

a ranked alphabet

arity 2



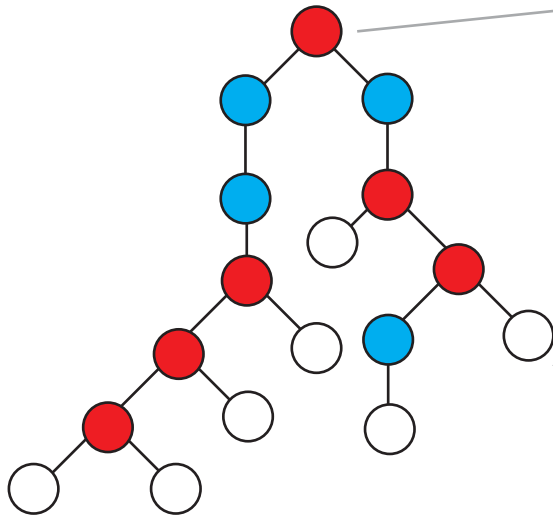
arity 1



arity 0



a tree



this node has a
label of arity 2,
and therefore it has
2 children

this node is
child 2
(children are
ordered)



A tree t over $\Sigma^{[2]}$



$\text{unfold}_1(t)$



$\text{unfold}_2(t)$





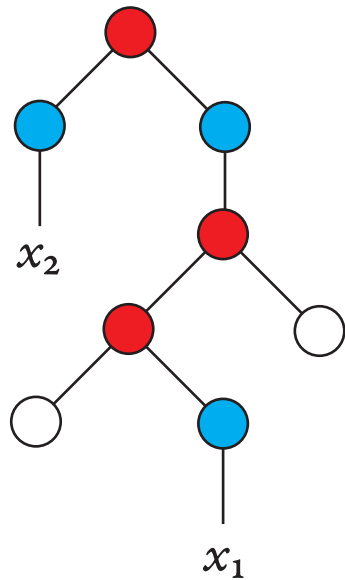
t



substitute(t)

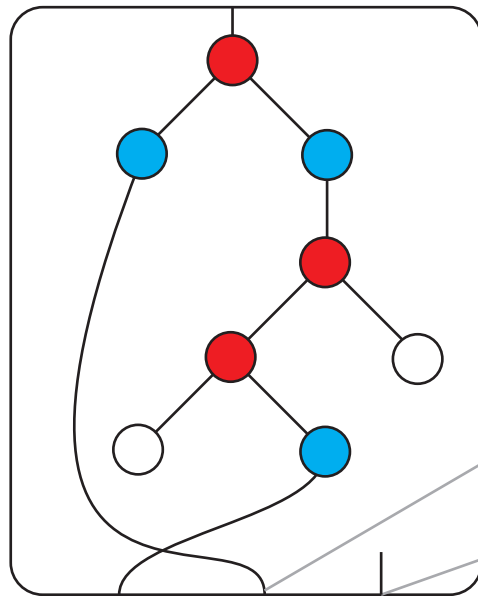






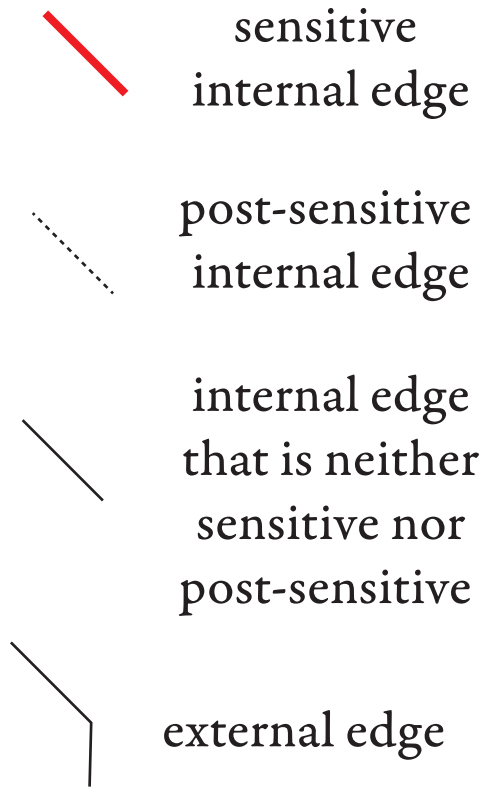
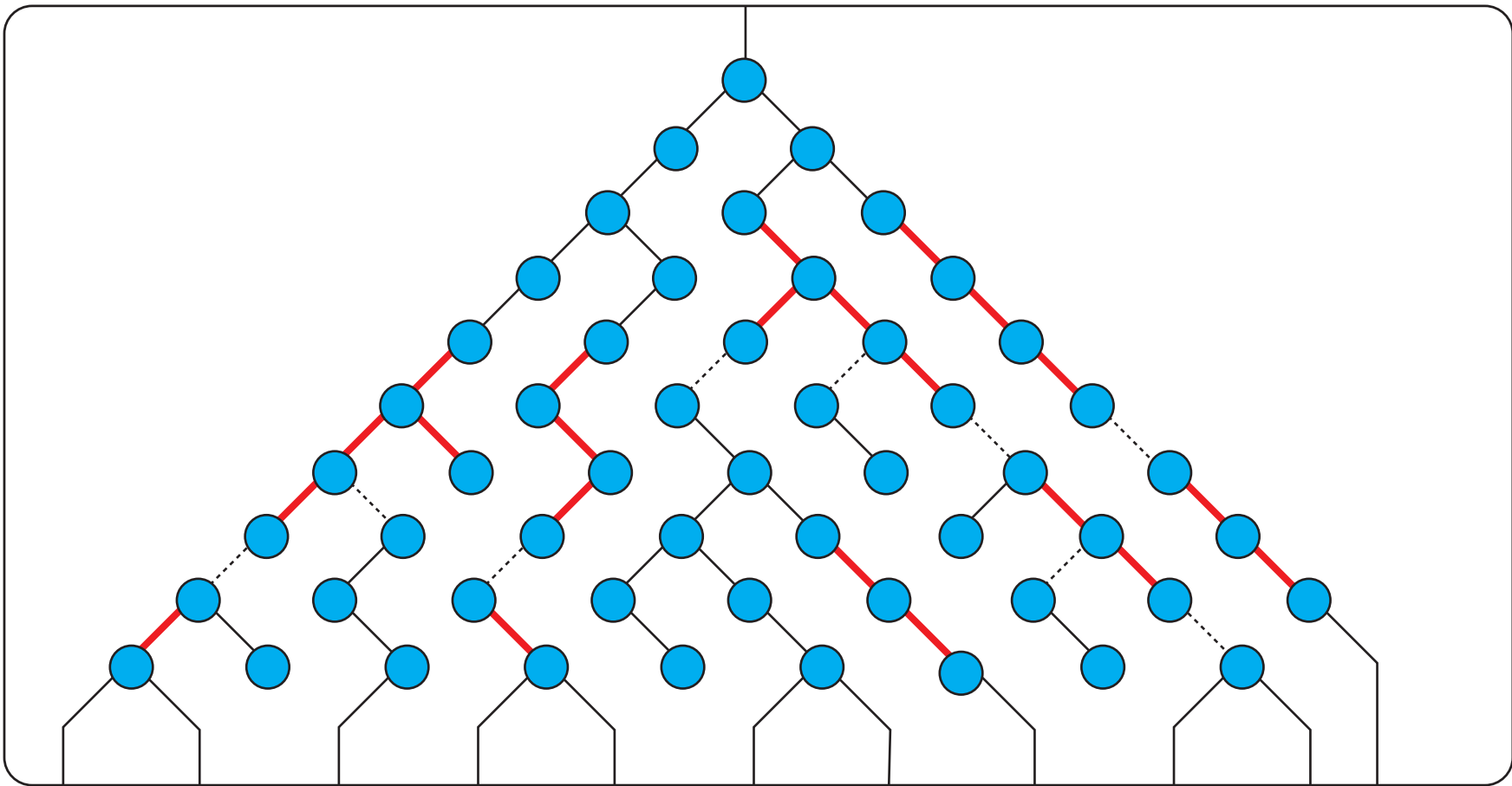
=

a term of arity 3



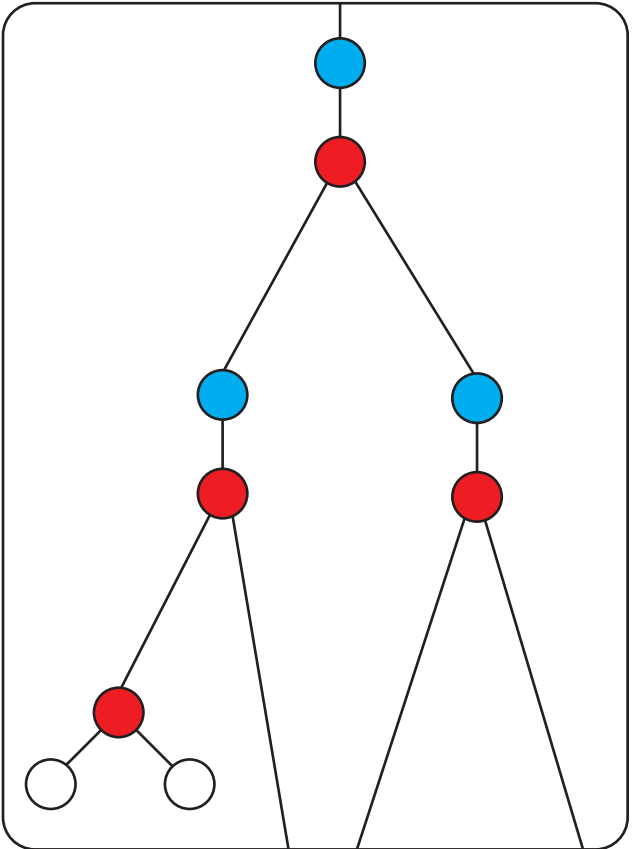
lines leaving at the bottom of the box
represent variables

dangling edges represent unused variables





\mapsto





a term



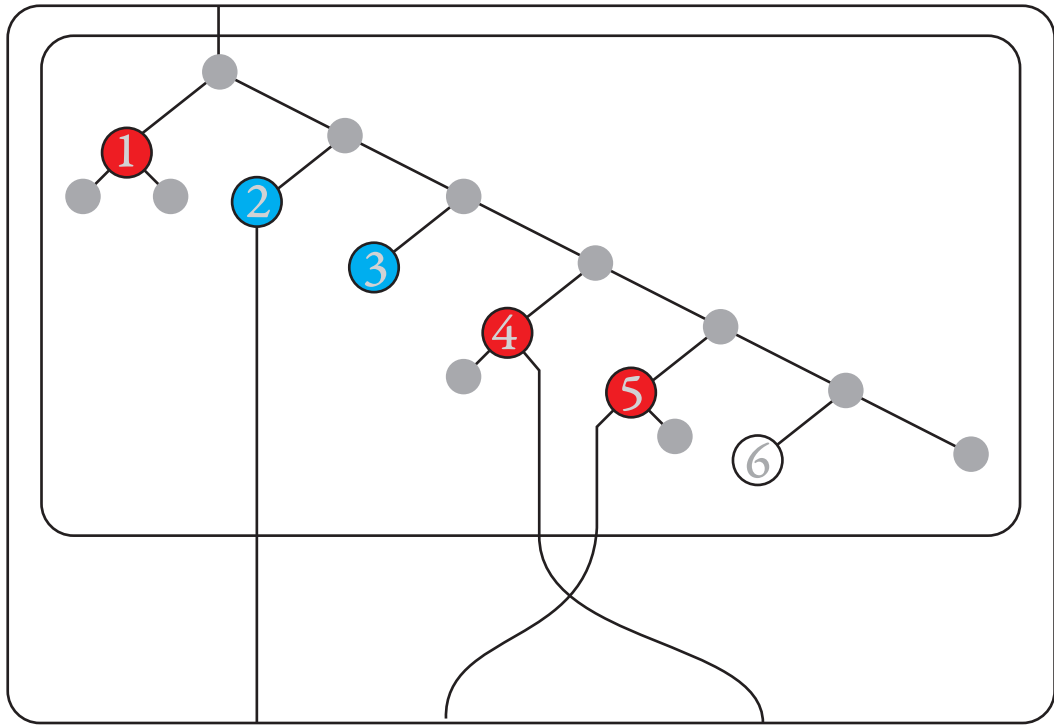
ancestor equivalence



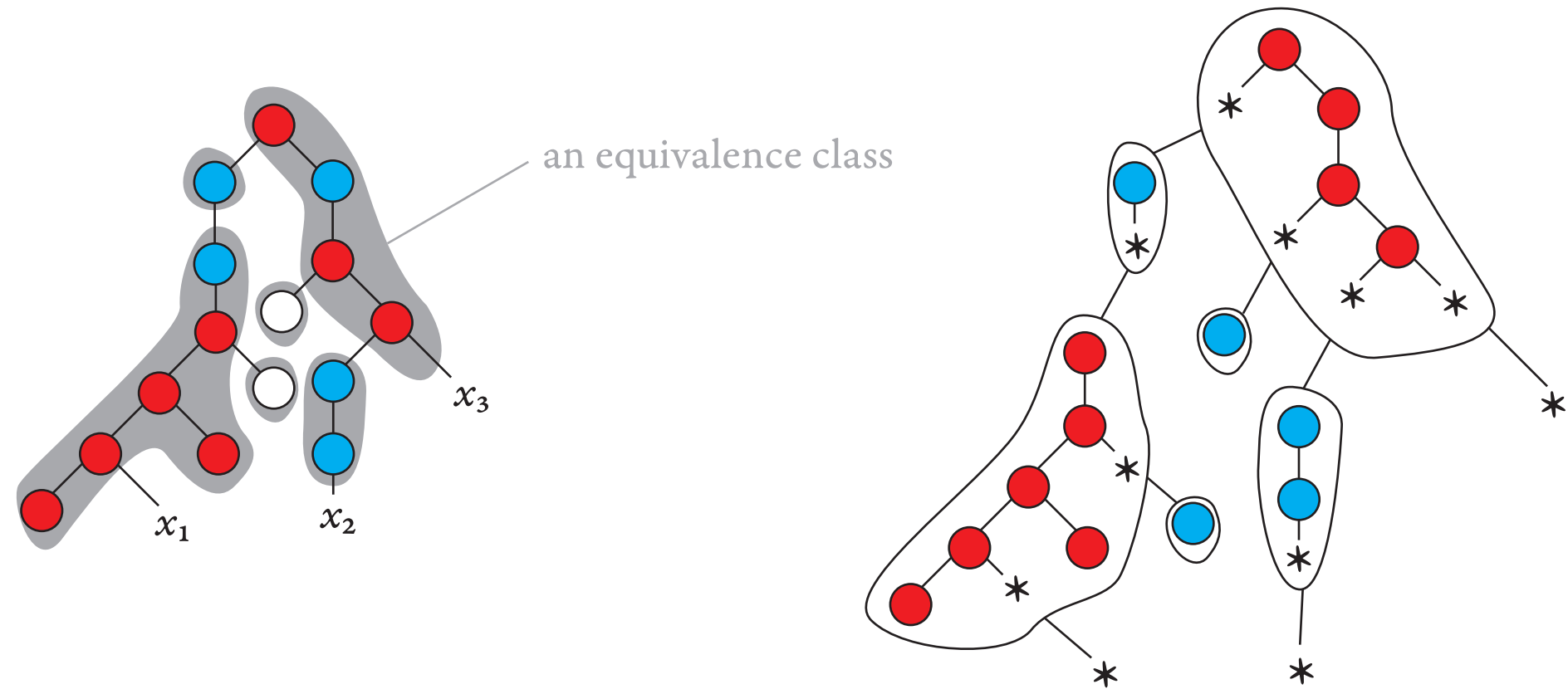
descendant equivalence



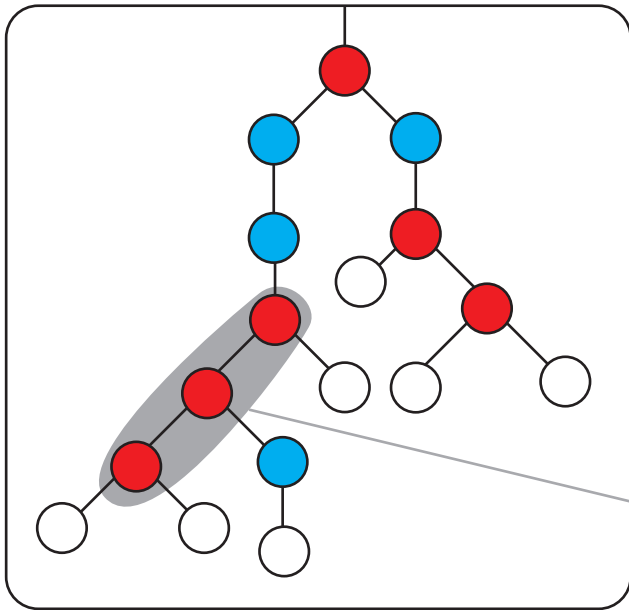




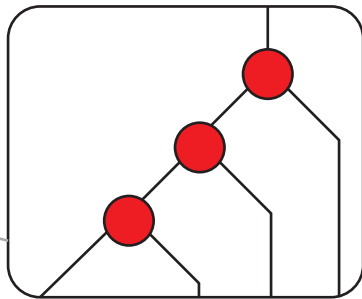
a factorisation equivalence



a tree



a term with
4 ports that
represents
part of the
tree





input alphabet

arity 2



arity 1



arity 0



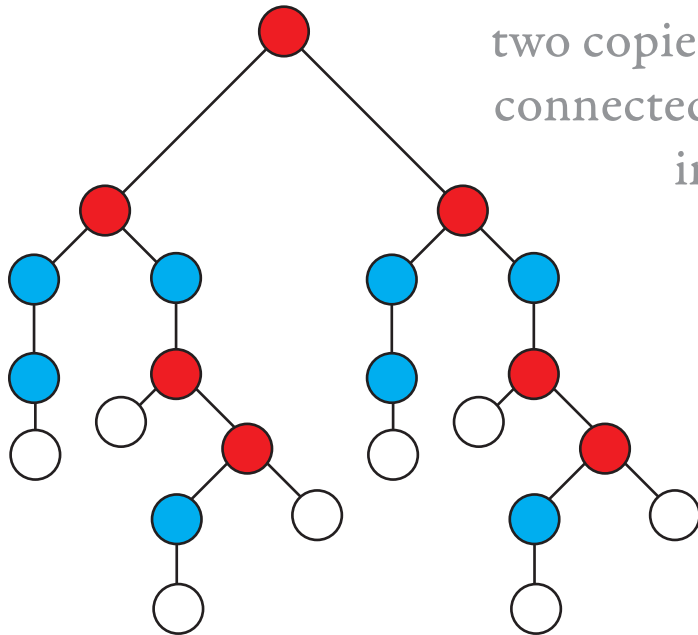
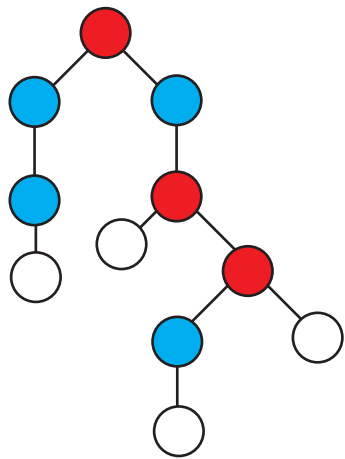
output alphabet

