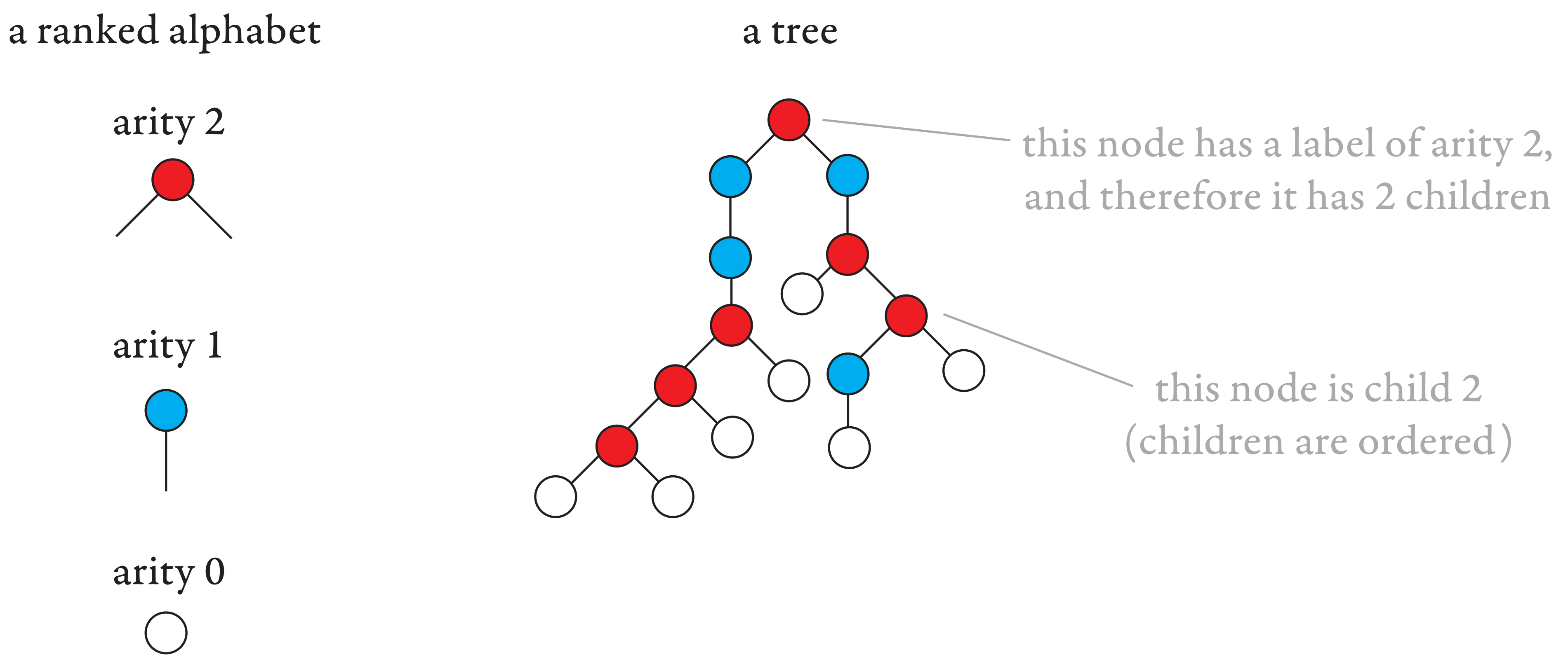
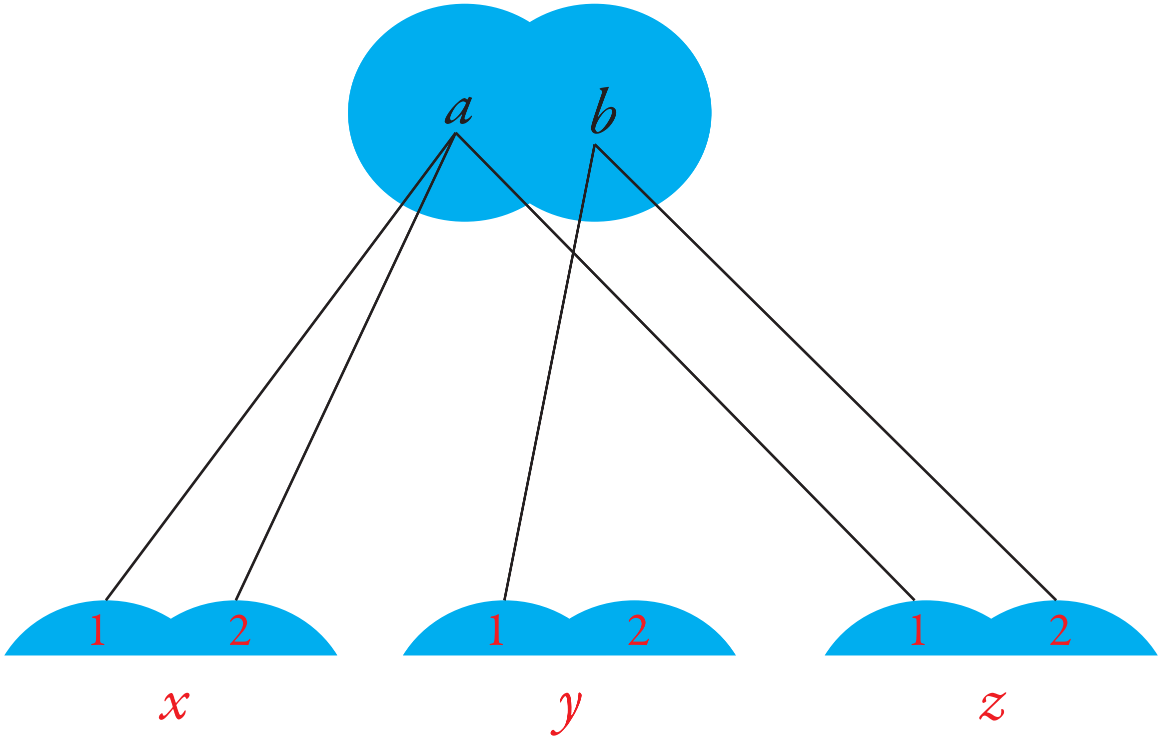
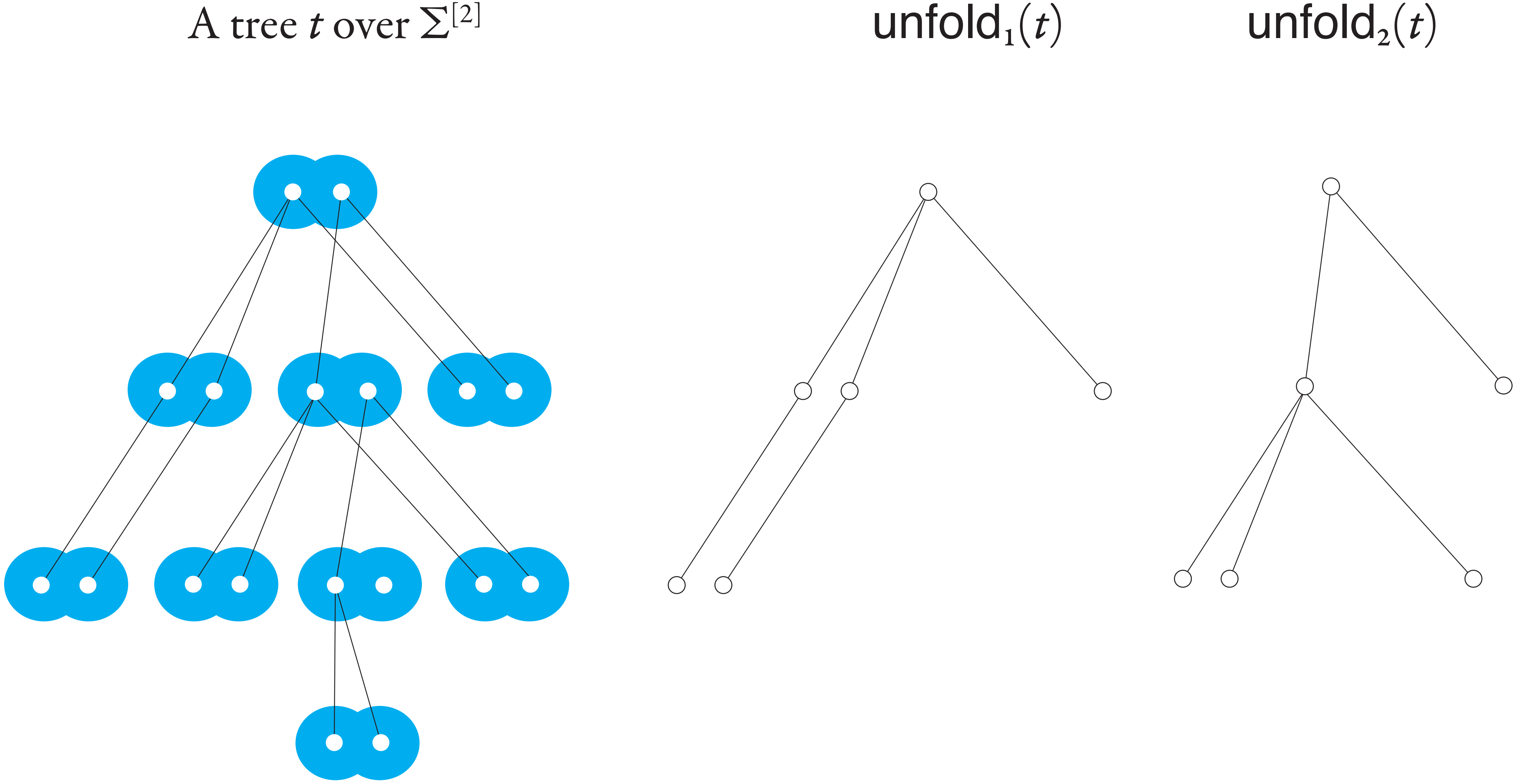
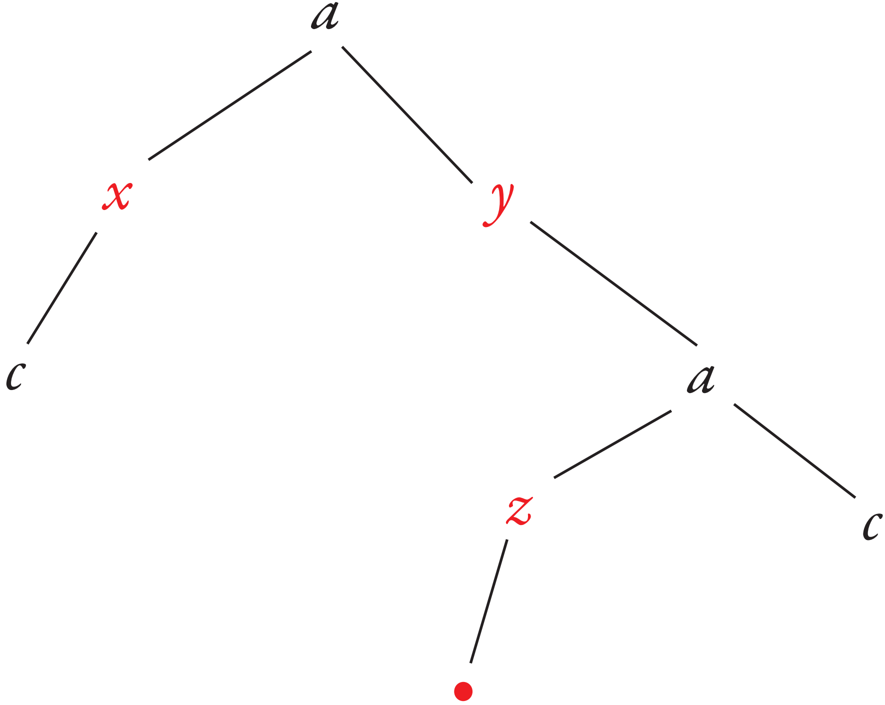
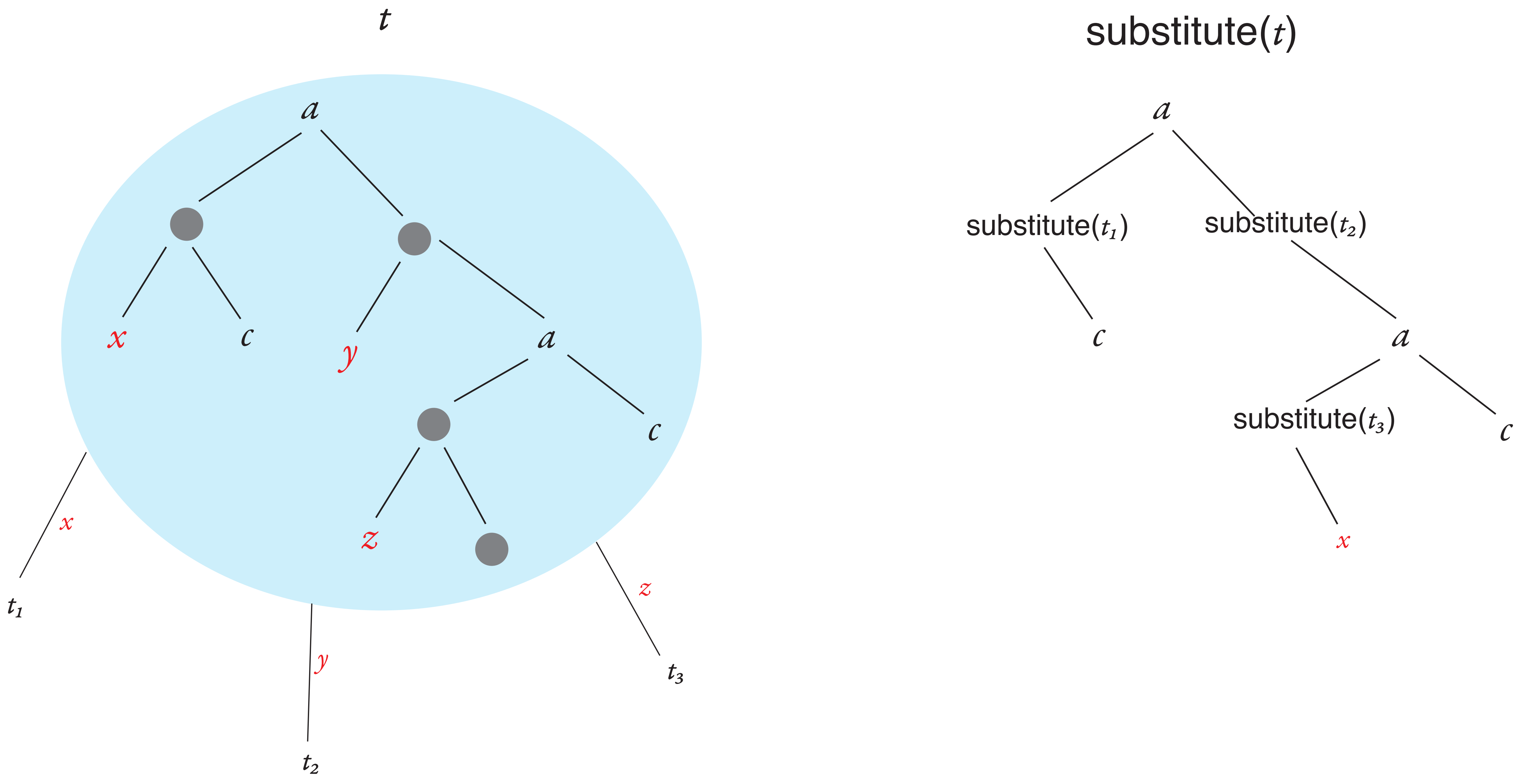

