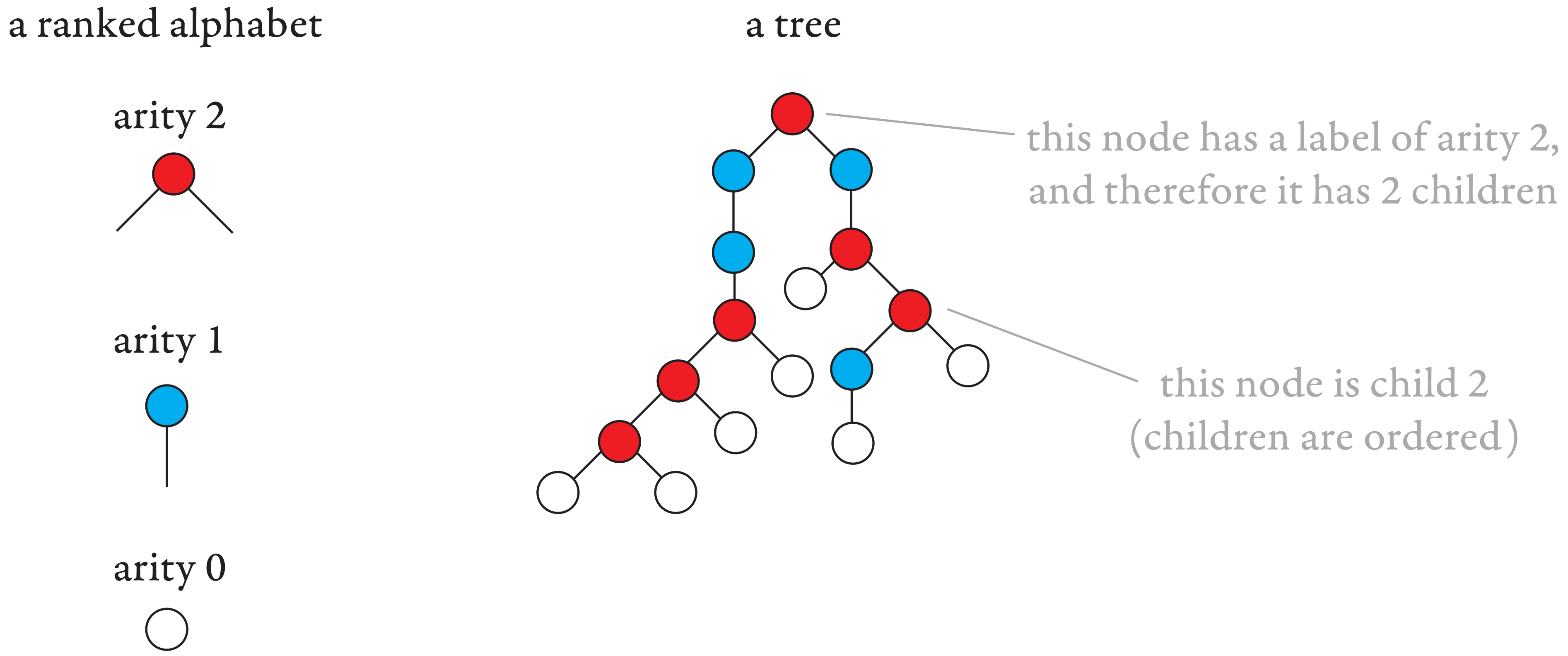
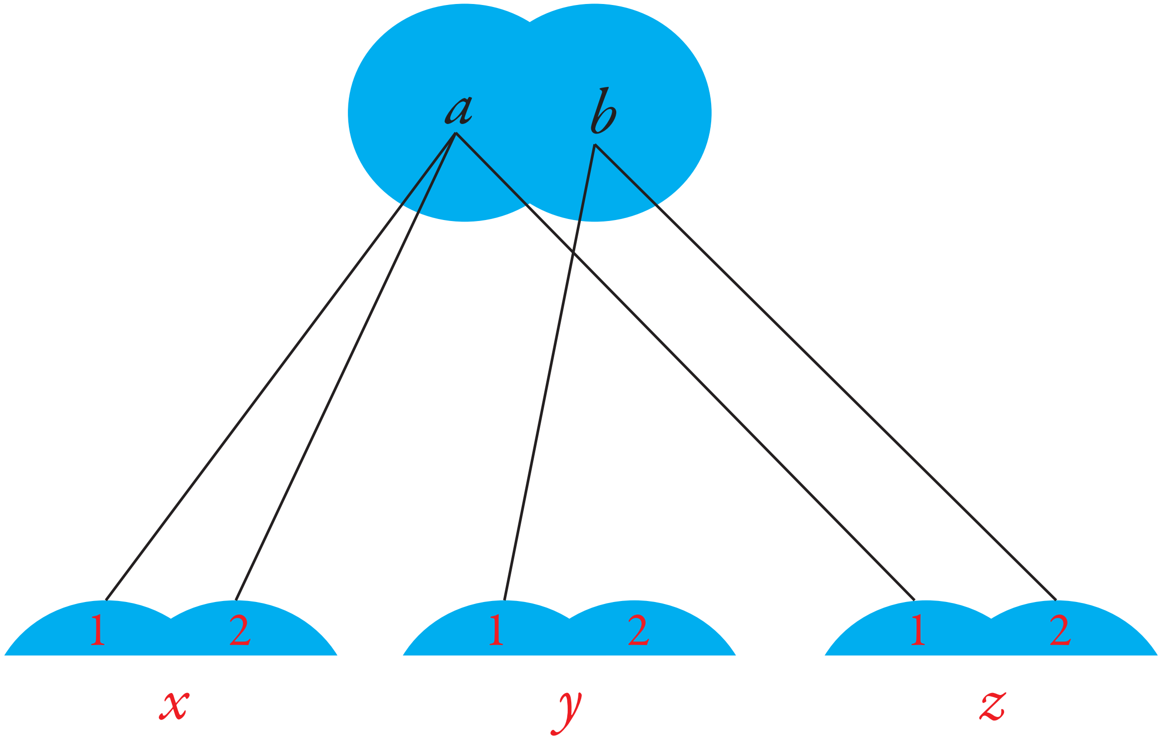
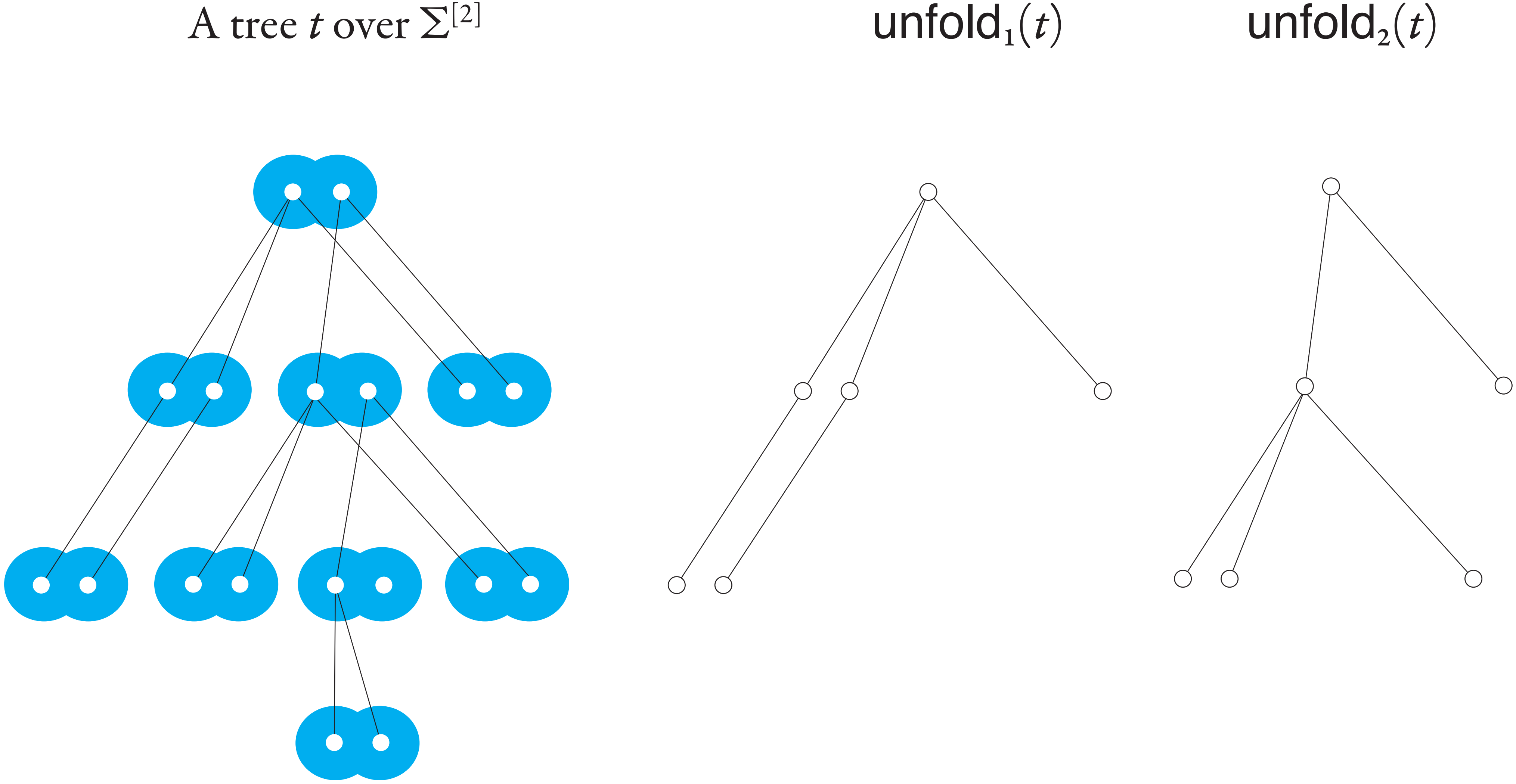
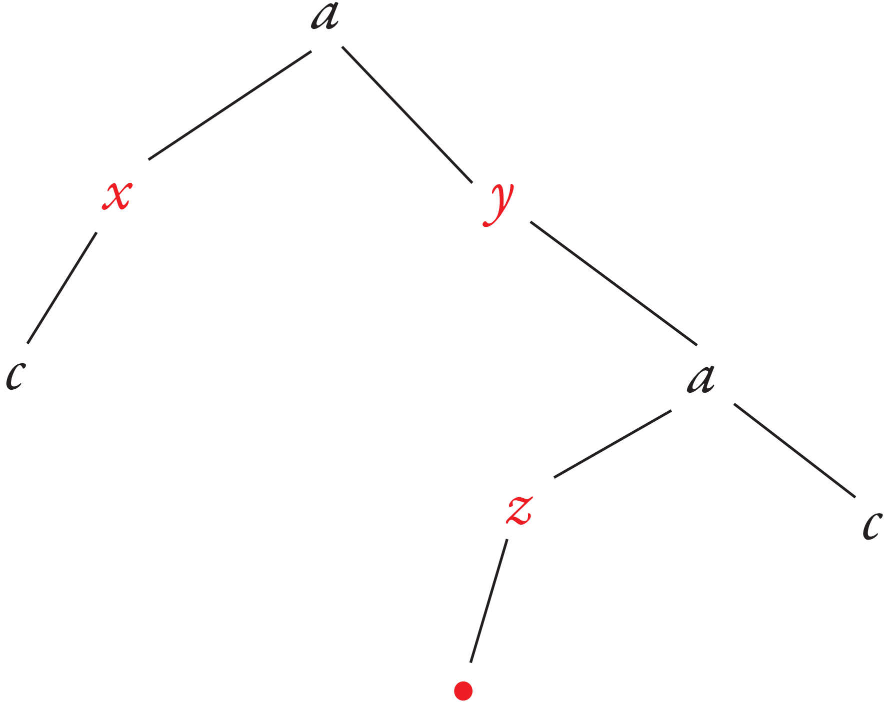
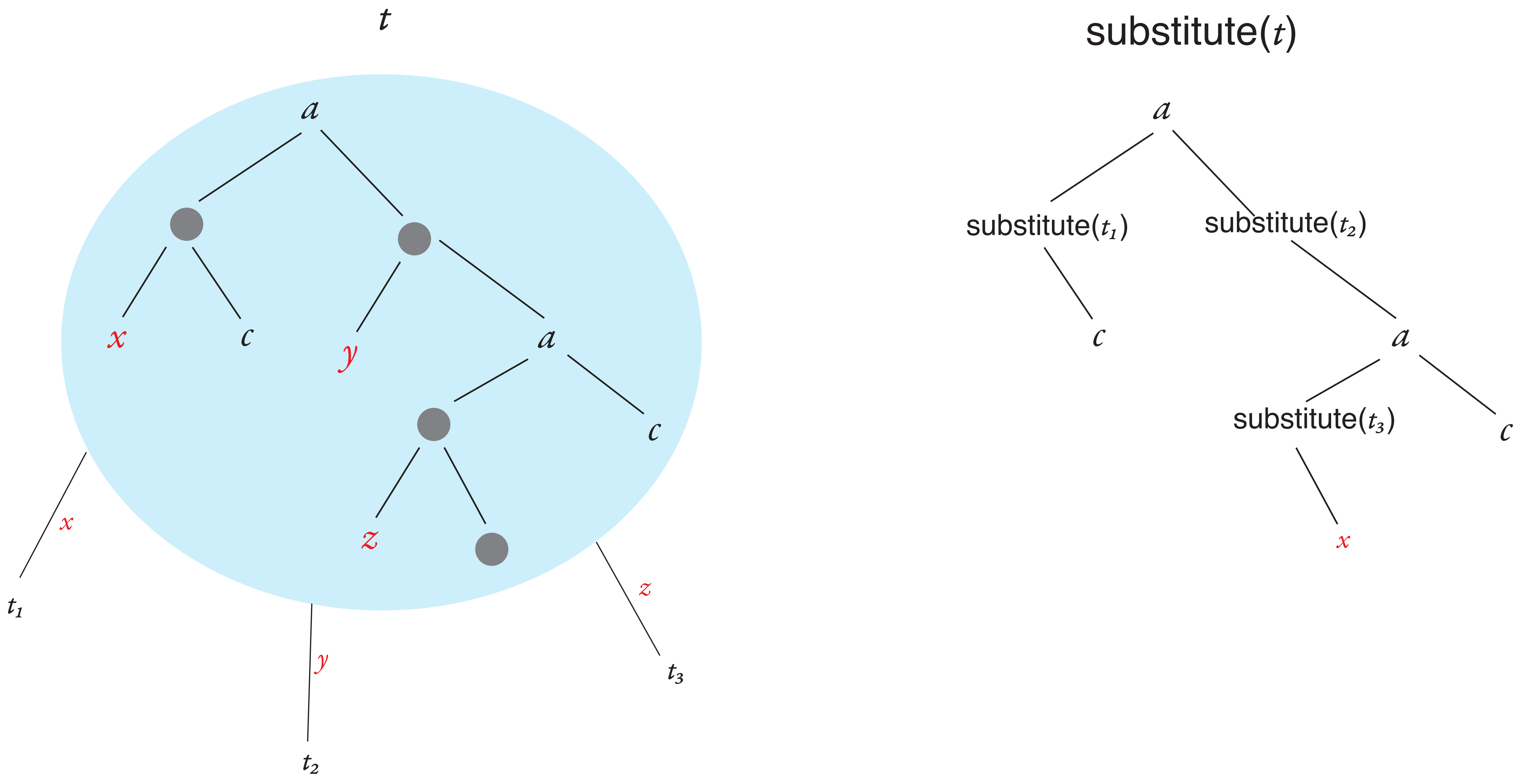
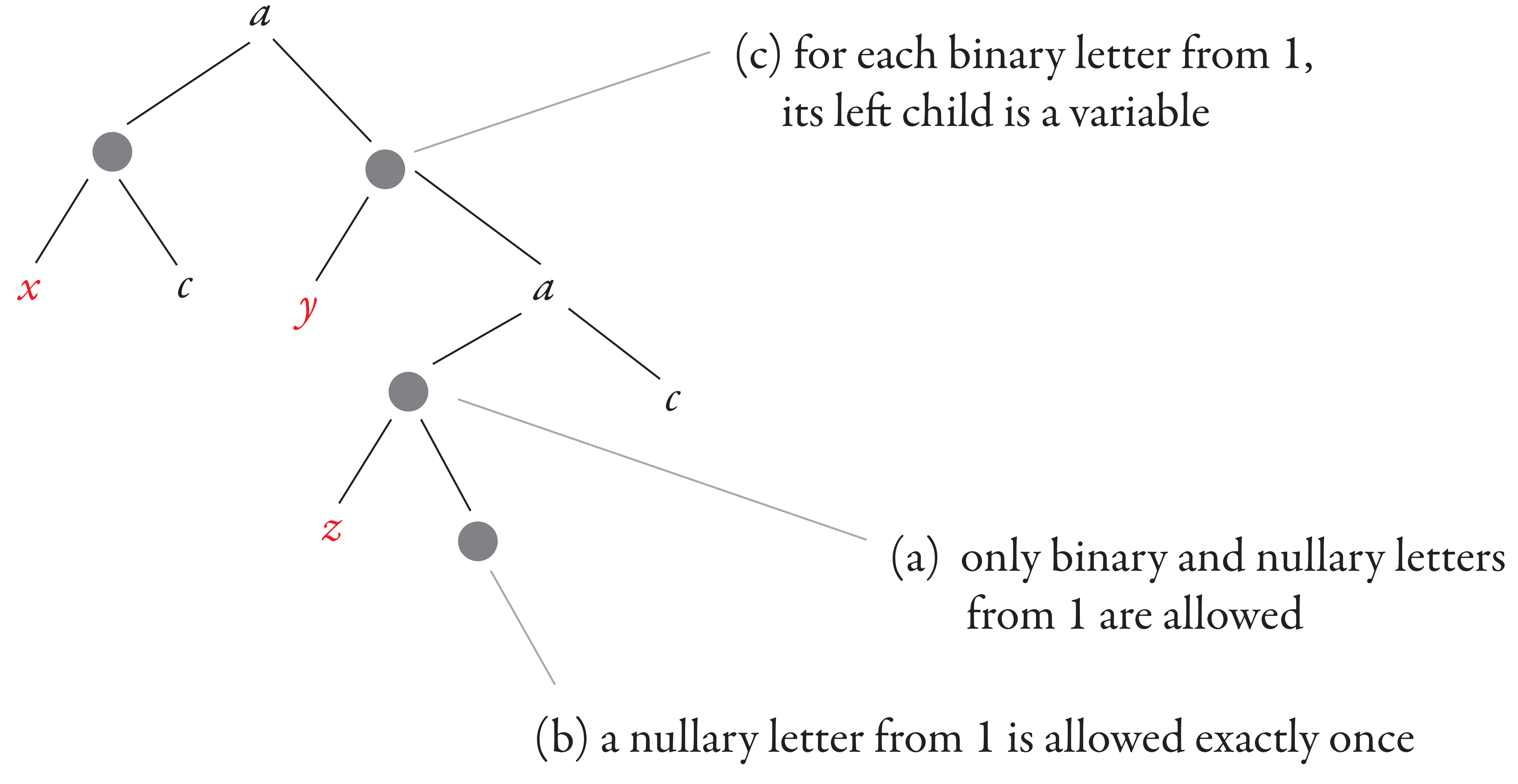

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)

