

a ranked alphabet

arity 2



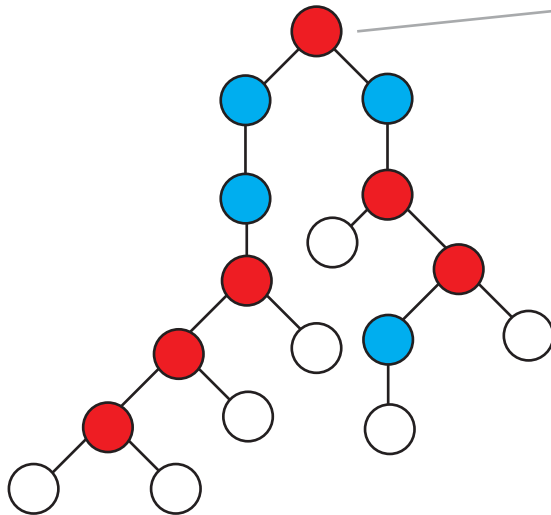
arity 1



arity 0



a tree



this node has a  
label of arity 2,  
and therefore it has  
2 children

this node is  
child 2  
(children are  
ordered)



A tree  $t$  over  $\Sigma^{[2]}$



$\text{unfold}_1(t)$



$\text{unfold}_2(t)$





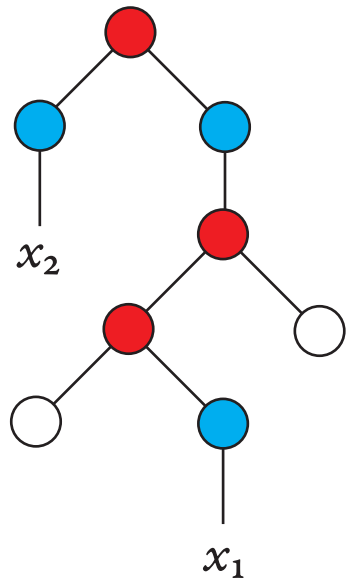
$t$



substitute( $t$ )

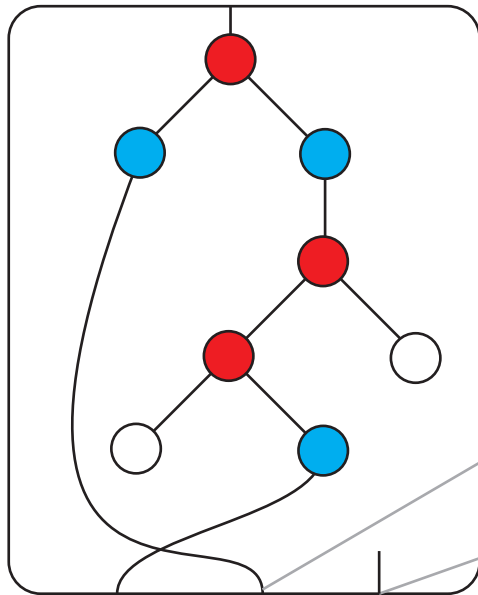






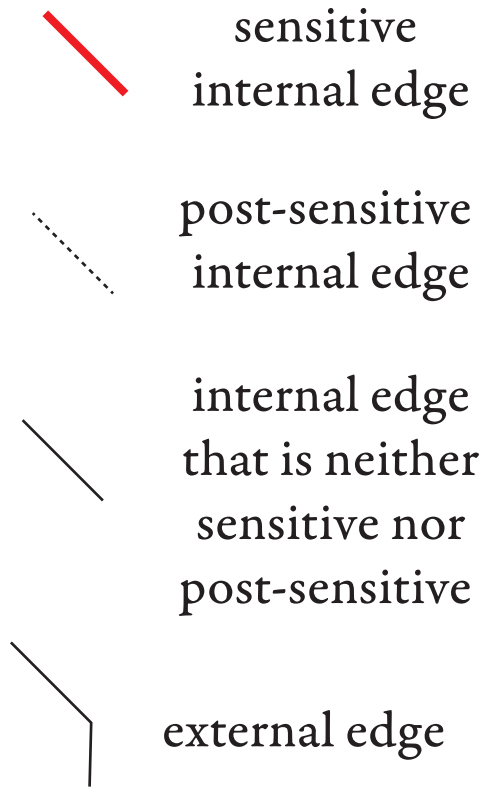
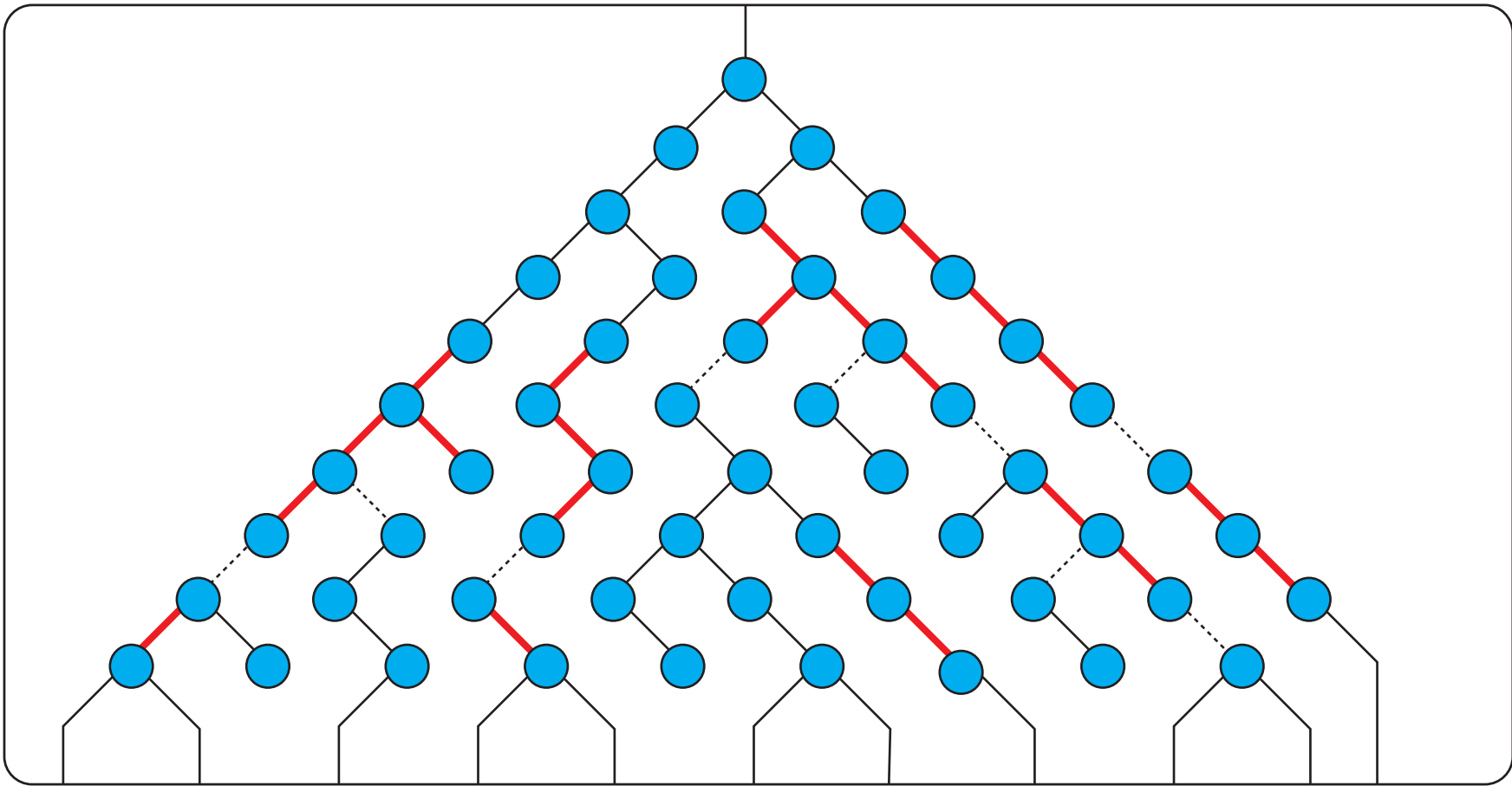
=

a term of arity 3



lines leaving at the bottom of the box  
represent variables

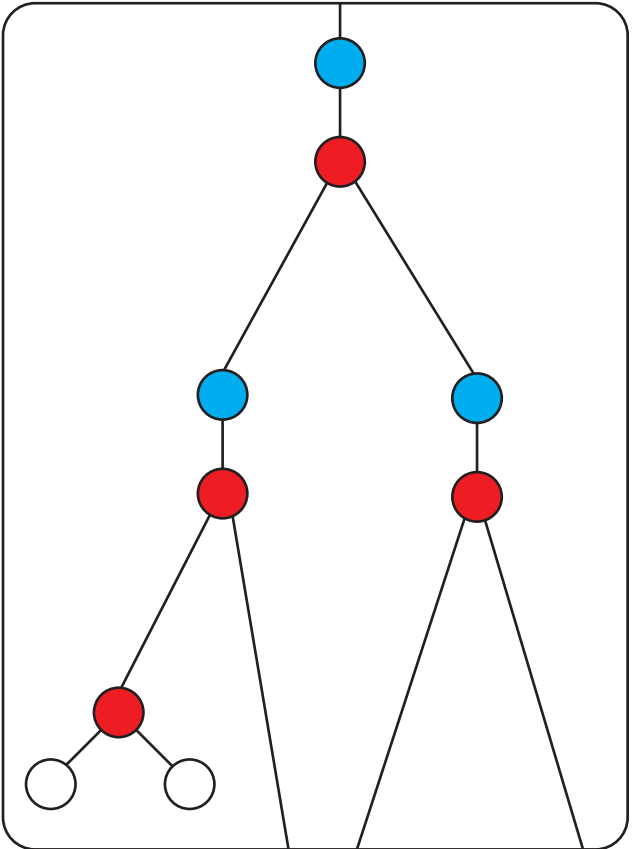
dangling edges represent unused variables







$\mapsto$





a term



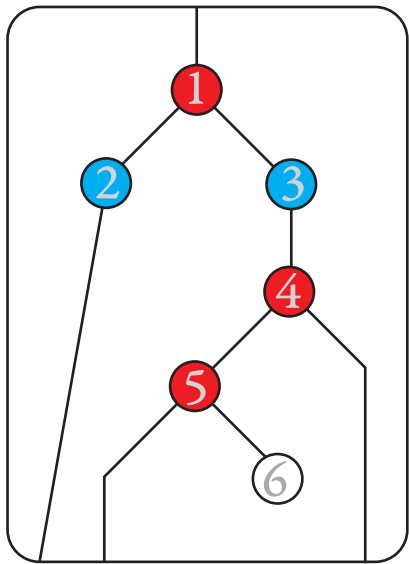
ancestor equivalence



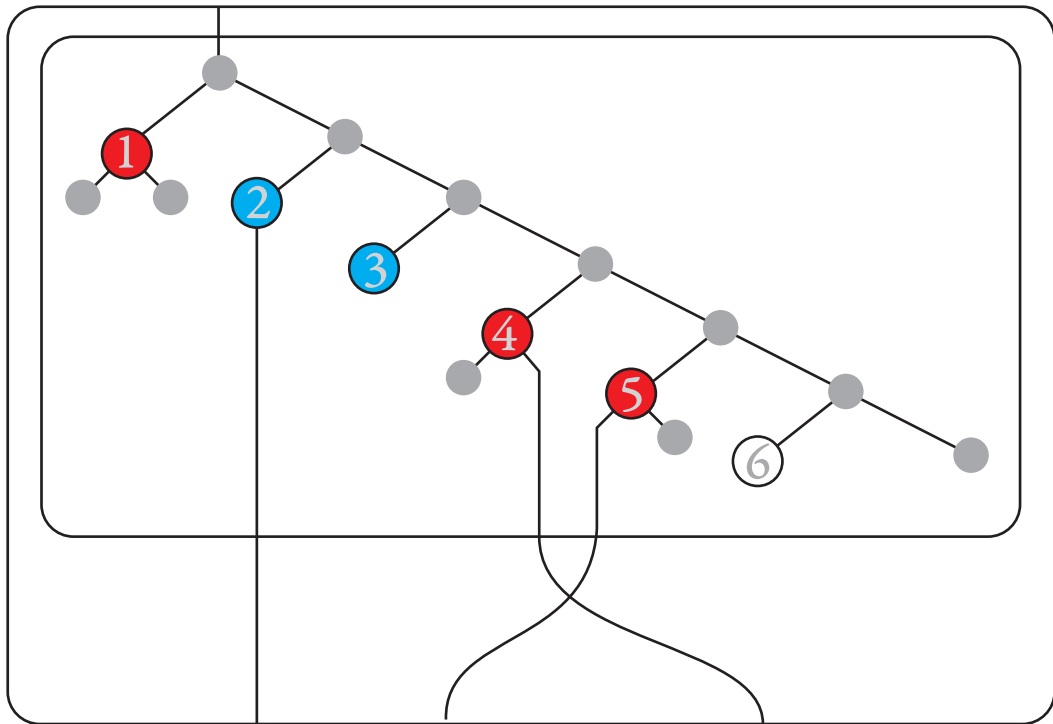
descendant equivalence



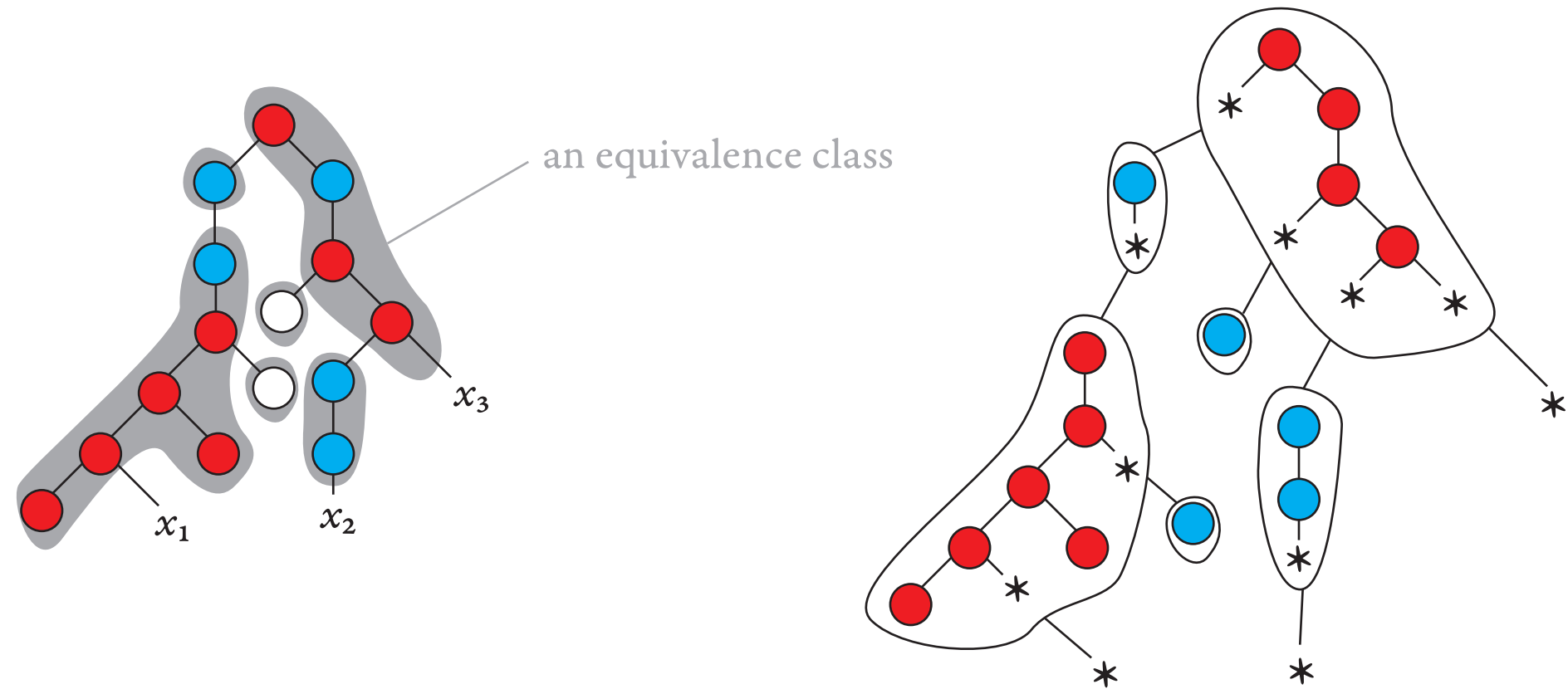




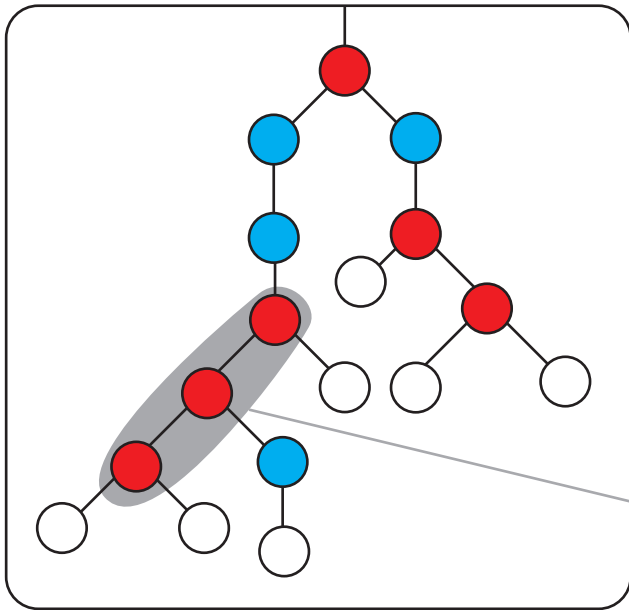
$\mapsto$



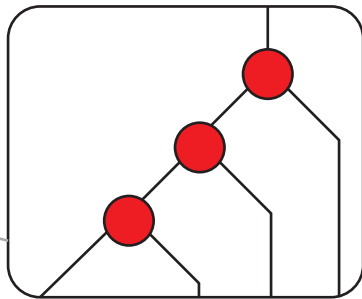
a factorisation equivalence



a tree



a term with  
4 ports that  
represents  
part of the  
tree







input alphabet

arity 2



arity 1



arity 0



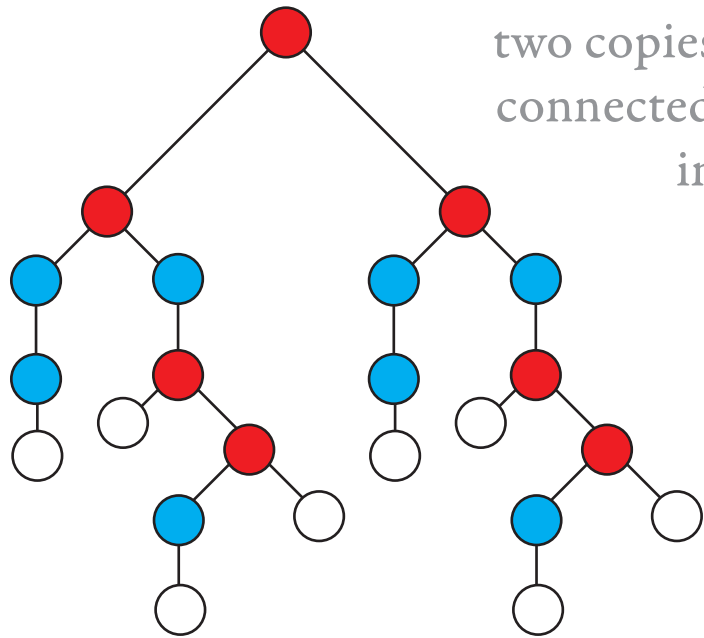
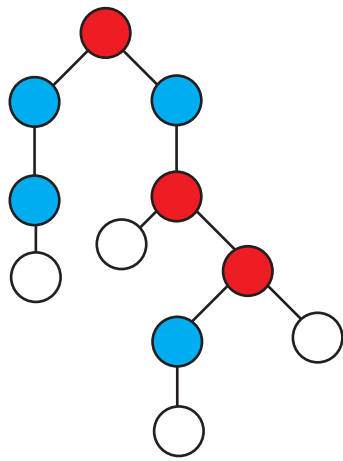
output alphabet