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)


a
b
1
2
1
2
1
2
x
y
z

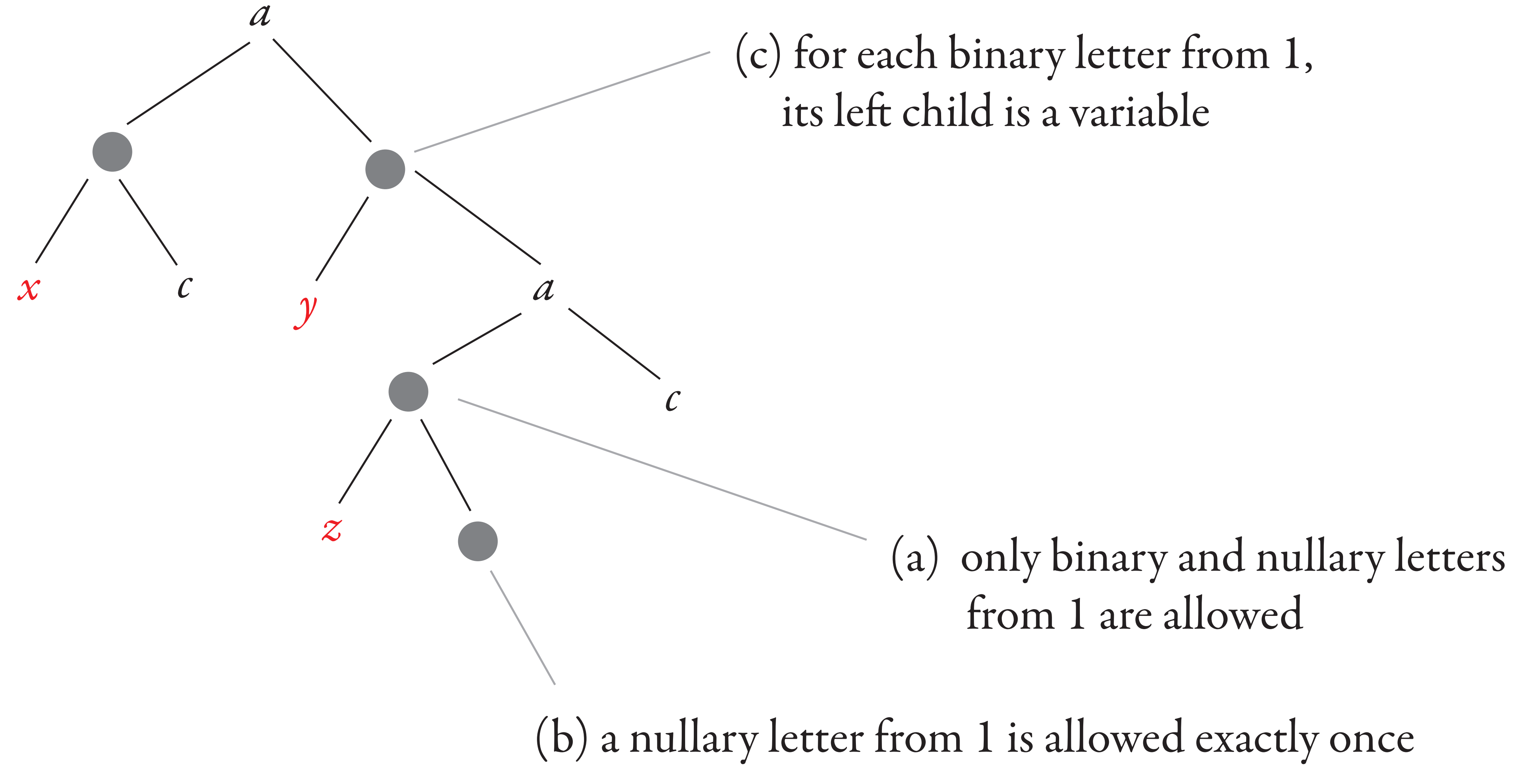


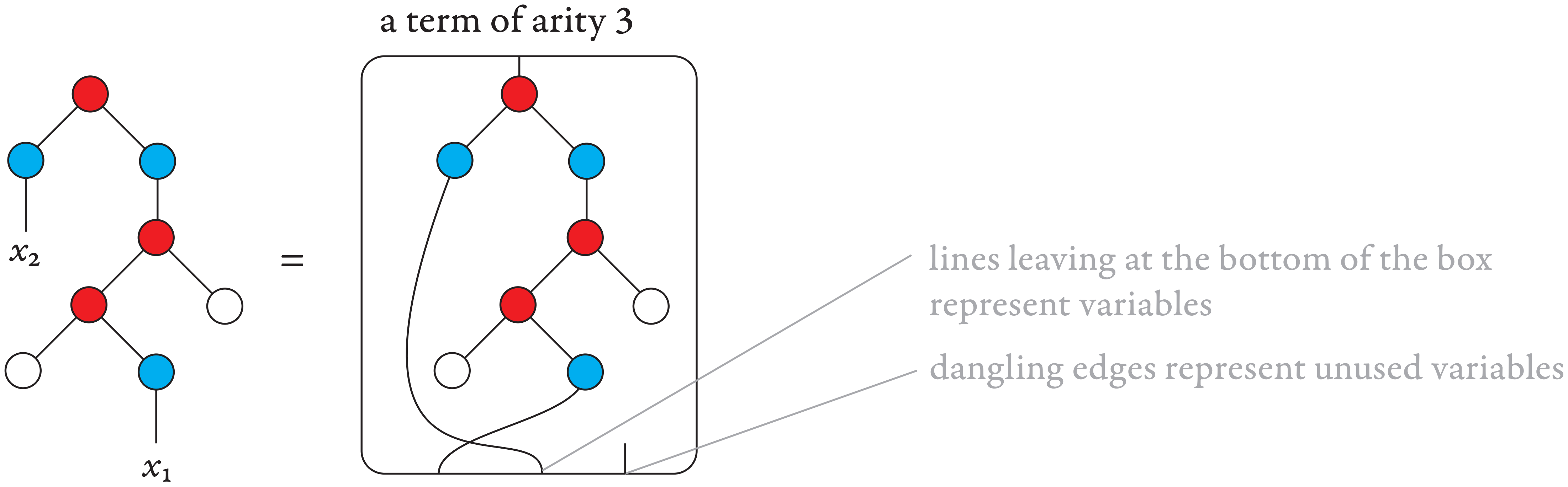
A tree t over $\Sigma^{[2]}$
$\mathsf{unfold}_1(t)$
$\mathsf{unfold}_2(t)$

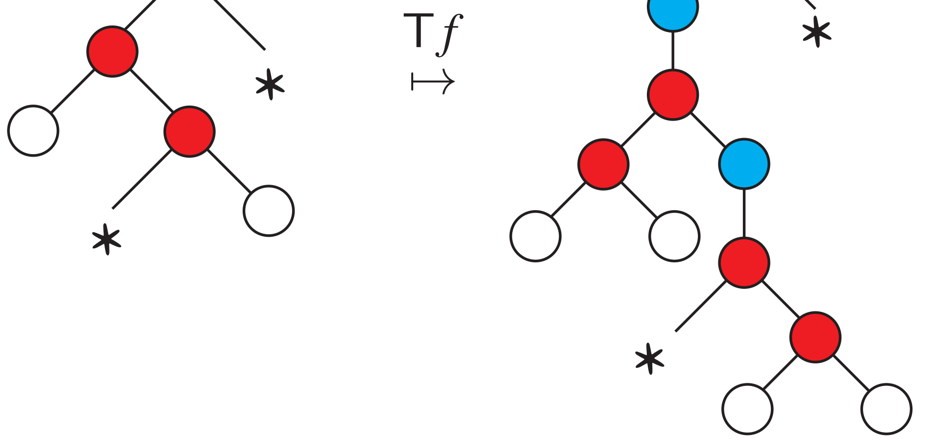

a
x
y
c
a
z
c

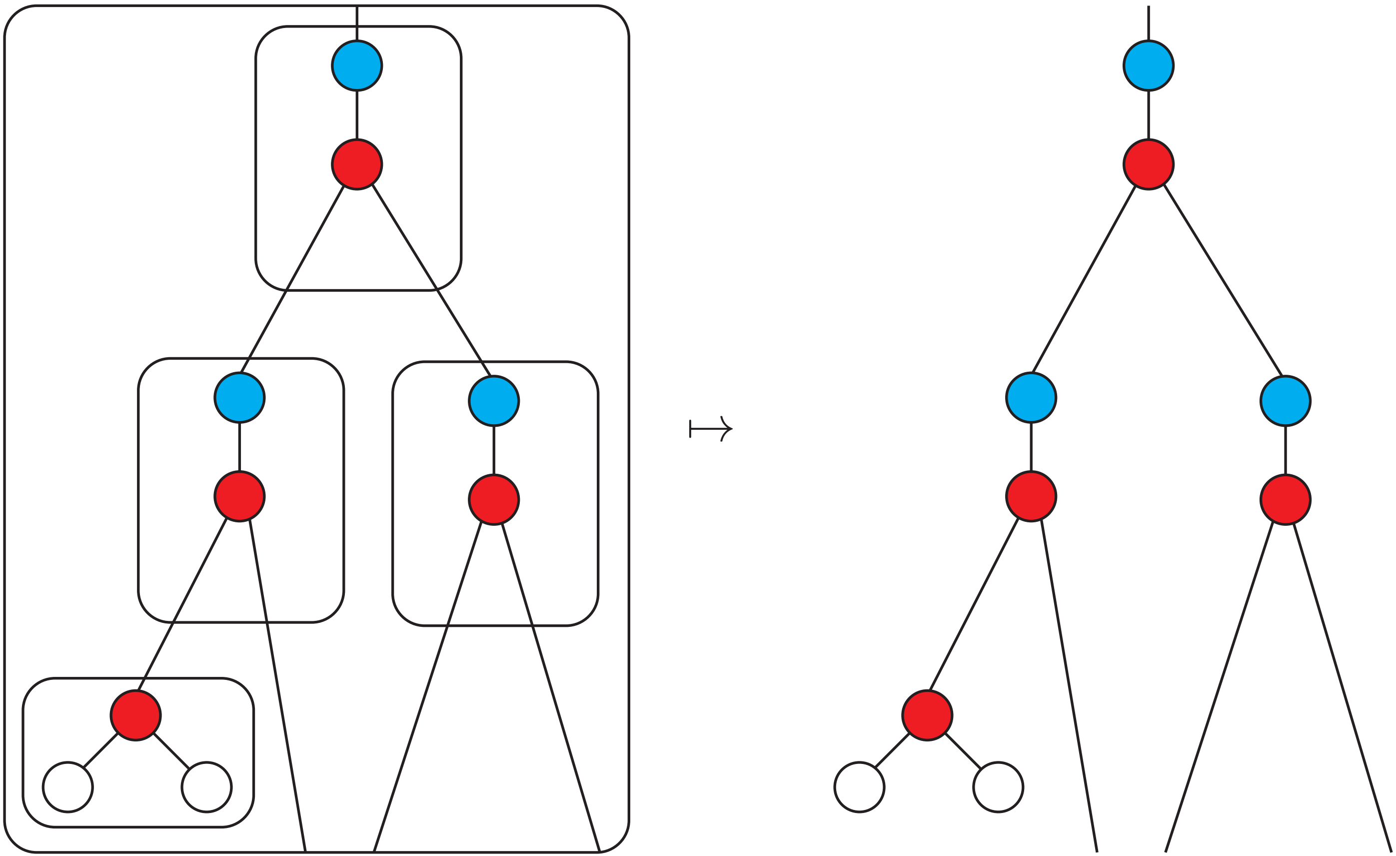


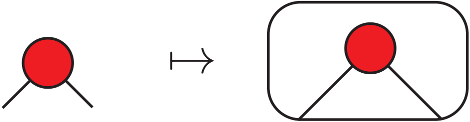
t
a
x
c
y
a
z
c
x
y
z
t1
t2
t3
substitute(t)
a
substitute(t1)
substitute(t2)
c
a
substitute(t3)
c
x

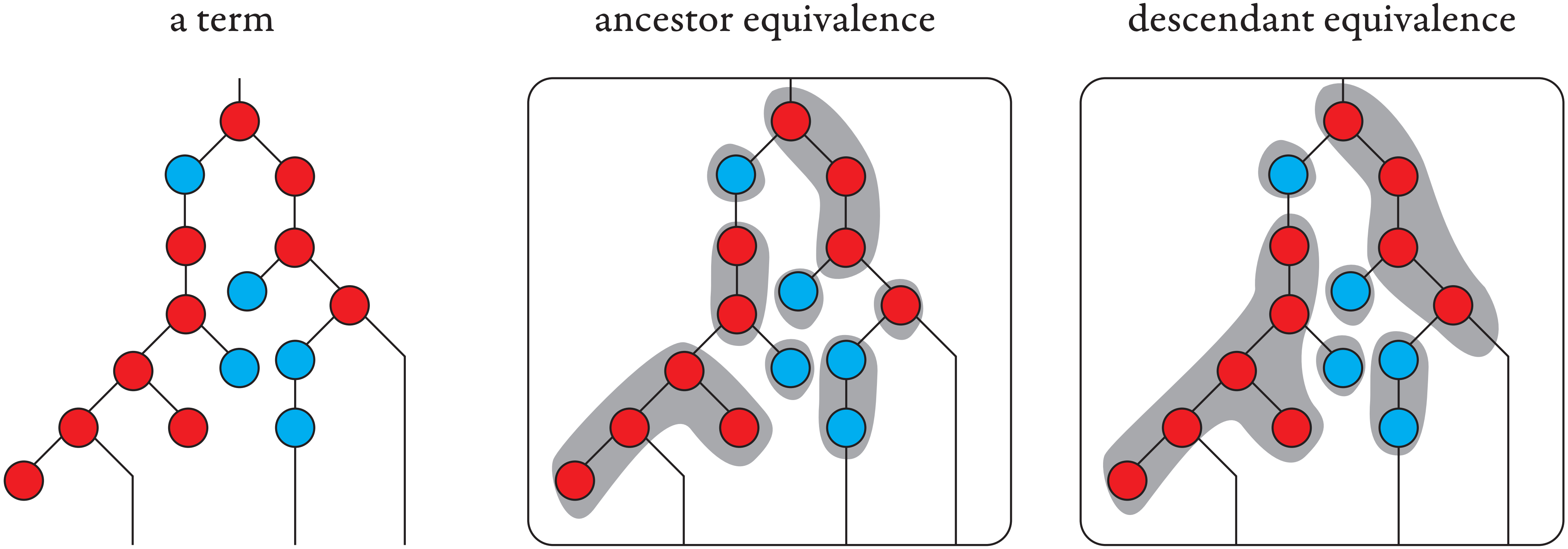

a
(c) for each binary letter from 1,
its left child is a variable
x
c
y
a
c
z
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

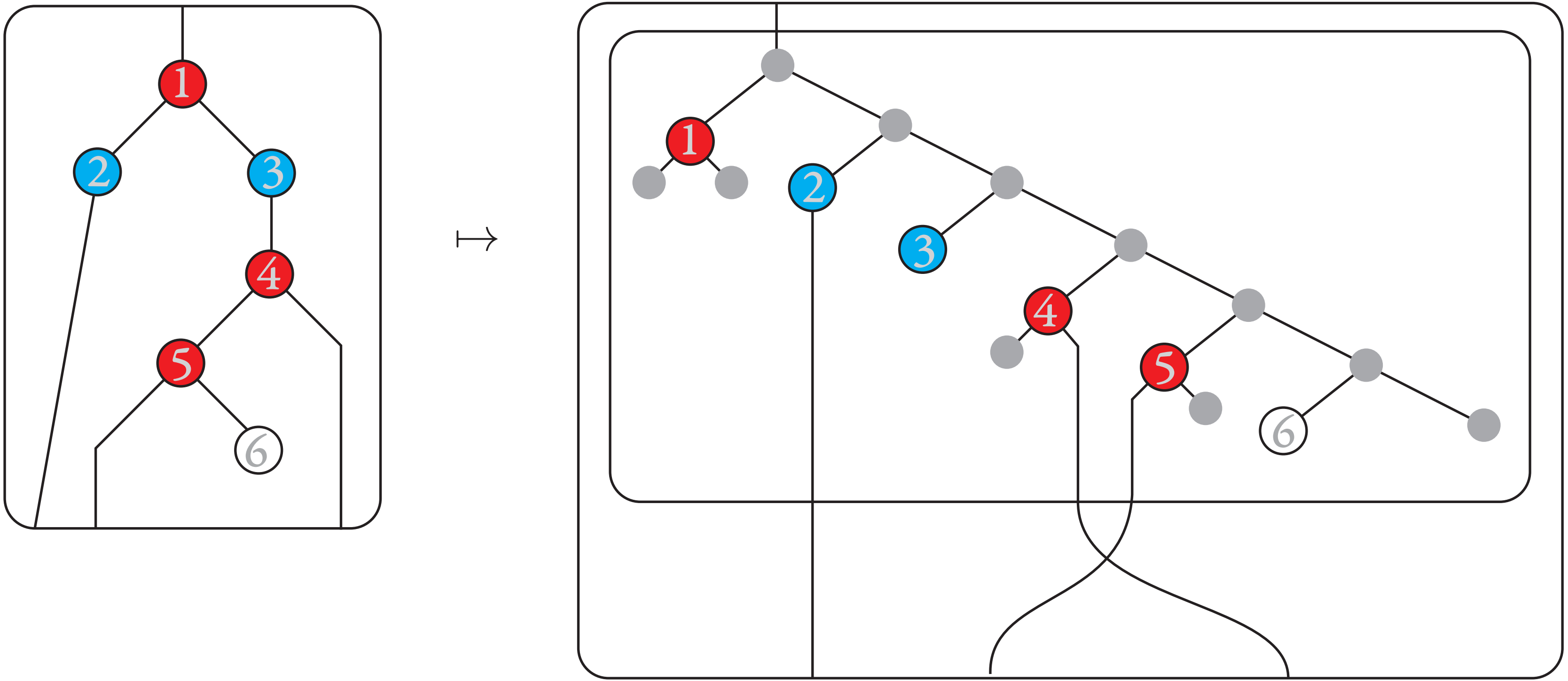

a term of arity 3
x_2
x_1
=
lines leaving at the bottom of the box represent variables
dangling edges represent unused variables