arity 2



arity 0





two copies of the input tree,
connected by a binary node
in the root





input alphabet

arity 2



arity 1



arity 0



output alphabet

arity 2



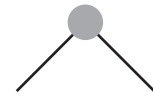
arity 1



arity 0

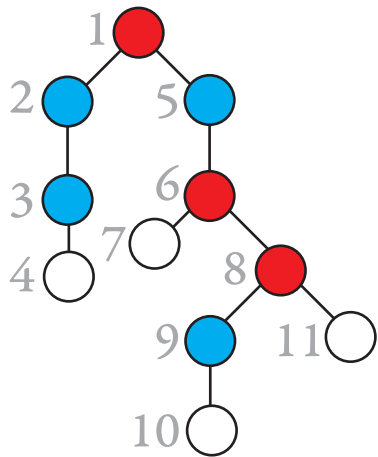


arity 2

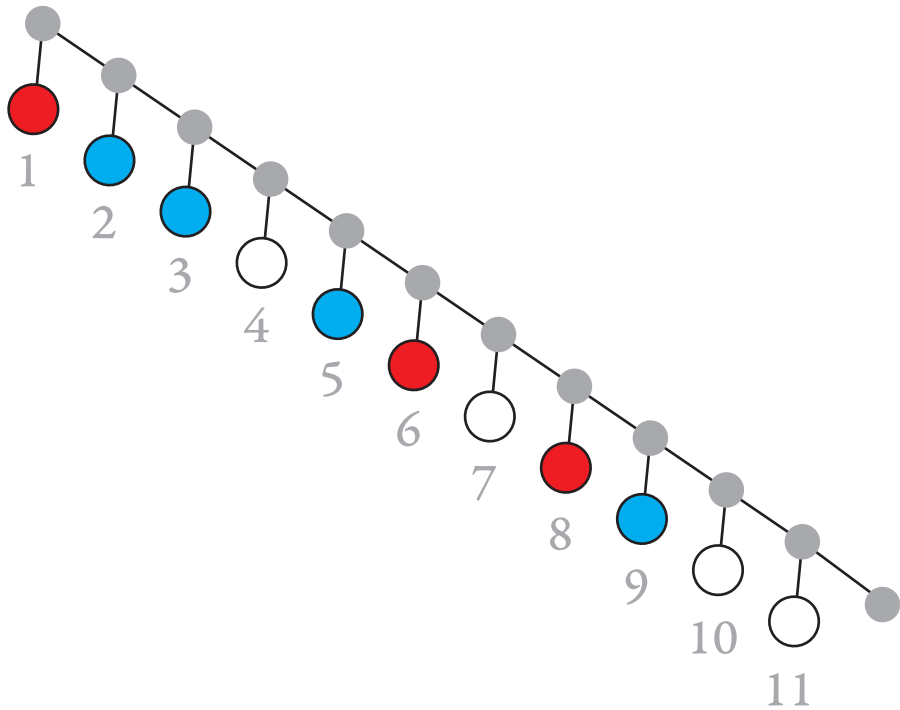


arity 0





\mapsto





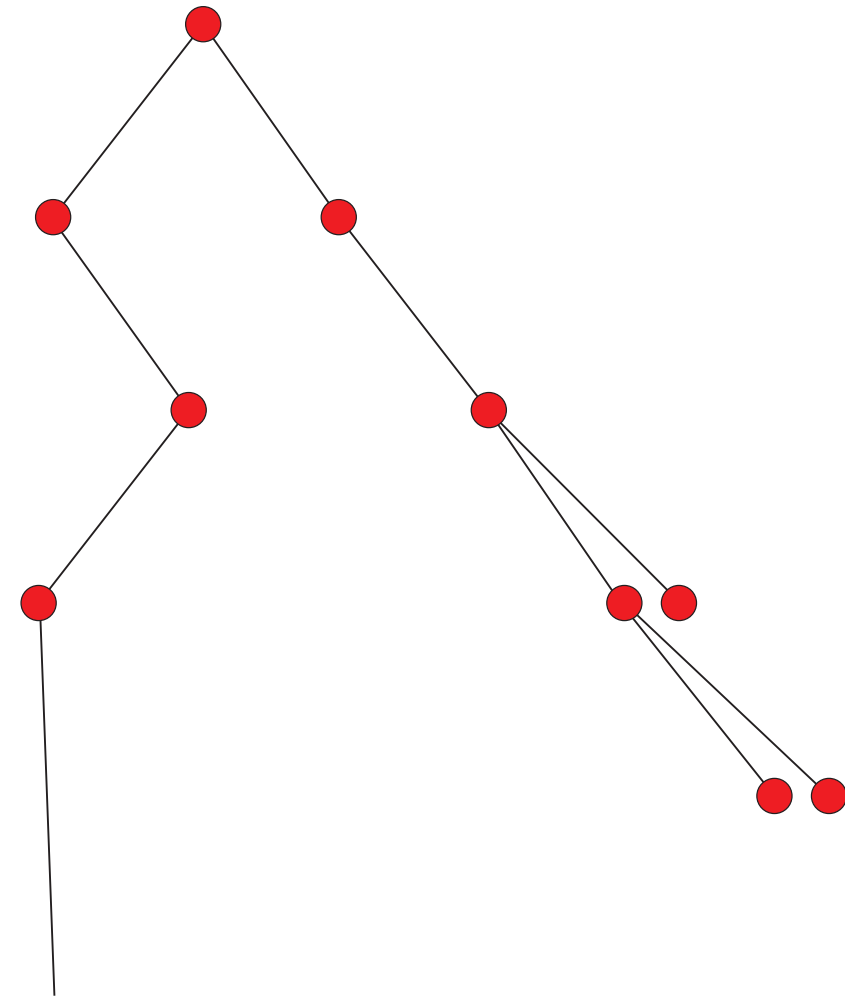
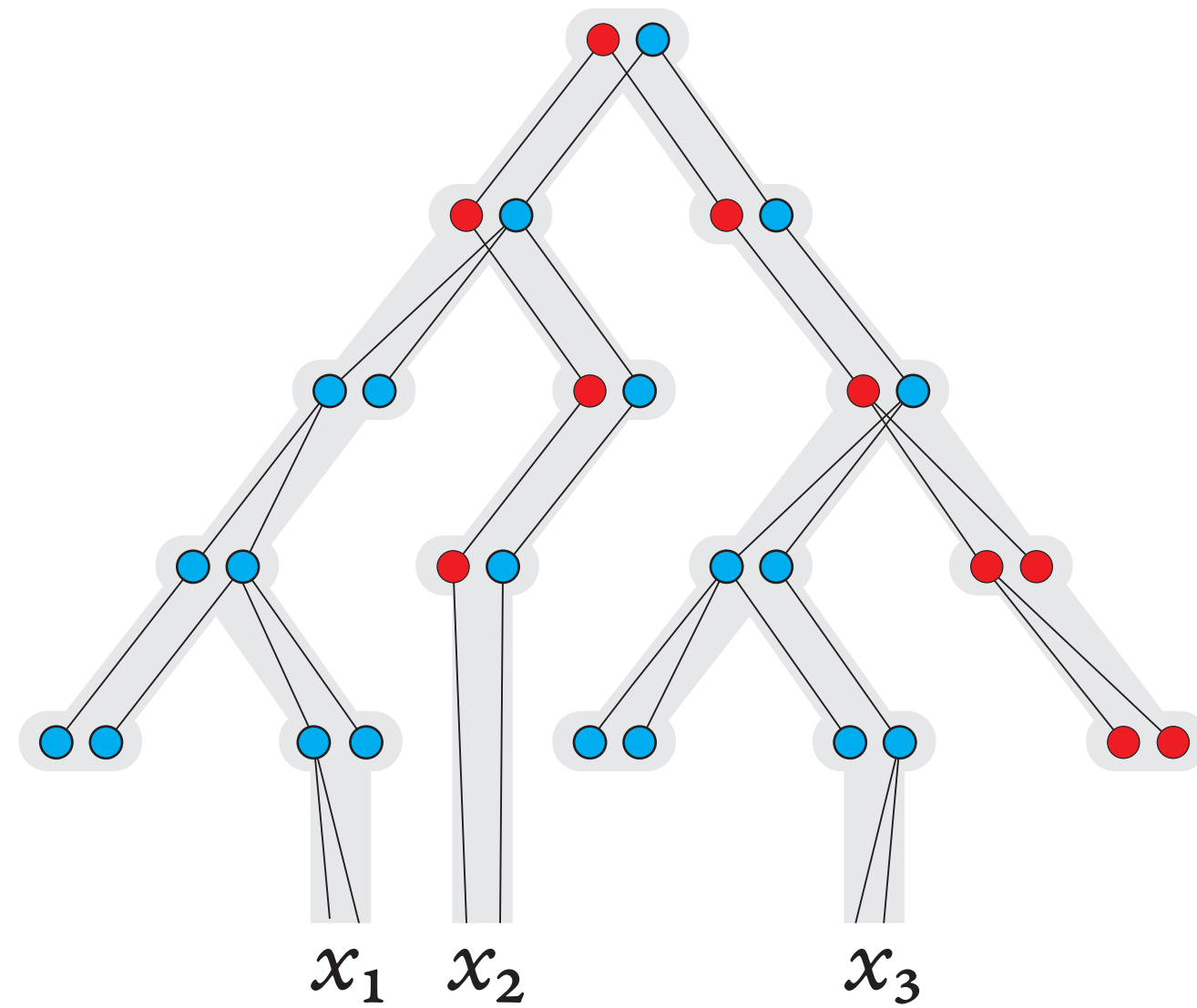


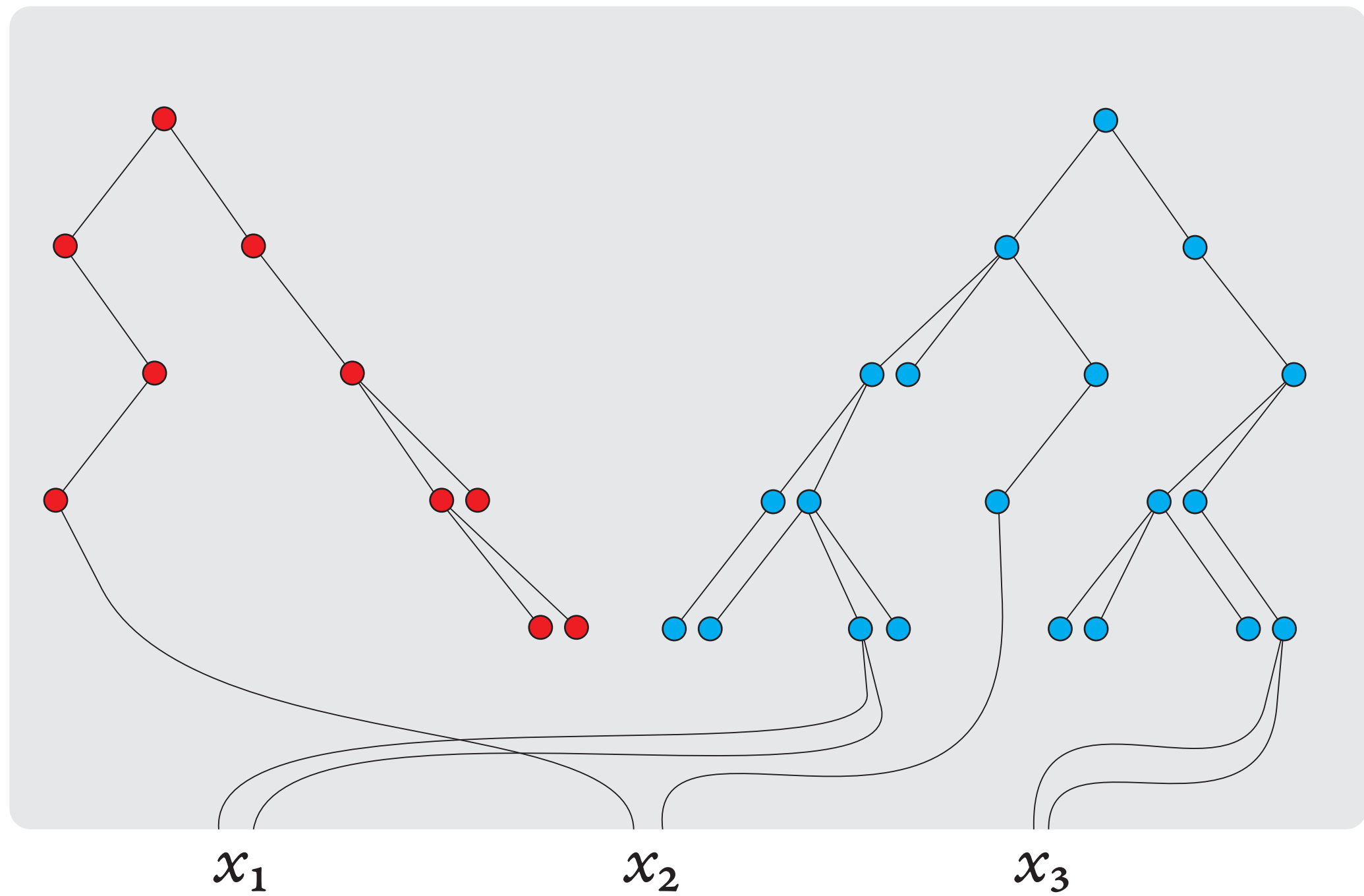
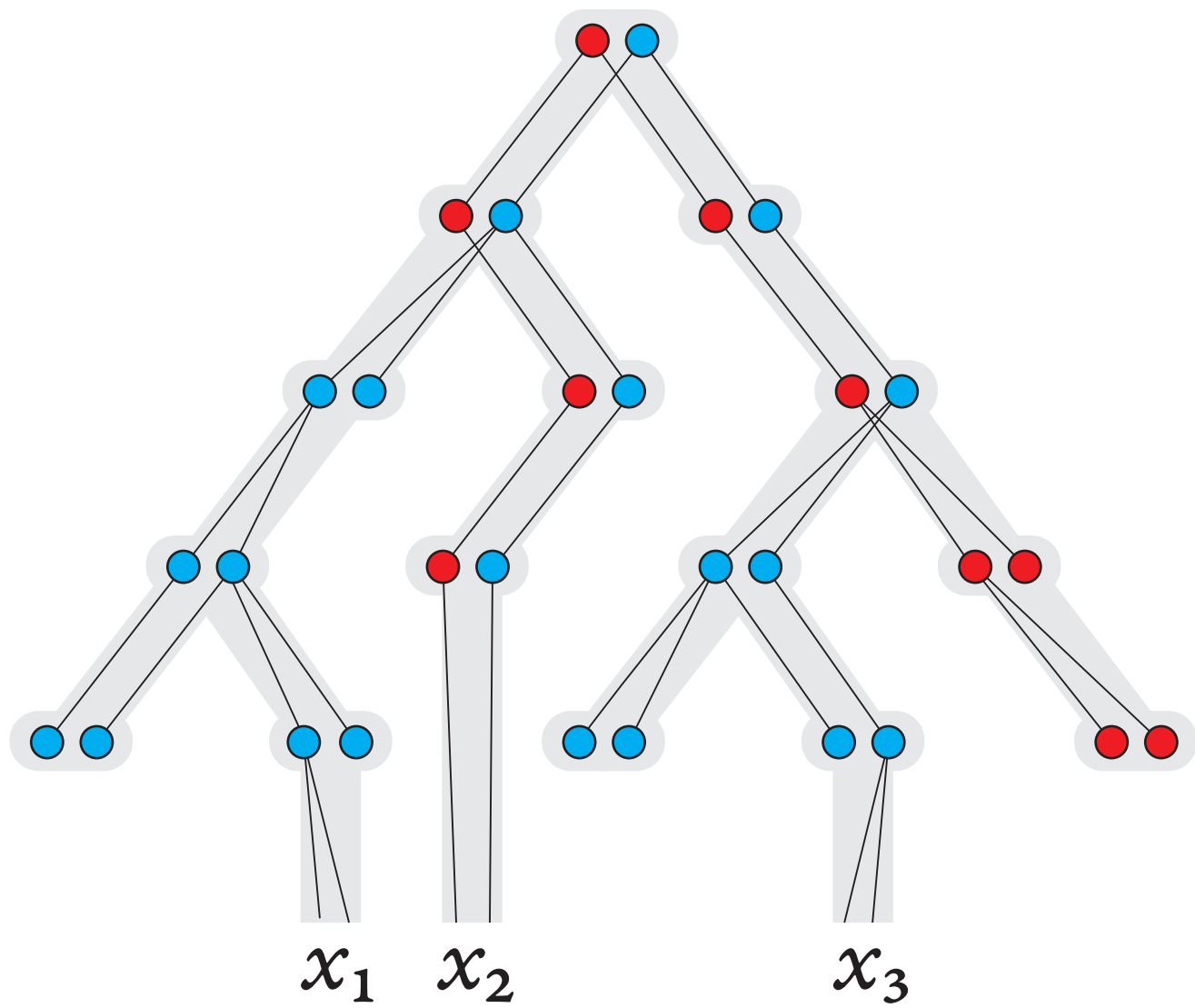
a term of arity 4



a term of arity 0





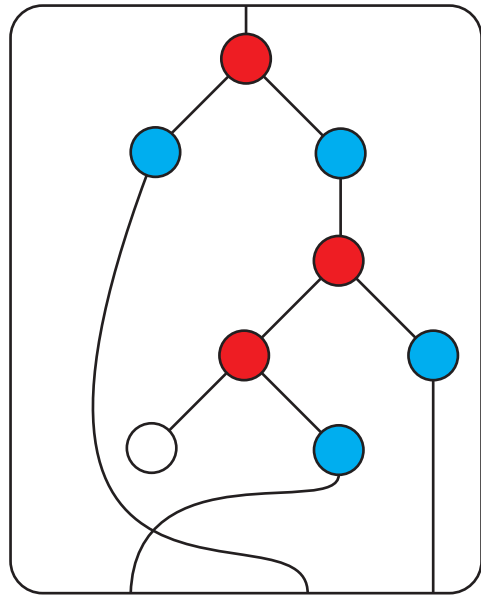




satisfies (*)

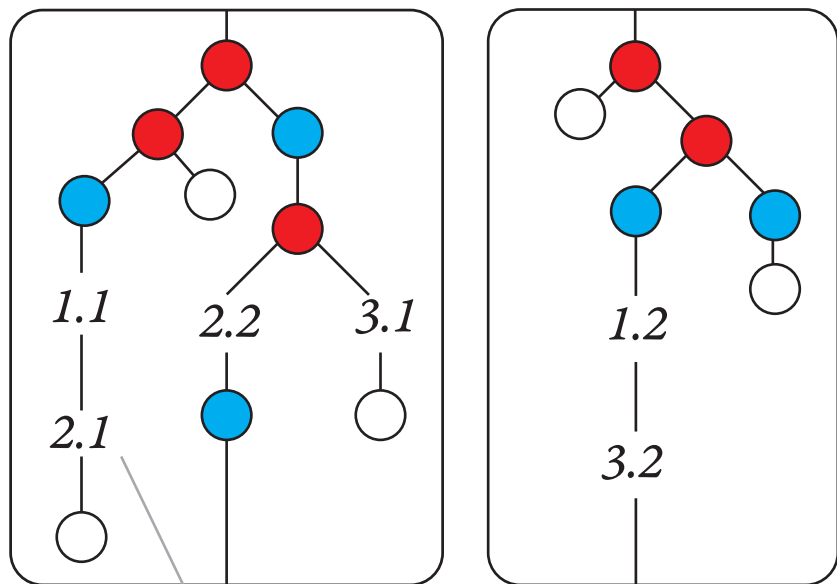
(*)

If the root has arity n ,
and $1 \leq i < j \leq n$, then
all ports of the j -th
subterm of the root are
after all ports of the
 i -th subterm of the root



violates (*)

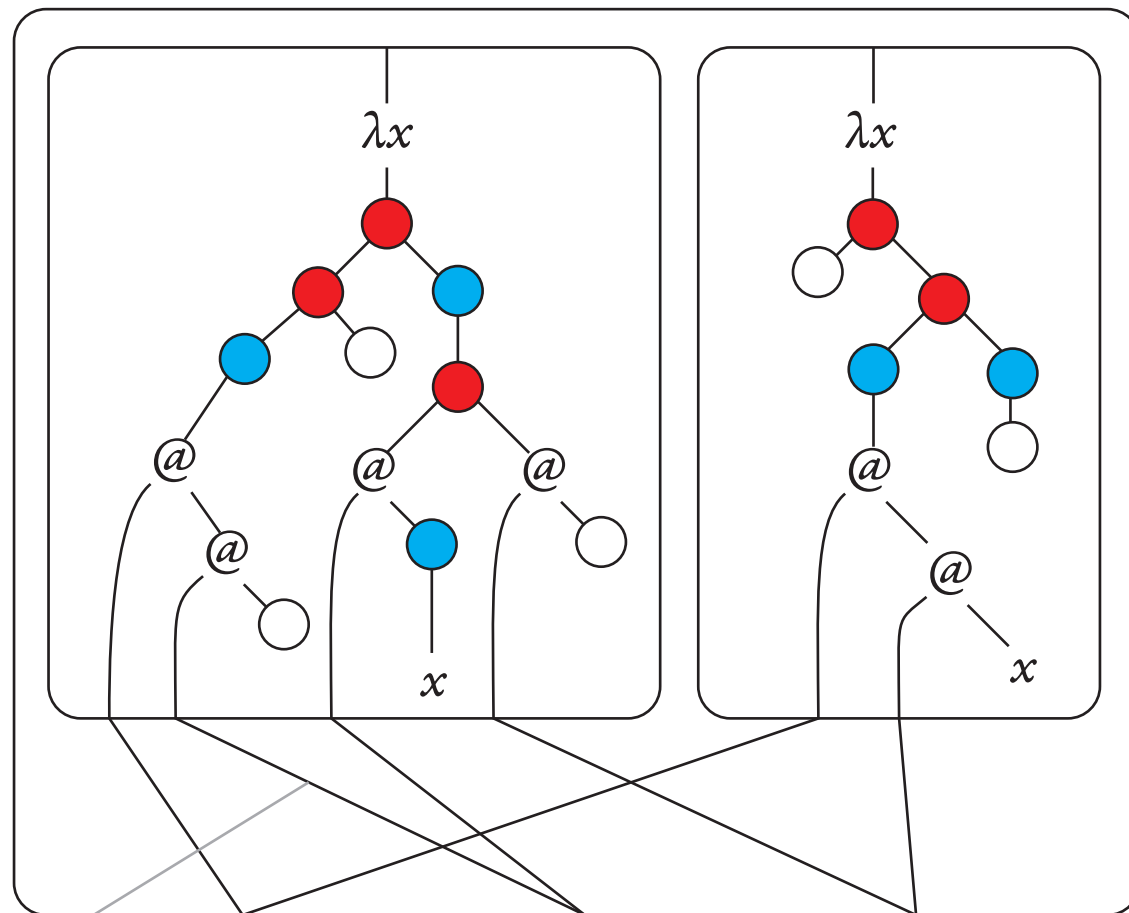
a register update



Variable $i.j$ represents register i in the j -th argument of the register update.

In the dual, this variable is mapped to the i -th edge which enters the j -th port of the reducer.

