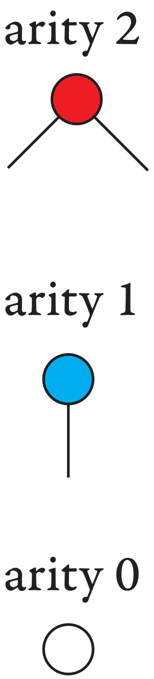
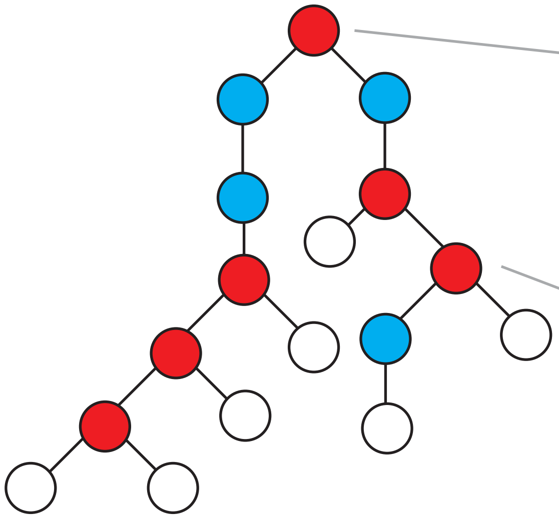
a ranked alphabet


arity 2

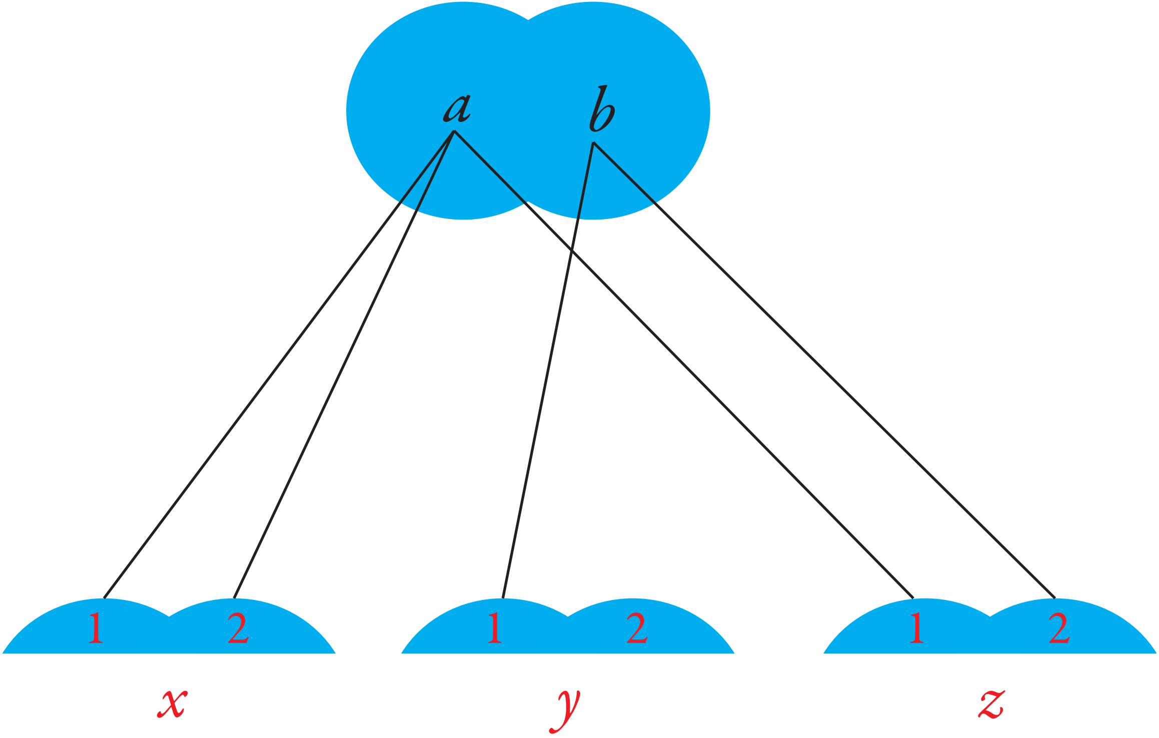
arity 1

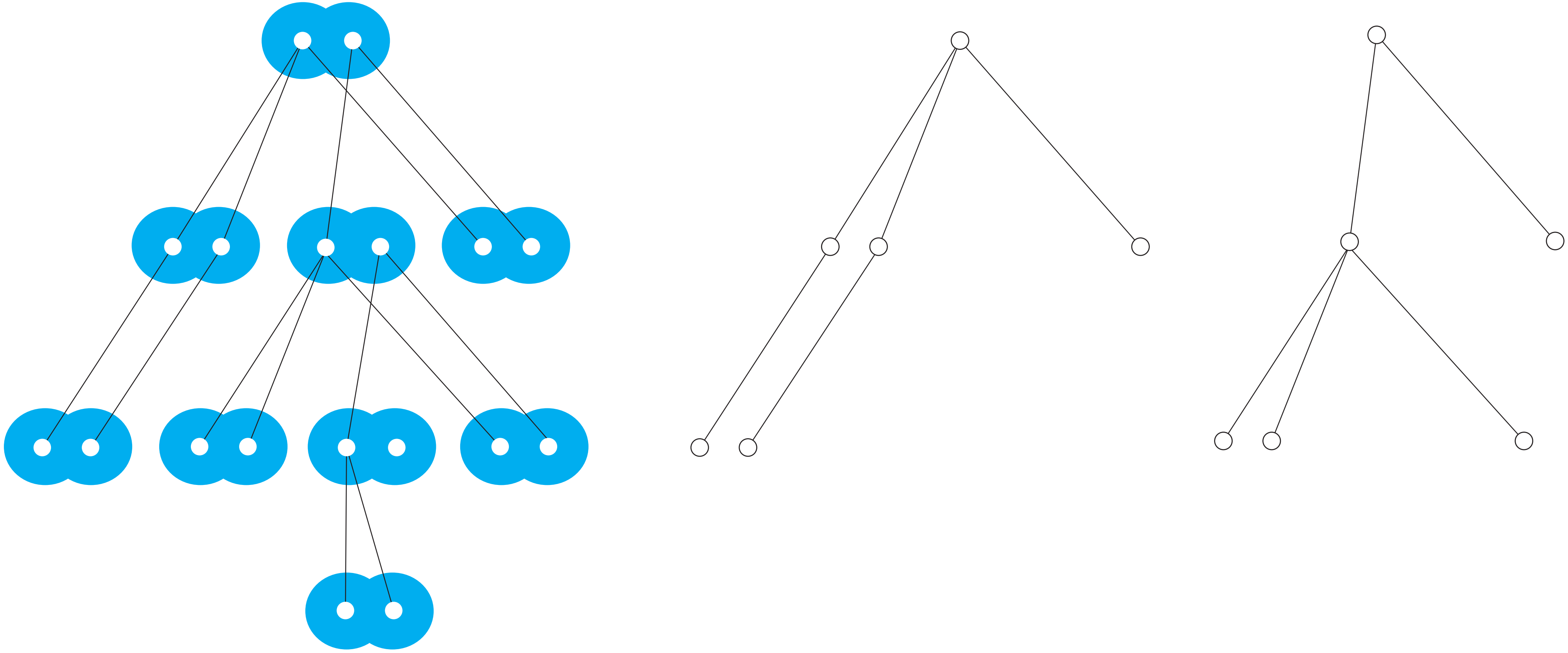
arity 0

a tree


this node has a label of arity 2,
and therefore it has 2 children

this node is child 2
(children are ordered)

