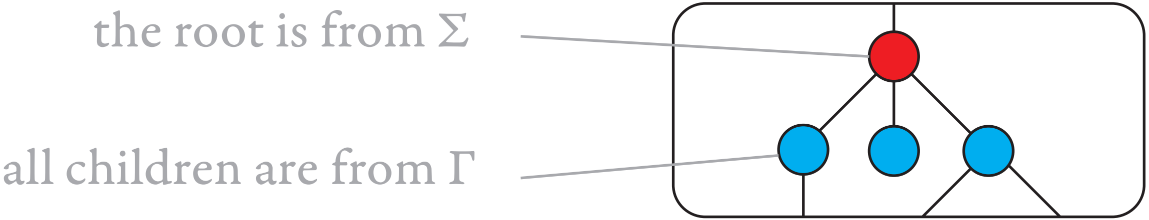

placeholder for the term
stored in the unique register
of the 2nd child
2
1
3

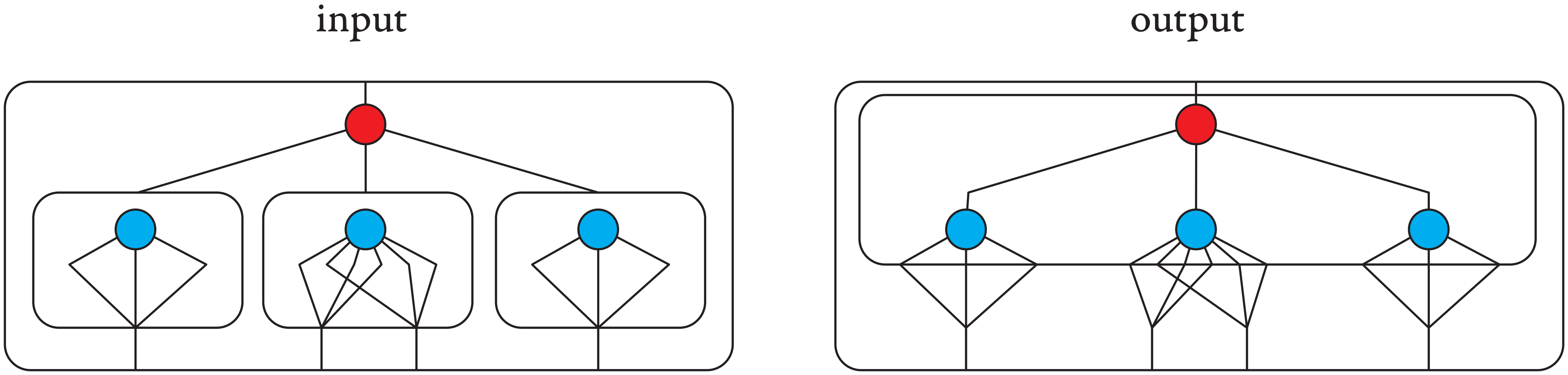

λx
variable x is bound in the root
@
@
@
x