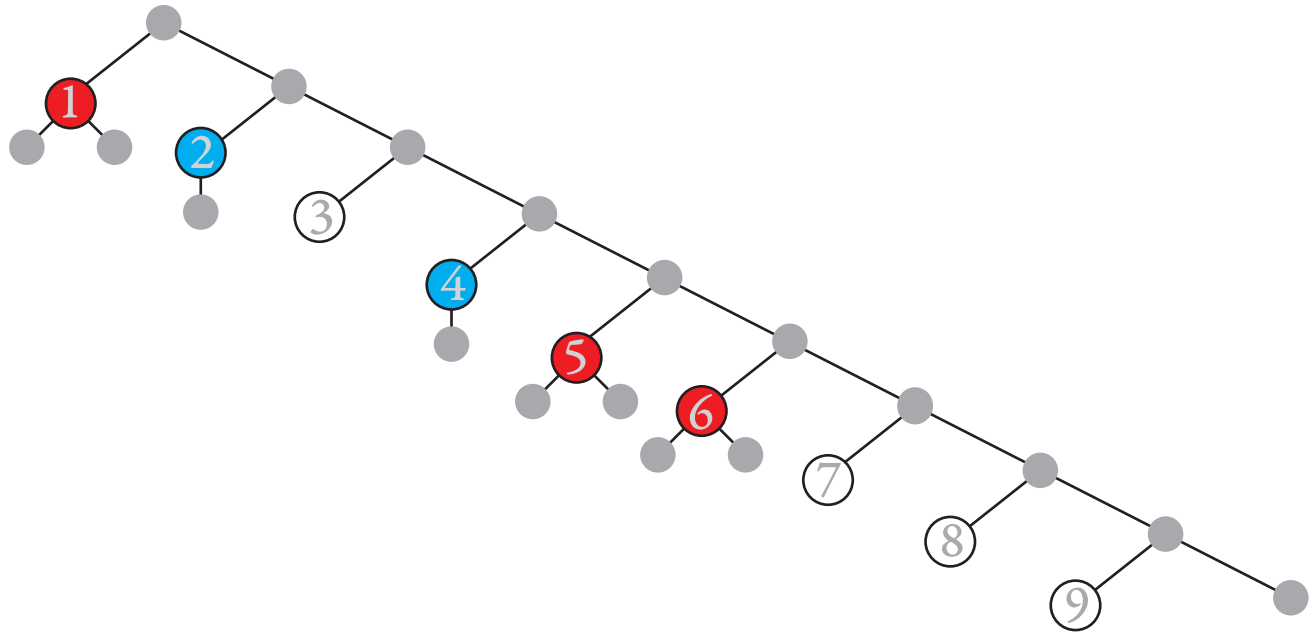
its dual



input

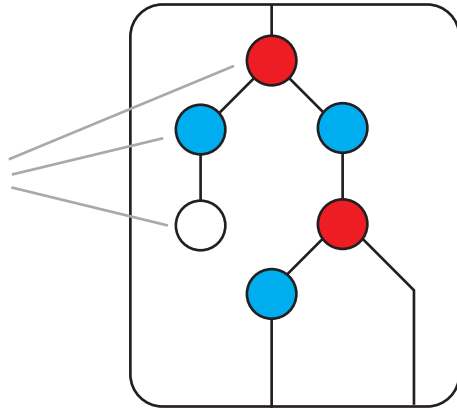


output

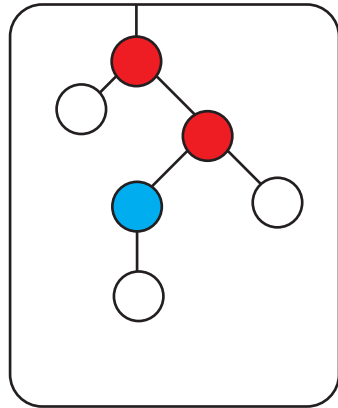


register r of arity 2

letters of the
output alphabet



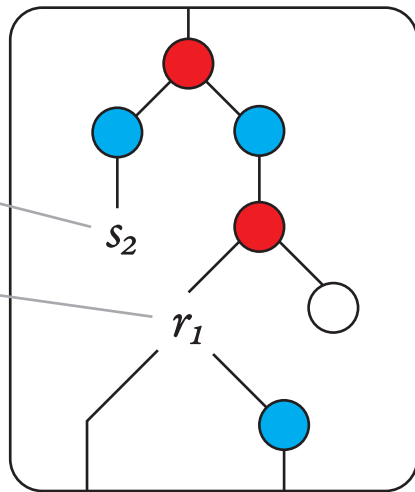
register s of arity 0



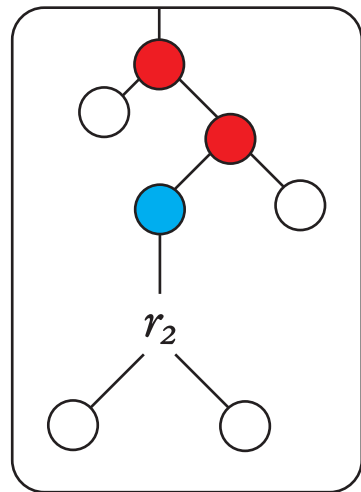
register r of arity 2

copy 2 of register s

copy 1 of register r

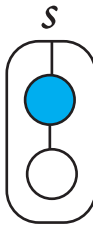


register s of arity 0









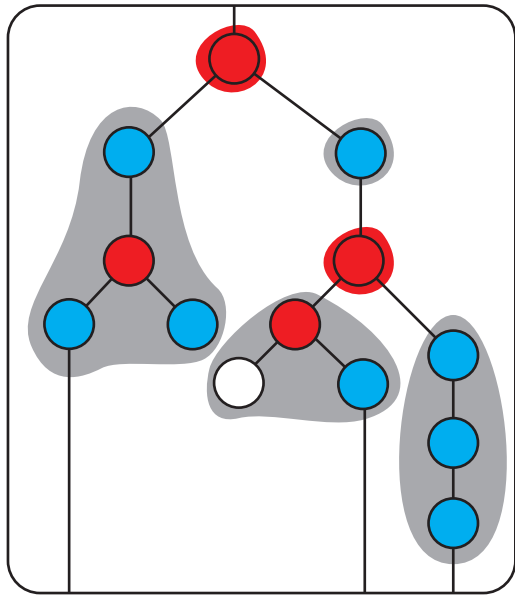




factors without
branching nodes

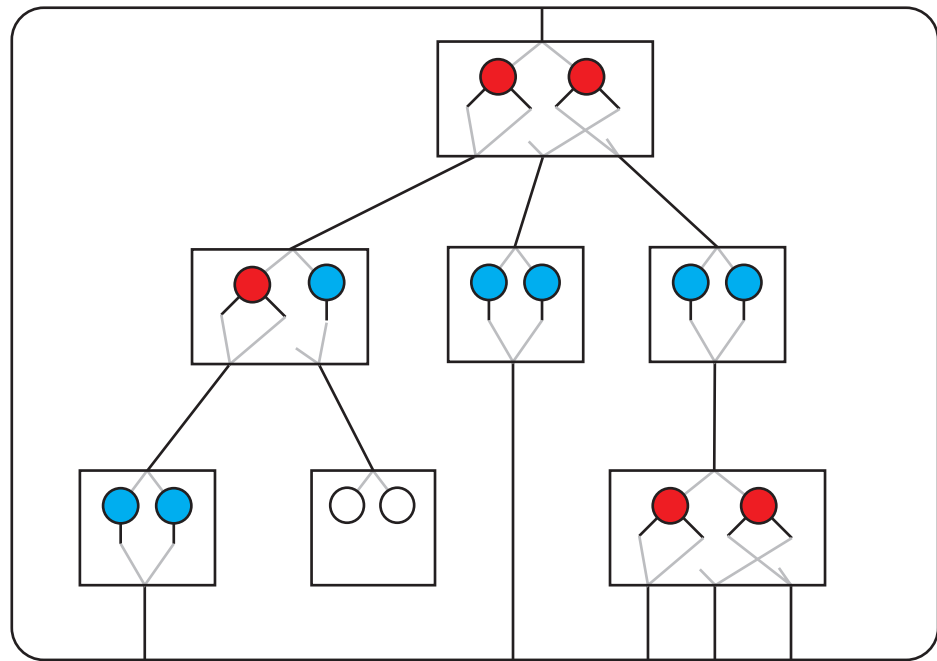


factors with
branching nodes

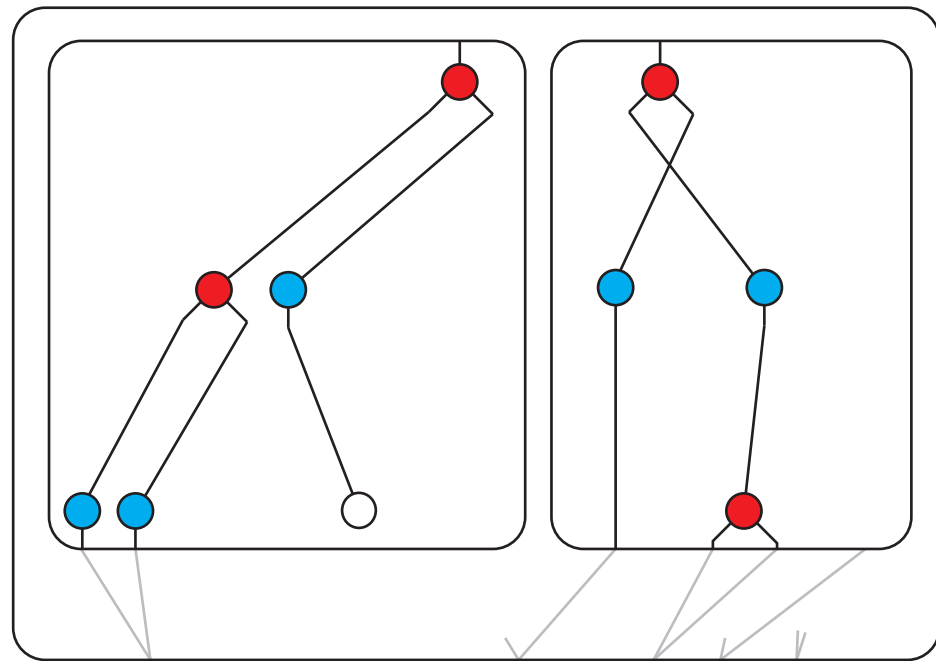


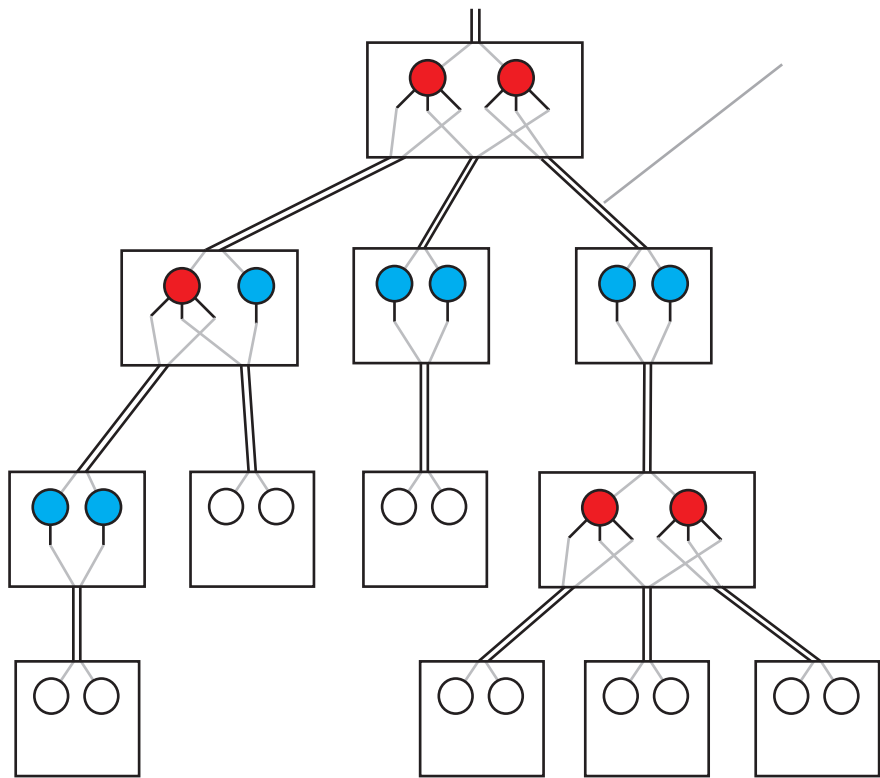


a term of matrix powers

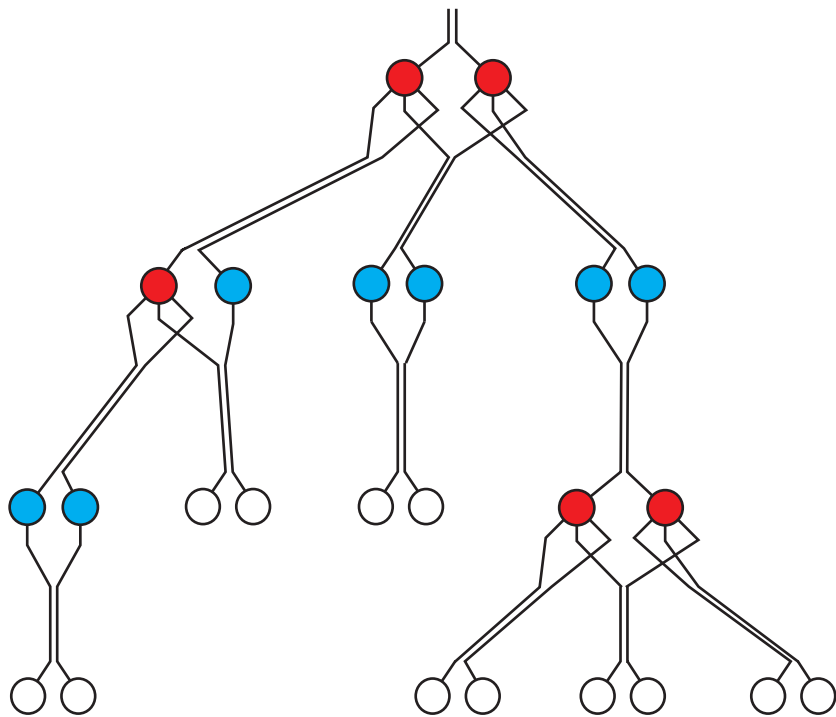


its term unfolding





\mapsto





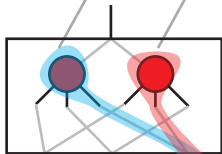
source 1 of e

source 2 of e

edge e

target 1 of e

target 2 of e



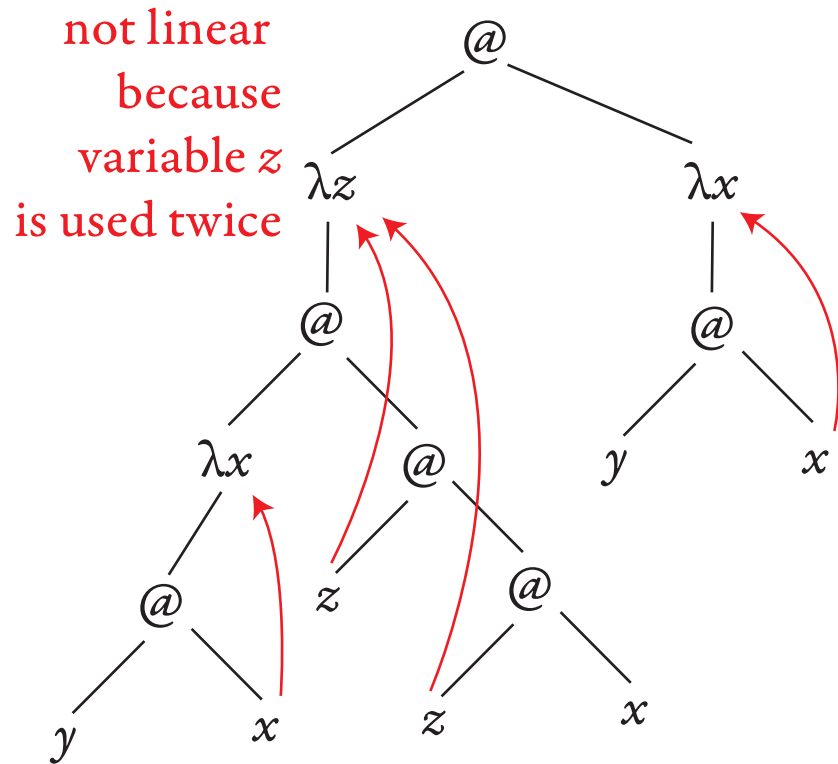
linear



we only count
variables used
in their scope

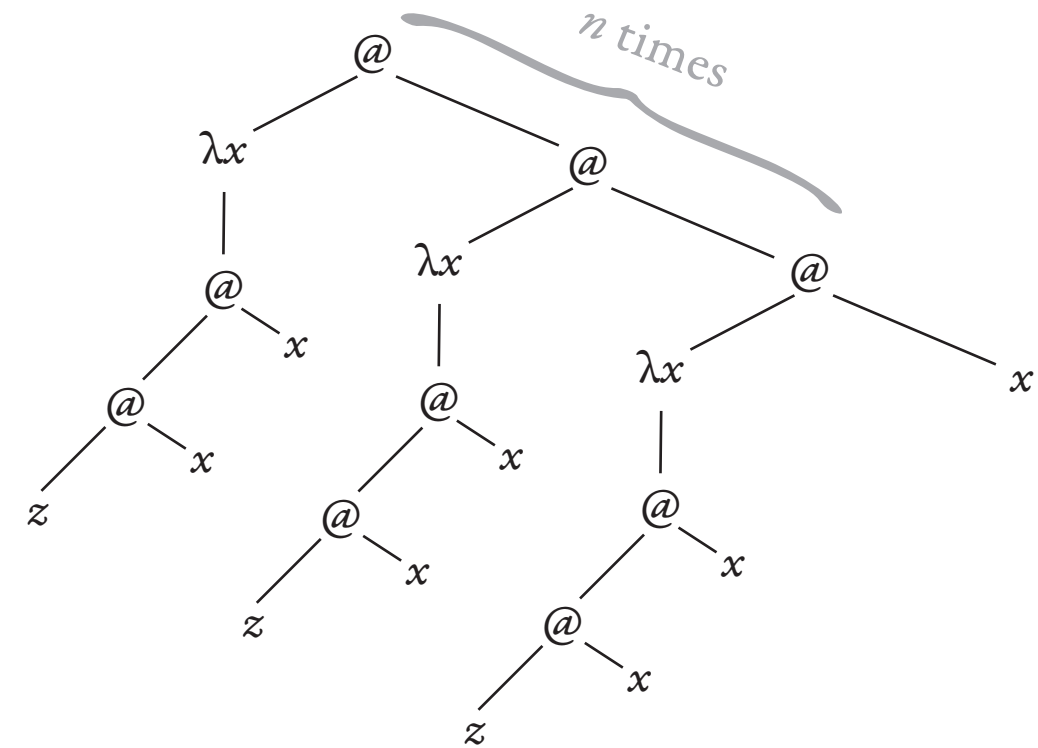
variable z can be used twice because it is free

