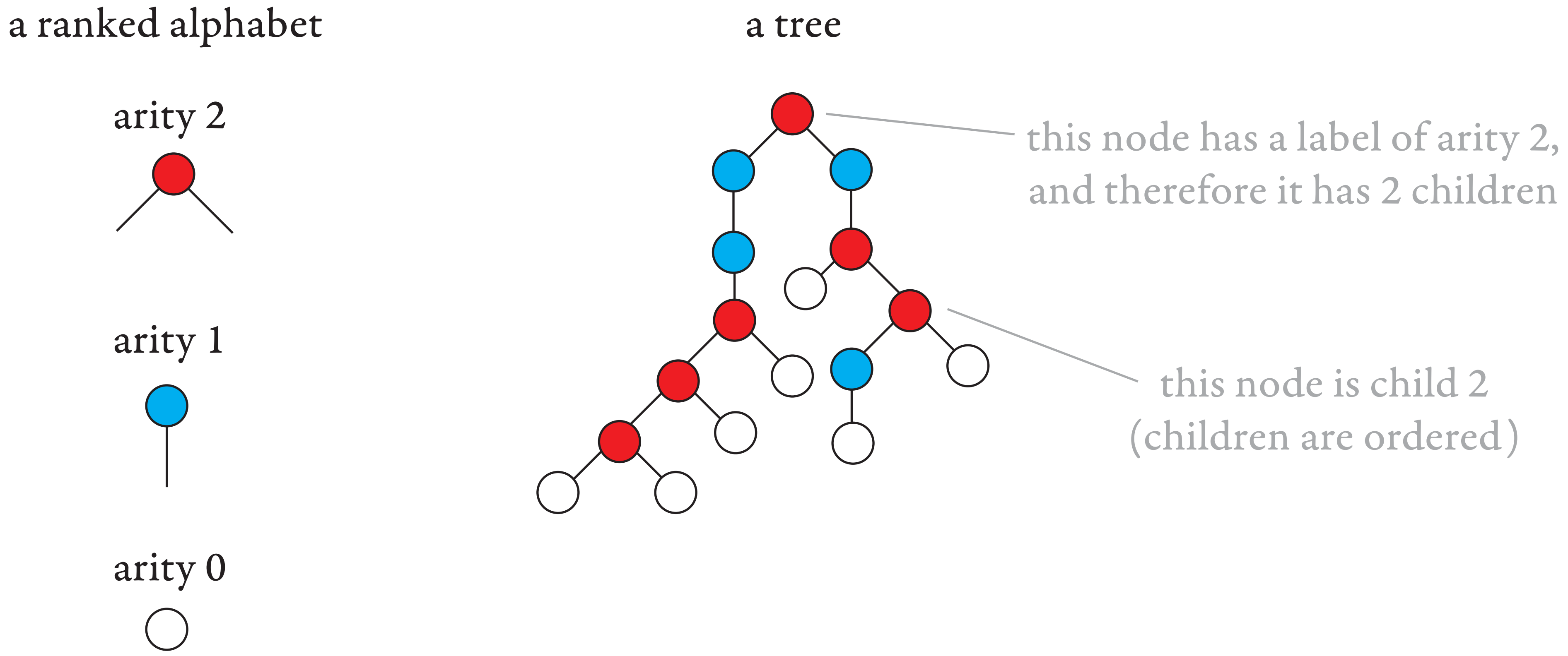
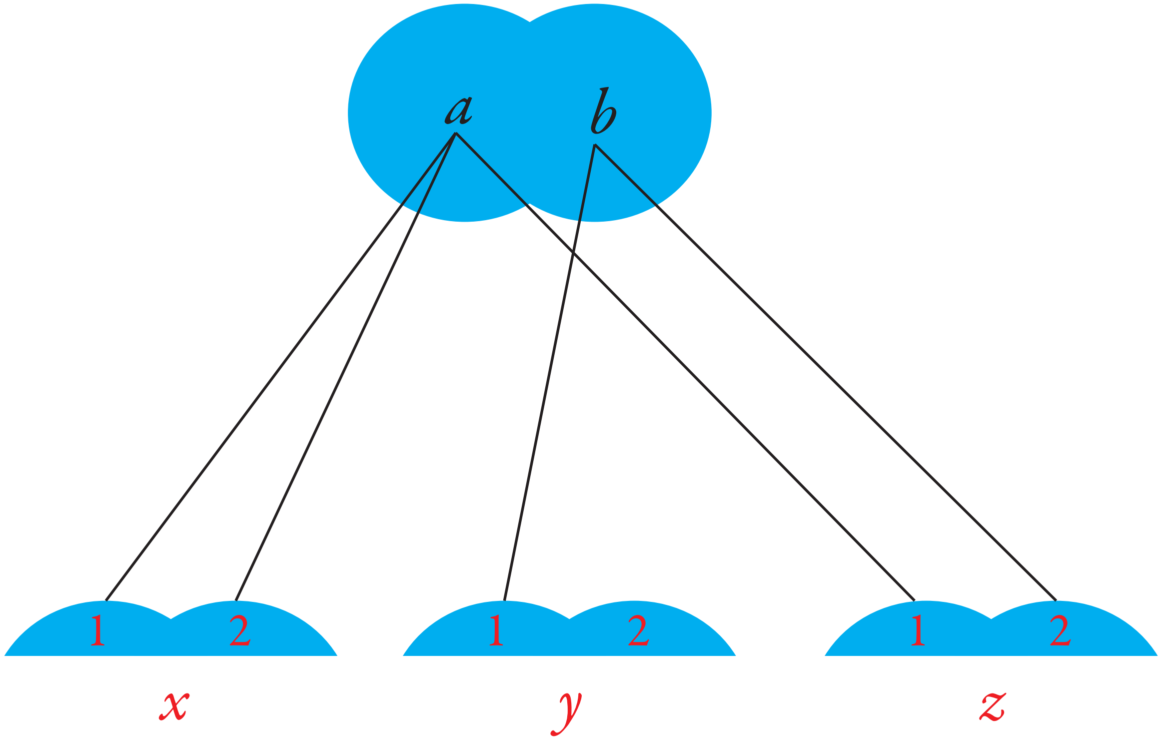