the original port is replaced by x

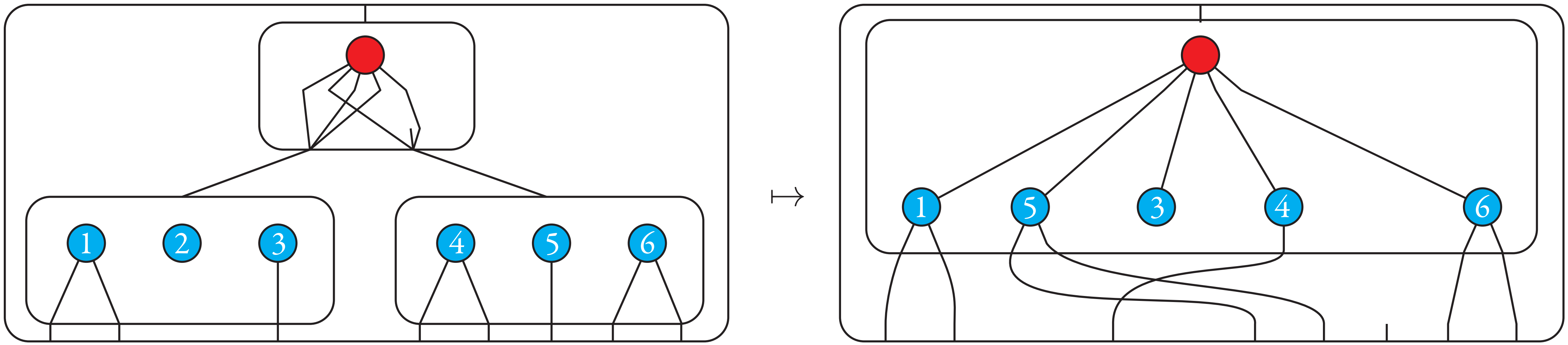


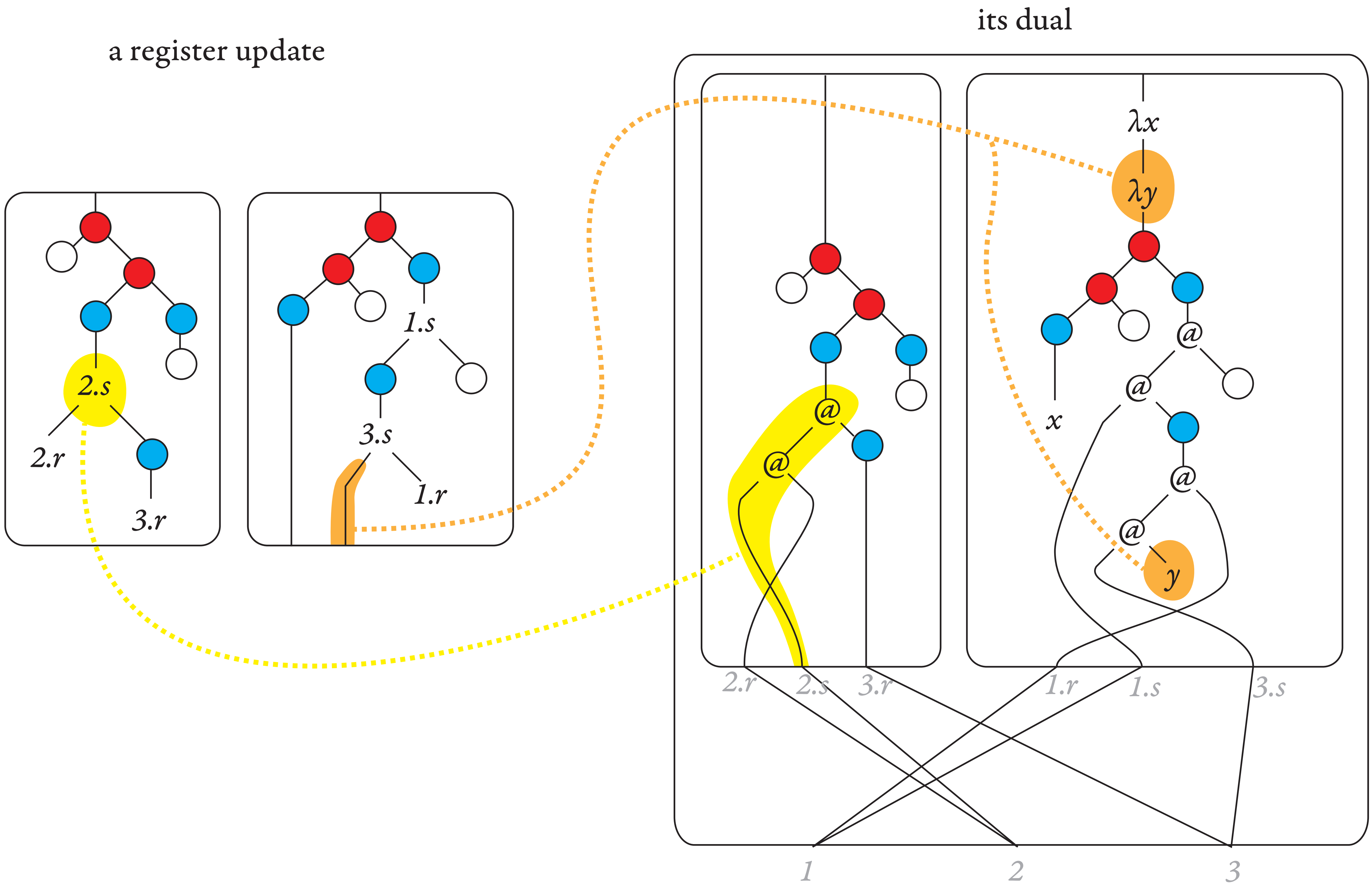




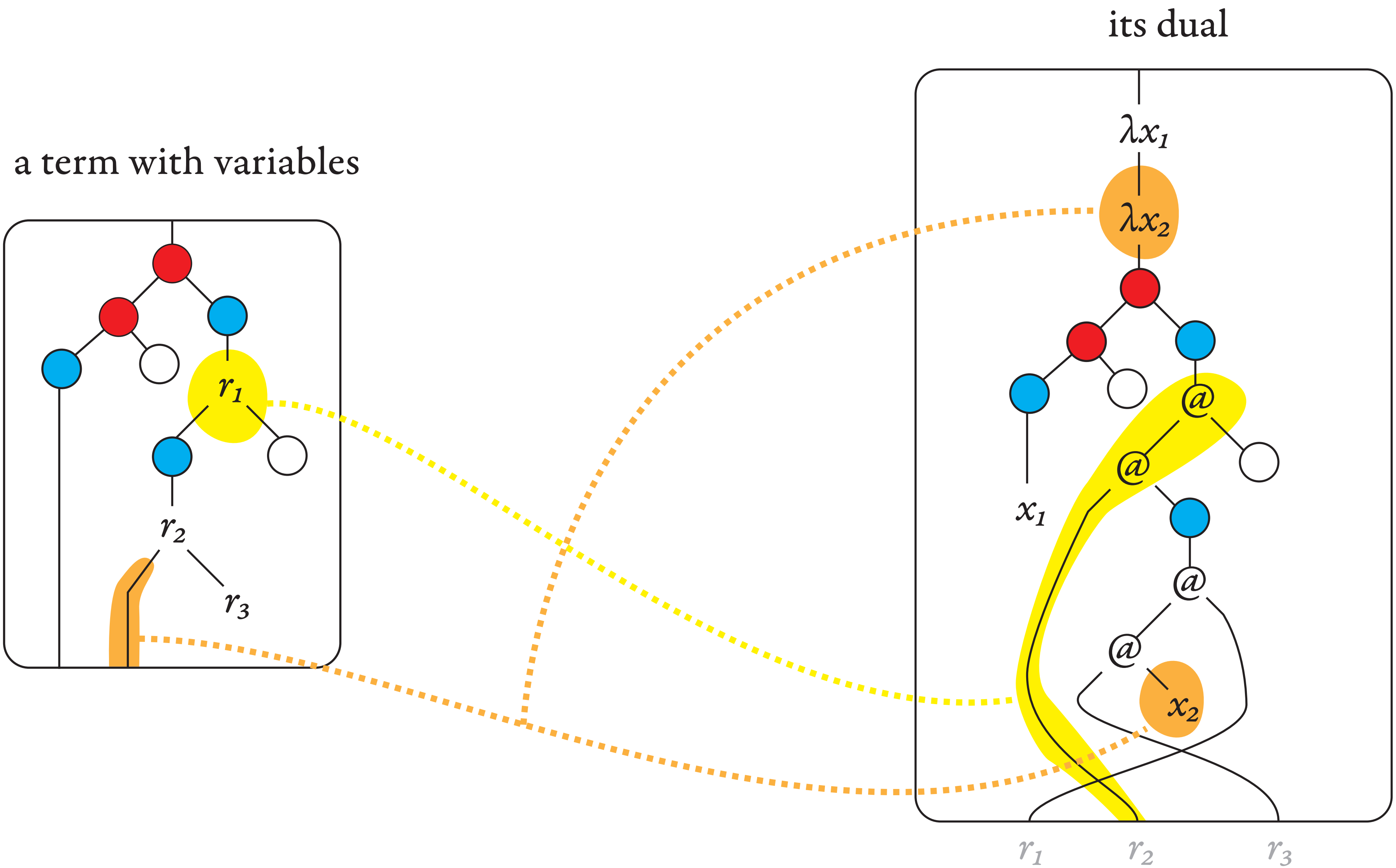

the root is from Σ
all children are from Γ