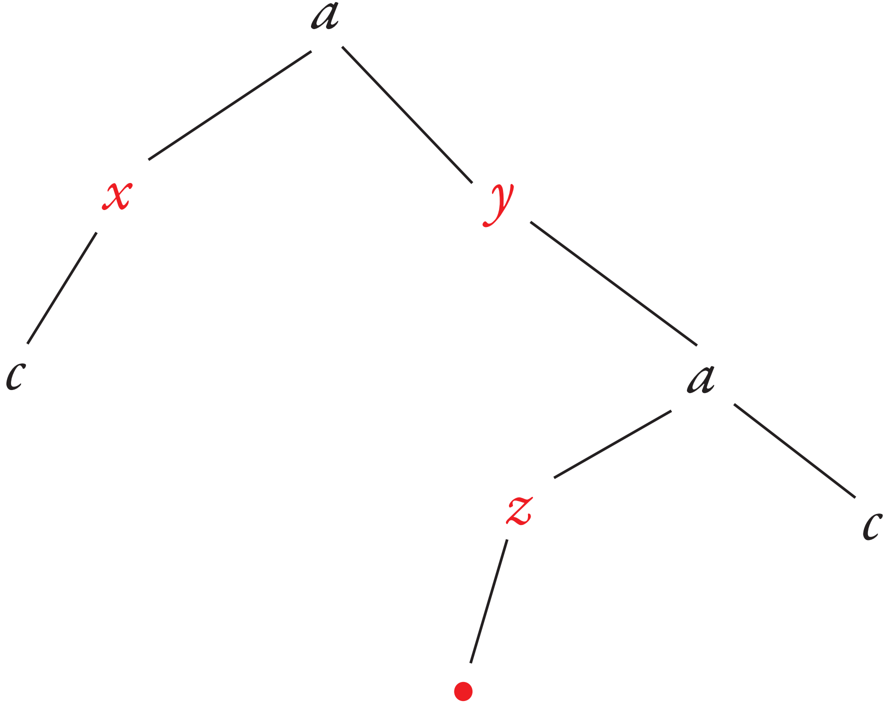

a
b
1
2
1
2
1
2
x
y
z

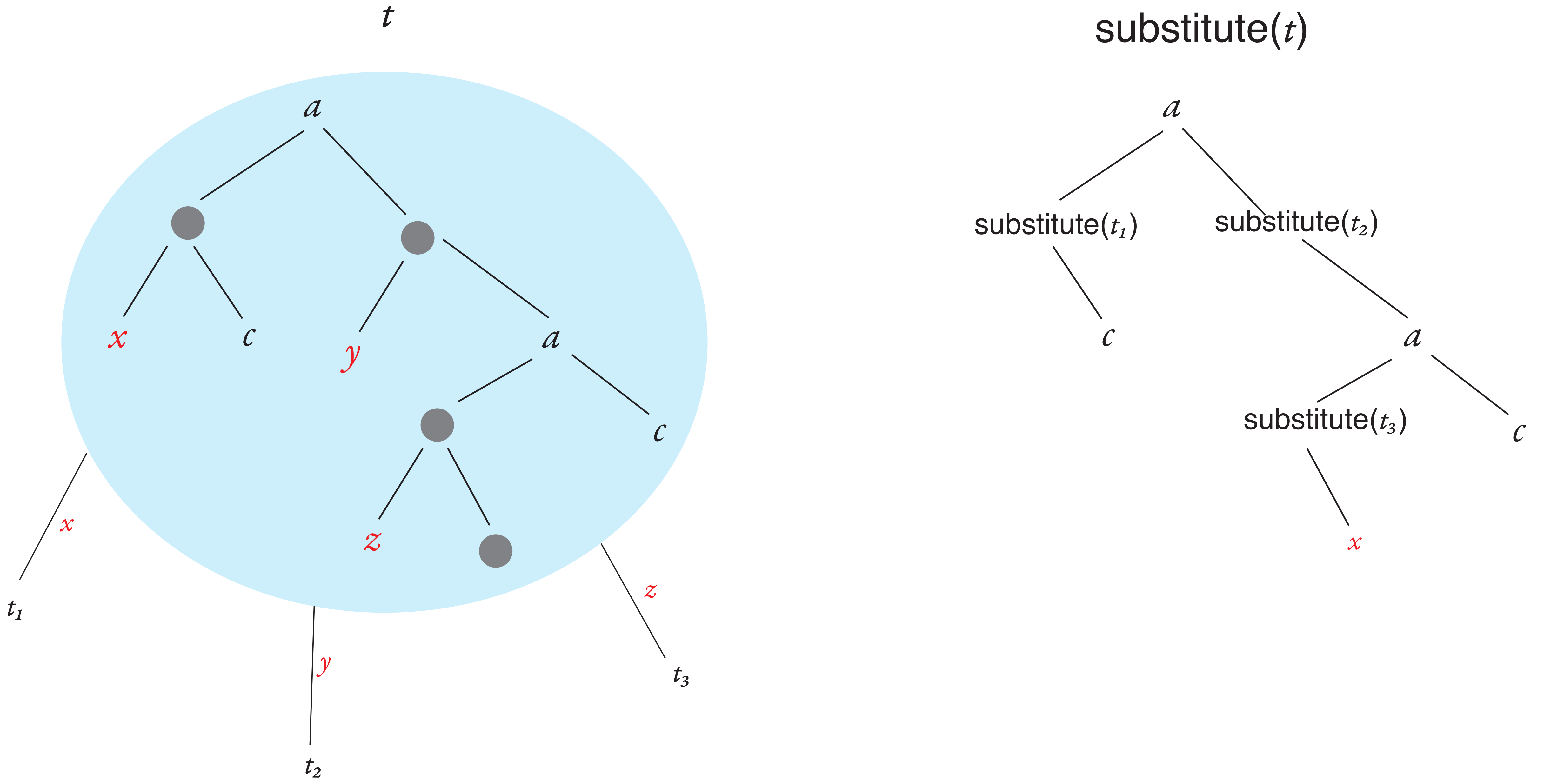


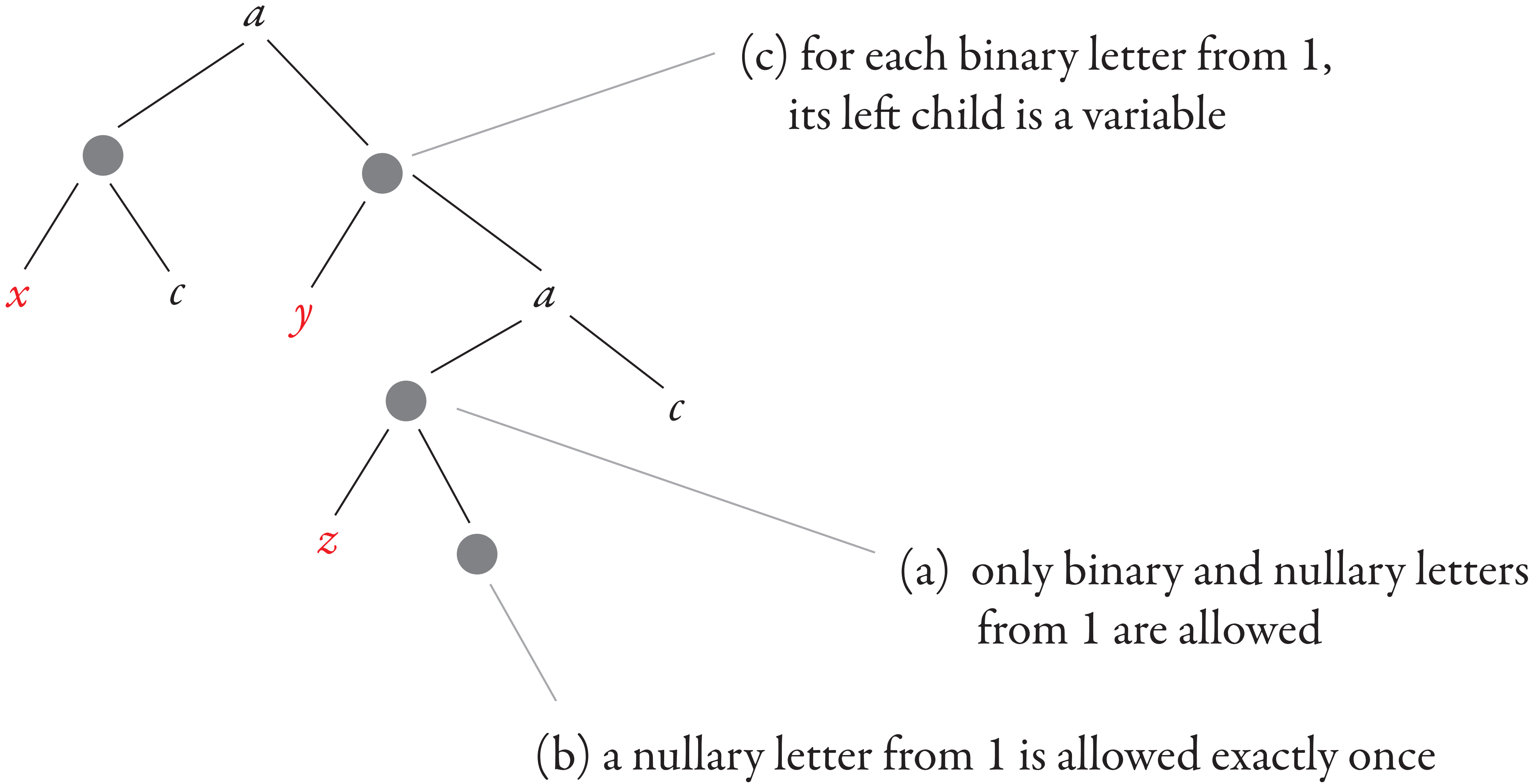
A tree t over $\Sigma^{[2]}$

$\mathsf{unfold}_1(t)$

$\mathsf{unfold}_2(t)$


a
x
y
c
a
z
c

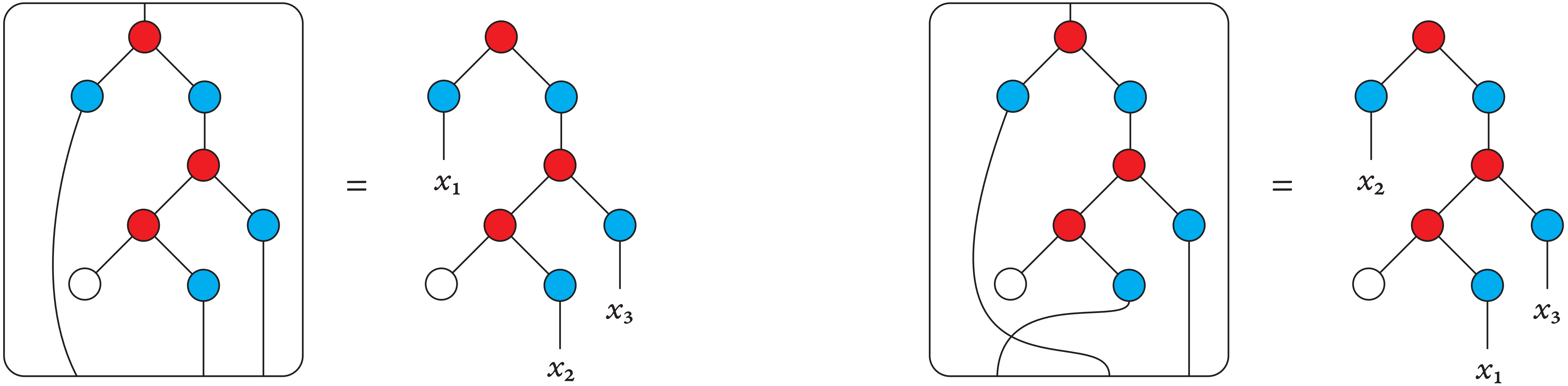

t
a
x
c
y
a
z
c
x
t_1
y
t_2
z
t_3
substitute(t)
a
substitute(t_1)
substitute(t_2)
c
a
substitute(t_3)
c
x

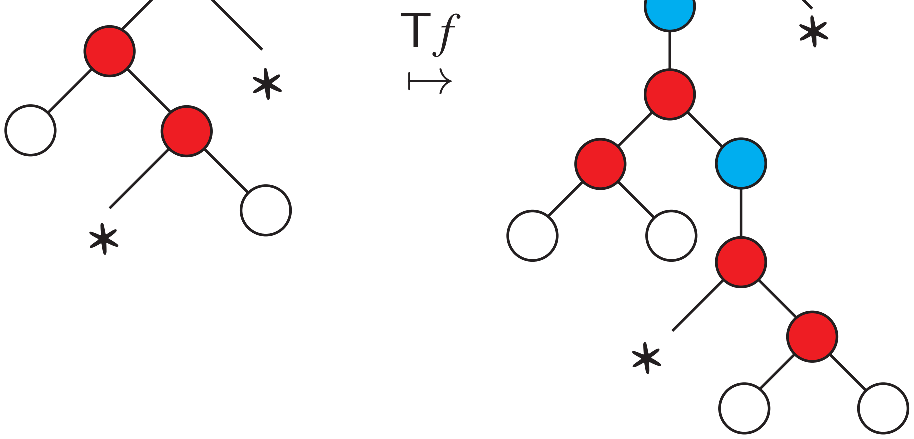


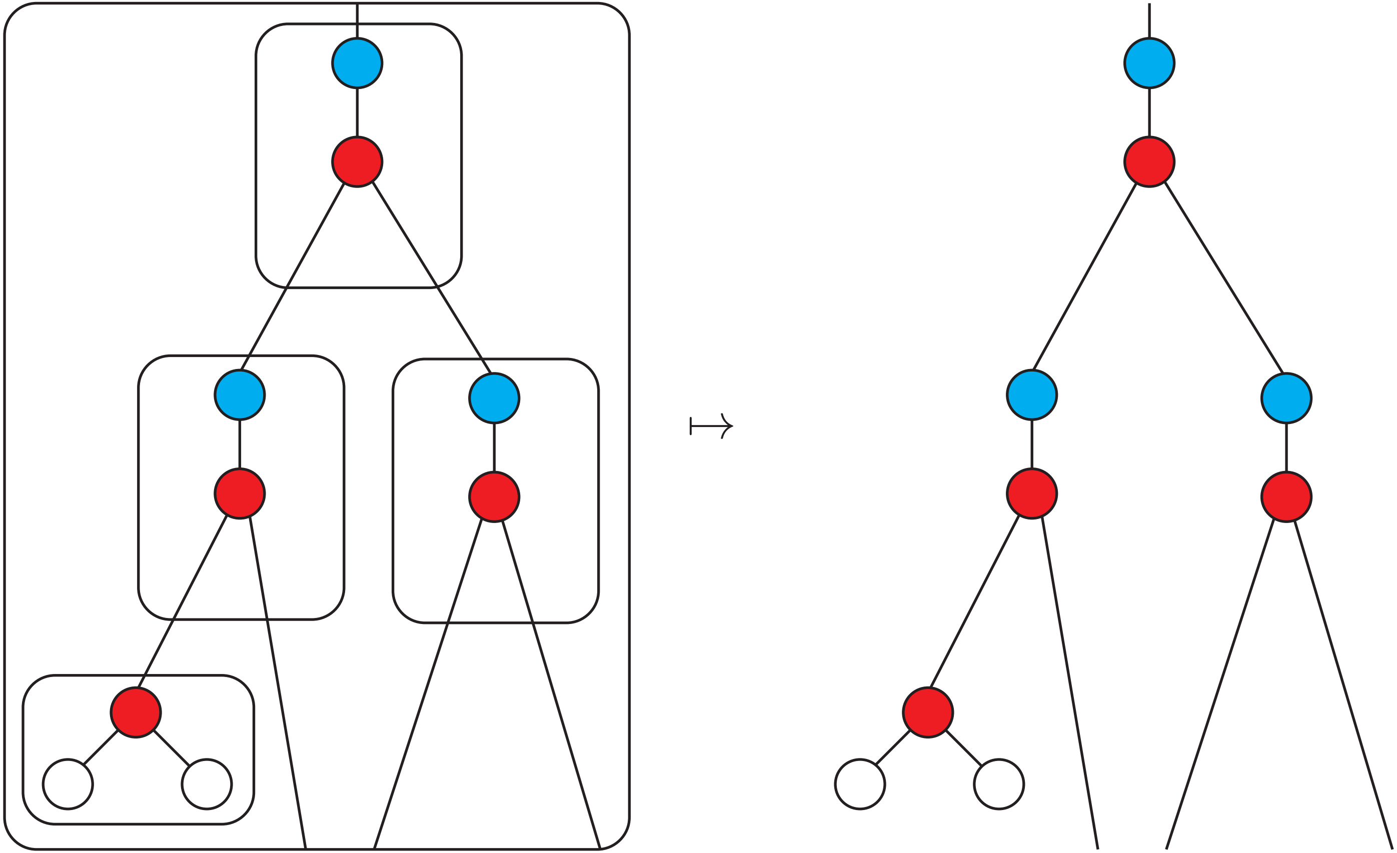
(c) for each binary letter from 1, its left child is a variable

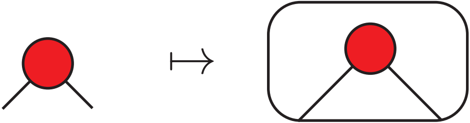
(a) only binary and nullary letters from 1 are allowed

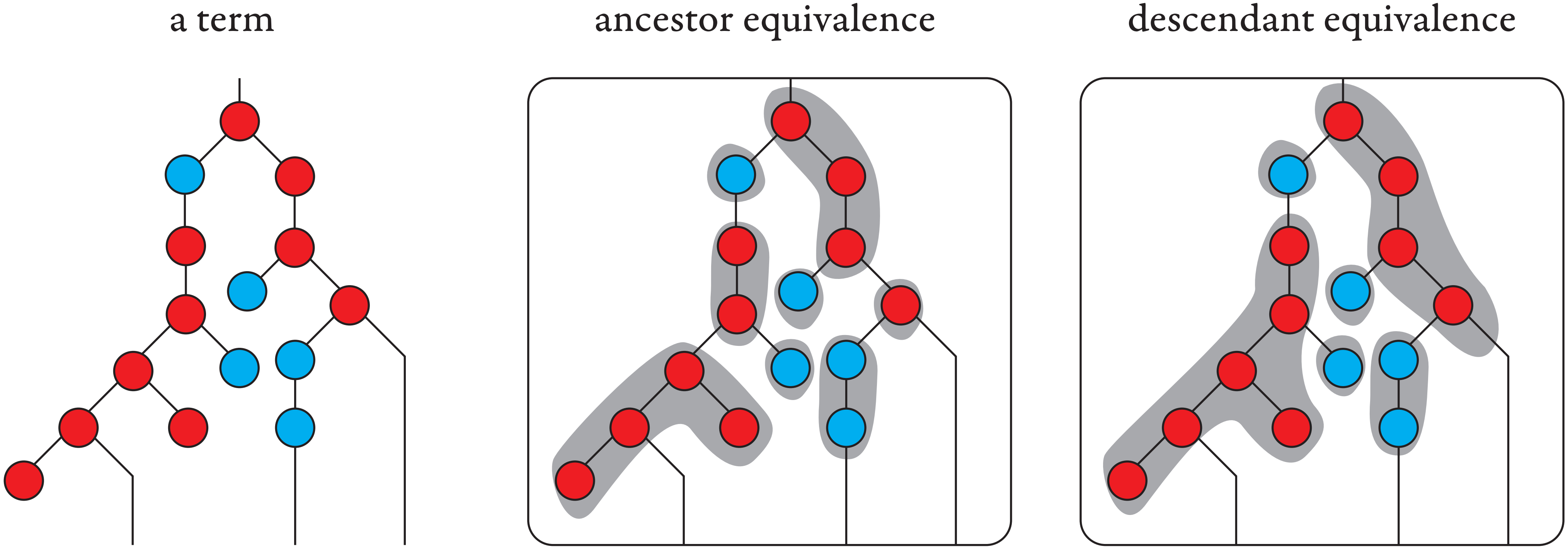
(b) a nullary letter from 1 is allowed exactly once