arity 2



arity 0





two copies of the input tree,  
connected by a binary node  
in the root





input alphabet

arity 2



arity 1



arity 0



output alphabet

arity 2



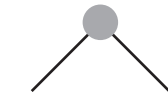
arity 1



arity 0

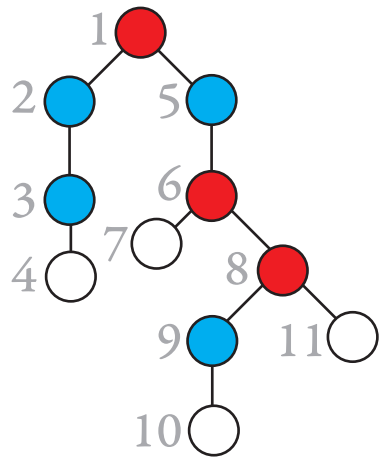


arity 2

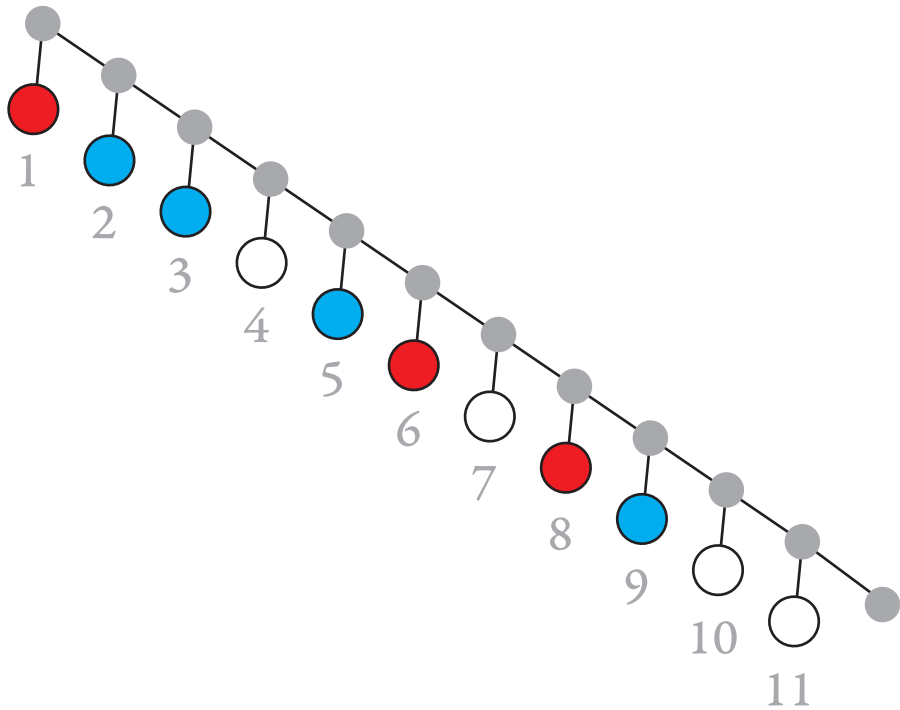


arity 0





$\mapsto$







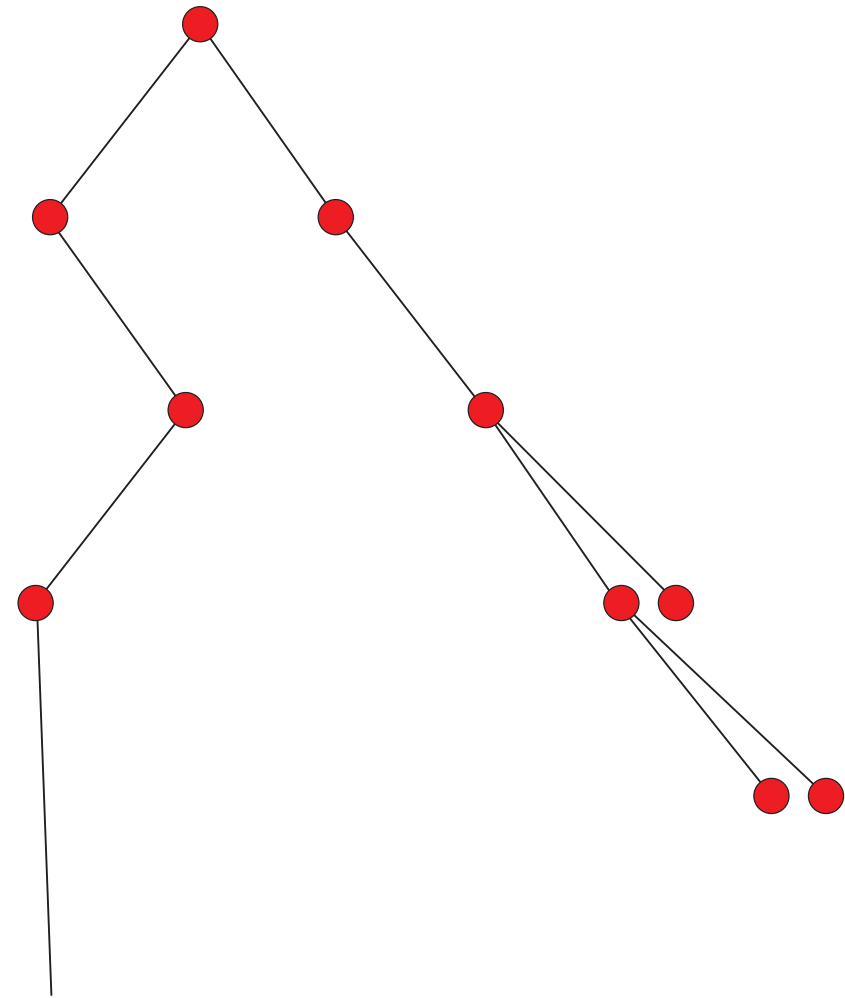
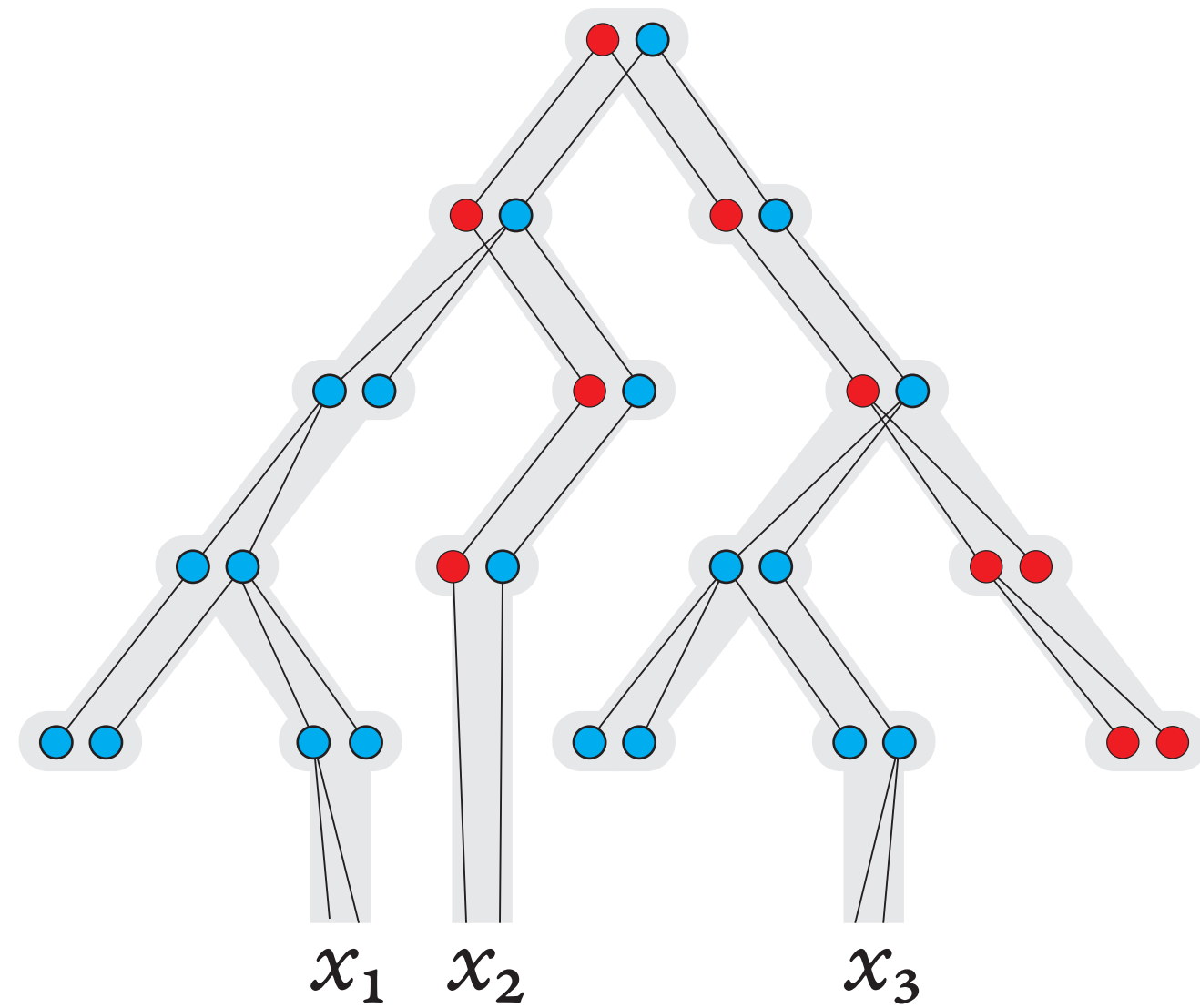


a term of arity 4



a term of arity 0





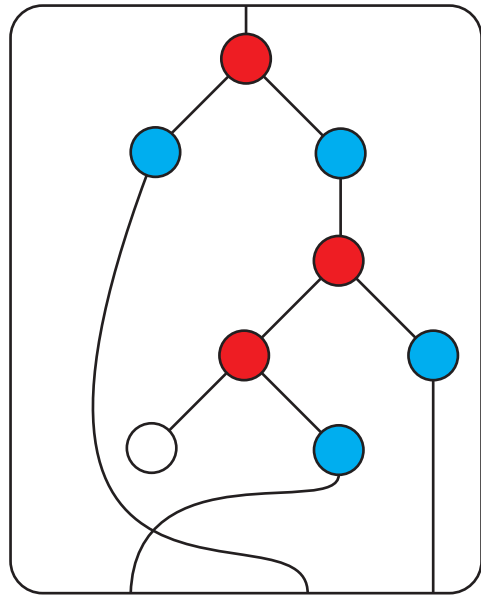




satisfies (\*)

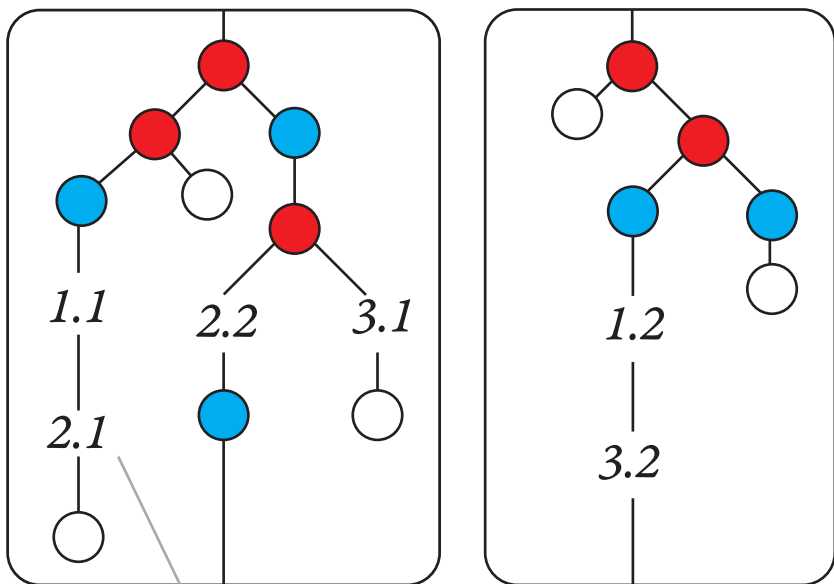
(\*)

If the root has arity  $n$ ,  
and  $1 \leq i < j \leq n$ , then  
all ports of the  $j$ -th  
subterm of the root are  
after all ports of the  
 $i$ -th subterm of the root



violates (\*)

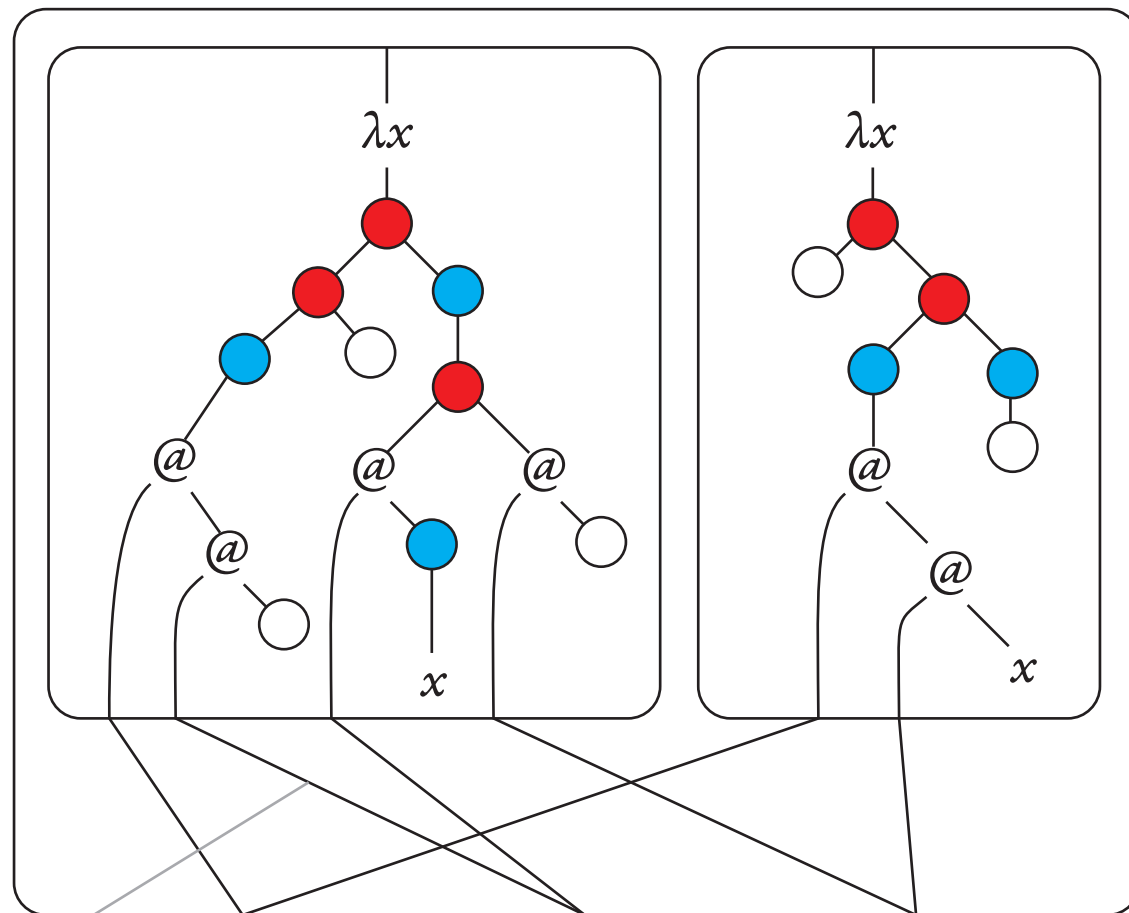
a register update



Variable  $i.j$  represents register  $i$  in the  $j$ -th argument of the register update.

In the dual, this variable is mapped to the  $i$ -th edge which enters the  $j$ -th port of the reducer.