$\mathsf{T}f$
$\mapsto$

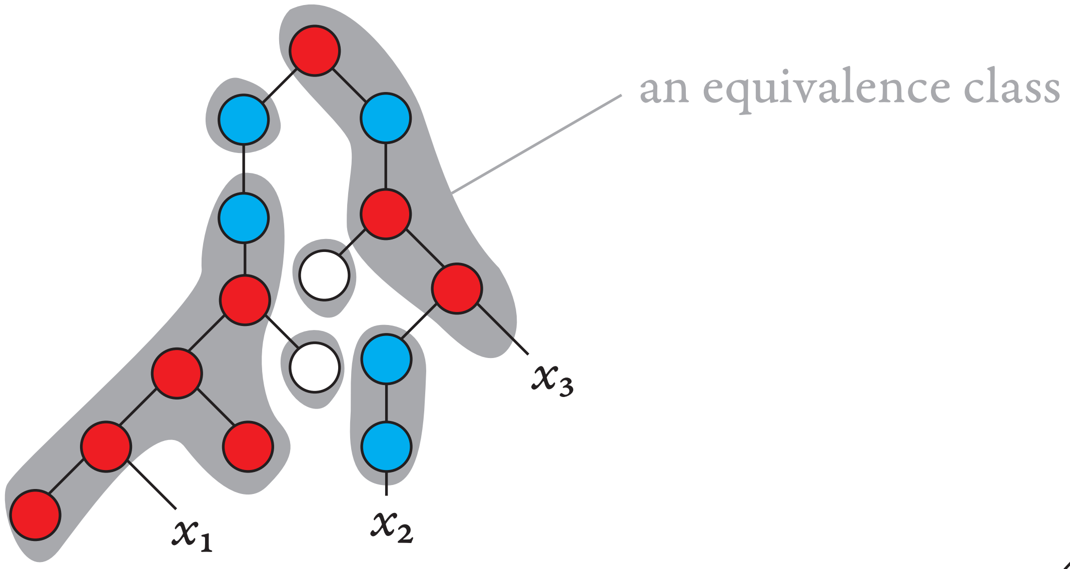


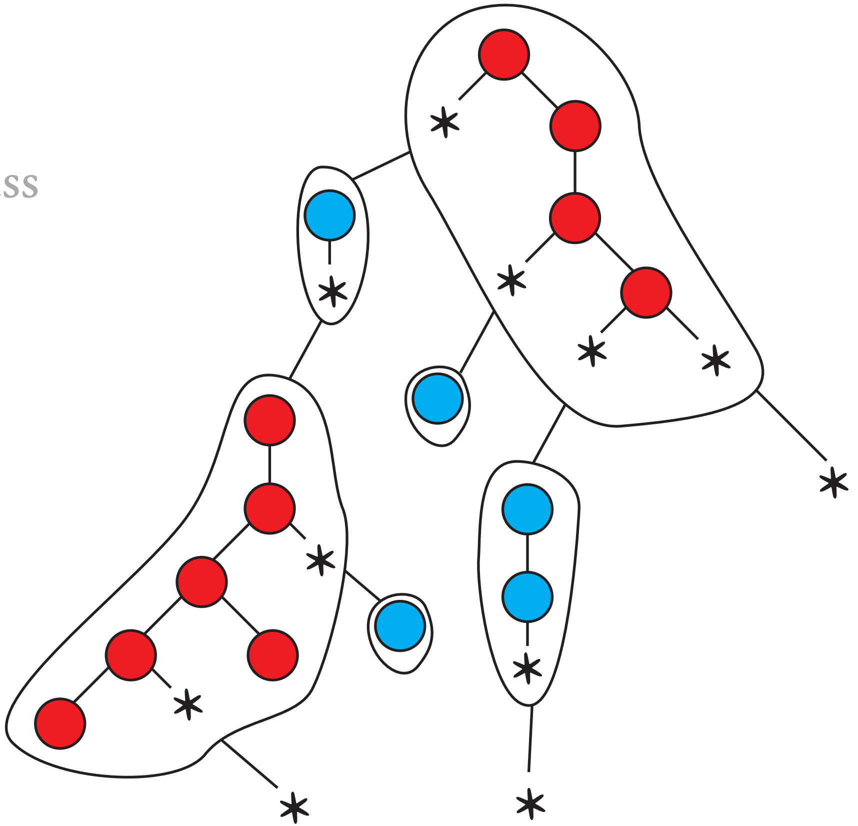


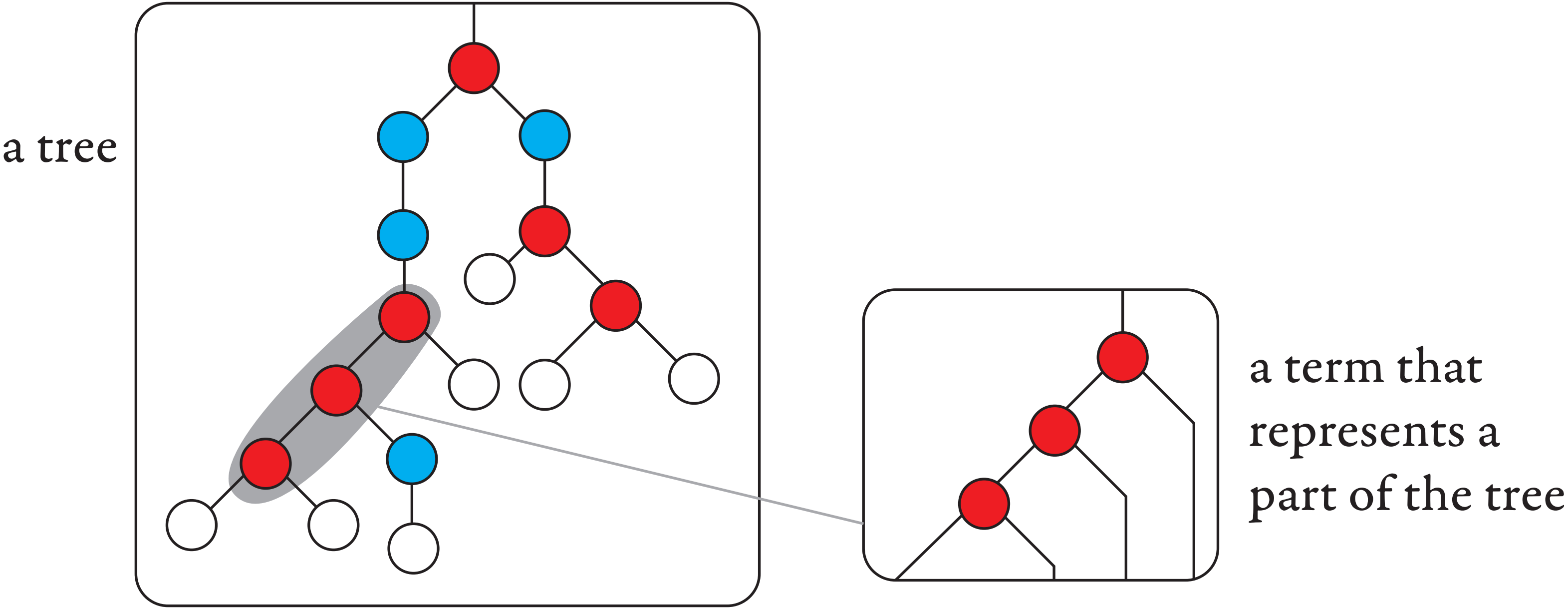

a term
ancestor equivalence
descendant equivalence

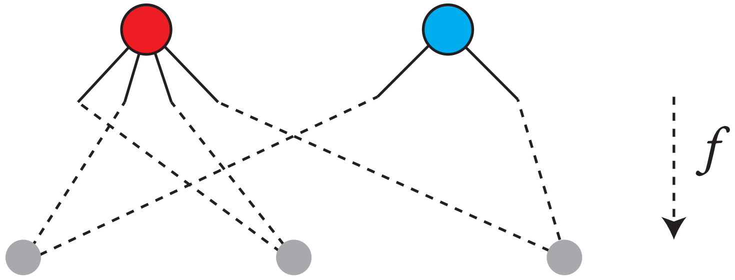

1
2
3
4
5
6
↦
1
2
3
4
5
6

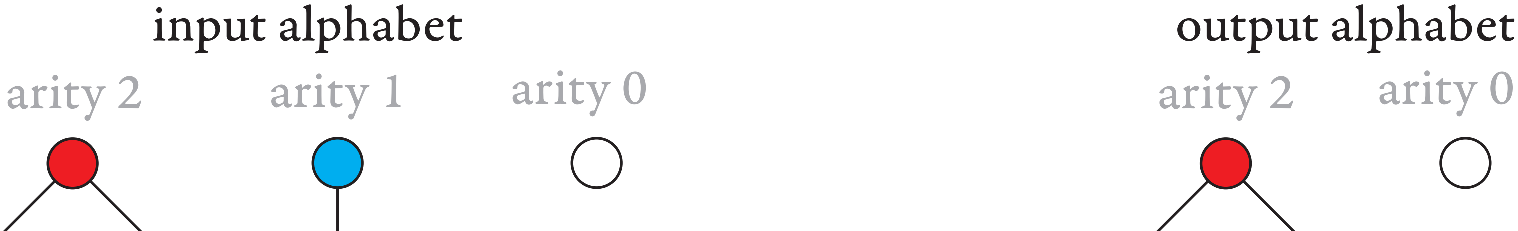
a factorisation equivalence

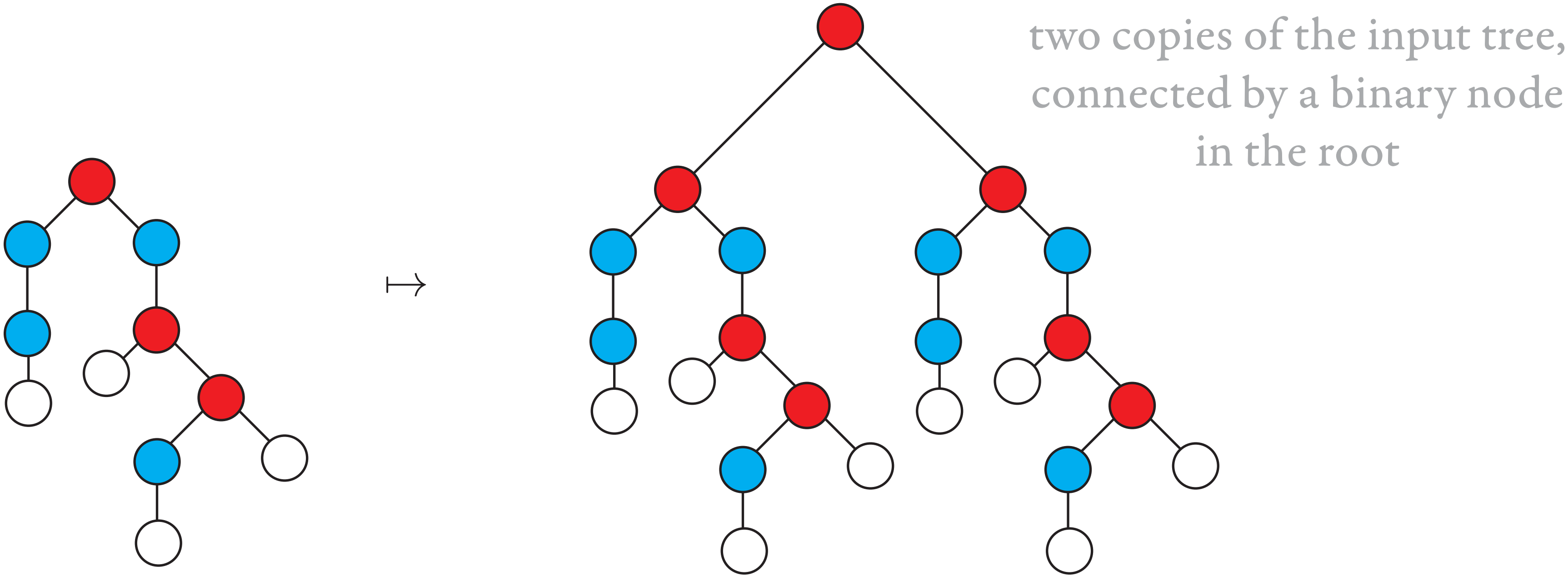


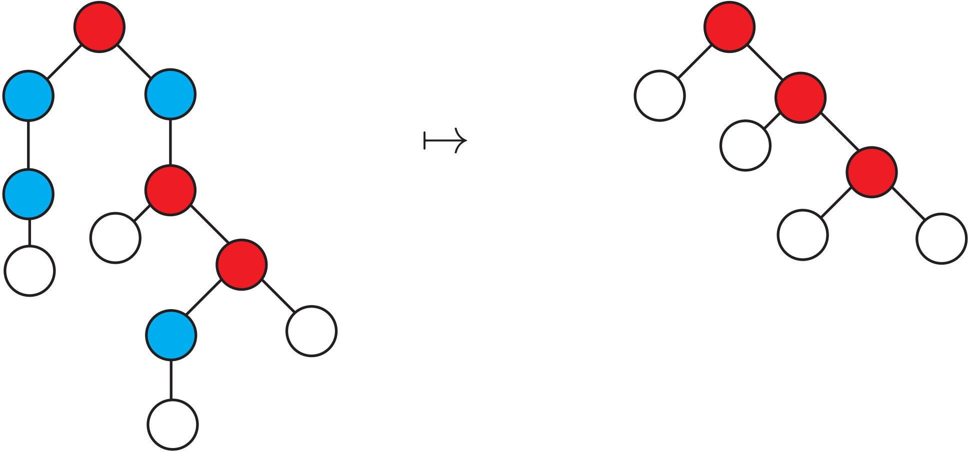


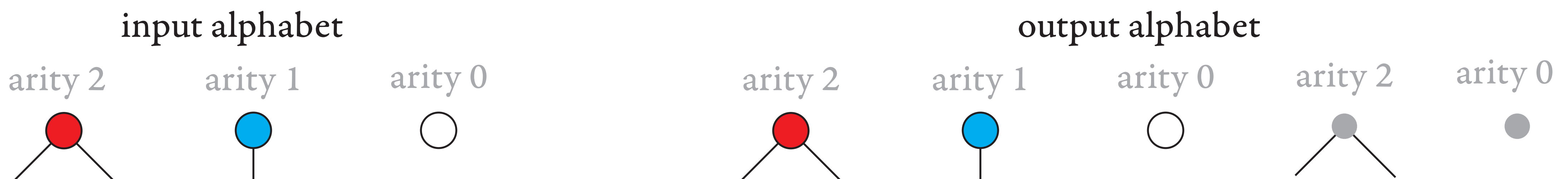

a tree
a term that
represents a
part of the tree

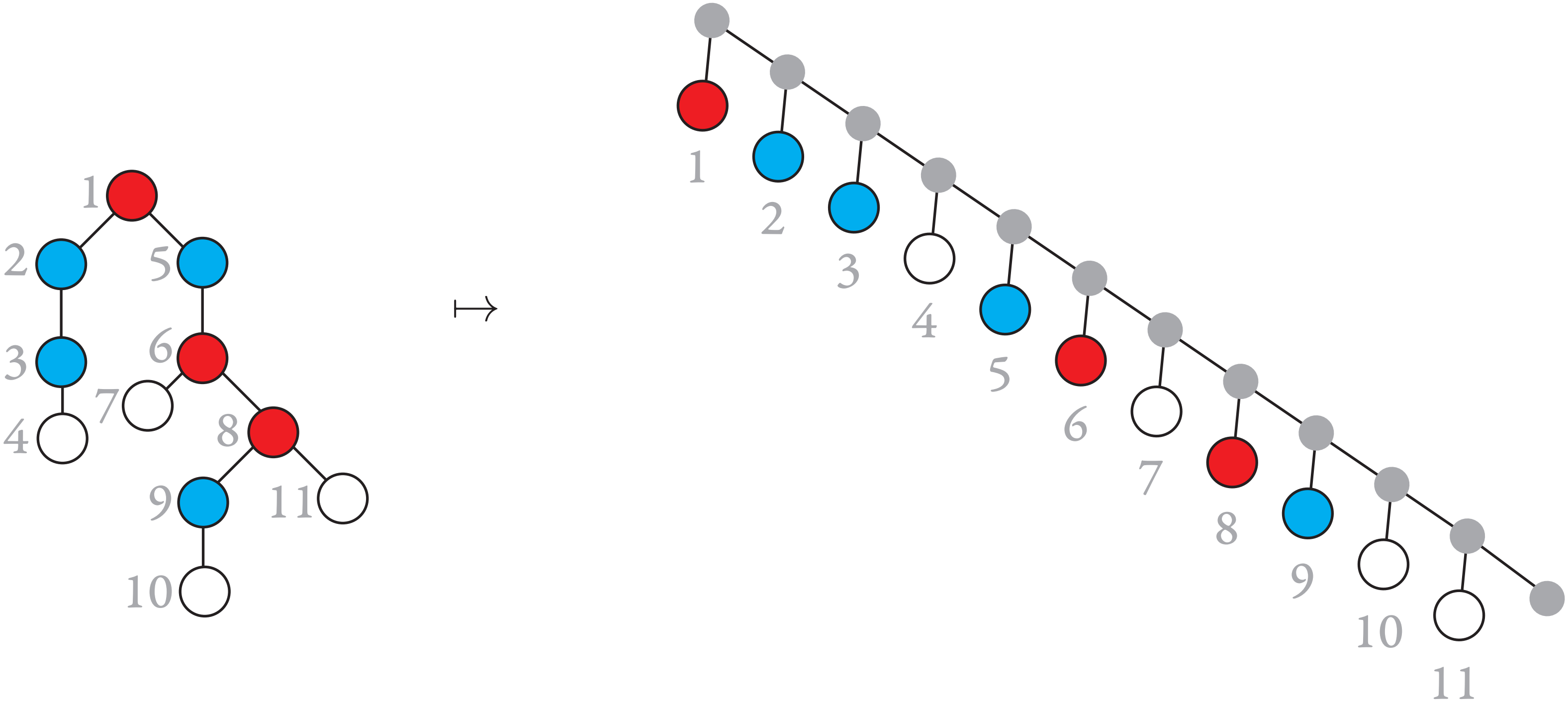

f


input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0


two copies of the input tree,
connected by a binary node
in the root

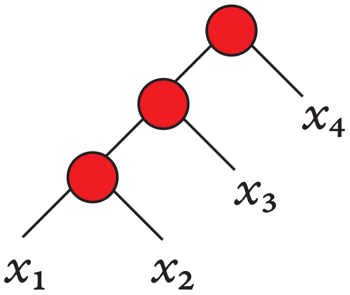



input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 0