not linear

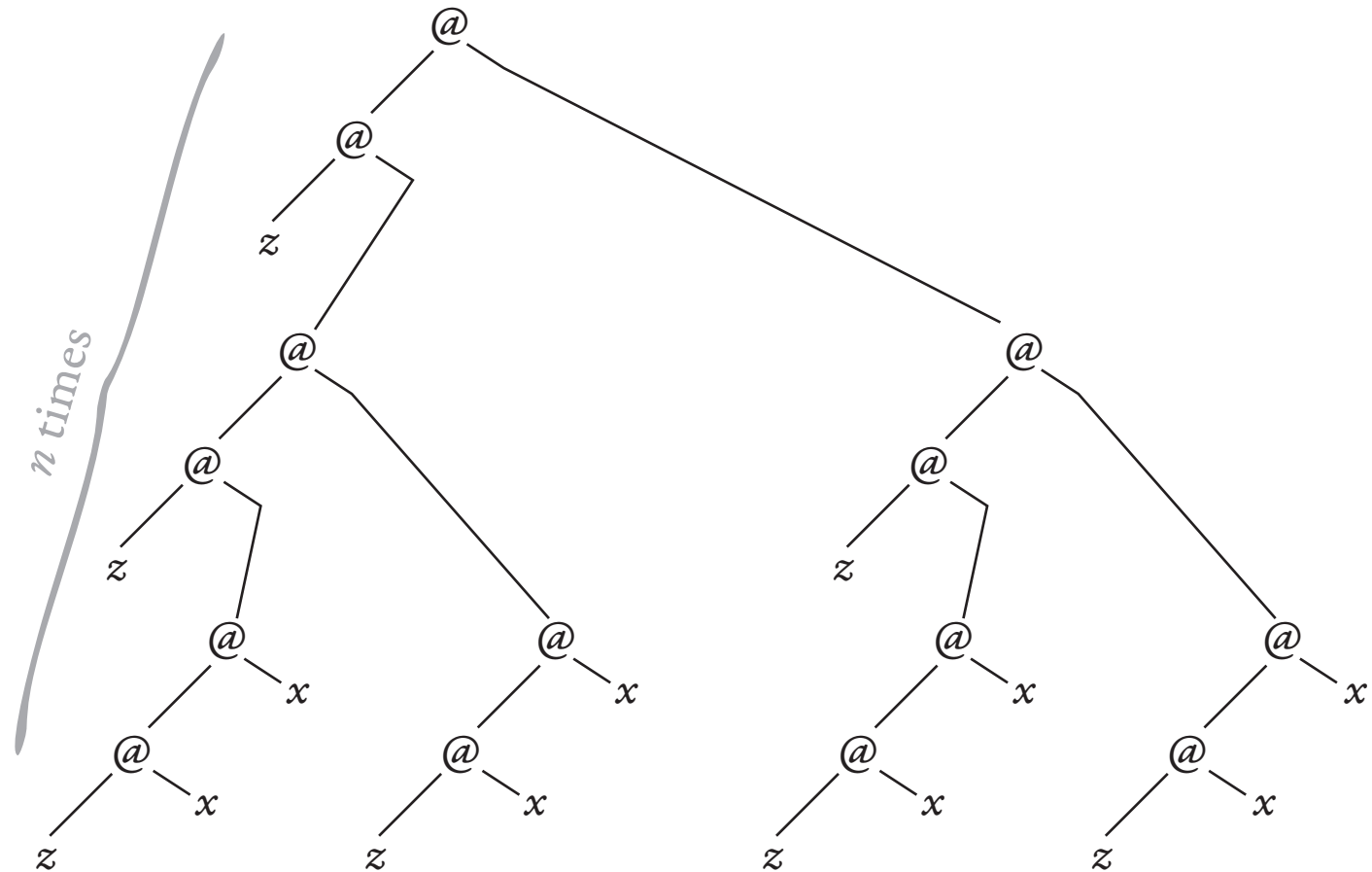


not linear
because
variable z
is used twice

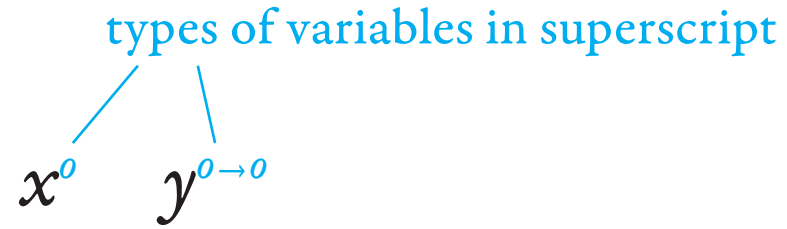
a λ -term of size $O(n)$



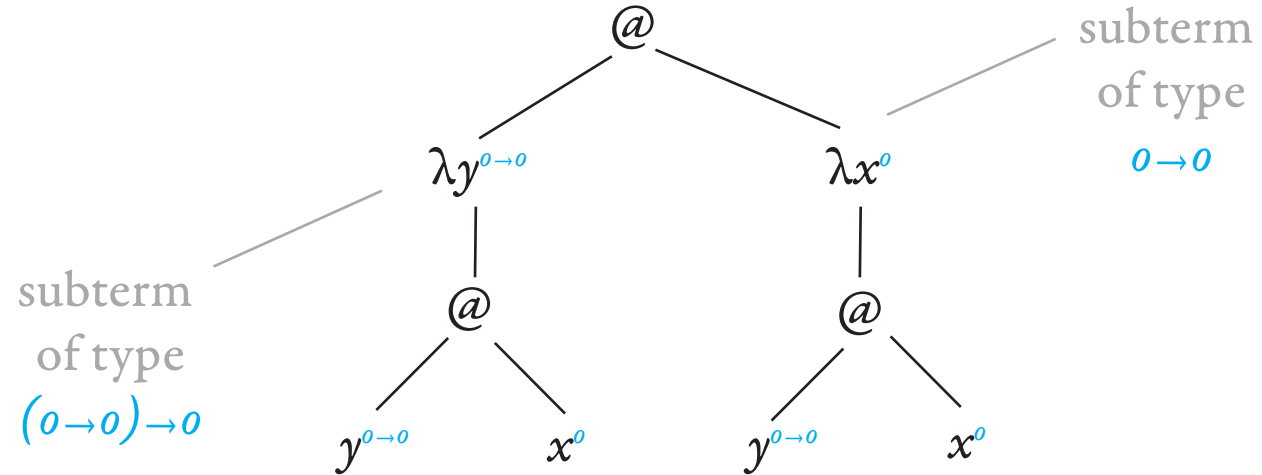
its normal form of size $O(2^n)$



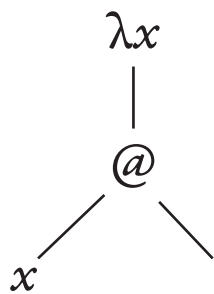
variables



λ -term of type o



@



$\lambda x.$

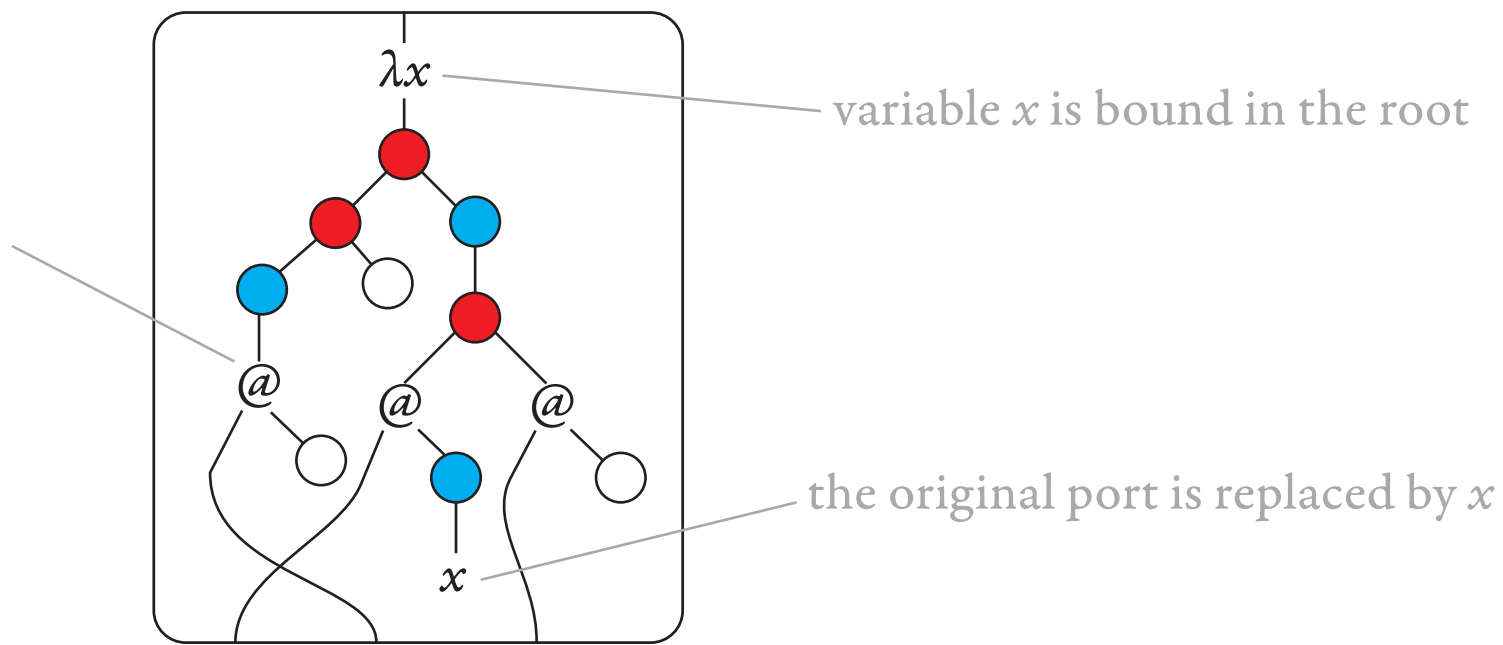


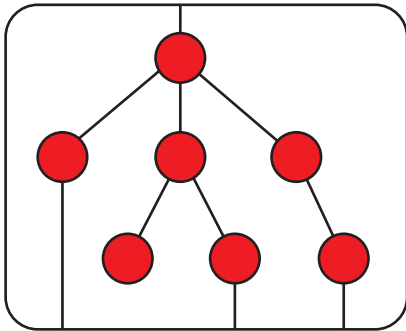
r



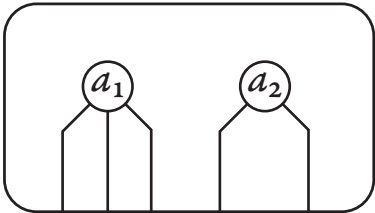
placeholder for the term
stored in the unique register
of the 2nd child

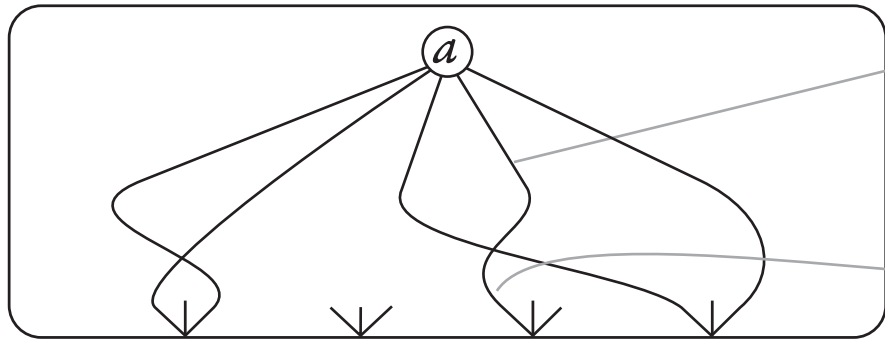






dangling edges
represent ports





port 4

$\Downarrow f$

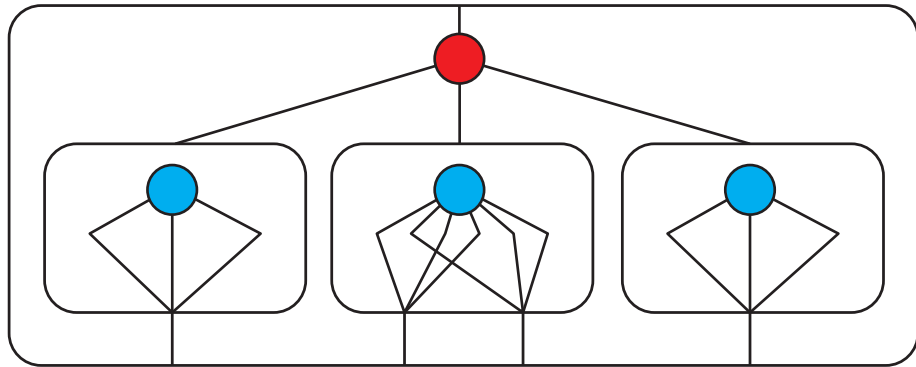
position 1, group 3

the root is from Σ

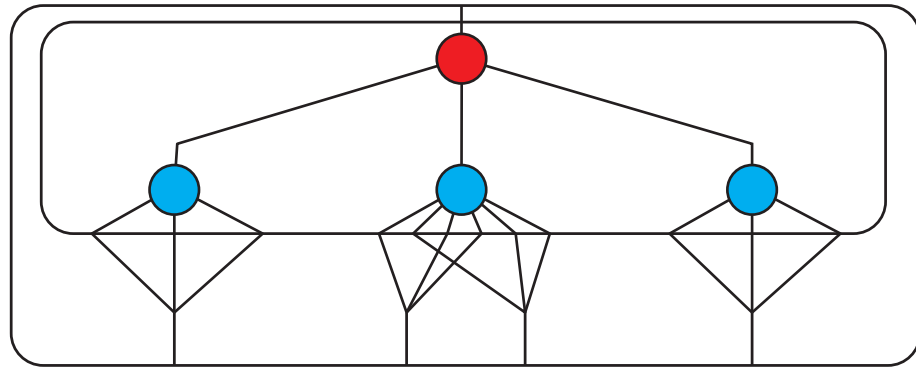
all children are from Γ

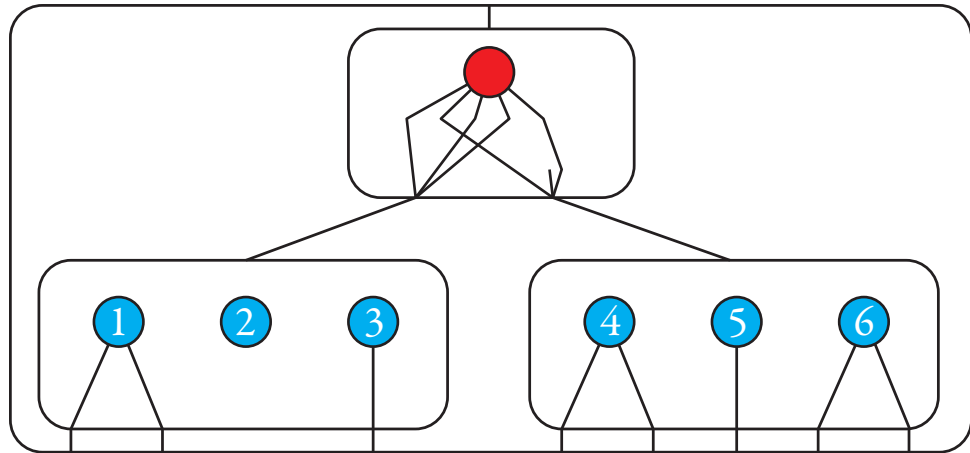


input

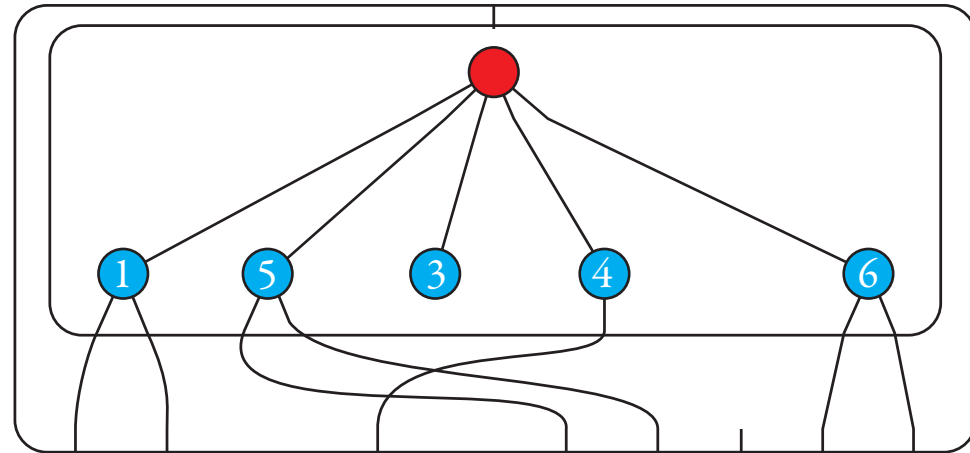


output

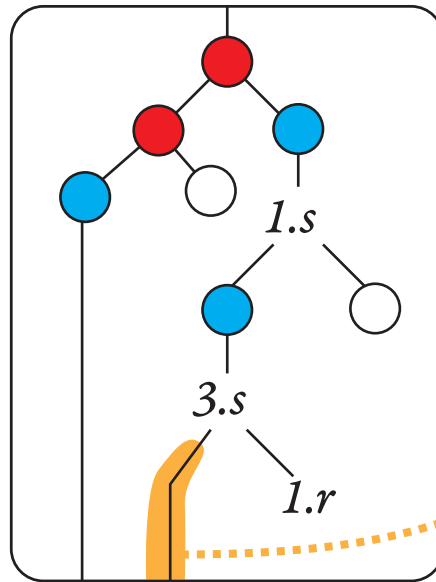
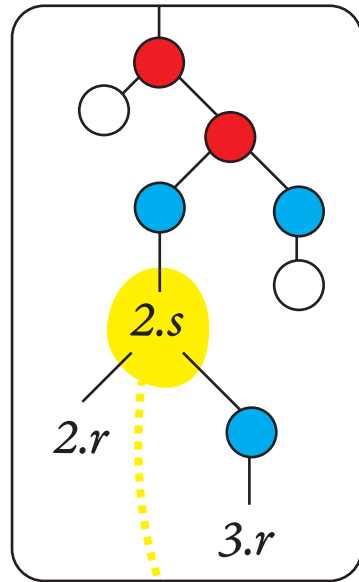




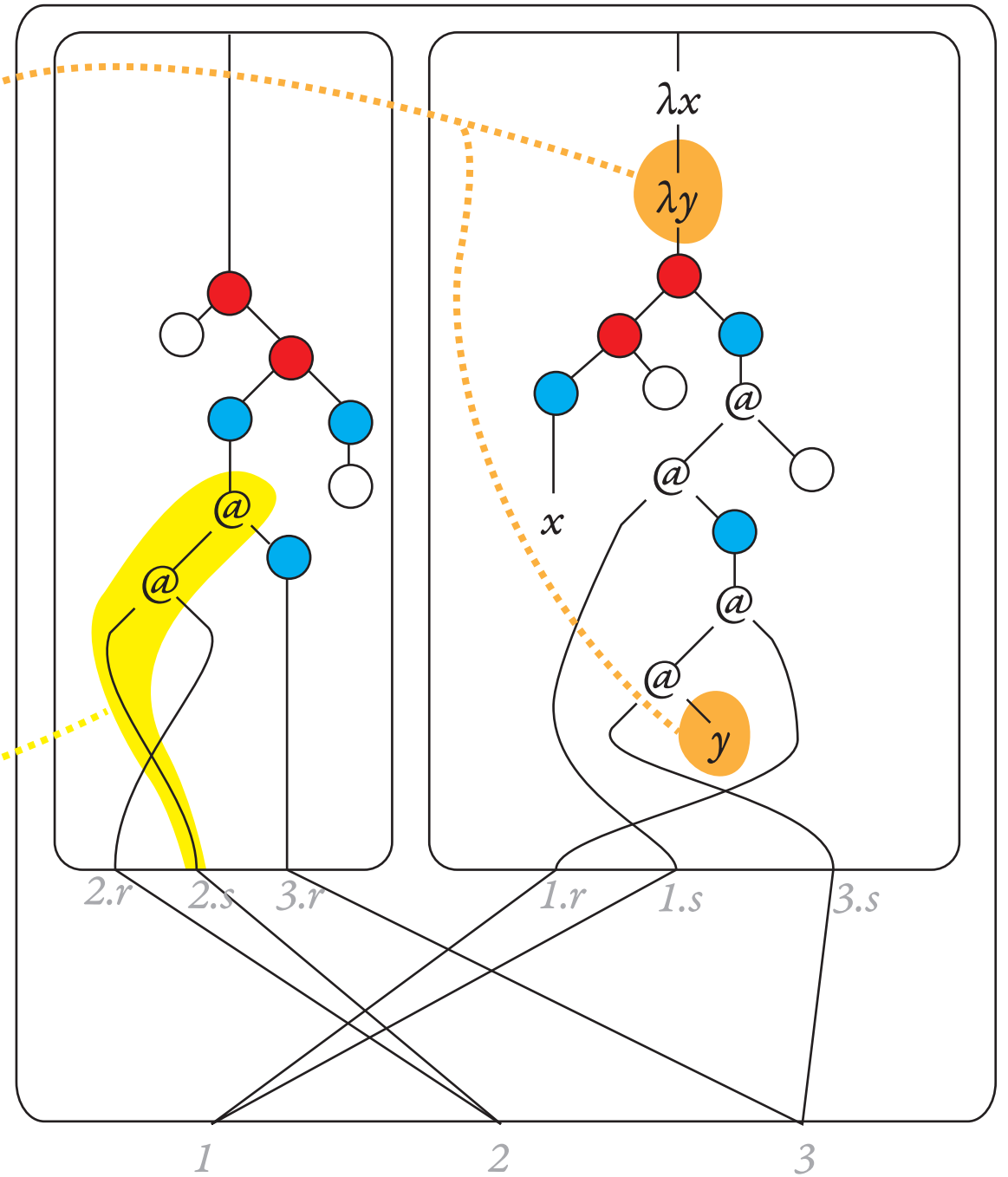
\mapsto



a register update



its dual

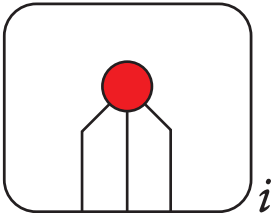


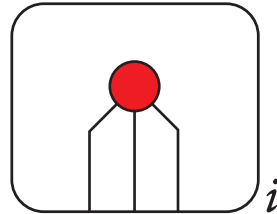
A diagram showing a hierarchical tree structure within a rounded rectangle. The root node is red. It has two children: a red node on the left and a blue node on the right. The red node has two children: a blue node on the left and a white node on the right. The blue node under the red node has one child: a blue node. The blue node at the top right has one child: a yellow circle labeled r_1 . This yellow circle has two children: a blue node on the left and a white node on the right. Below the blue node under r_1 , there is a label r_2 next to a thick orange vertical bar. From this orange bar, a dashed orange line extends downwards and to the right. Another dashed orange line starts from the right side of the yellow circle r_1 and extends towards the right edge of the frame.[illegible]

a non-port node is represented by a variable, corresponding to the label, applied to the children of the node