=
x_1
x_2
x_3
=
x_2
x_3
x_1


Tf
↦
*
*
*
*



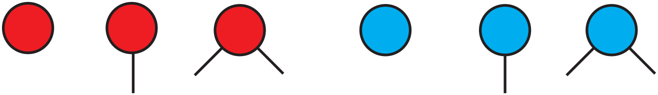


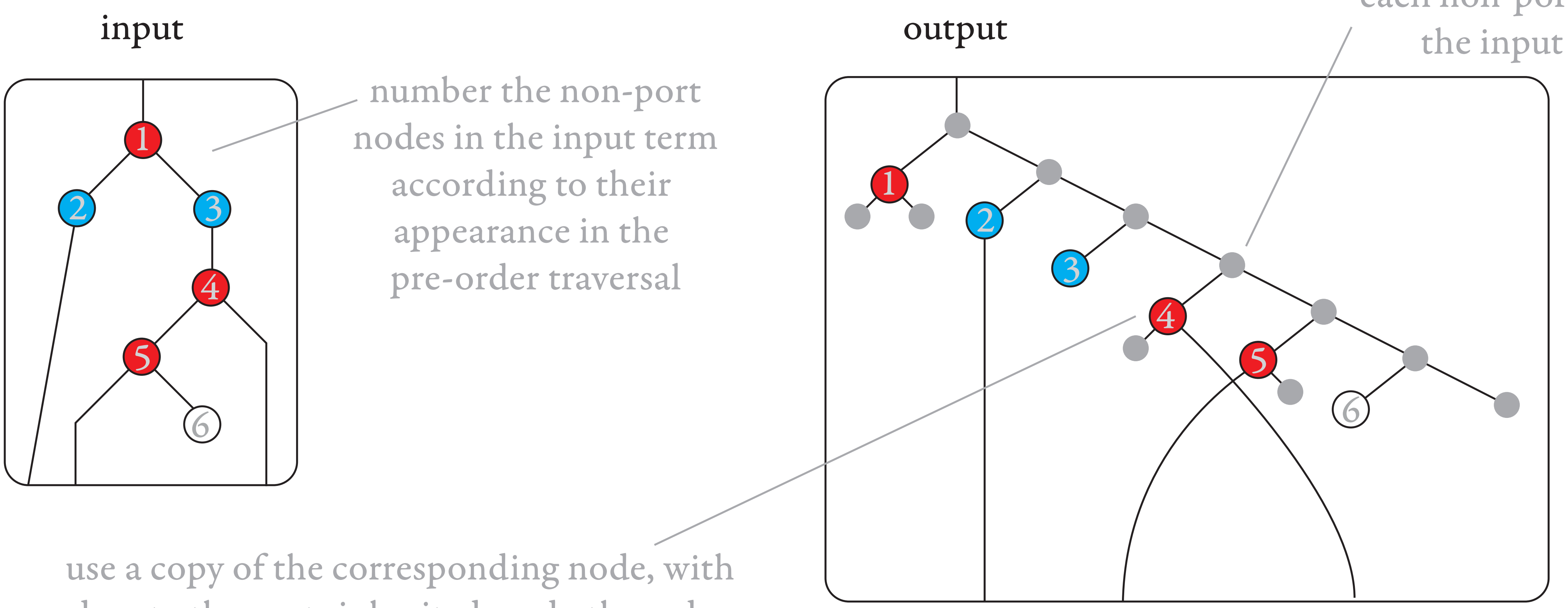


a term

ancestor equivalence

descendant equivalence



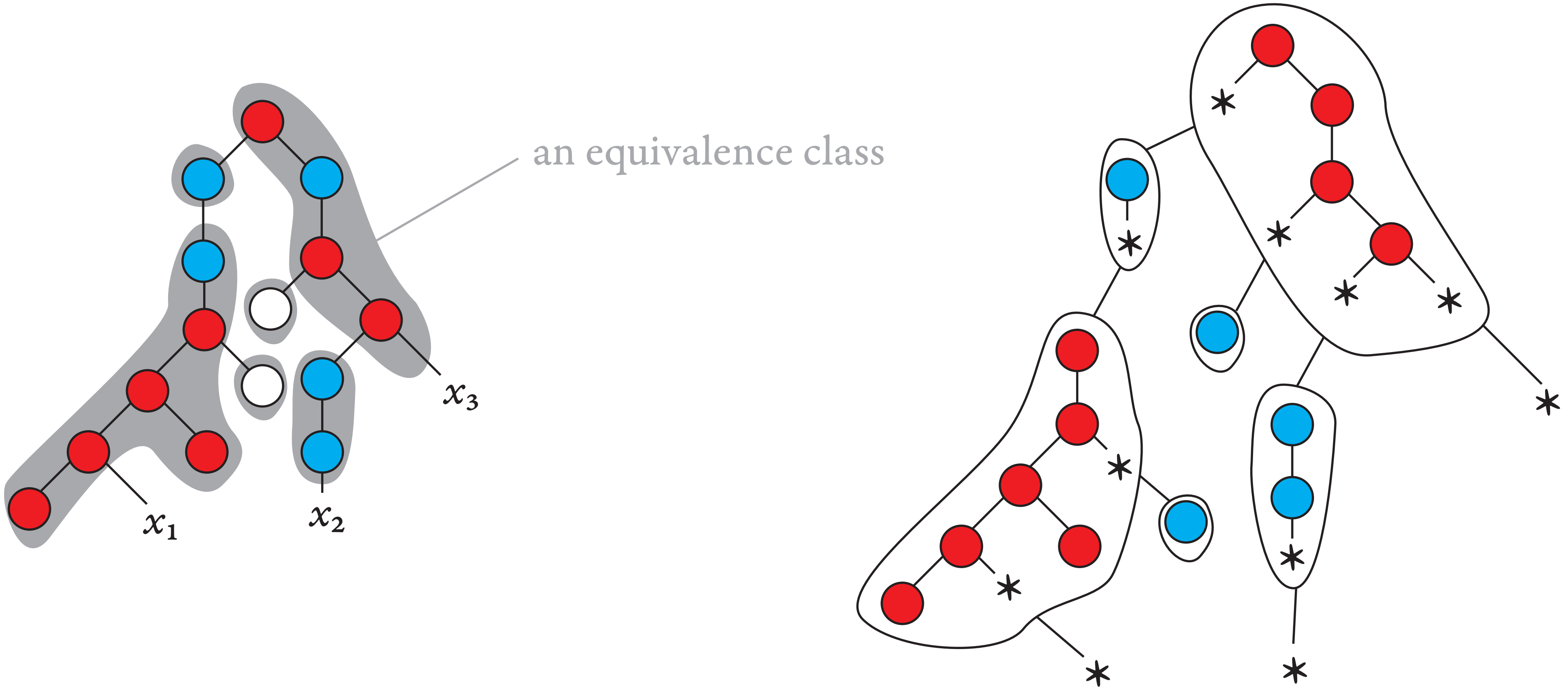


number the non-port nodes in the input term according to their appearance in the pre-order traversal

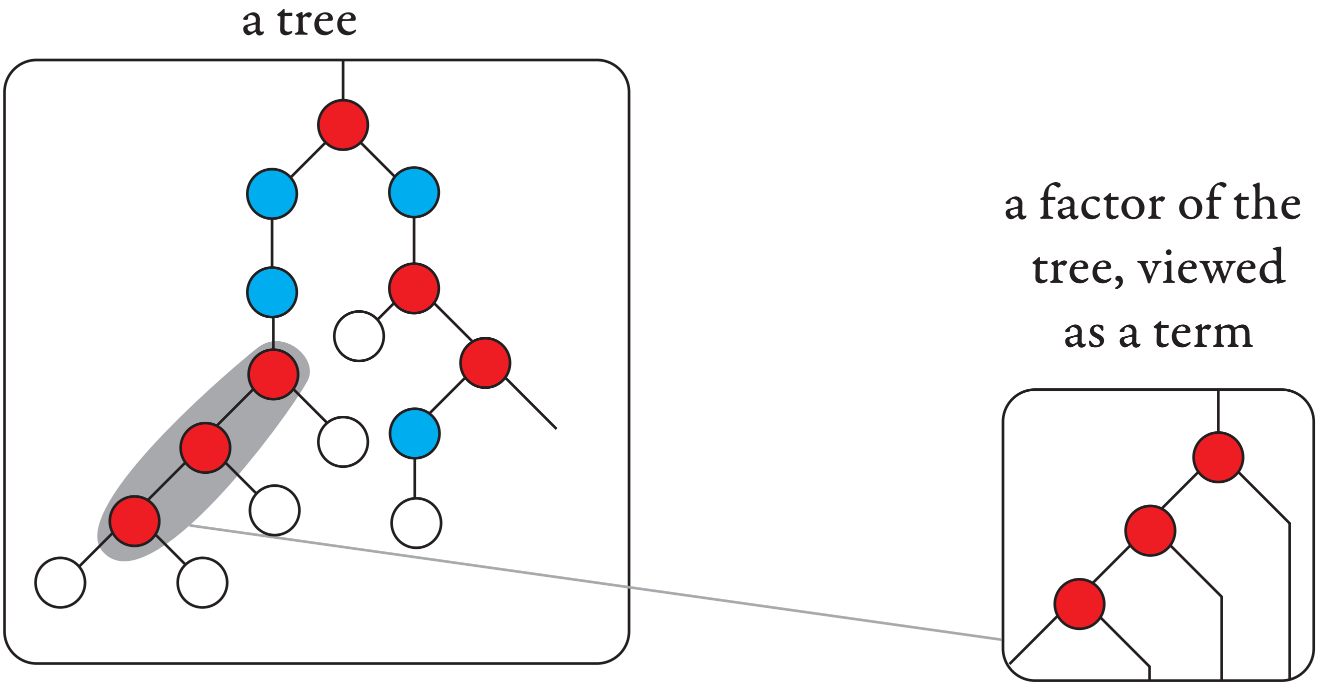
create a binary node for each non-port node in the input term

use a copy of the corresponding node, with edges to the ports inherited, and other edges plugged by ●