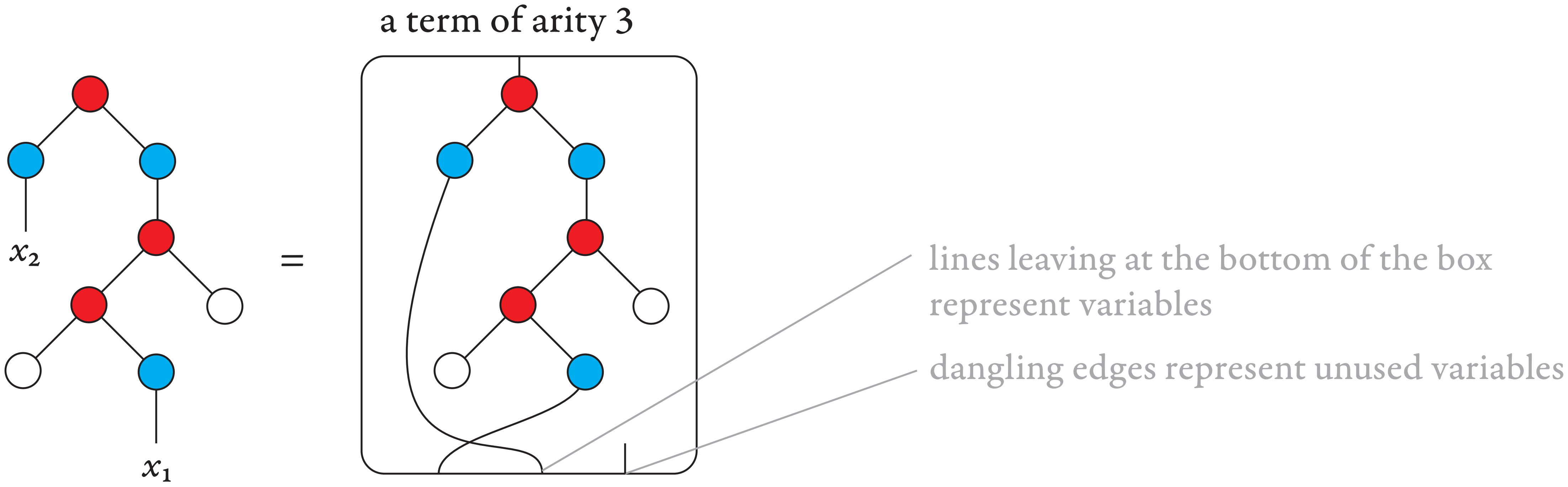

a
b
1
2
1
2
1
2
x
y
z


A tree t over $\Sigma^{[2]}$
$\mathsf{unfold}_1(t)$
$\mathsf{unfold}_2(t)$

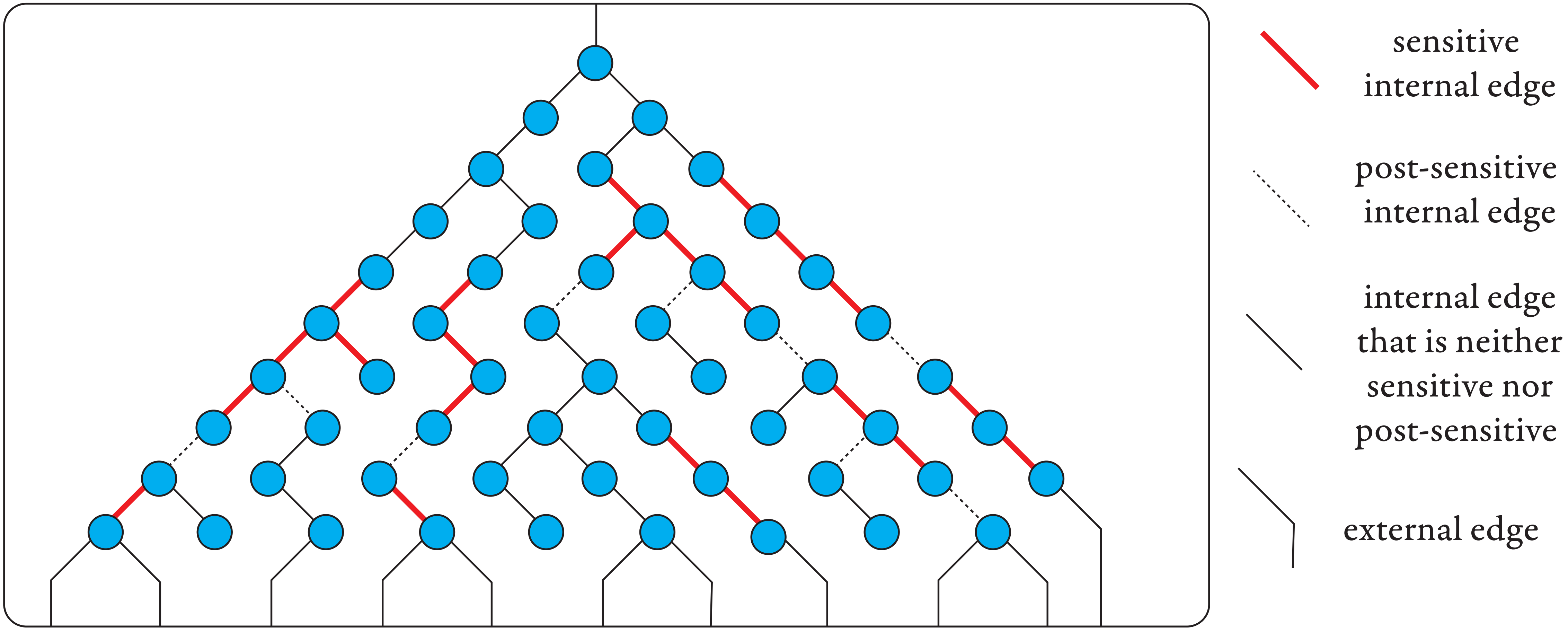

a
x
y
c
a
z
c

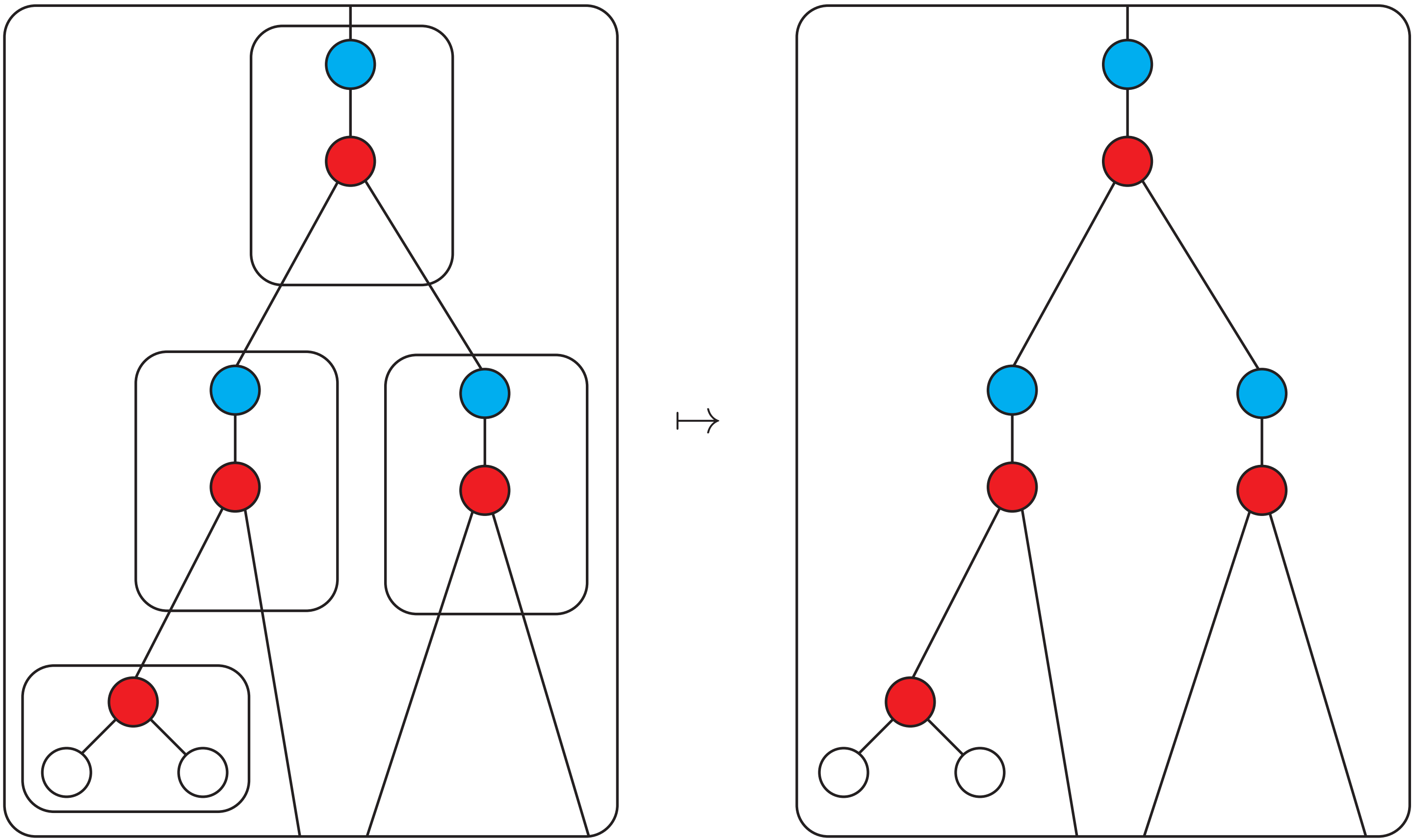

t
a
x
c
y
a
z
c
x
y
z
t1
t2
t3
substitute(t)
a
substitute(t1)
substitute(t2)
c
a
substitute(t3)
c
x

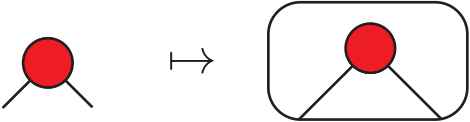

a
x
c
y
a
z
c
(c) for each binary letter from 1,
its left child is a variable
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

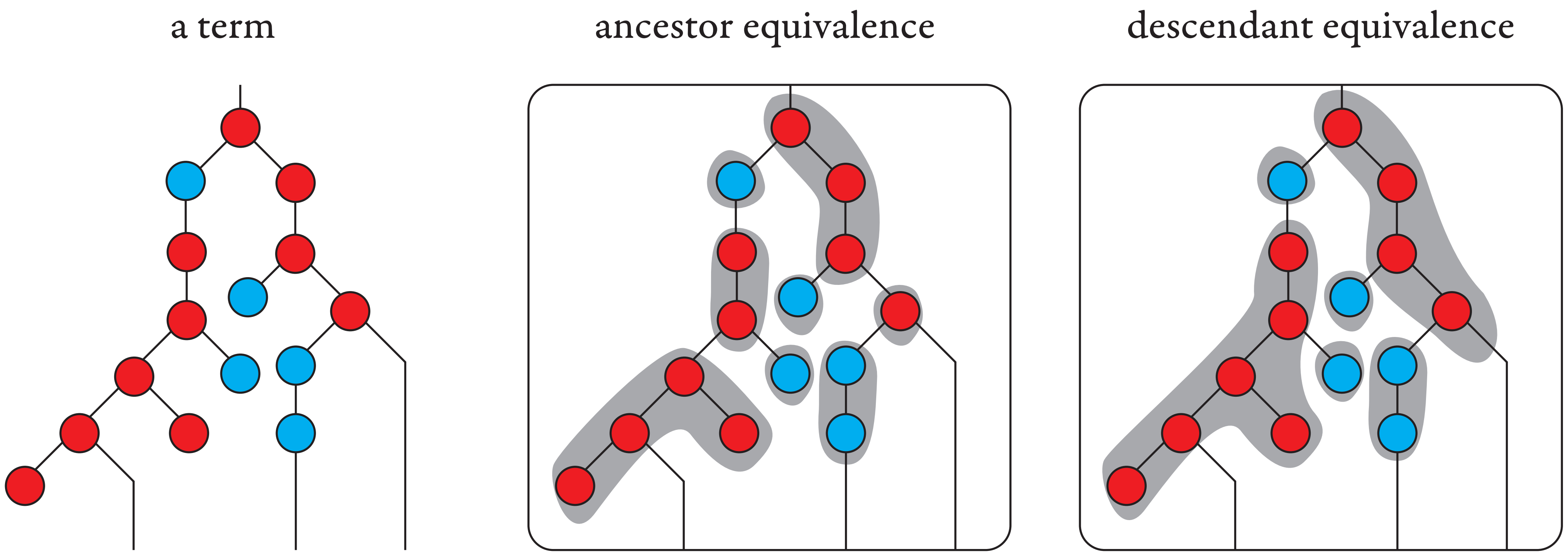


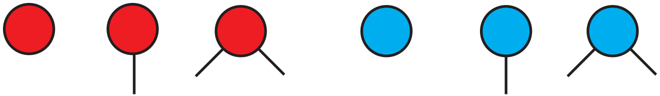
a term of arity 3
x_2
x_1
=
lines leaving at the bottom of the box represent variables
dangling edges represent unused variables

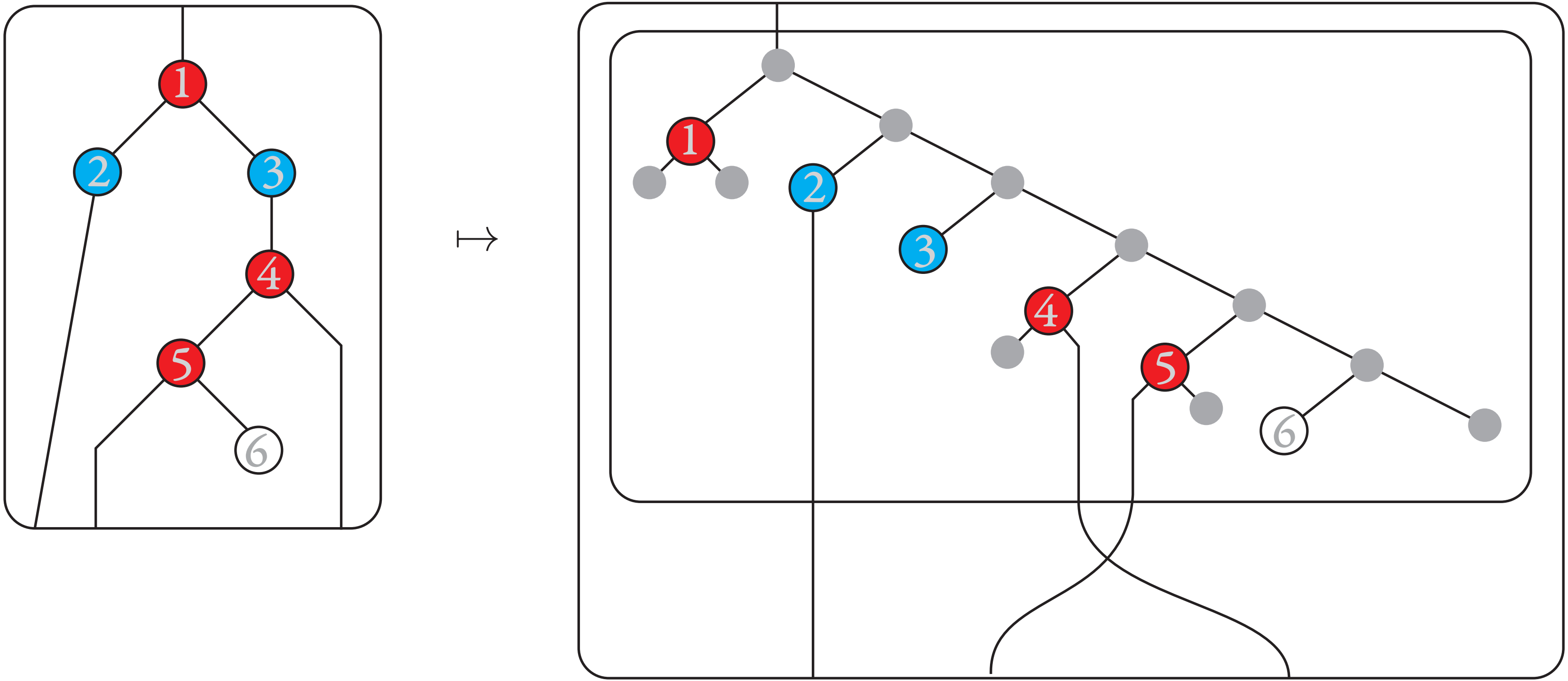

sensitive
internal edge
post-sensitive
internal edge
internal edge
that is neither
sensitive nor
post-sensitive
external edge