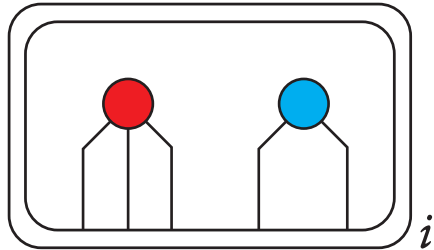
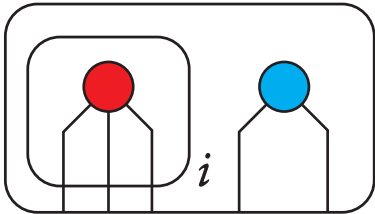
- the variables representing the ports are bound outside

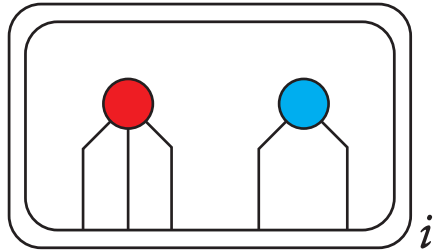
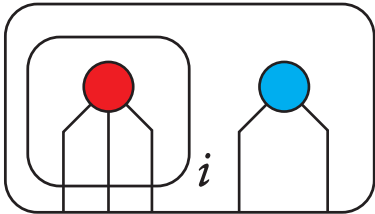
the i -th port is represented by a variable x_i of type \mathbf{o}

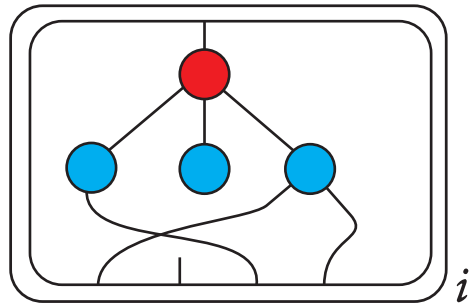
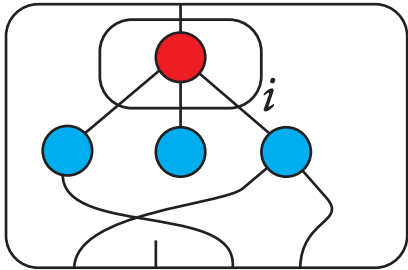


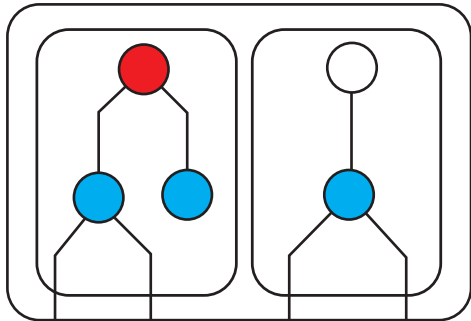
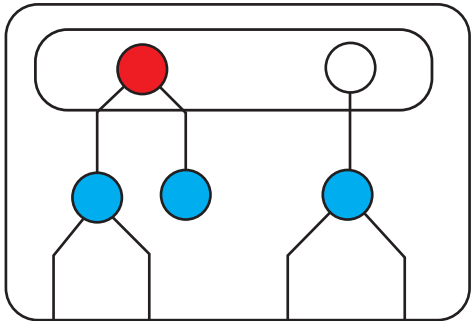


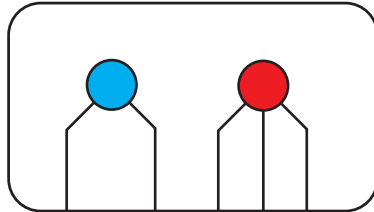




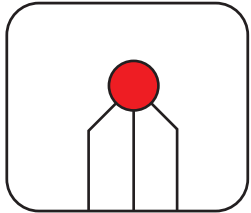


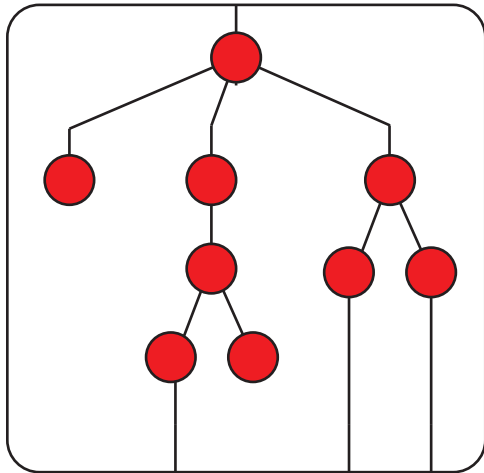
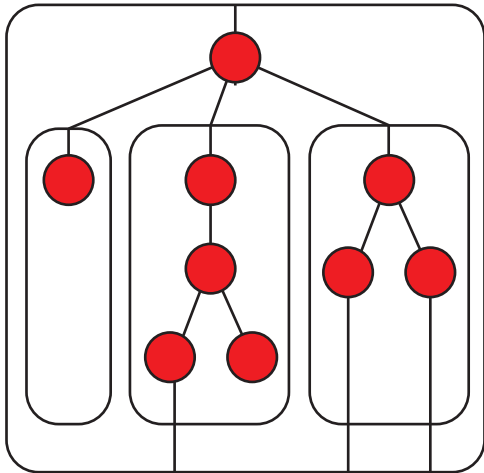


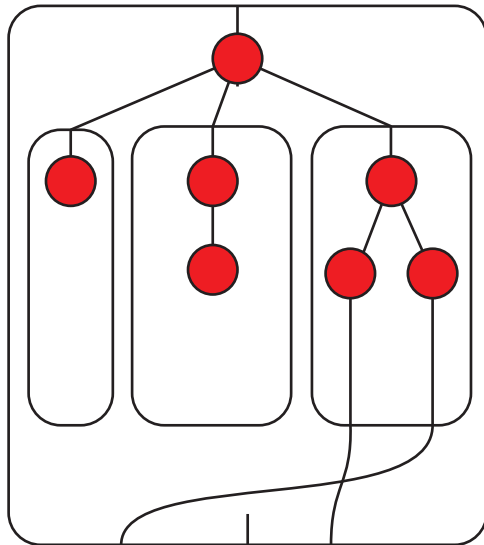
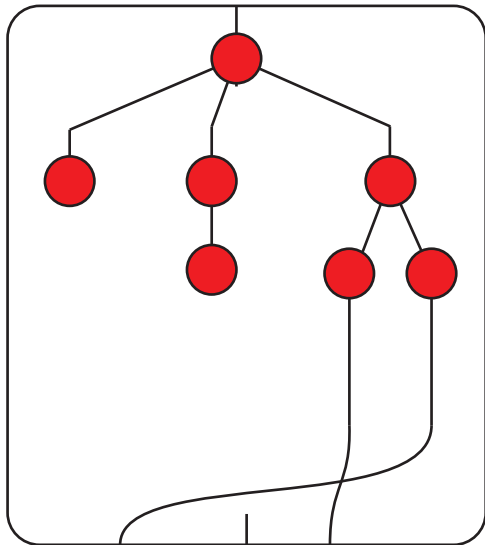


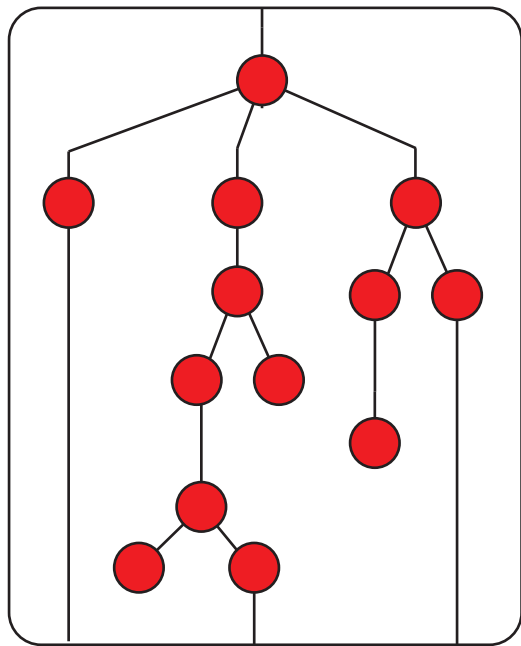


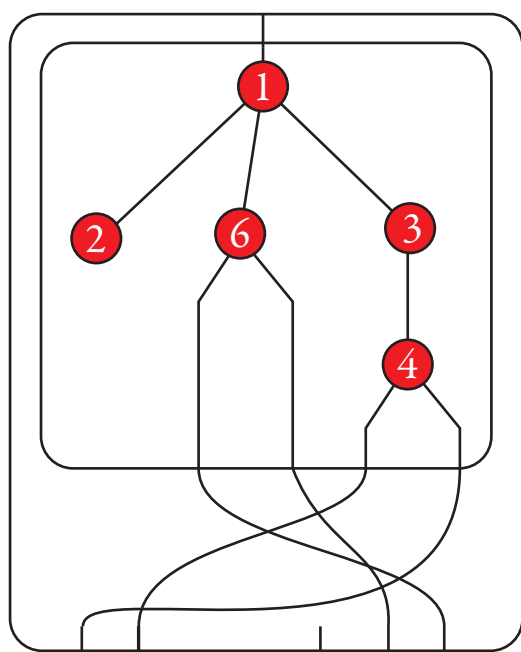


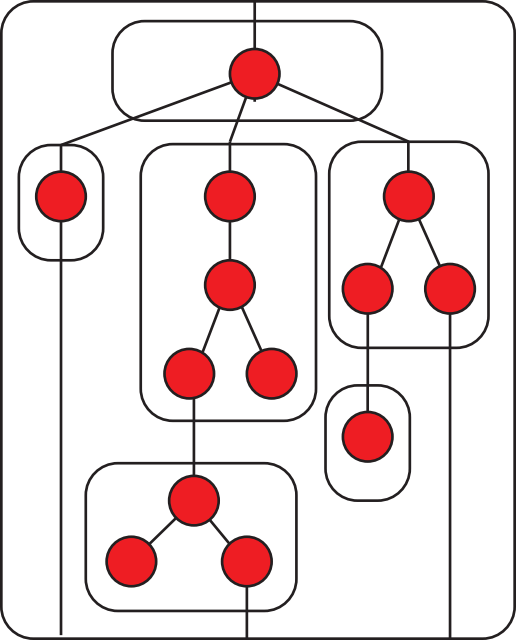




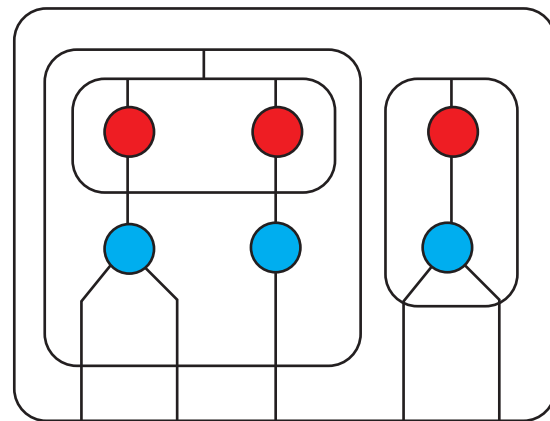
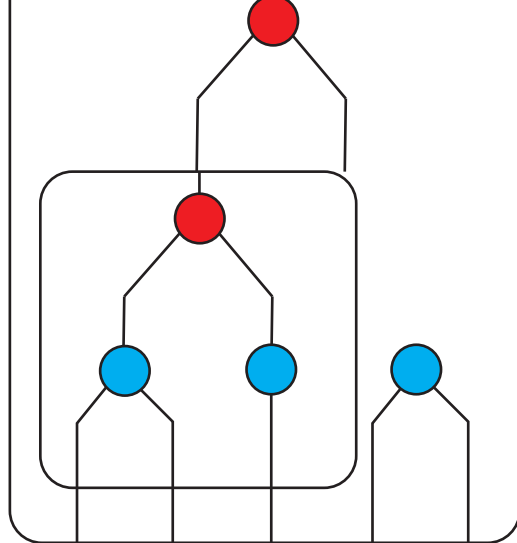
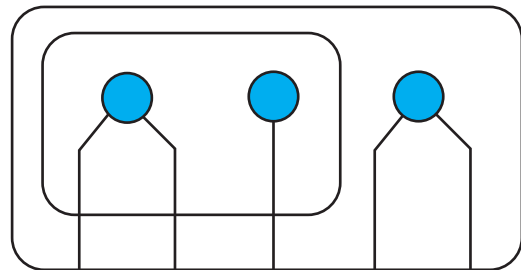


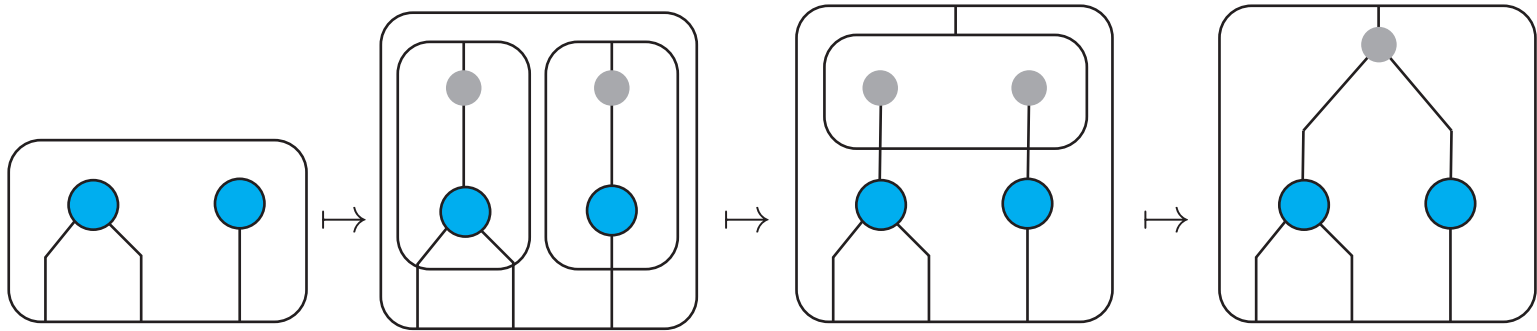






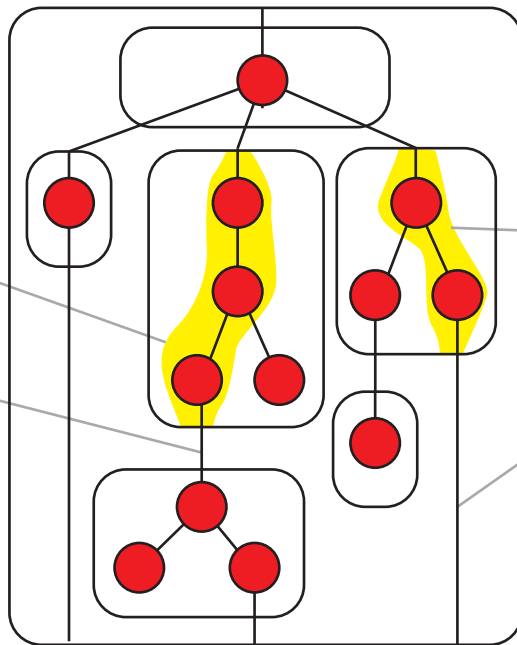




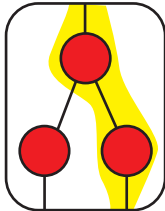




the subbranch
corresponding to
an internal edge

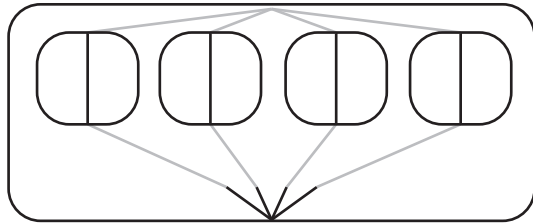


the subbranch
corresponding to
an external edge



a branch can be visualised as
a term with a distinguished
root-to-port path

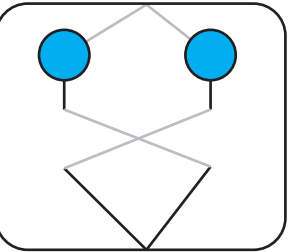




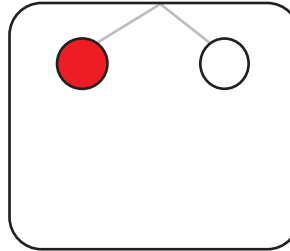
a tuple of k identity terms
with all their ports folded
into one

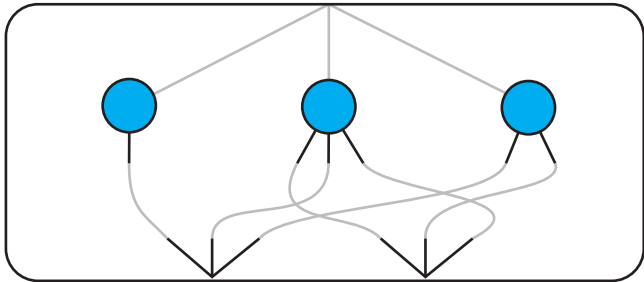
$$\Sigma = \{ \text{blue circle with stem}, \text{red circle}, \text{white circle} \}$$