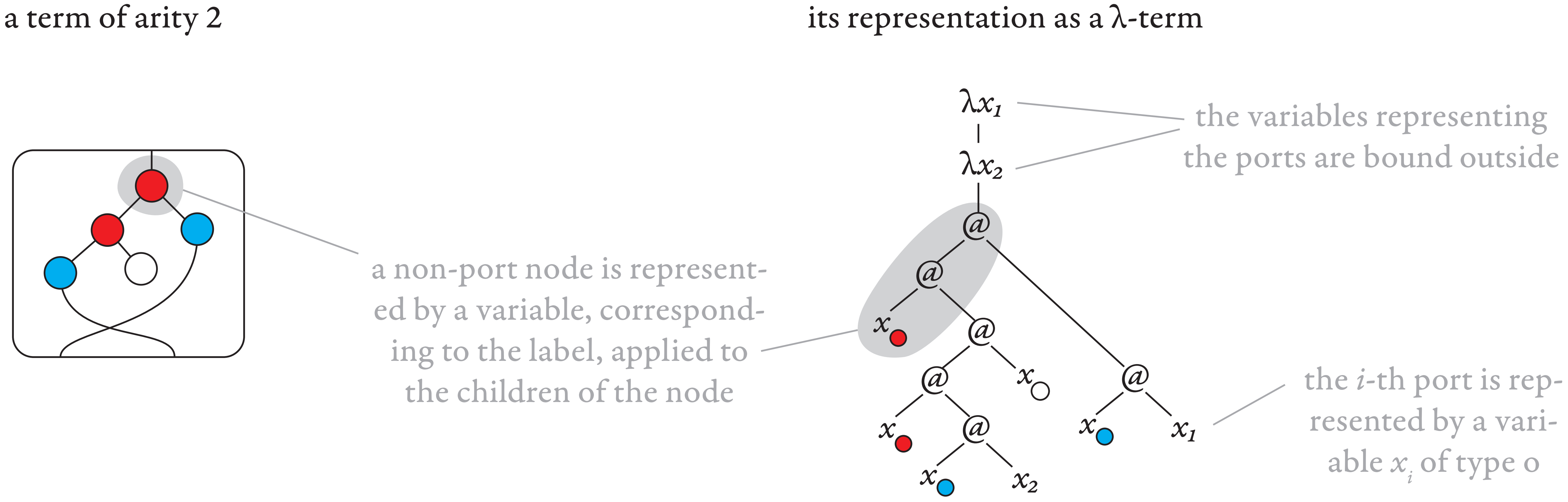

input
output


1
2
3
4
5
6
↦
1
5
3
4
6



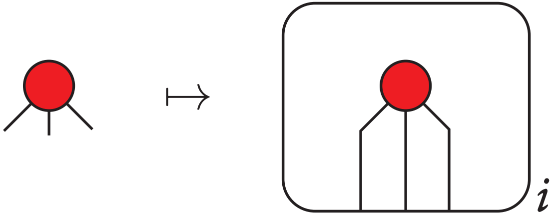
a register update
its dual
2.s
2.r
3.r
1.s
3.s
1.r
λx
λy
@
@
@
x
@
@
@
y
2.r
2.s
3.r
1.r
1.s
3.s
1
2
3