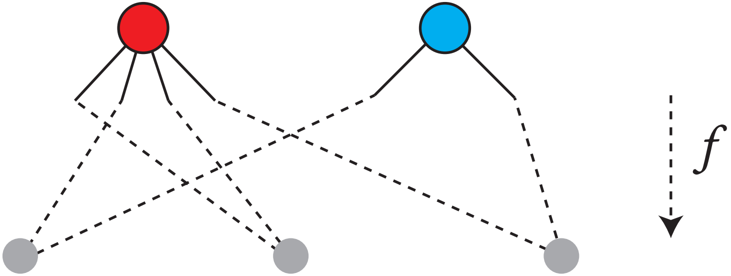
a factorisation equivalence



an equivalence class



a tree
a factor of the tree, viewed as a term


f

input alphabet

arity 2

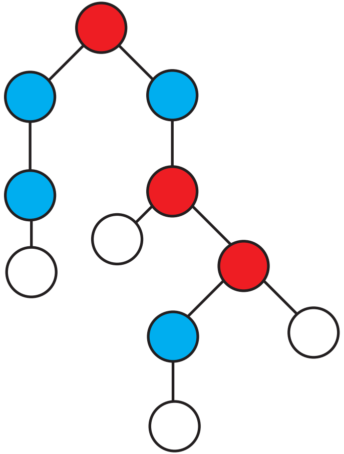
arity 1

arity 0

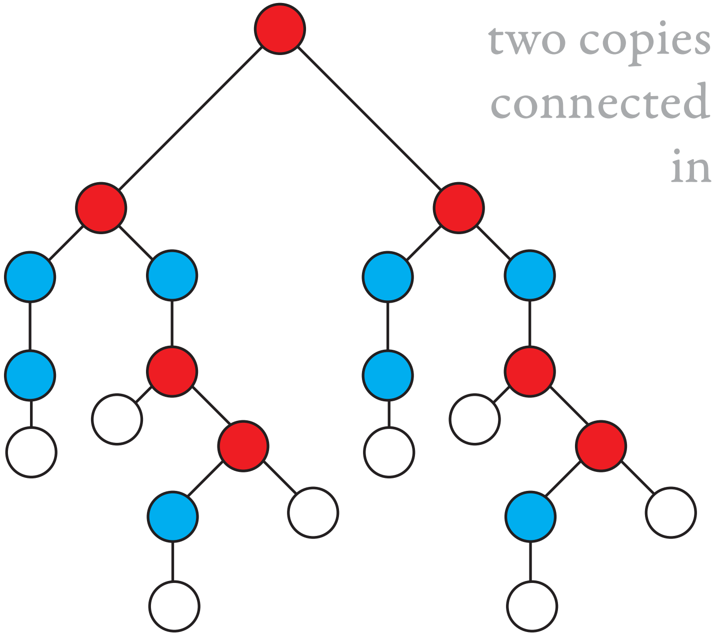
output alphabet

arity 2

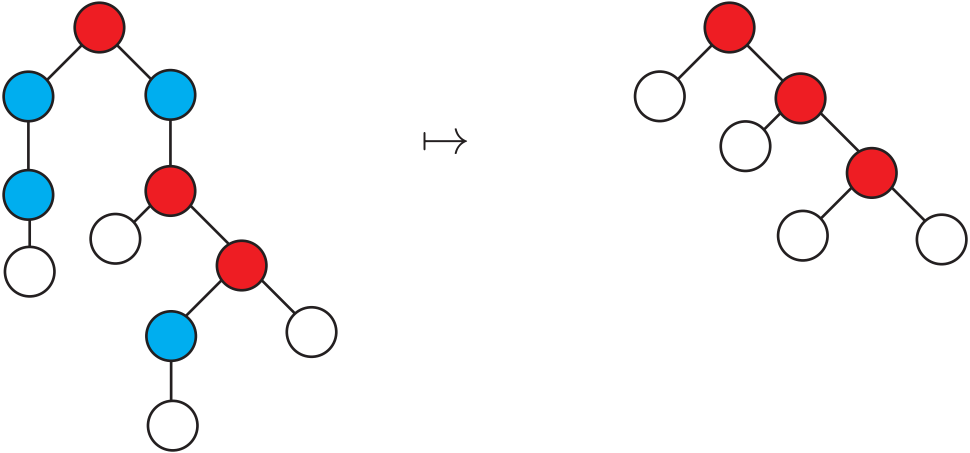
arity 0

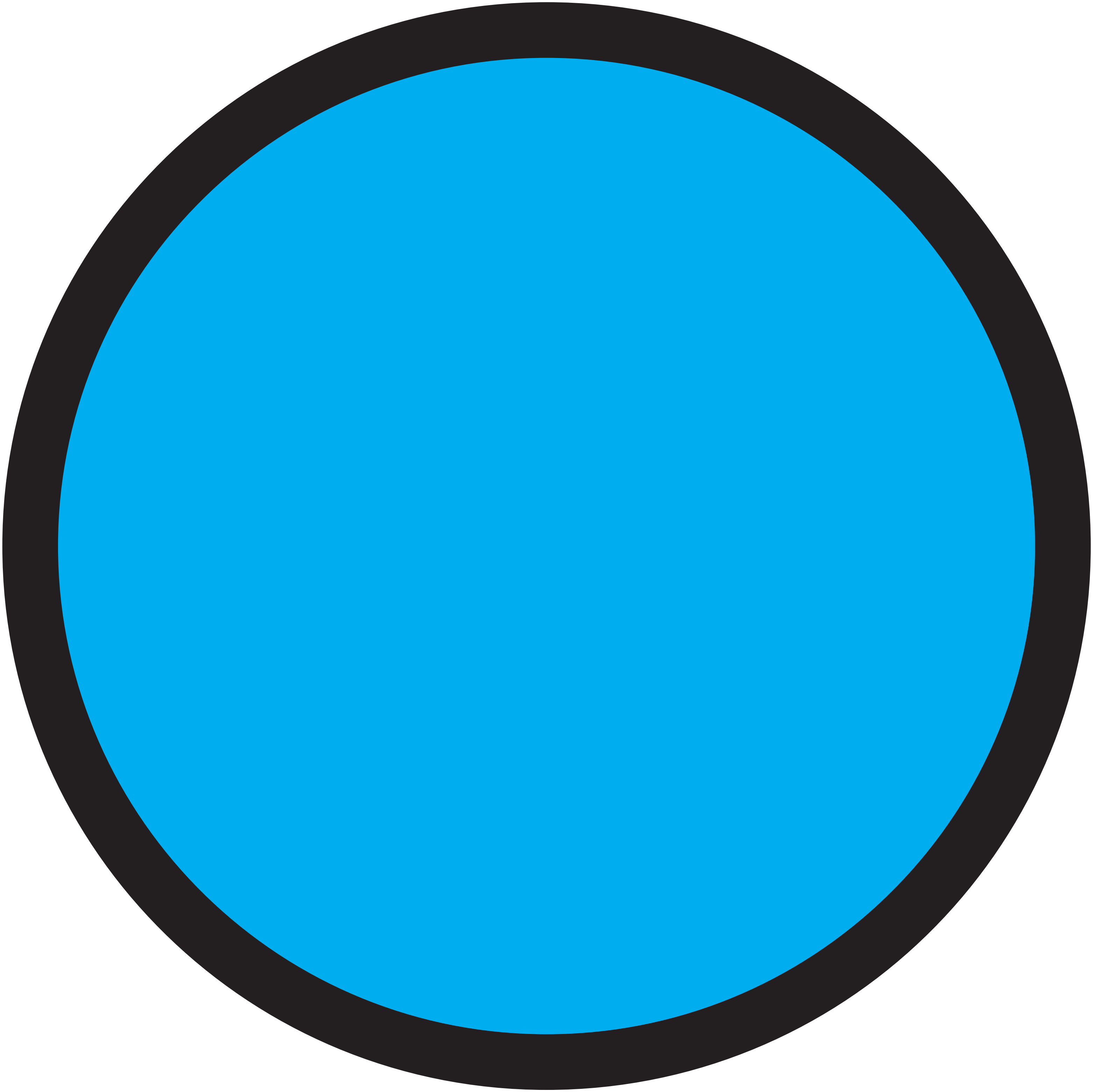


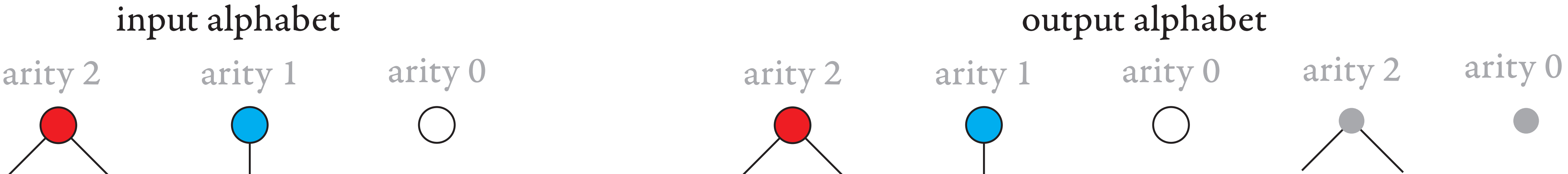
$\mapsto$

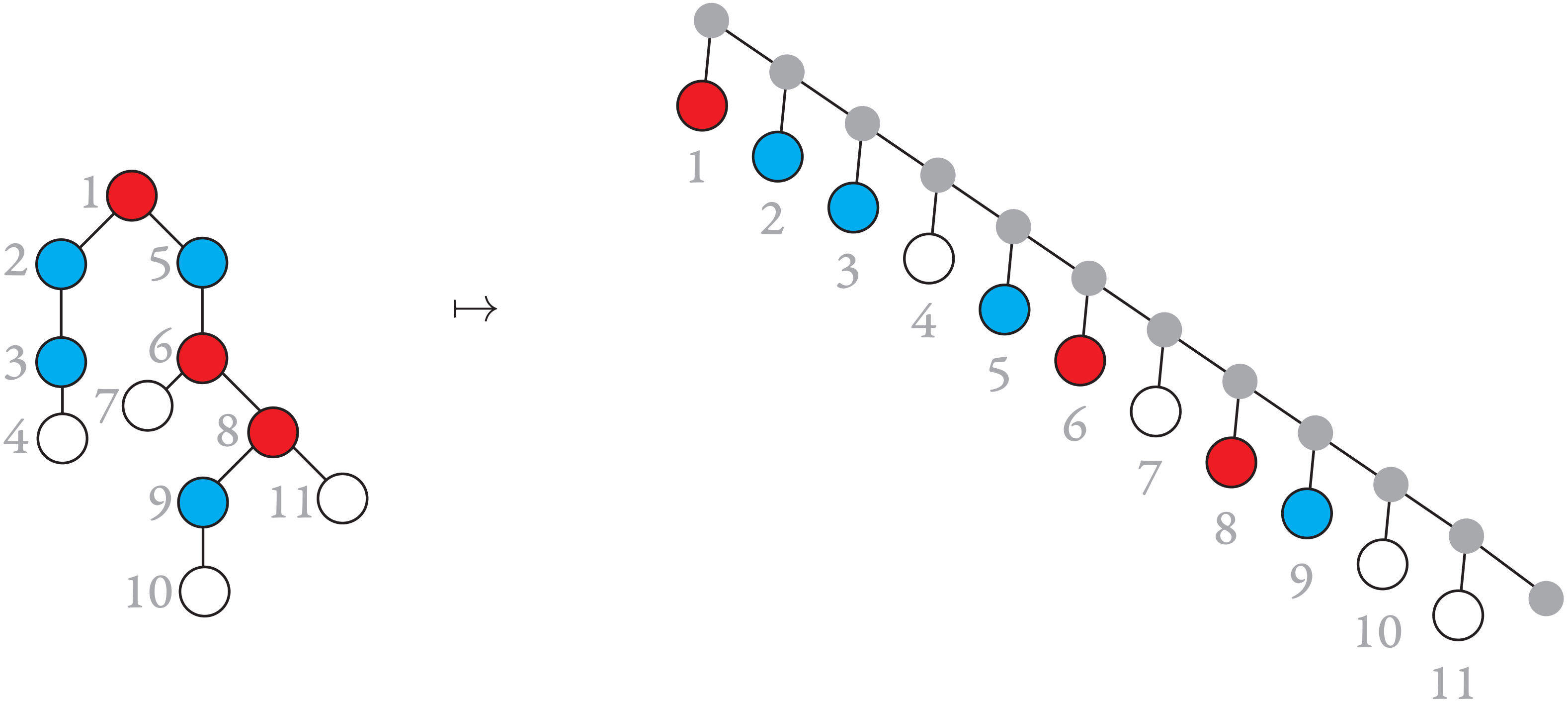


two copies of the input tree,
connected by a binary node
in the root

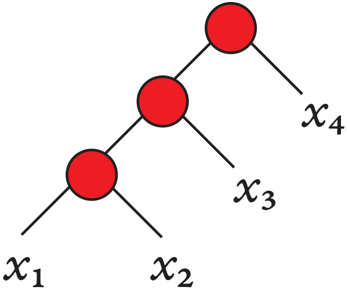




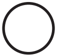