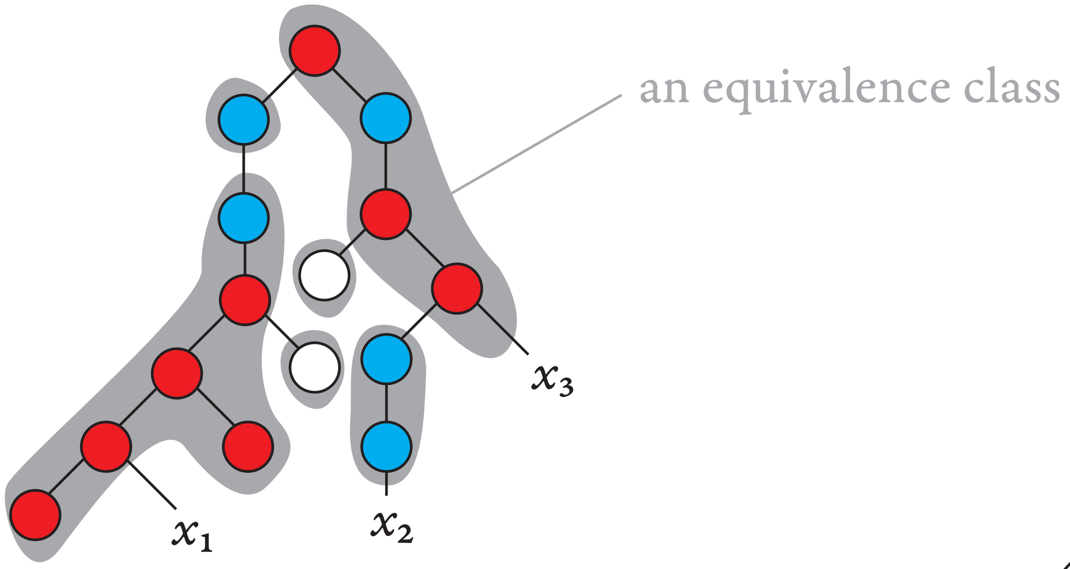


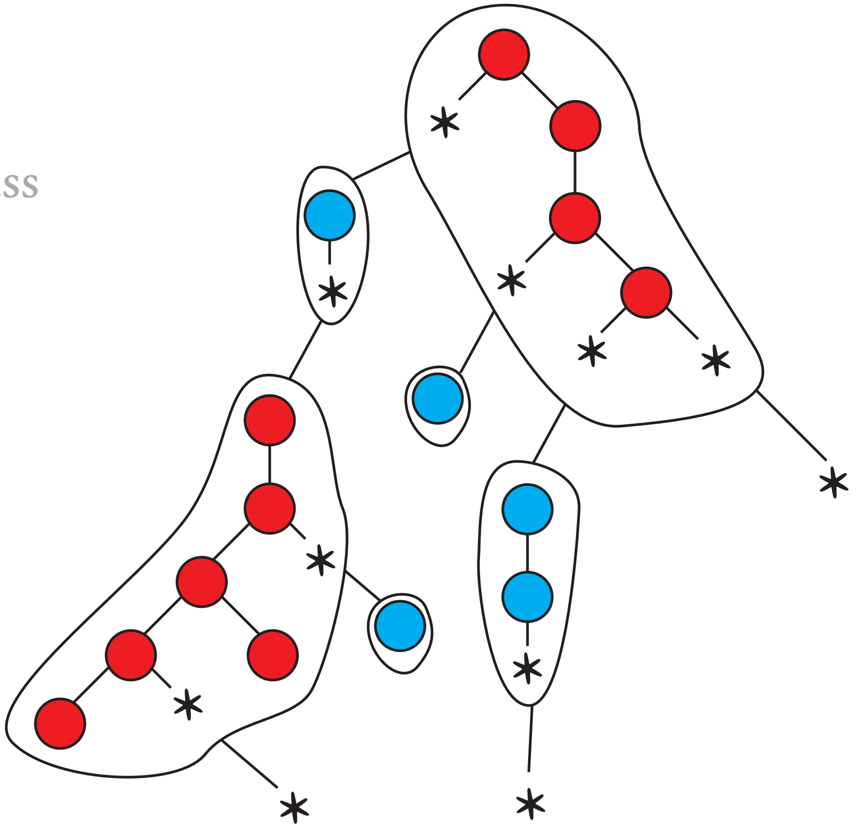

a term
ancestor equivalence
descendant equivalence

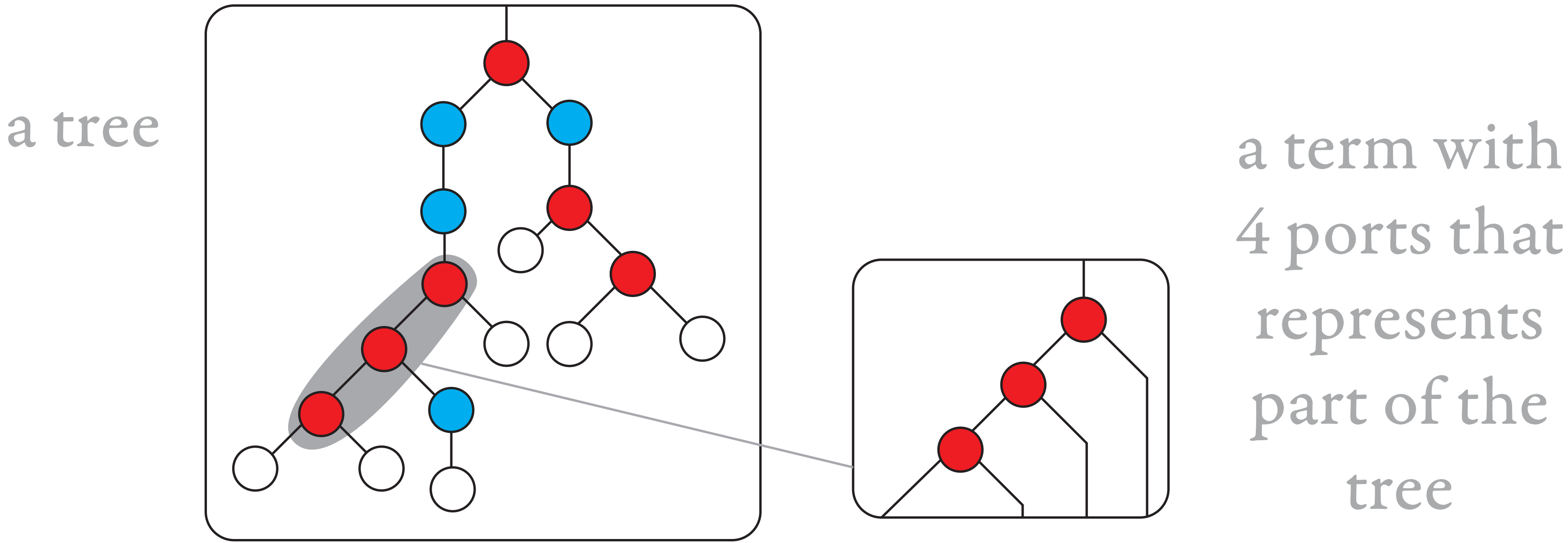


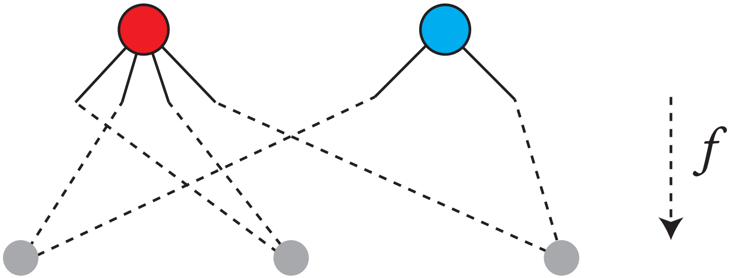

1
2
3
4
5
6
1
2
3
4
5
6

a factorisation equivalence

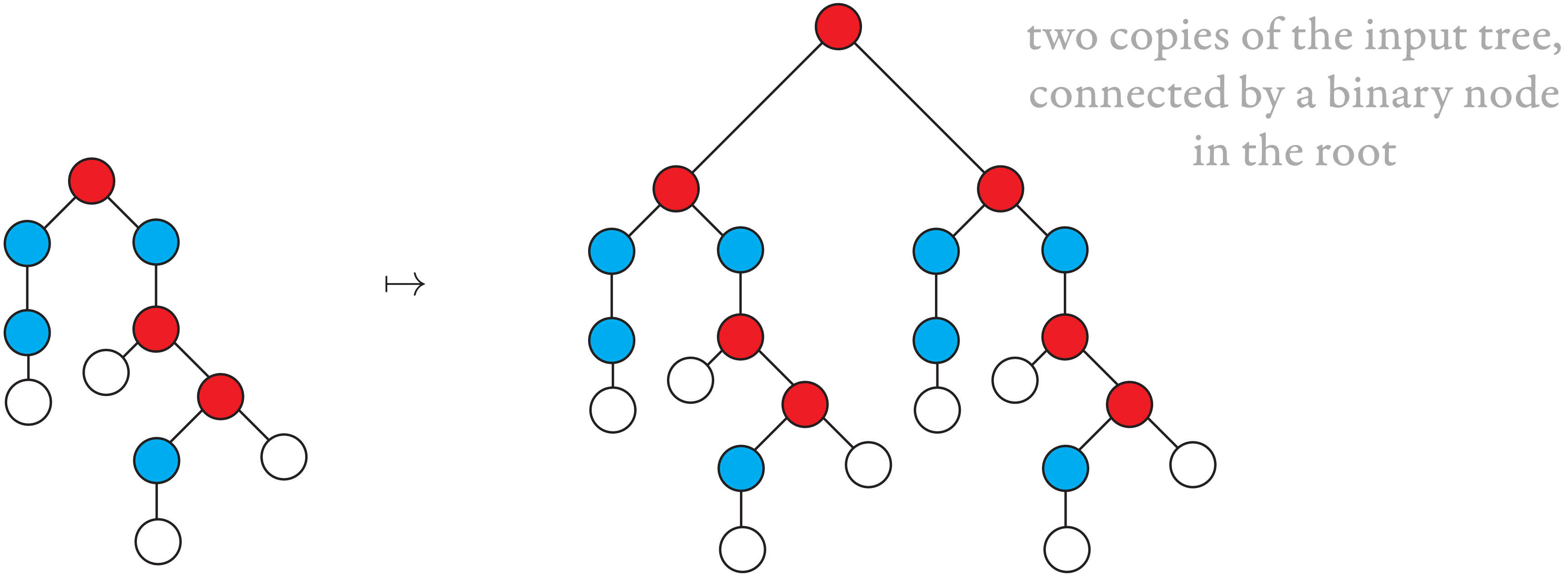


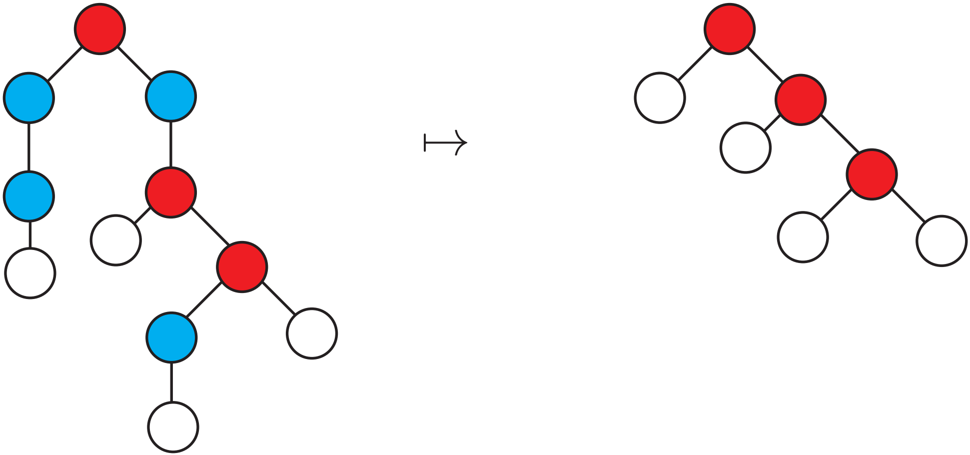


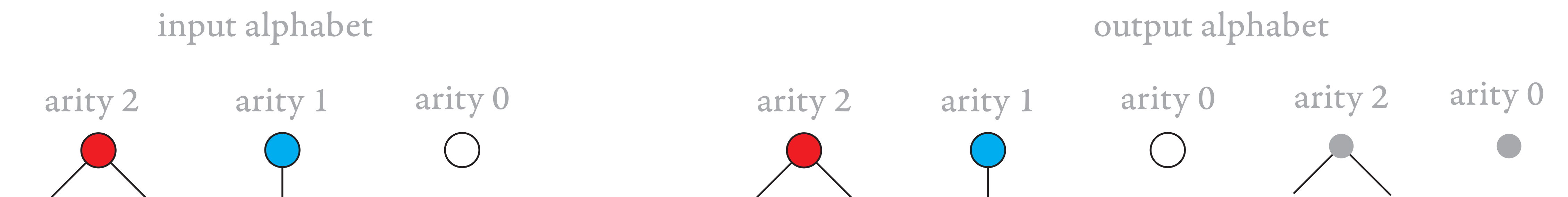

a tree
a term with 4 ports that represents part of the tree

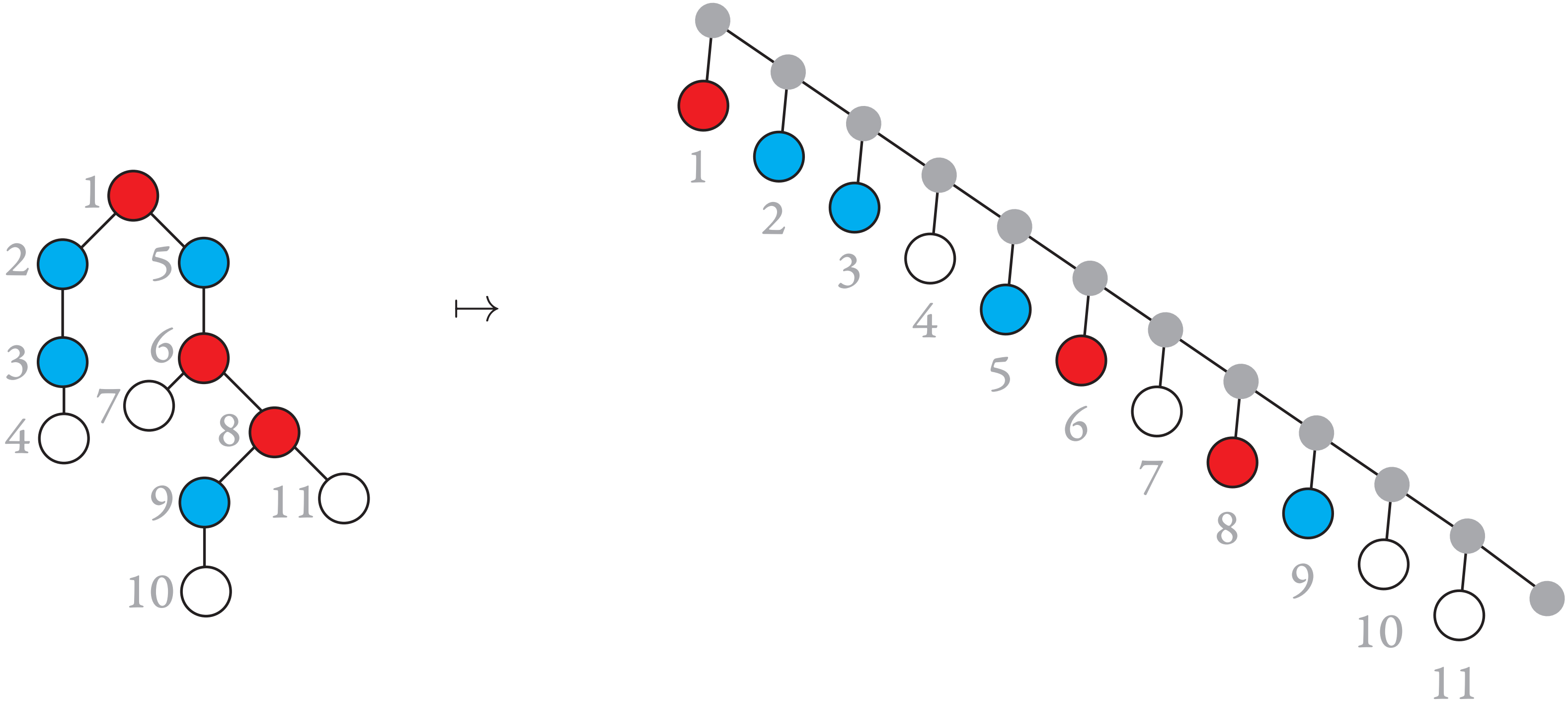

f


input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0

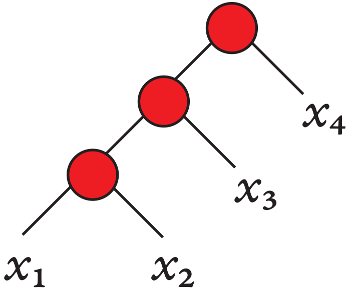

two copies of the input tree,
connected by a binary node
in the root



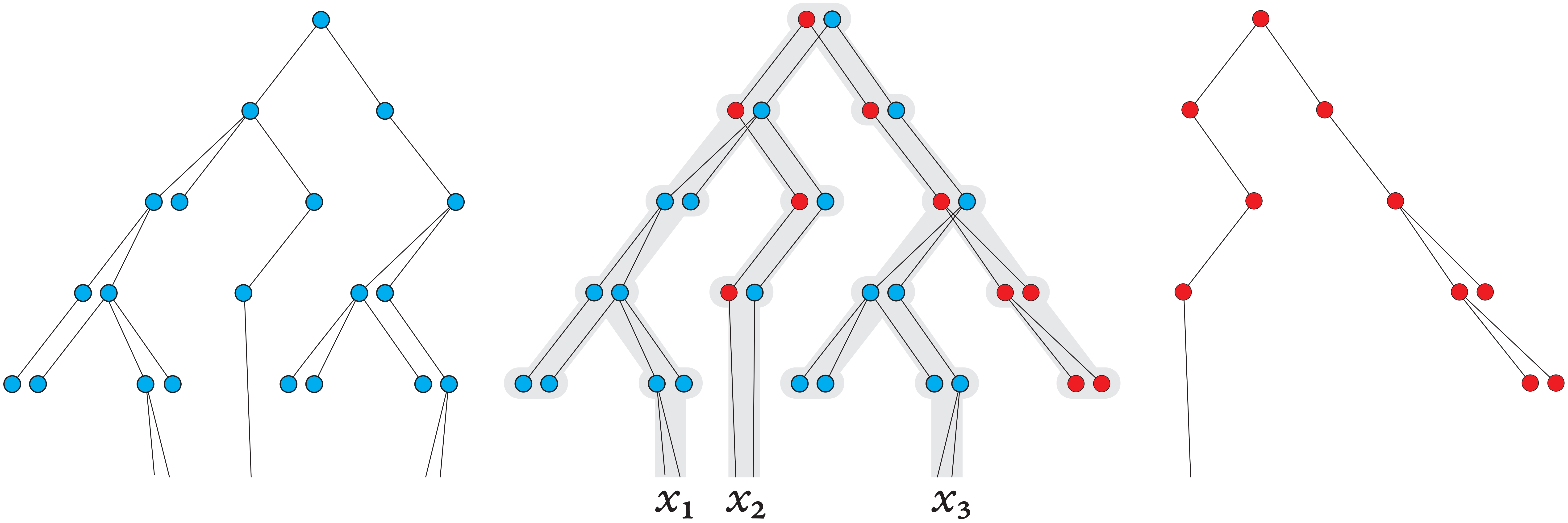

input alphabet
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 1
arity 0
arity 2
arity 0

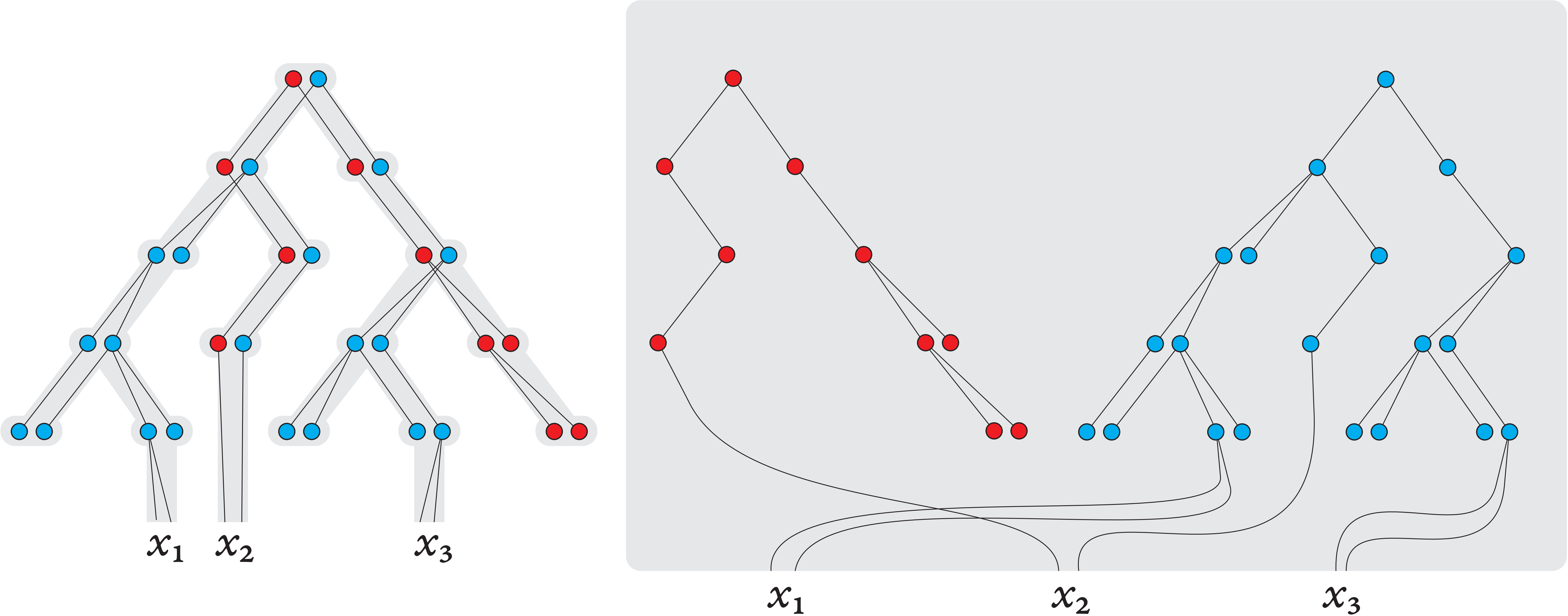

1
2
5
3
6
7
4
8
9
11
10
↦
1
2
3
4
5
6
7
8
9
10
11

a term of arity 4

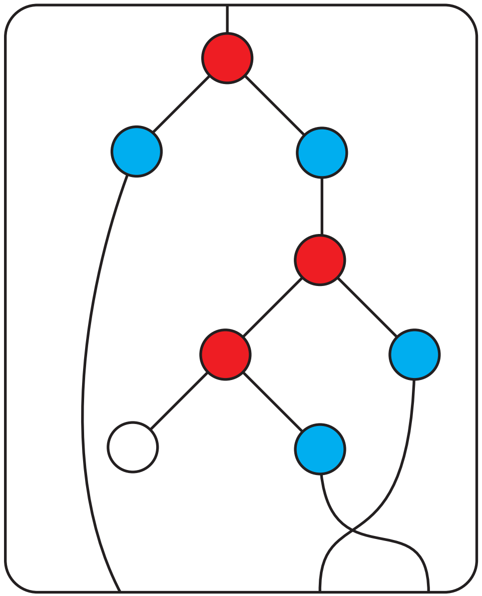


a term of arity 0


x_1 x_2 x_3



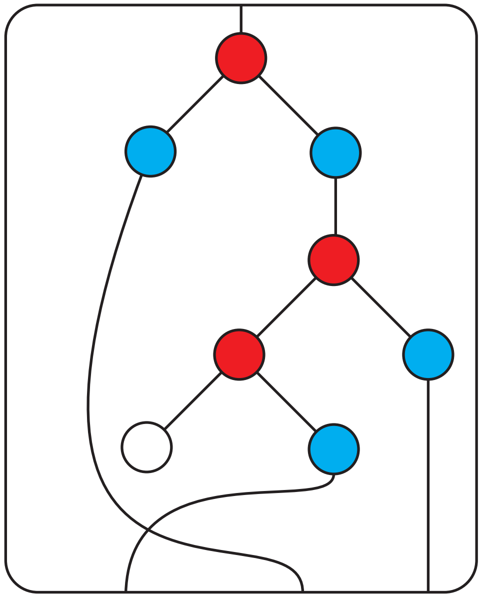
x_1
x_2
x_3
x_1
x_2
x_3



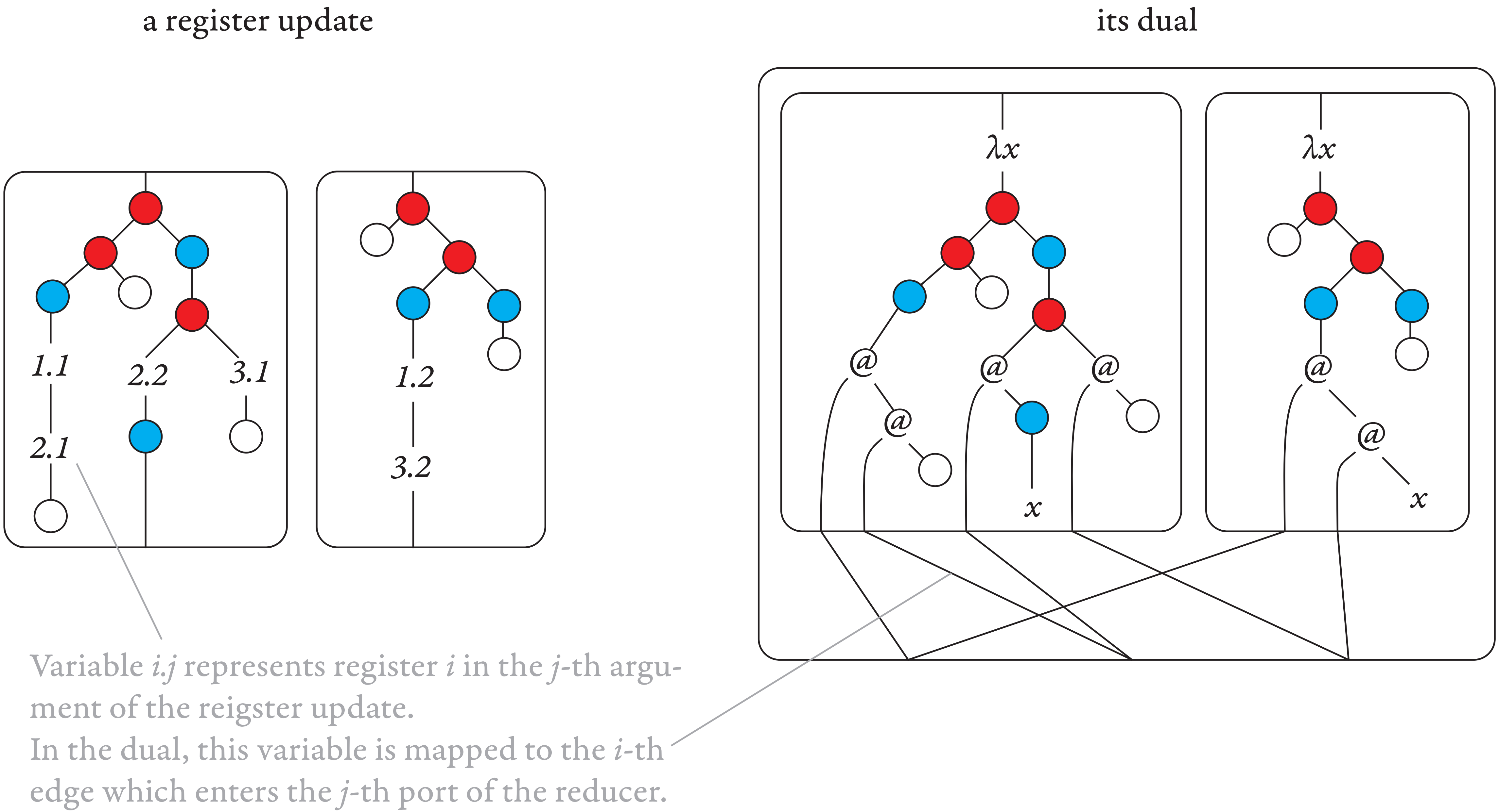
satisfies (*)

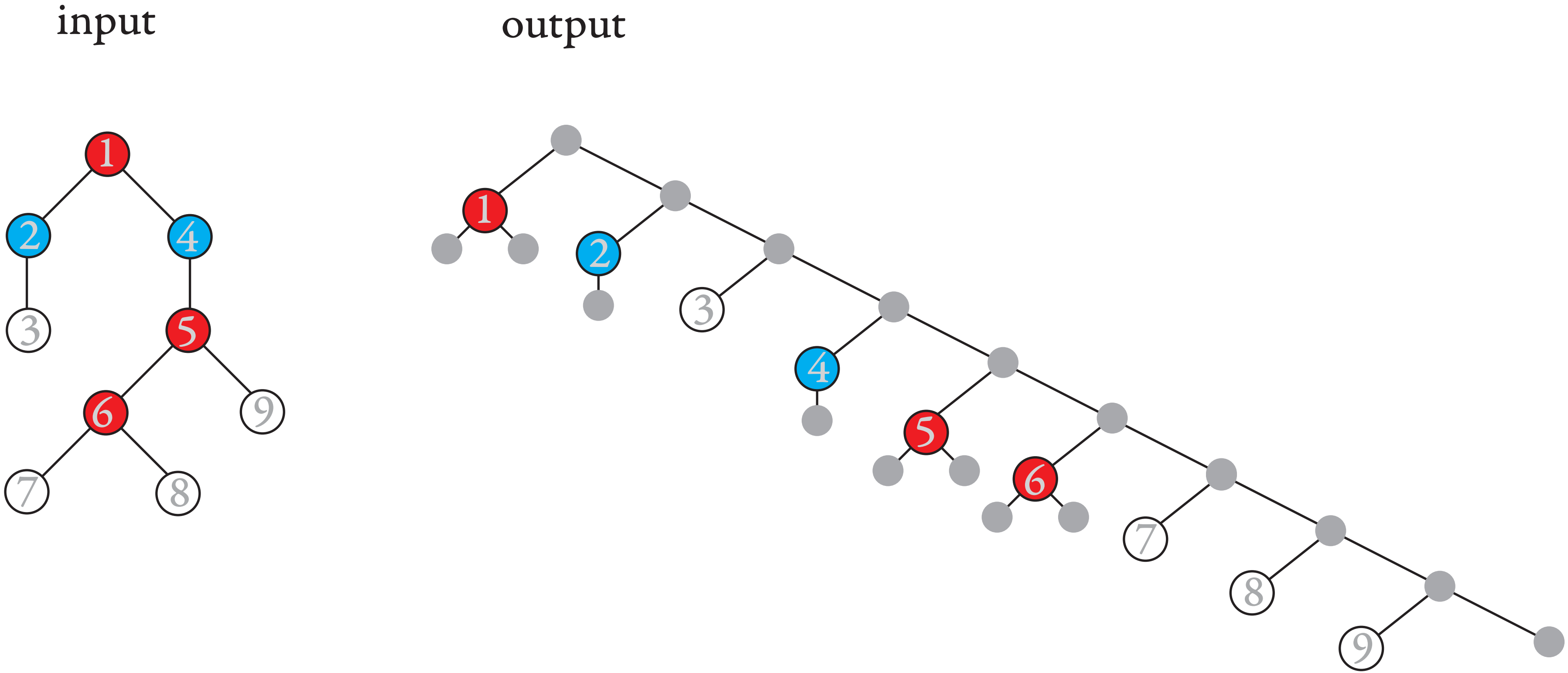
(*)