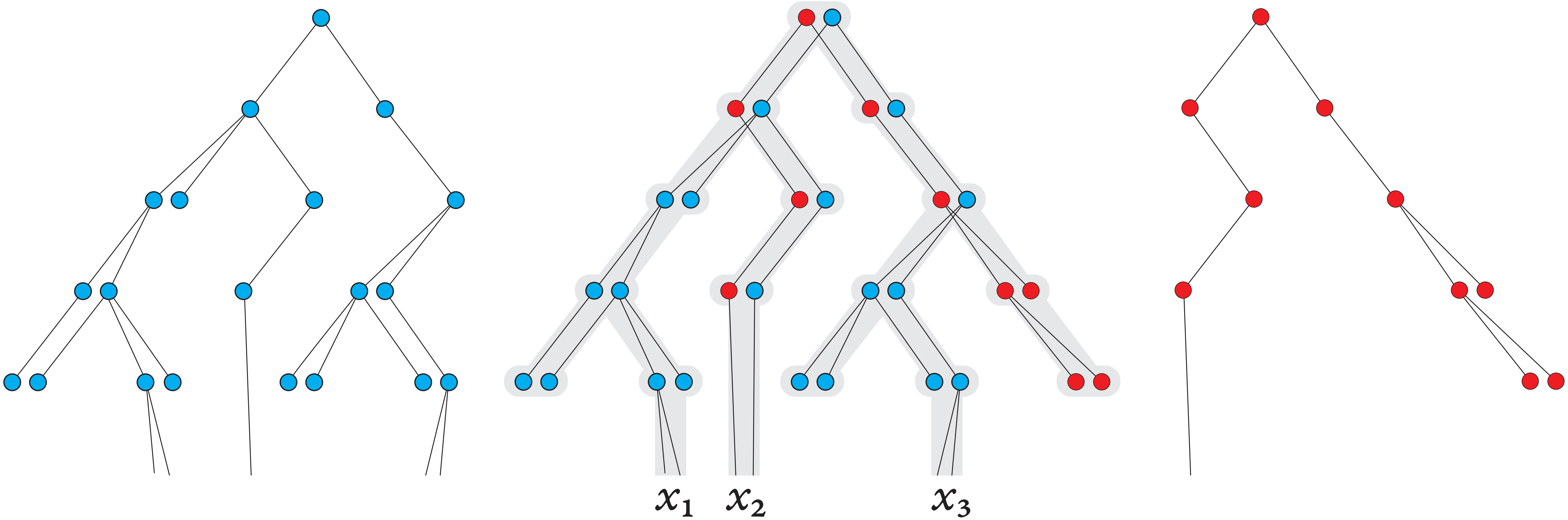


1
2
5
3
6
7
4
8
9
11
10
↦
1
2
3
4
5
6
7
8
9
10
11

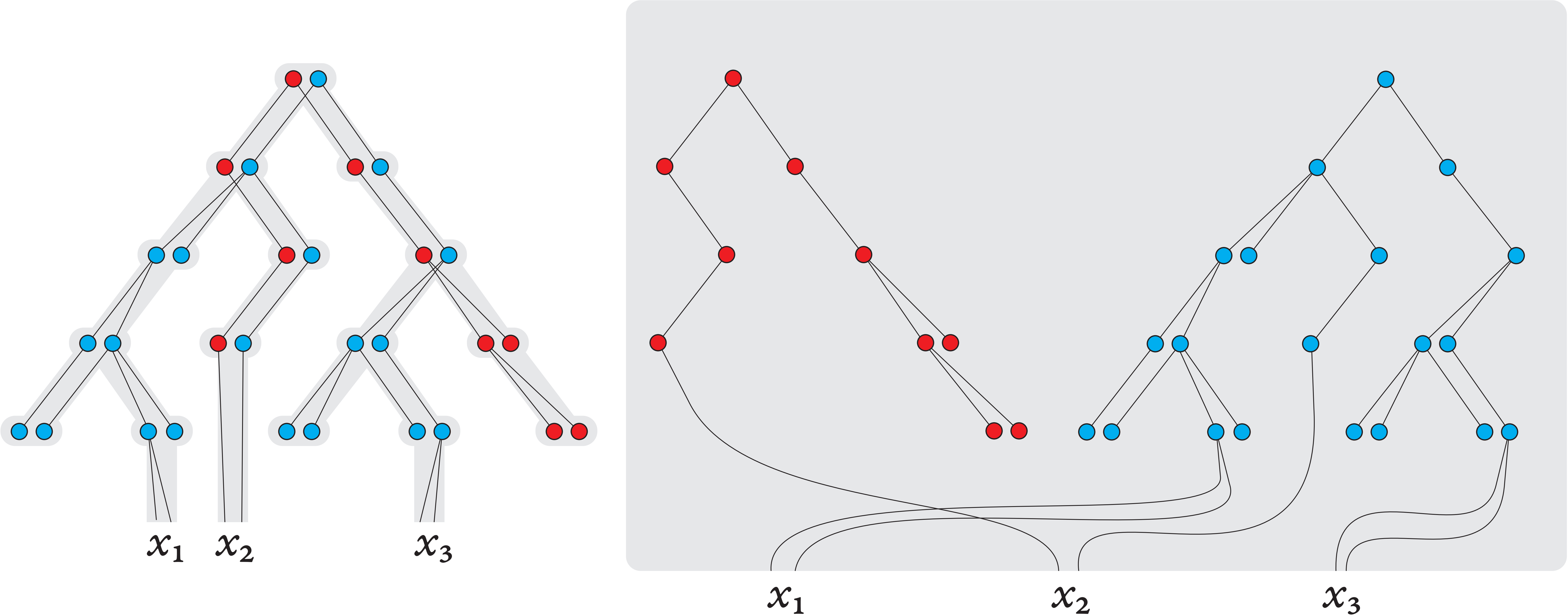
a term of arity 4

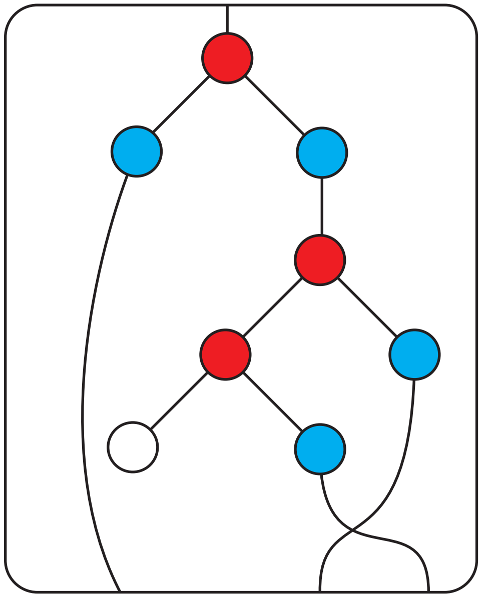


a term of arity 0



x_1
x_2
x_3

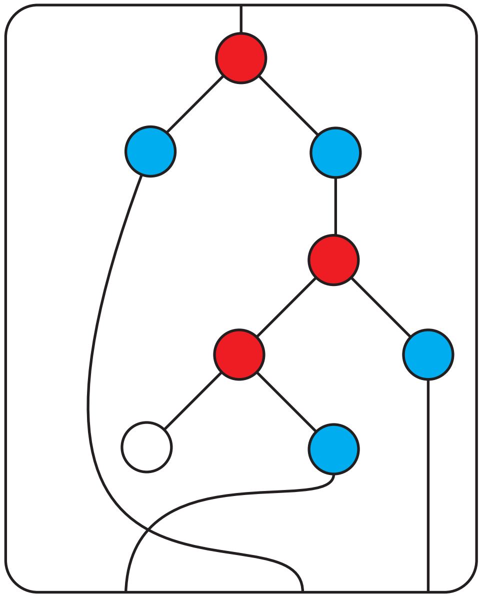

x_1 x_2 x_3
x_1 x_2 x_3



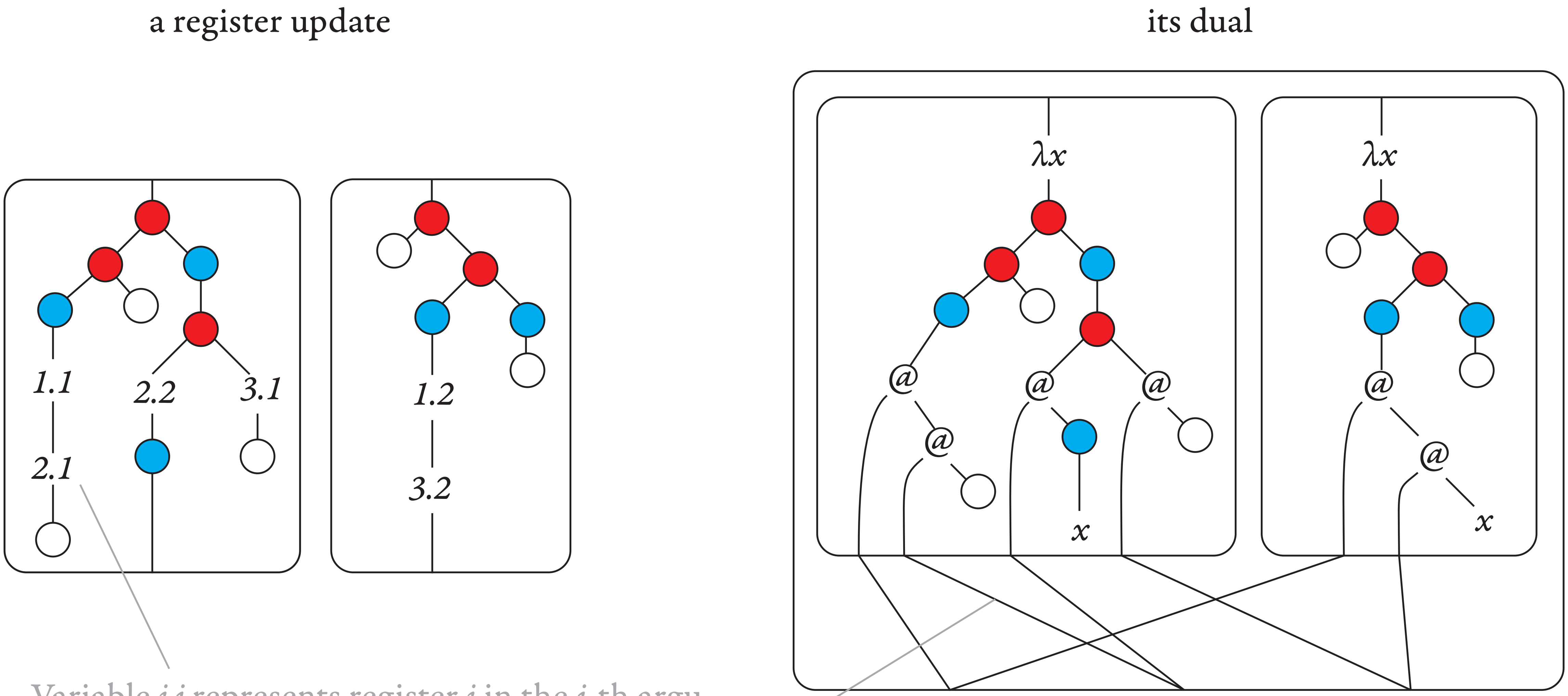
satisfies (*)

(*)

If the root has arity n, and $1 \leq i < j \leq n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root

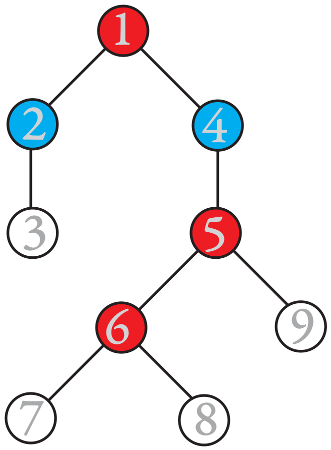


violates (*)

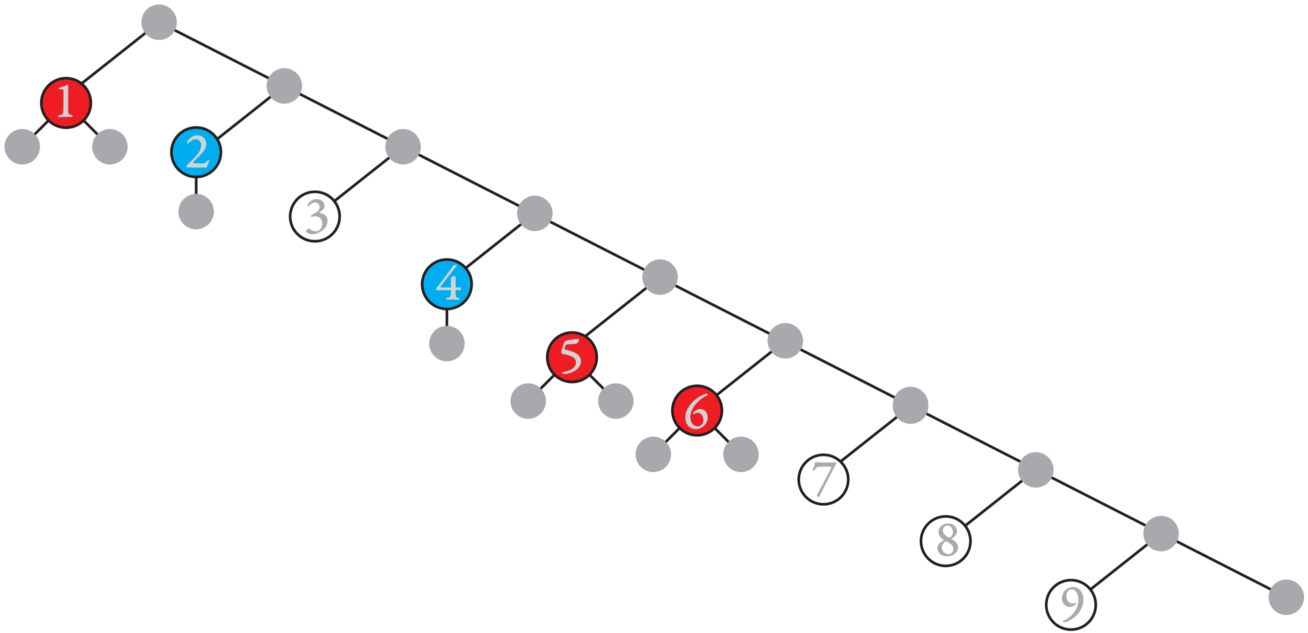


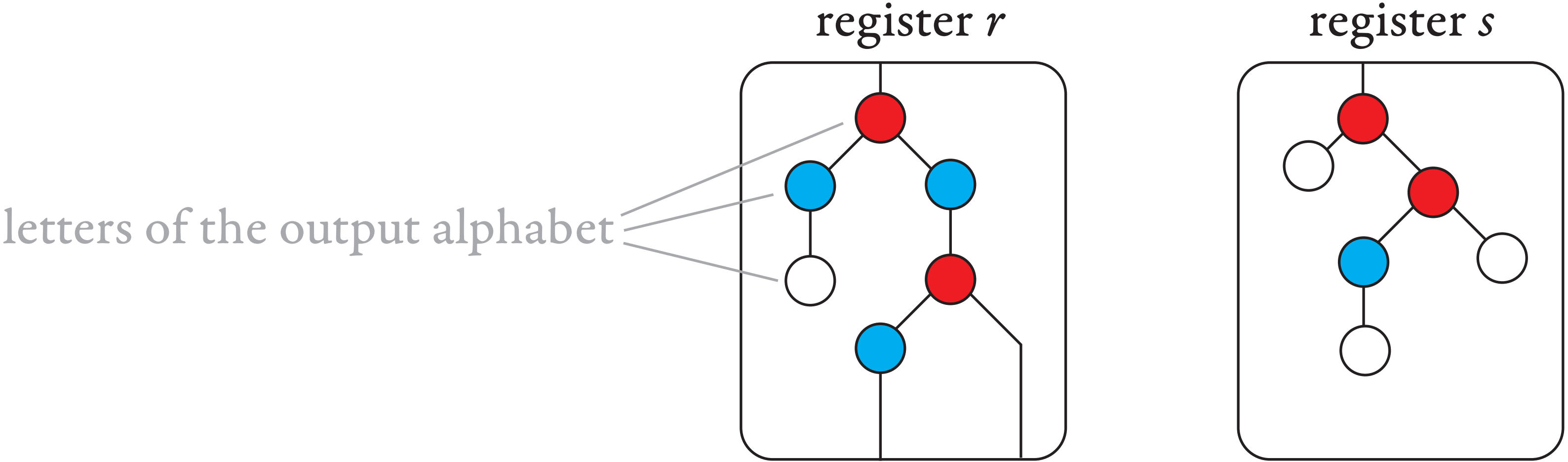
Variable *i.j* represents register *i* in the *j*-th argument of the reigster update.
In the dual, this variable is mapped to the *i*-th edge which enters the *j*-th port of the reducer.