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 0

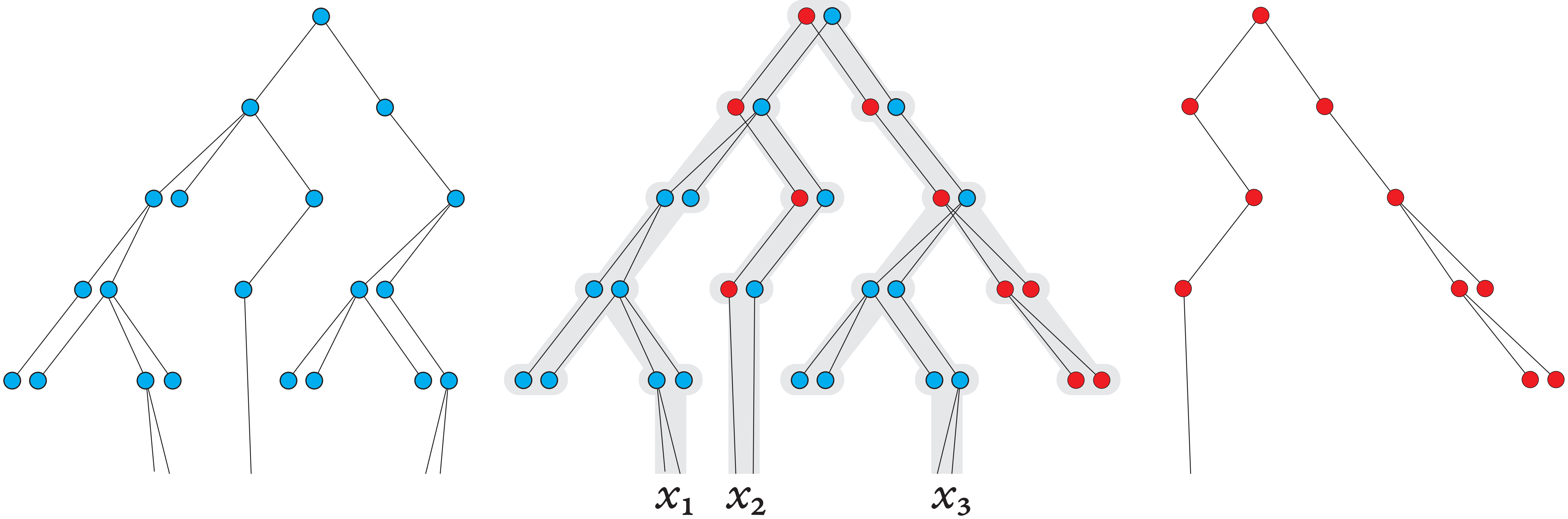

1
2
3
4
5
6
7
8
9
10
11
↦
1
2
3
4
5
6
7
8
9
10
11

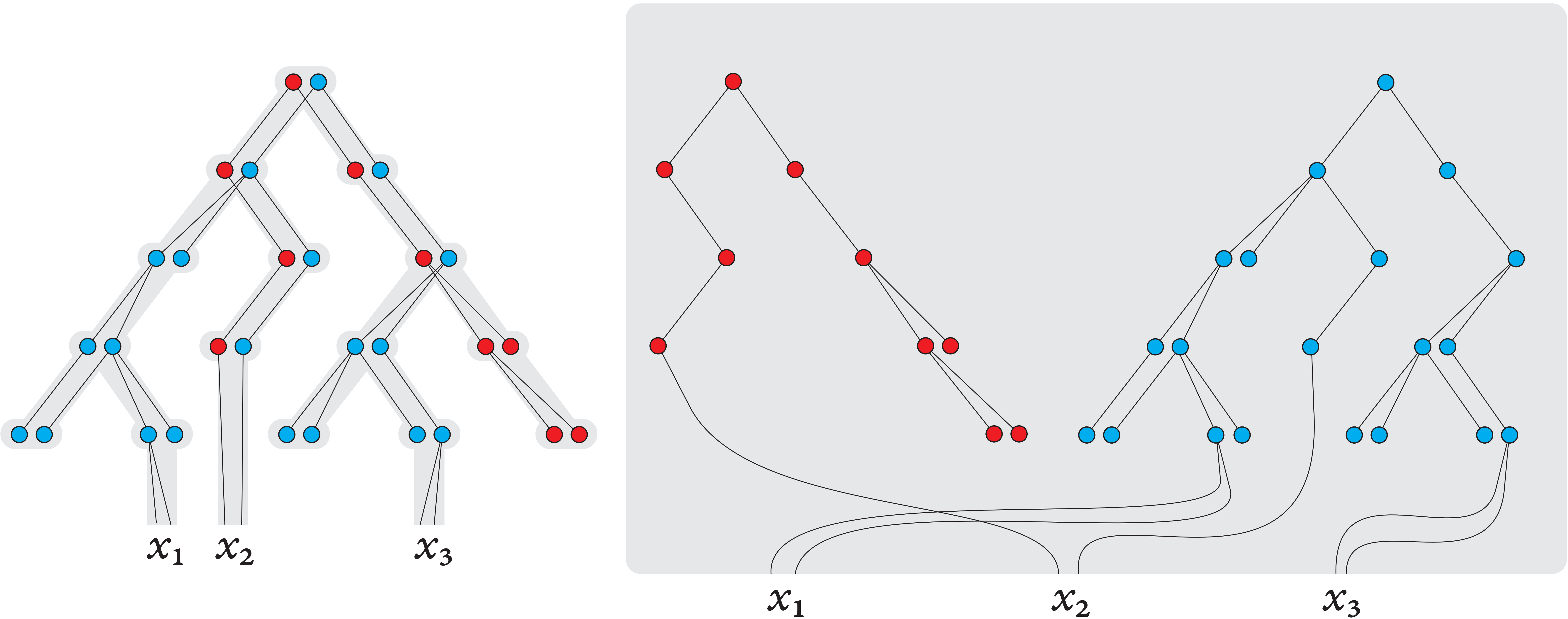
a term of arity 4

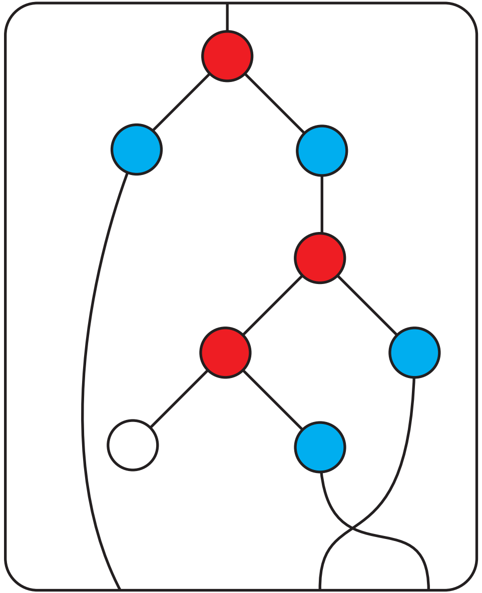


a term of arity 0




x_1
x_2
x_3

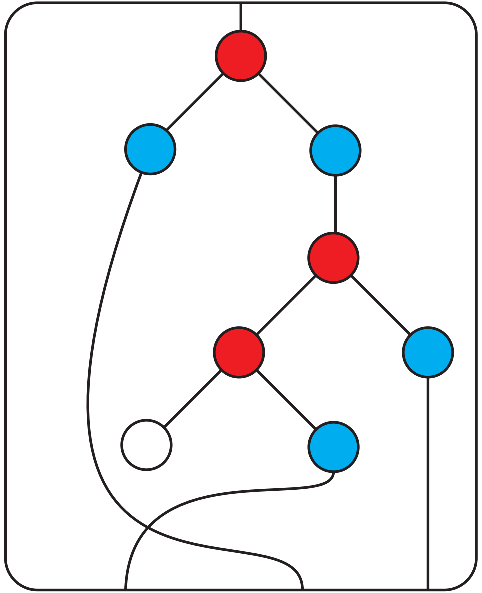

x_1
x_2
x_3
x_1
x_2
x_3



satisfies (*)

(*)

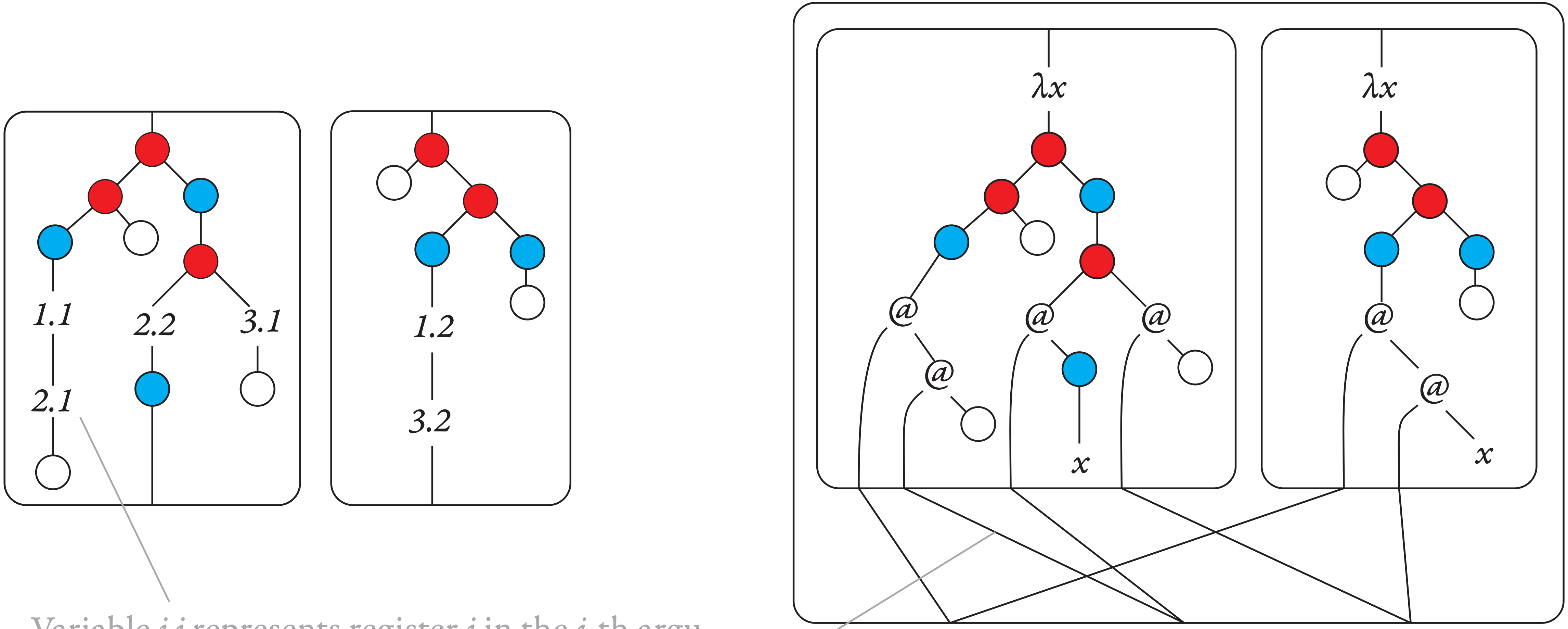
If the root has arity n,
and $1 \le i < j \le n$, then
all ports of the j-th
subterm of the root are
after all ports of the
i-th subterm of the root



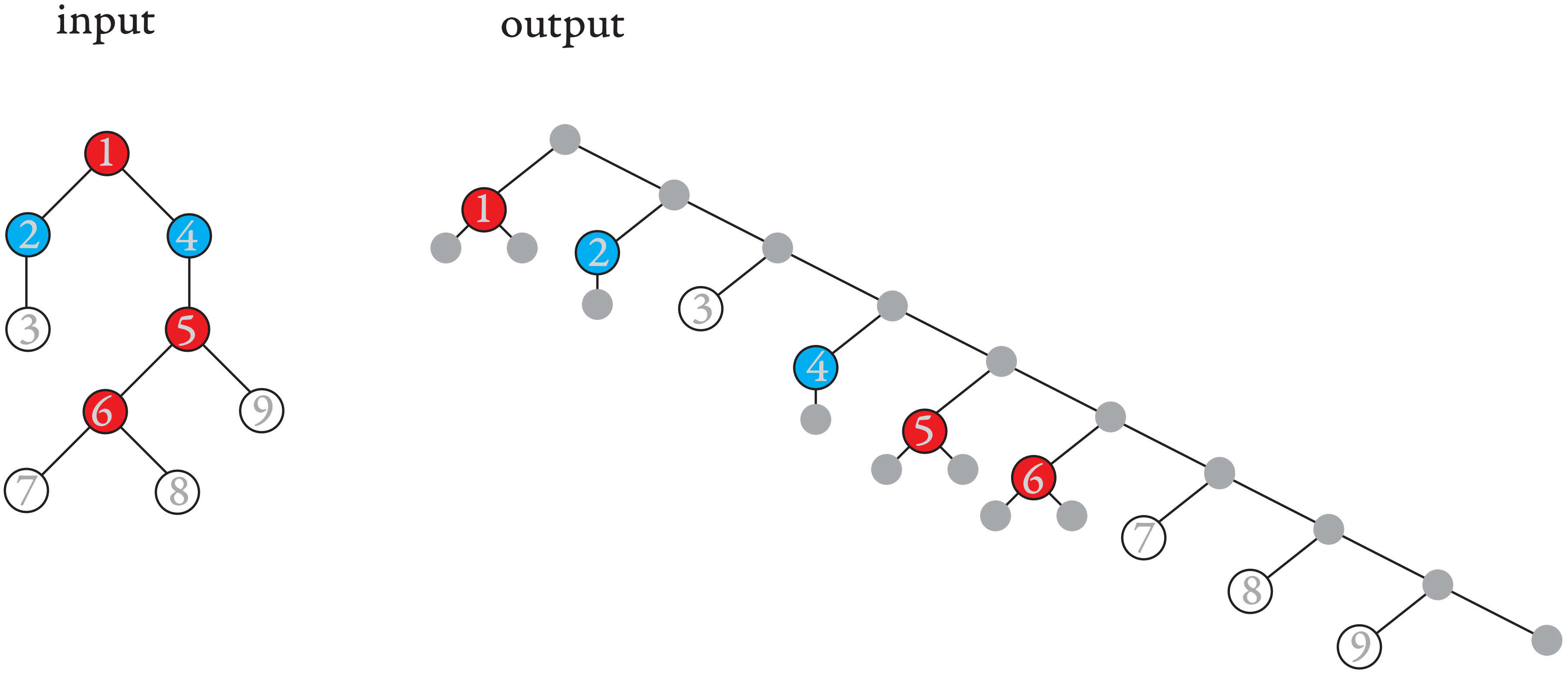
violates (*)