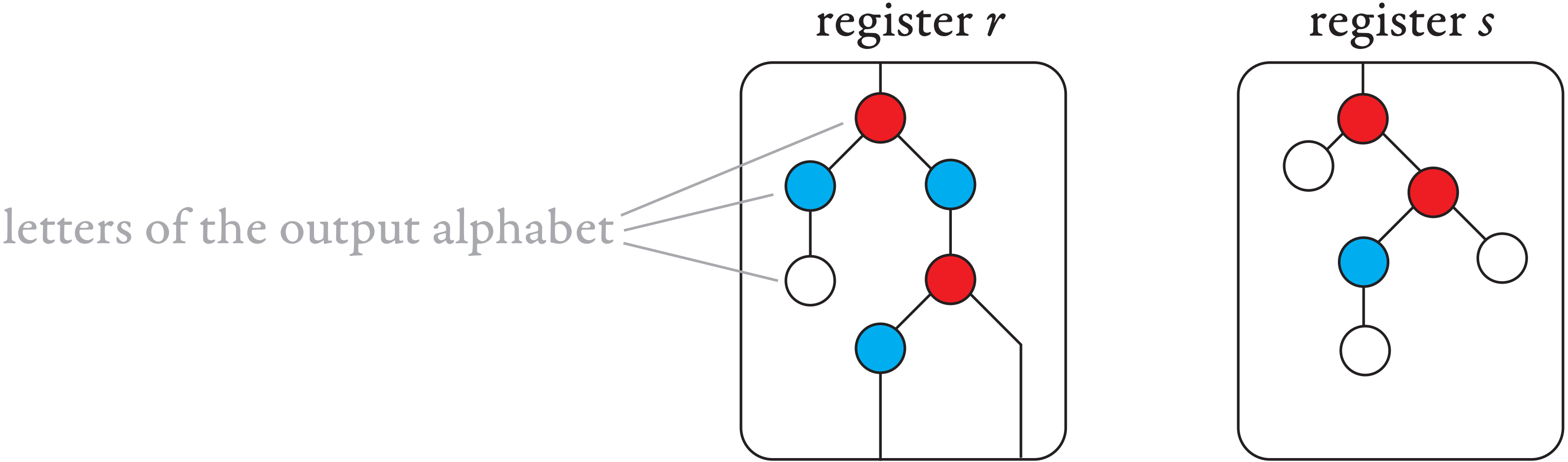
If the root has arity n, and $1 \leq i < j \leq n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root

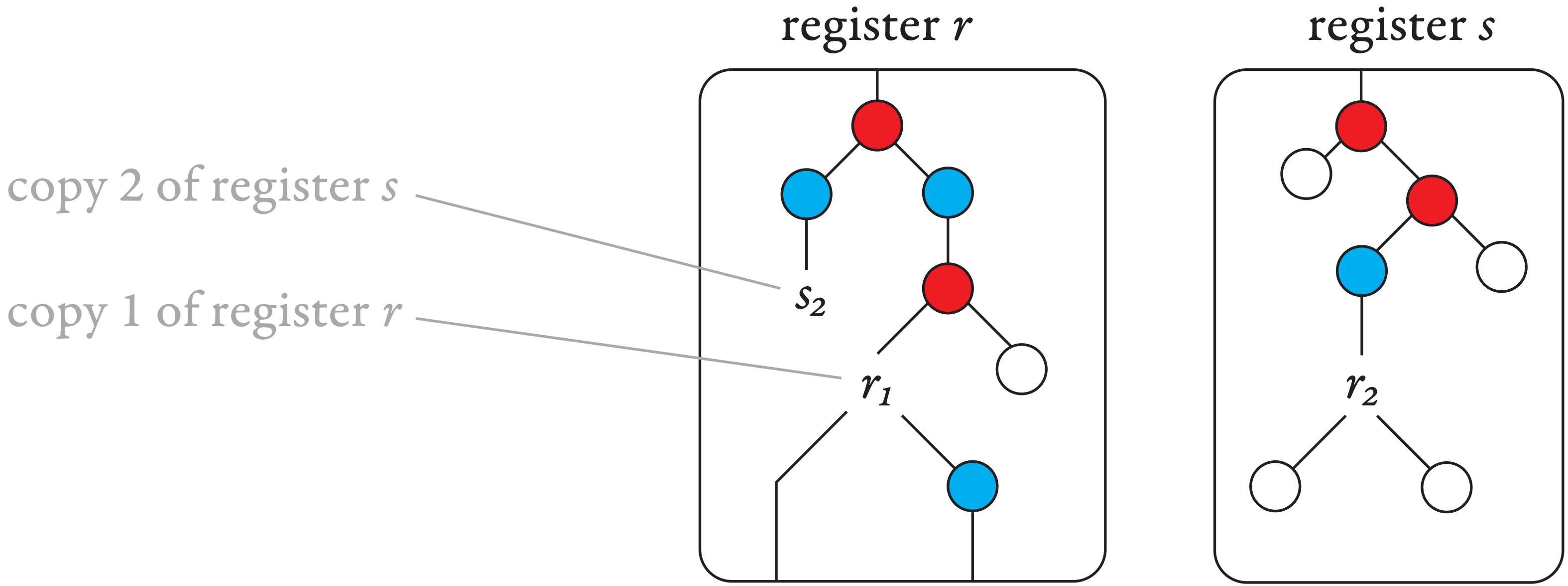


violates (*)

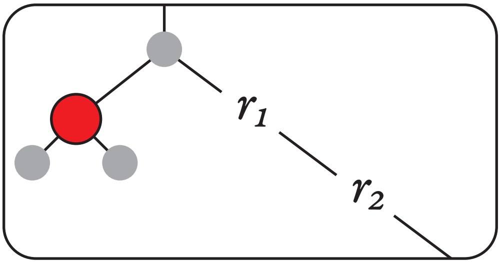

a register update
its dual
1.1
2.2
3.1
2.1
1.2
3.2
λx
λx
@
@
@
@
@
@
x
x
Variable i.j represents register i in the j-th argument of the reigster update.
In the dual, this variable is mapped to the i-th edge which enters the j-th port of the reducer.