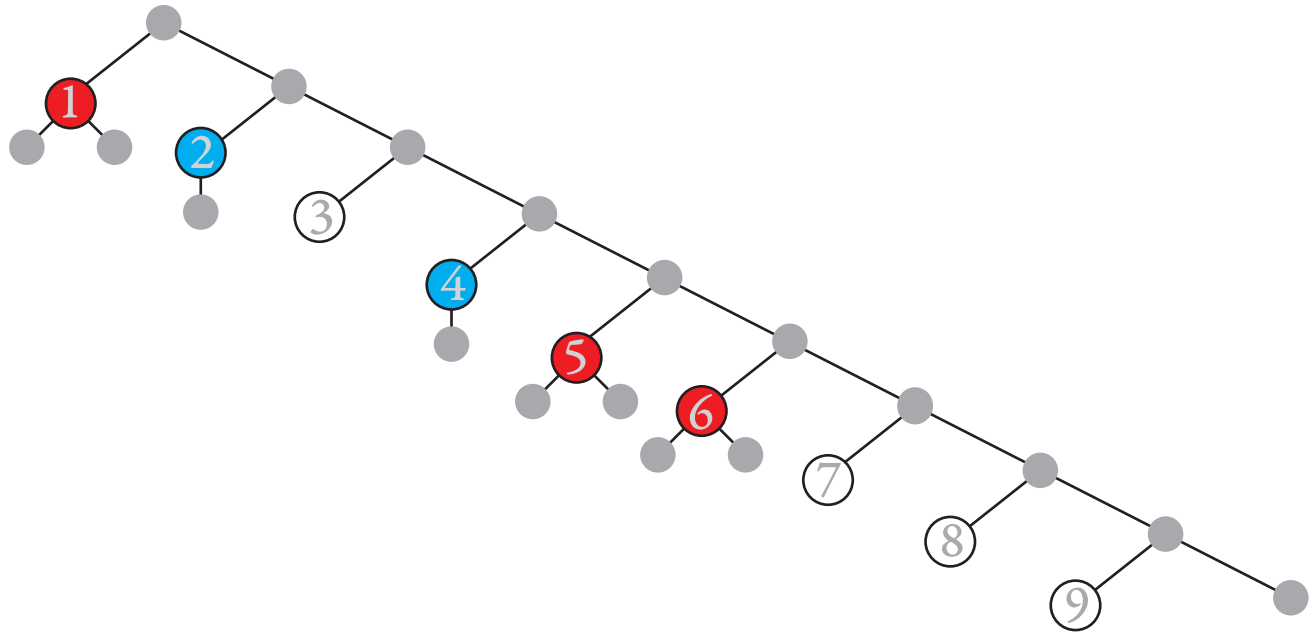
its dual



input



output

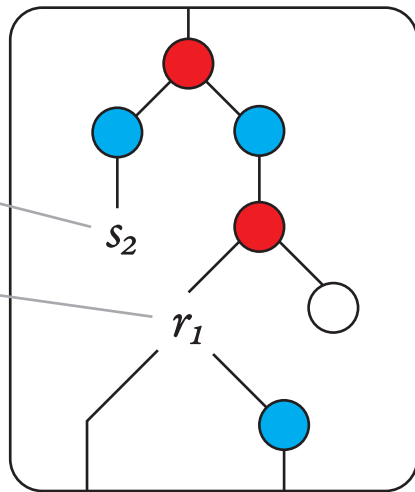




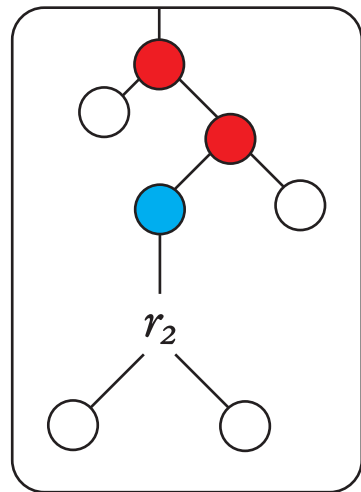
register  $r$  of arity 2

copy 2 of register  $s$

copy 1 of register  $r$



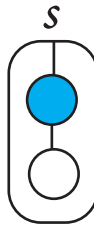
register  $s$  of arity 0











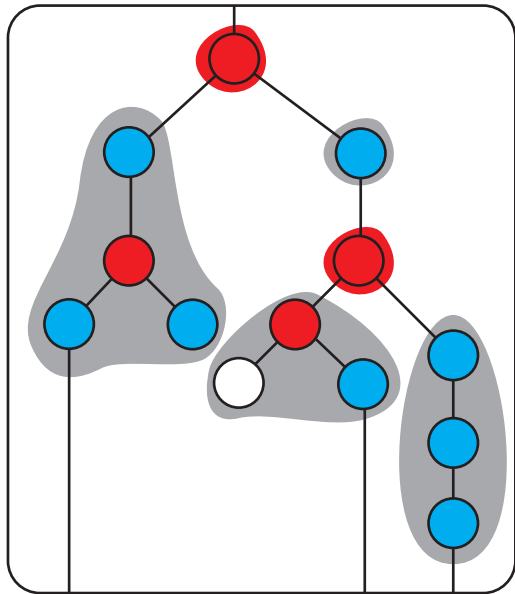




factors without  
branching nodes

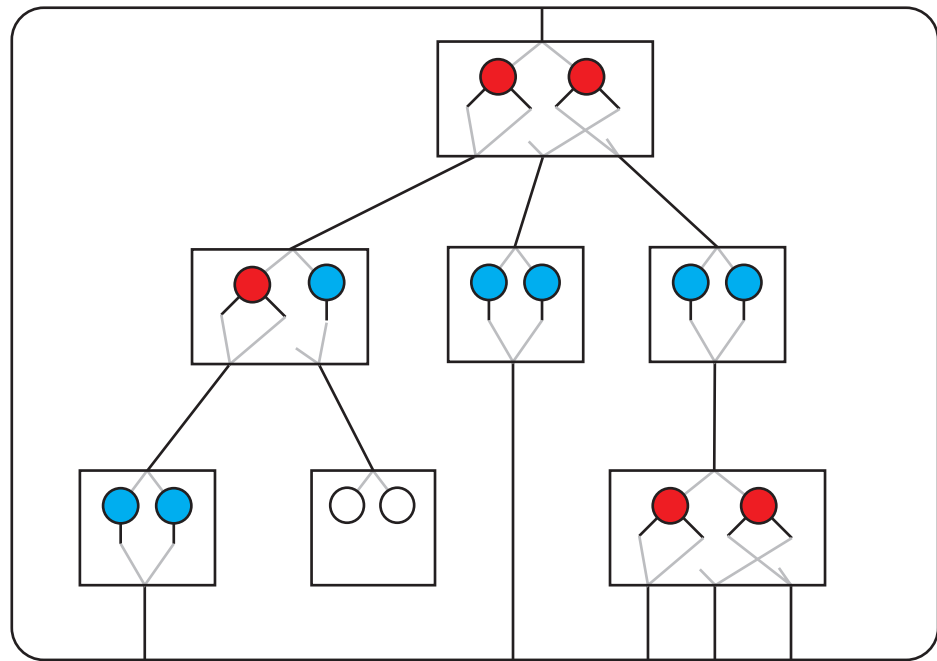


factors with  
branching nodes

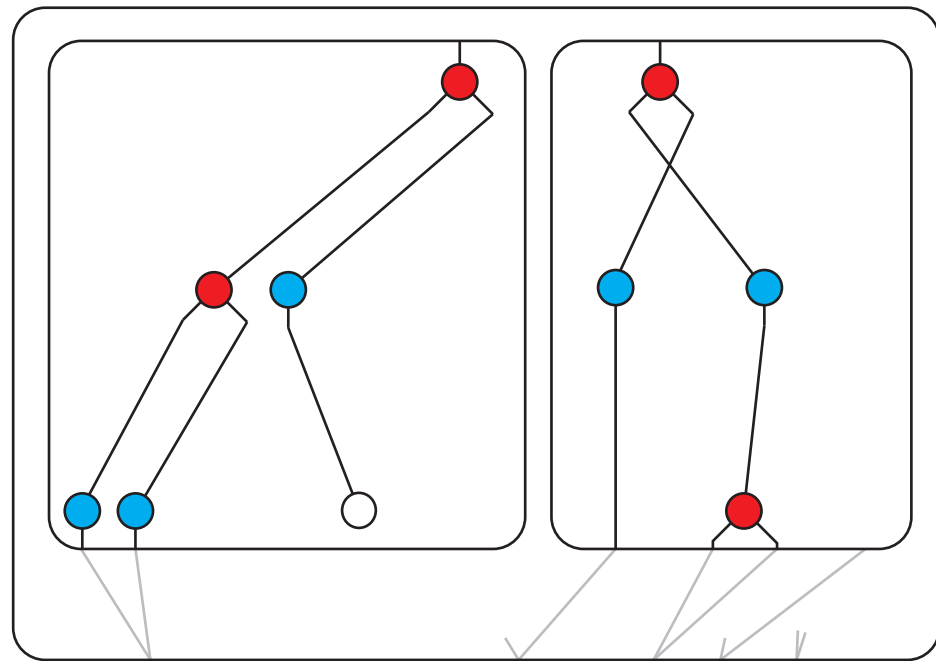


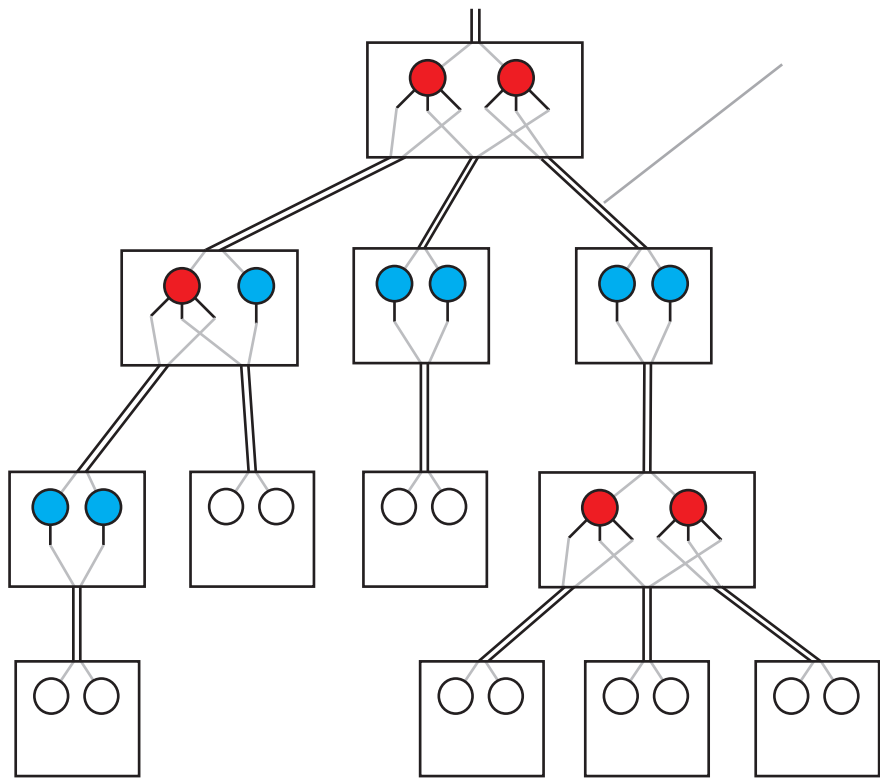


a term of matrix powers

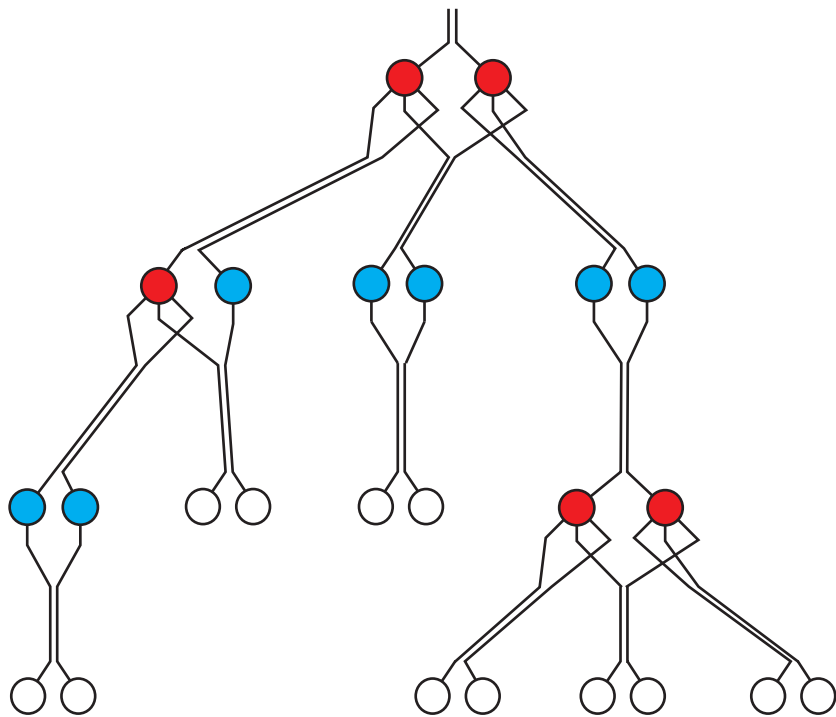


its term unfolding





$\mapsto$







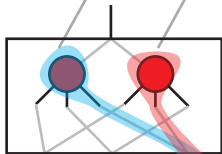
source 1 of  $e$

source 2 of  $e$

edge  $e$

target 1 of  $e$

target 2 of  $e$



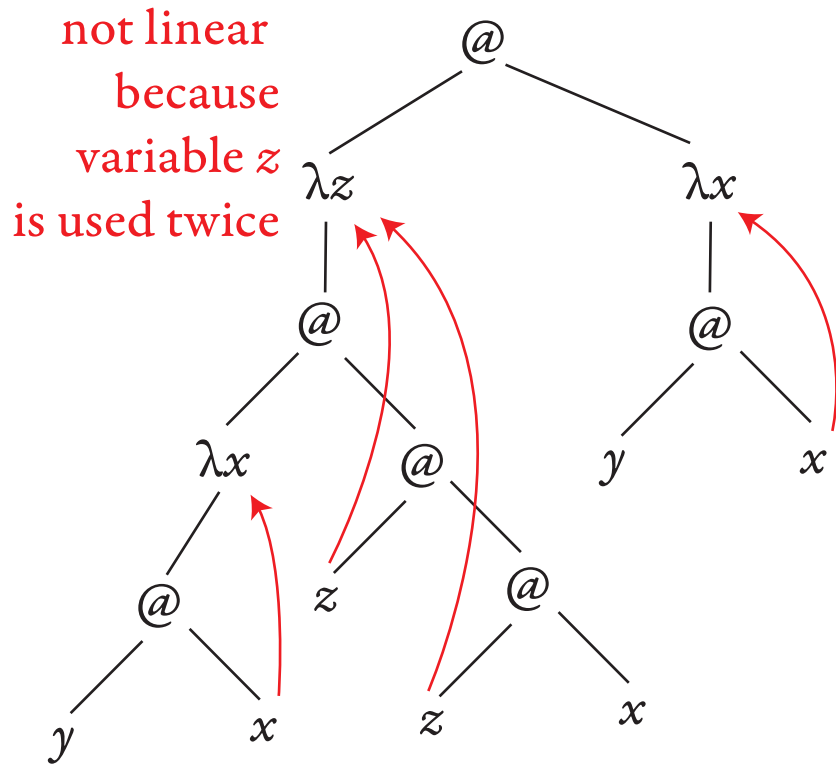
linear



we only count  
variables used  
in their scope

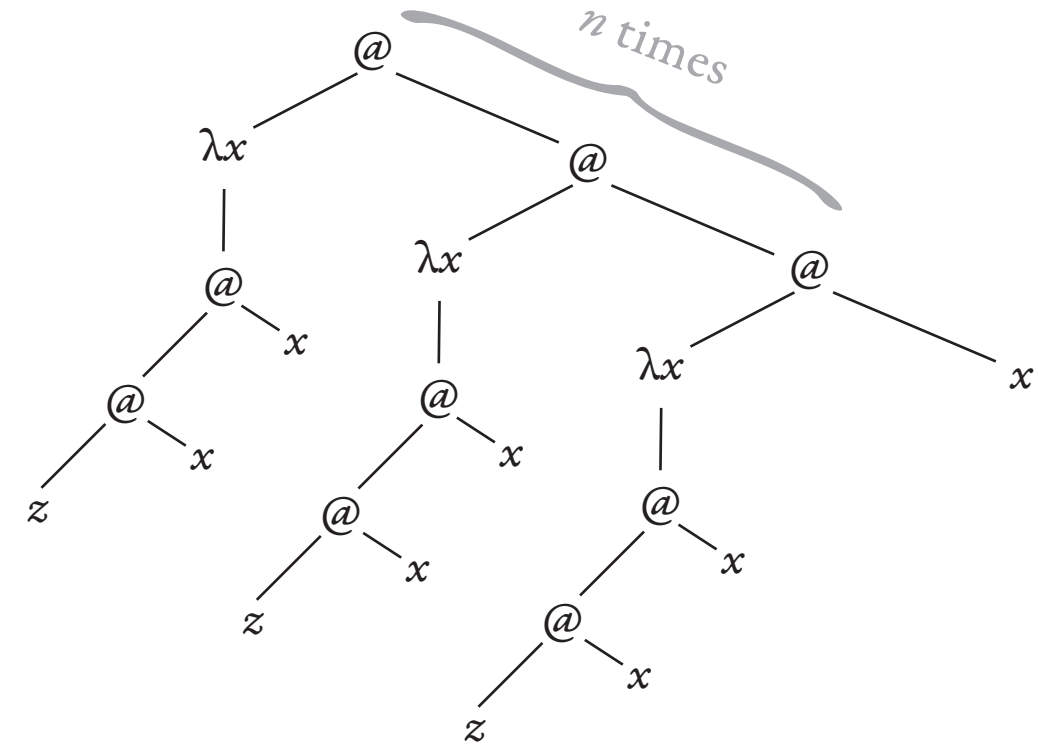
variable  $z$  can be used twice because it is free