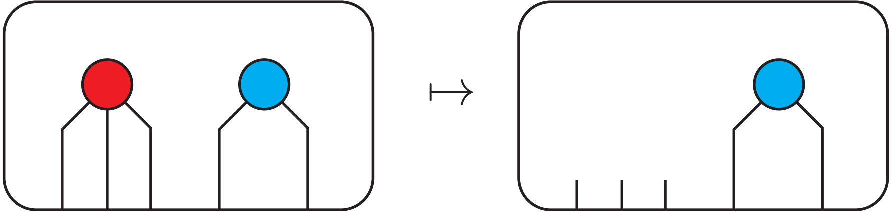

a term with variables
r_1
r_2
r_3
its dual
λx_1
λx_2
@
@
@
@
x_1
x_2
r_1
r_2
r_3

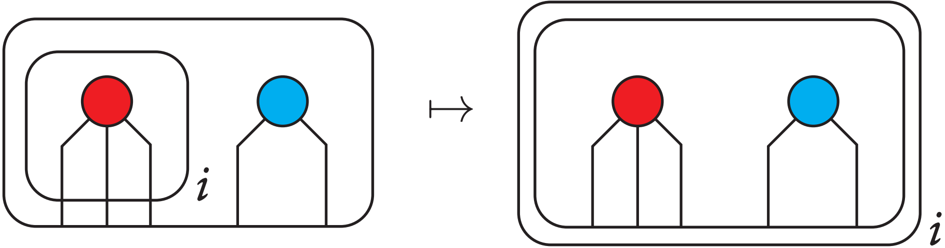


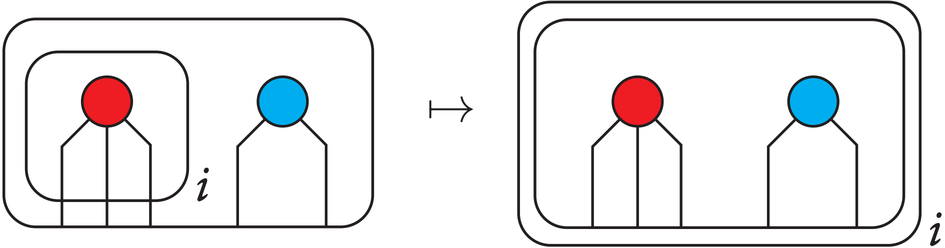
a term of arity 2
its representation as a λ-term
λx_1
λx_2
the variables representing the ports are bound outside
@
x
x_1
x_2
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
the i-th port is represented by a variable x_i of type o

i


i




i
i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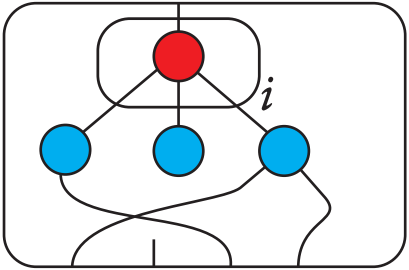

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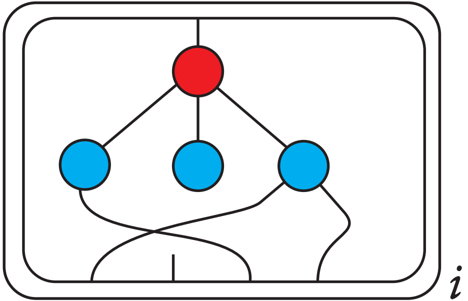