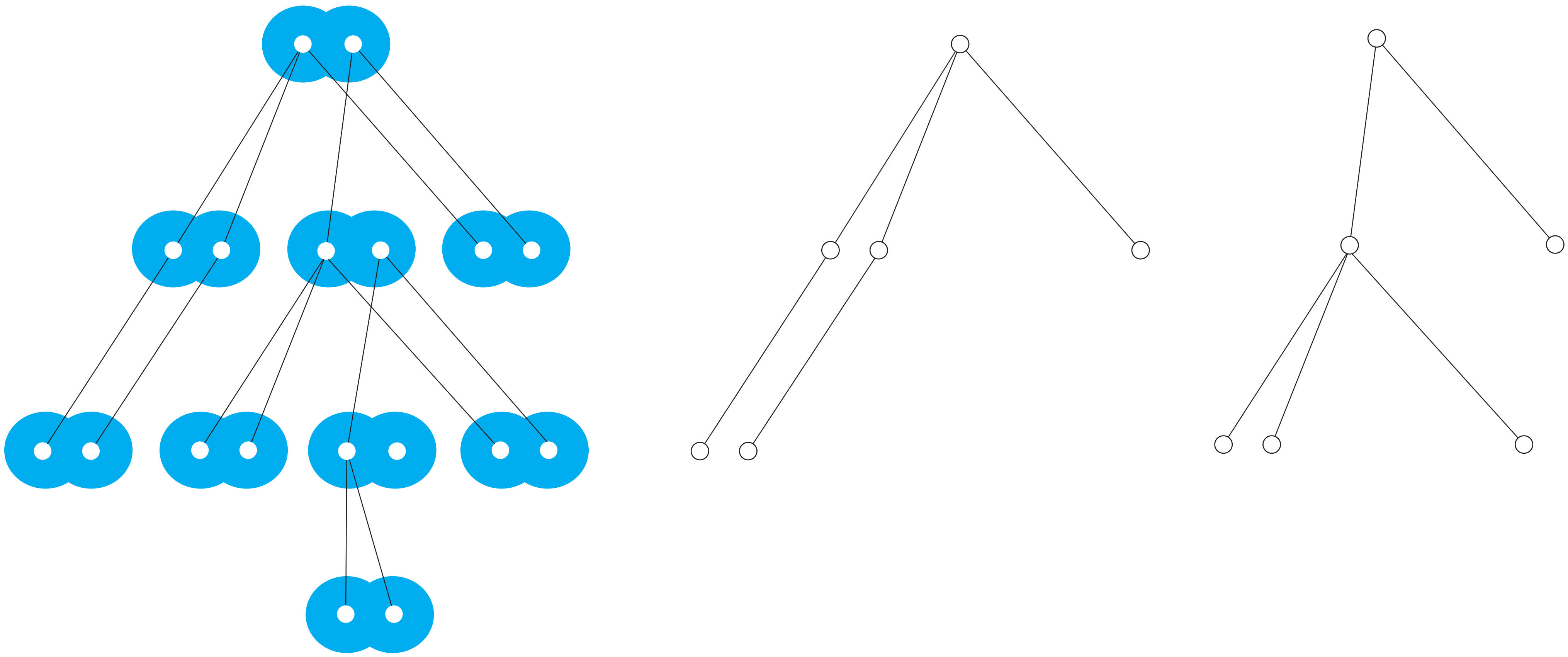
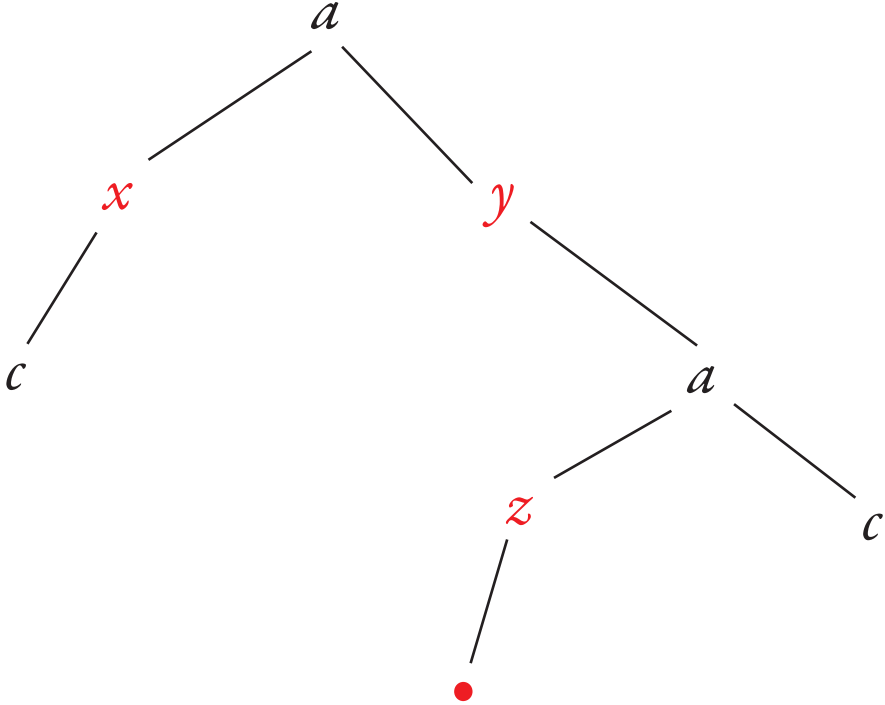