not linear

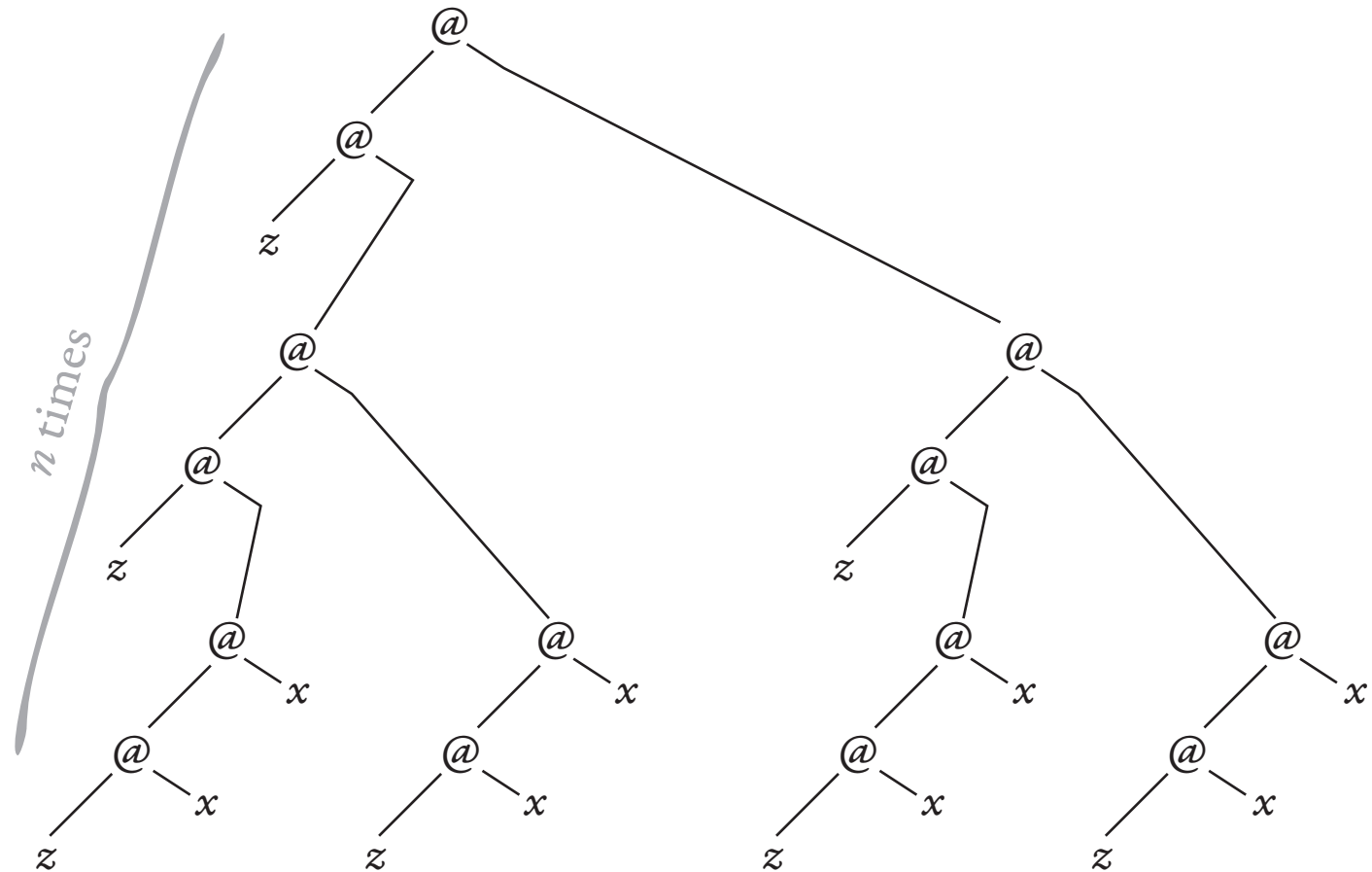


not linear  
because  
variable  $z$   
is used twice

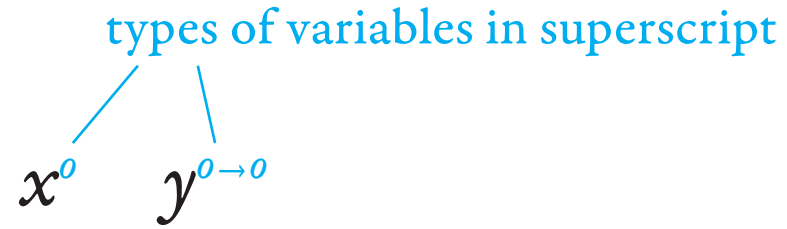
a  $\lambda$ -term of size  $O(n)$



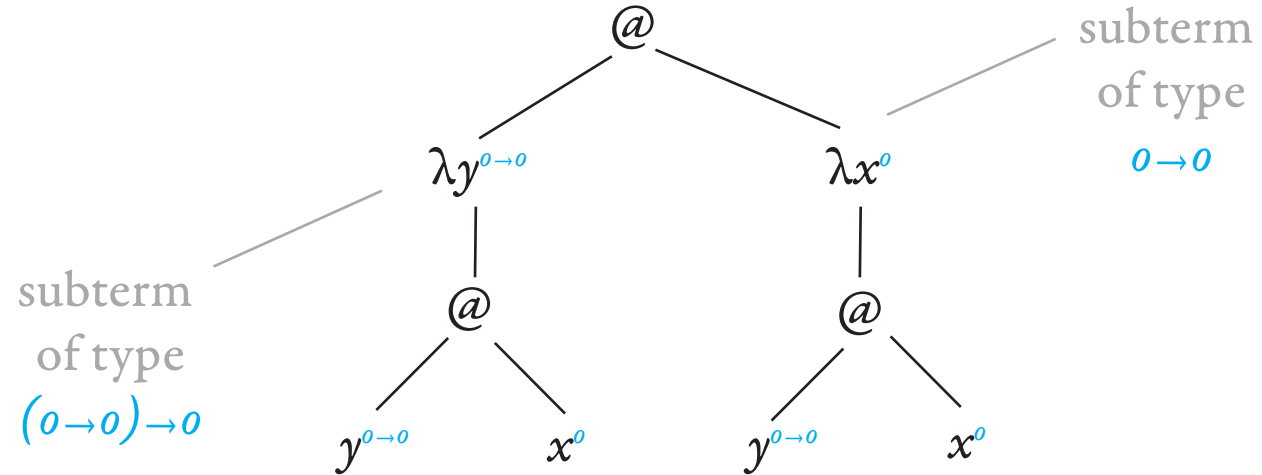
its normal form of size  $O(2^n)$



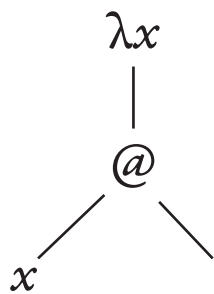
variables



$\lambda$ -term of type  $o$



@



$\lambda x.$



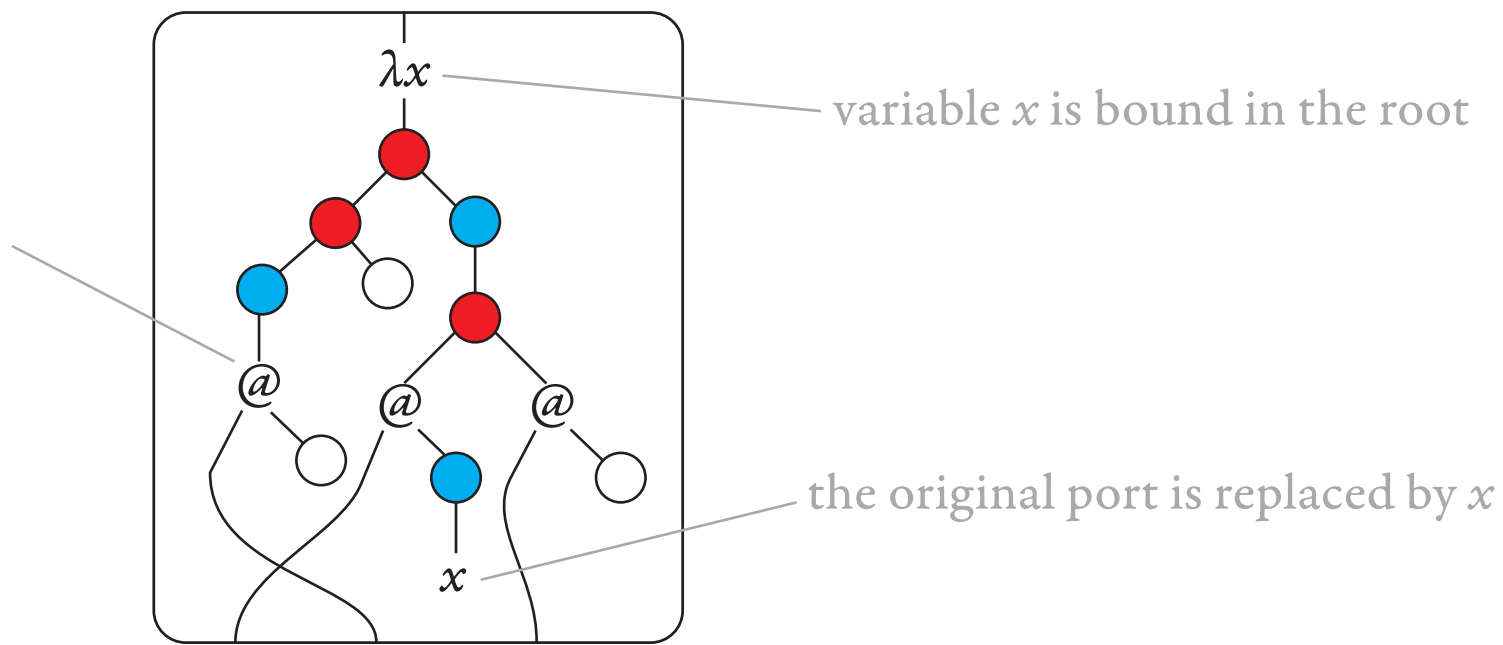
*r*

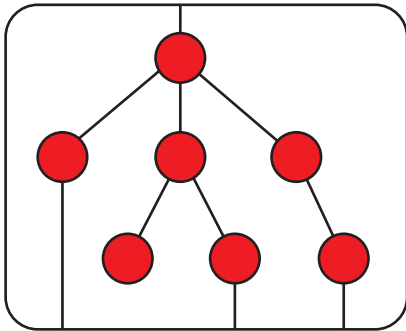




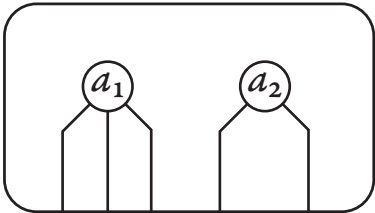
placeholder for the term  
stored in the unique register  
of the 2nd child

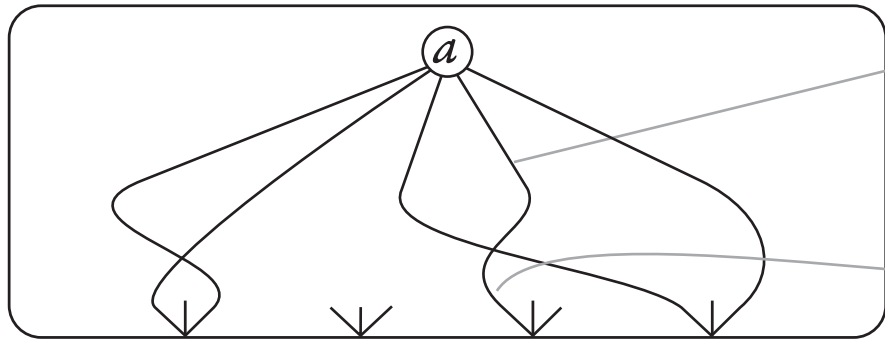






dangling edges  
represent ports





port 4

$\Downarrow f$

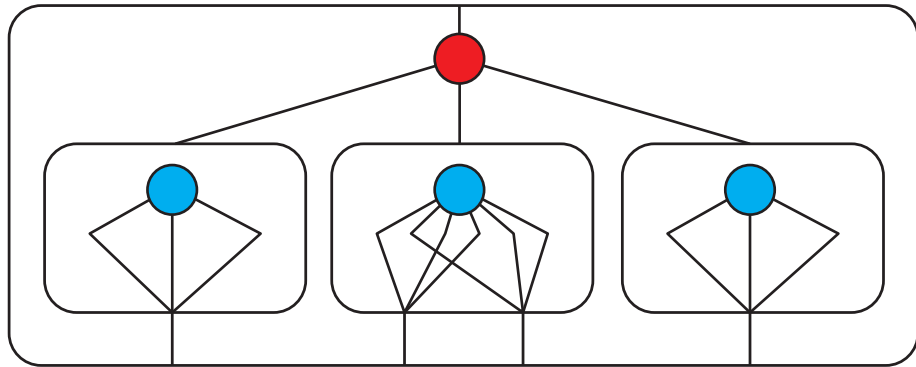
position 1, group 3

the root is from  $\Sigma$

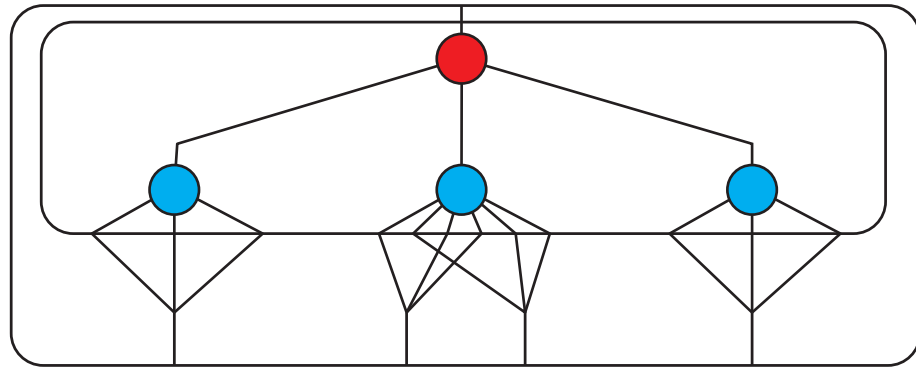
all children are from  $\Gamma$

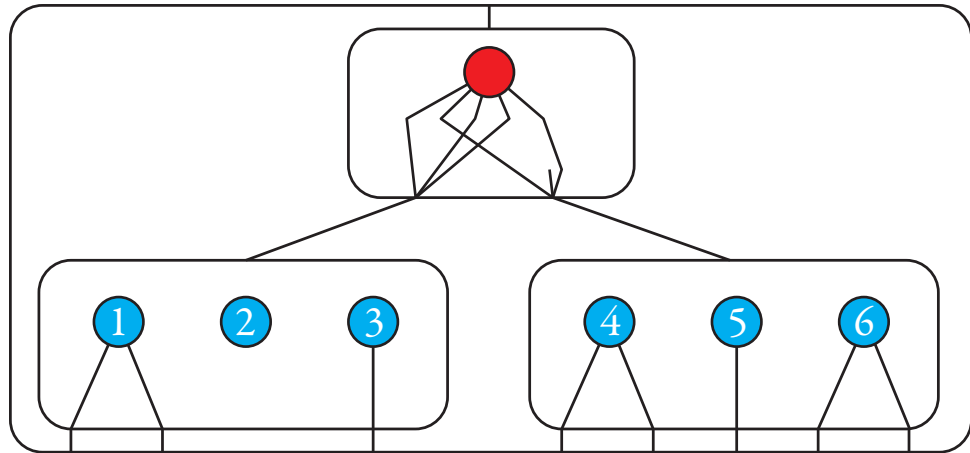


input

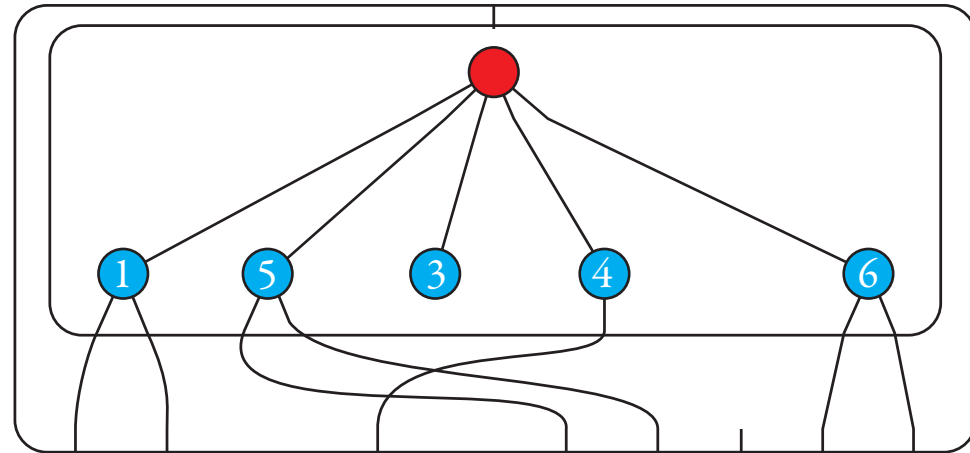


output



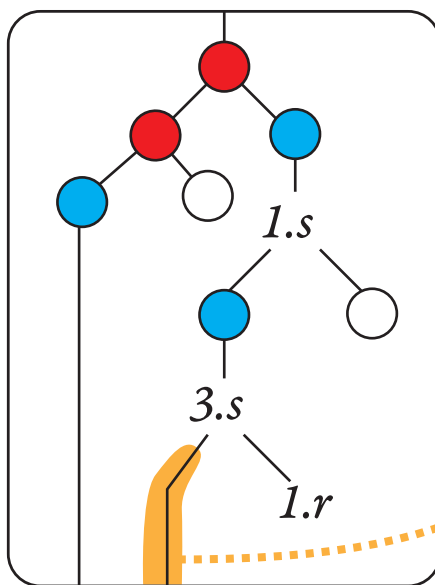
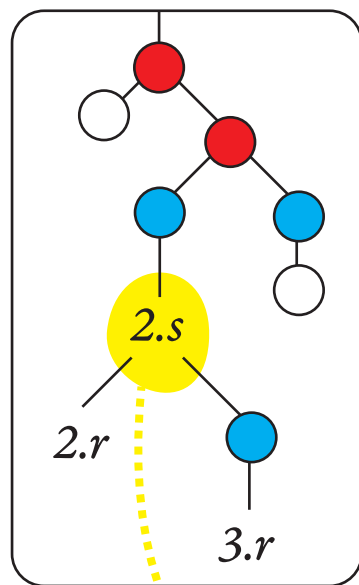


$\mapsto$

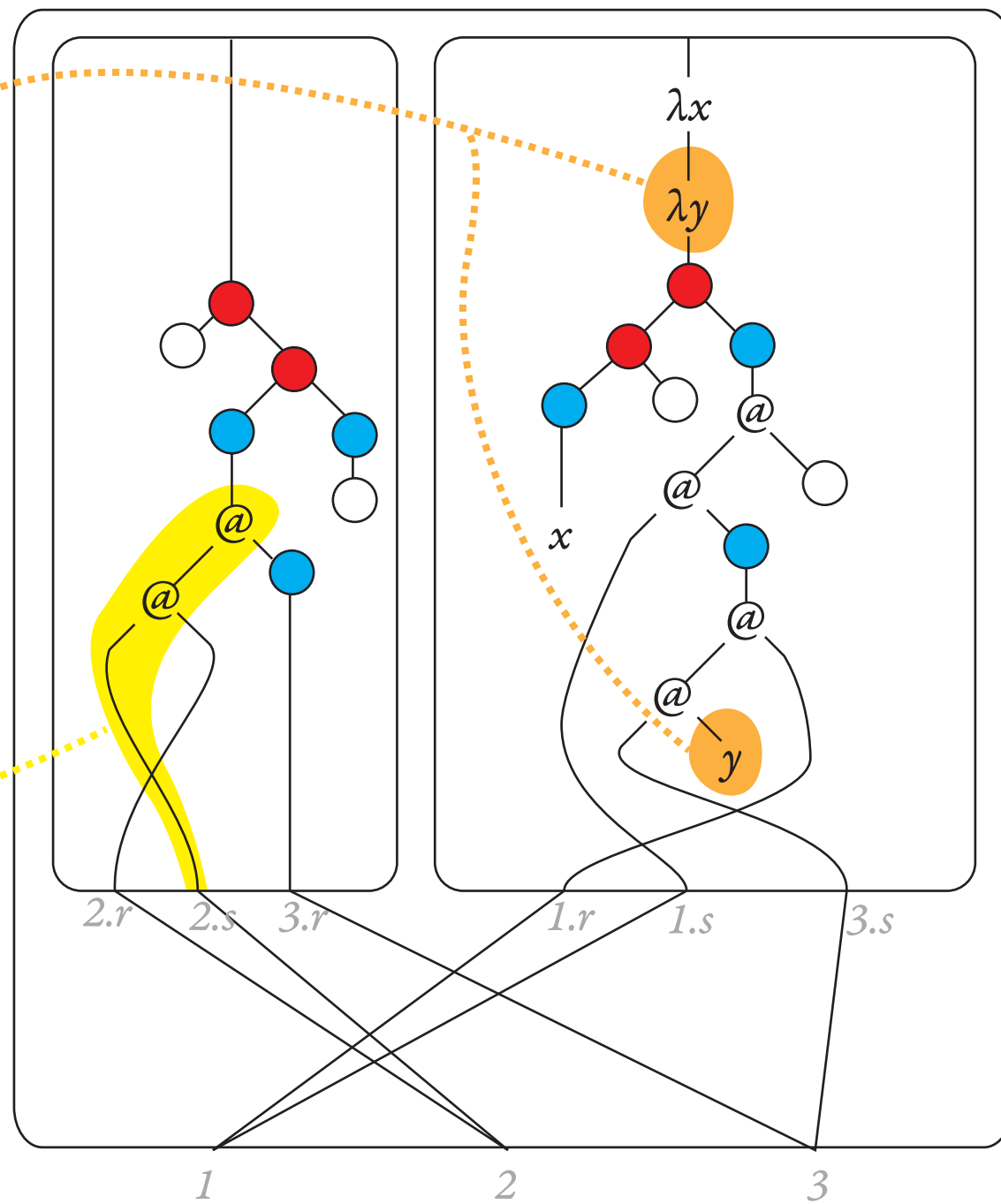




a register update



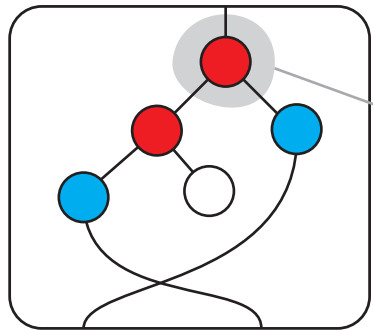
its dual



The diagram shows a binary tree structure. The root node is red. Its left child is red, and its right child is blue. The red node's left child is blue, and its right child is white. The blue node's left child is blue, and its right child is white. A yellow circle labeled  $r_1$  highlights the blue node that is the right child of the root. An orange shape labeled  $r_2$  highlights the subtree rooted at the blue node that is the left child of the root. A dashed orange line labeled  $r_3$  indicates a path or boundary starting from the orange shape and extending to the right.

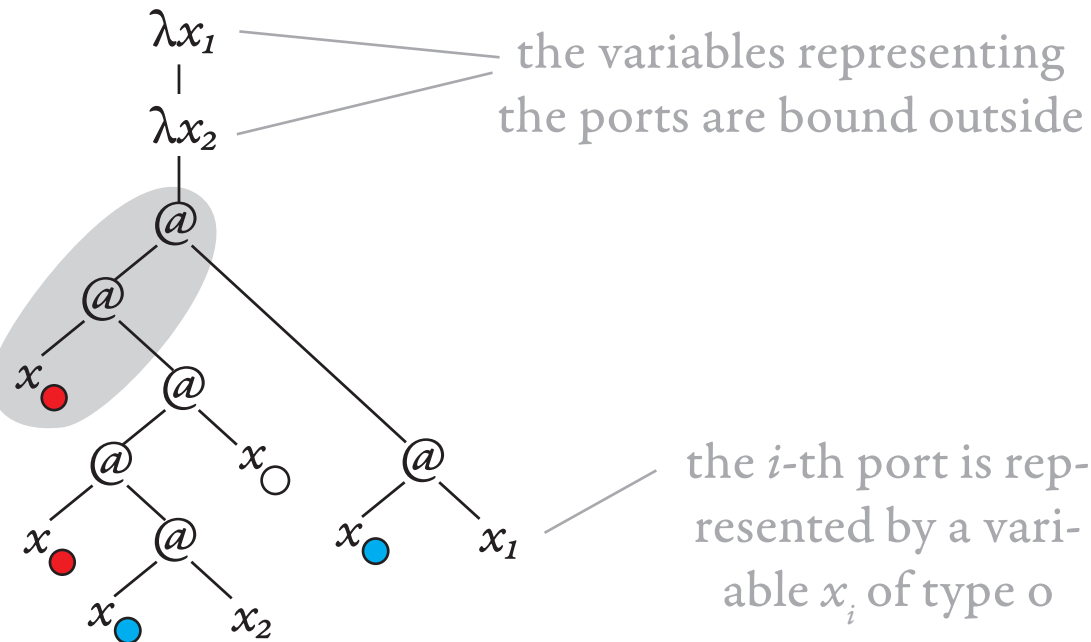
[illegible]

a term of arity 2

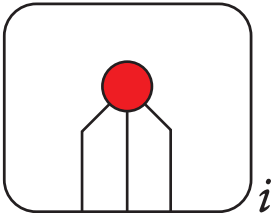


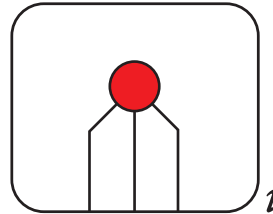
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node

its representation as a  $\lambda$ -term

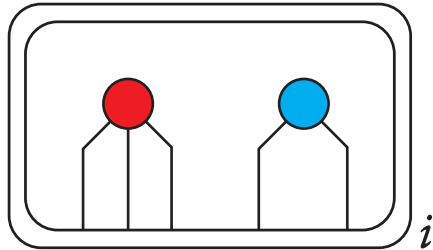
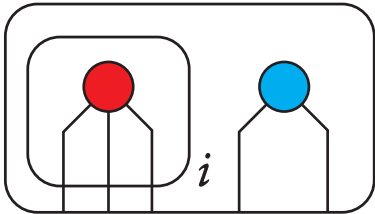