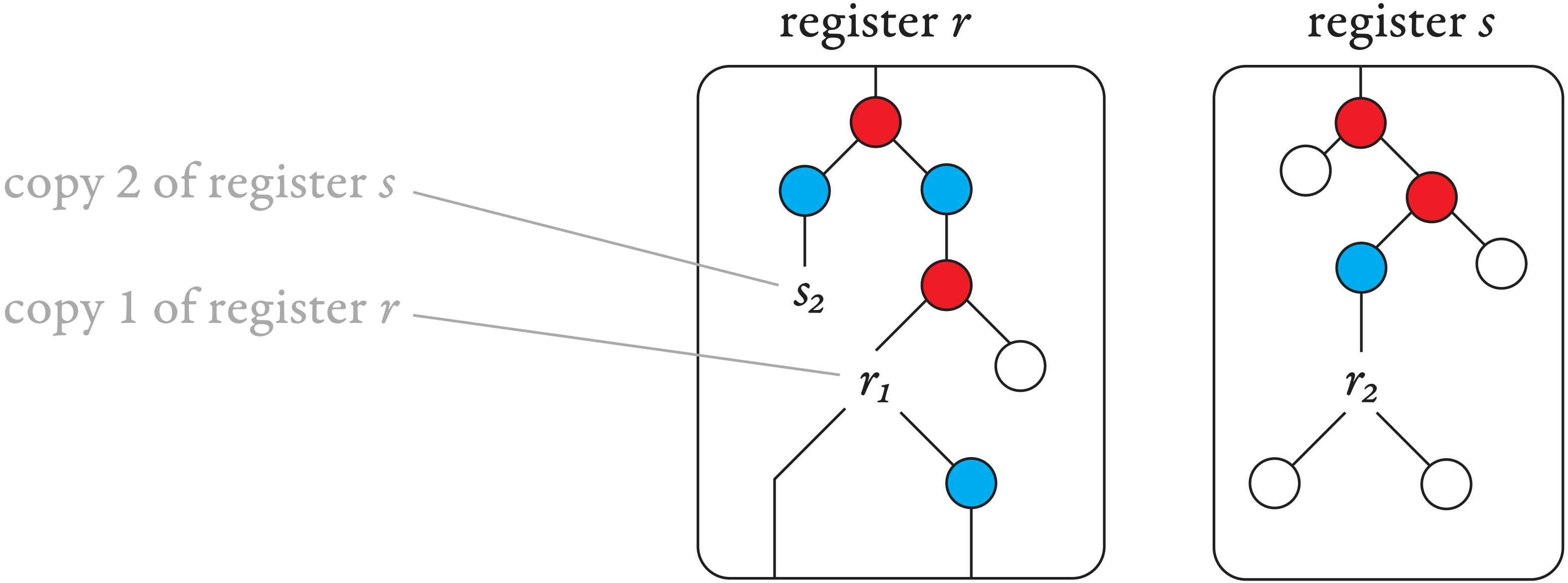
input



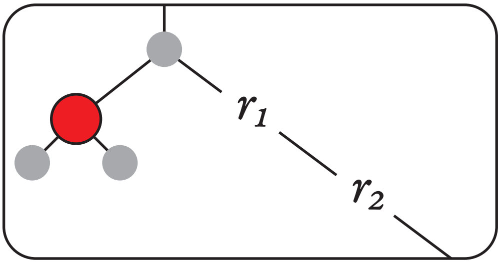
output



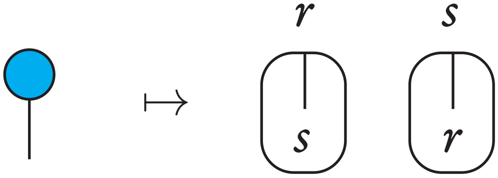

register *r*
register *s*
letters of the output alphabet

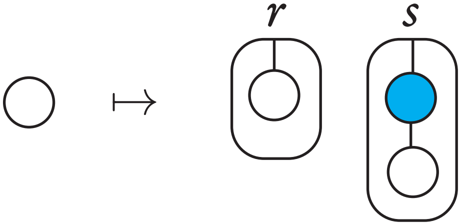

register r
register s
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2

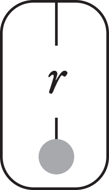

r
r
r
s

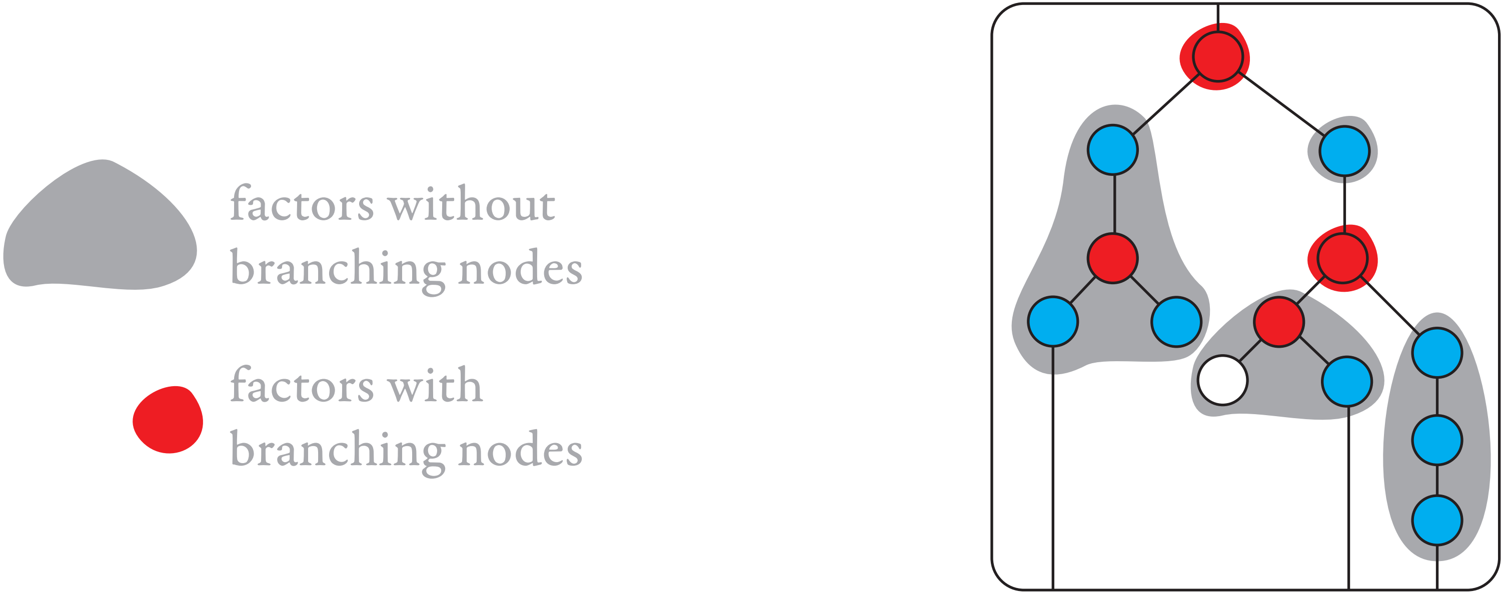


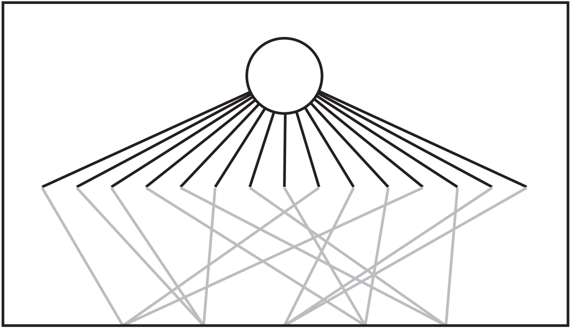
r_1
r_2


r
s
s
r

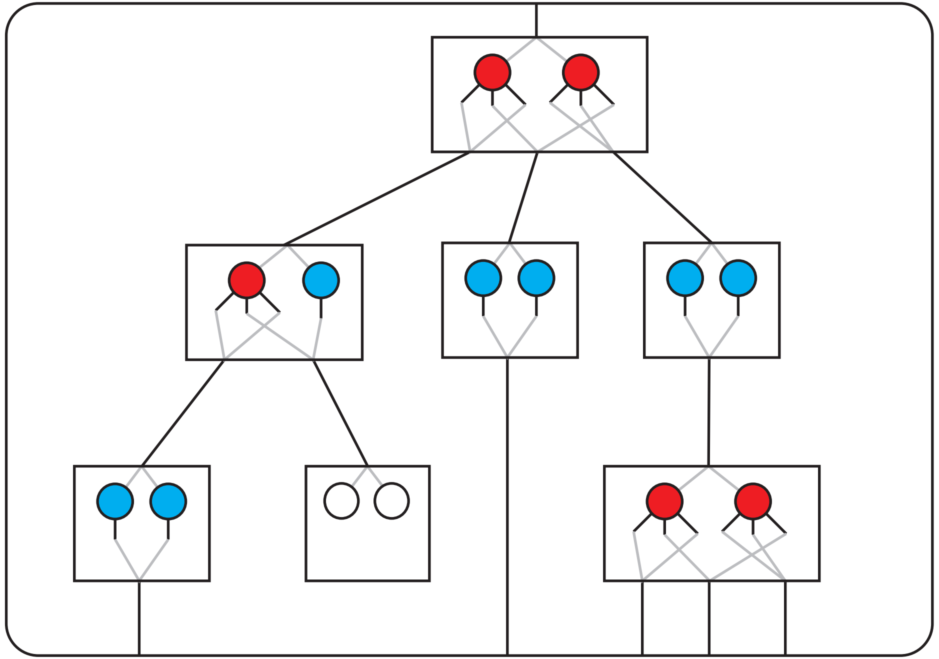

r
s


r

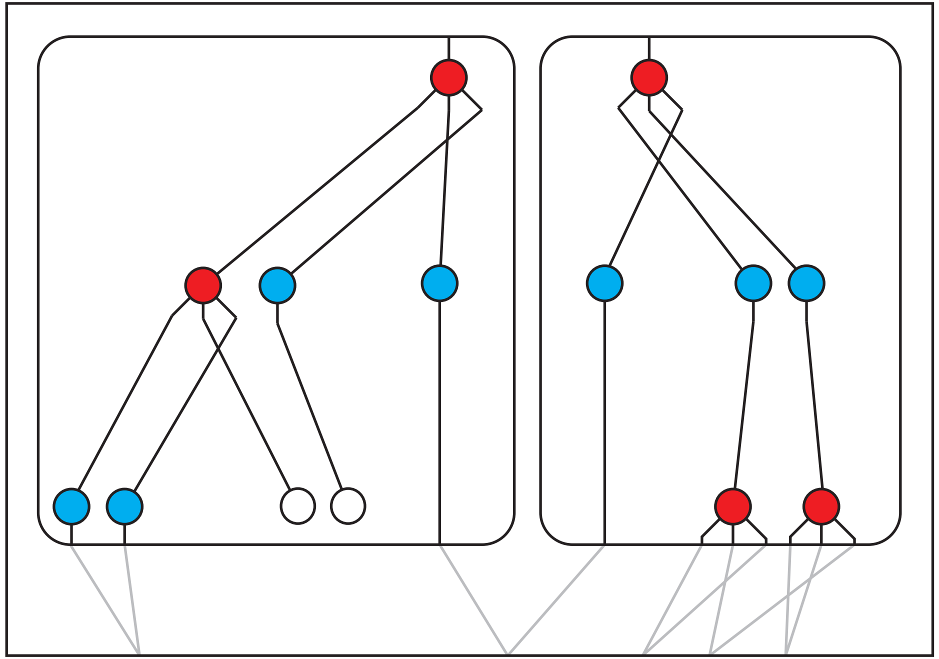

factors without
branching nodes
factors with
branching nodes

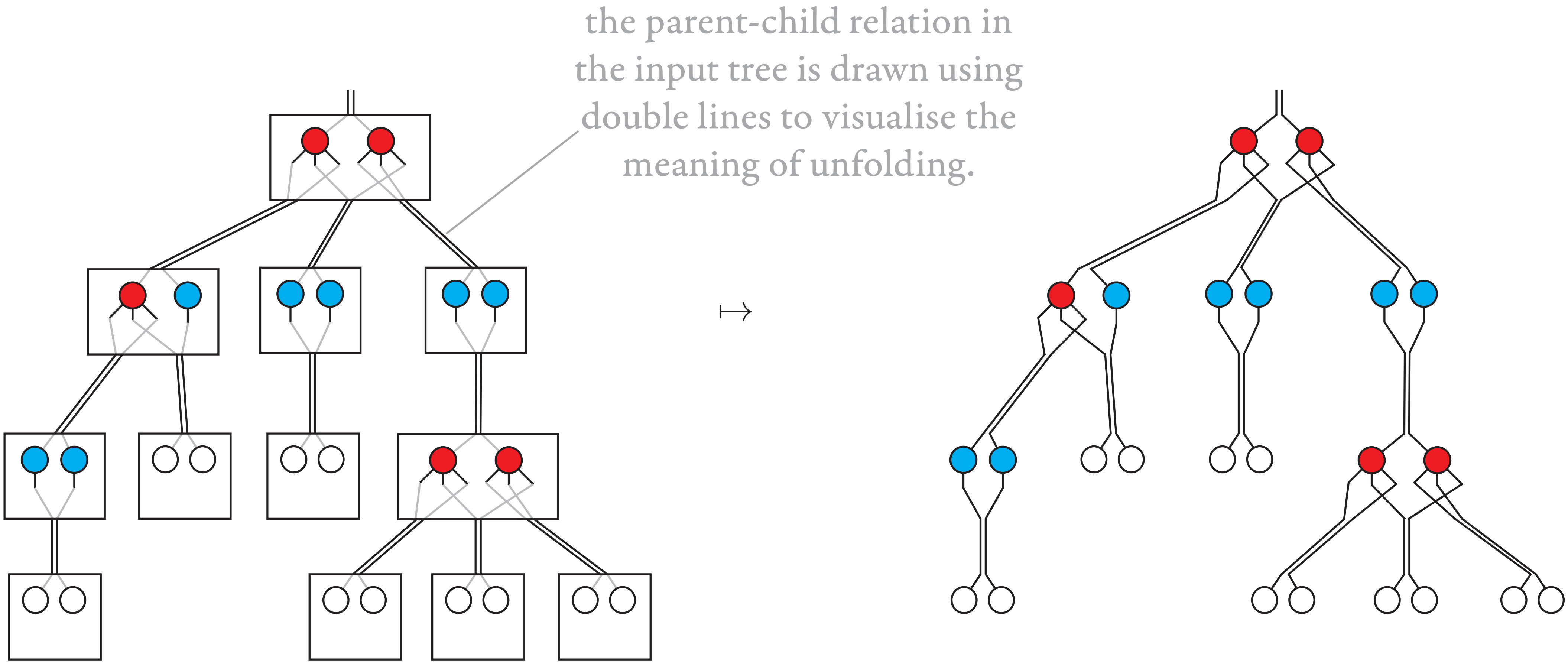


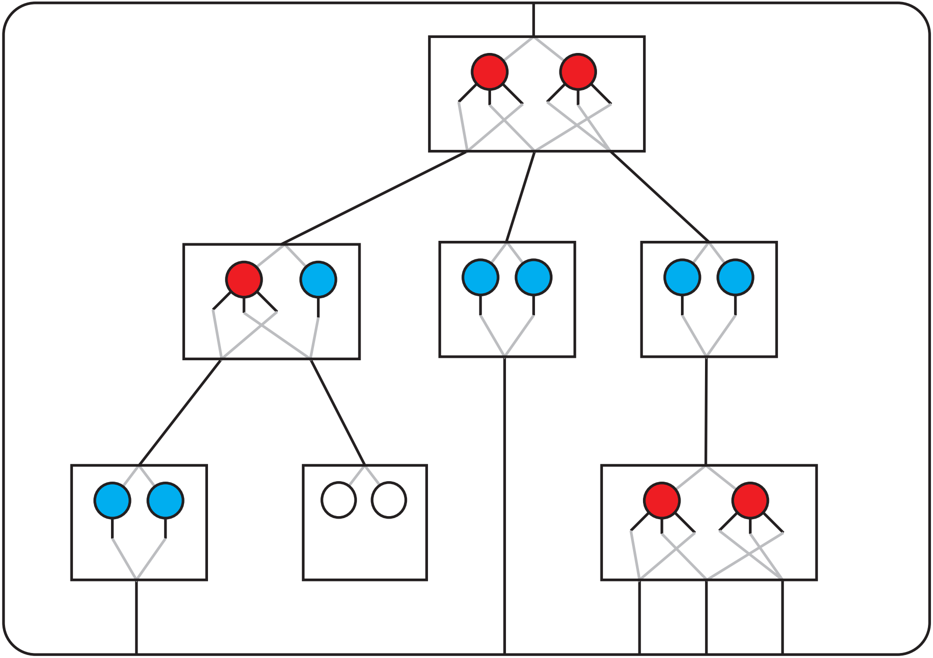
input

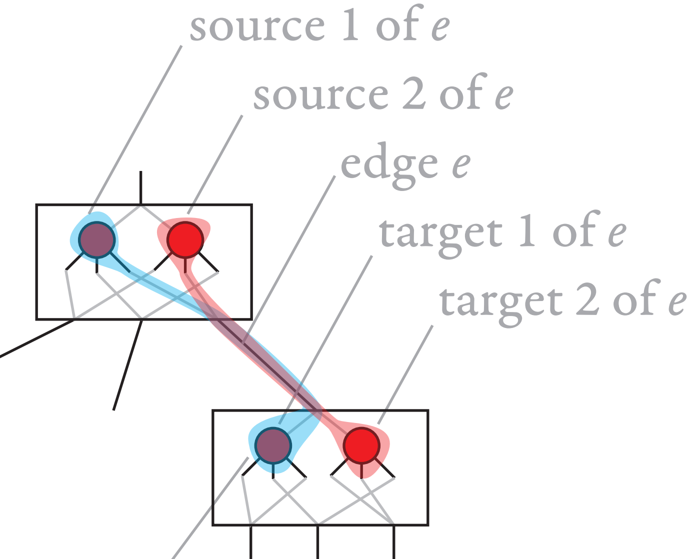


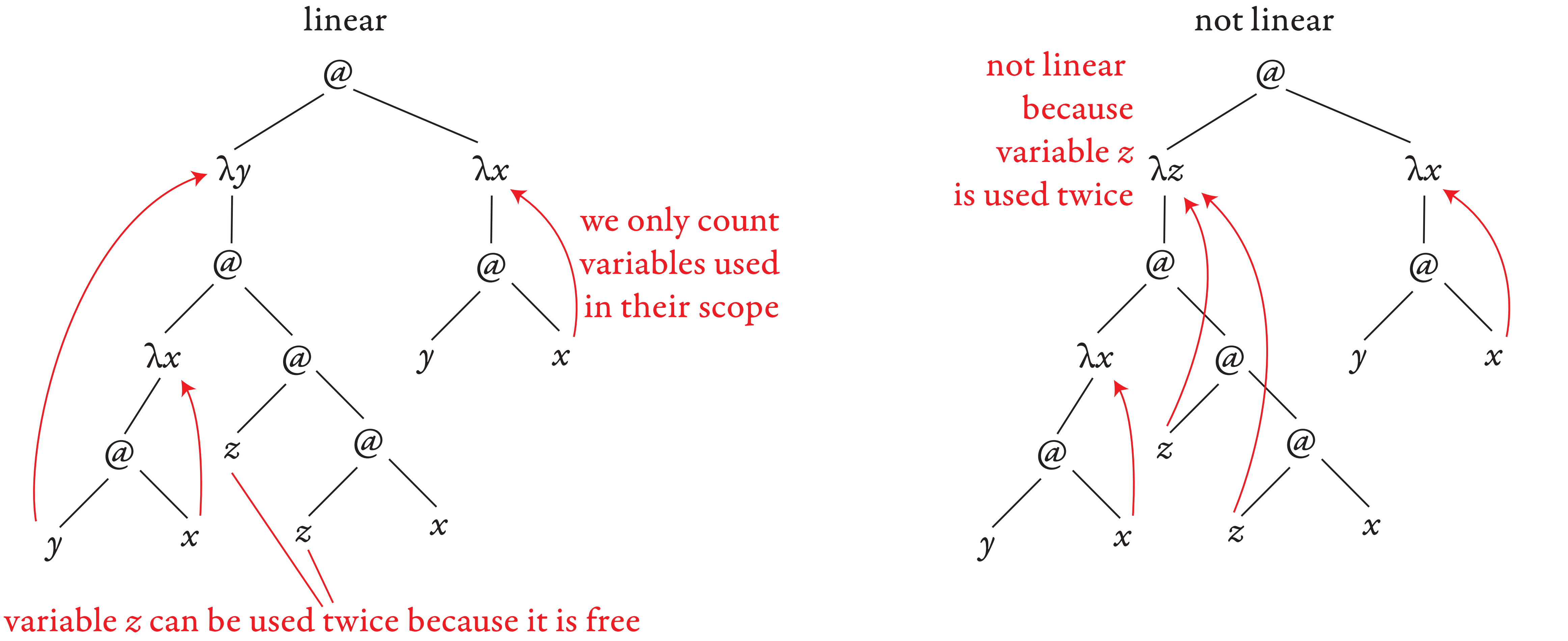
output