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2,
and therefore it has 2 children
this node is child 2
(children are ordered)

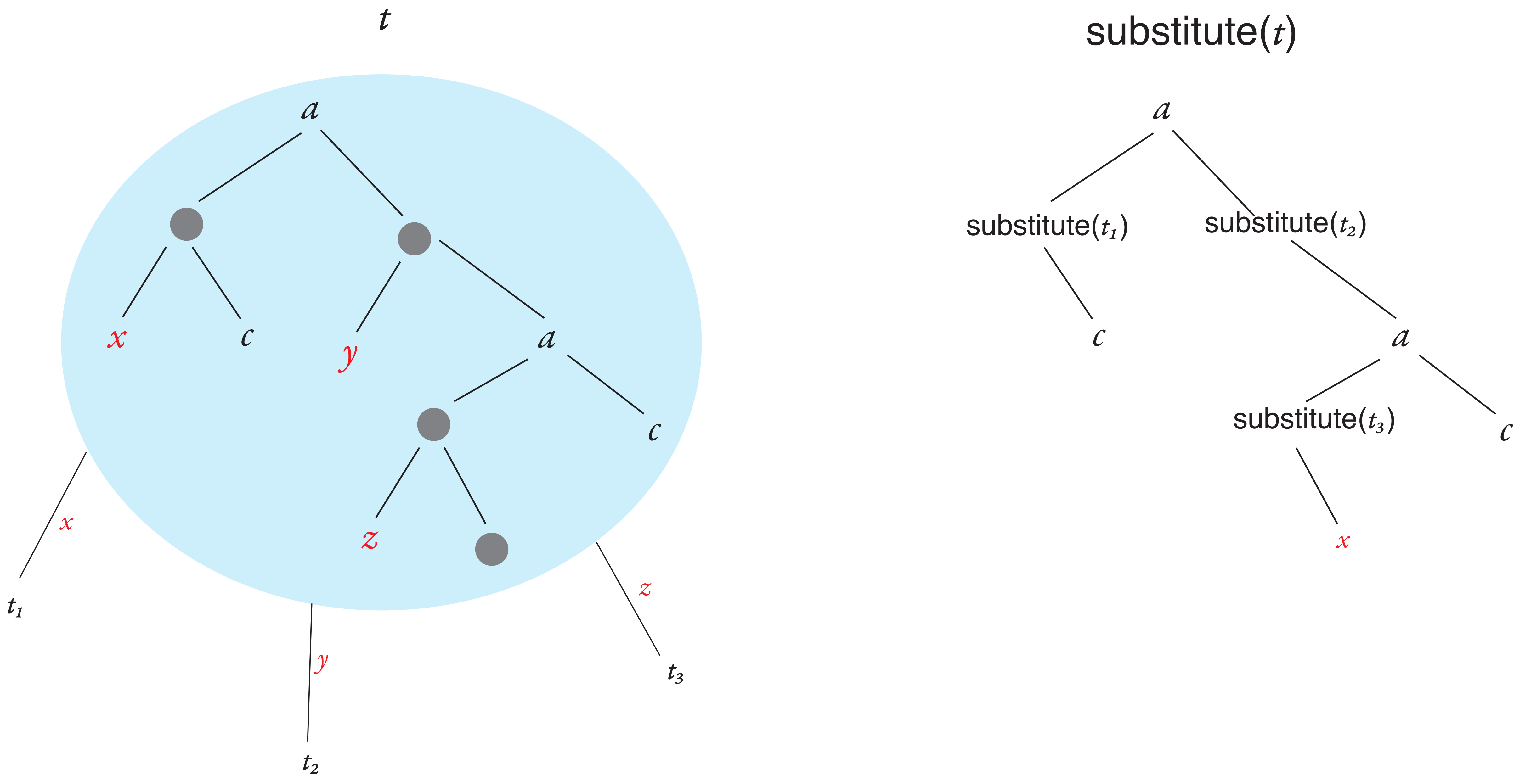


a
b
1
2
1
2
1
2
x
y
z

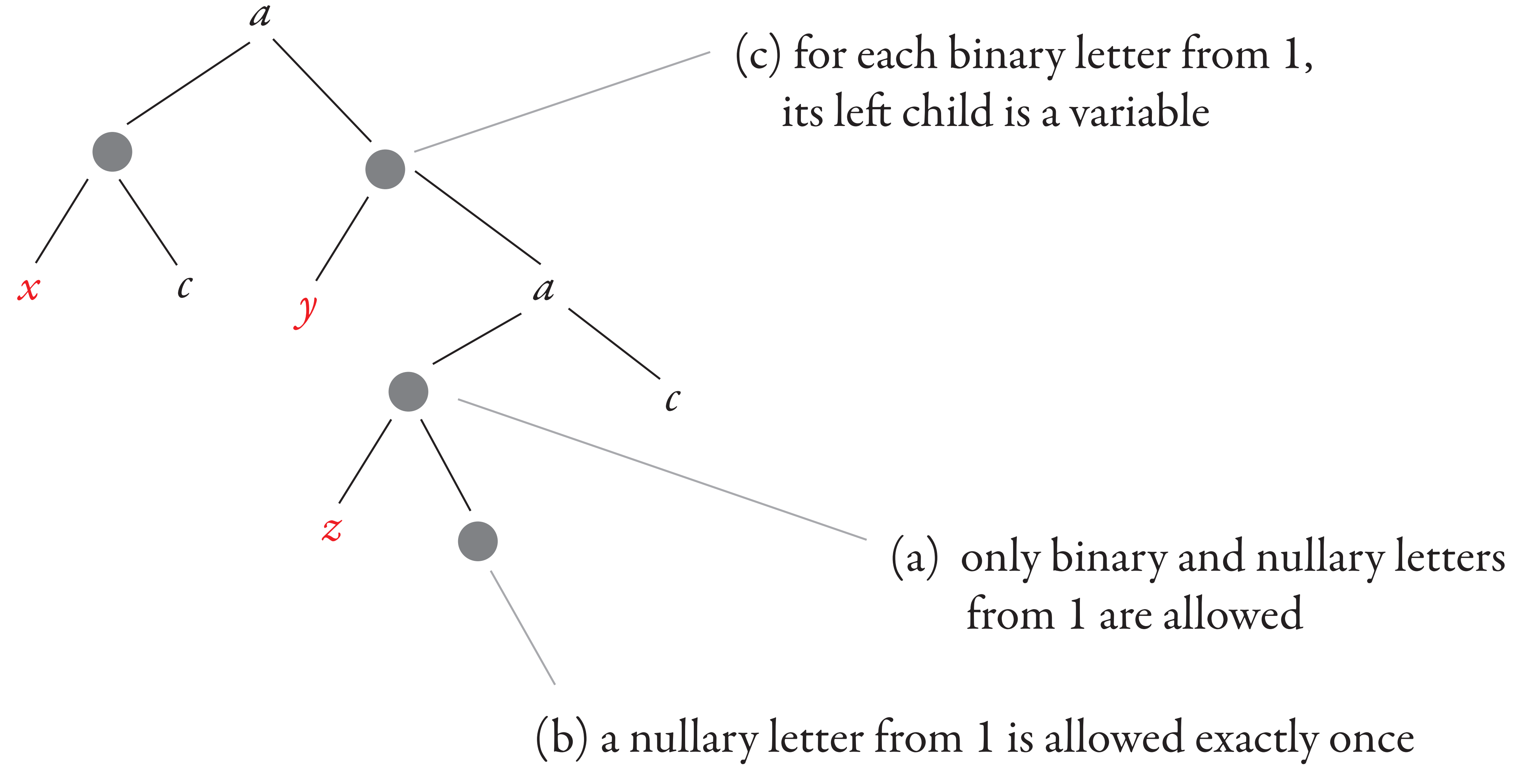
A tree t over $\Sigma^{[2]}$ $\quad$ $\mathsf{unfold}_1(t)$ $\quad$ $\mathsf{unfold}_2(t)$