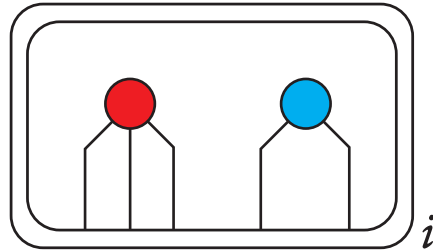
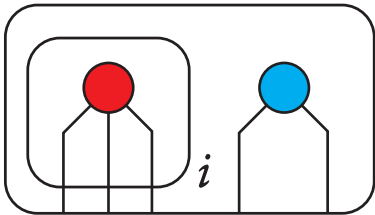


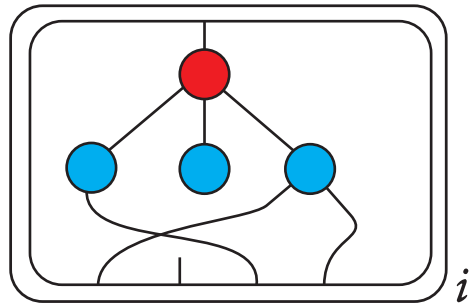
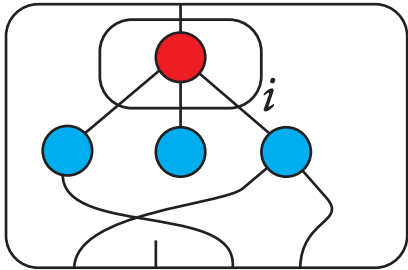


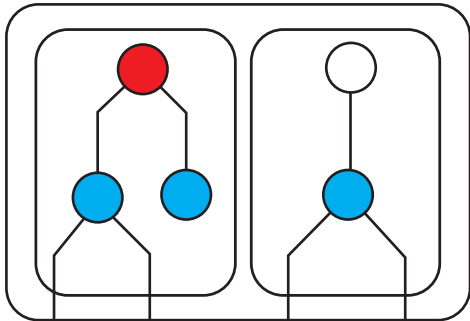
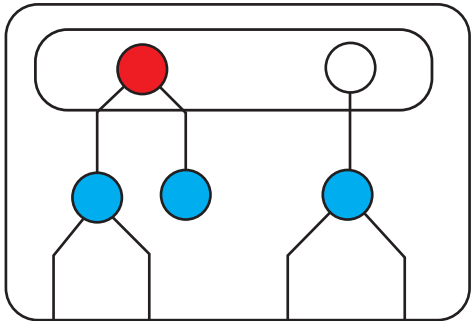


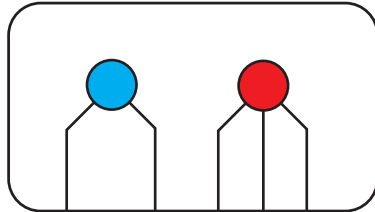




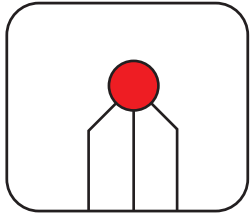


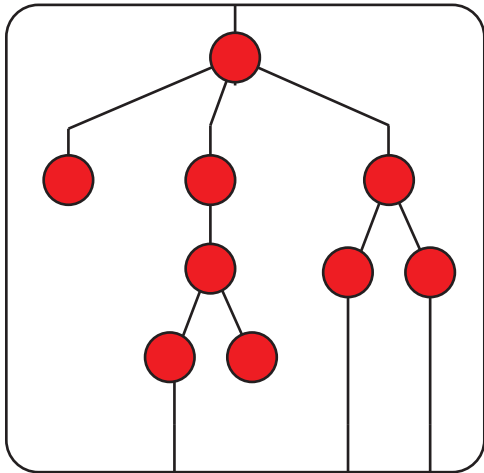
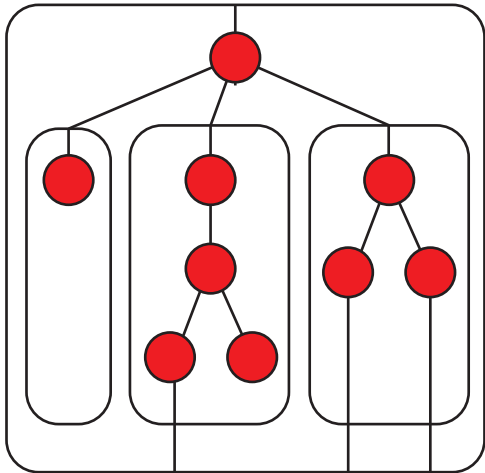


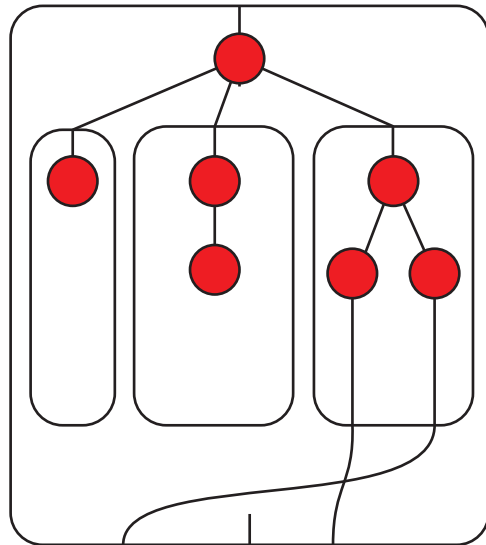
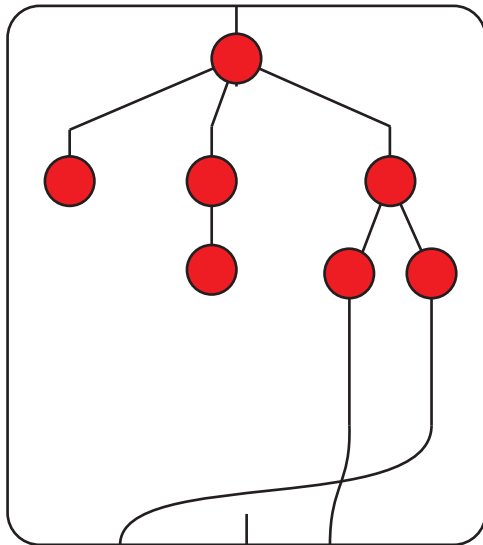




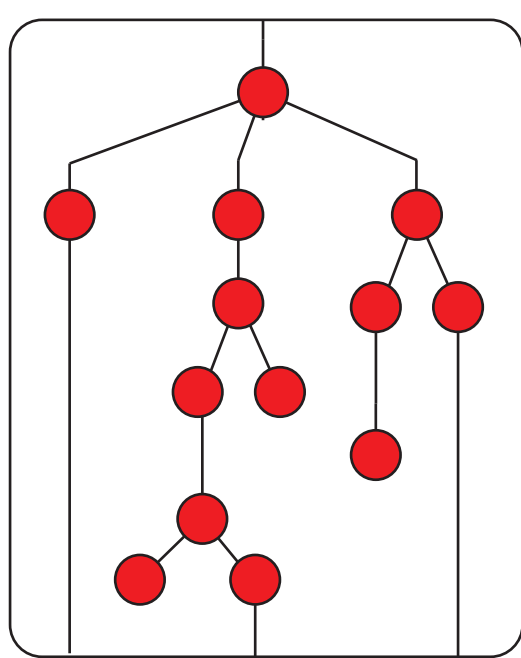
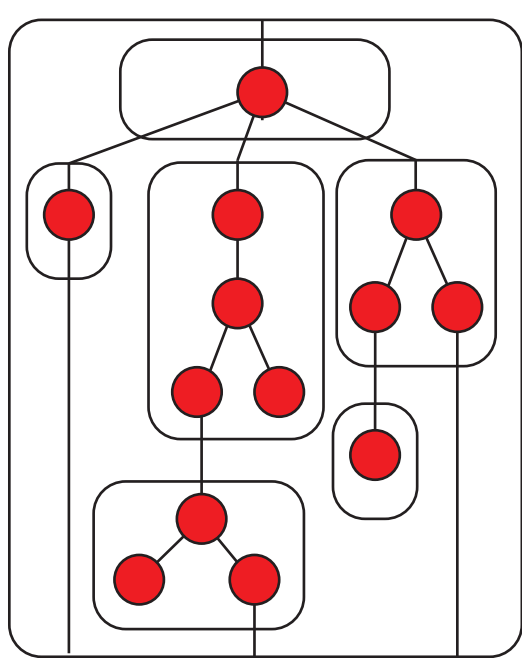


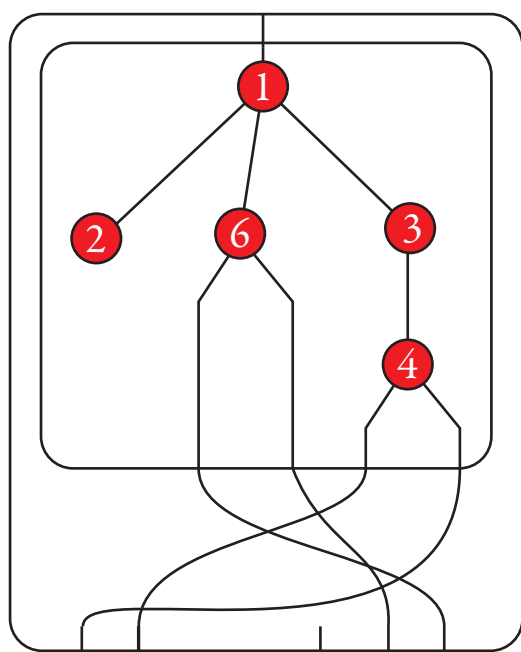


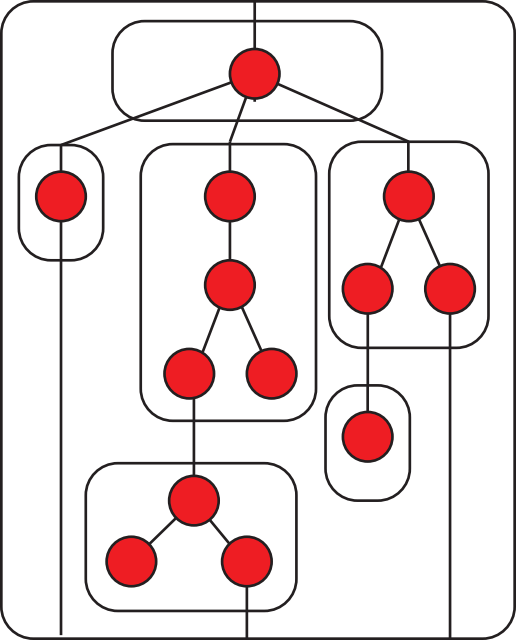




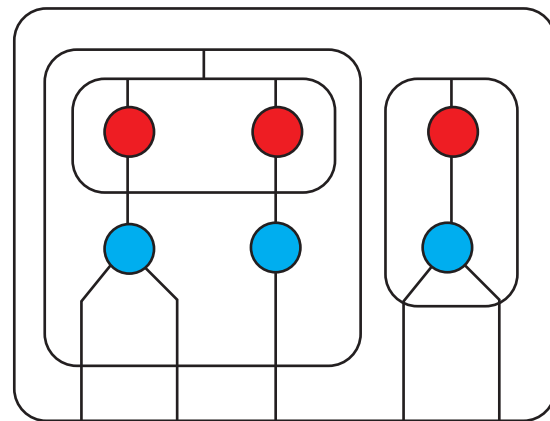
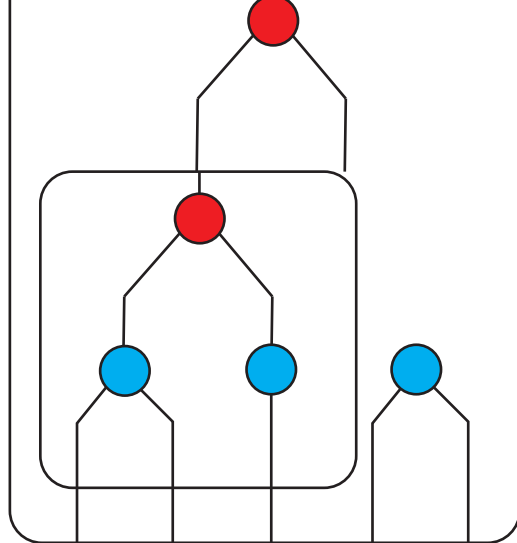
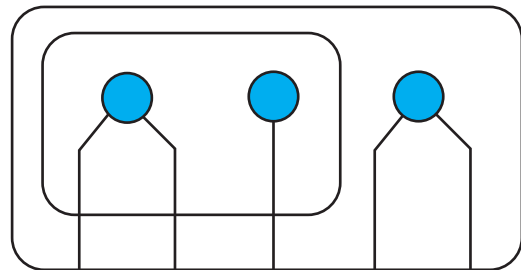


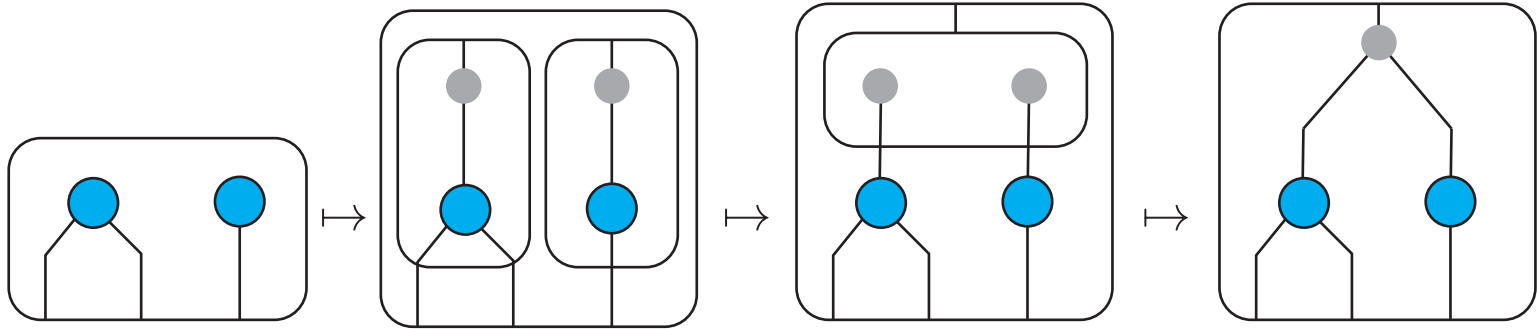






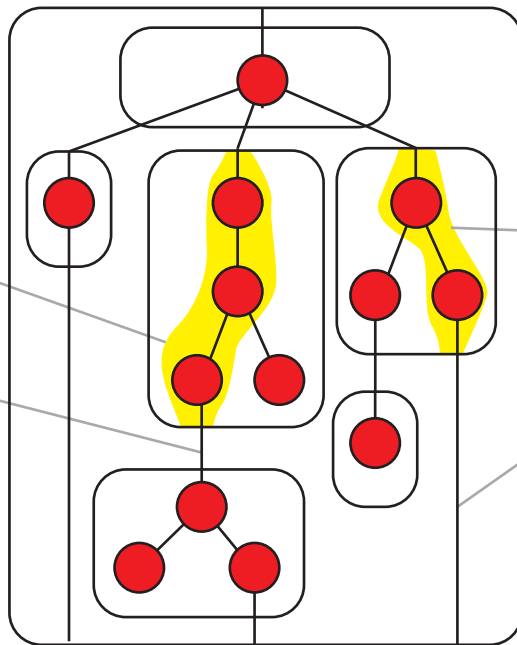






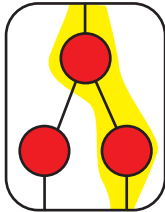


the subbranch  
corresponding to  
an internal edge



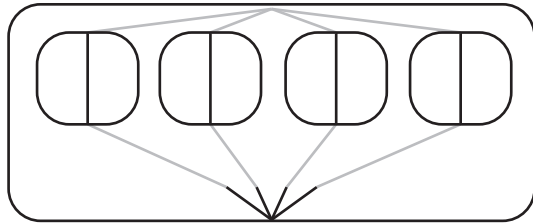
the subbranch  
corresponding to  
an external edge





a branch can be visualised as  
a term with a distinguished  
root-to-port path

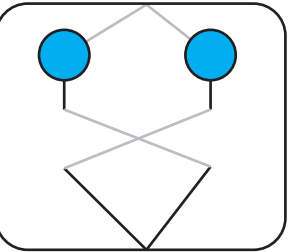




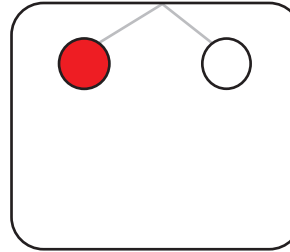
a tuple of  $k$  identity terms  
with all their ports folded  
into one

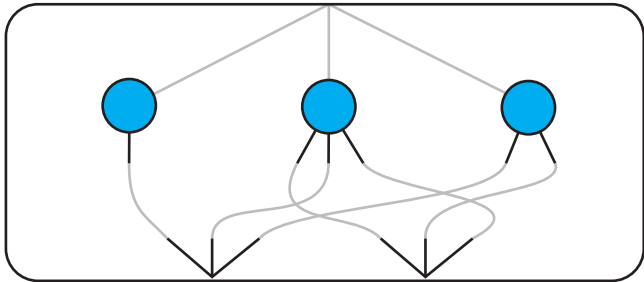
$$\Sigma = \{ \text{blue circle with stem}, \text{red circle}, \text{white circle} \}$$