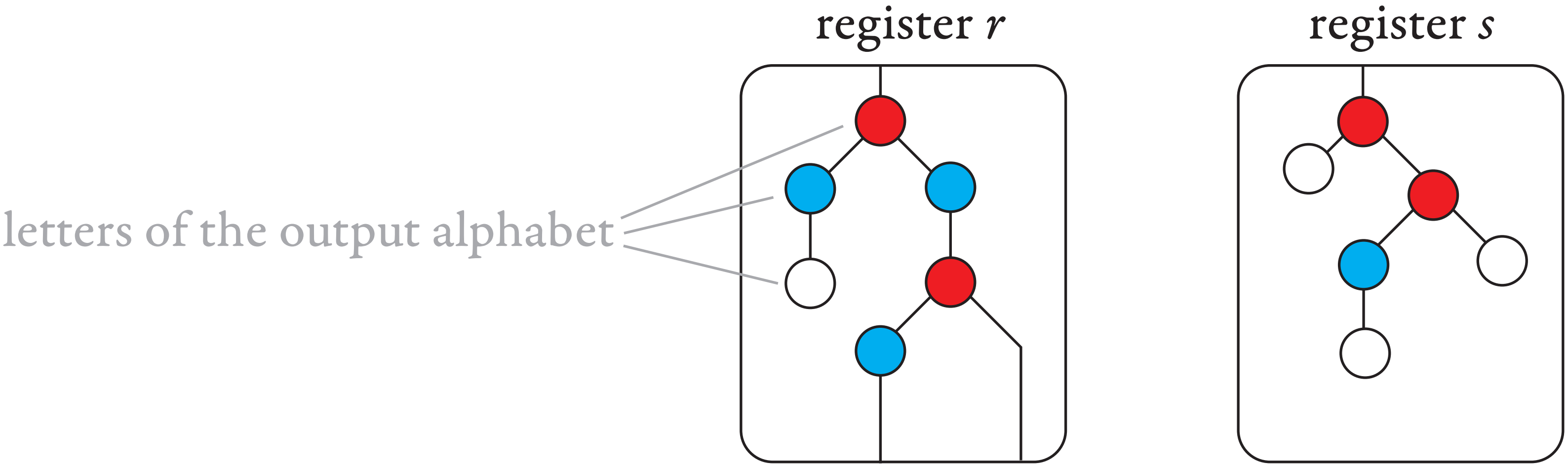
a register update

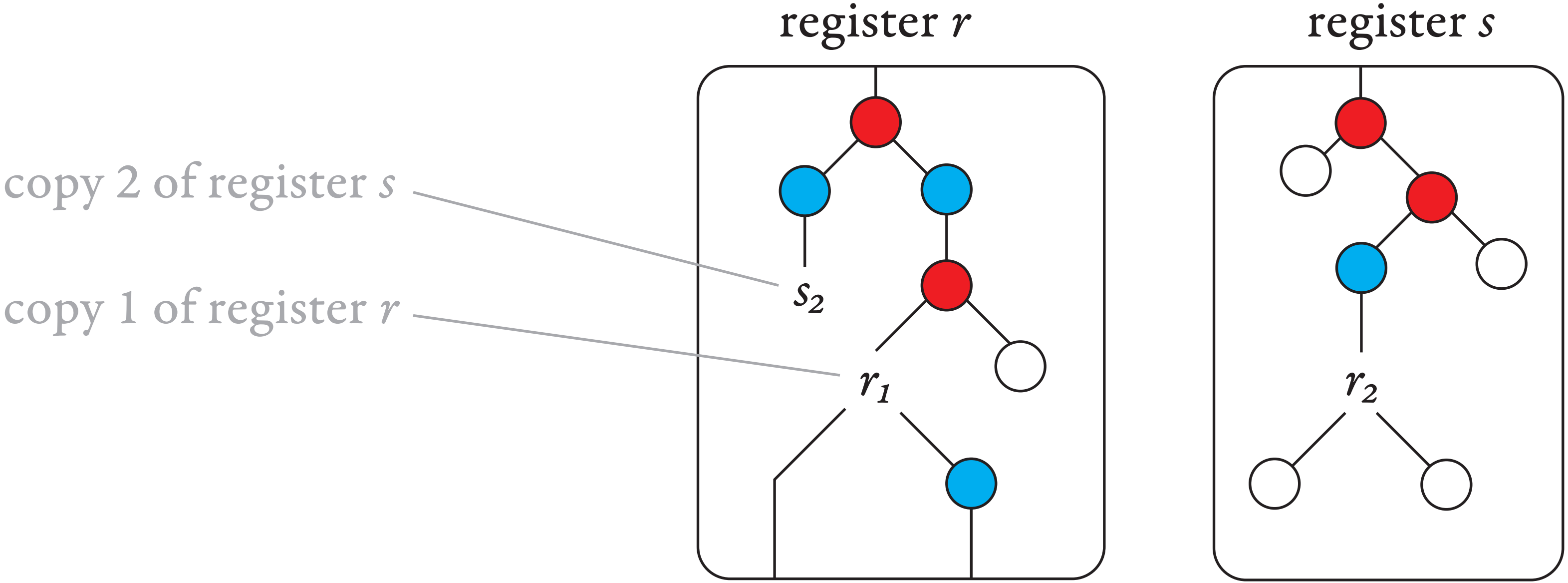
its dual

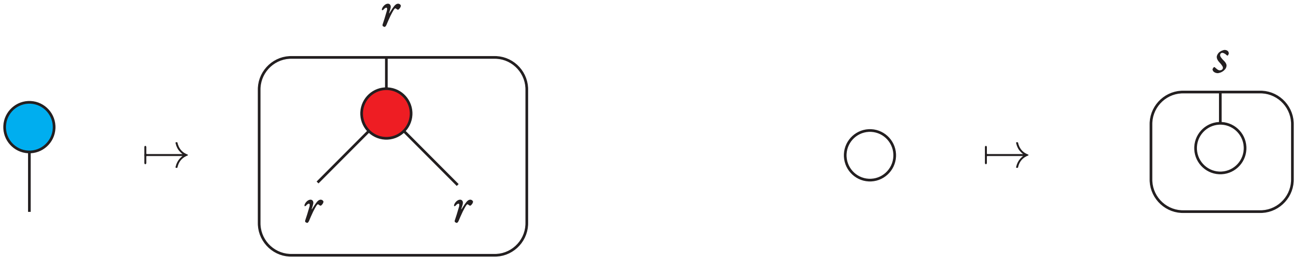


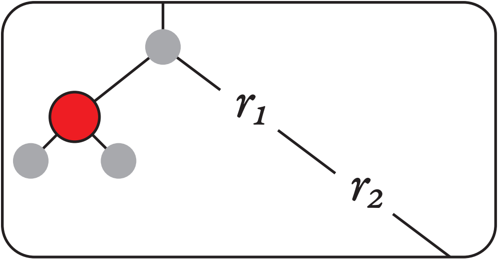
Variable *i.j* represents register *i* in the *j*-th argument of the reigster update.
In the dual, this variable is mapped to the *i*-th edge which enters the *j*-th port of the reducer.