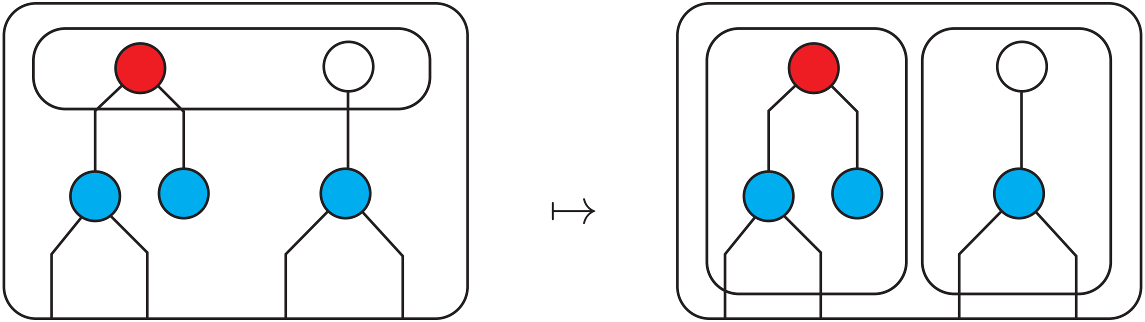

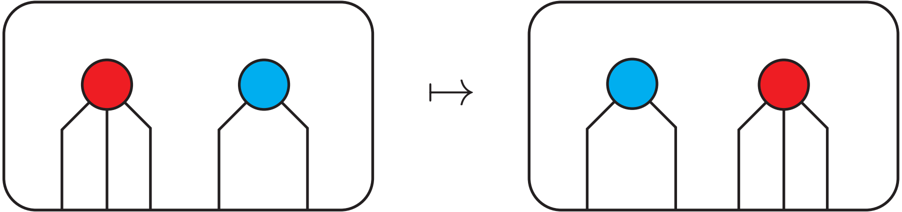

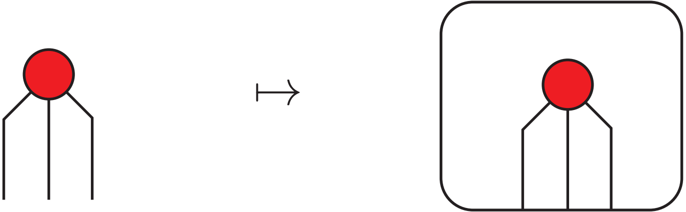

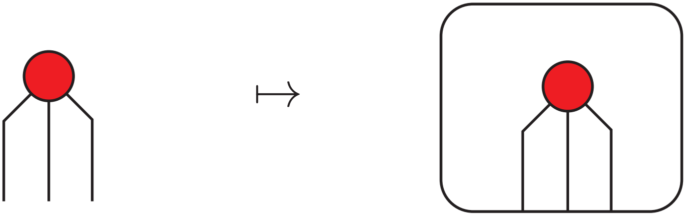

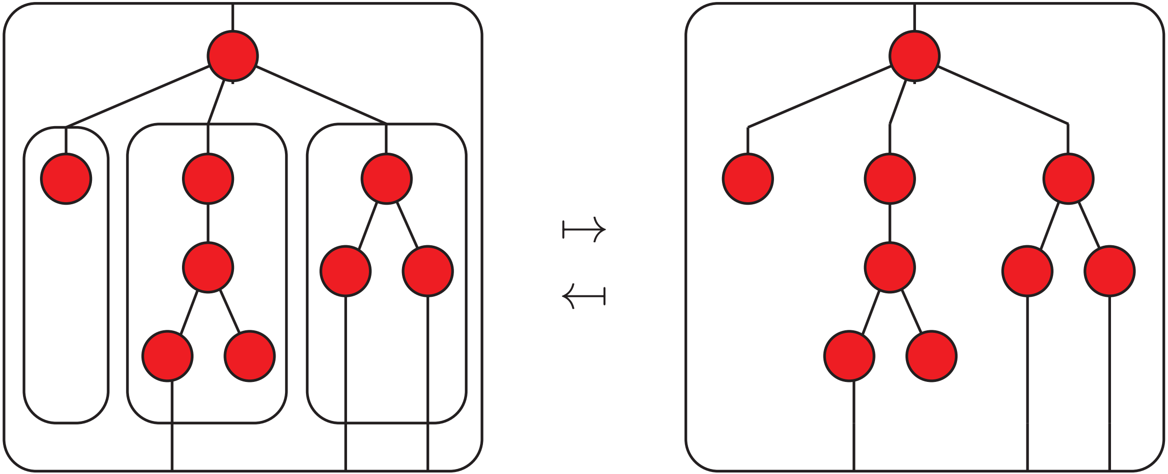