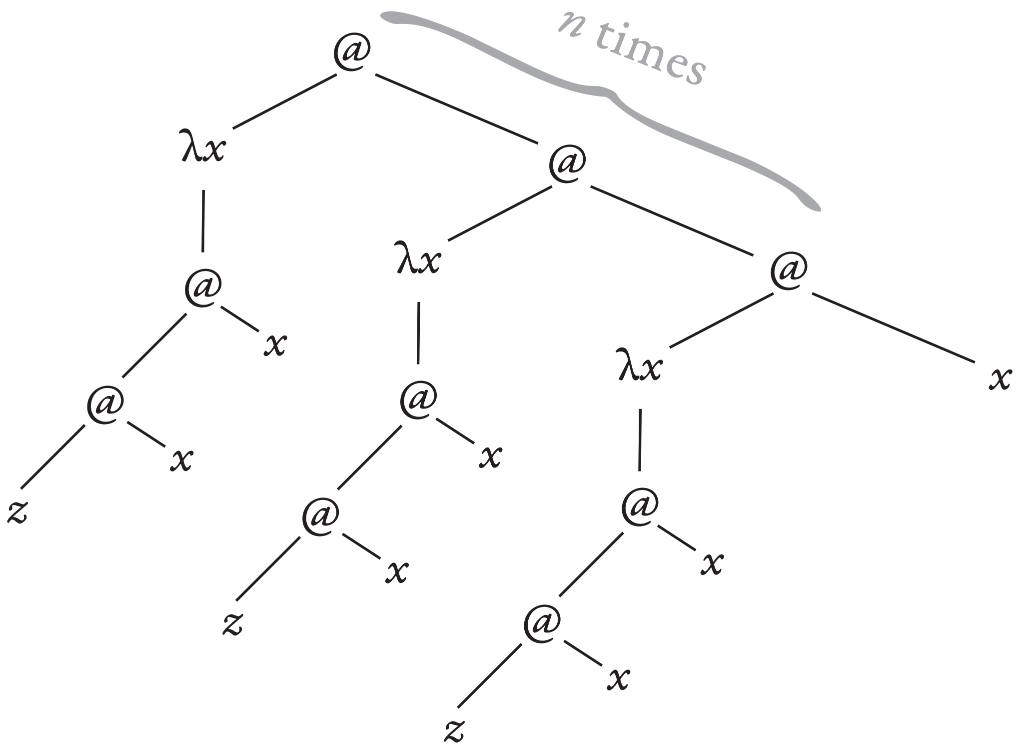

the parent-child relation in
the input tree is drawn using
double lines to visualise the
meaning of unfolding.
↦



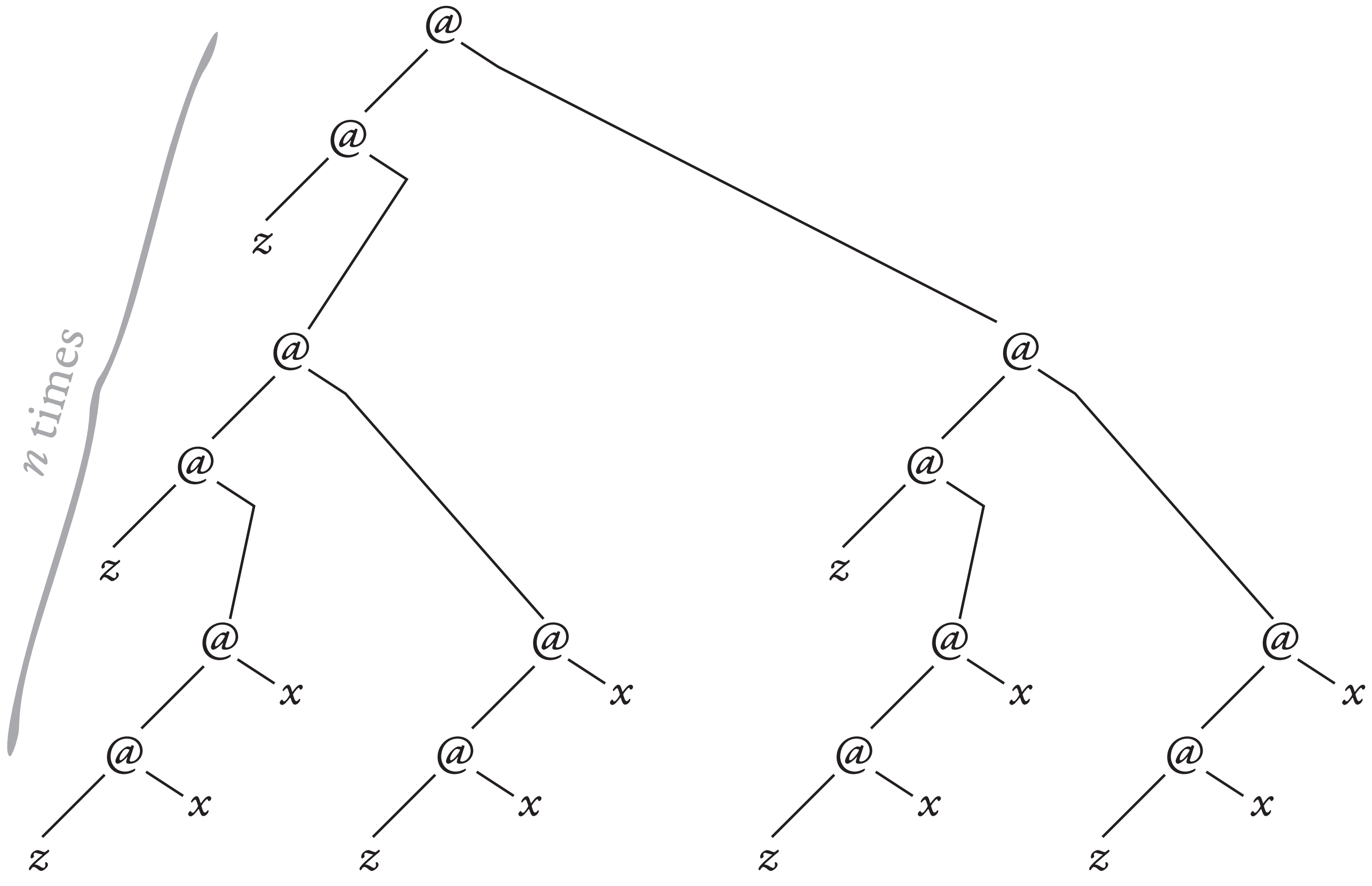

source 1 of *e*
source 2 of *e*
edge *e*
target 1 of *e*
target 2 of *e*


linear
not linear
not linear because variable z is used twice
we only count variables used in their scope
variable z can be used twice because it is free

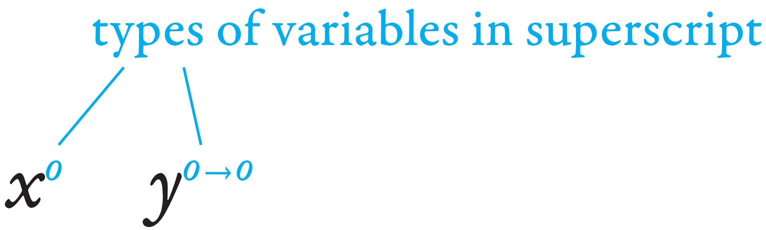
a λ-term of size $O(n)$



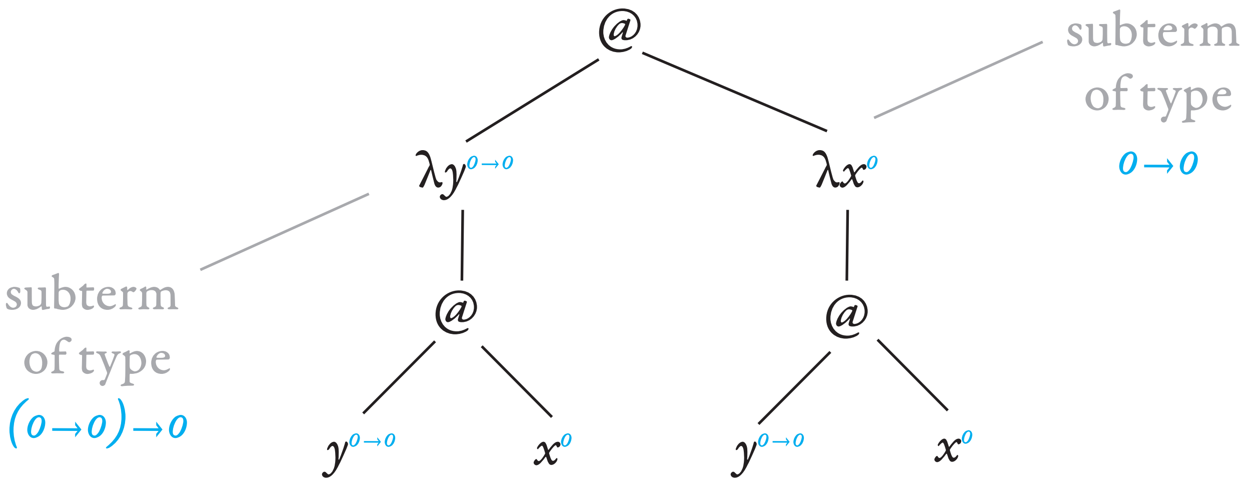
its normal form of size $O(2^n)$



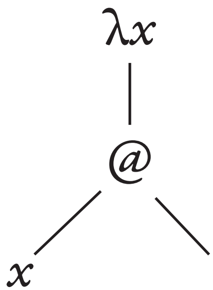
variables



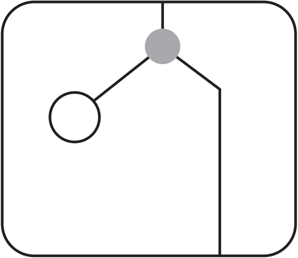
λ-term of type o

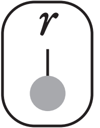


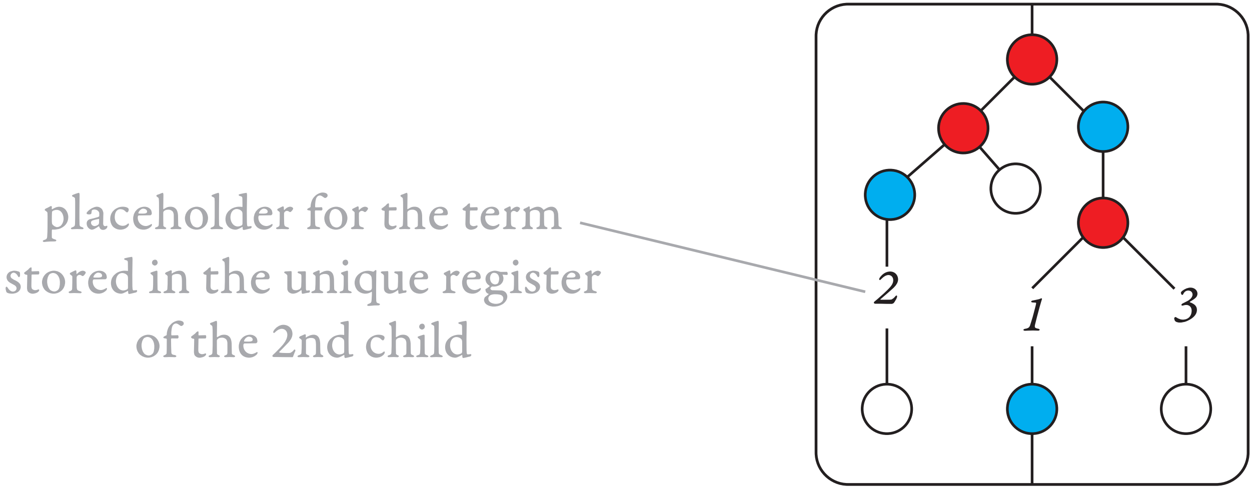
@

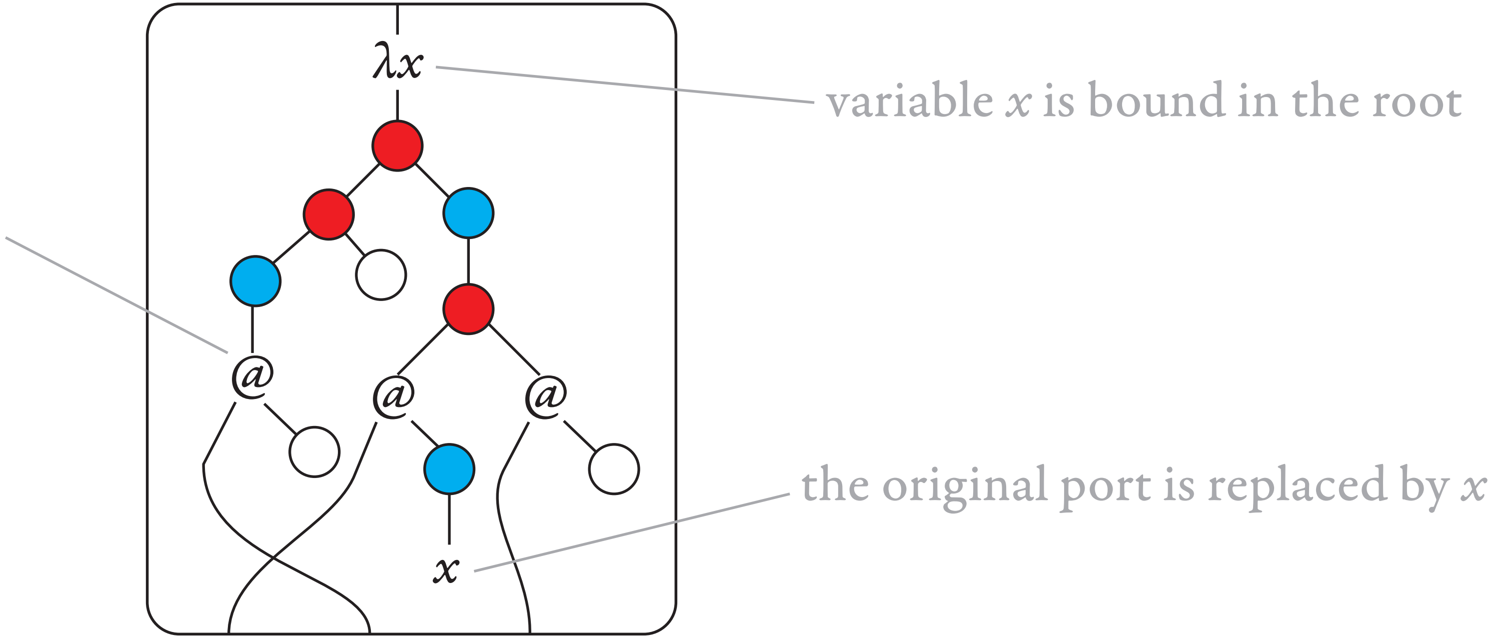


λx.