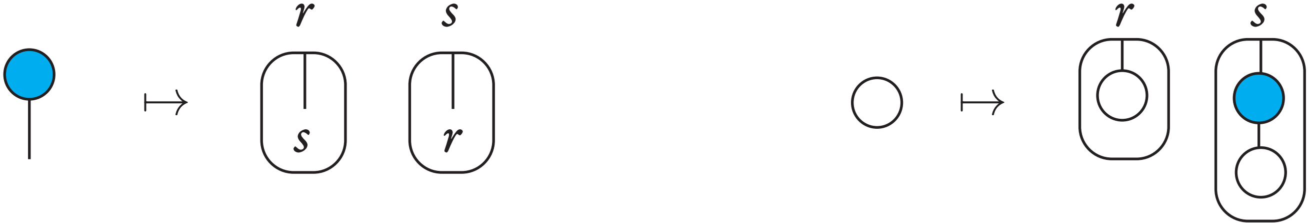

input
output
1
2
4
3
5
6
9
7
8
1
2
3
4
5
6
7
8
9

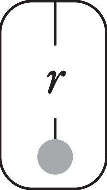

register *r*
register *s*
letters of the output alphabet


register r
register s
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2


r
r
r
s

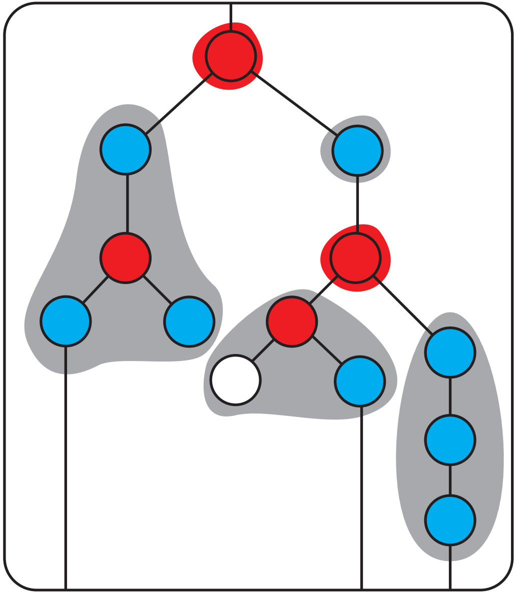

r_1
r_2

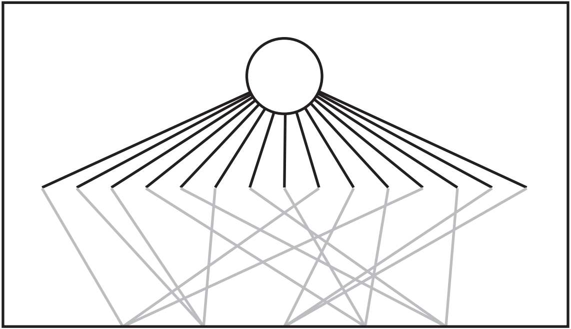

r
s
s
r
r
s

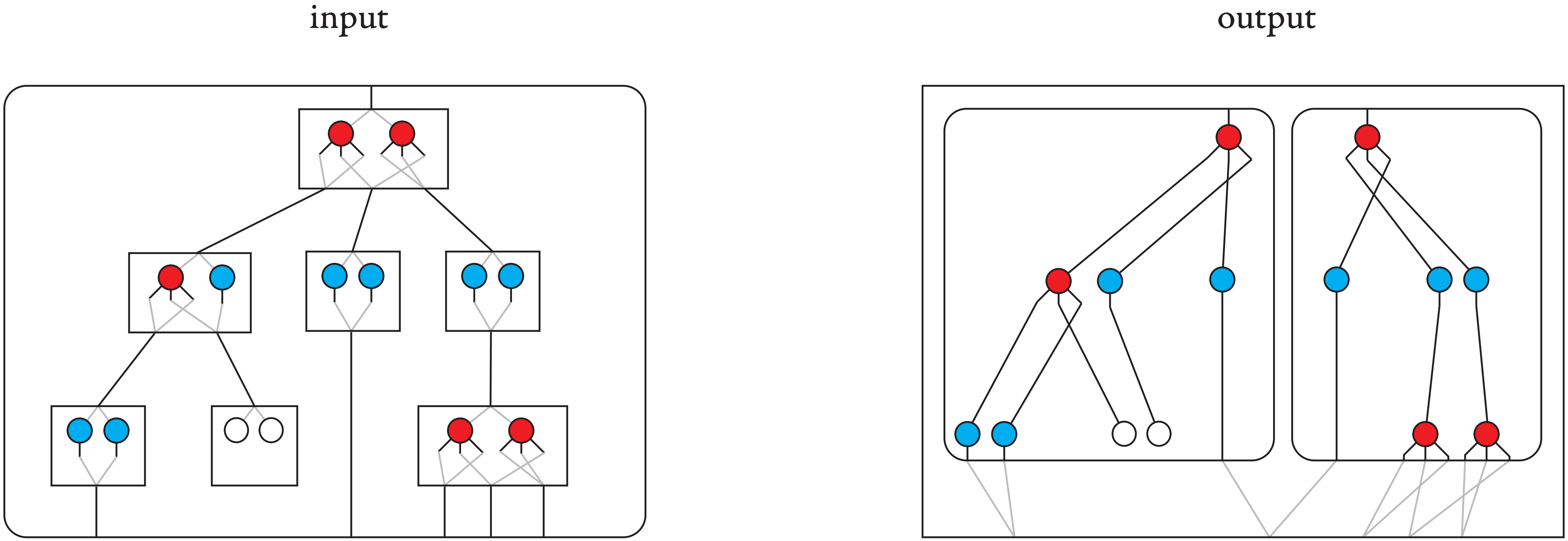

r

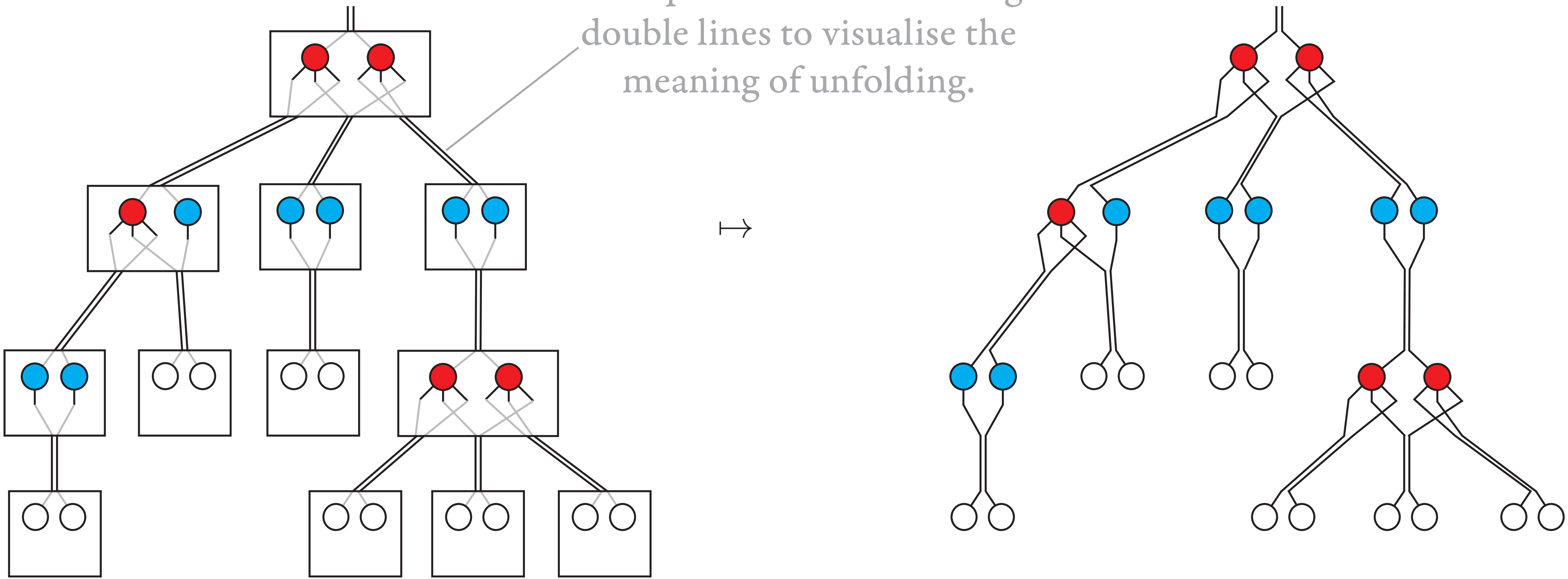
▨ factors without branching nodes

🟥 factors with branching nodes

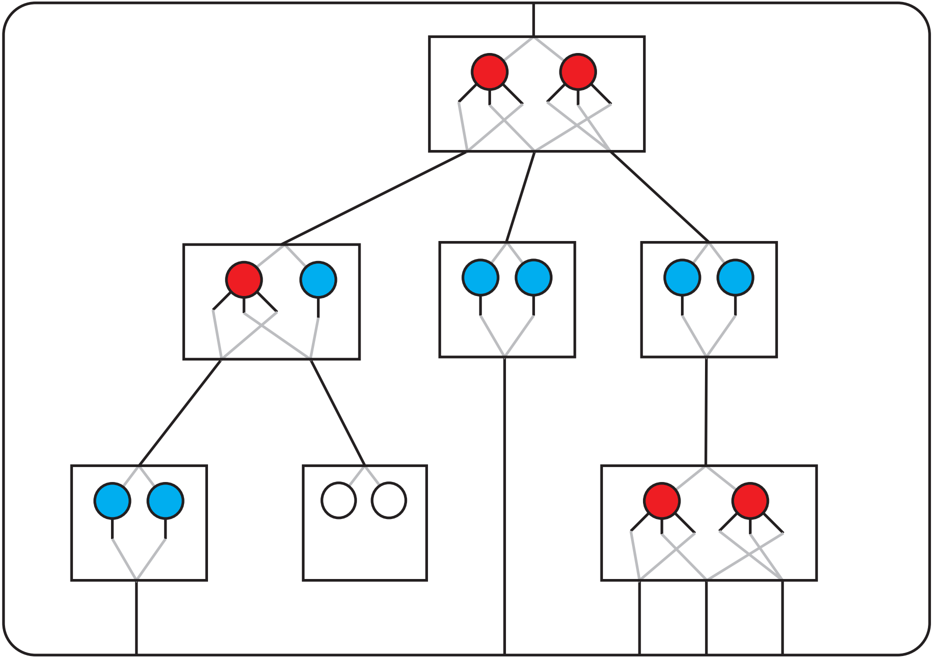


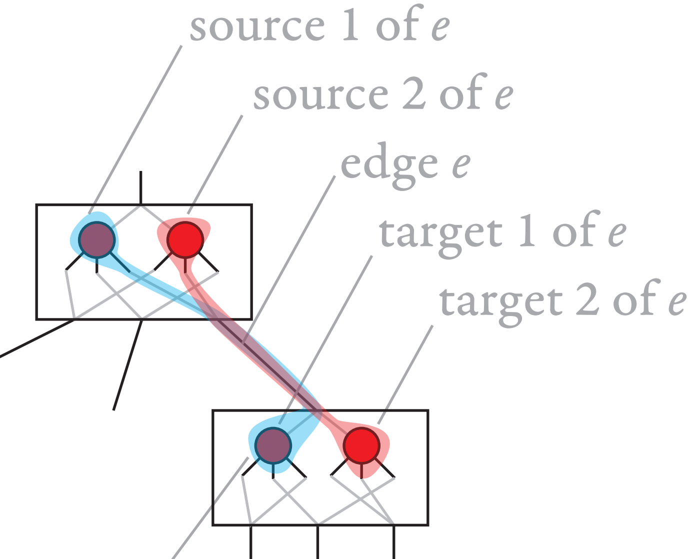


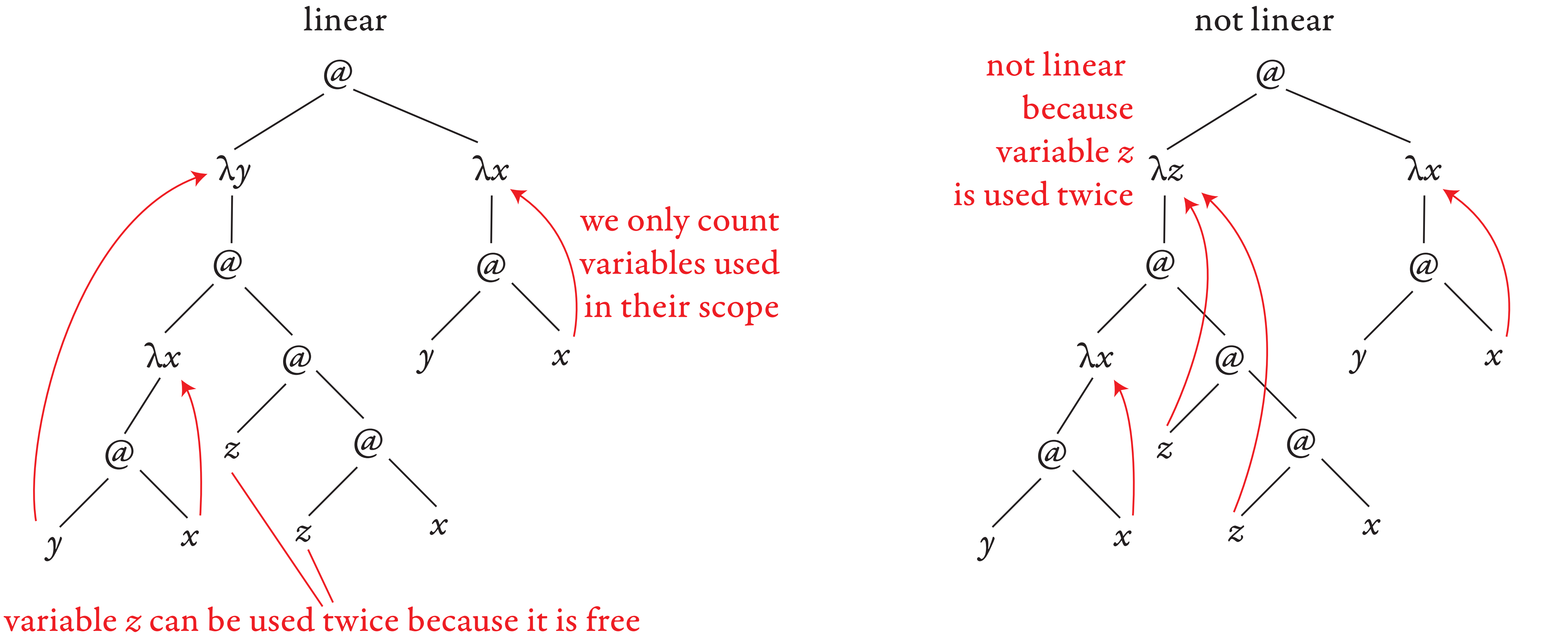

input
output



the parent-child relation in the input tree is drawn using double lines to visualise the meaning of unfolding.




source 1 of *e*
source 2 of *e*
edge *e*
target 1 of *e*
target 2 of *e*



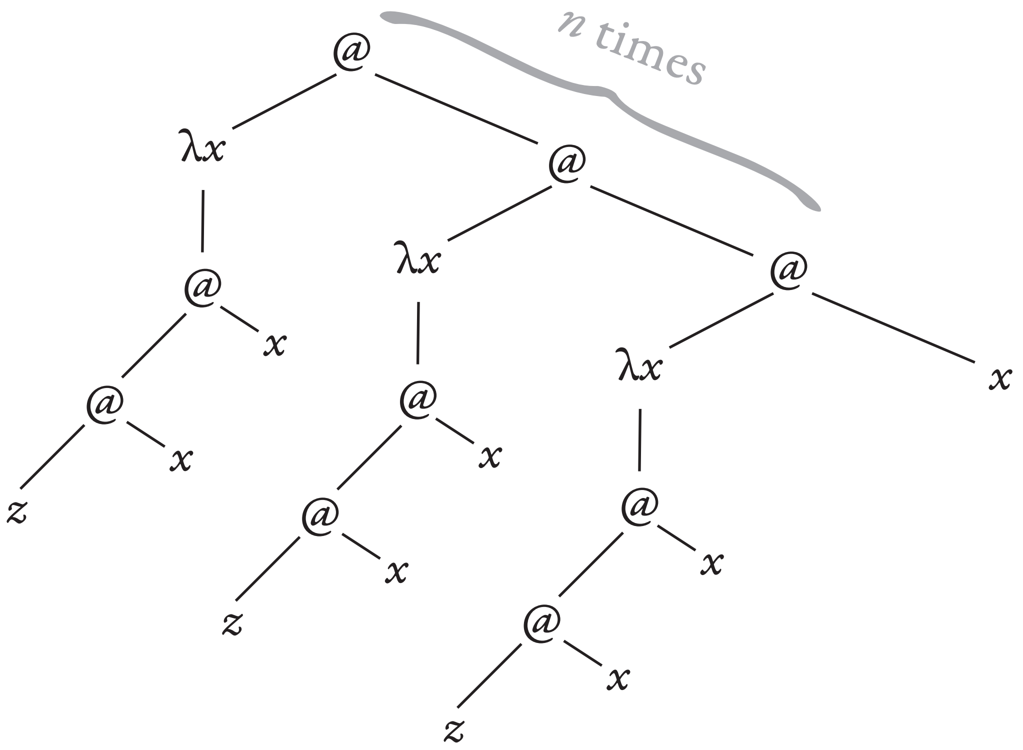
linear

not linear

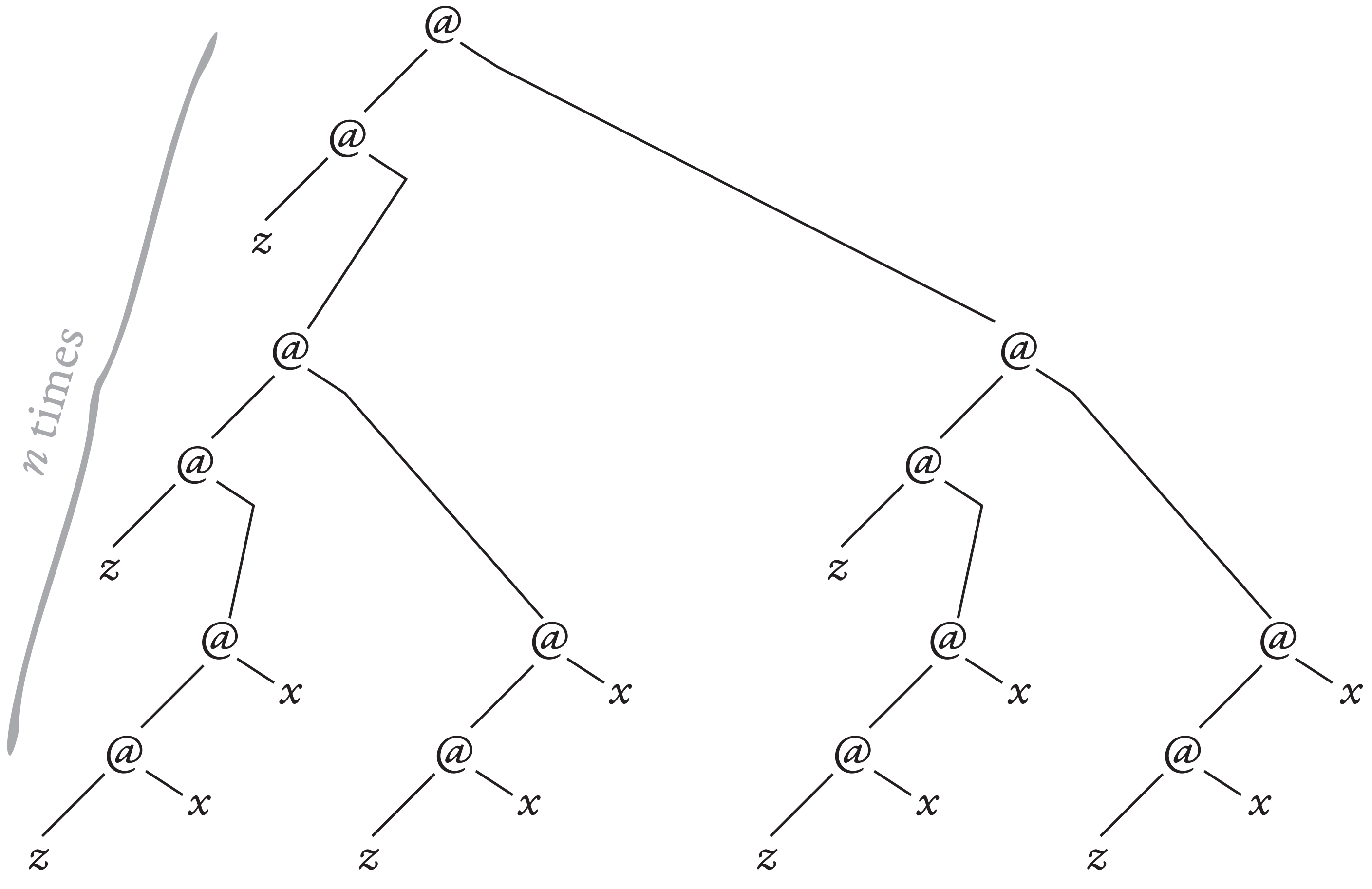
not linear because variable z is used twice

we only count variables used in their scope

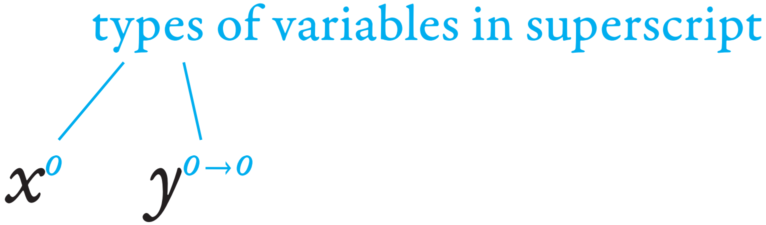
variable z can be used twice because it is free a λ-term of size $O(n)$