input
output
1
2
3
4
5
6
7
8
9
1
2
3
4
5
6
7
8
9

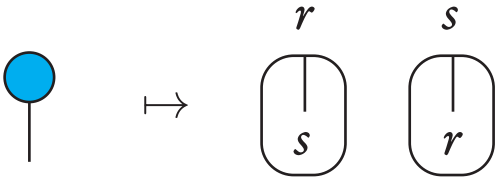

register r
register s
letters of the output alphabet


register r
register s
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2

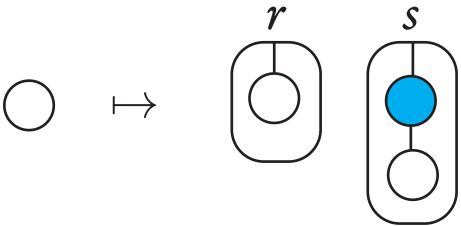

r
r
r
s



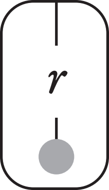
r_1
r_2

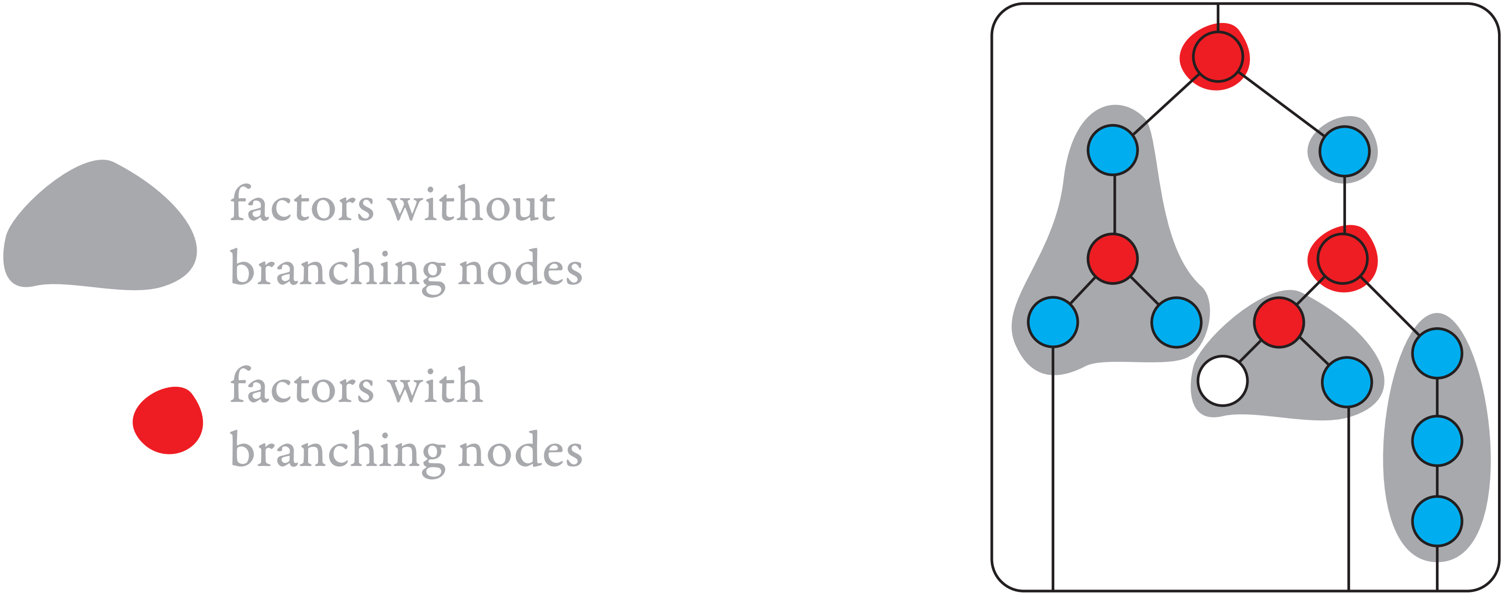


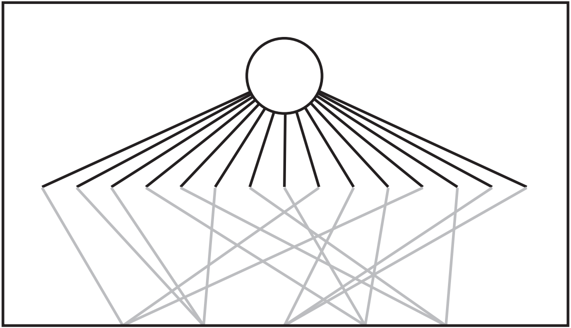
r
s
s
r



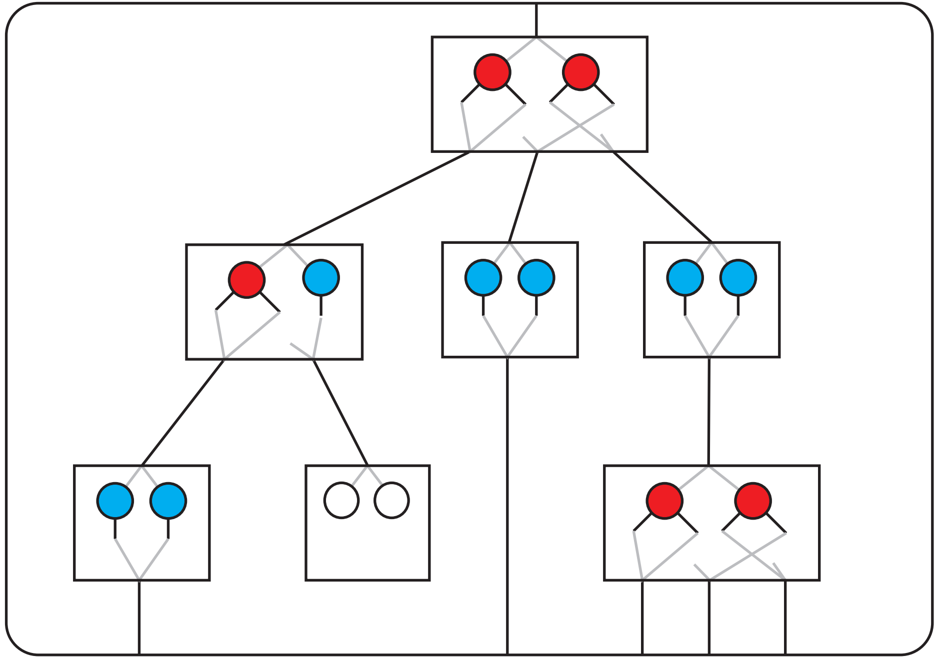
r
s


r

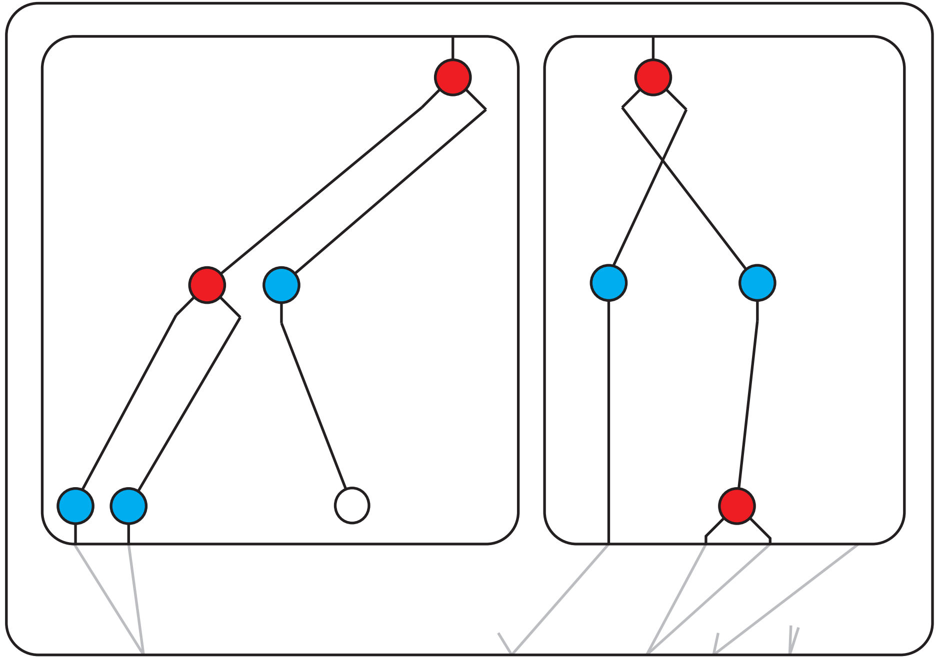

factors without branching nodes
factors with branching nodes

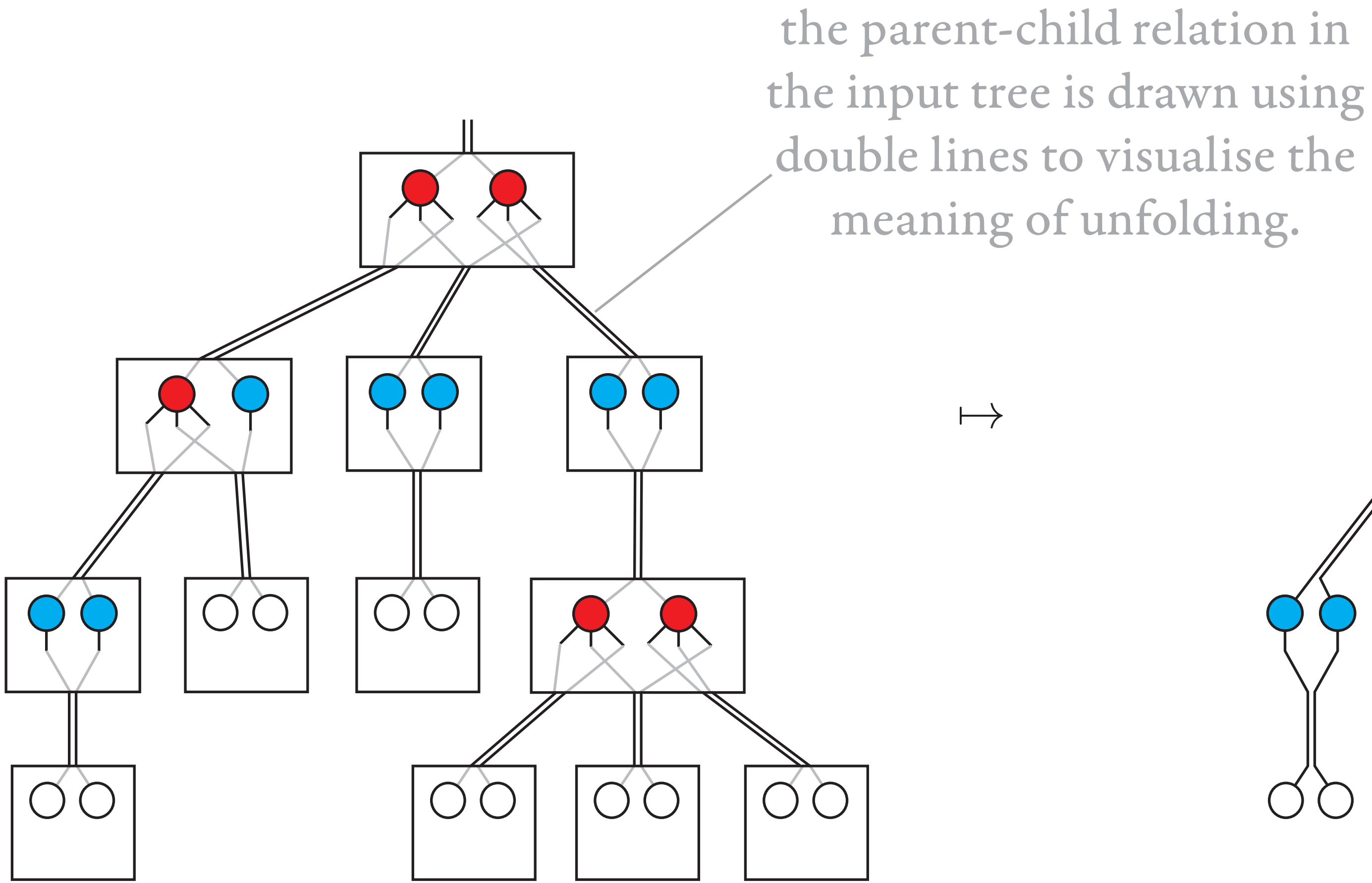


a term of matrix powers

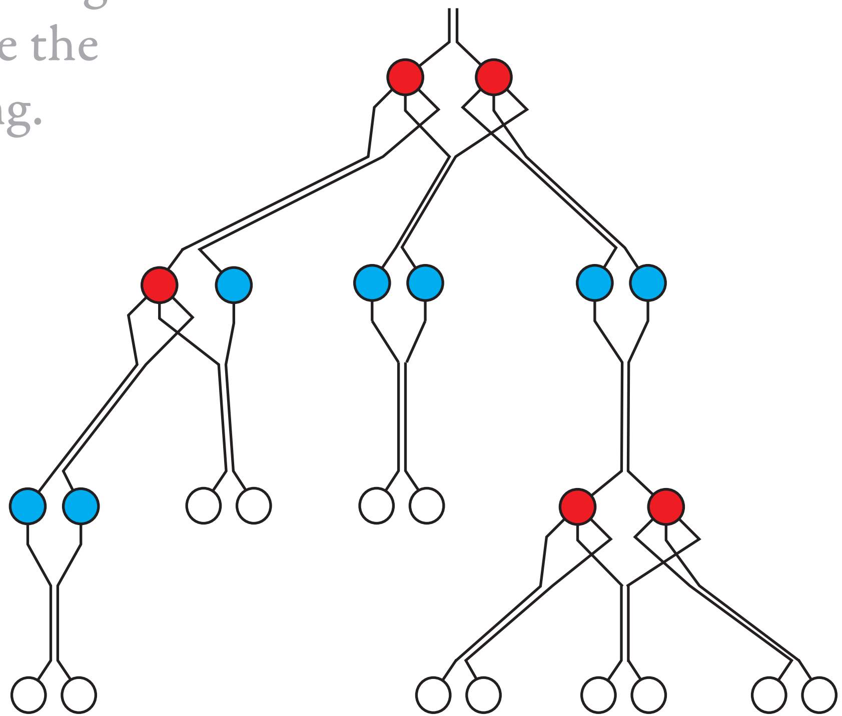


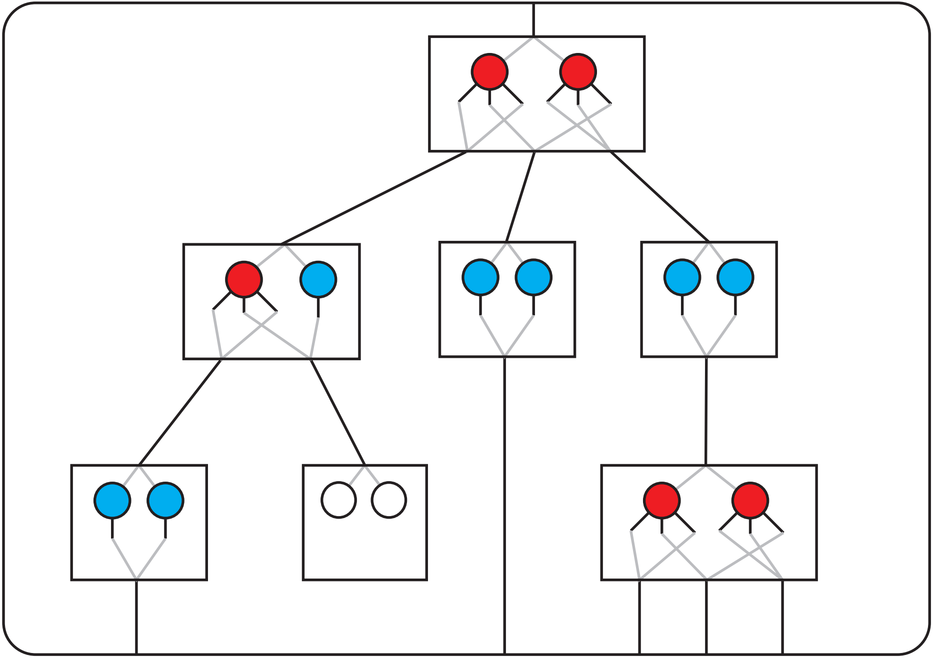
its term unfolding

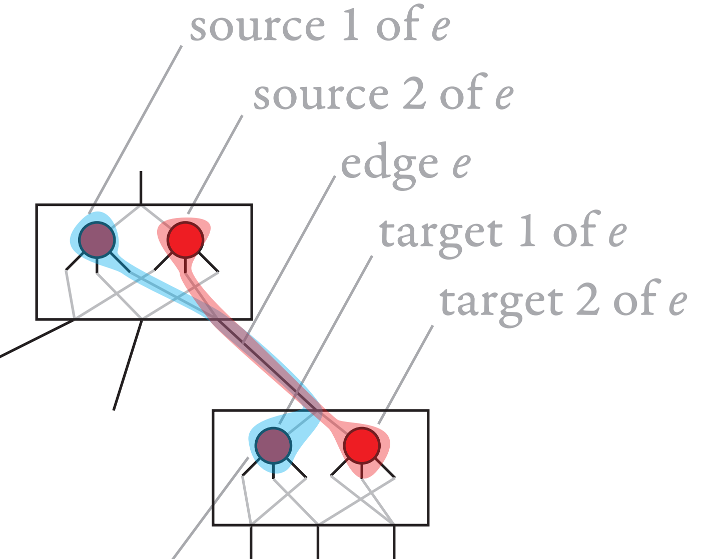


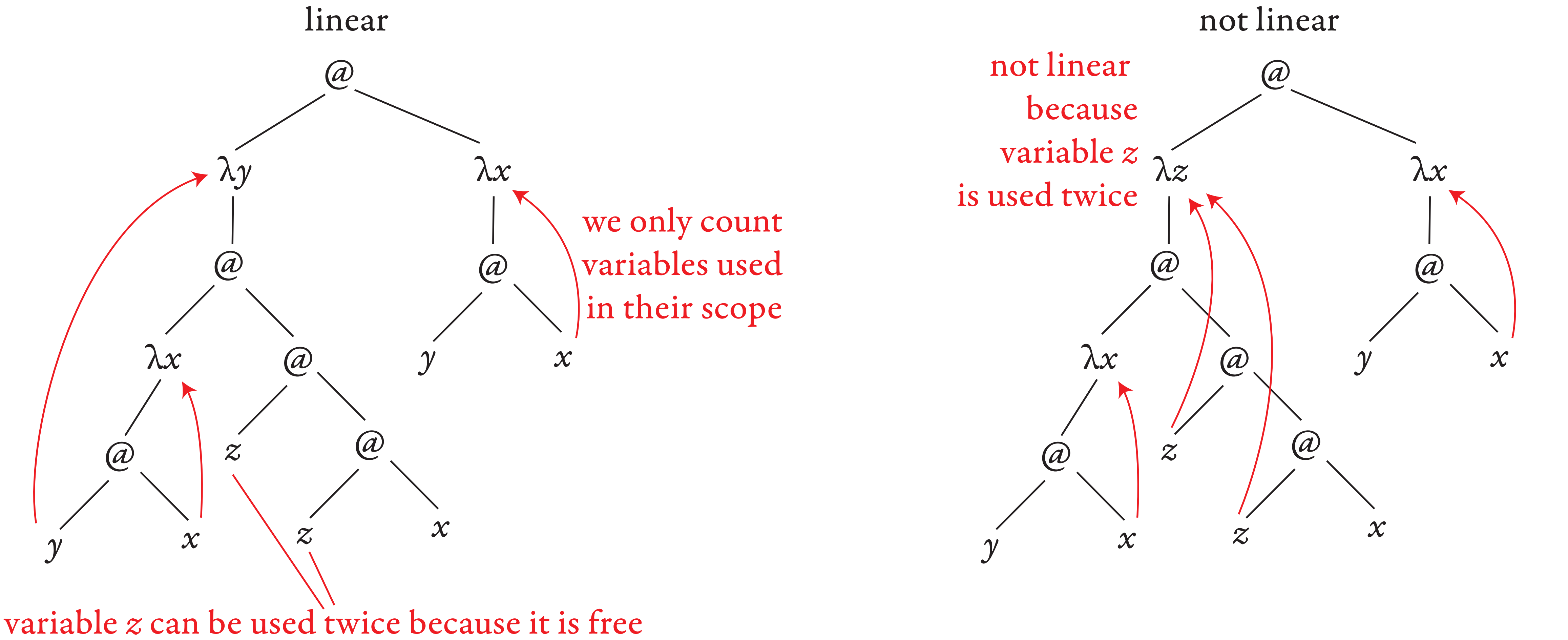

the parent-child relation in the input tree is drawn using double lines to visualise the meaning of unfolding.

$\mapsto$

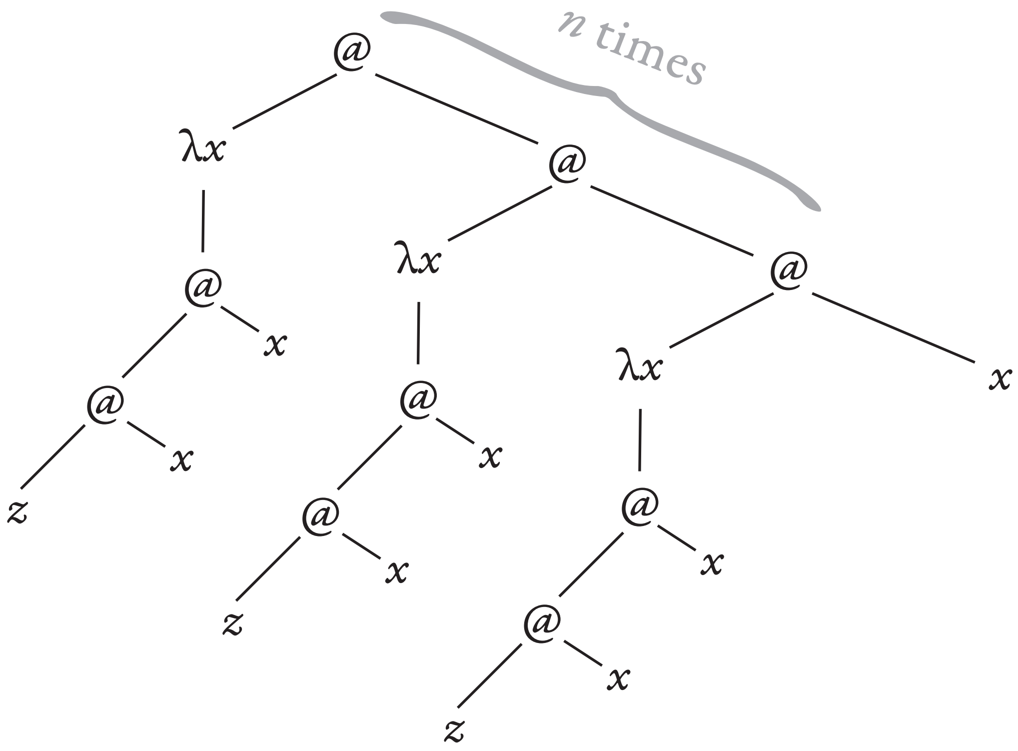




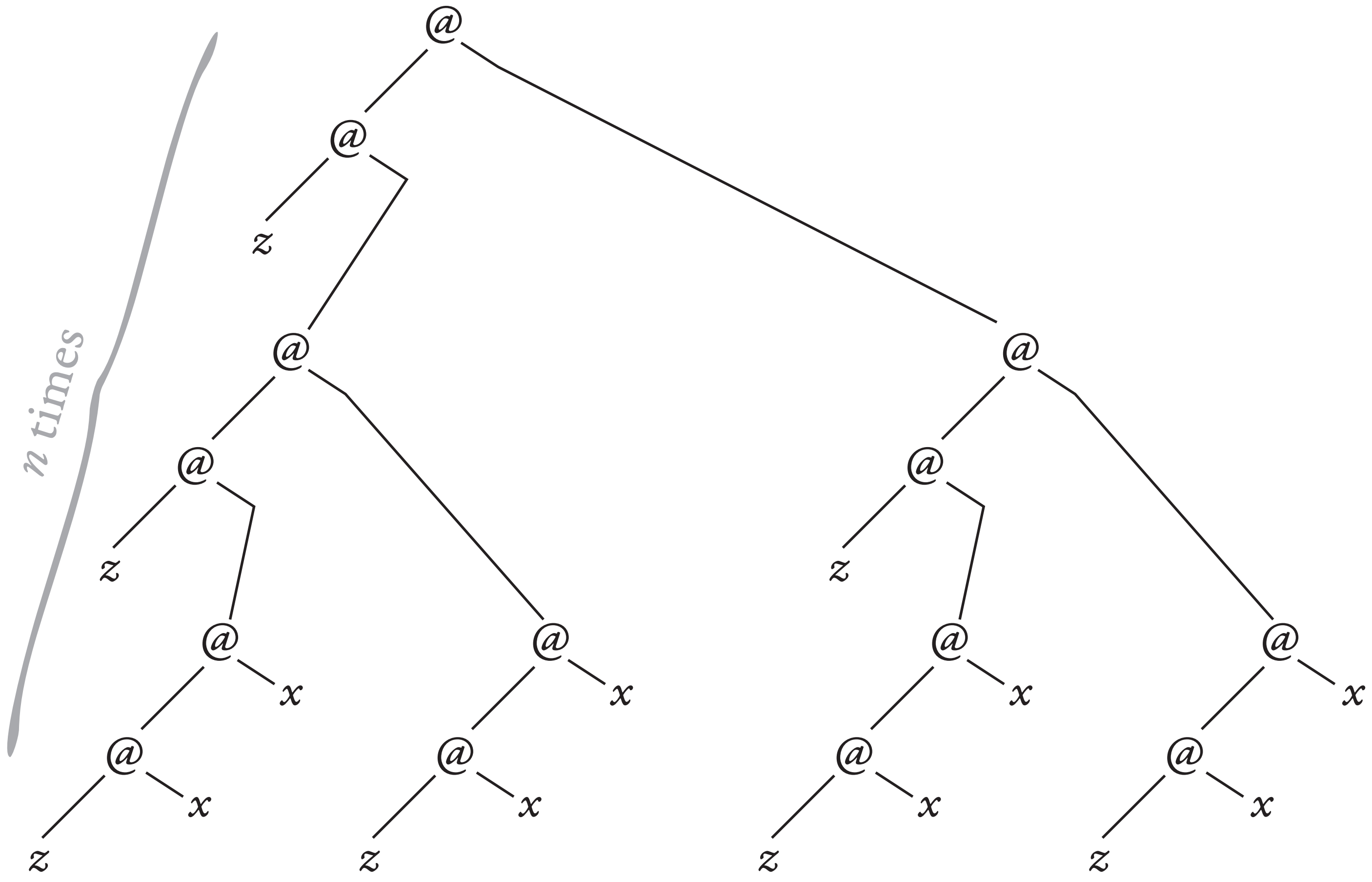

source 1 of *e*
source 2 of *e*
edge *e*
target 1 of *e*
target 2 of *e*

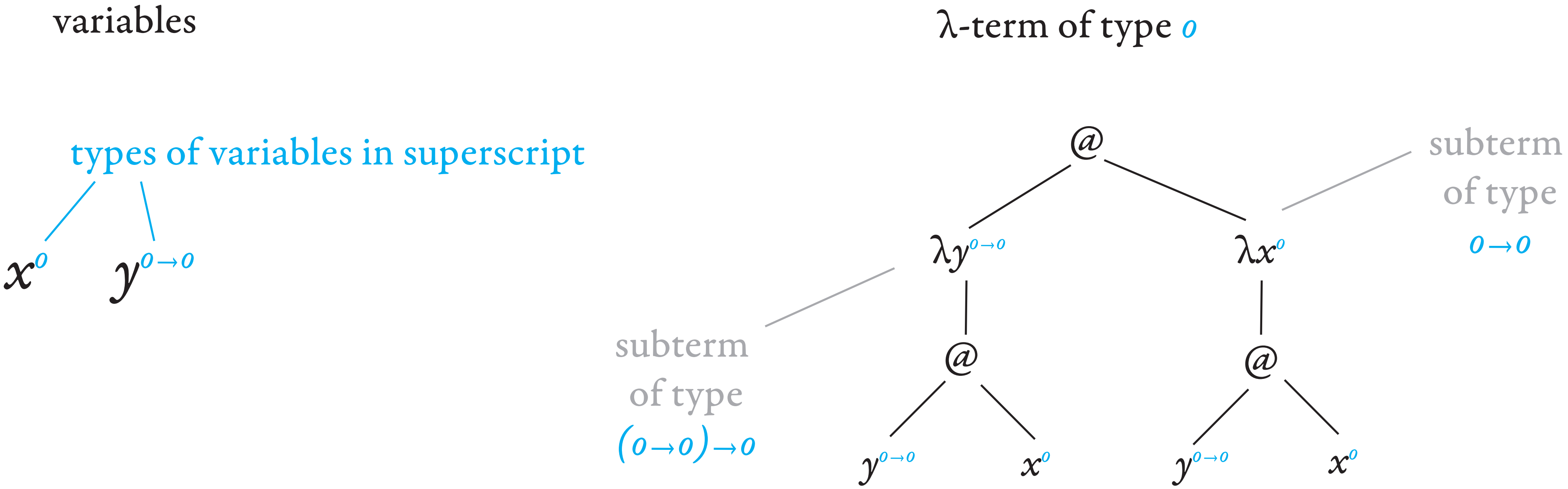

linear
not linear
we only count
variables used
in their scope
variable z can be used twice because it is free
not linear
because
variable z
is used twice

a λ-term of size $O(n)$

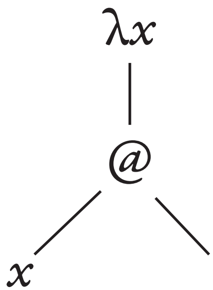


its normal form of size $O(2^n)$

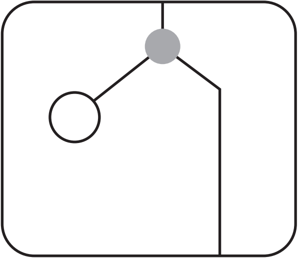


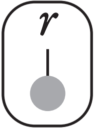

variables
types of variables in superscript
x^o
$y^{o\to o}$
λ-term of type o
@
$\lambda y^{o\to o}$
λx^o
subterm of type $o\to o$
subterm of type $(o\to o)\to o$
@
@
$y^{o\to o}$
x^o
$y^{o\to o}$
x^o

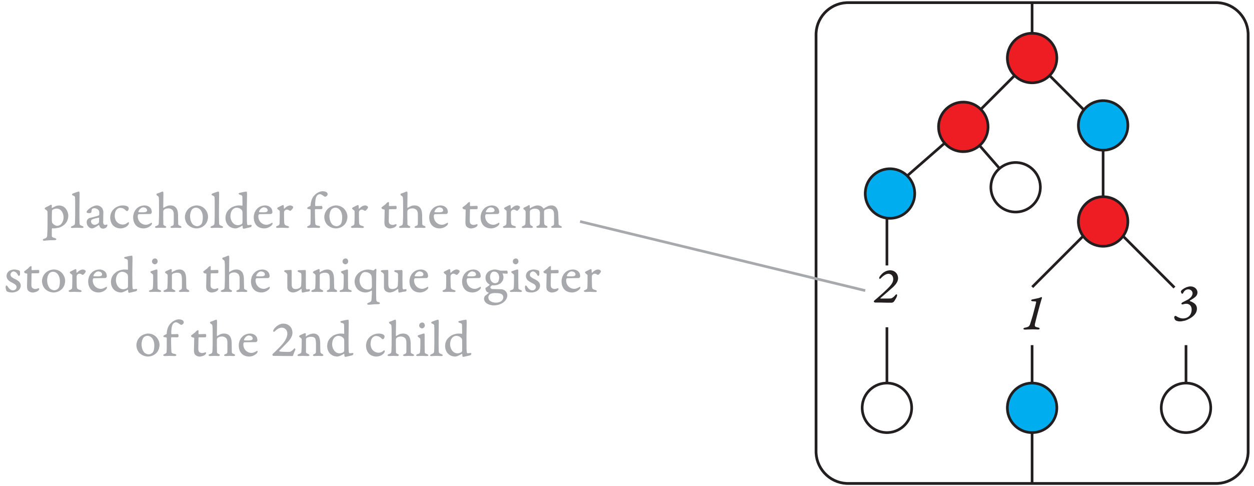
$@$

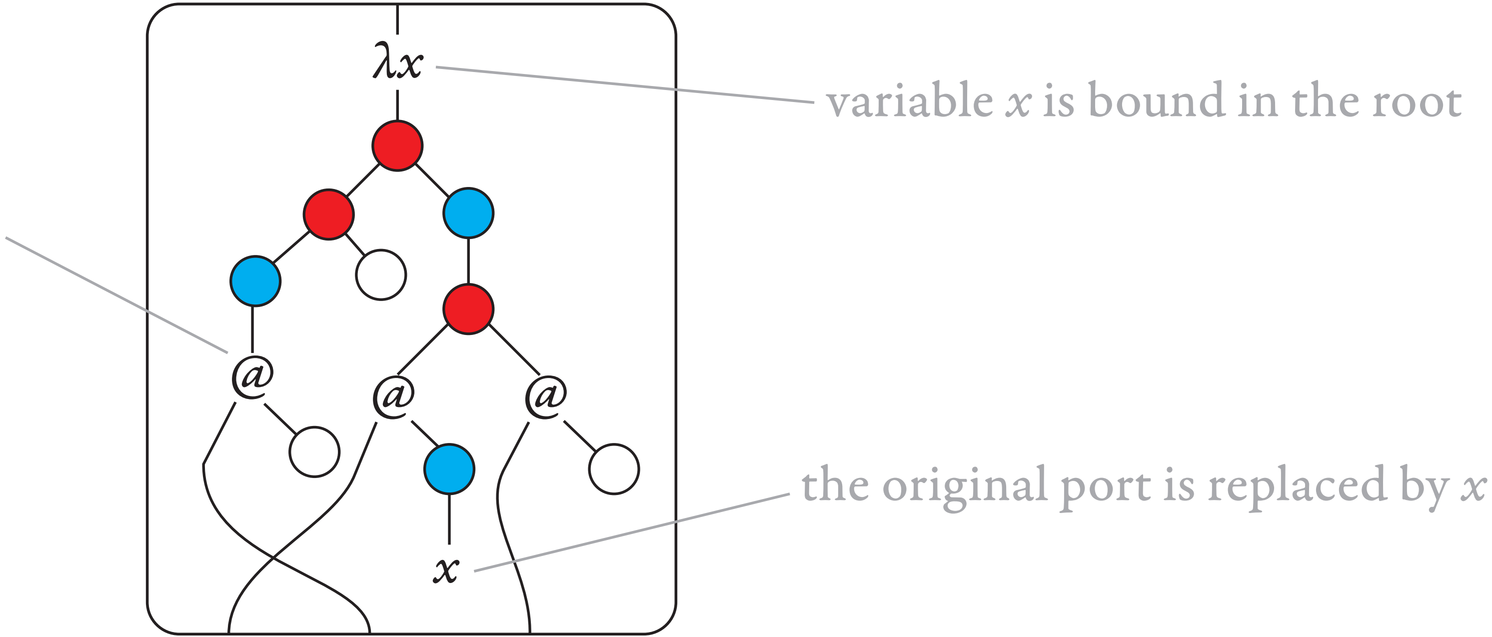


$\lambda x.$

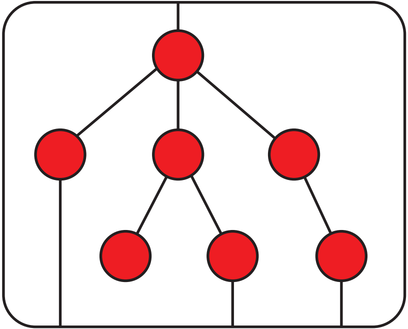



r

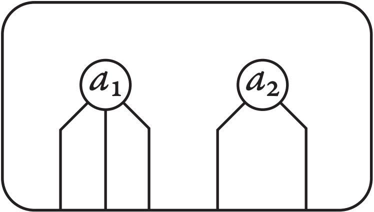

placeholder for the term stored in the unique register of the 2nd child
2
1
3



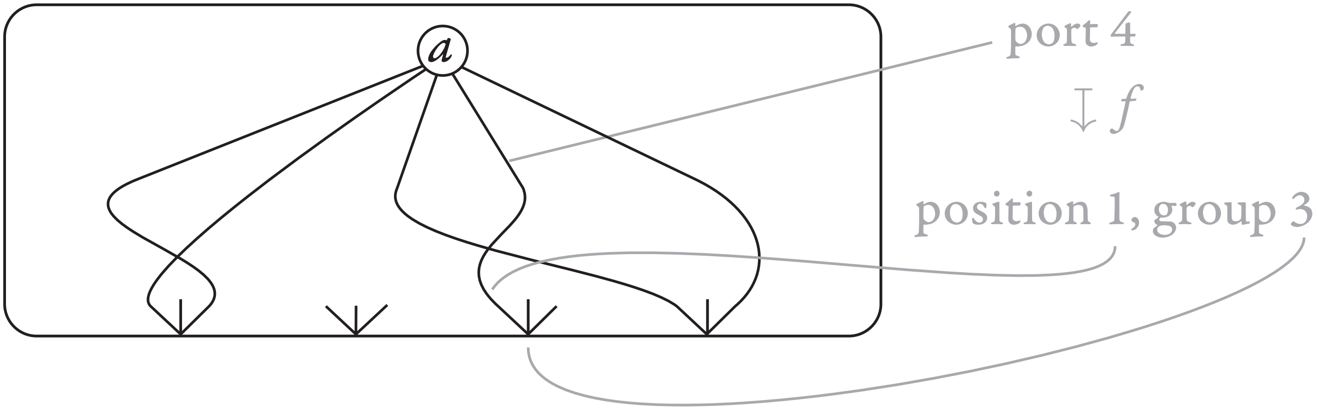
λx
variable x is bound in the root
@
@
@
x
the original port is replaced by x