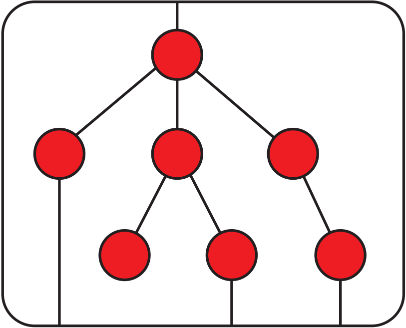


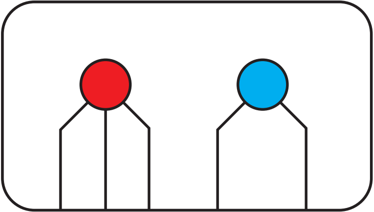

r

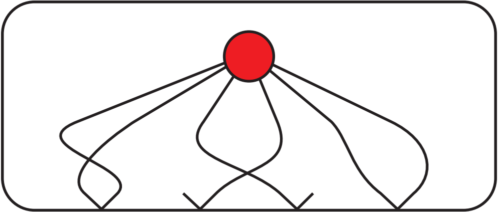

placeholder for the term
stored in the unique register
of the 2nd child
2
1
3

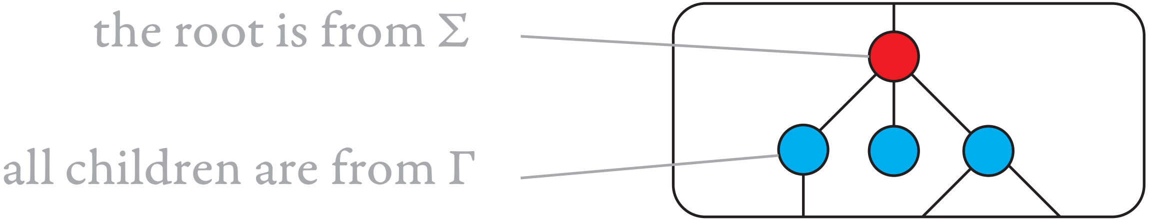


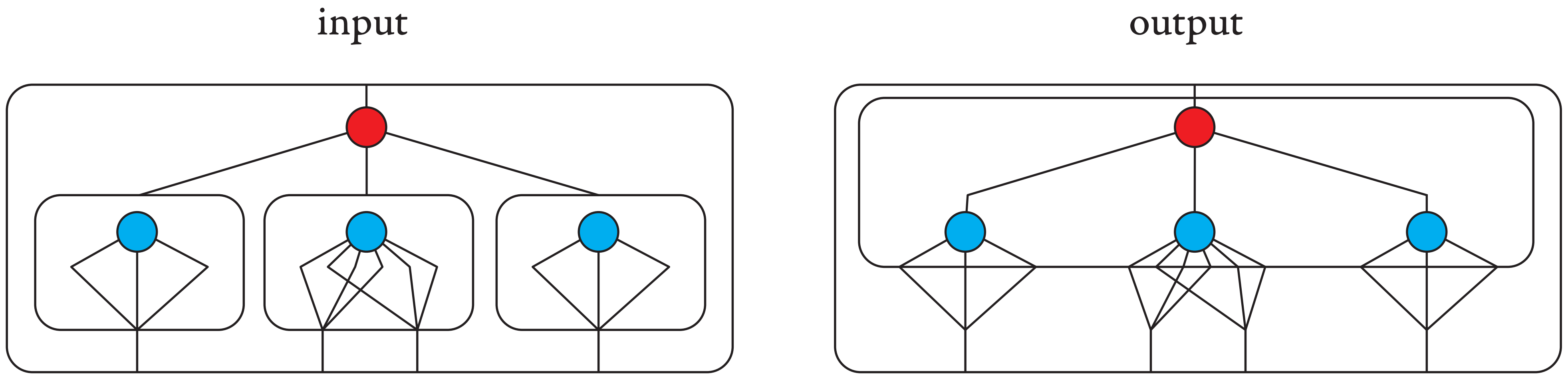
λx
@
@
@
x
variable x is bound in the root
the original port is replaced by x

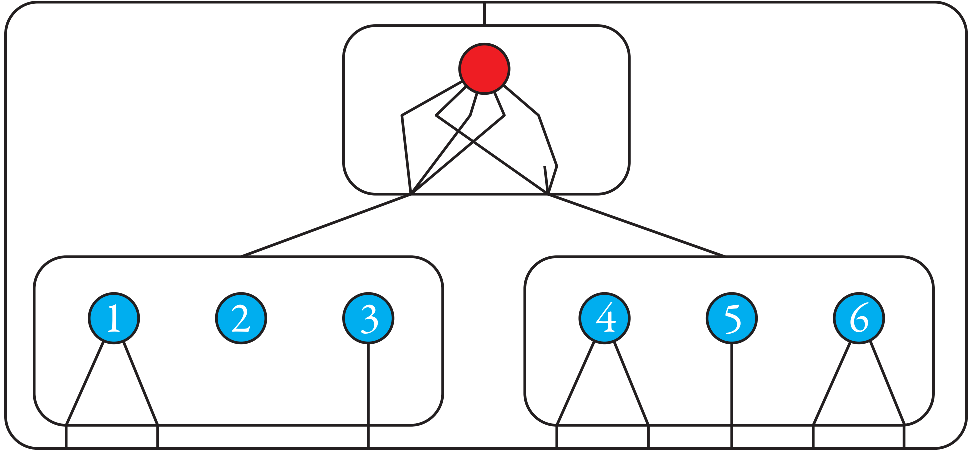




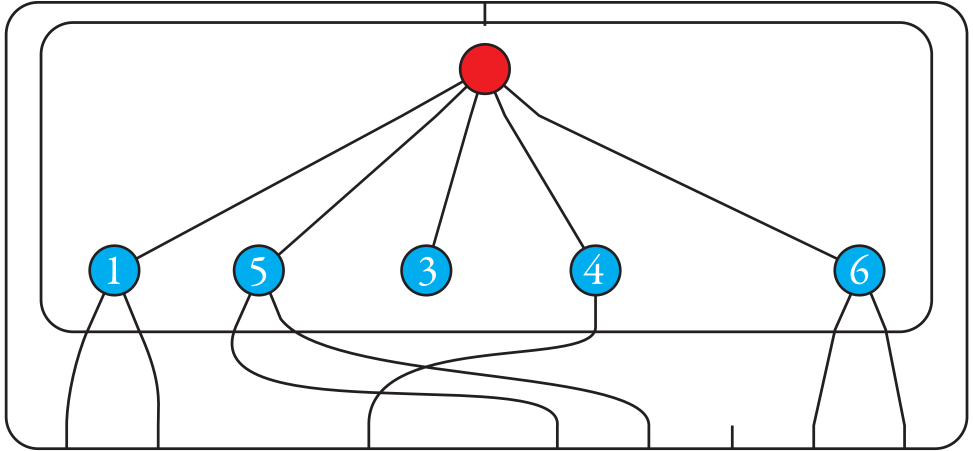


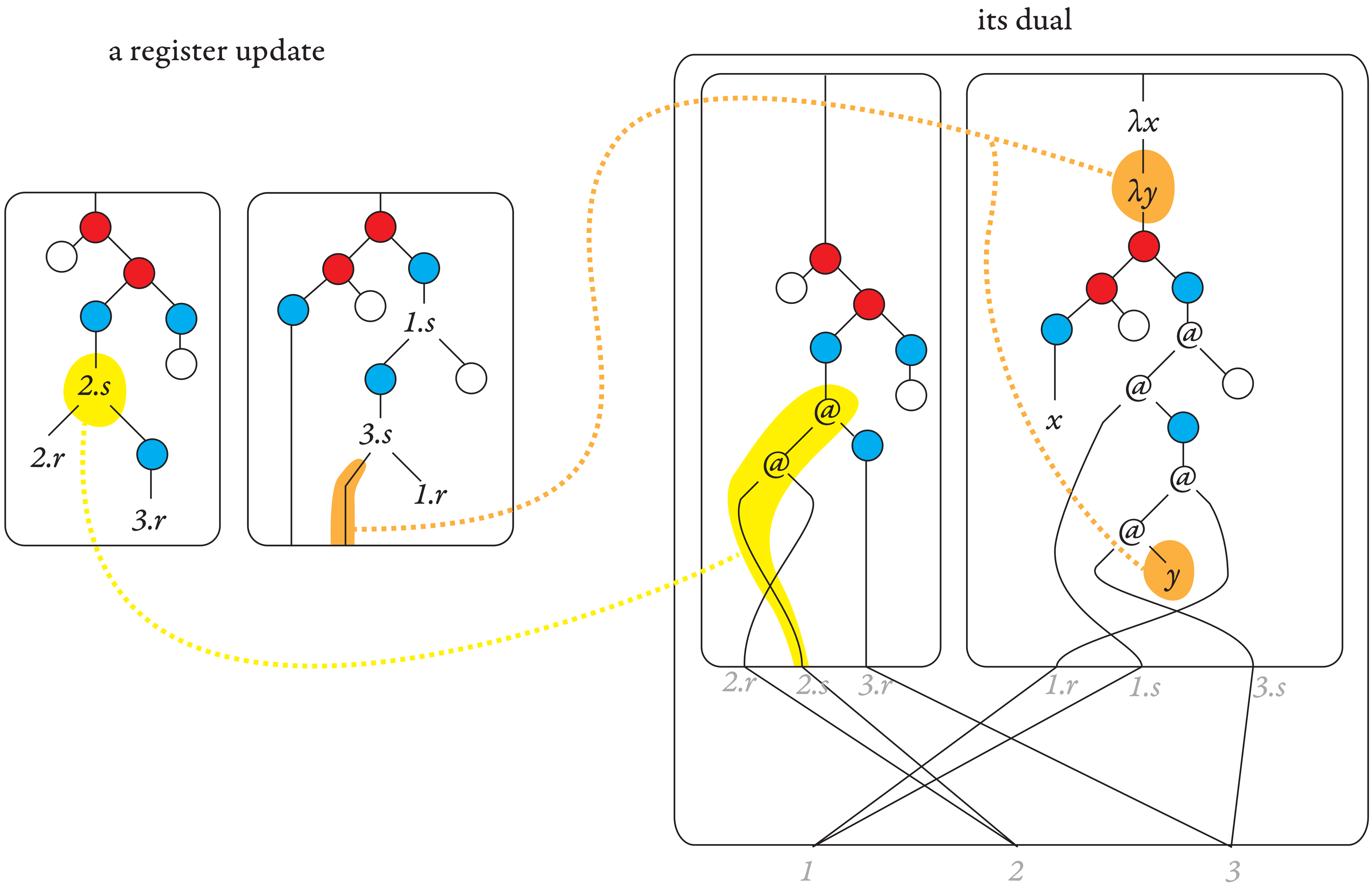

the root is from Σ
all children are from Γ


input
output

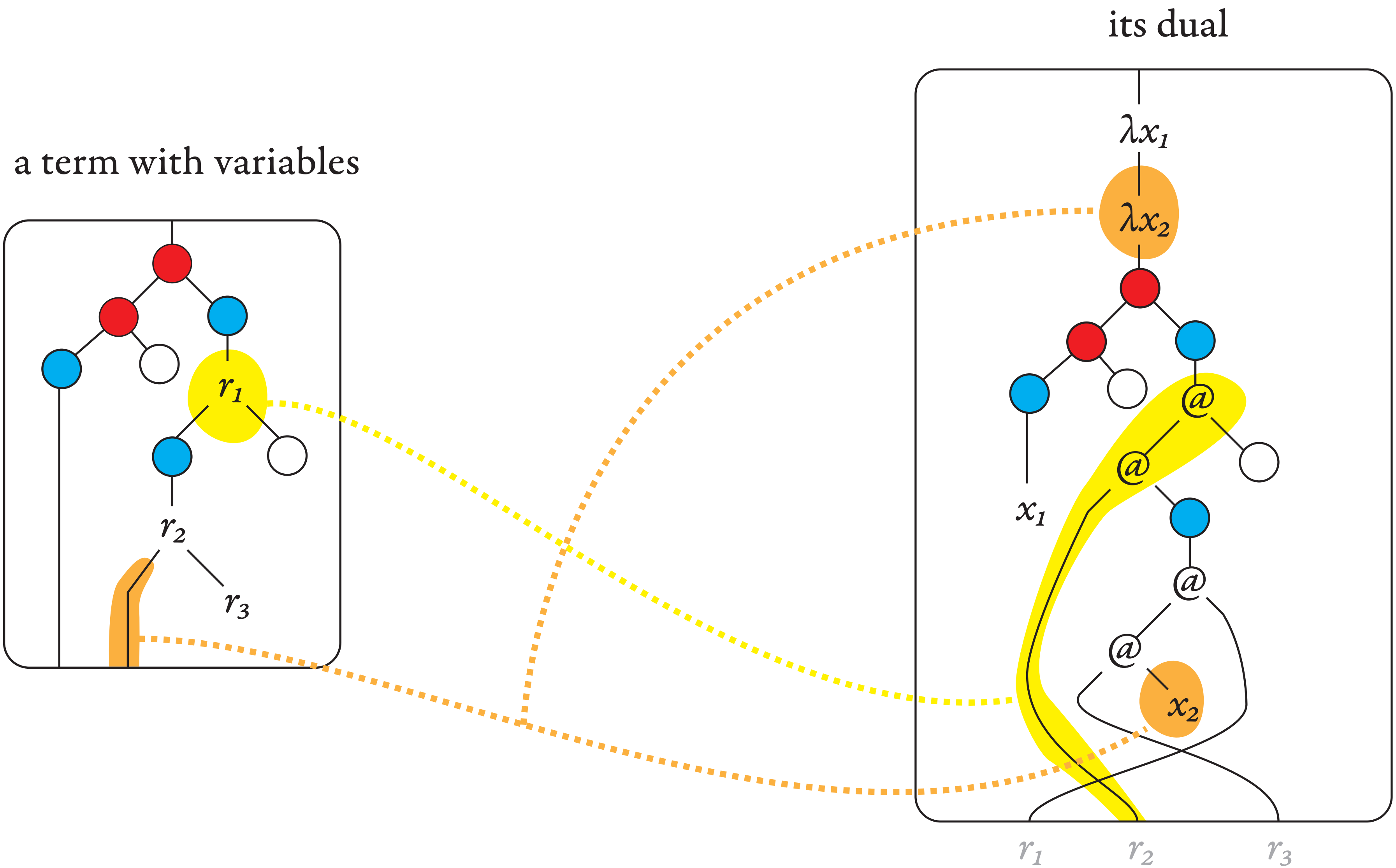


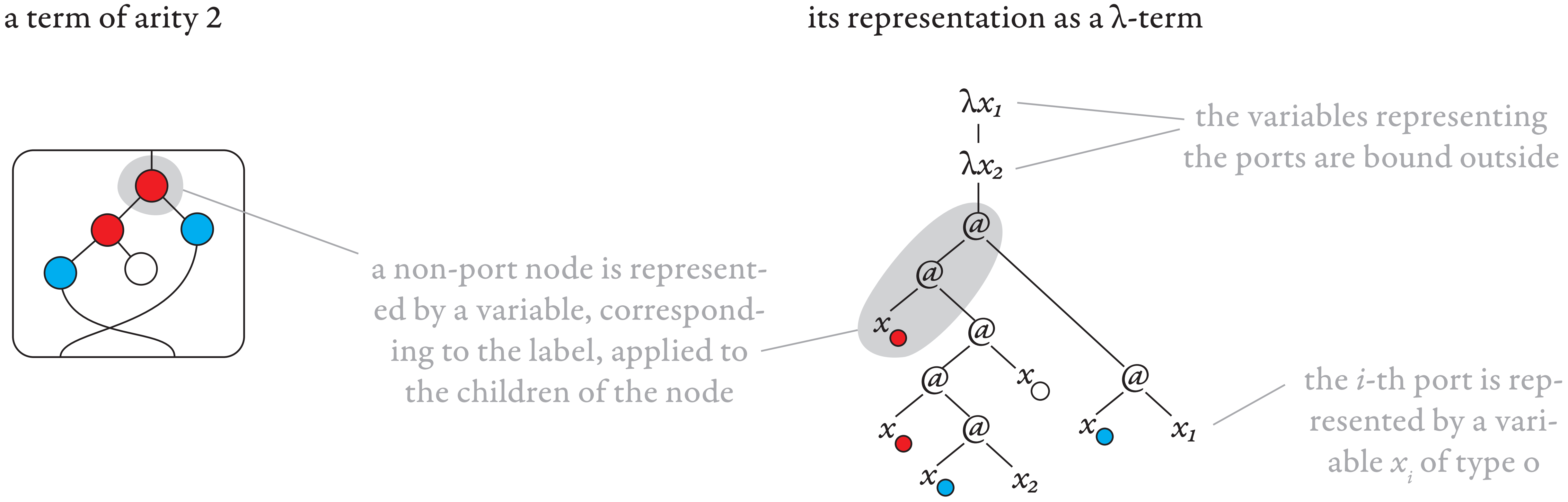
$\mapsto$





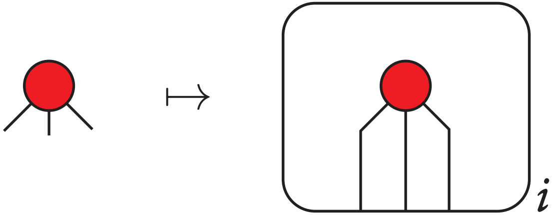
a register update
its dual
2.s
2.r
3.r
1.s
3.s
1.r
λx
λy
@
x
y
2.r
2.s
3.r
1.r
1.s
3.s
1
2
3