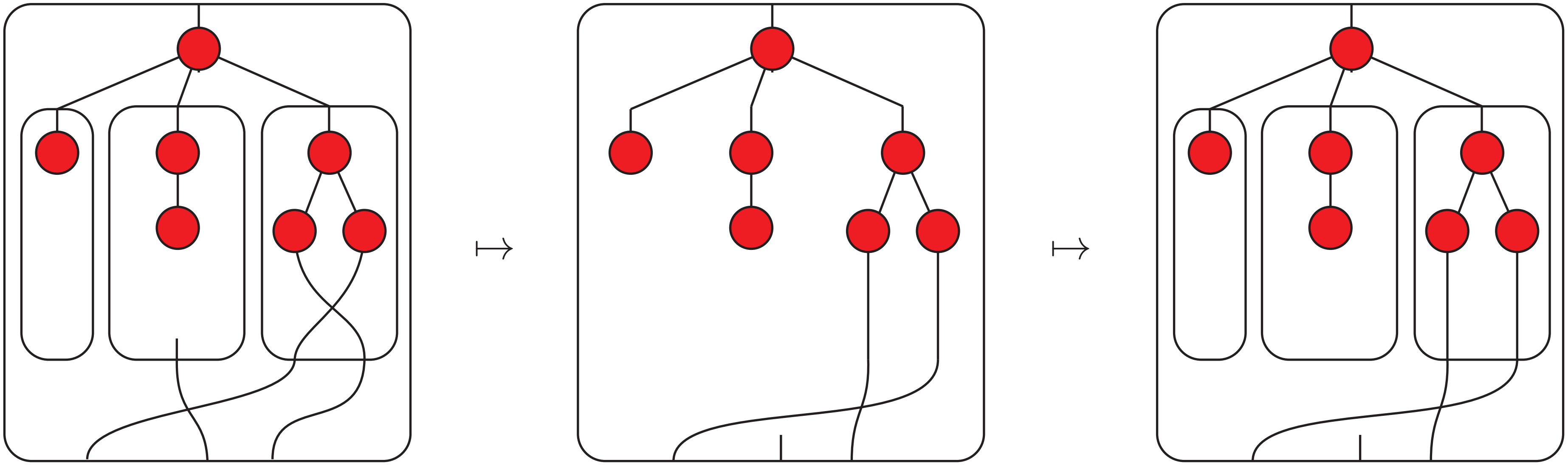

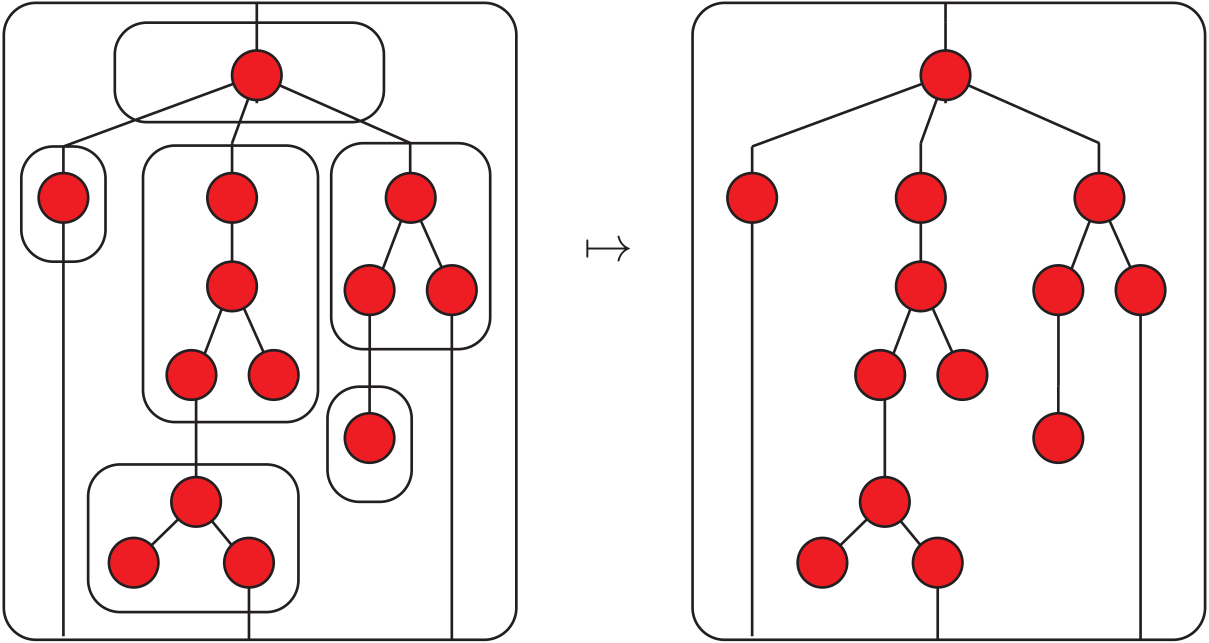

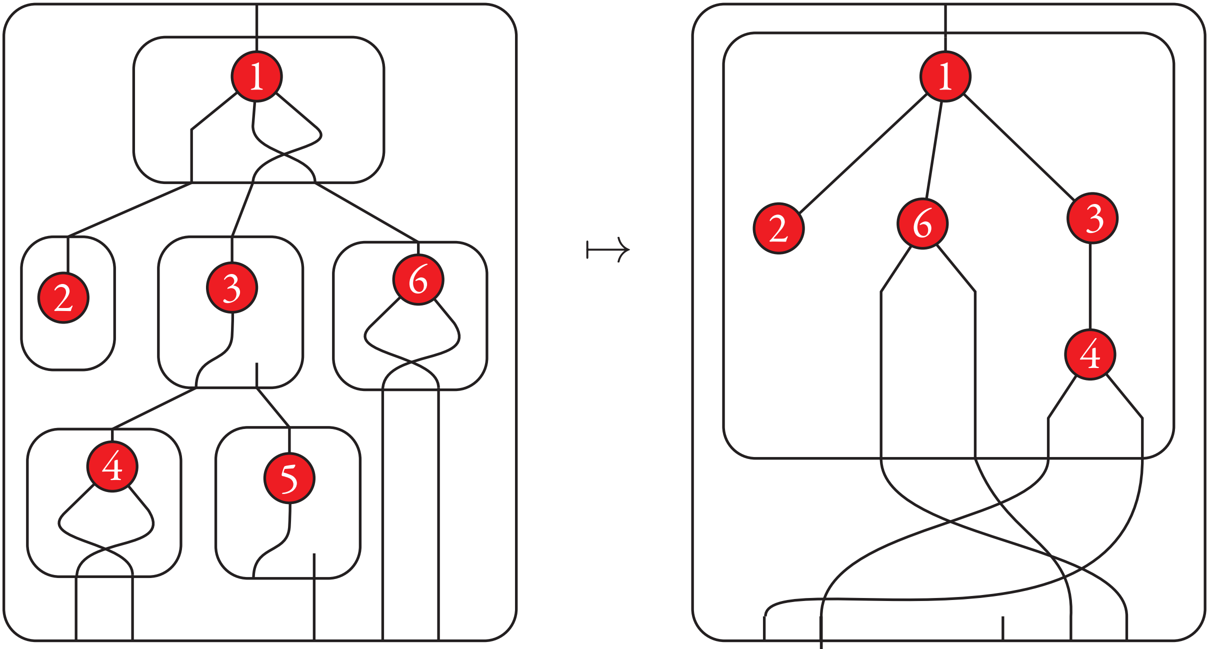
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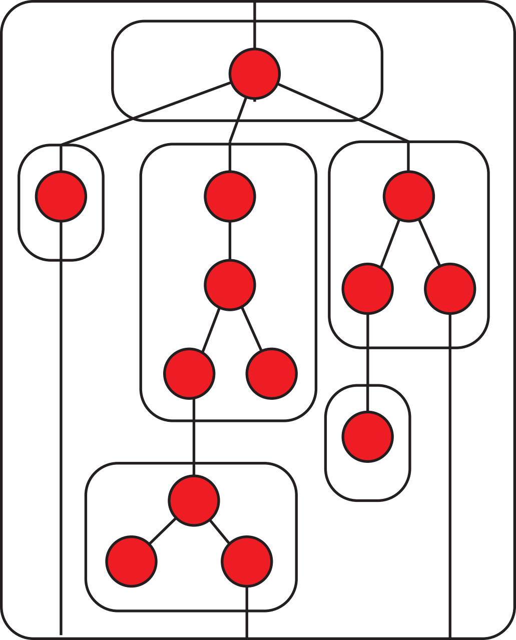