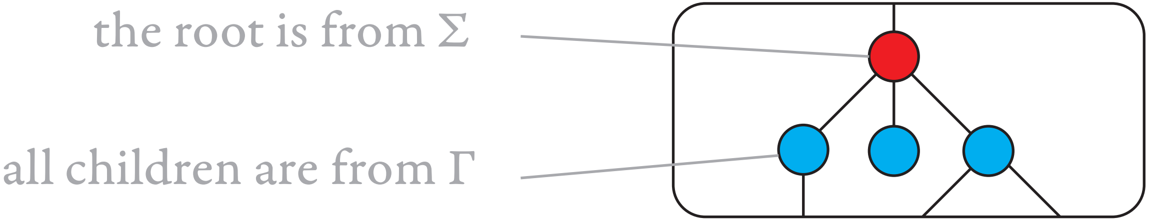
dangling edges
represent ports

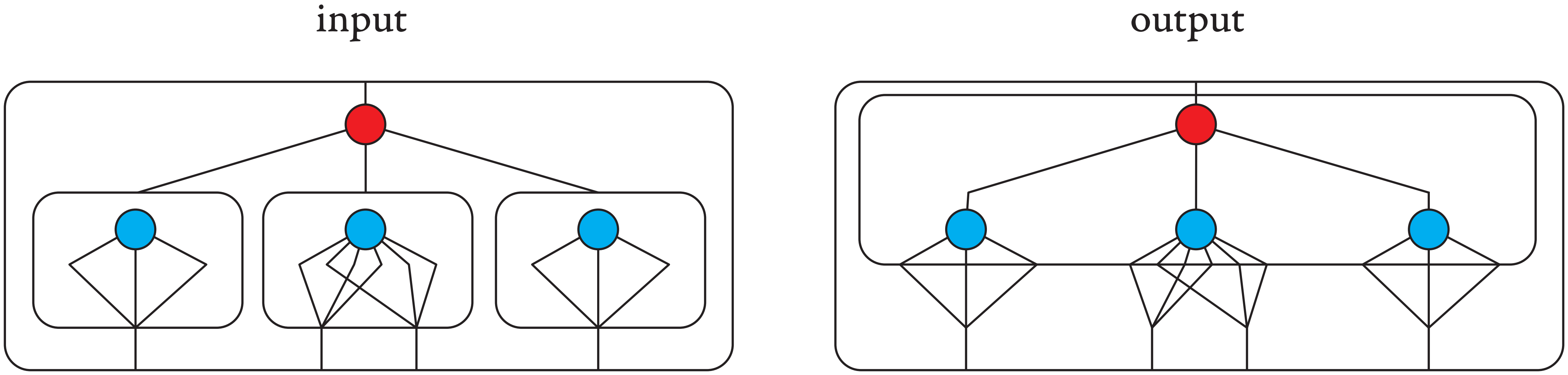


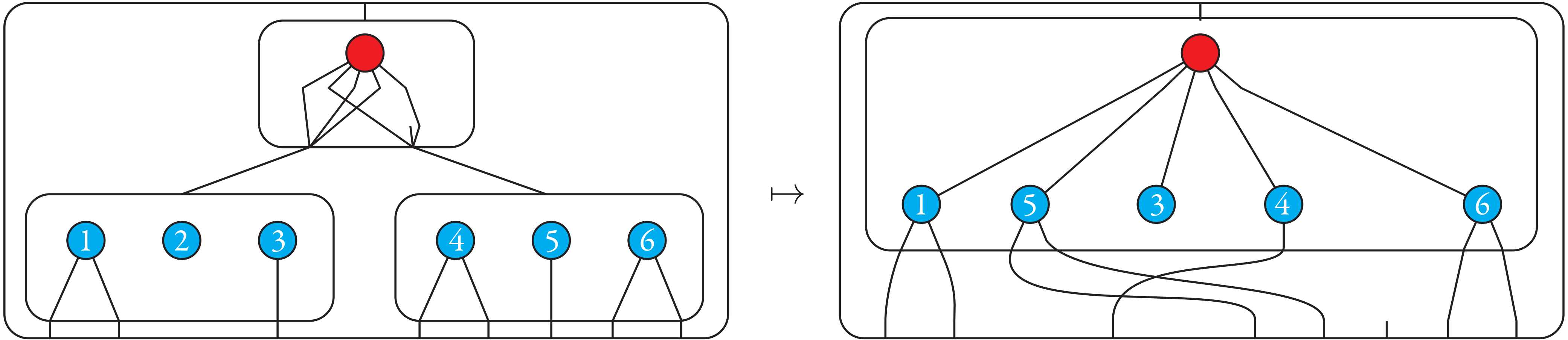
a_1
a_2



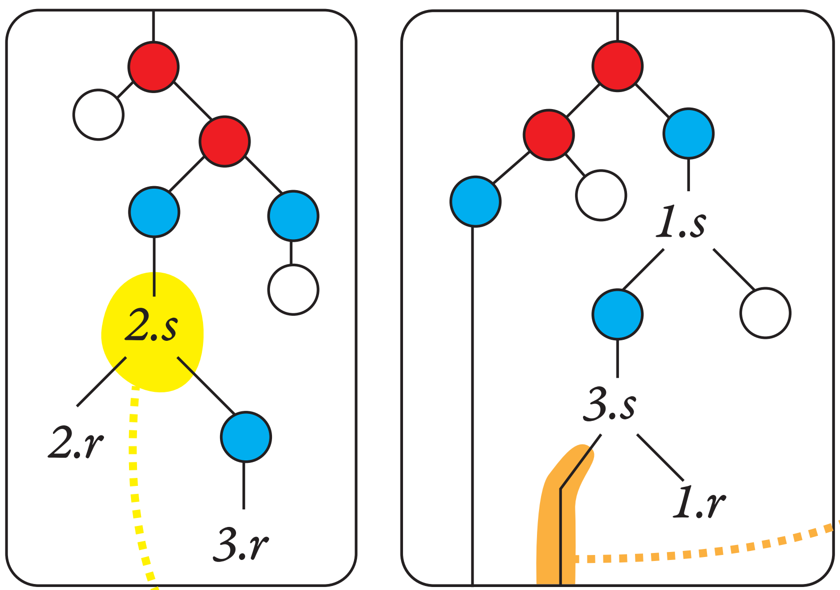
a
port 4
$\downarrow f$
position 1, group 3


the root is from Σ
all children are from Γ

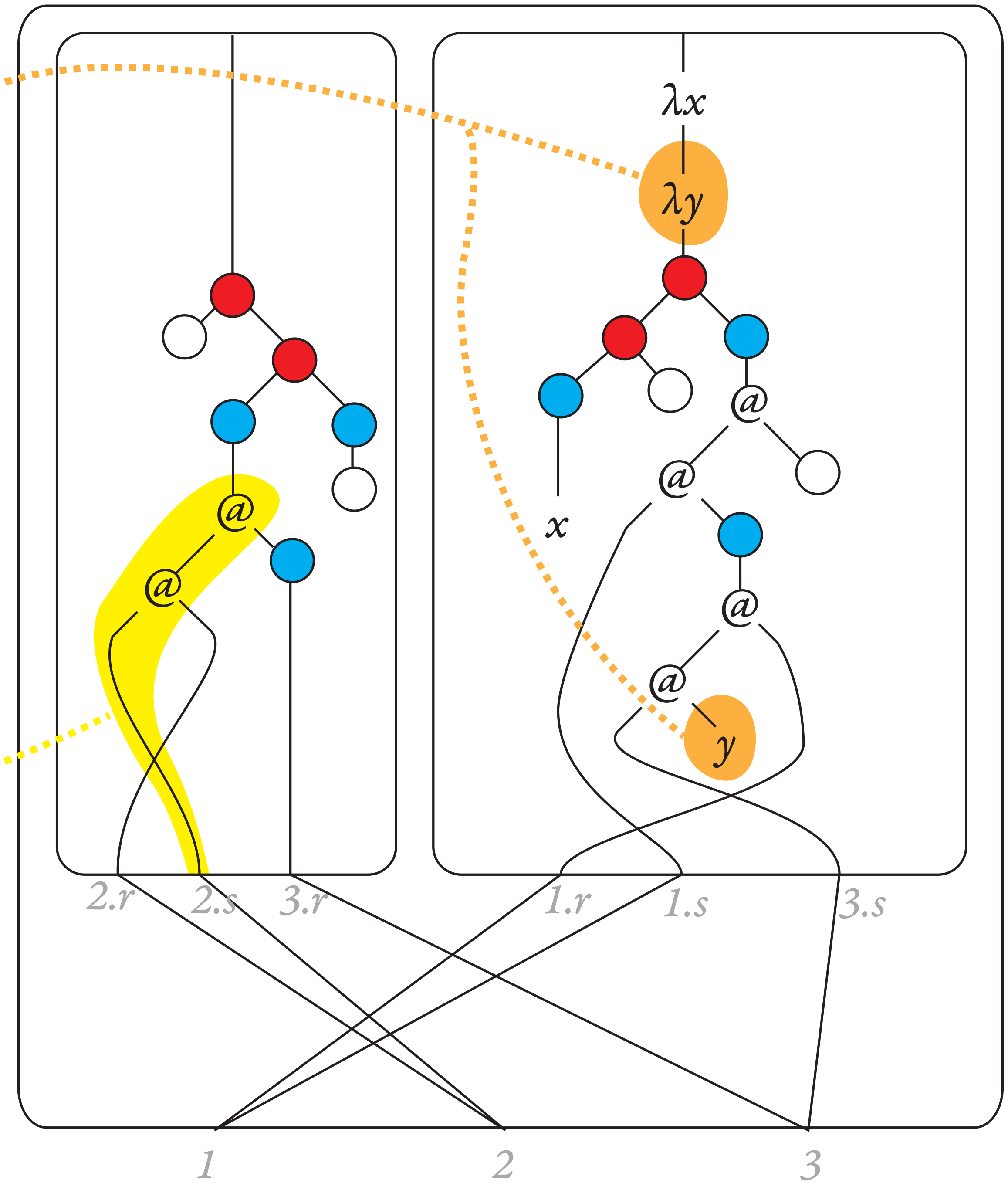

input
output

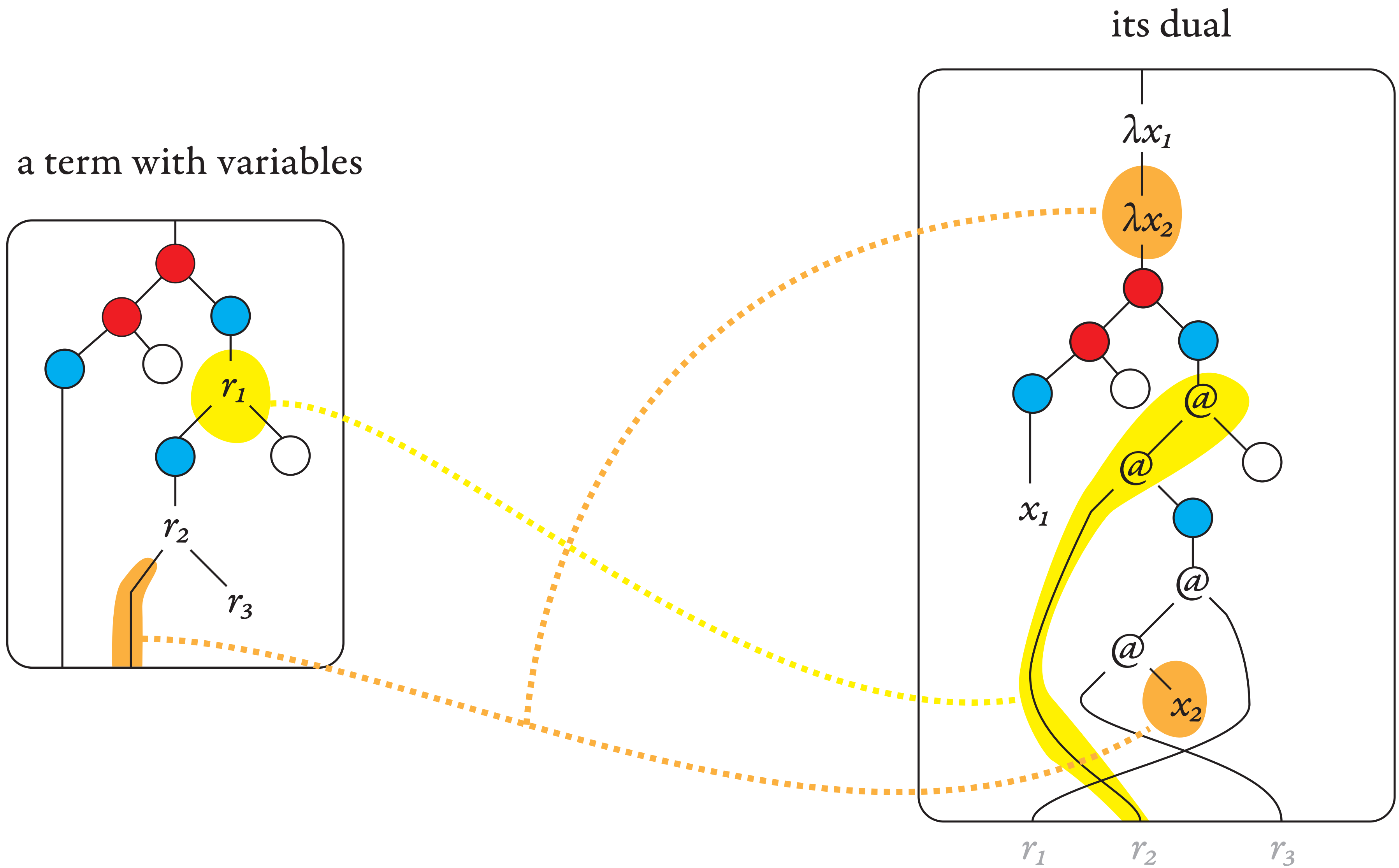

1 2 3 4 5 6
1 5 3 4 6

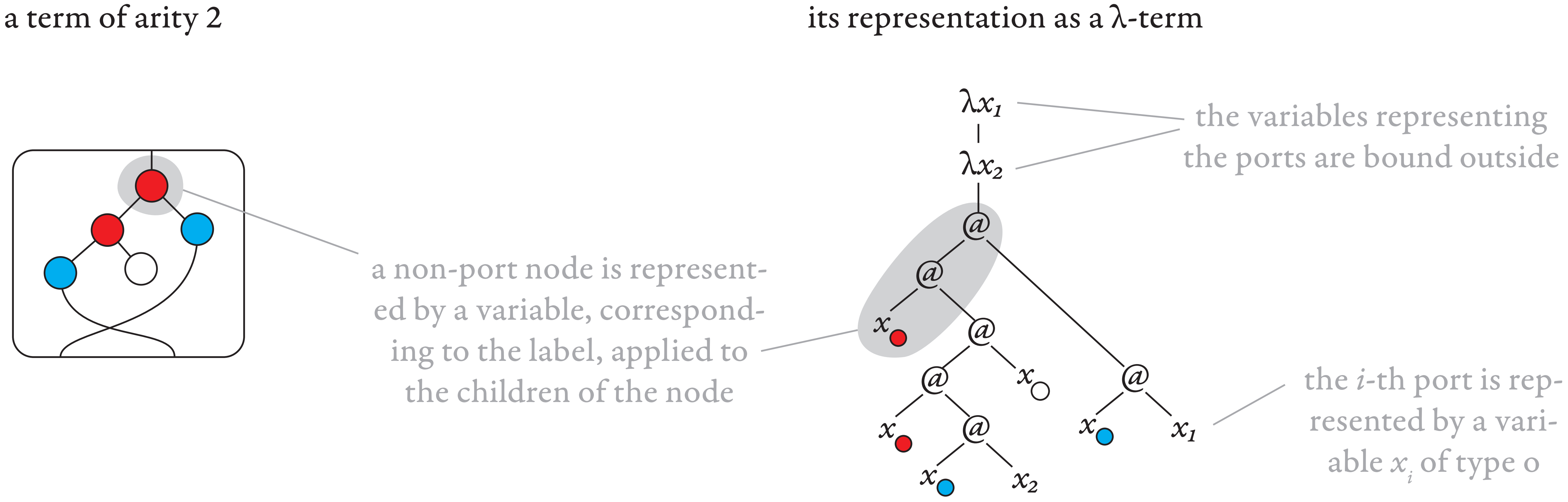
a register update



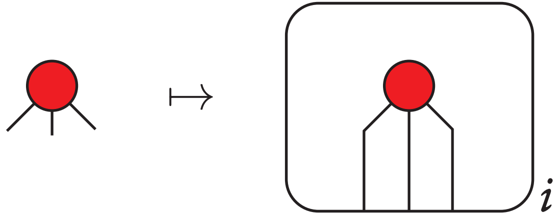
its dual

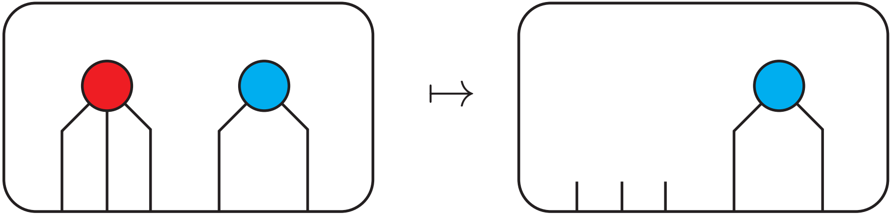


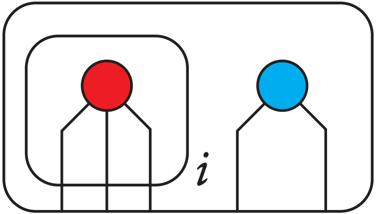

a term with variables
r_1
r_2
r_3
its dual
λx_1
λx_2
@
@
@
@
x_1
x_2
r_1
r_2
r_3


a term of arity 2
its representation as a λ-term
λx1
λx2
the variables representing the ports are bound outside
@
@
@
@
@
@
x
x
x
x
x
x1
x2
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
the i-th port is represented by a variable xi of type o

i


$\mapsto$
i





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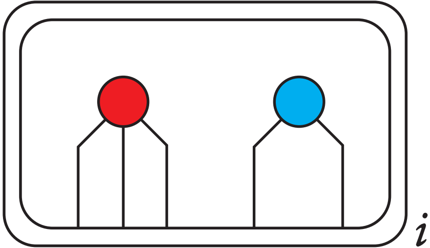

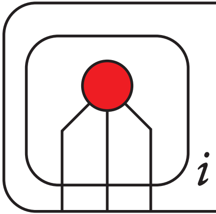

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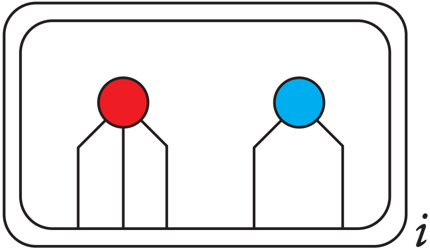
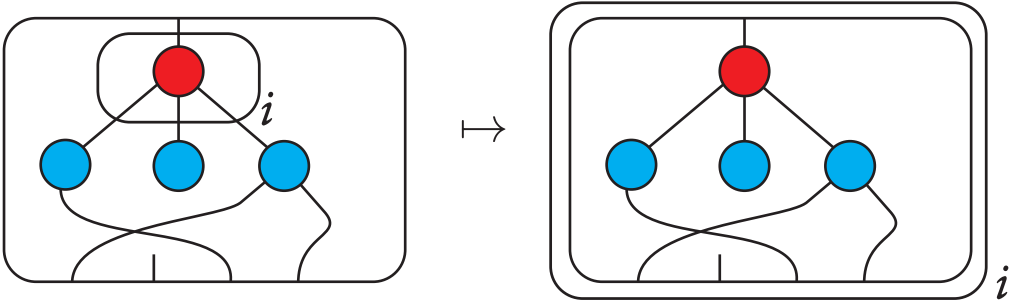
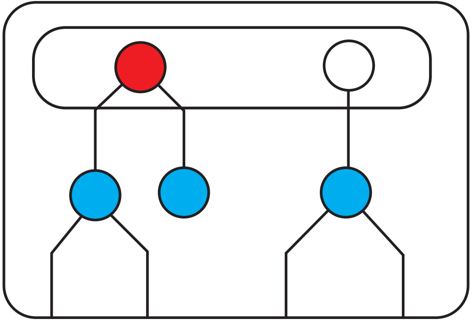
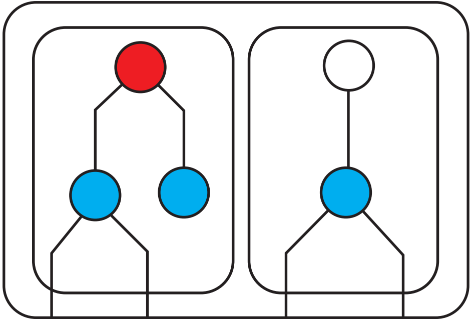
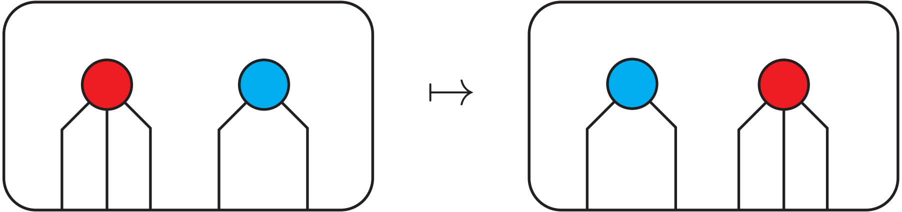