$$a \in \Sigma^{[2]}$$

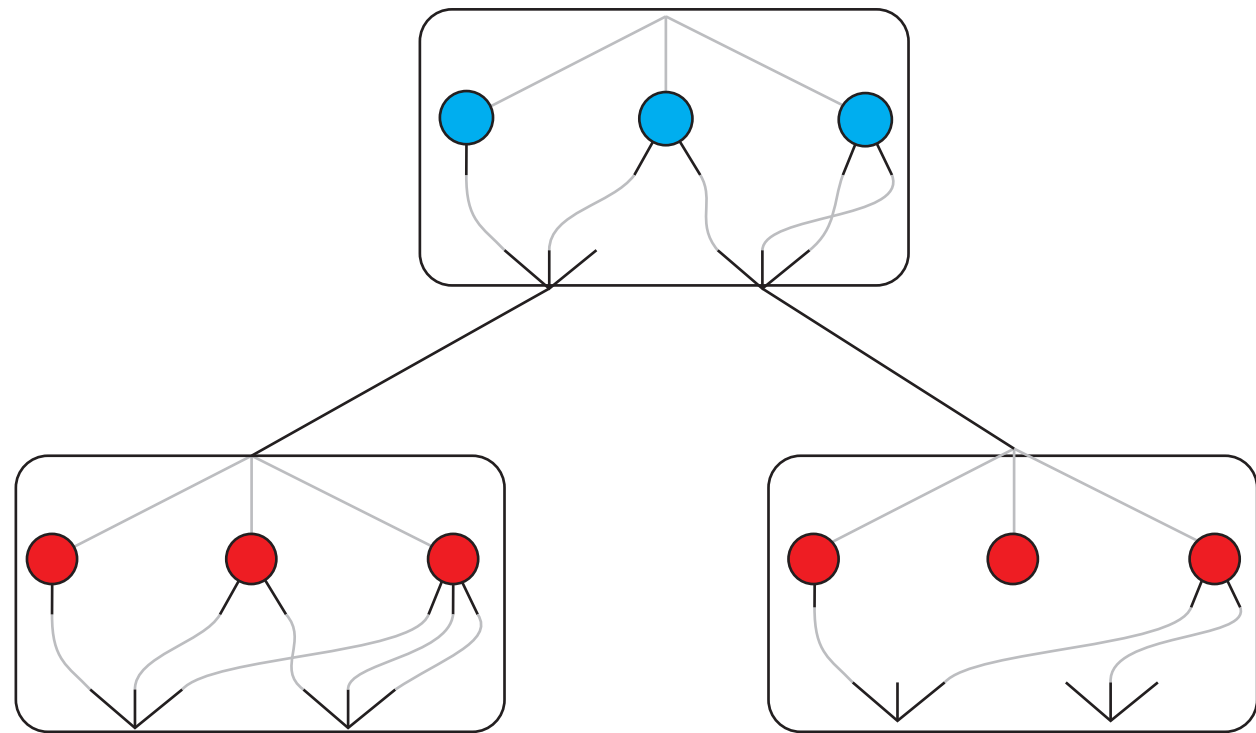


$$b \in \Sigma^{[2]}$$

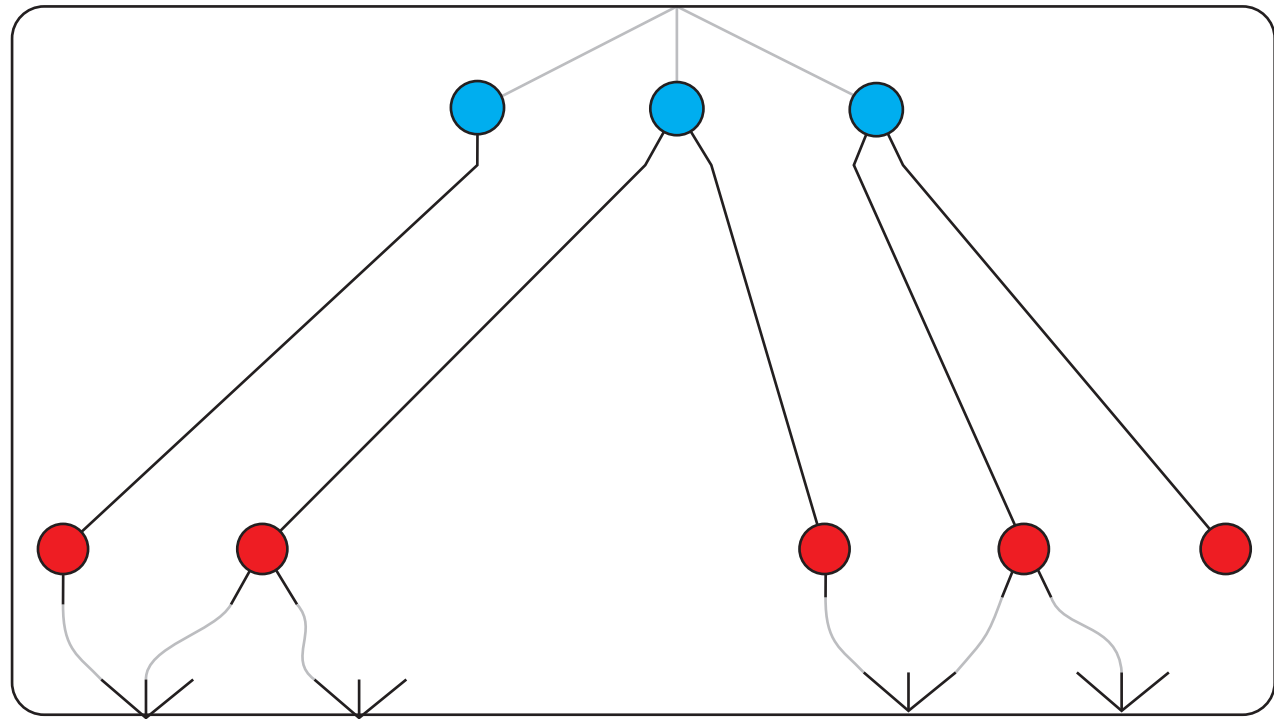


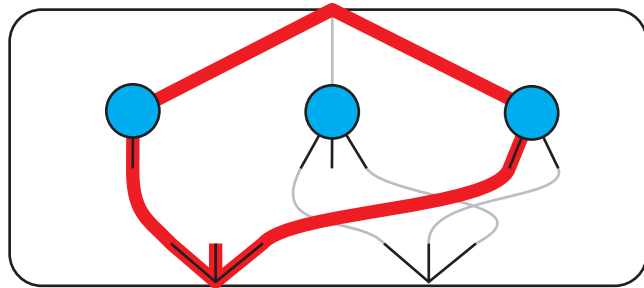


a shallow term of matrix powers



its shallow unfolding





twist of port 1

1

2

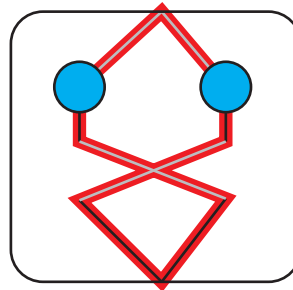
3



1

2

3



twist of port 1

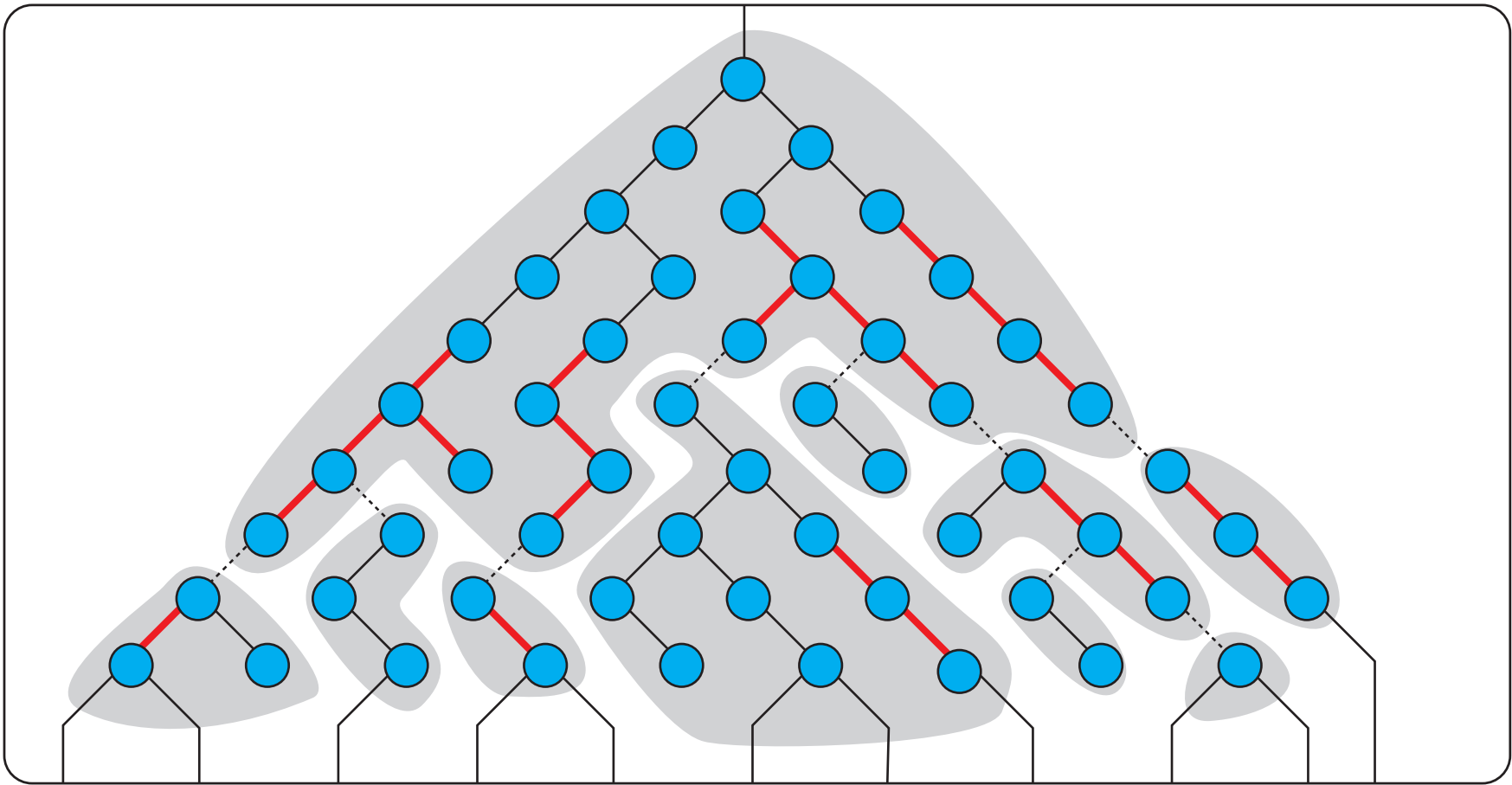
1

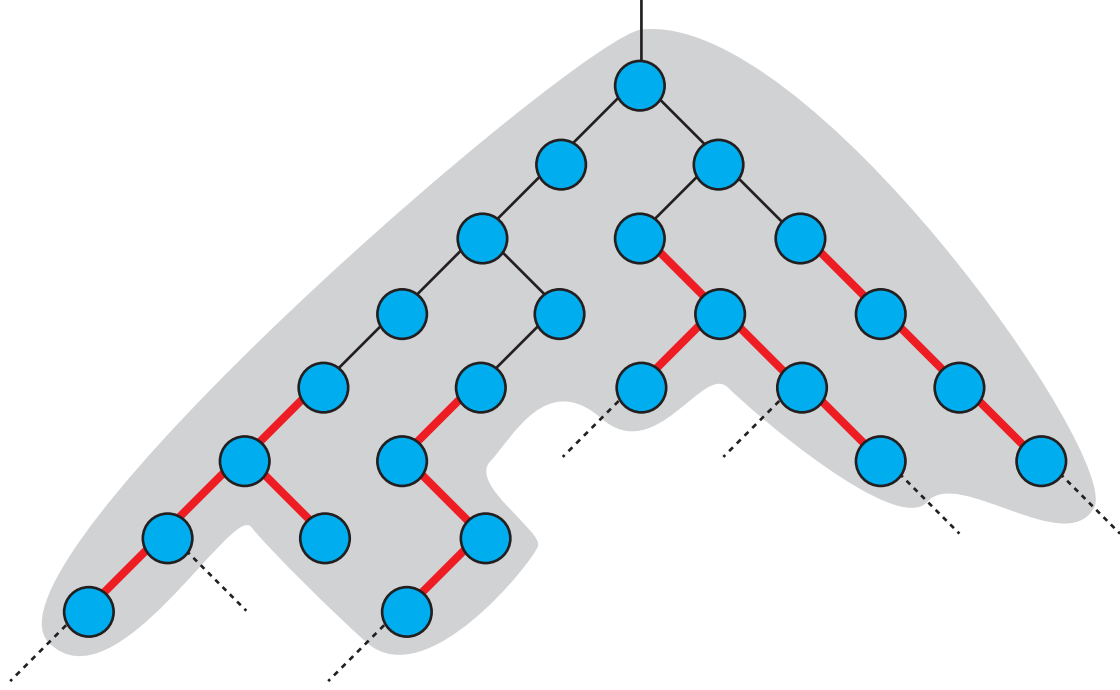
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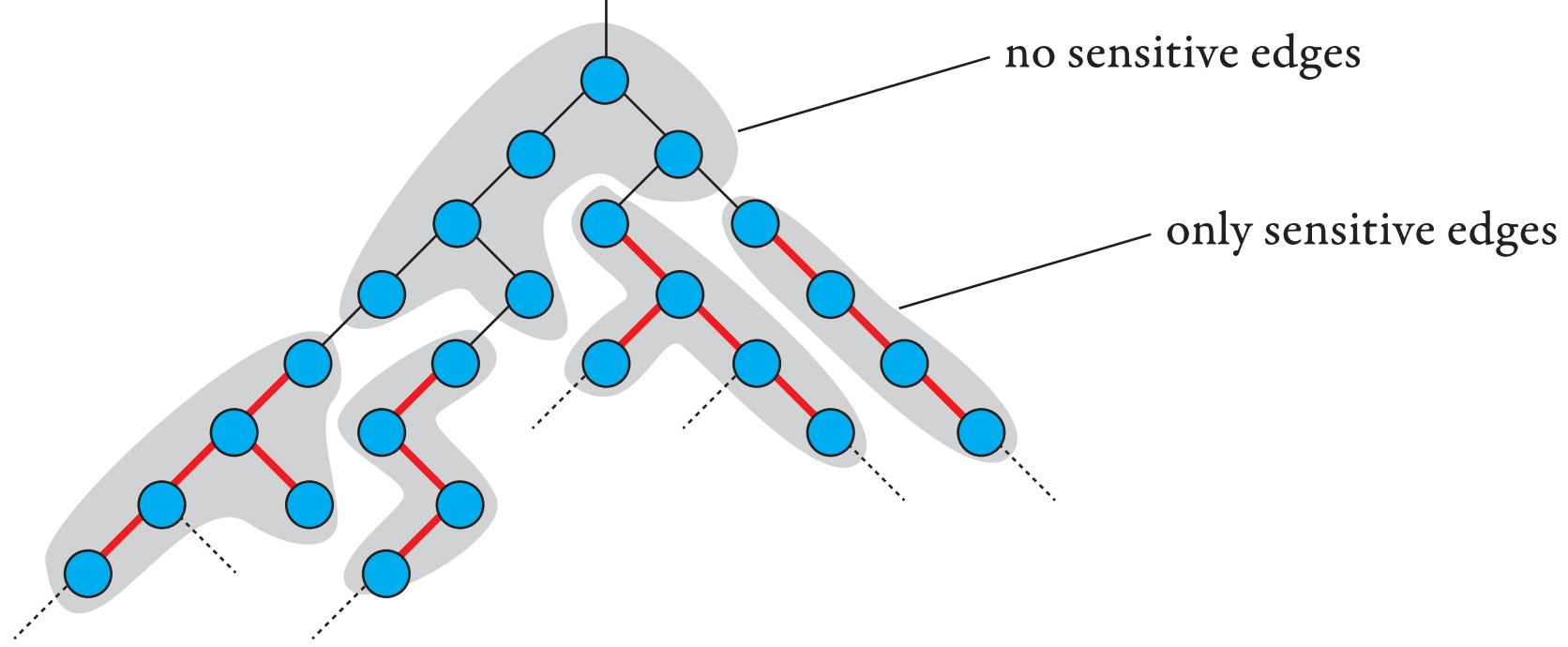


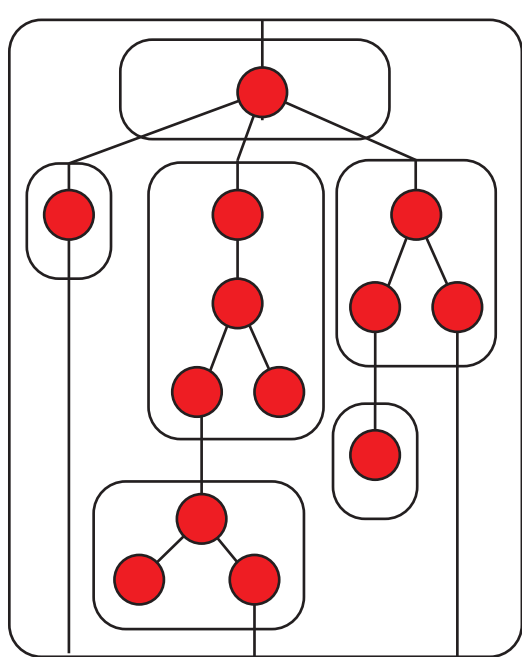
1

2

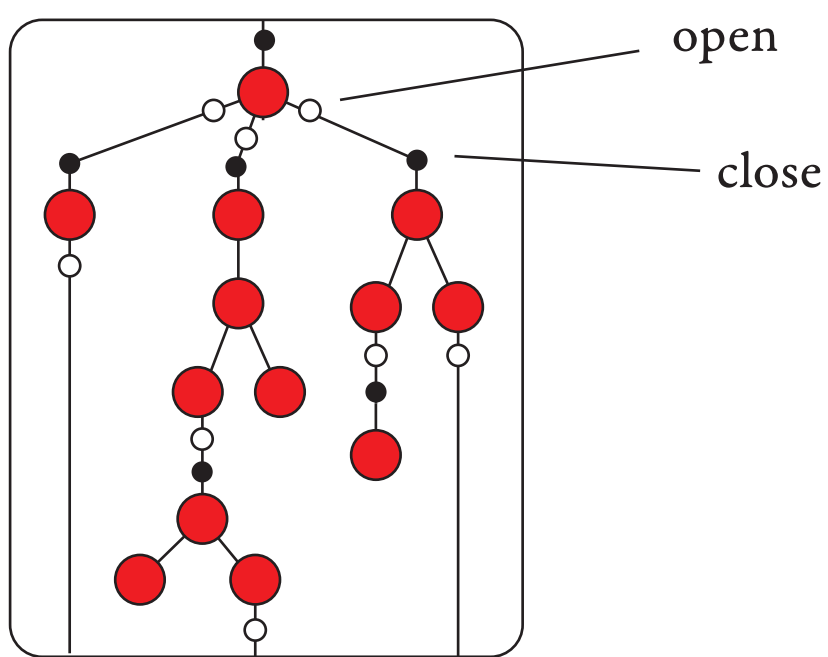






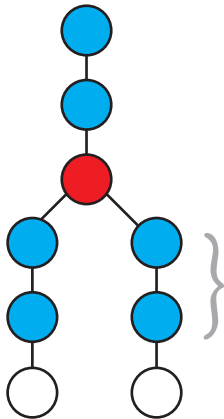
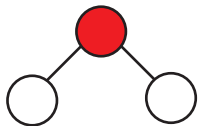


\mapsto



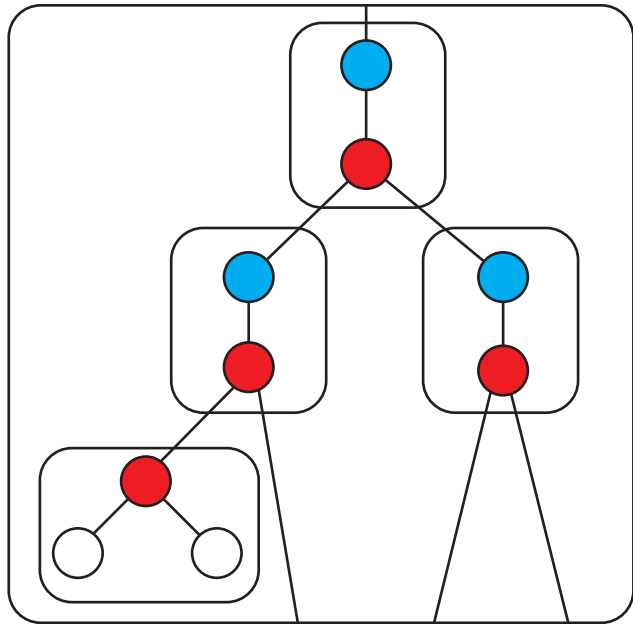




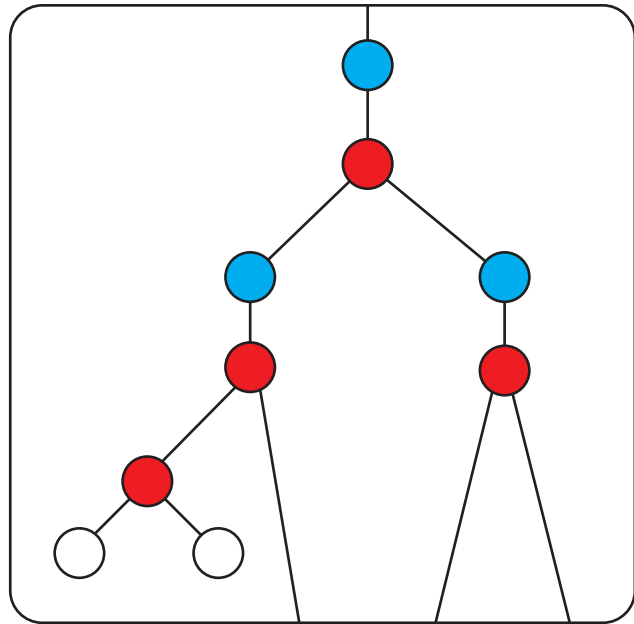


$$k - 1 = 2$$



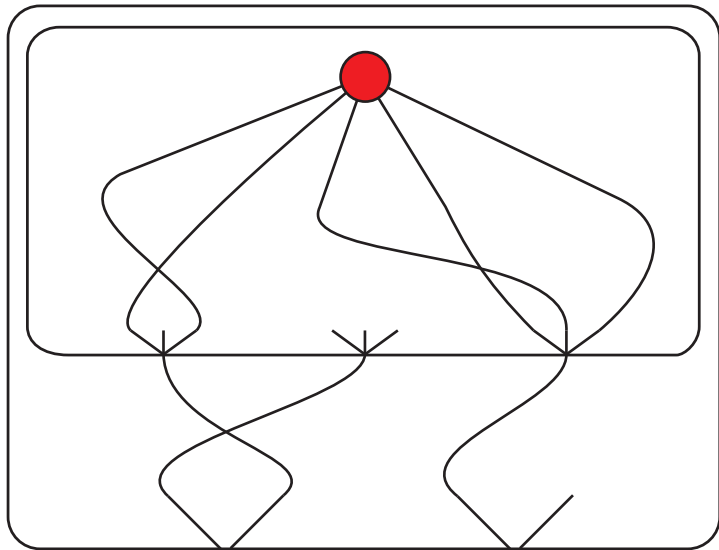


\mapsto

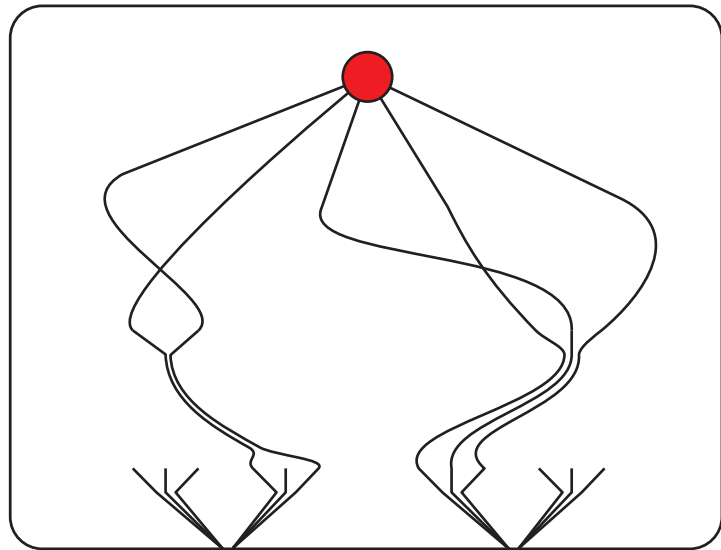


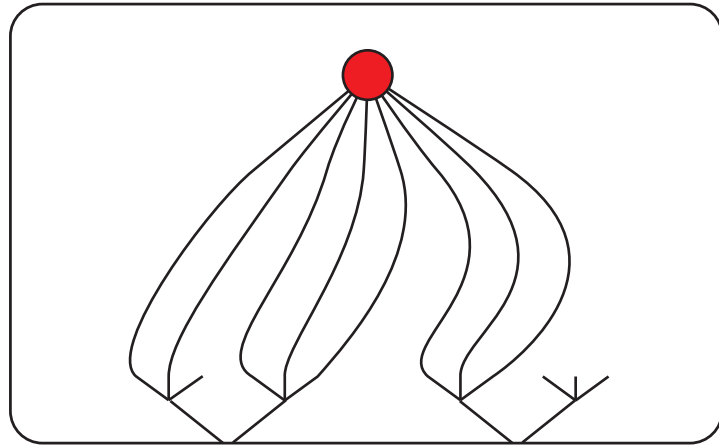
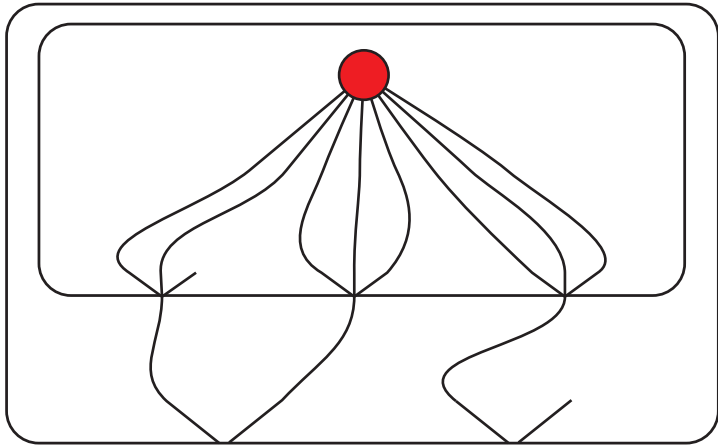