$$a \in \Sigma^{[2]}$$

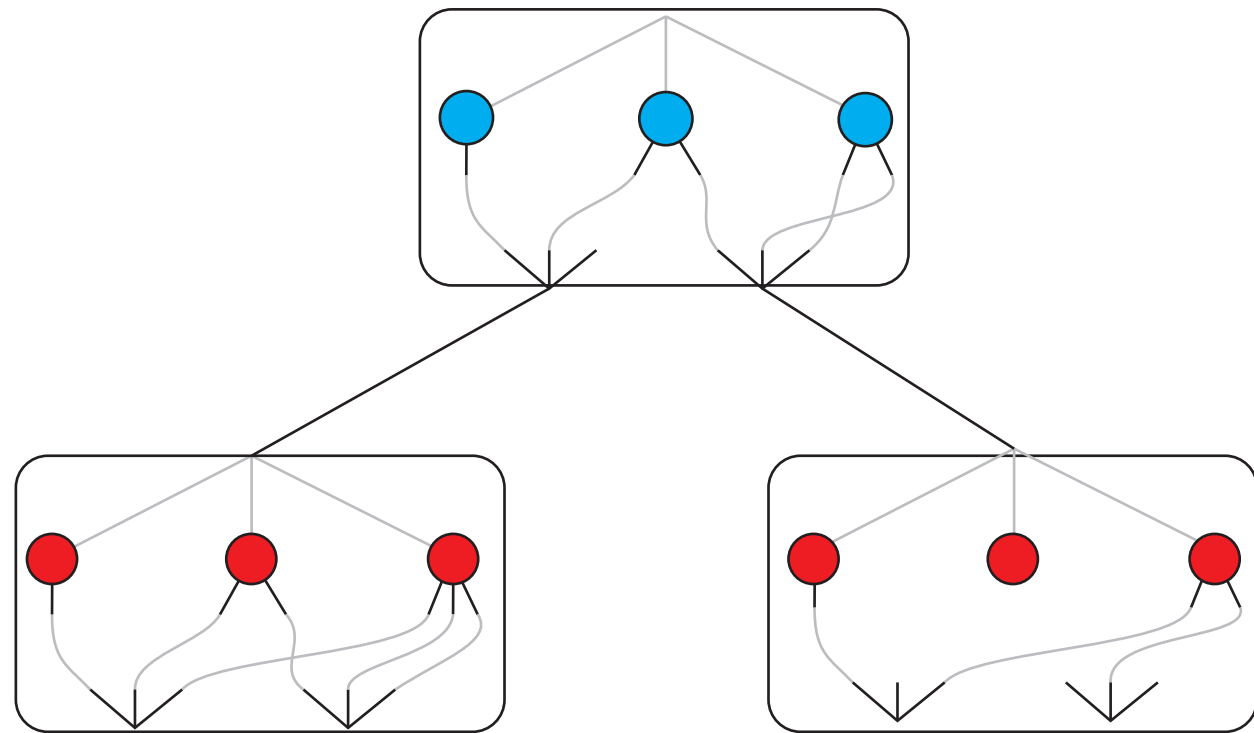


$$b \in \Sigma^{[2]}$$

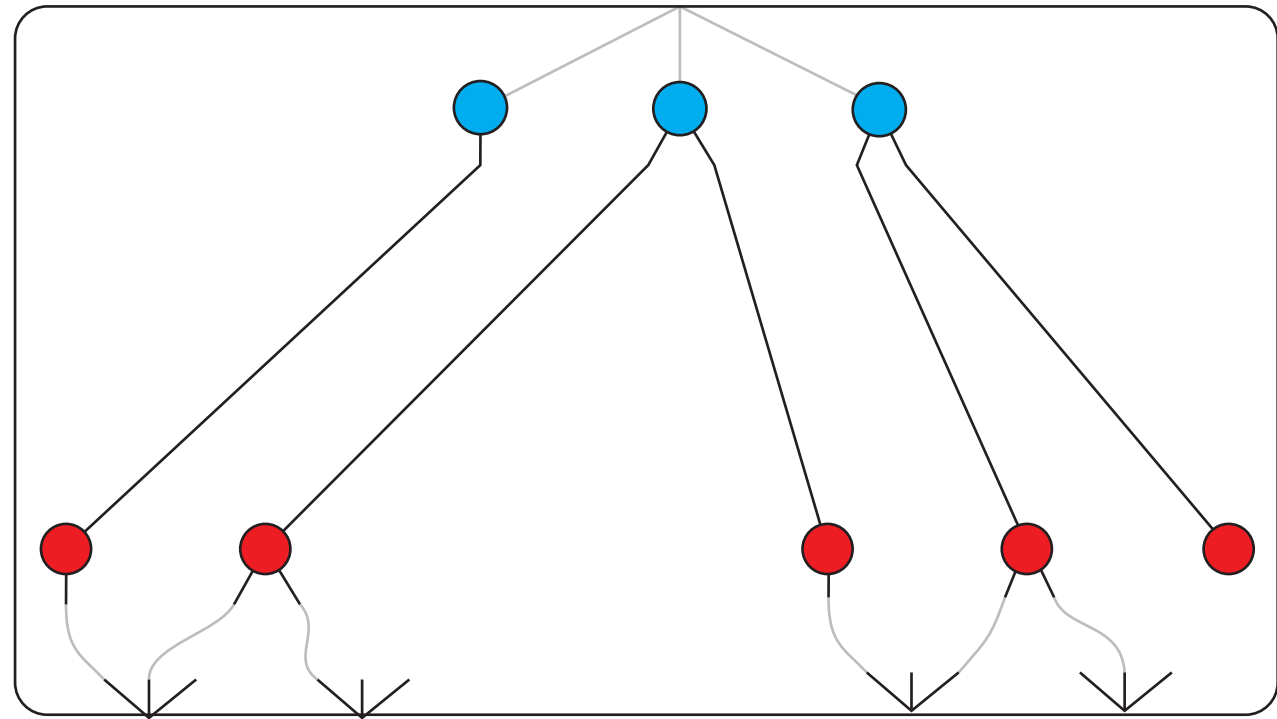


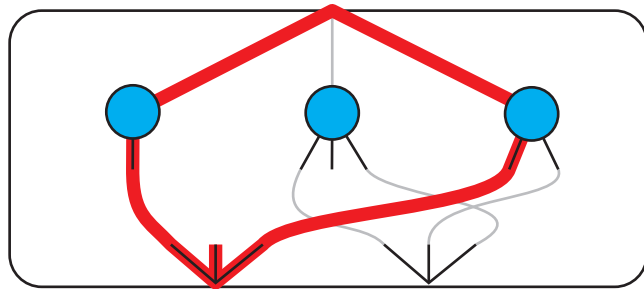


a shallow term of matrix powers



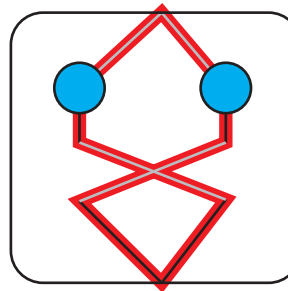
its shallow unfolding





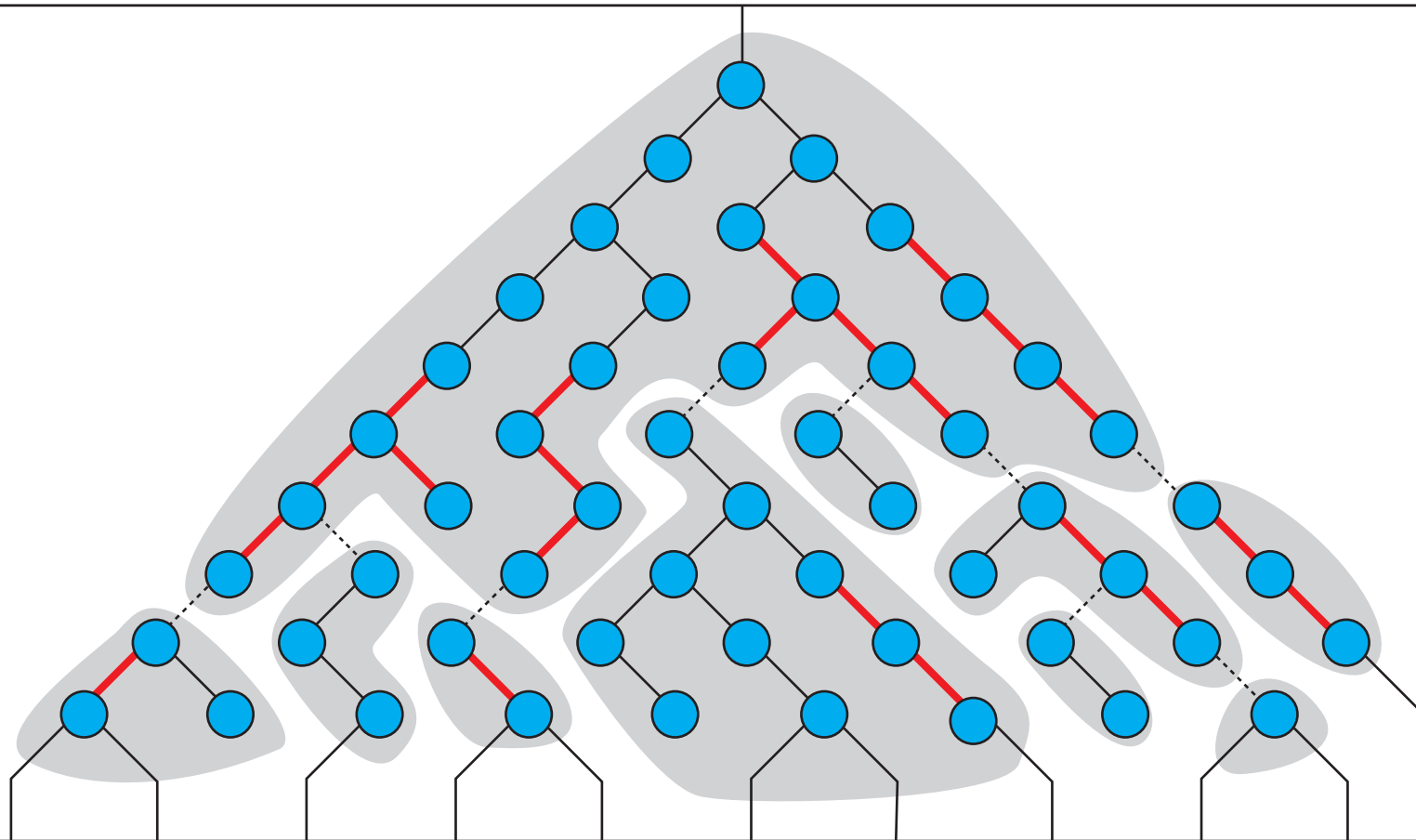
twist of port 1

1	2	3
↑		↑
1	2	3

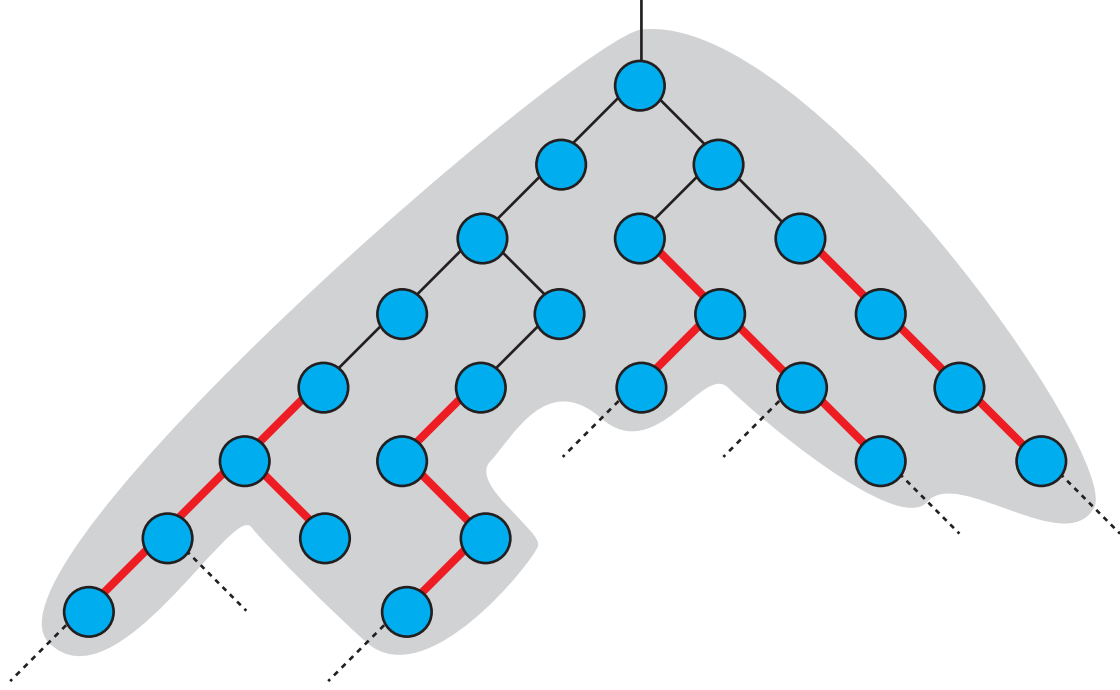


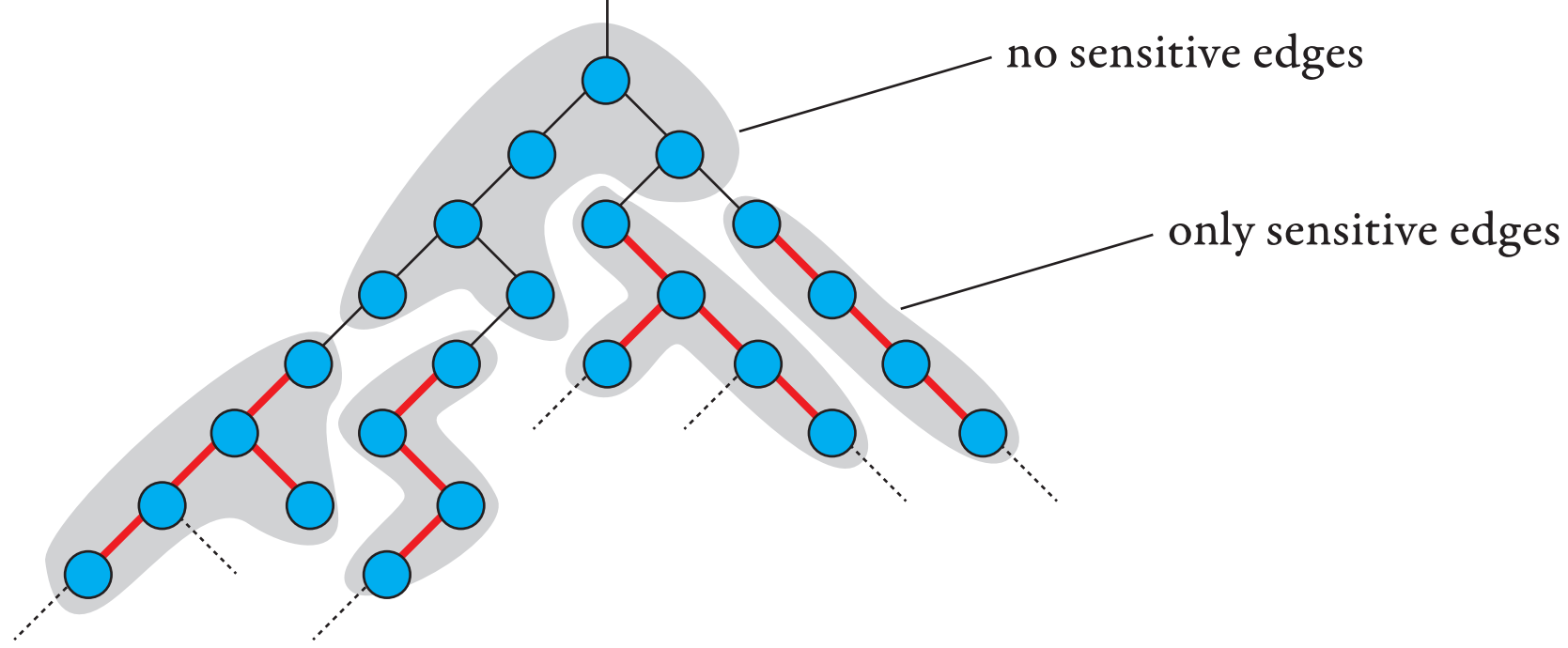
twist of port 1

1	2
↗	↖
1	2





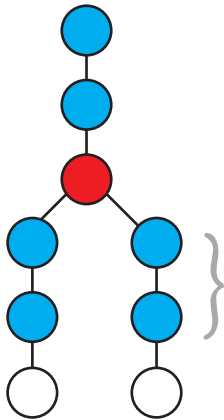
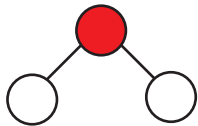










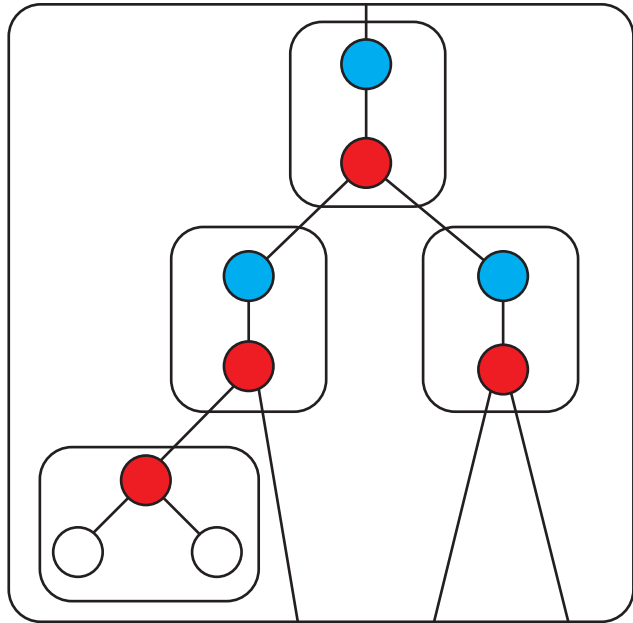


$$k - 1 = 2$$

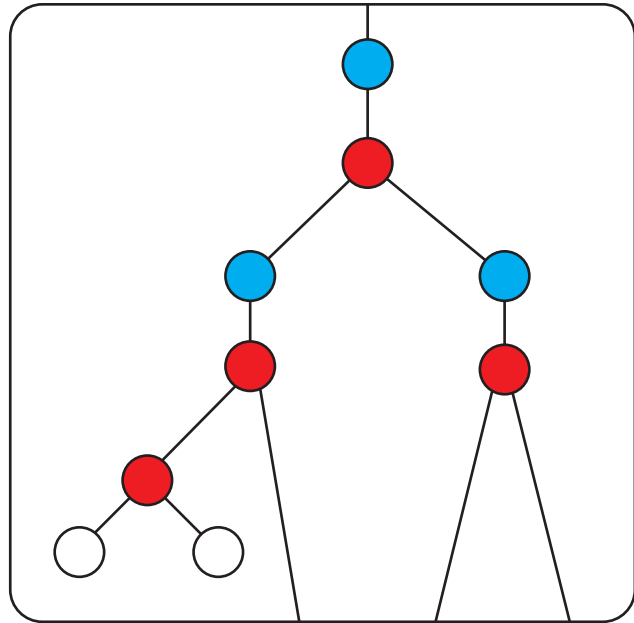






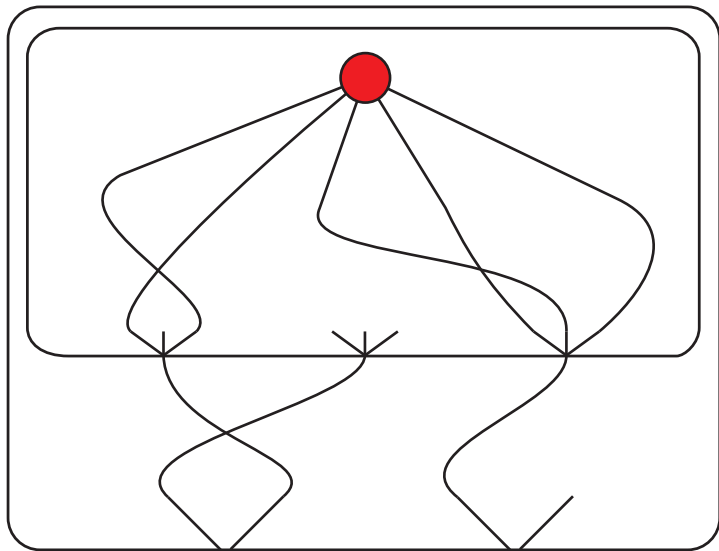


$\mapsto$

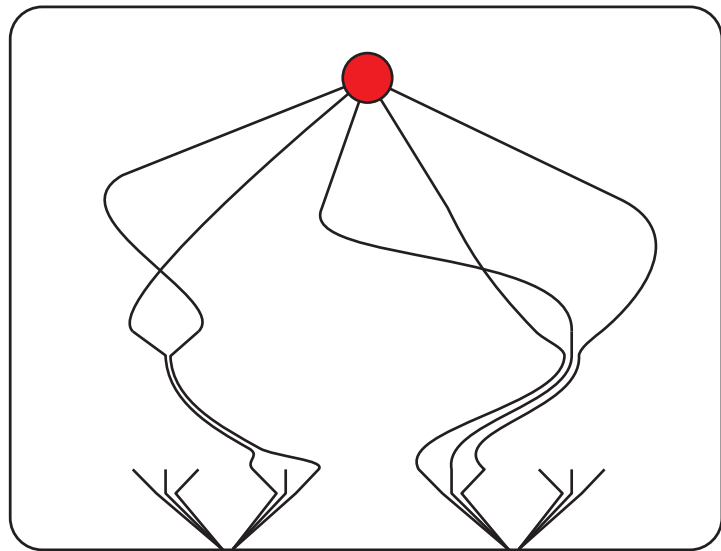


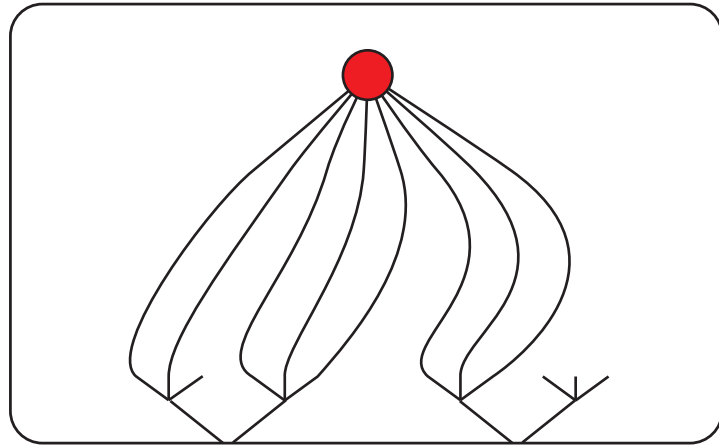
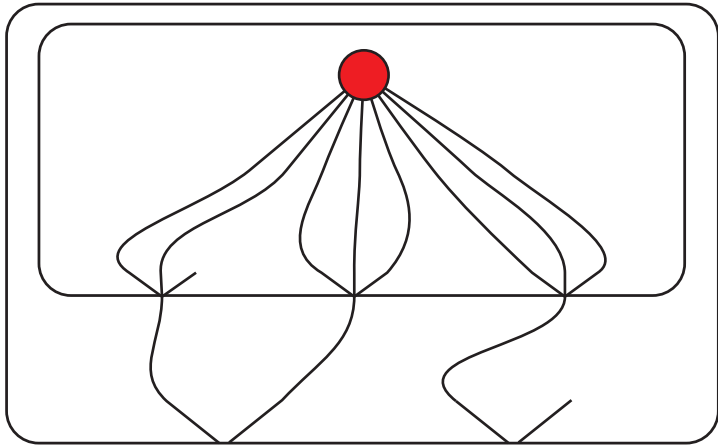