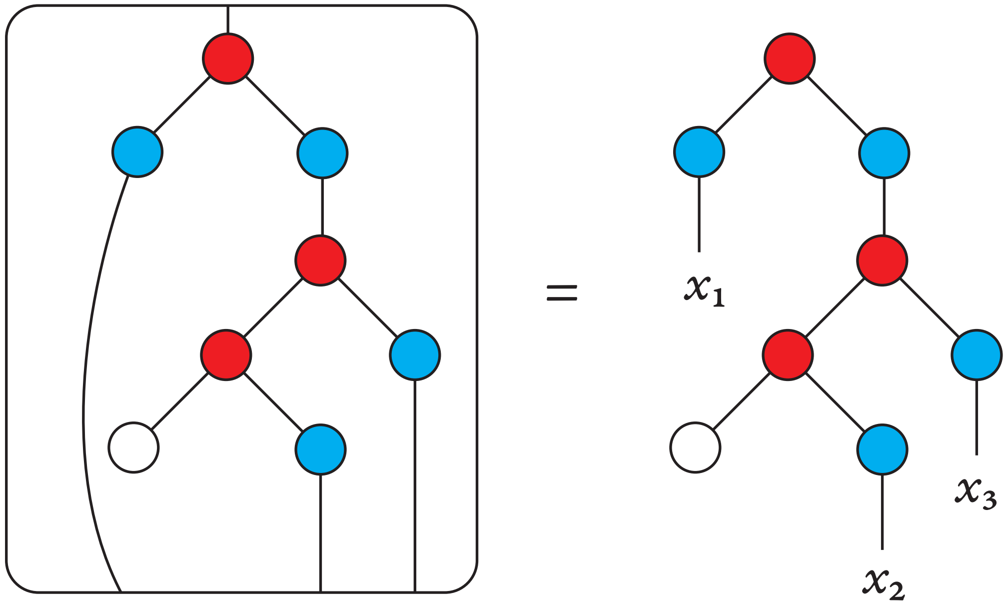

a
x
y
c
a
z
c



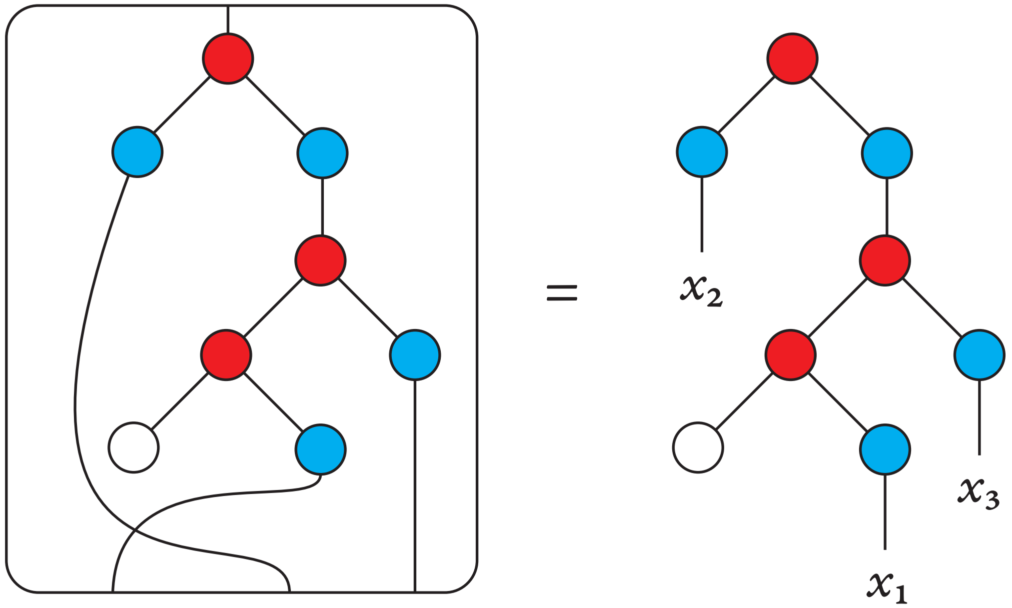
t
a
x
c
y
a
c
z
x
y
z
t1
t2
t3
substitute(t)
a
substitute(t1)
substitute(t2)
c
a
substitute(t3)
c
x



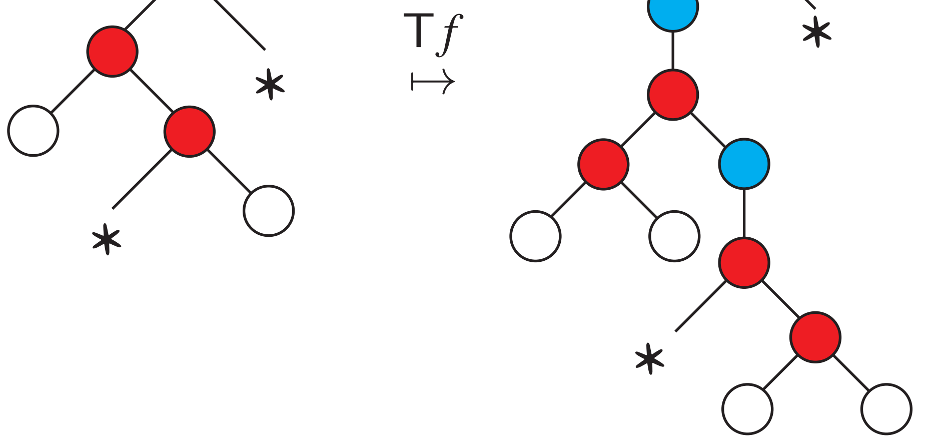
a
(c) for each binary letter from 1,
its left child is a variable
x
c
y
a
c
z
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