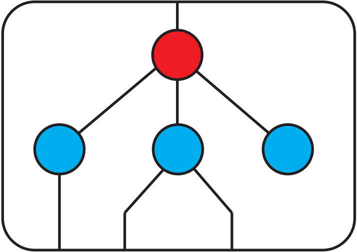
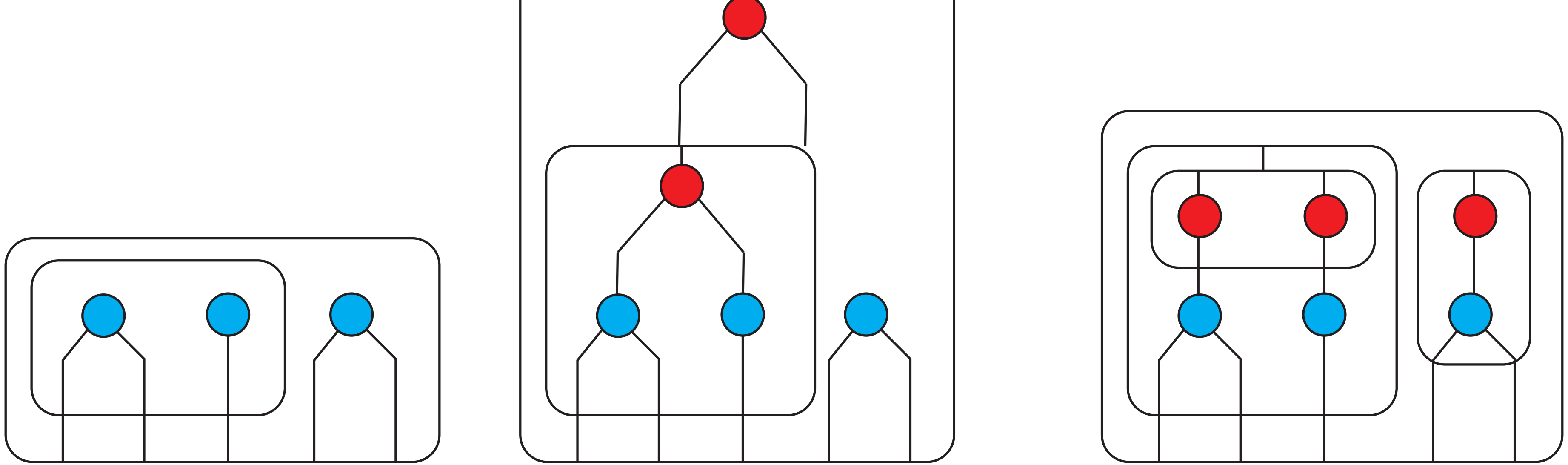
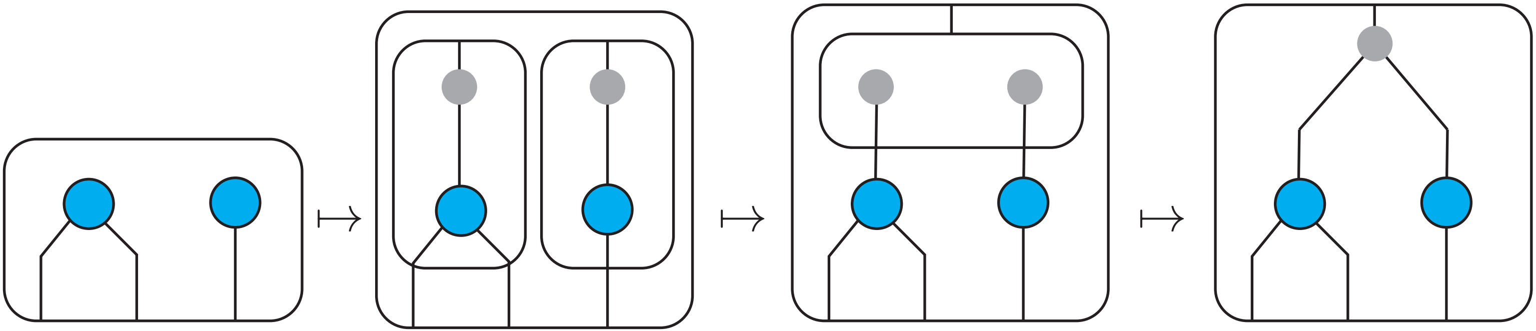
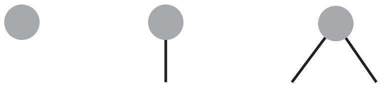
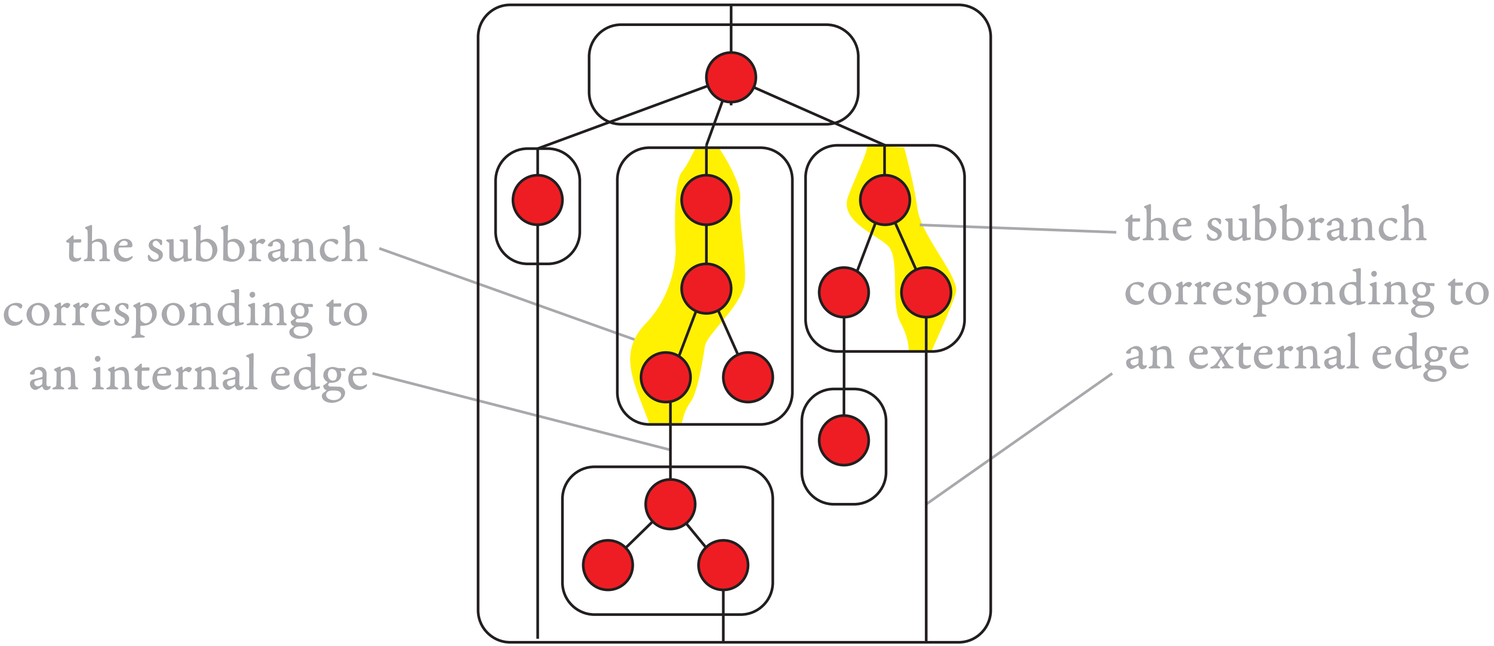