a term with variables
its dual
r1
r2
r3
λx1
λx2
@
x1
x2
r1
r2
r3

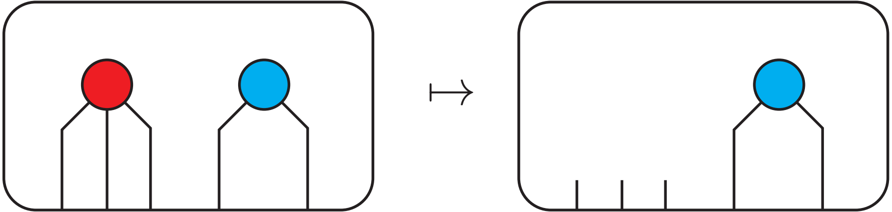


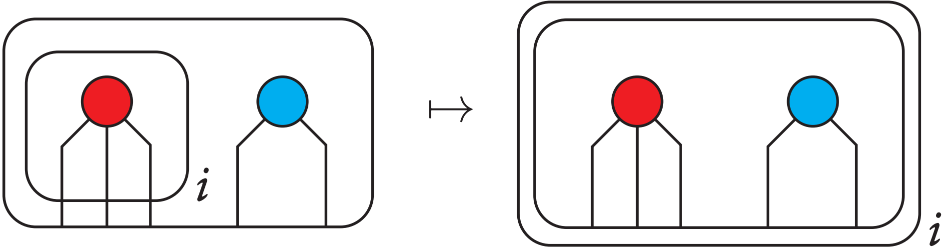
a term of arity 2
its representation as a λ-term
λx_1
λx_2
the variables representing the ports are bound outside
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
the i-th port is represented by a variable x_i of type o
@
x_1
x_2

i



i





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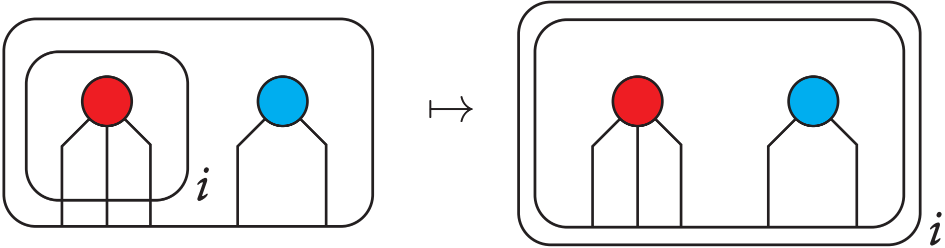

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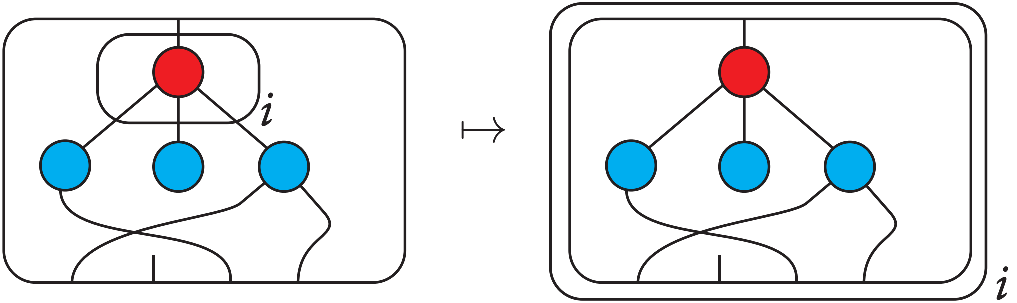

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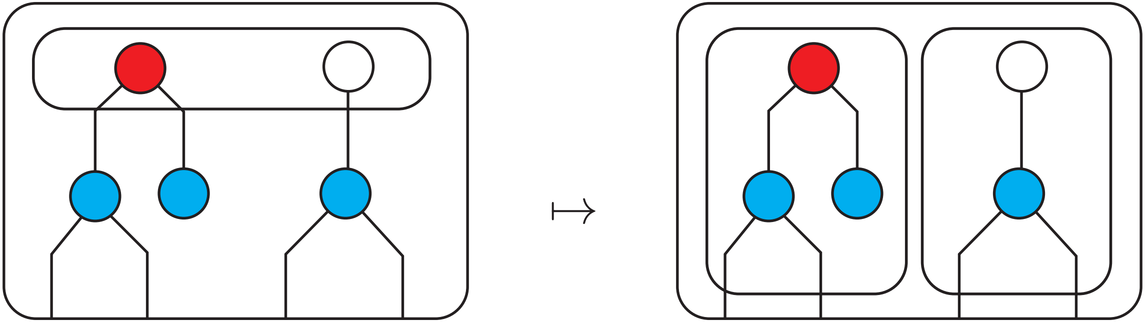

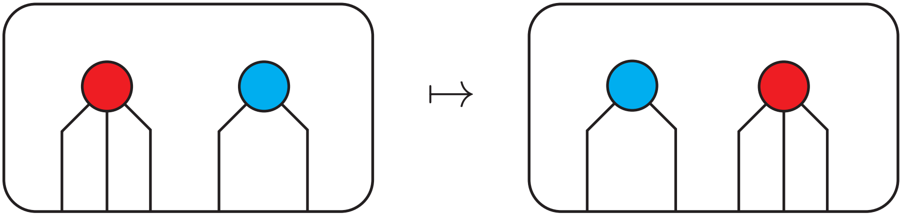

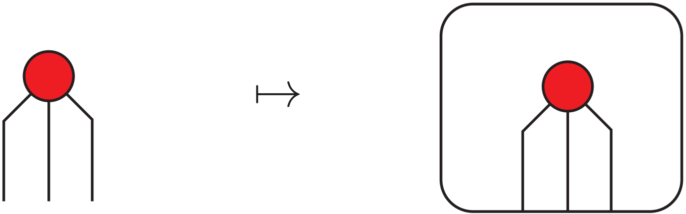

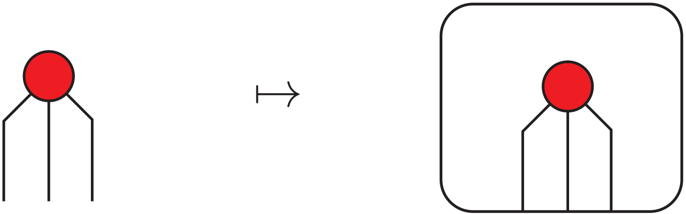

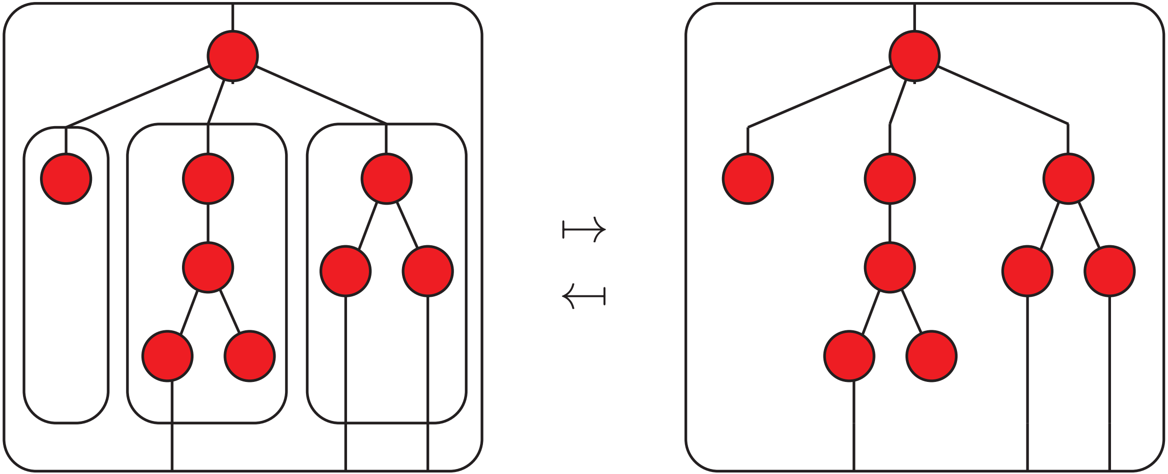

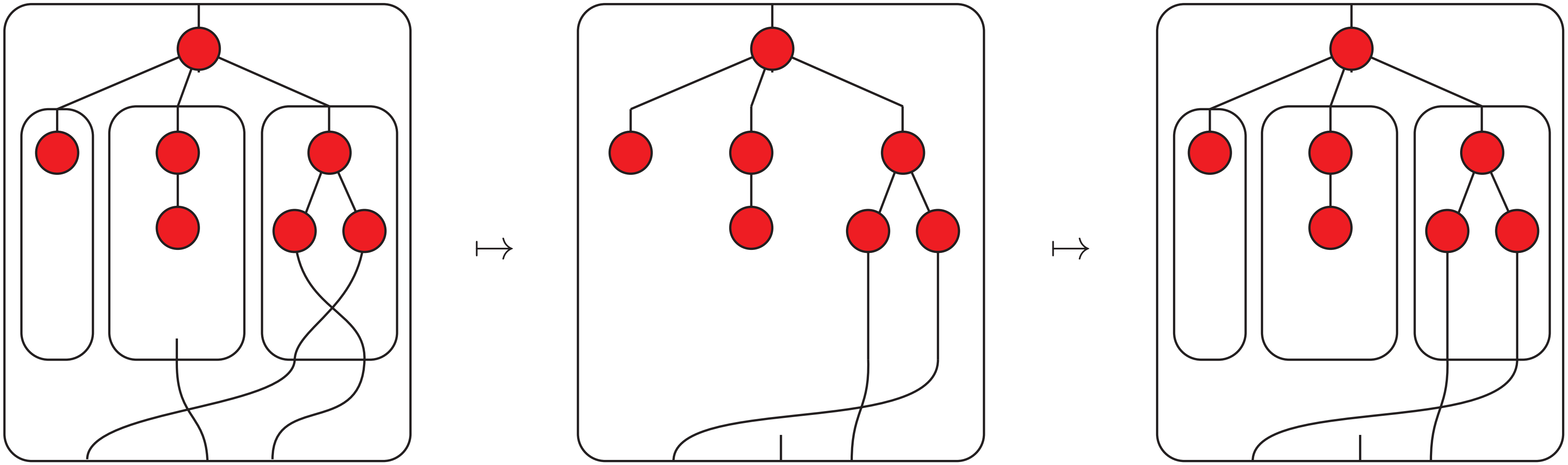

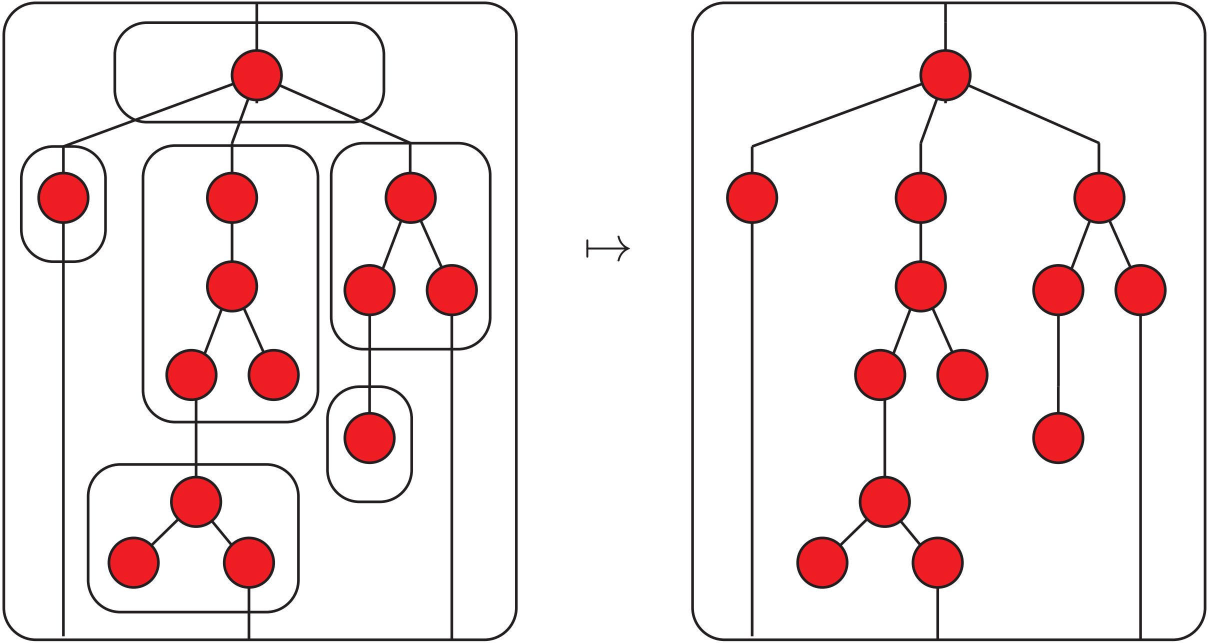

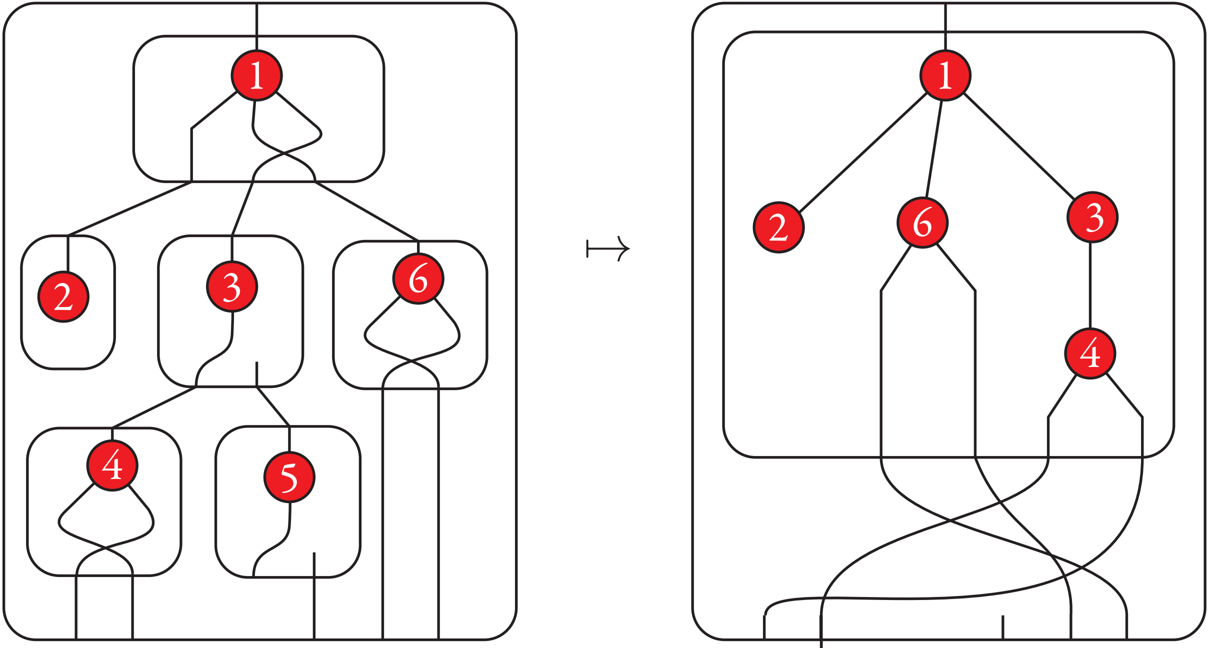
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