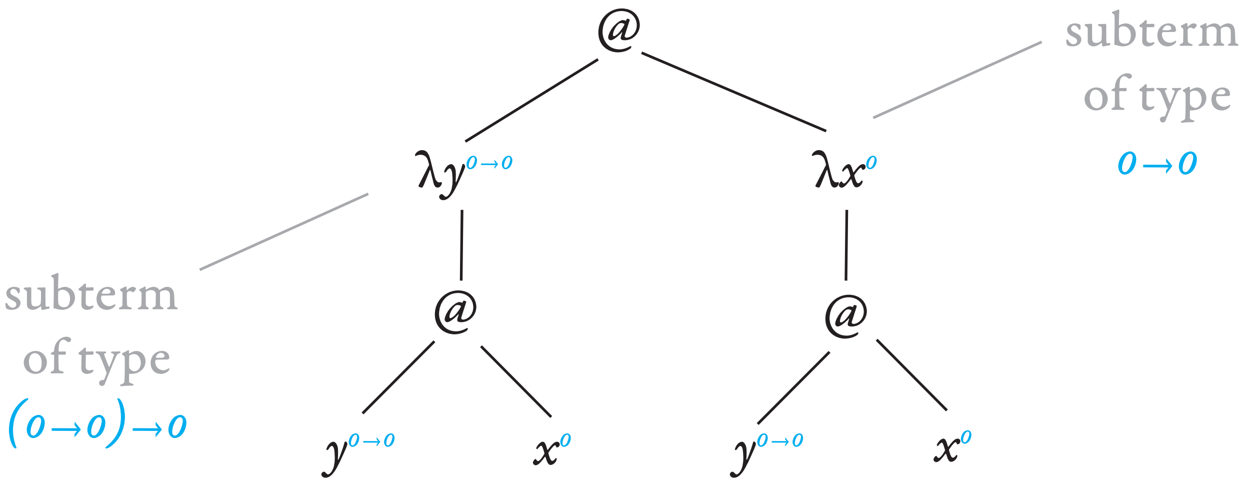
its normal form of size $O(2^n)$



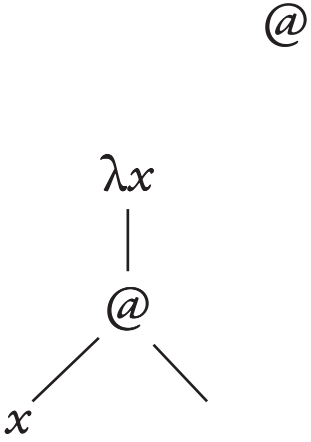
variables


types of variables in superscript

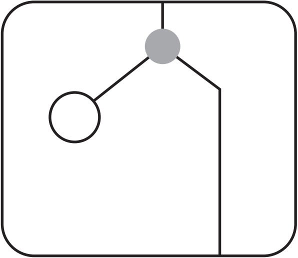
λ-term of type o

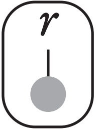

subterm of type $(o\to o)\to o$

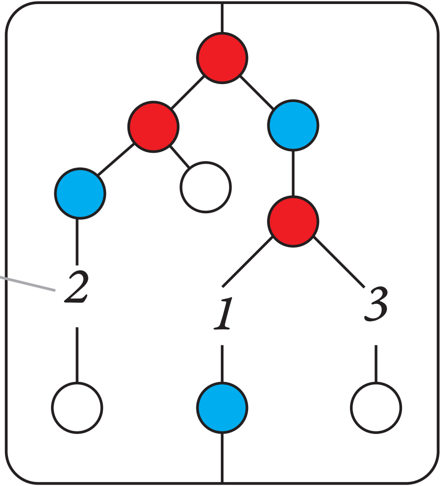
subterm of type $o\to o$



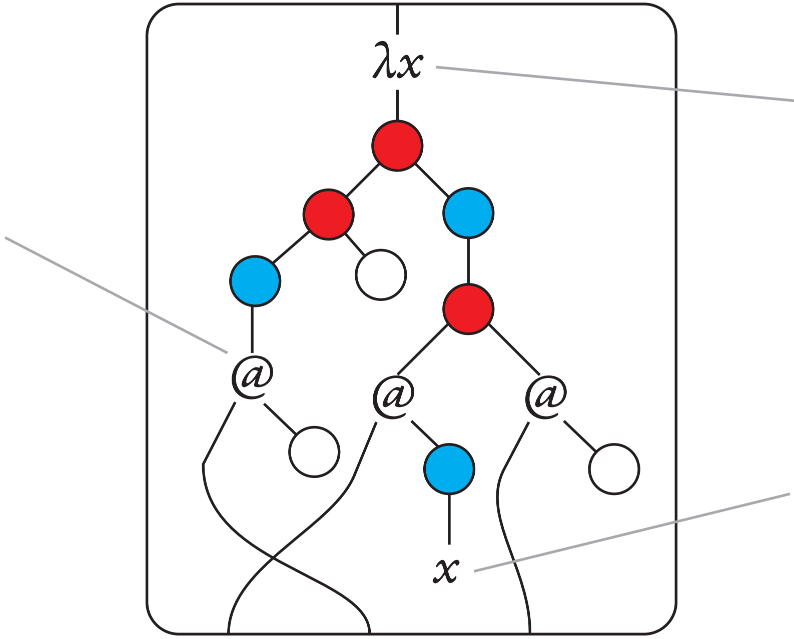
$\lambda x.$




r

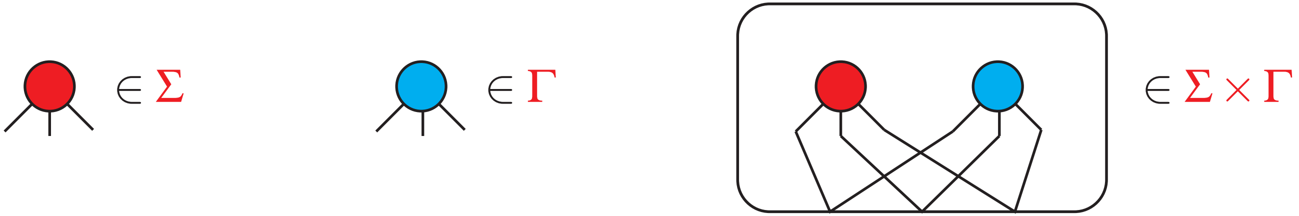


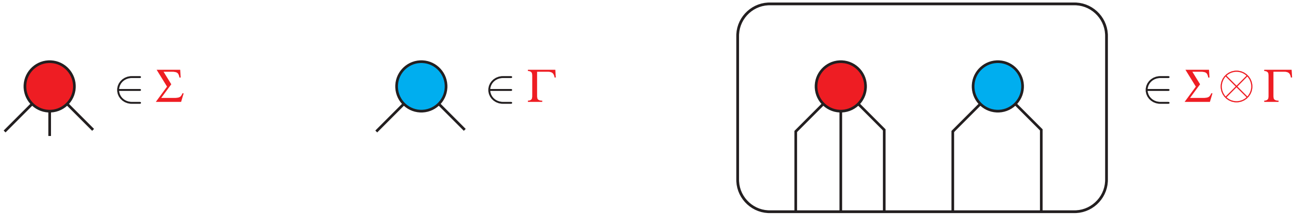
placeholder for the term stored in the unique register of the 2nd child



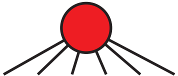
variable *x* is bound in the root

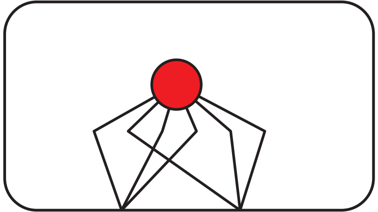
the original port is replaced by *x*

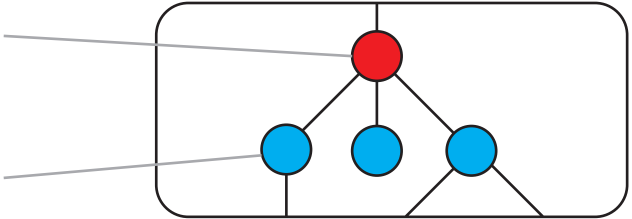

∈ Σ
∈ Γ
∈ Σ × Γ



$\in \Sigma$
$\in \Gamma$
$\in \Sigma \otimes \Gamma$

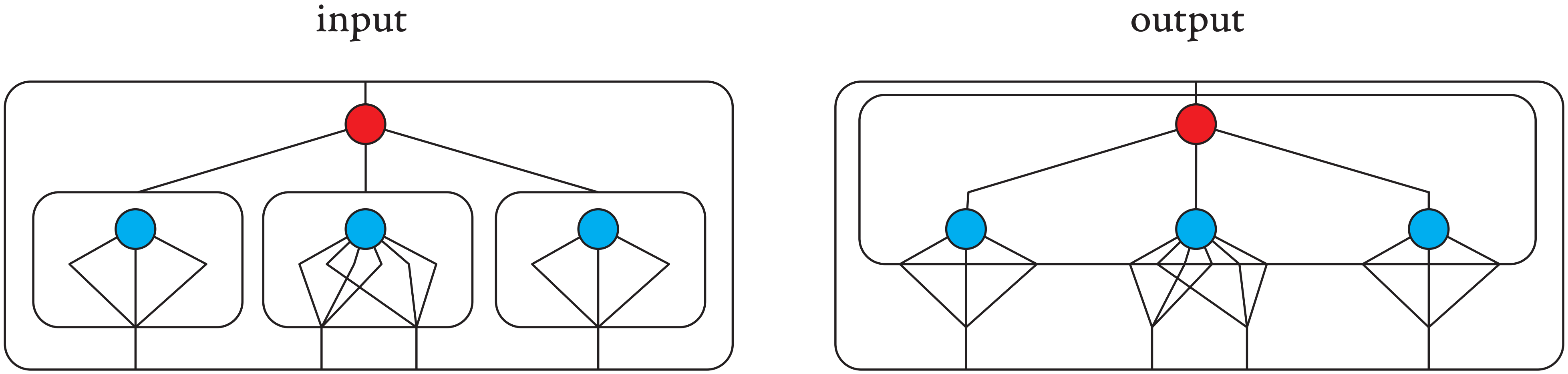
 $\in \Sigma$

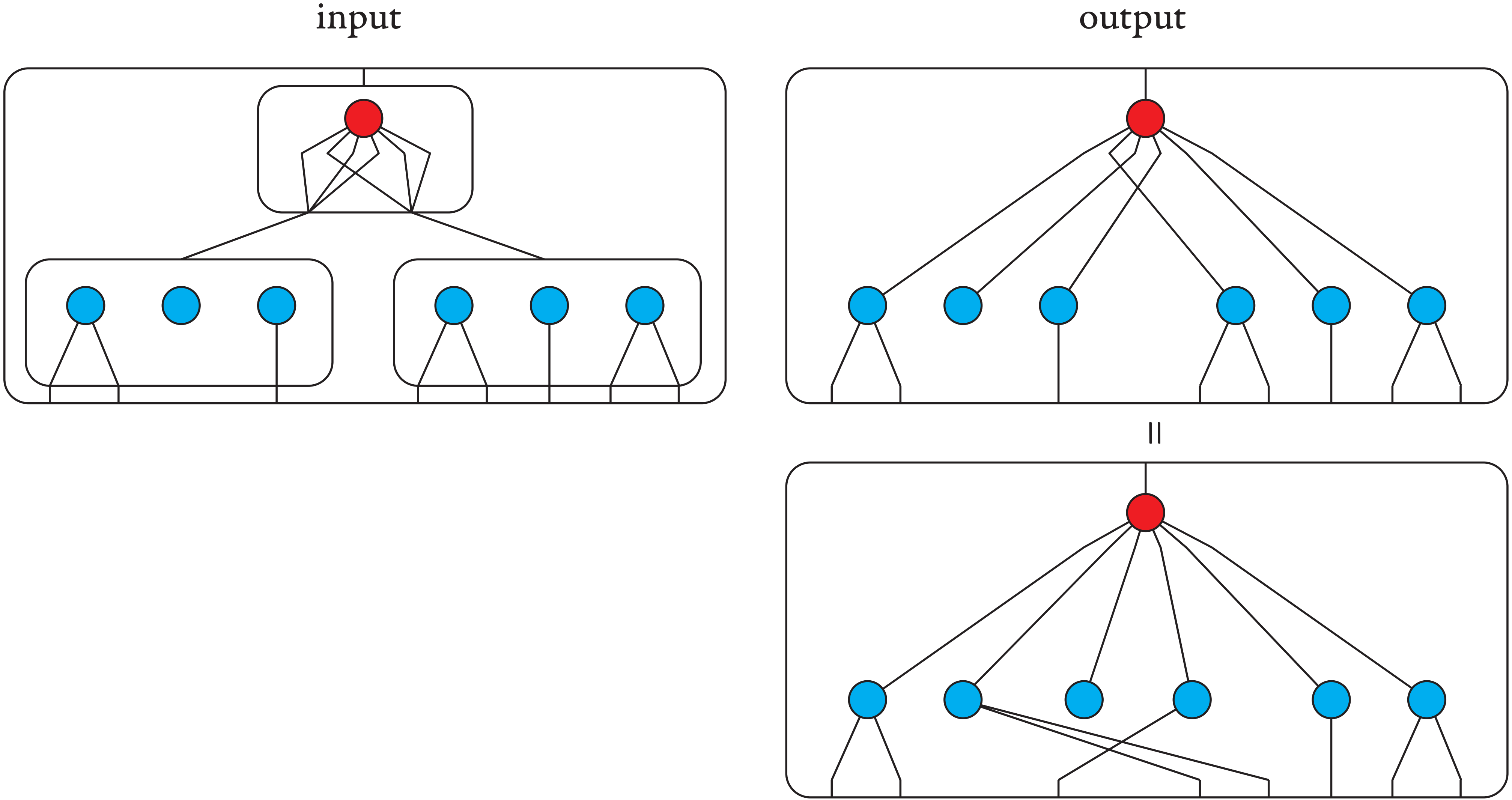
 $\in F_3\Sigma$



the root is from Σ

all children are from Γ


input
output


input
output
=