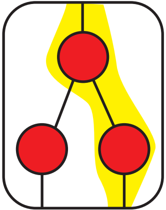




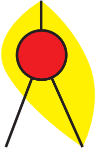


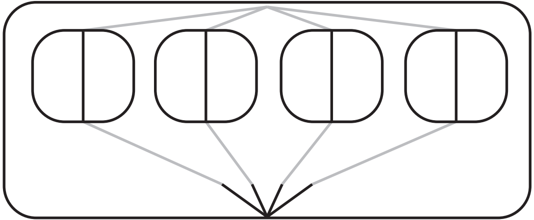



the subbranch corresponding to an internal edge
the subbranch corresponding to an external edge

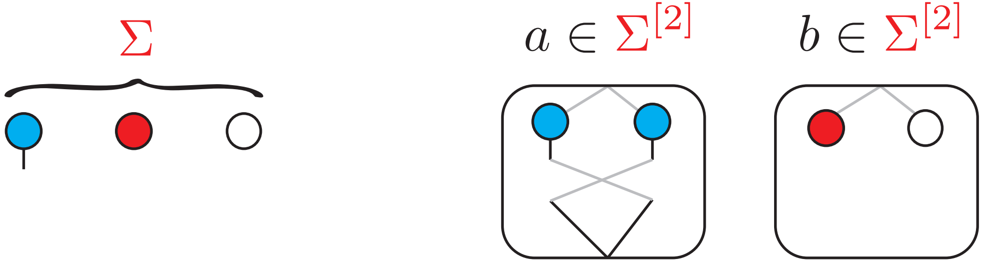


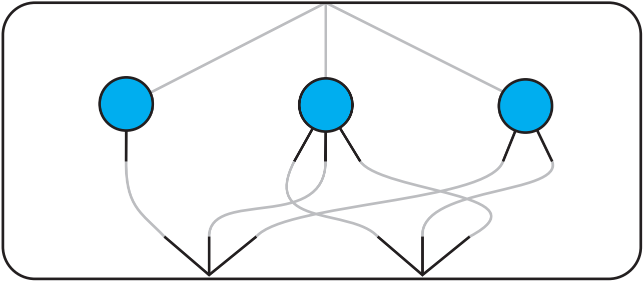
a branch can be visualised as
a term with a distinguished
root-to-port path



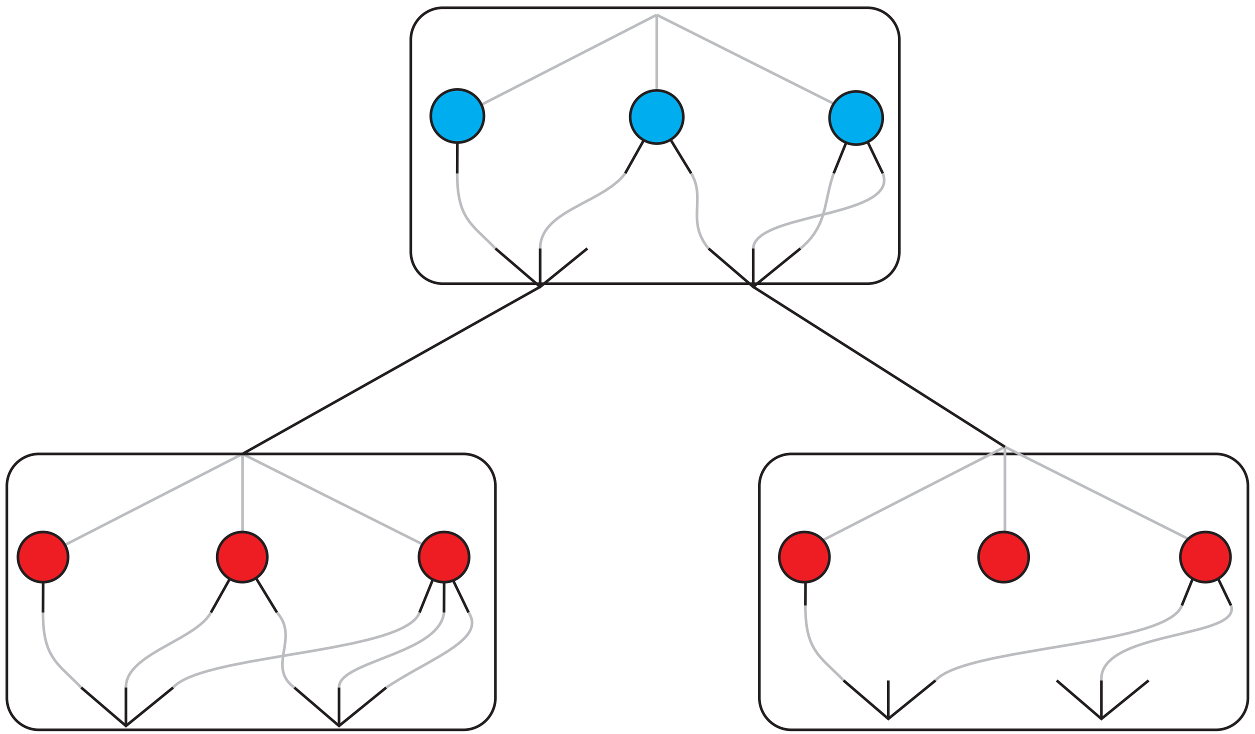


a tuple of *k* identity terms with all their ports folded into one


Σ
$a \in \Sigma^{[2]}$
$b \in \Sigma^{[2]}$



a shallow term of matrix powers



its shallow unfolding