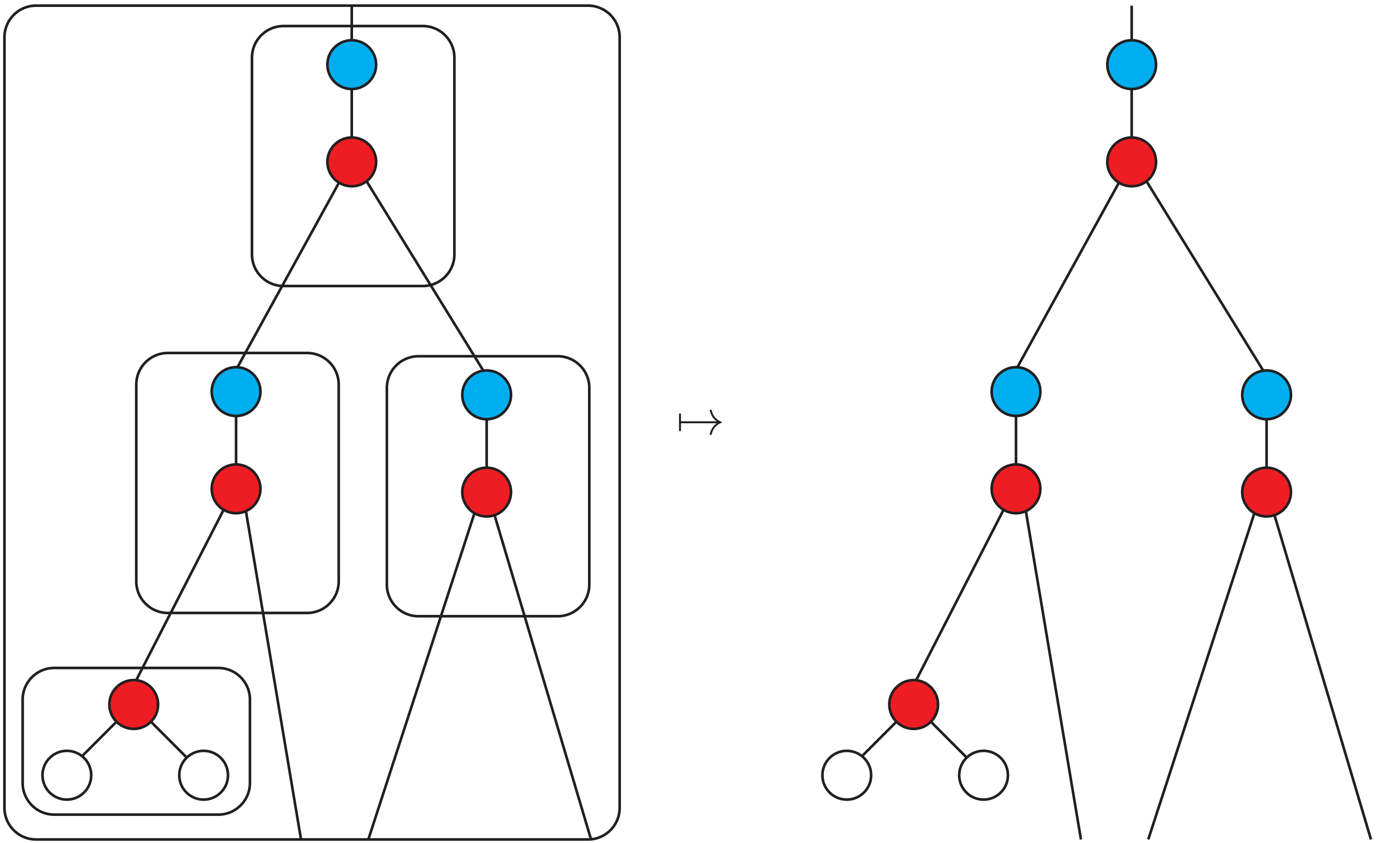


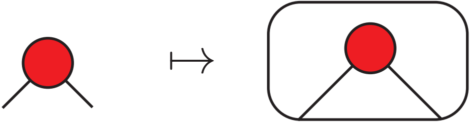
=
x_1
x_2
x_3

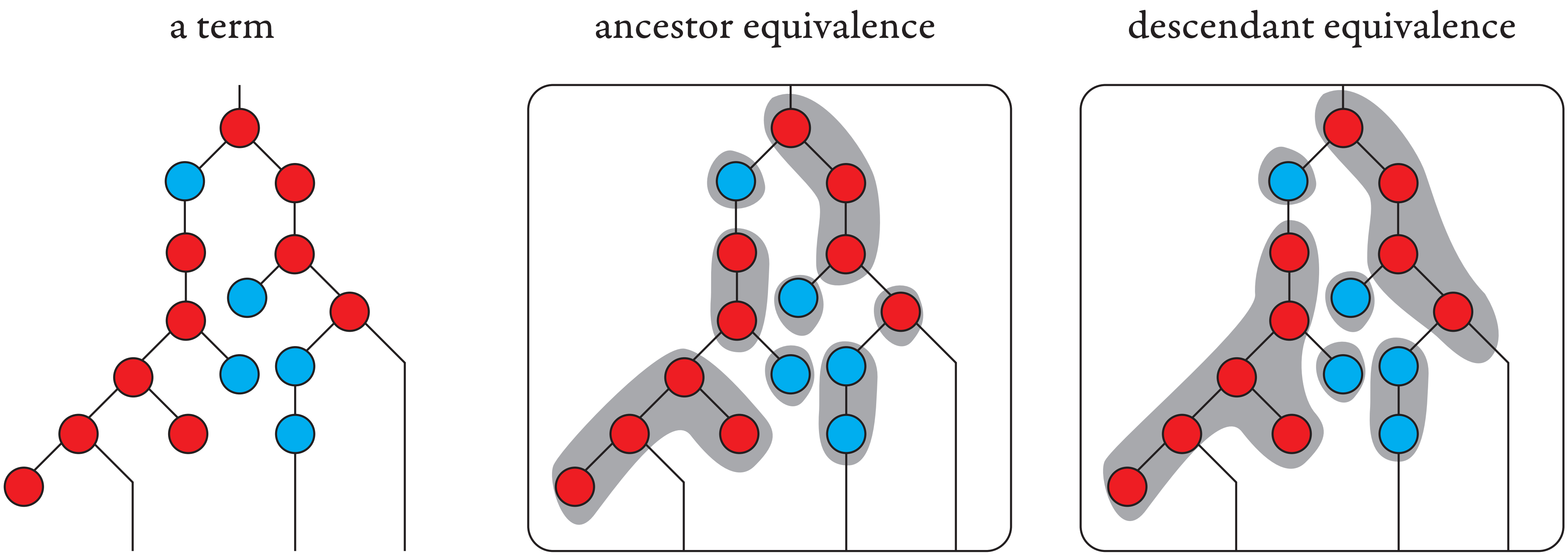


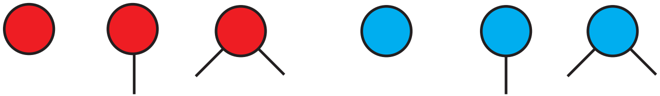
=
x_2
x_1
x_3

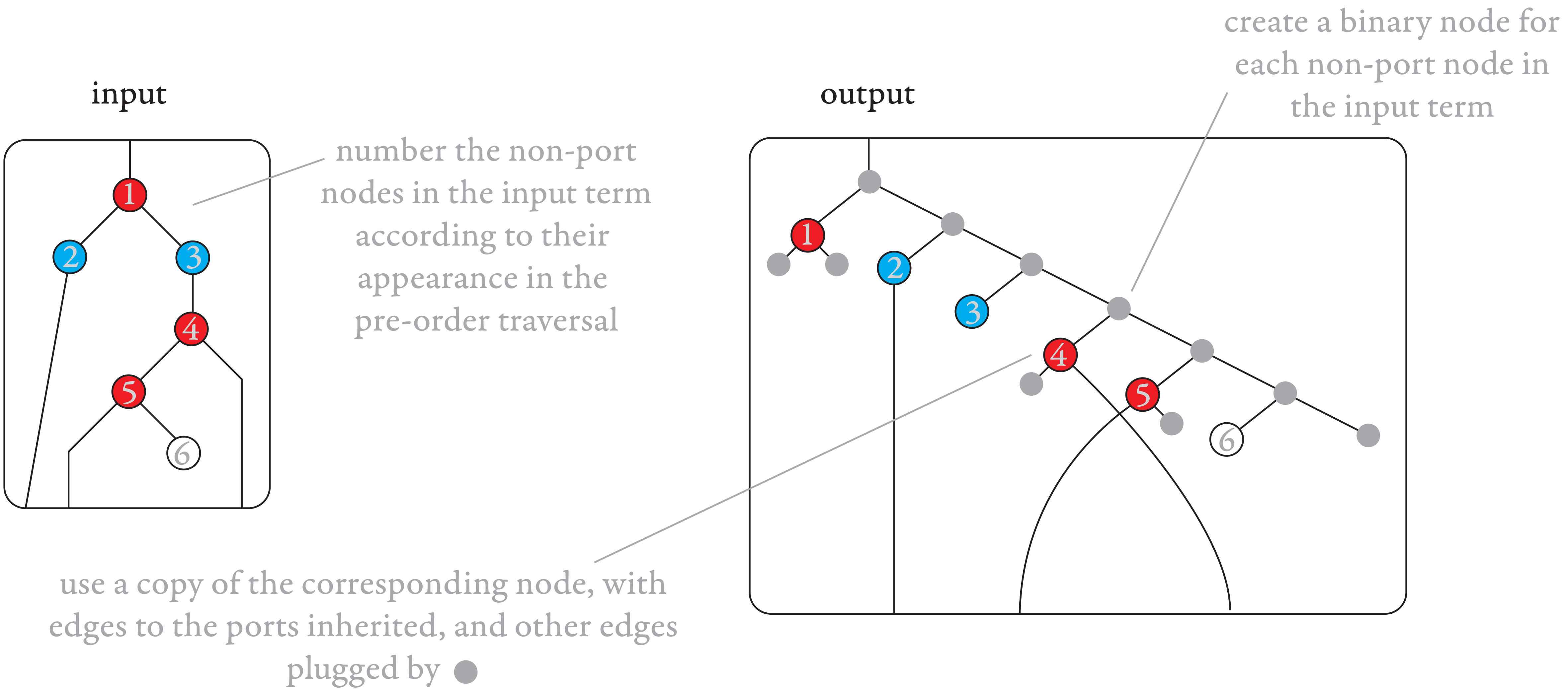

$\mathsf{T}f$