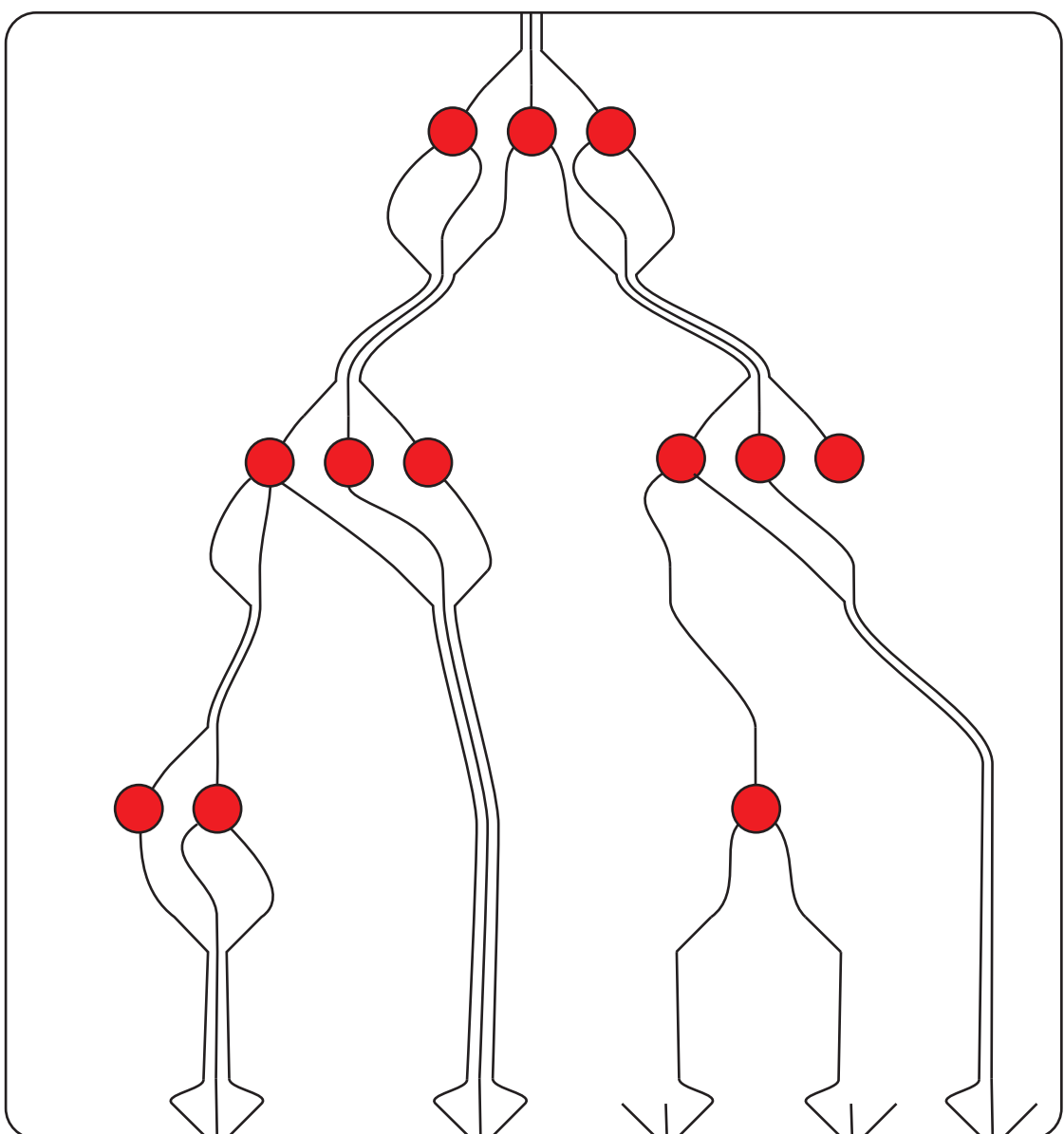
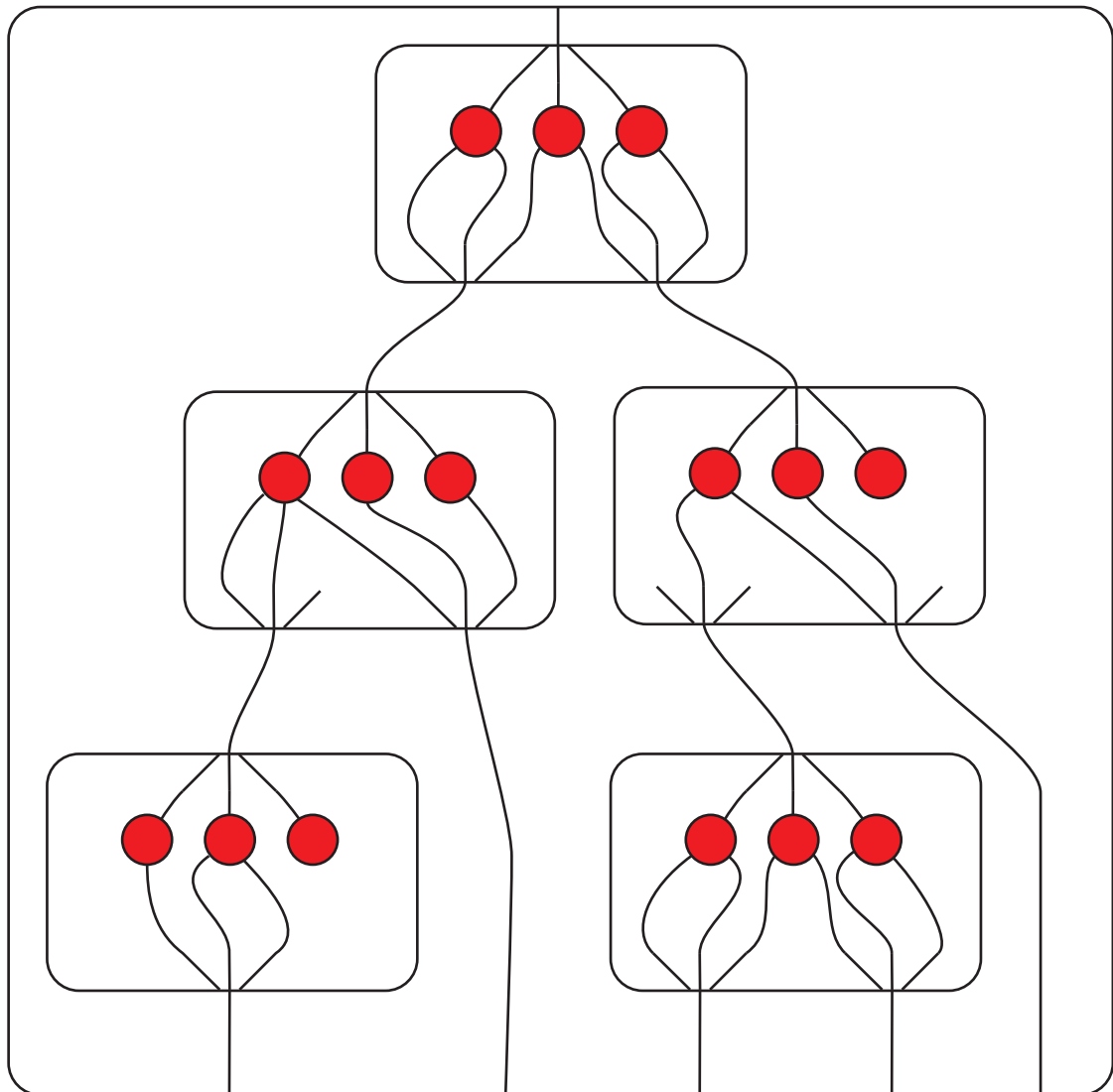
$F_2 F_3 \Sigma$



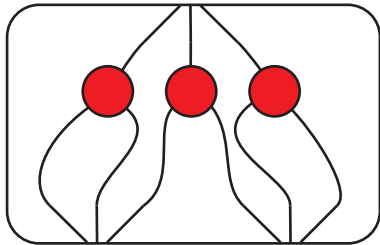
$F_6 \Sigma$



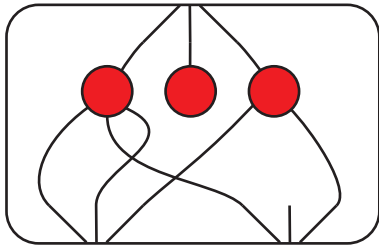




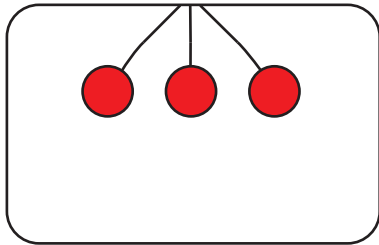
arity 2



arity 2



arity 0

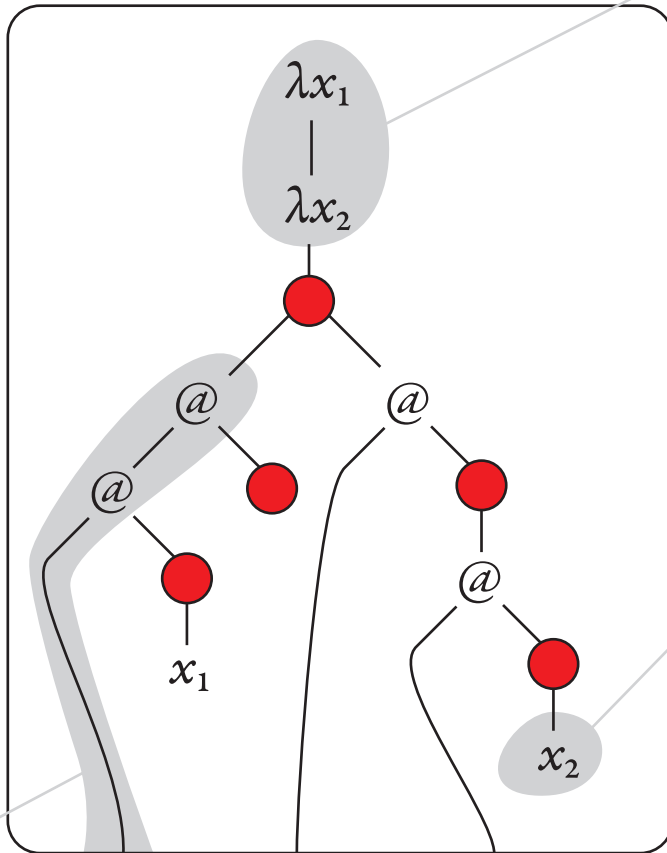
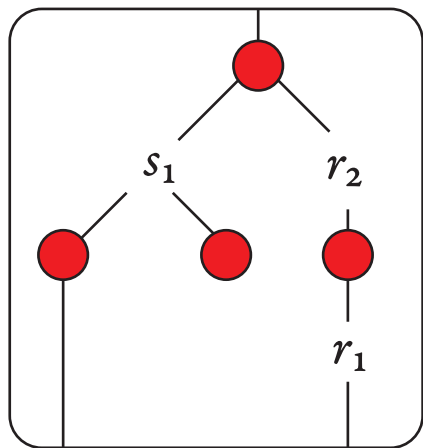


a term t with
placeholders

its λ -representation

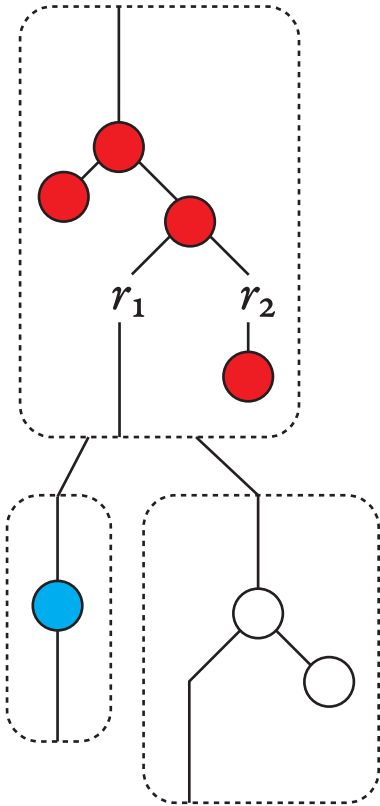
one bound
variable
for each port
of t

each port of t
is replaced by
a corresponding
variable



each placeholder of t is replaced by
a port applied to its children using $@$

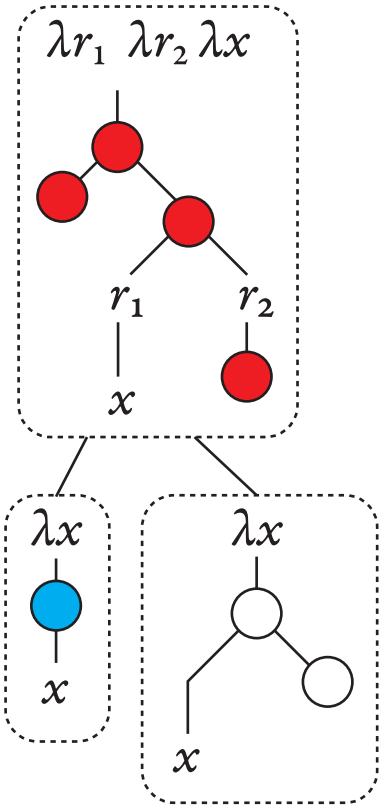
tree of register updates



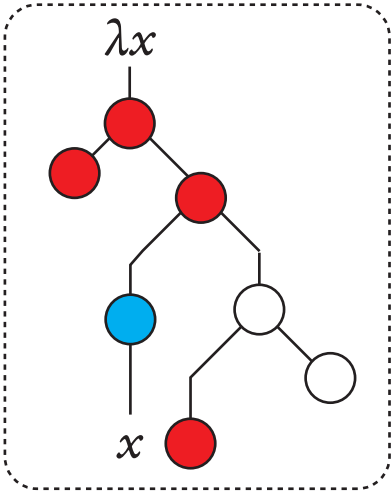
represent
as a λ -term



λ -term



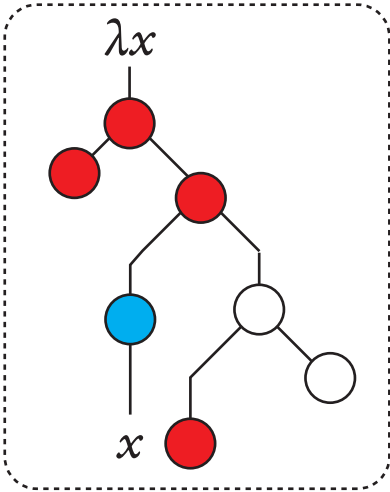
evaluate \Downarrow



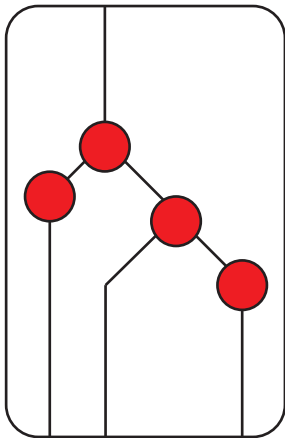
represent
as a λ -term



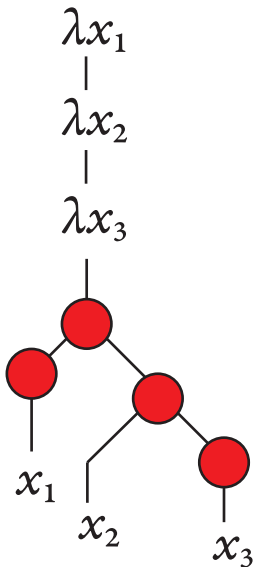
\Downarrow evaluate

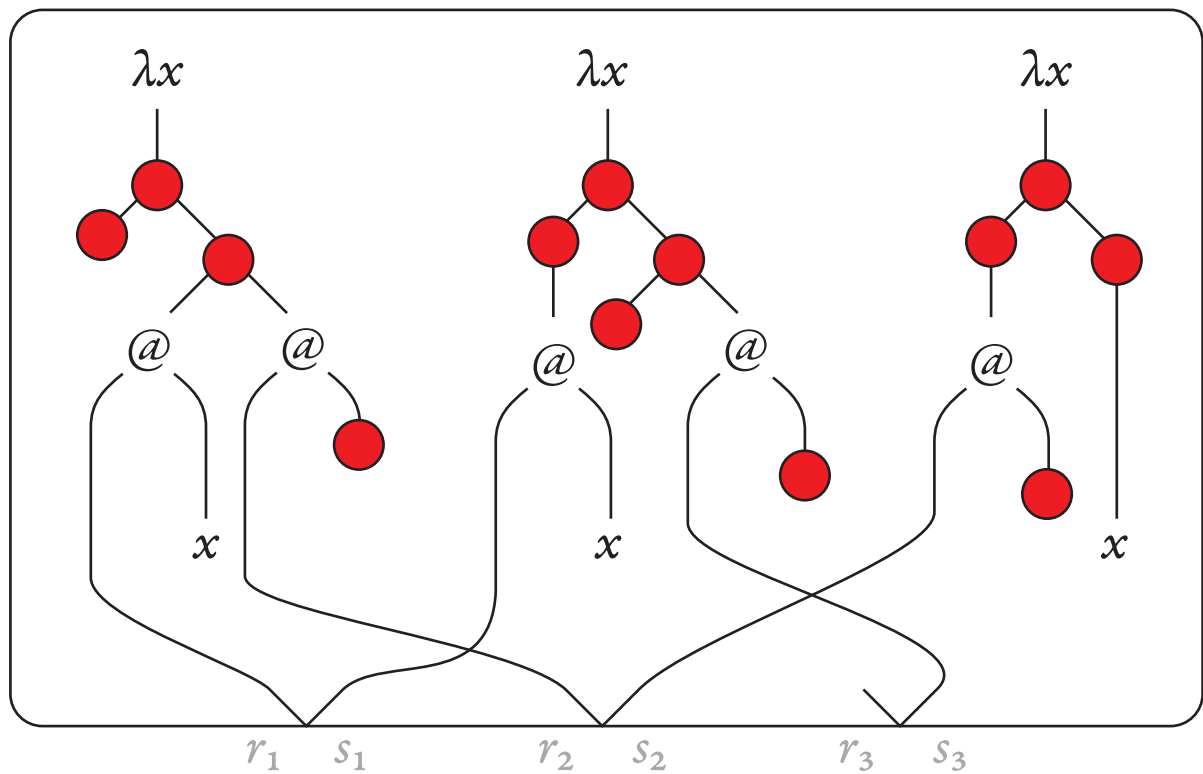
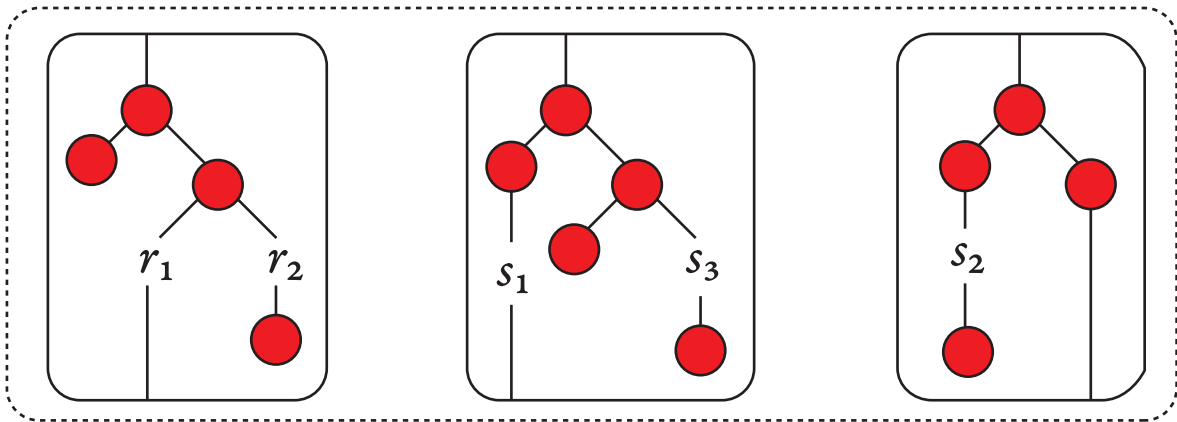