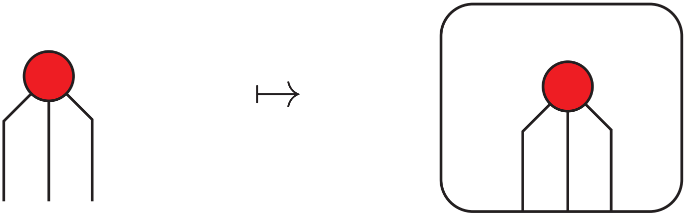

i
i

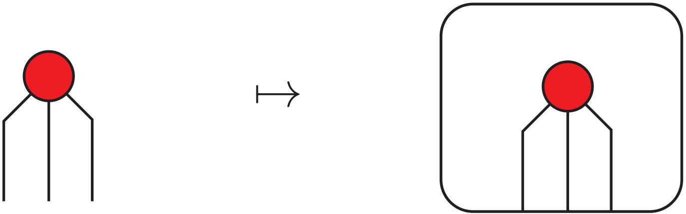


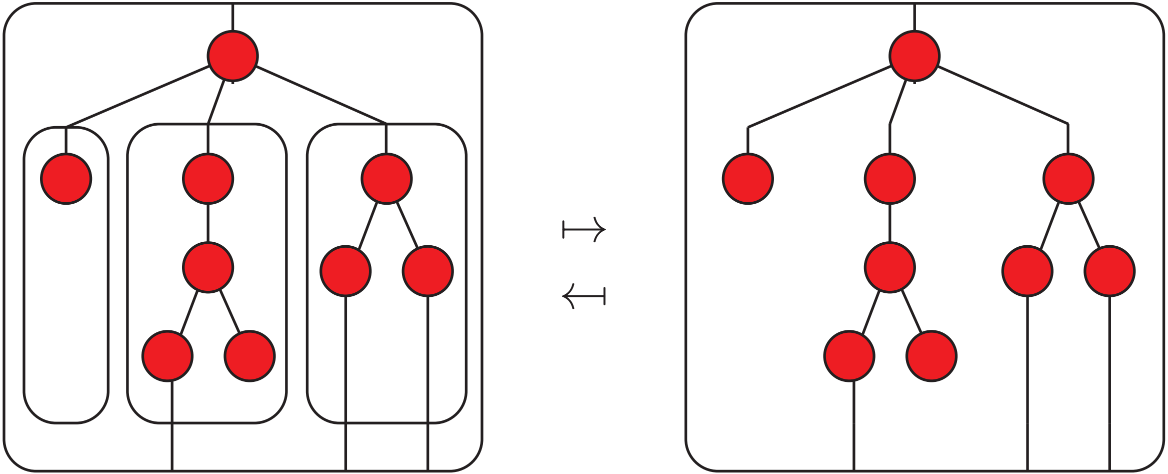
$$\mapsto$$

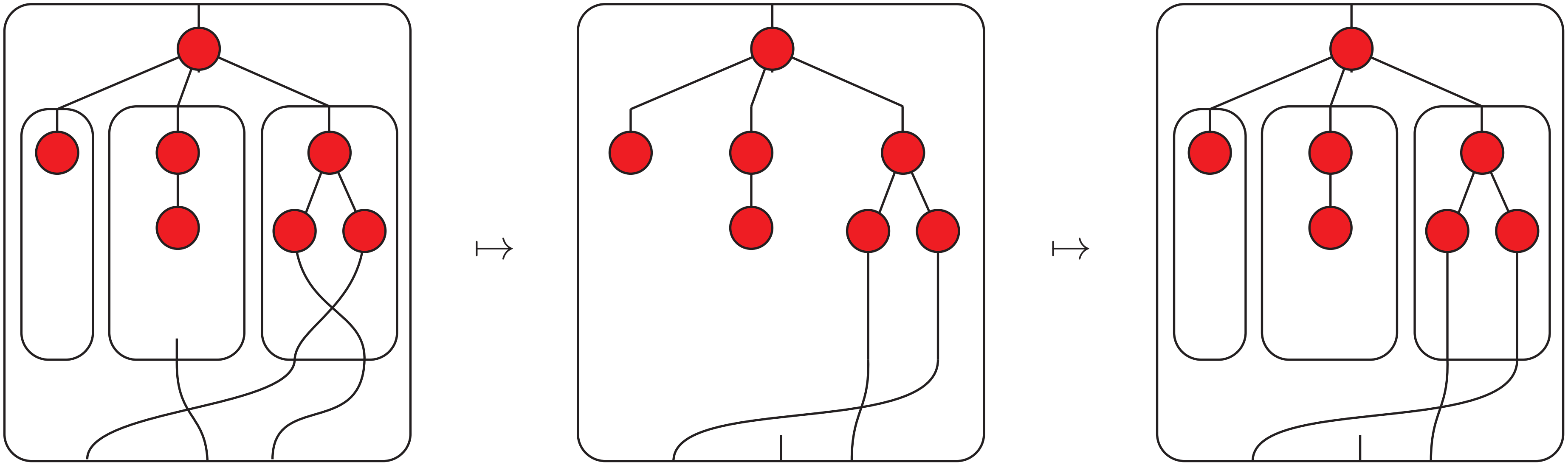


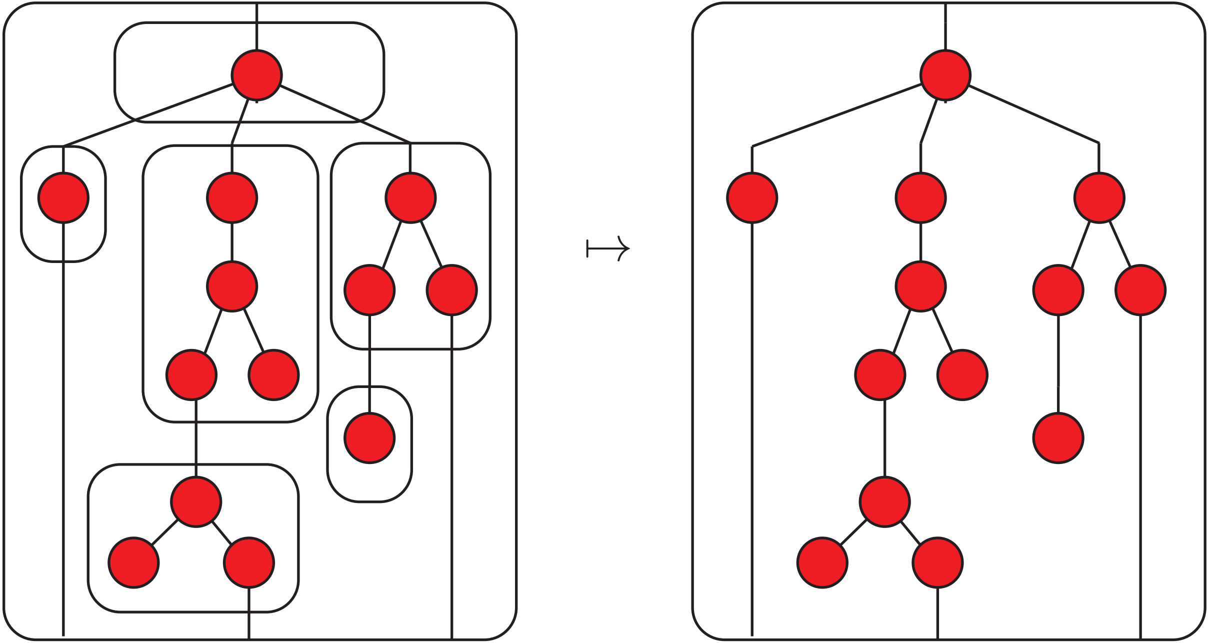


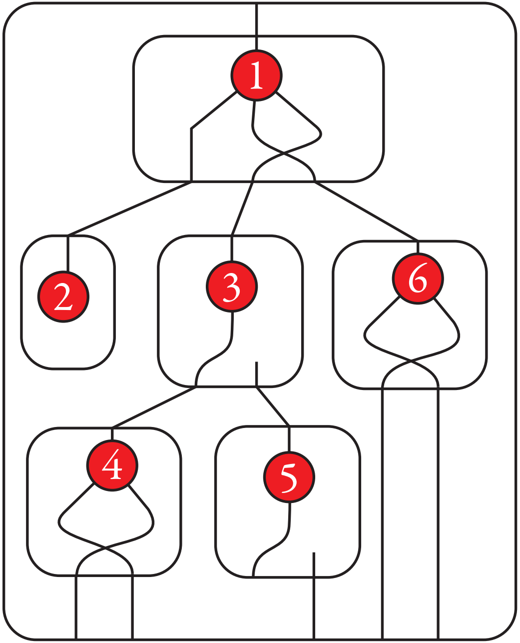




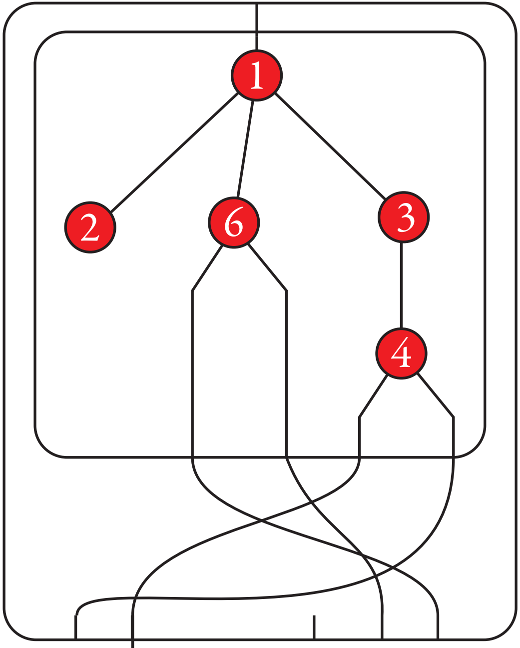


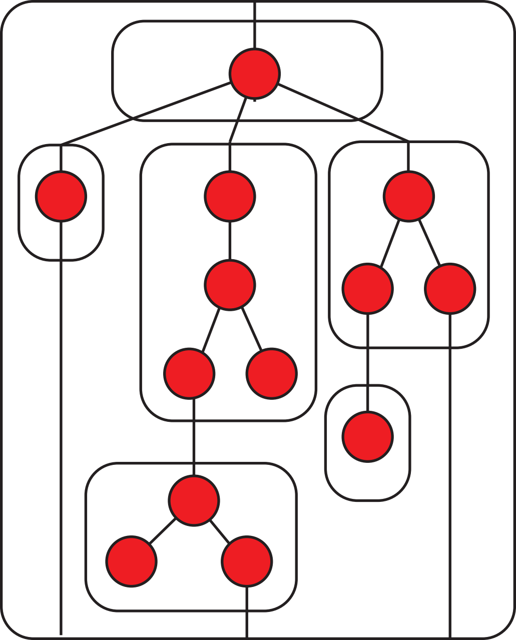


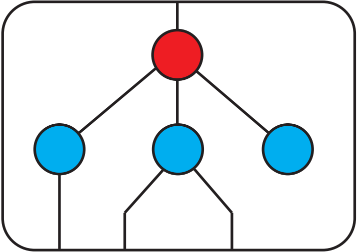


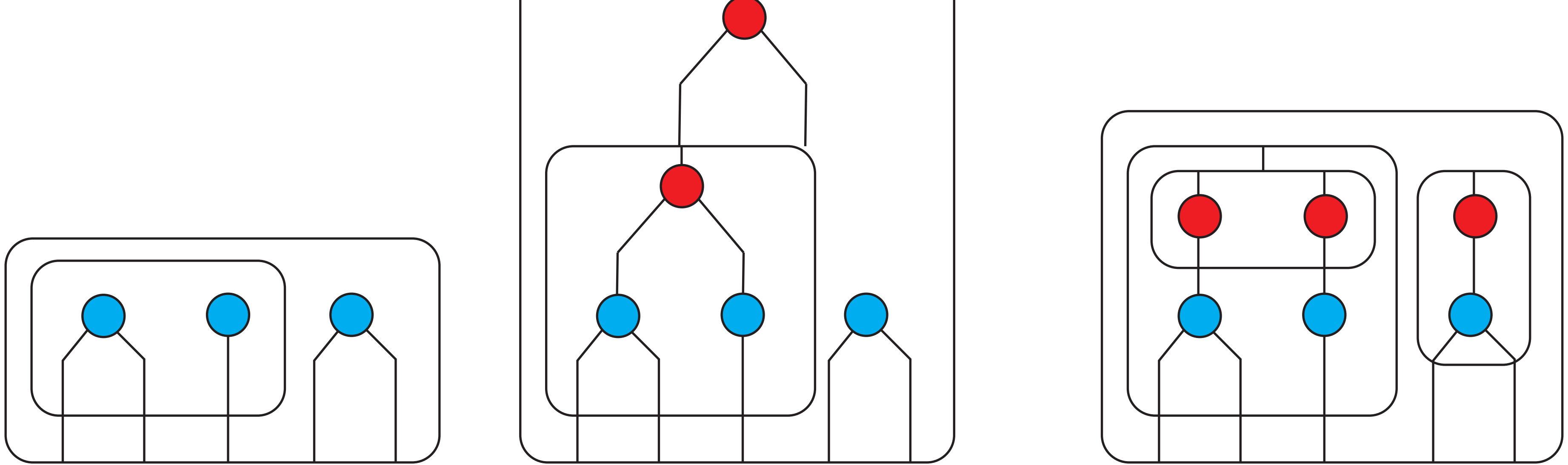


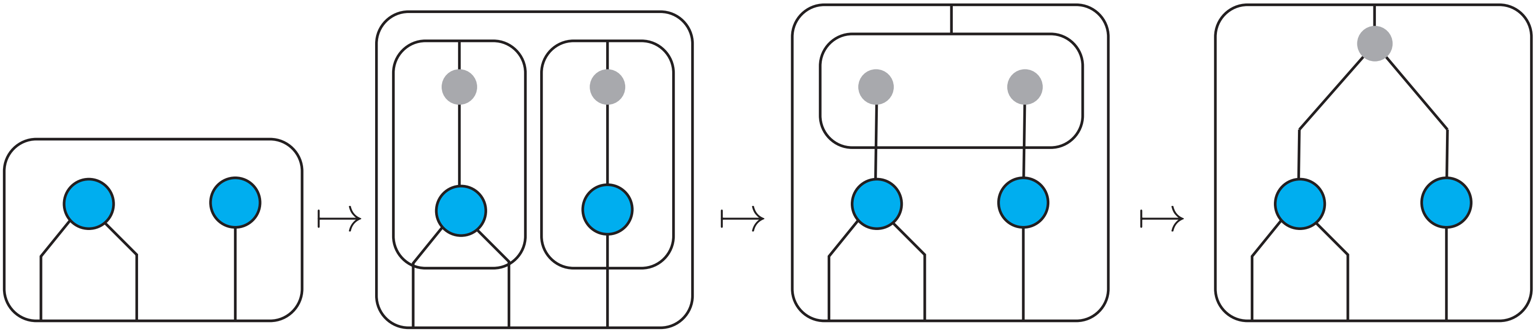
$\mapsto$

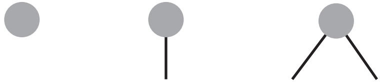


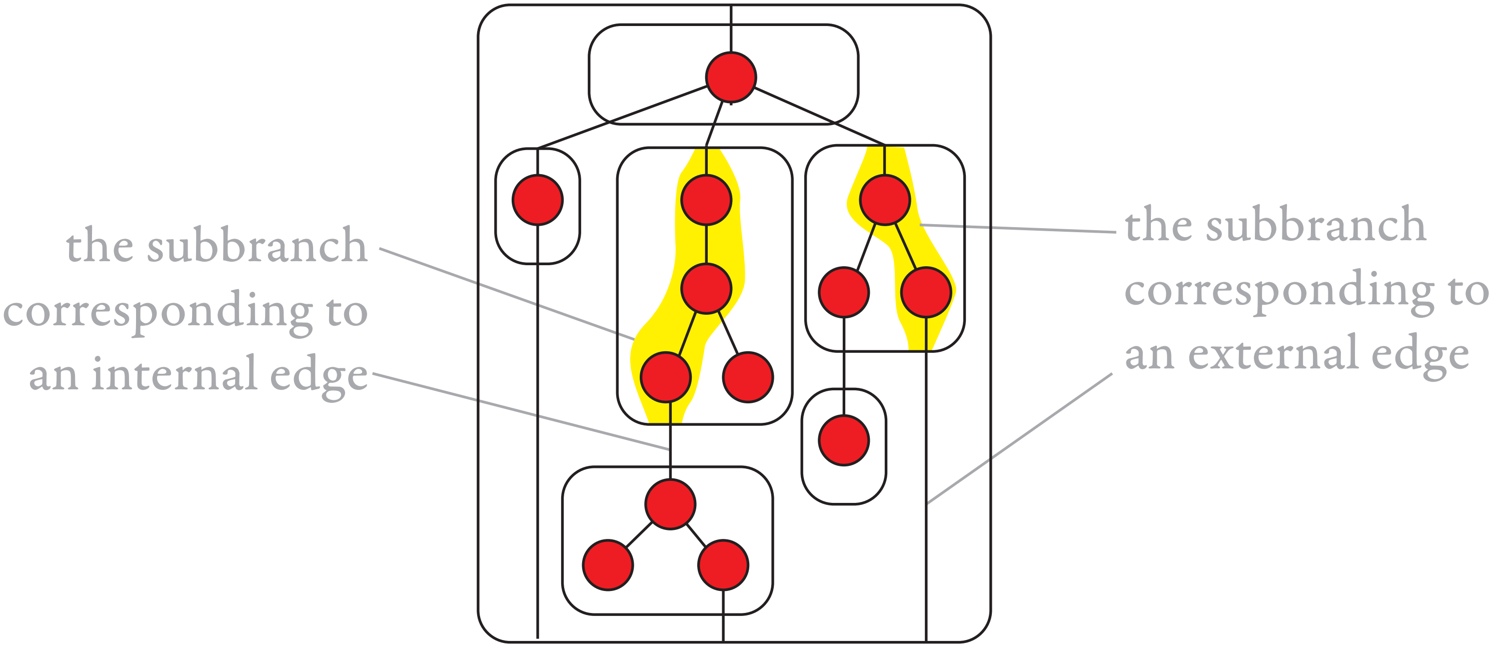


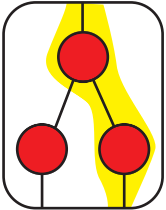




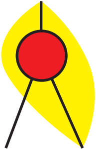


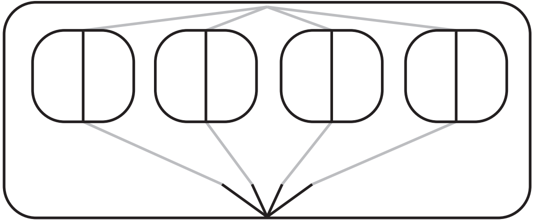



the subbranch
corresponding to
an internal edge
the subbranch
corresponding to
an external edge

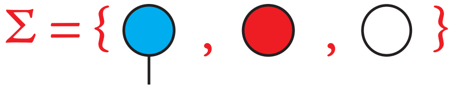


a branch can be visualised as
a term with a distinguished
root-to-port path

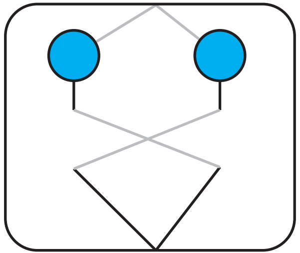




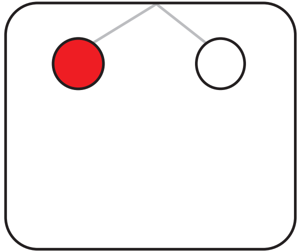
a tuple of *k* identity terms with all their ports folded into one