$\mapsto$



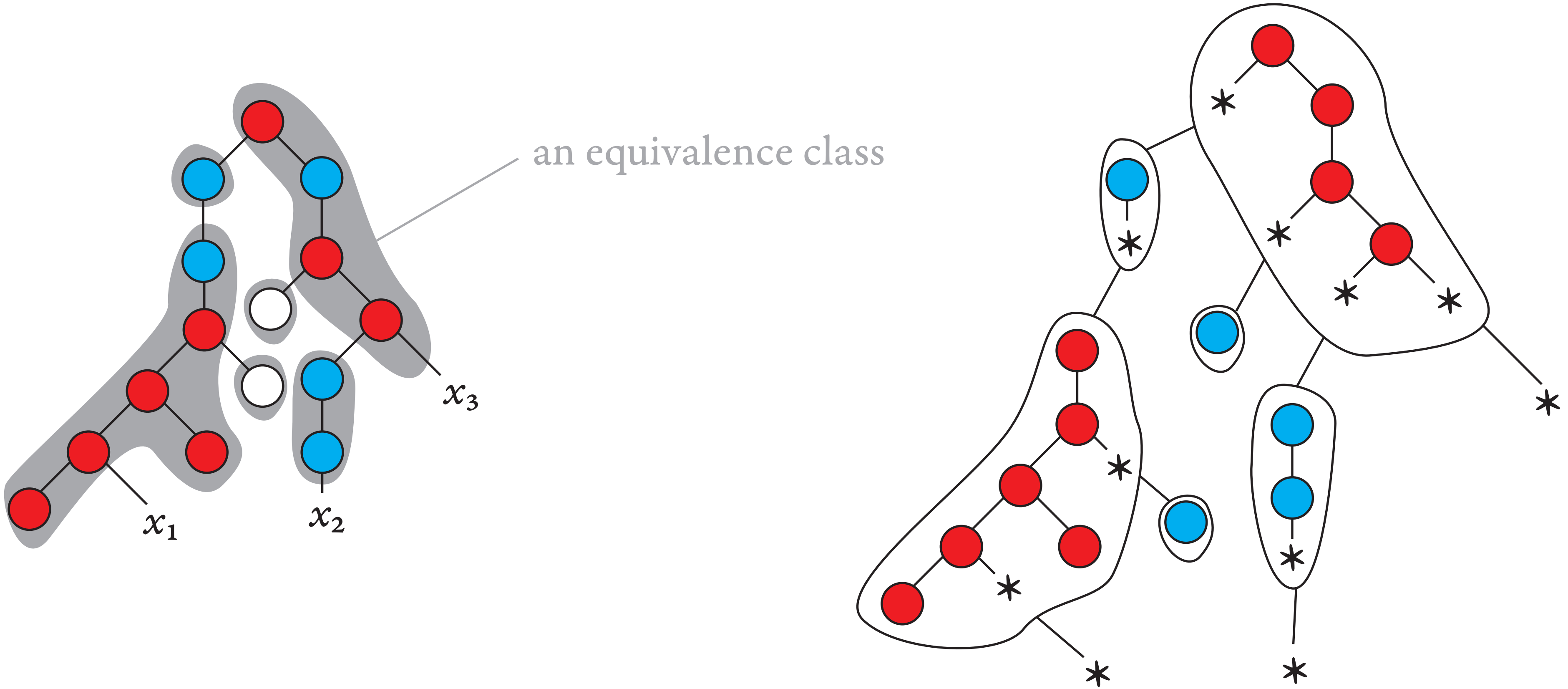


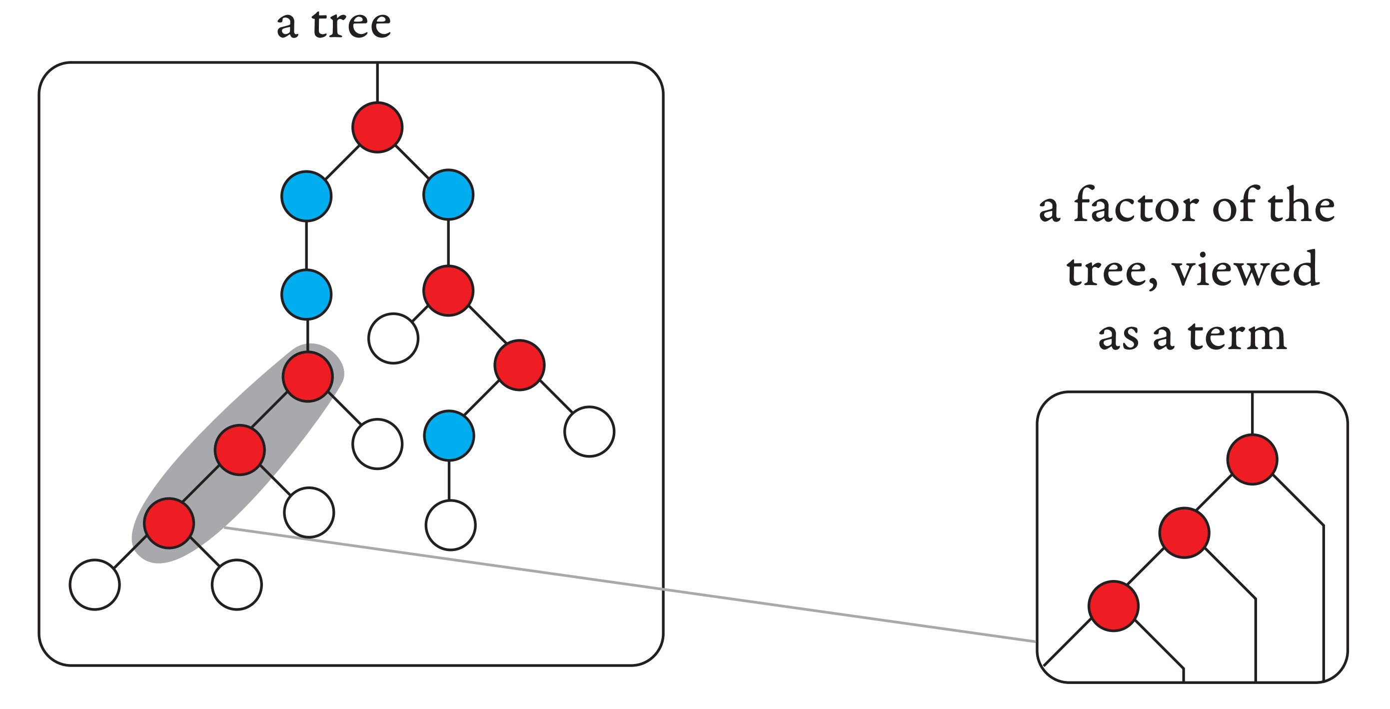

a term
ancestor equivalence
descendant equivalence



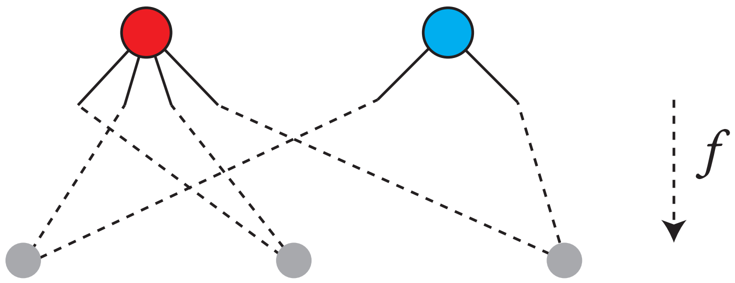

input
1
2
3
4
5
6
output
1
2
3
4
5
6
number the non-port nodes in the input term according to their appearance in the pre-order traversal
create a binary node for each non-port node in the input term
use a copy of the corresponding node, with edges to the ports inherited, and other edges plugged by