a term

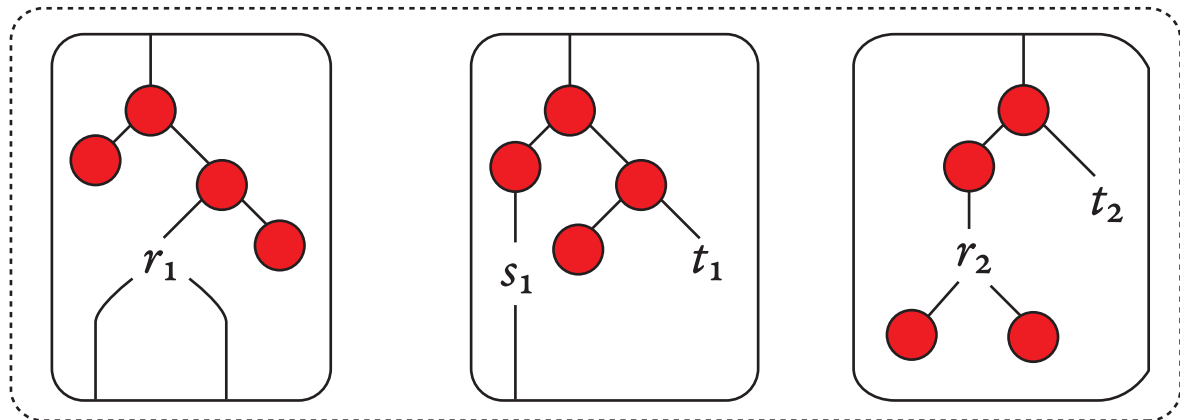


its λ -representation

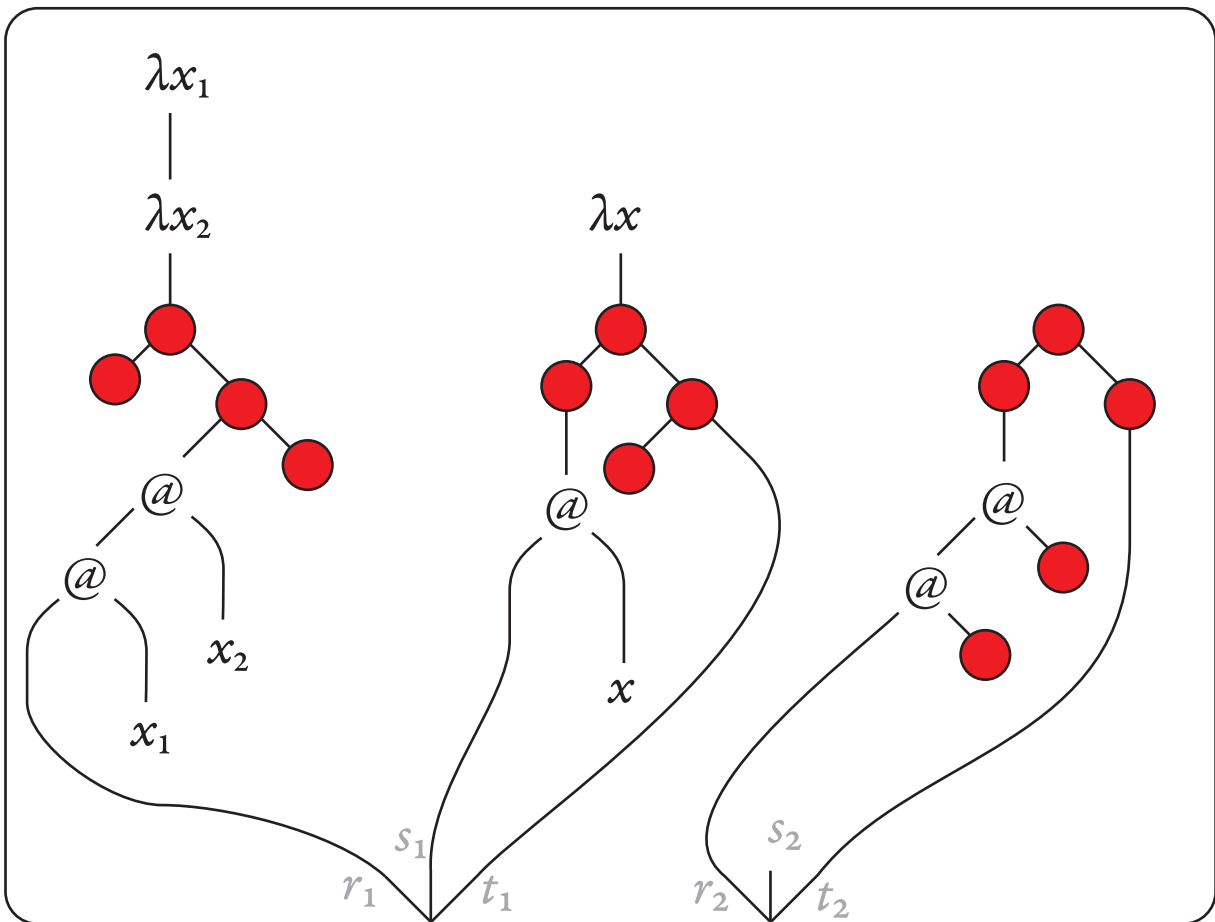


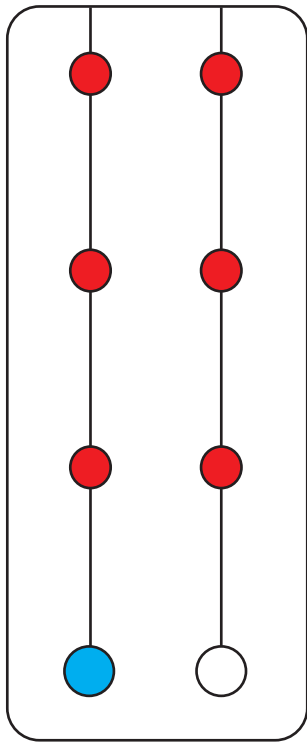
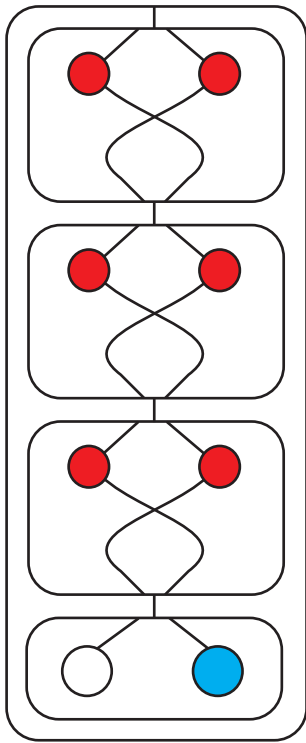


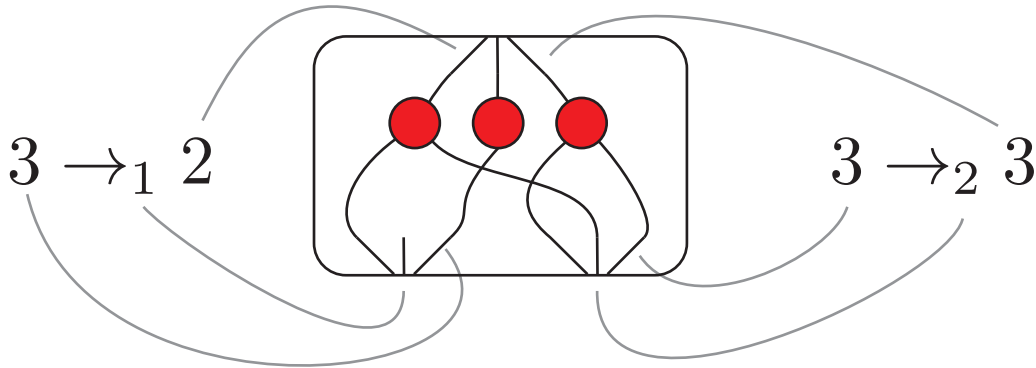
a register update

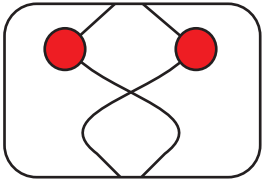


its λ -representation

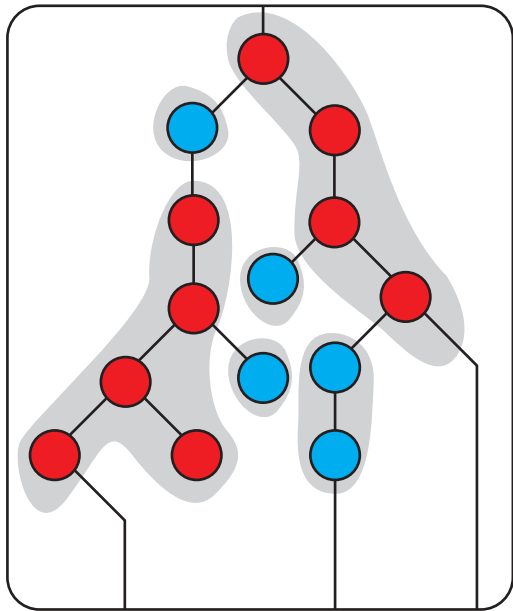




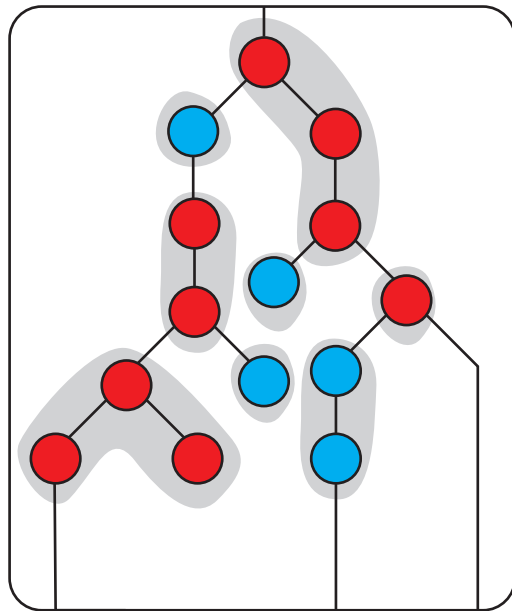


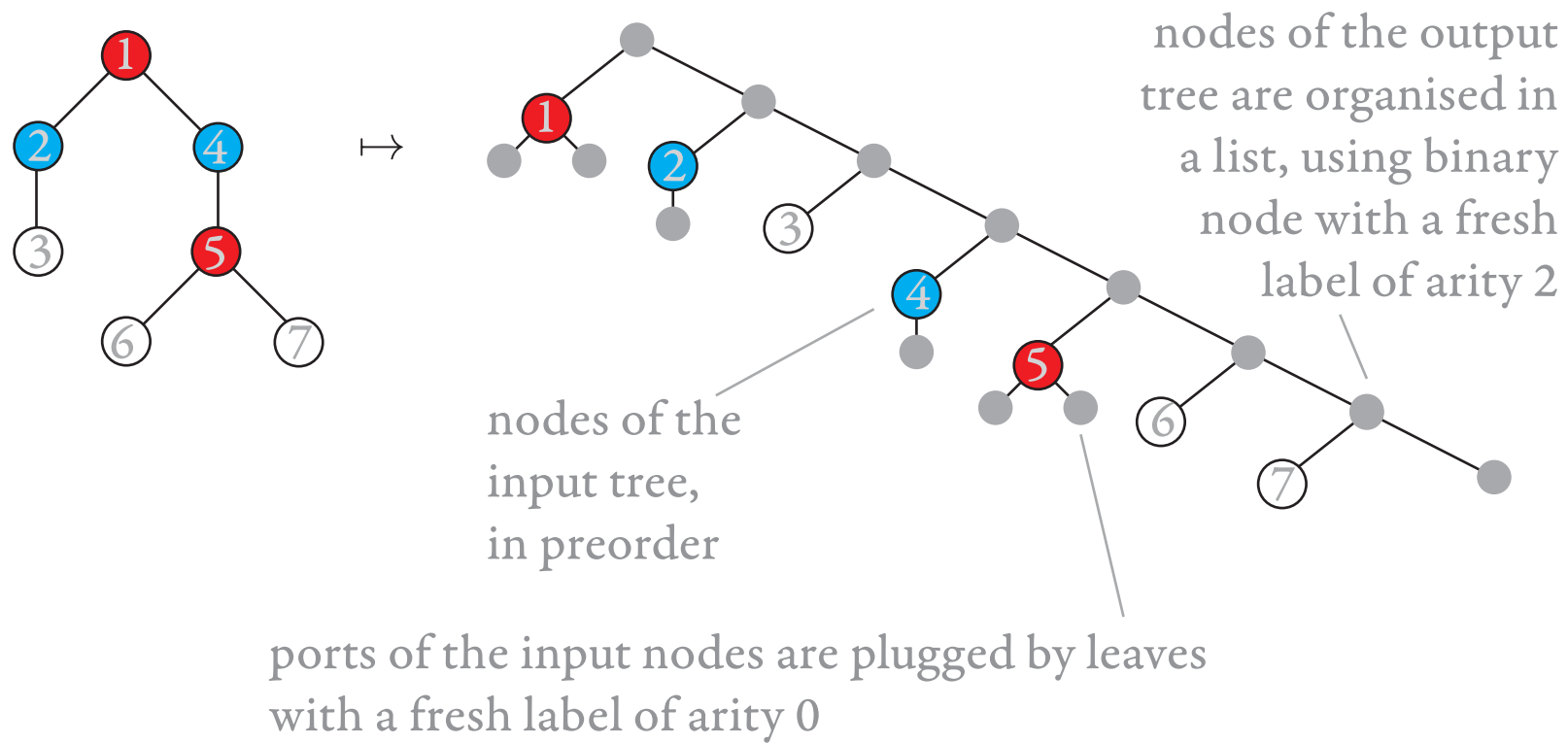


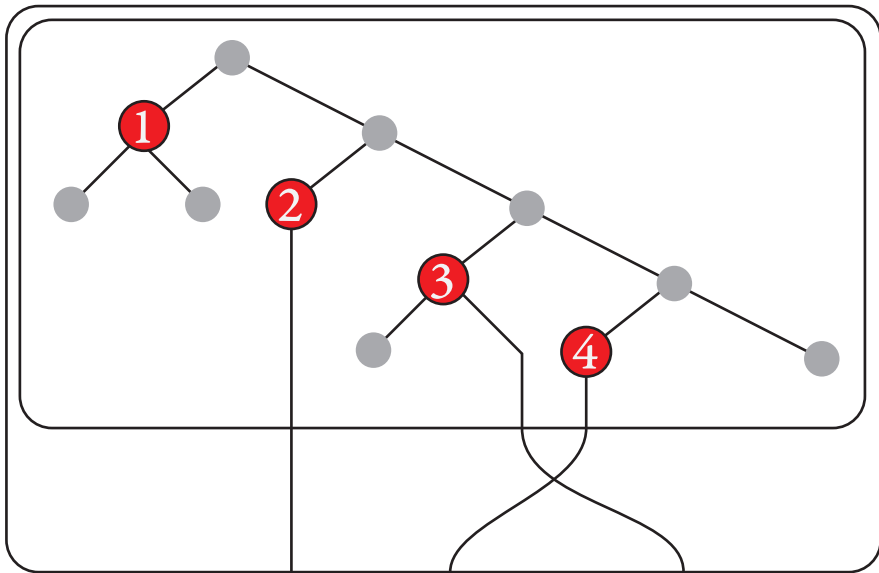
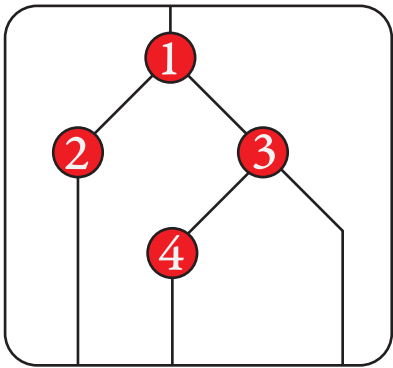
\uparrow -equivalence



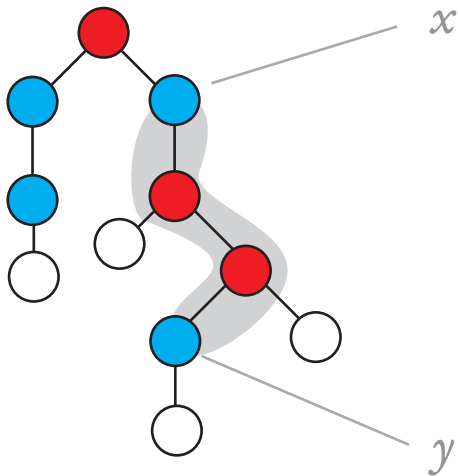
\downarrow -equivalence



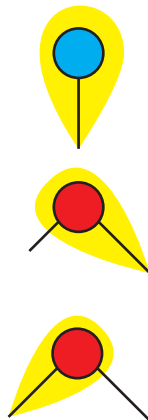




tree with a path
from x to y



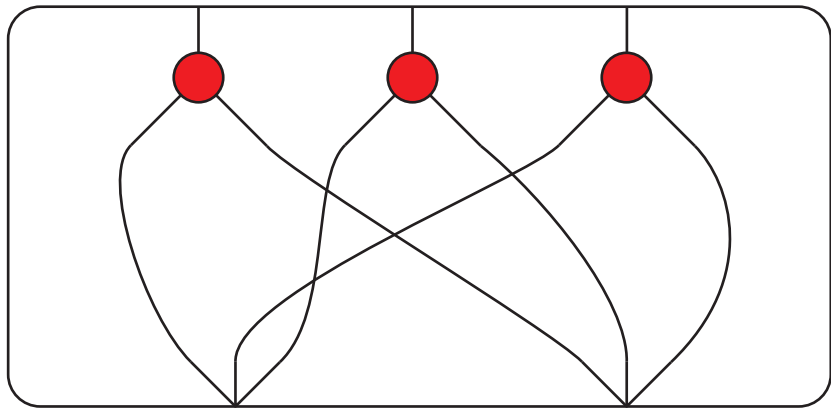
word corresponding
to the path

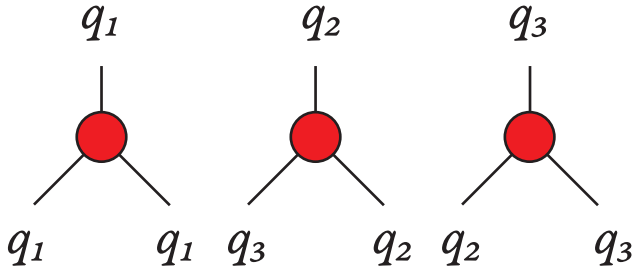


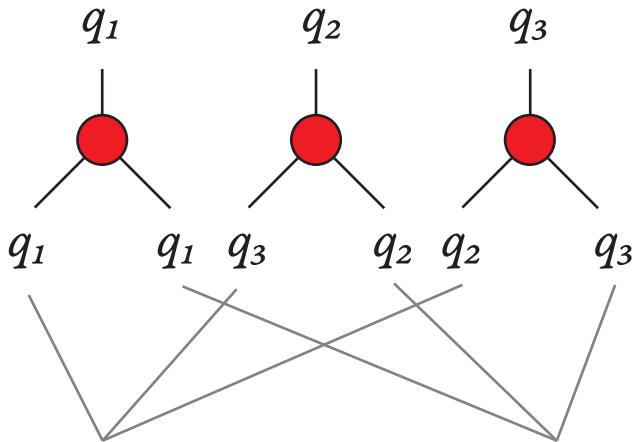
copy 1
of a

copy 2
of a

copy 3
of a

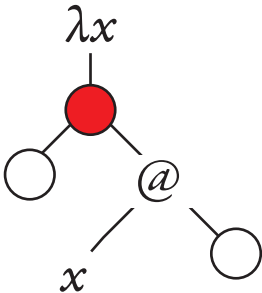




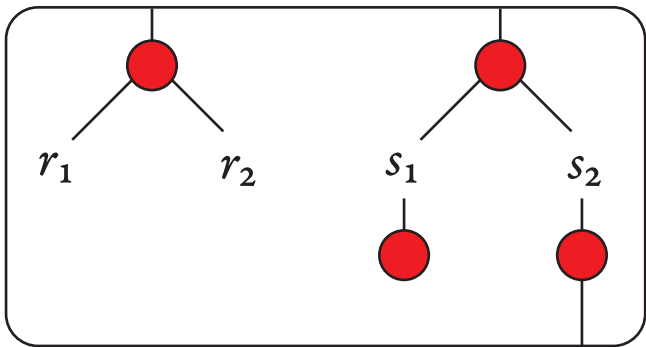


each state appears
exactly once as
a first child

each state appears
exactly once as
a second child



a register valuation



its λ -representation