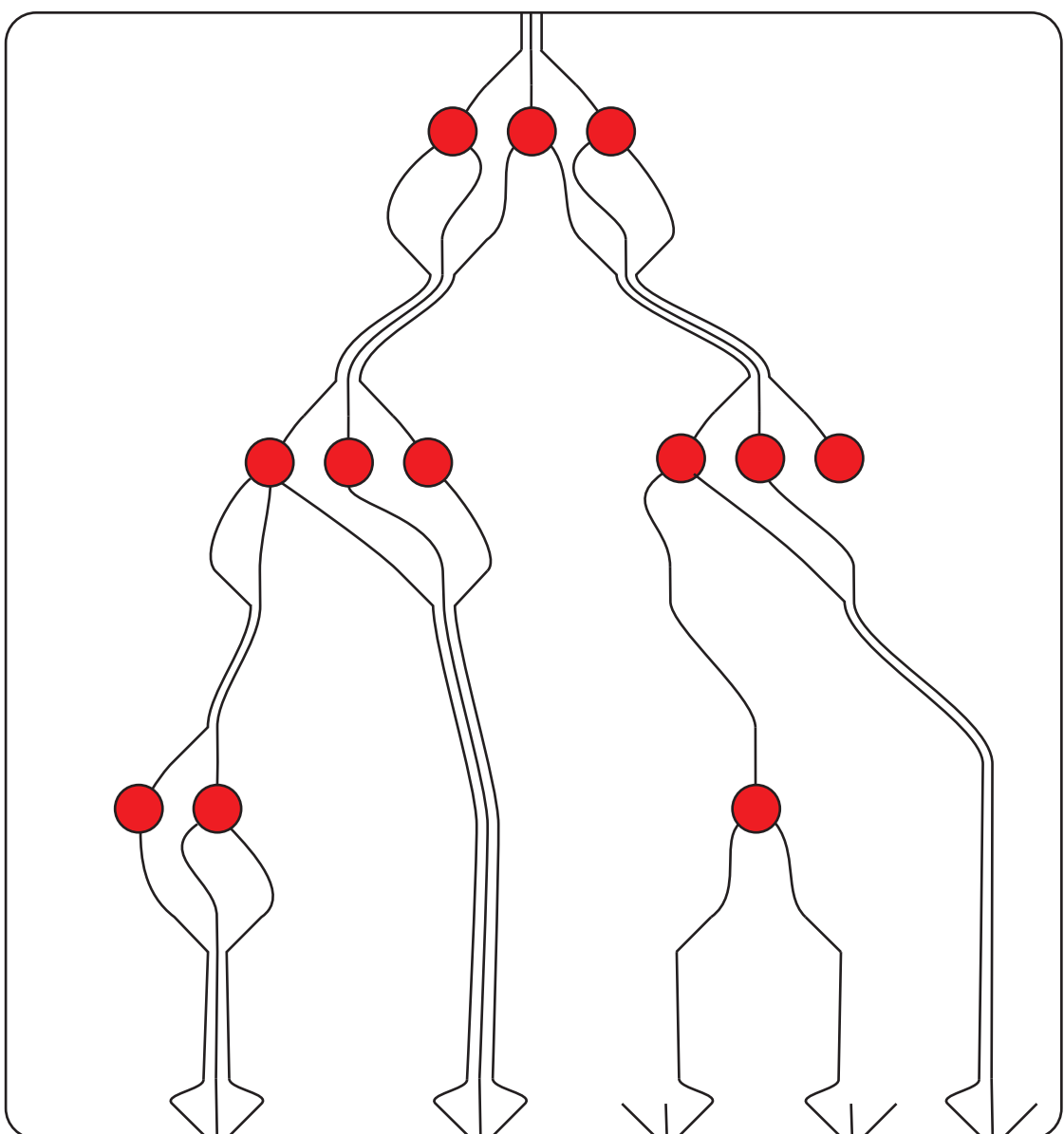
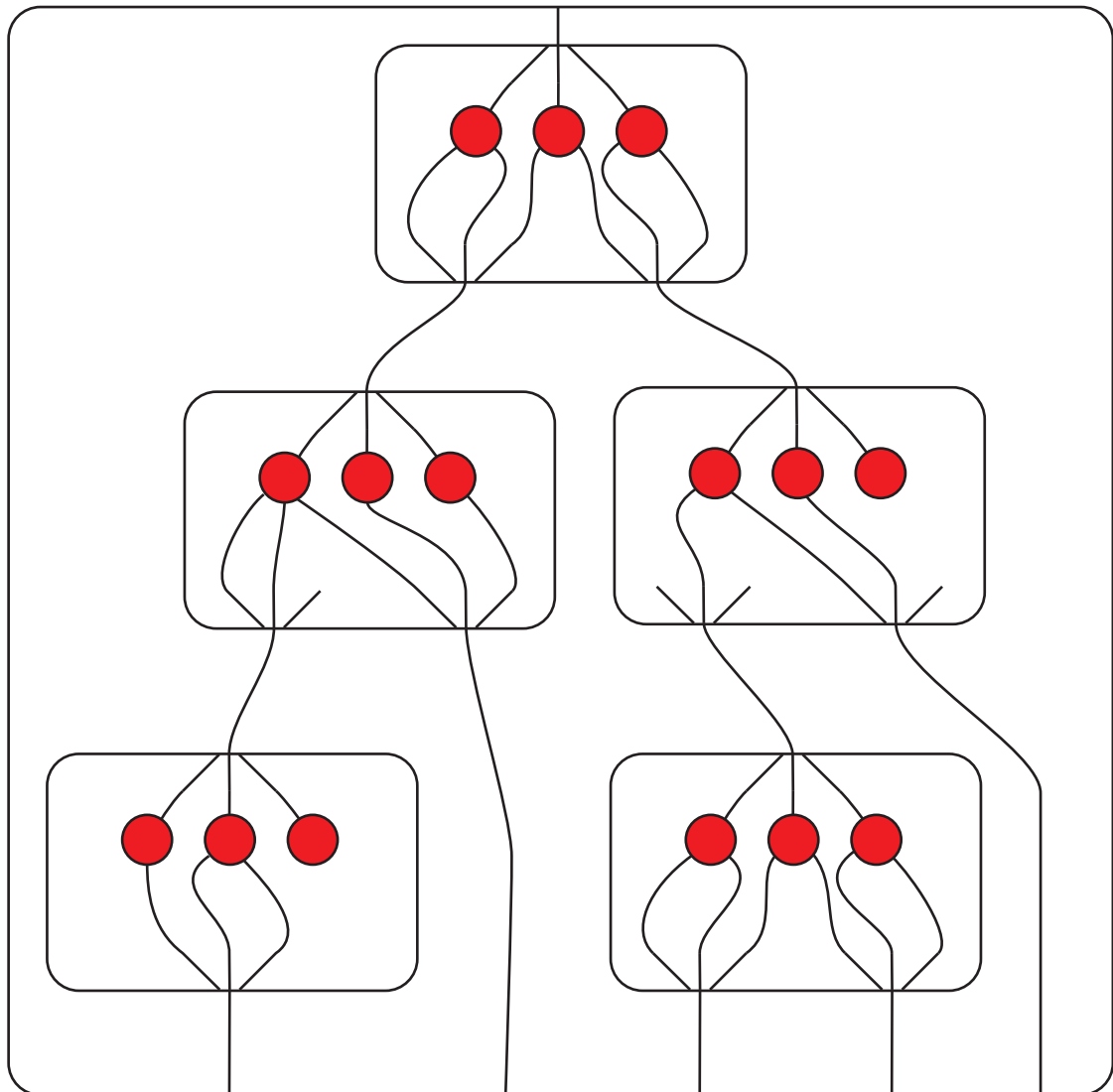
$F_2 F_3 \Sigma$



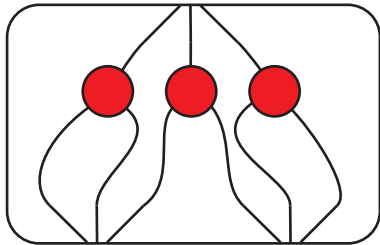
$F_6 \Sigma$



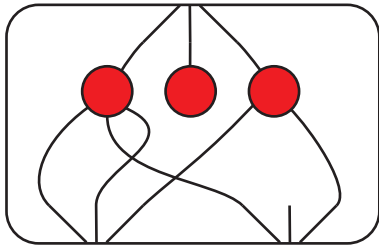




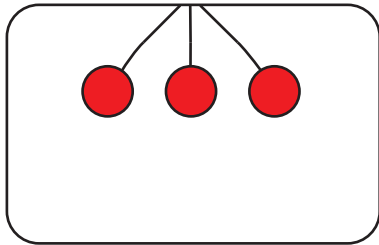
arity 2

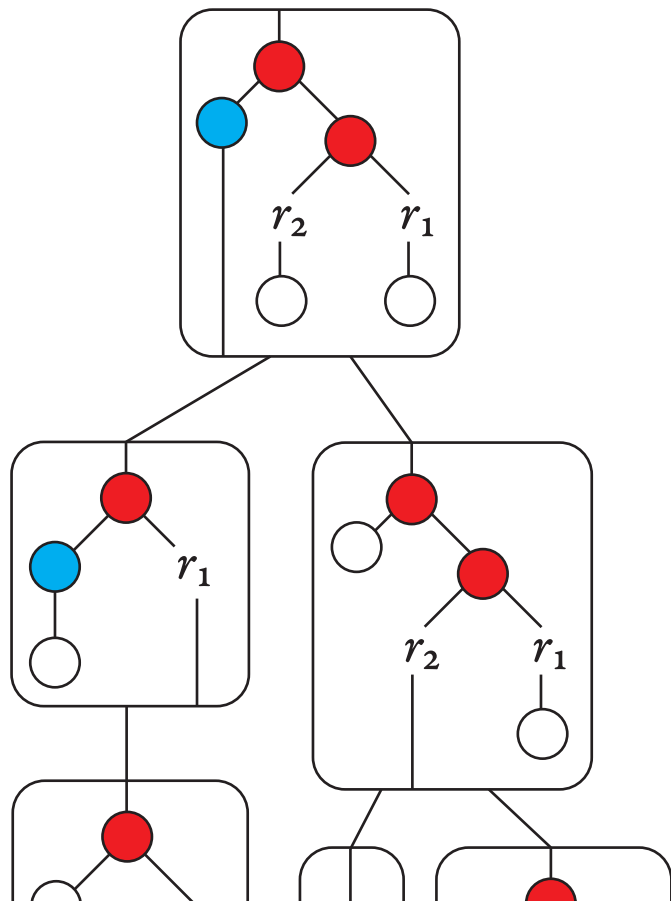


arity 2

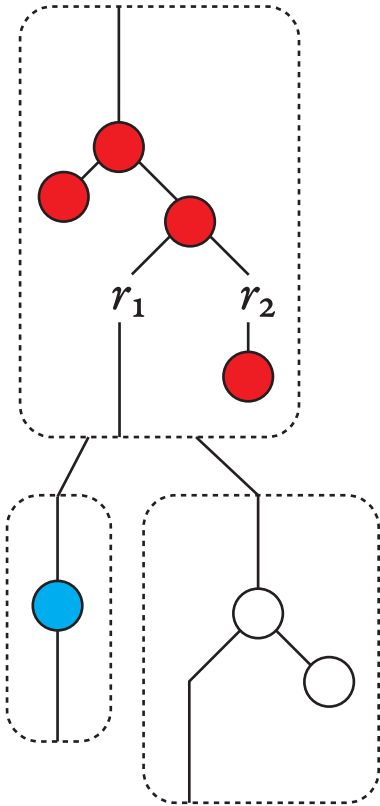


arity 0





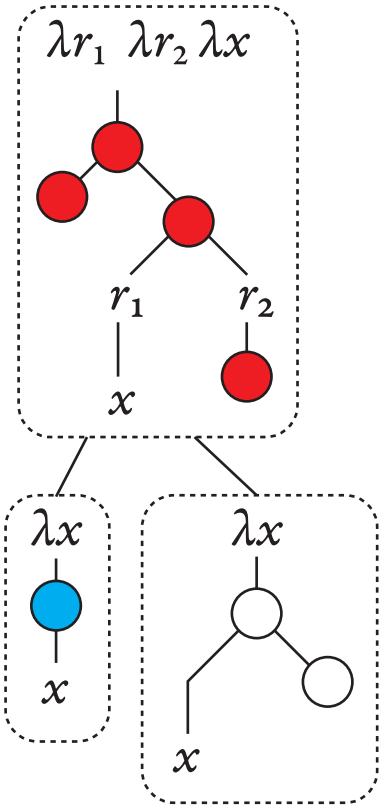
tree of register updates



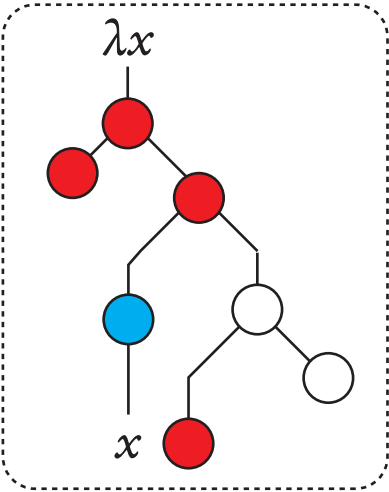
represent  
as a  $\lambda$ -term



$\lambda$ -term



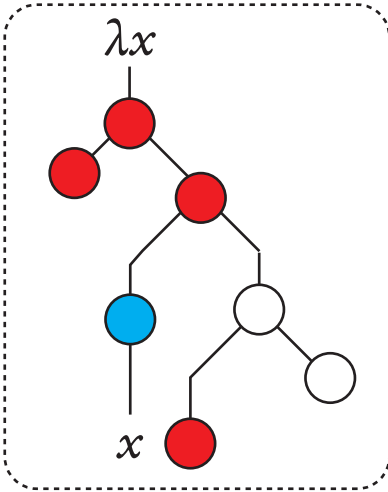
evaluate  $\Downarrow$



represent  
as a  $\lambda$ -term

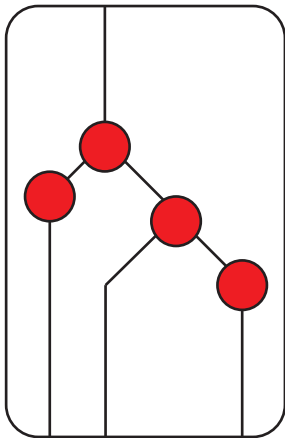


$\Downarrow$  evaluate

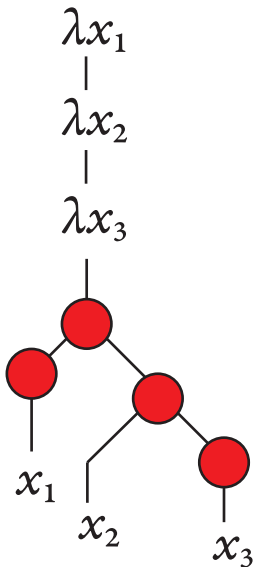


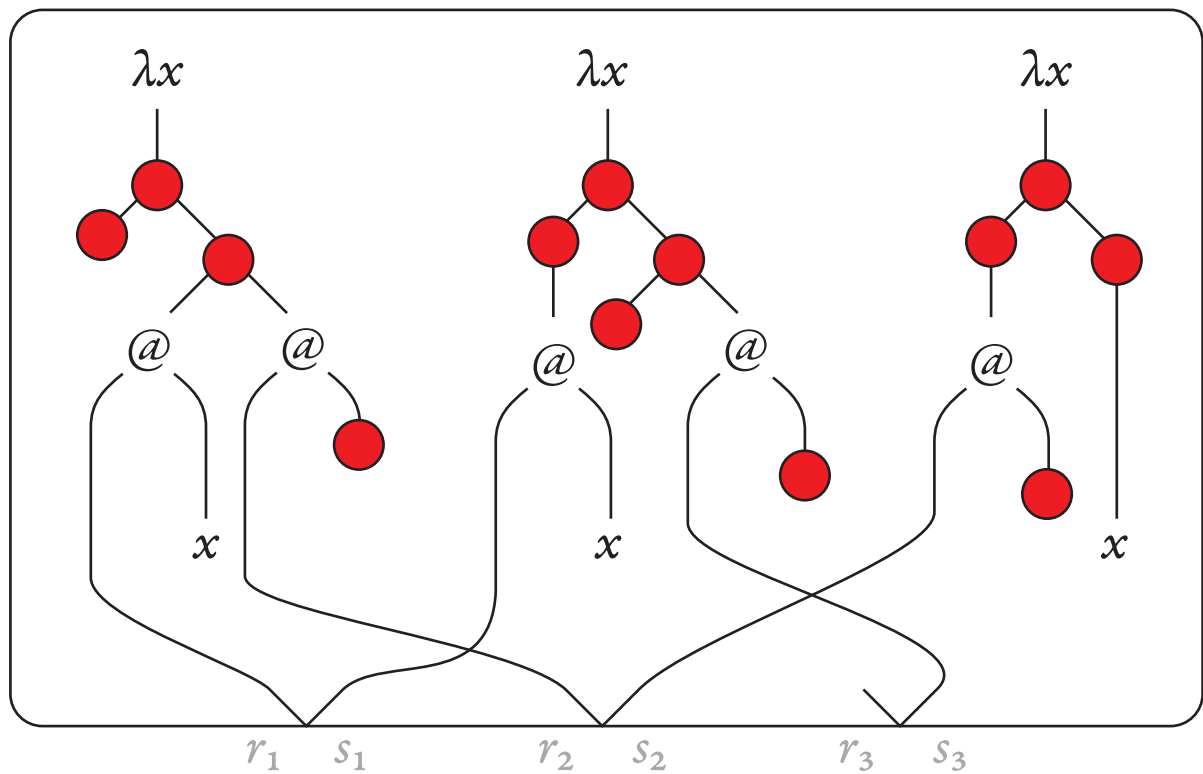
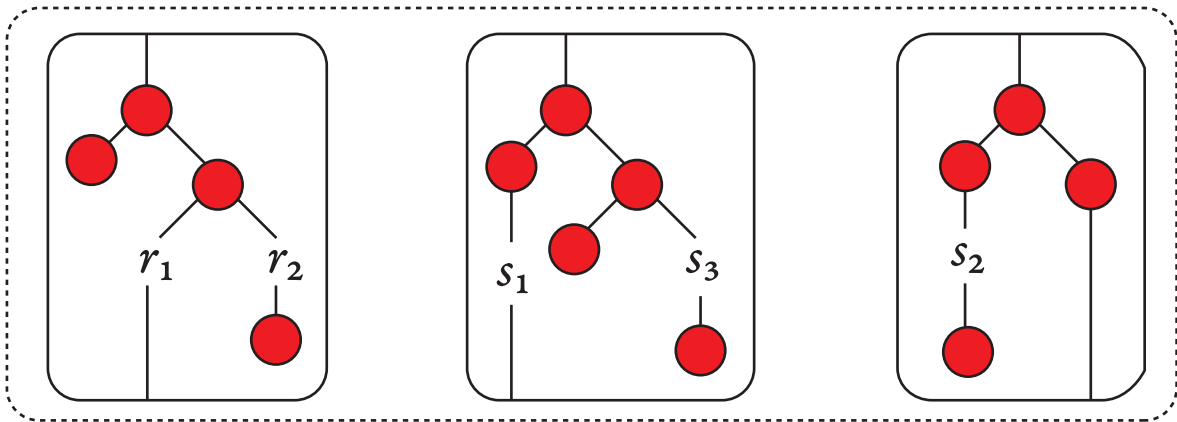