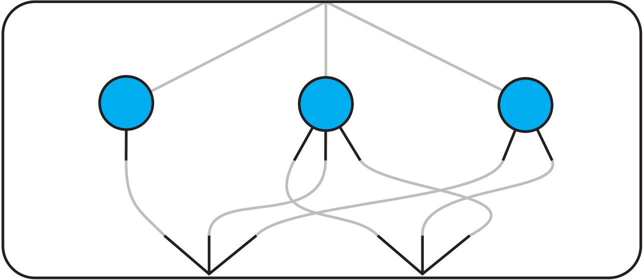


$a \in \Sigma^{[2]}$



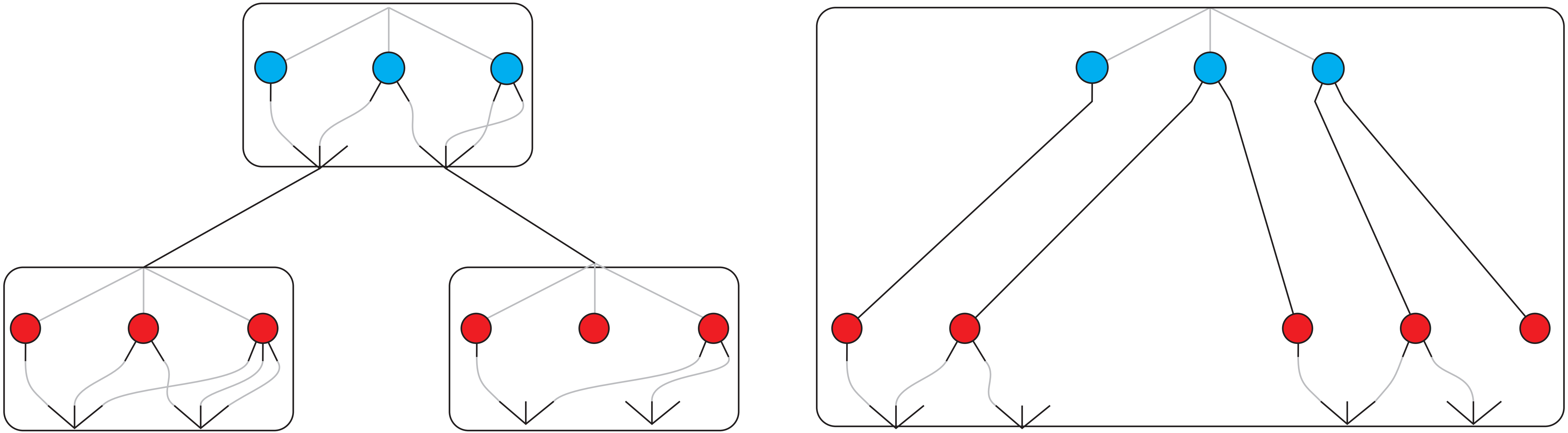
$b \in \Sigma^{[2]}$

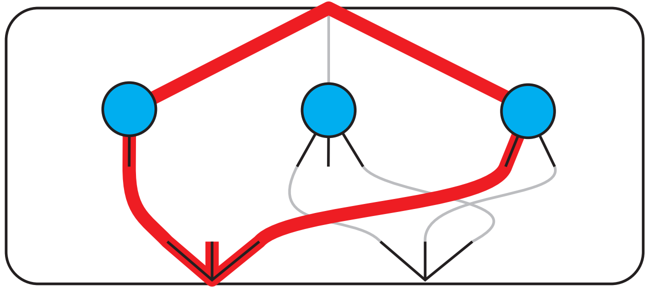




a shallow term of matrix powers

its shallow unfolding

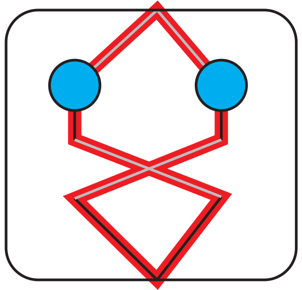




twist of port 1

1 2 3

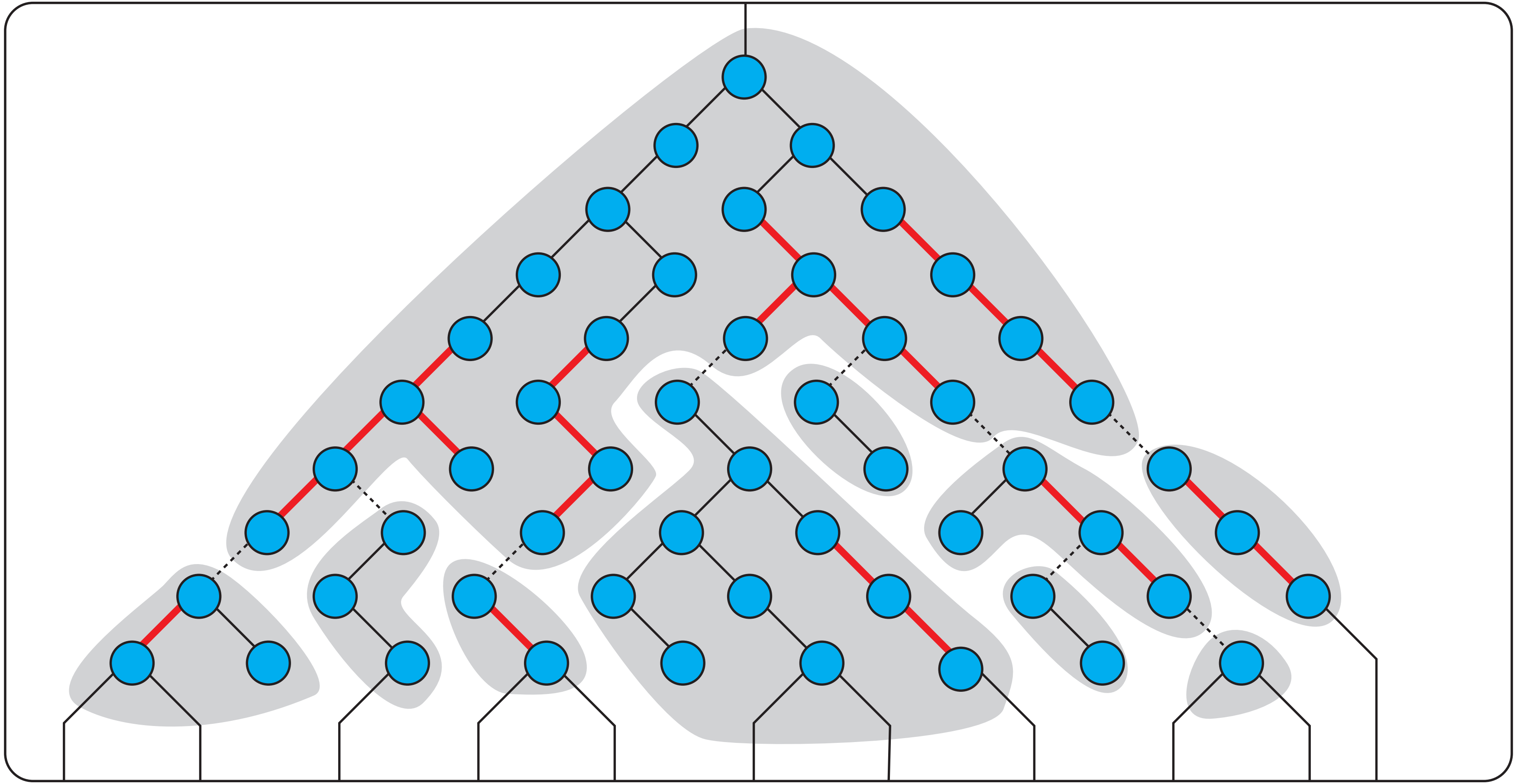
1 2 3

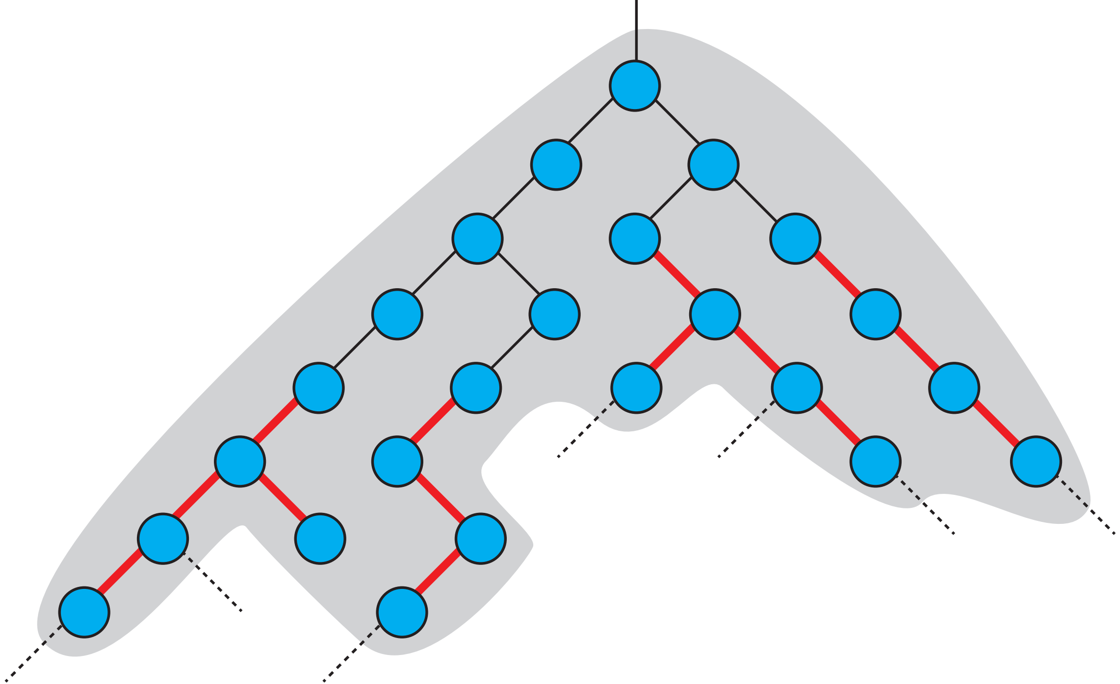


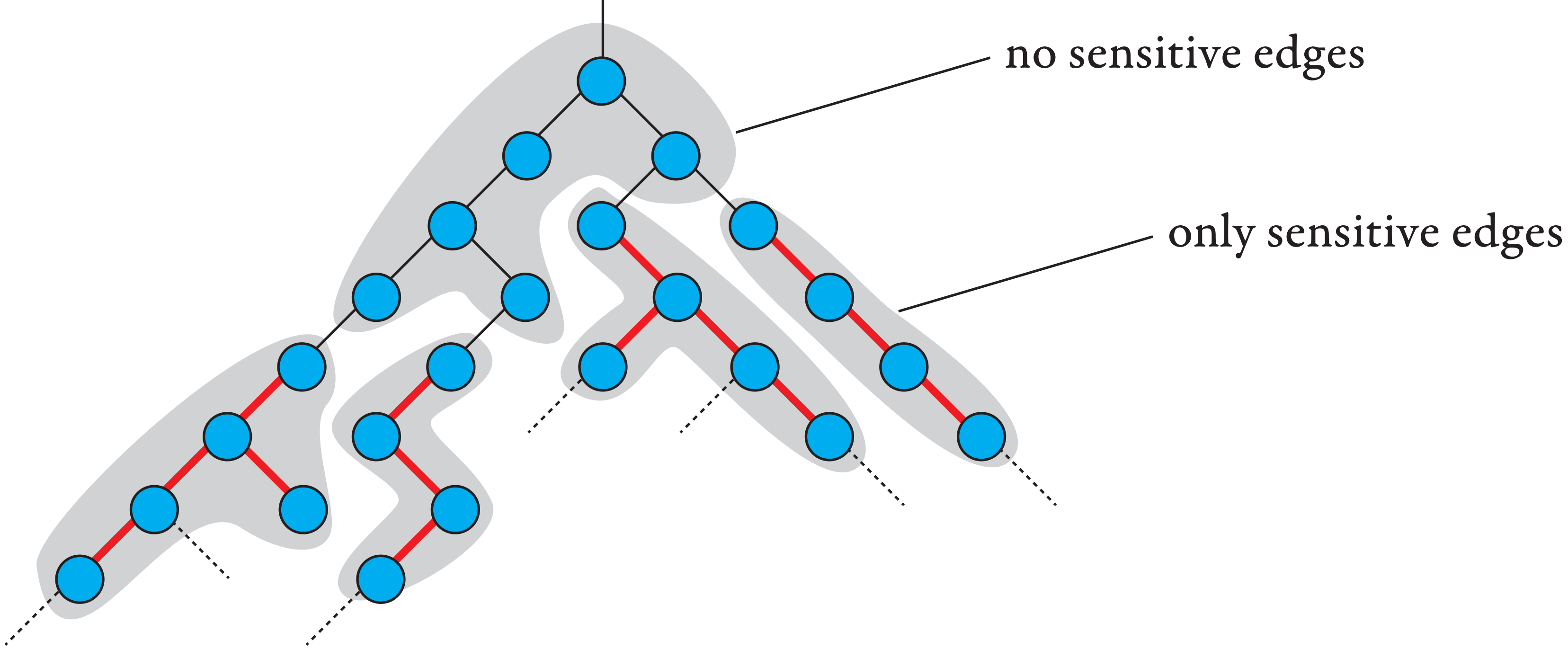
twist of port 1

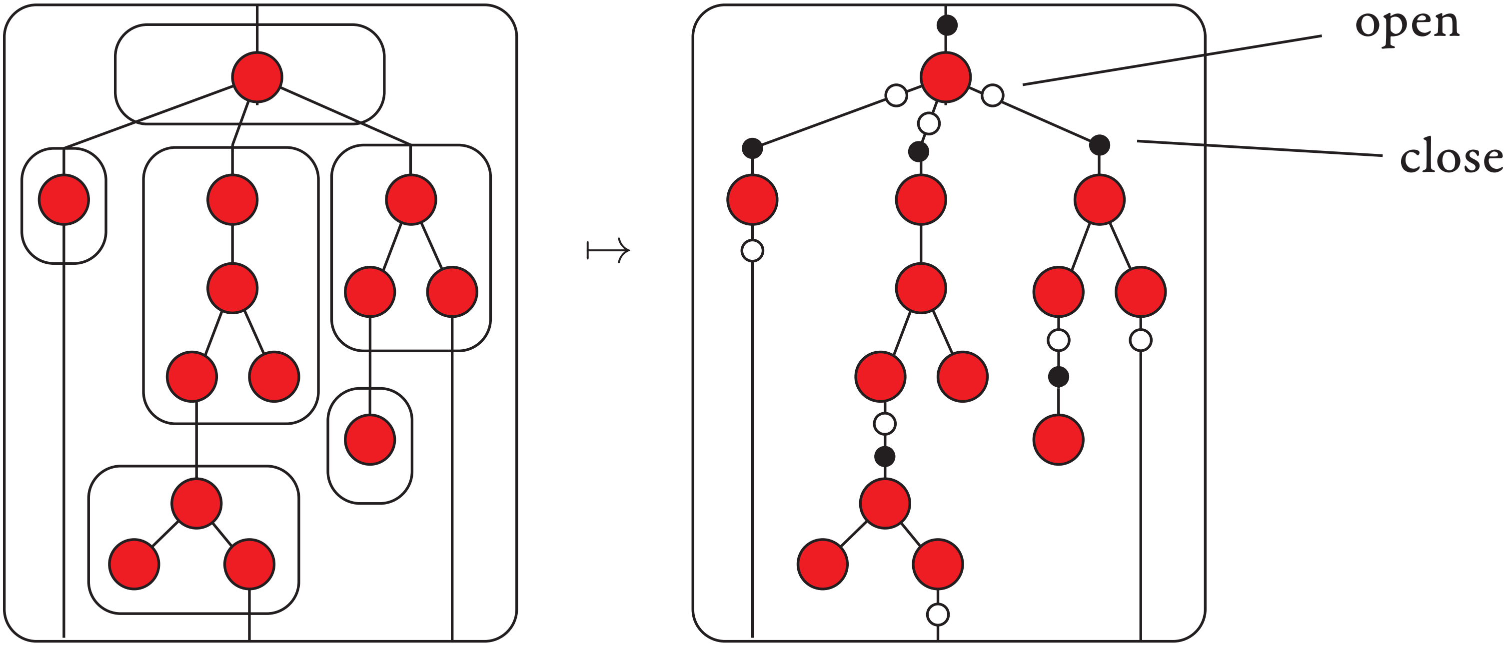
1 2

1 2

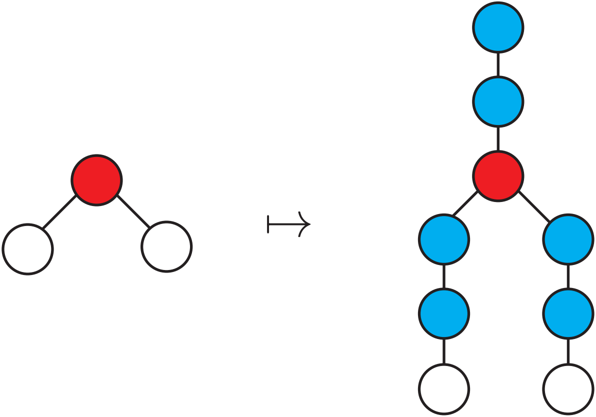


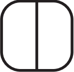


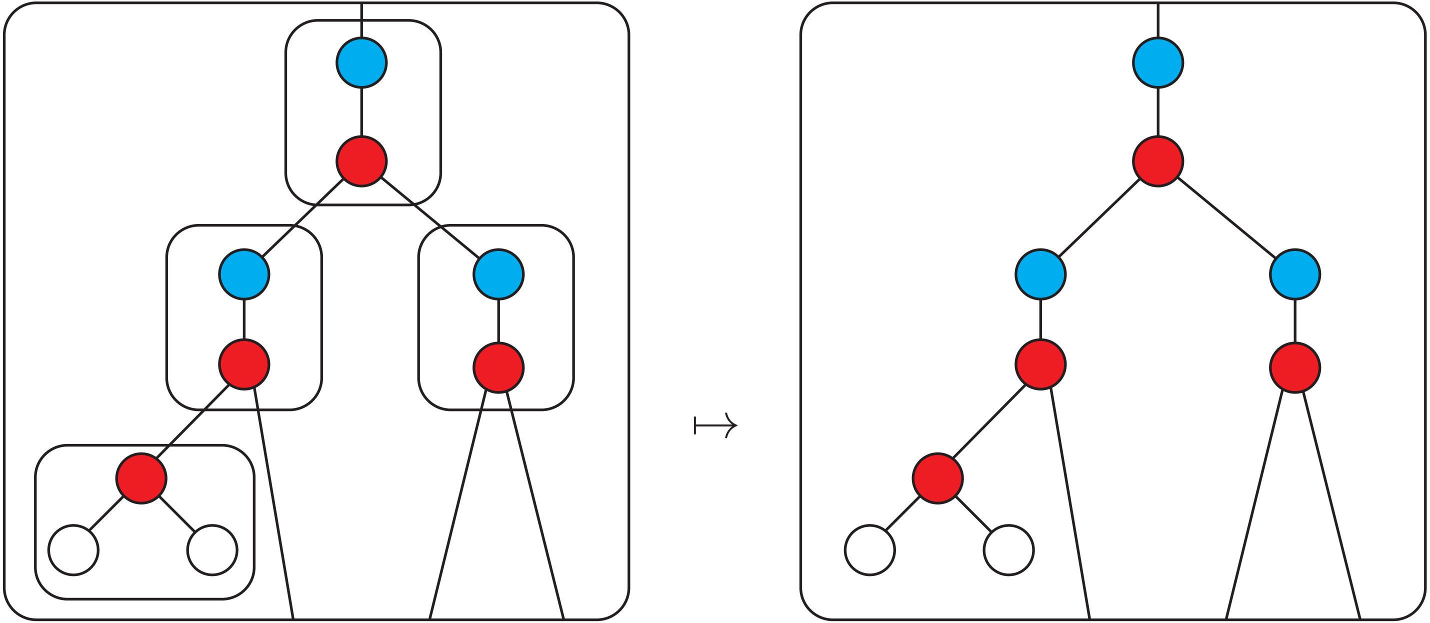

no sensitive edges
only sensitive edges

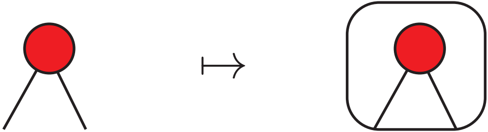


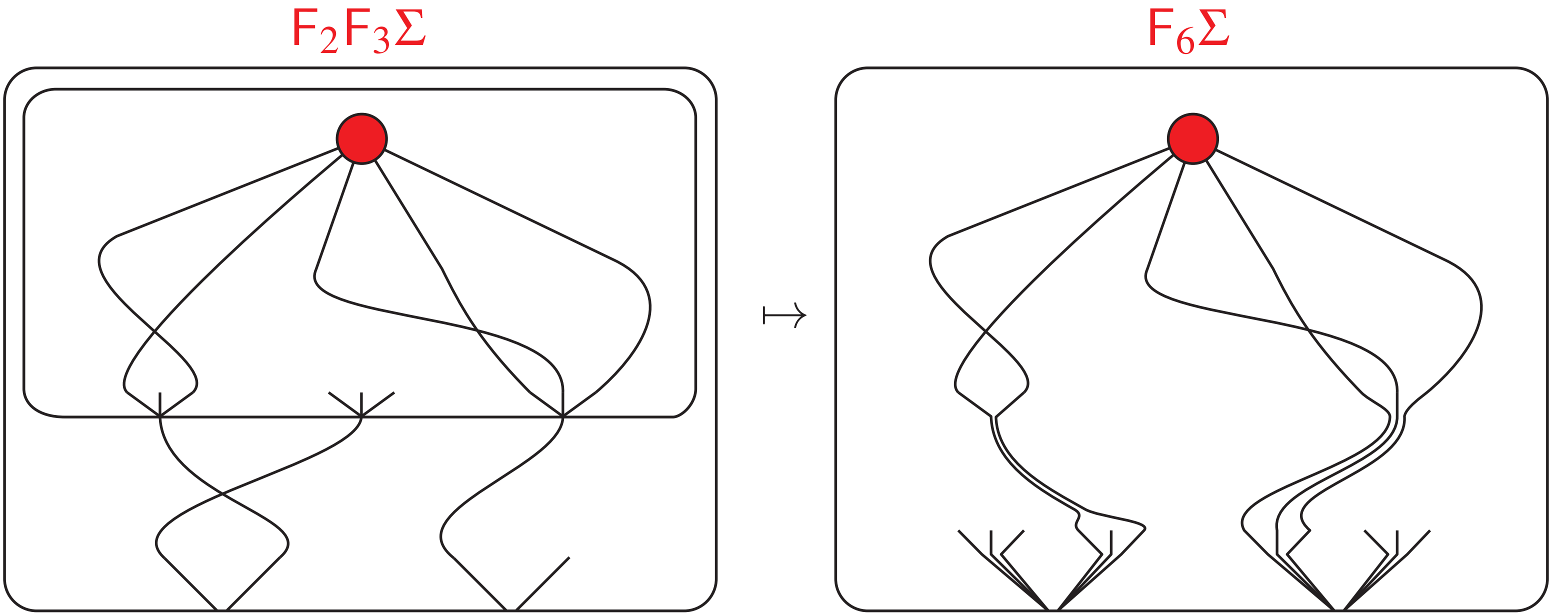
$\mapsto$
open
close



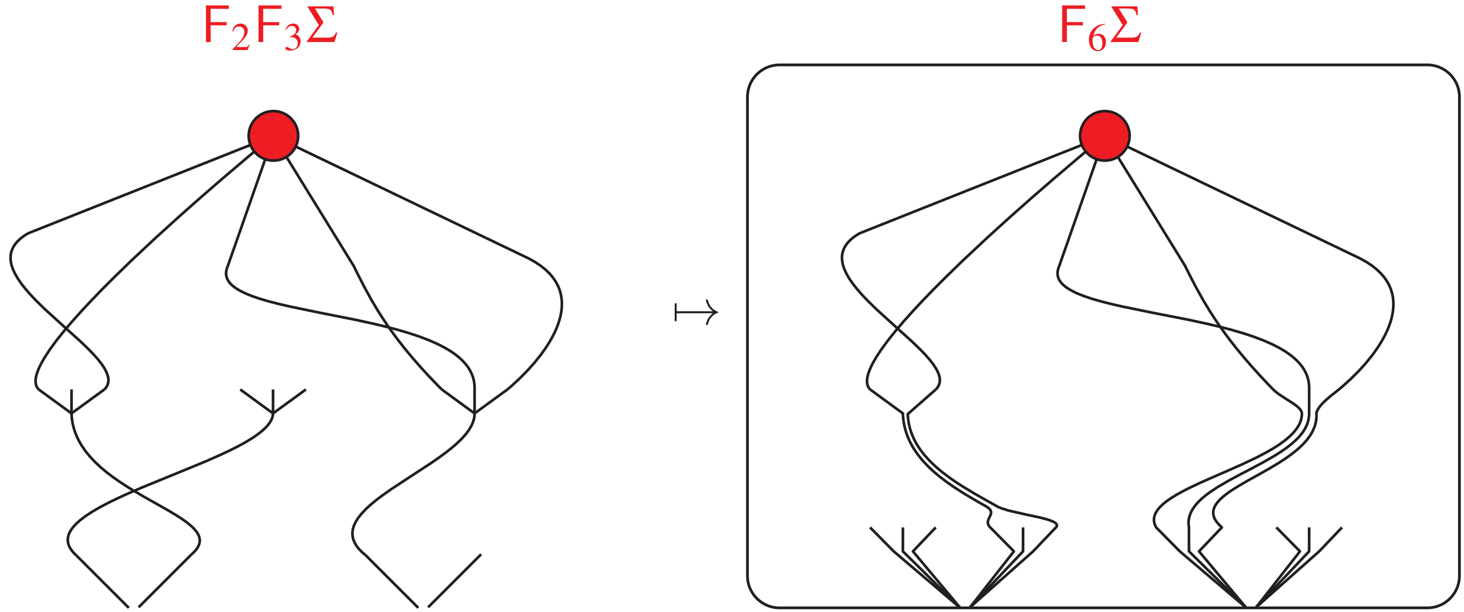








$F_2F_3\Sigma$
$\mapsto$
$F_6\Sigma$


$F_2F_3\Sigma$
$\mapsto$
$F_6\Sigma$