a term

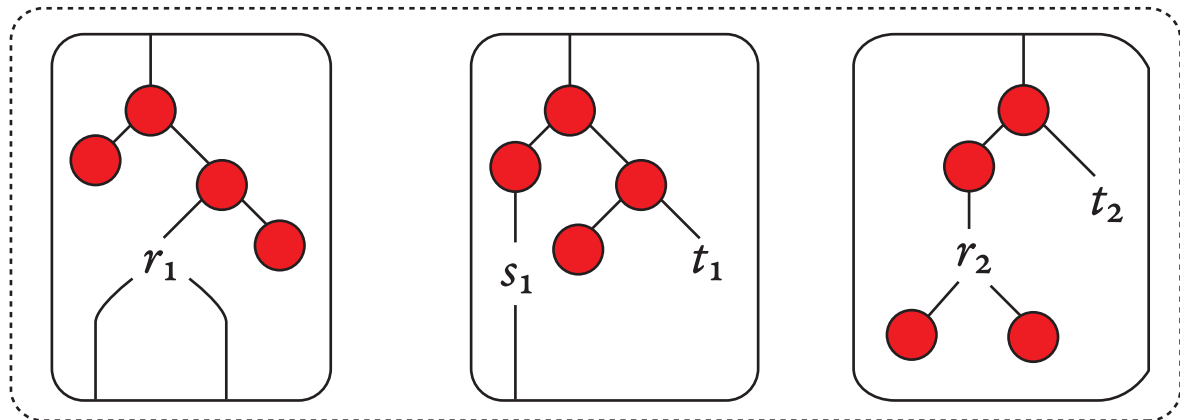


its  $\lambda$ -representation

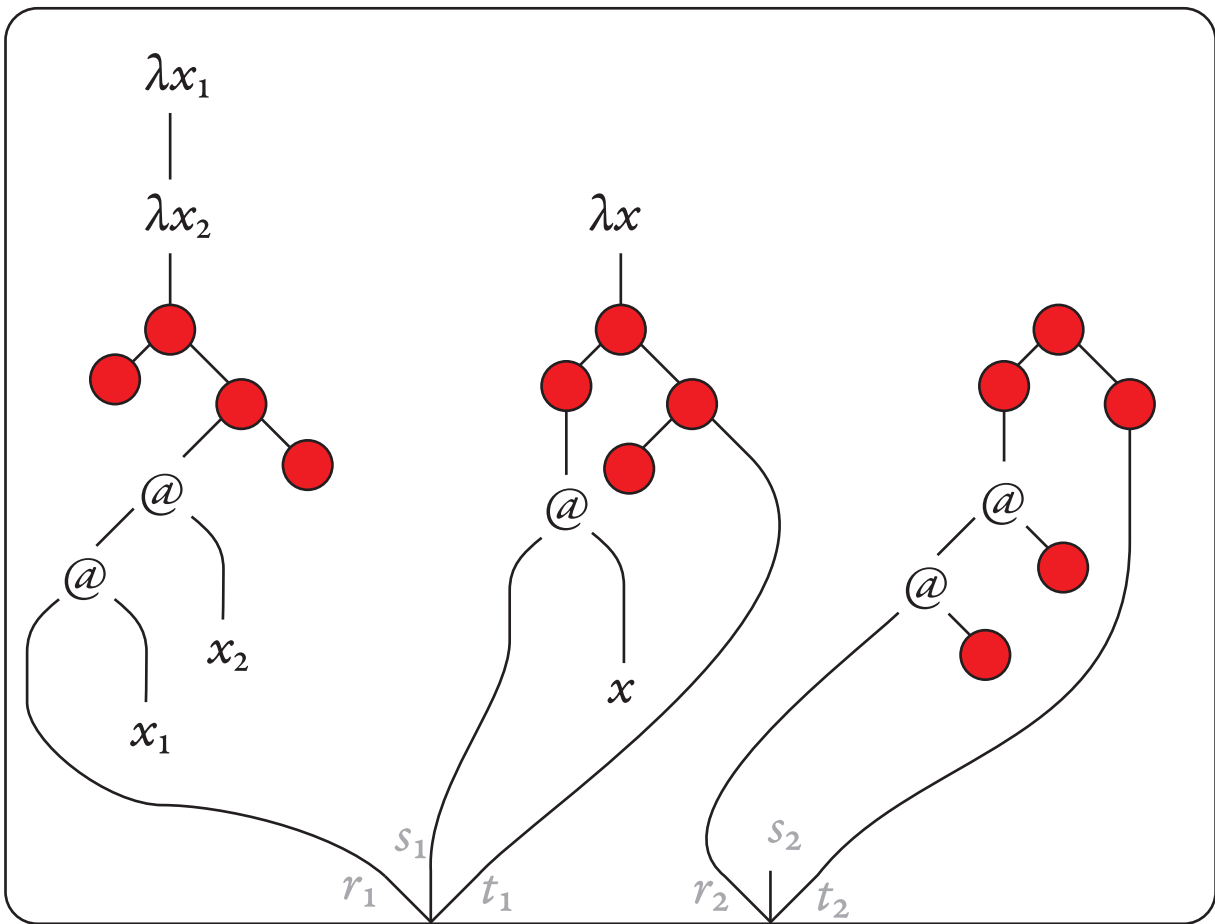


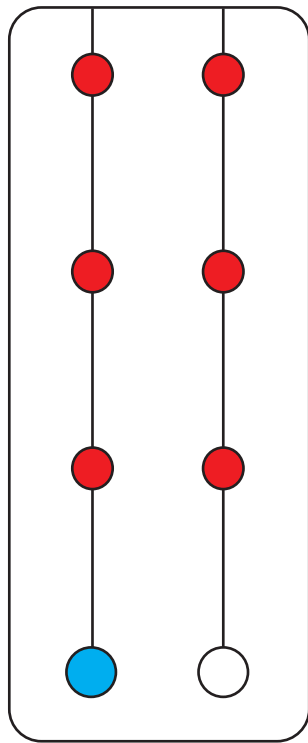
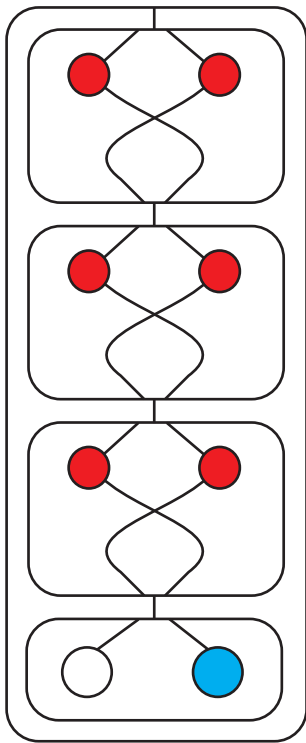


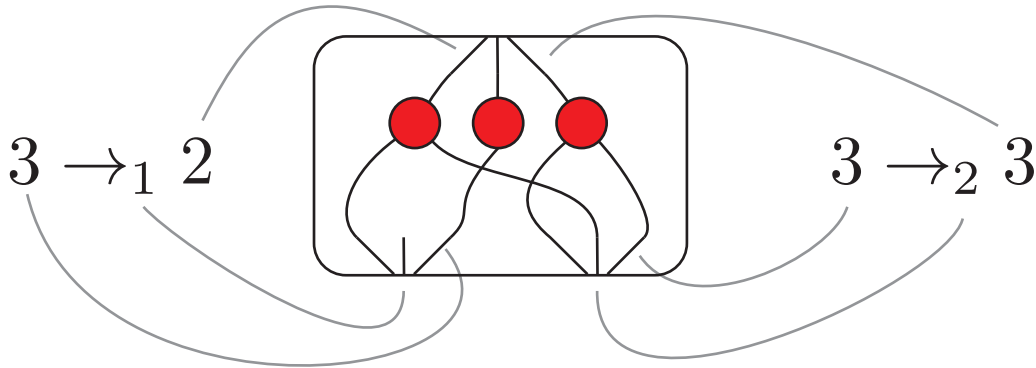
a register update

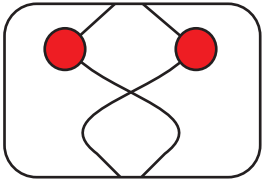


its  $\lambda$ -representation

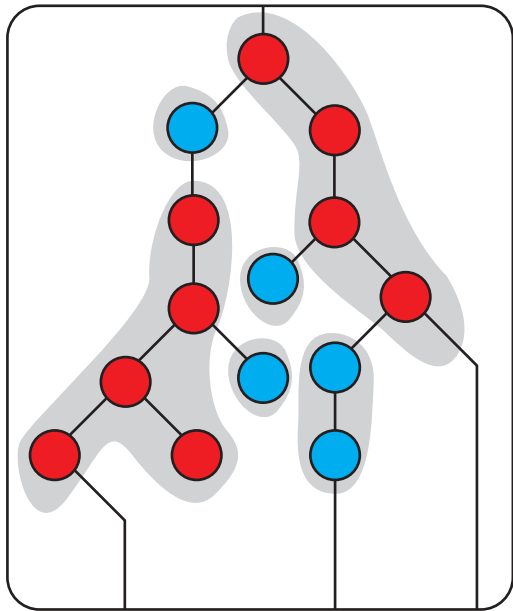




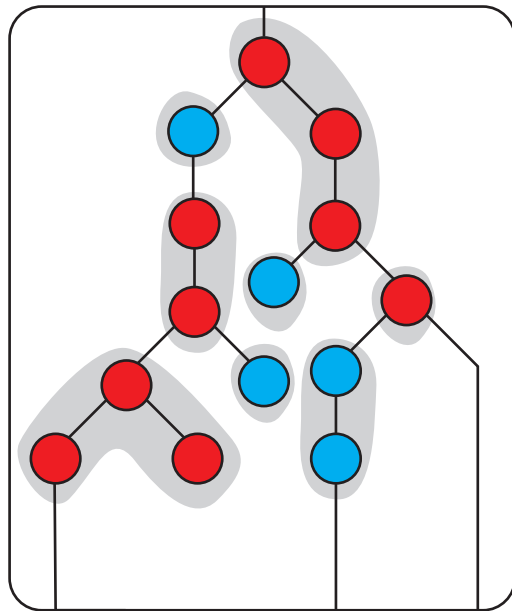


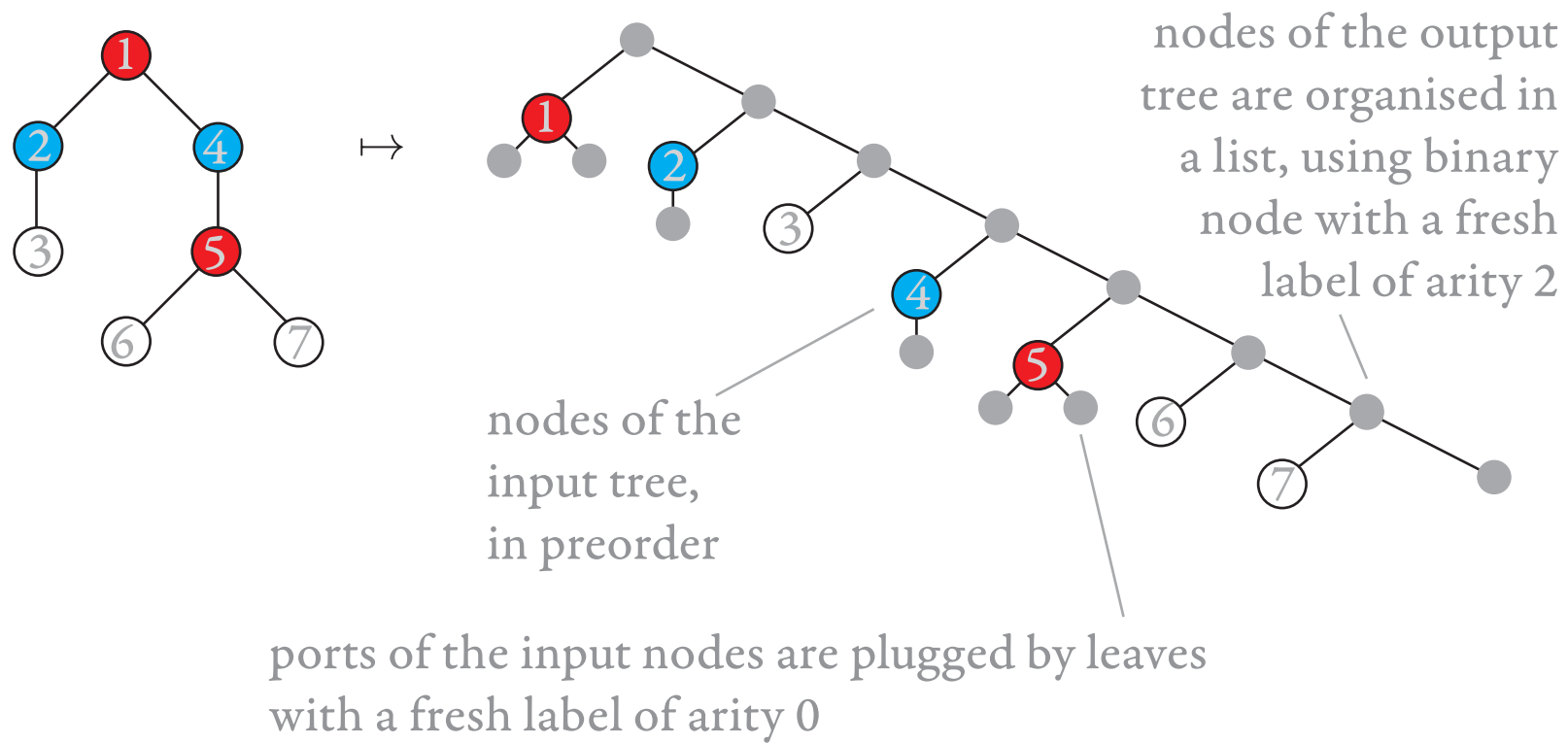


$\uparrow$ -equivalence

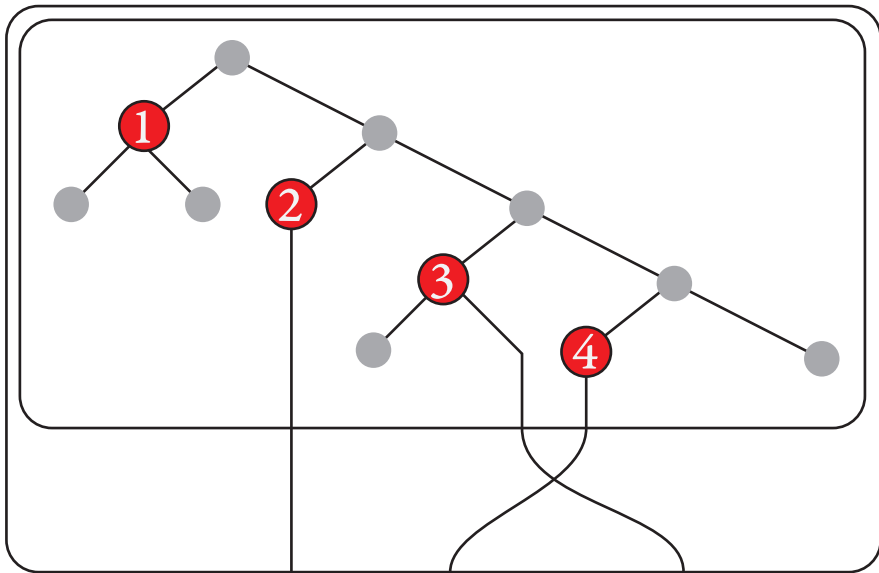
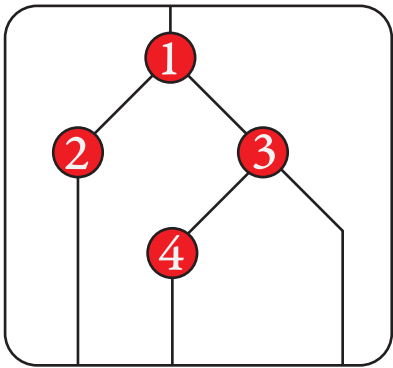


$\downarrow$ -equivalence

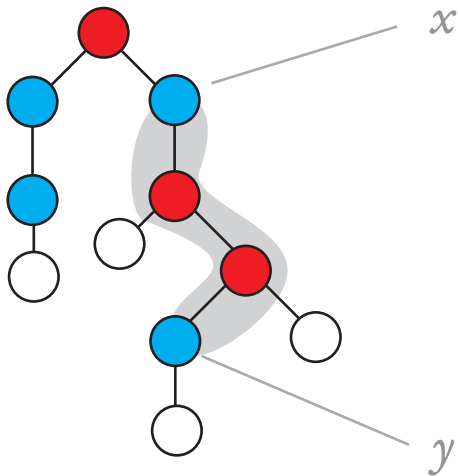




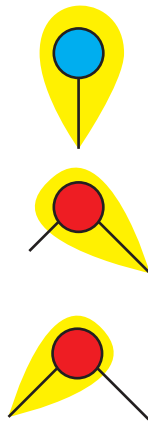




tree with a path  
from  $x$  to  $y$



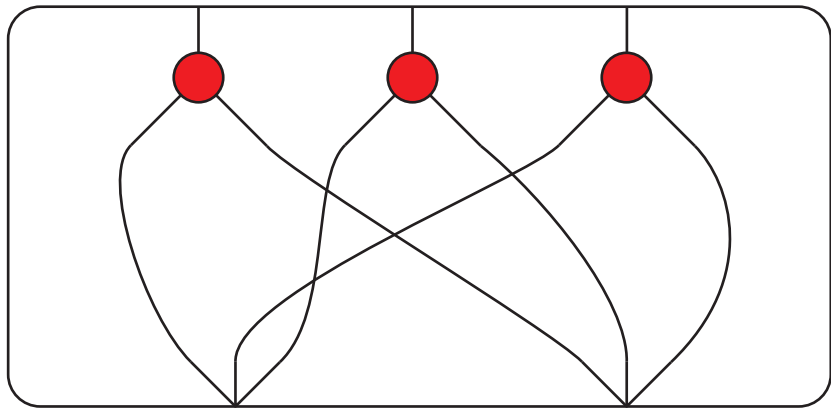
word corresponding  
to the path

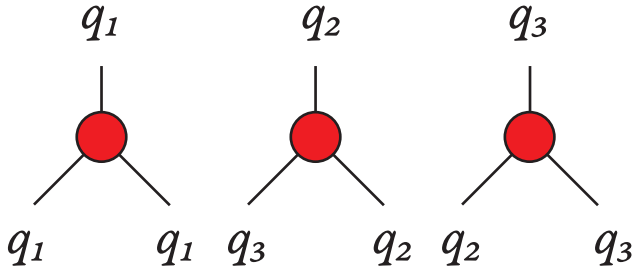


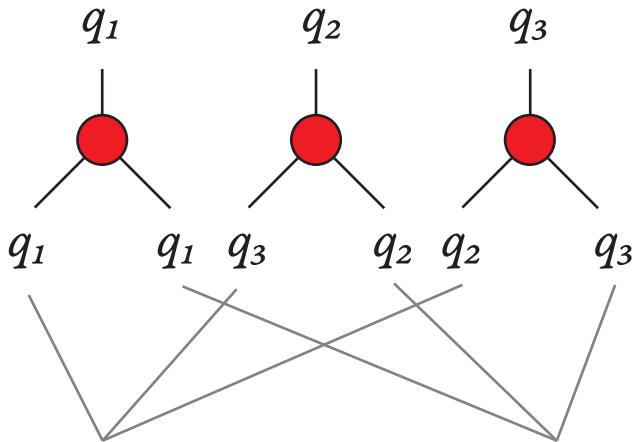
copy 1  
of  $a$

copy 2  
of  $a$

copy 3  
of  $a$







each state appears  
exactly once as  
a first child

each state appears  
exactly once as  
a second child