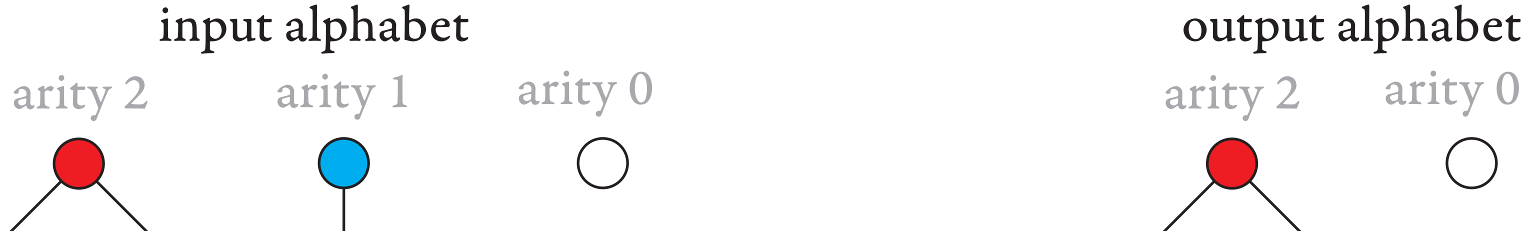
a factorisation equivalence

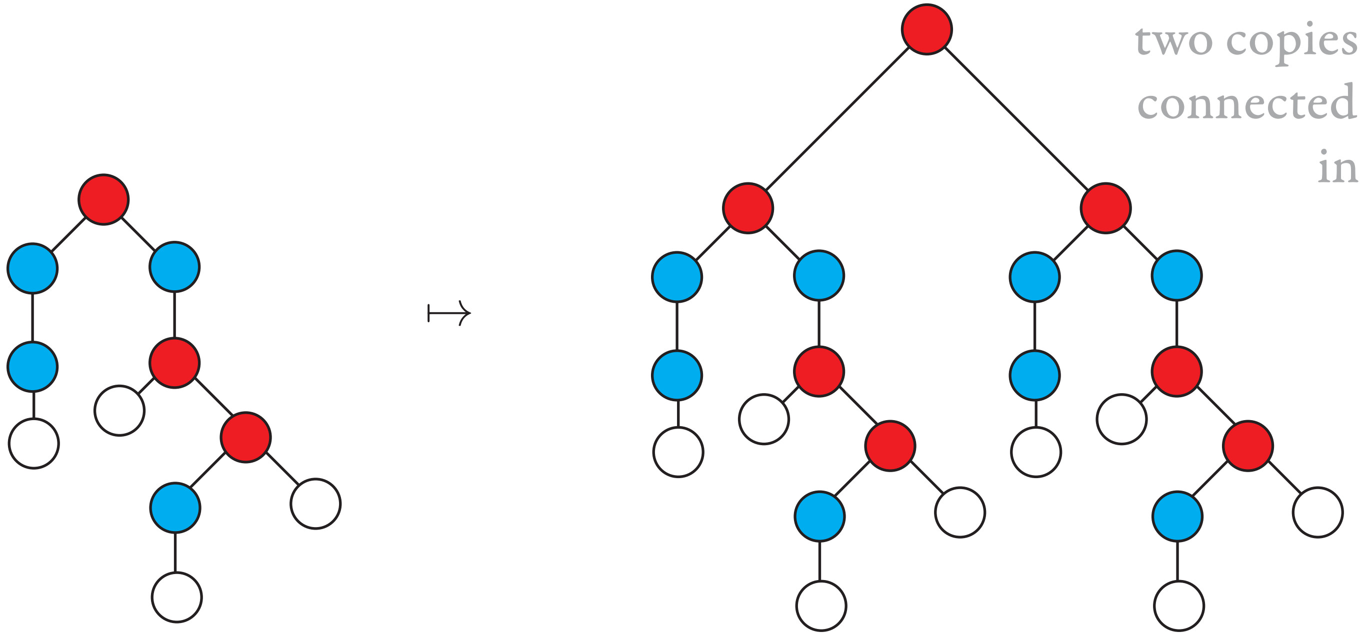




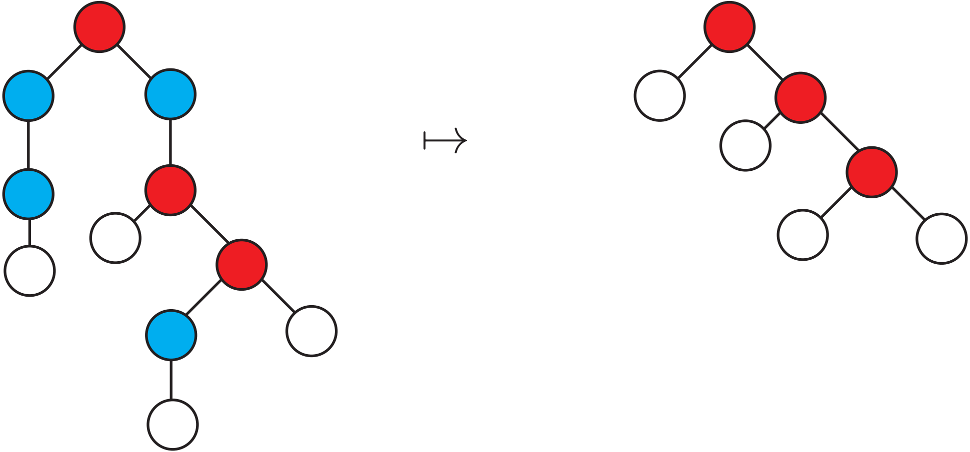
a tree
a factor of the tree, viewed as a term

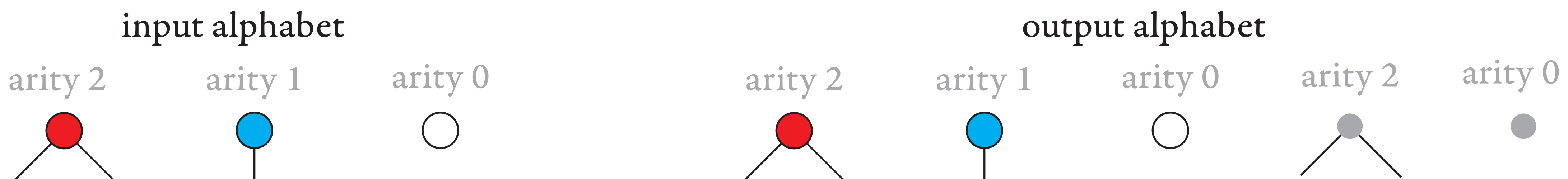

f

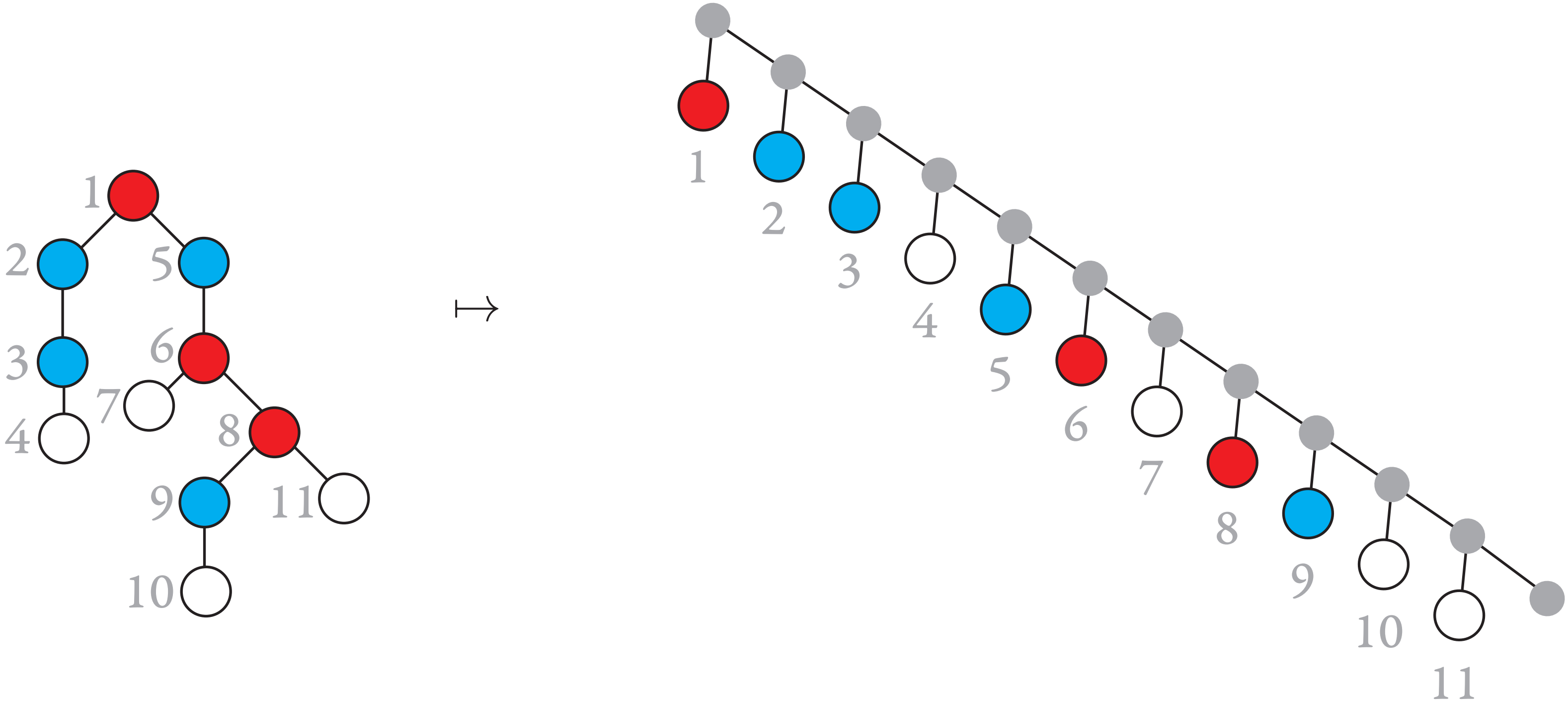

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0



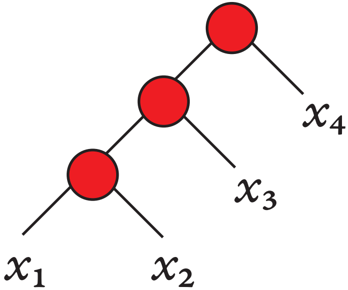
two copies of the input tree,
connected by a binary node
in the root



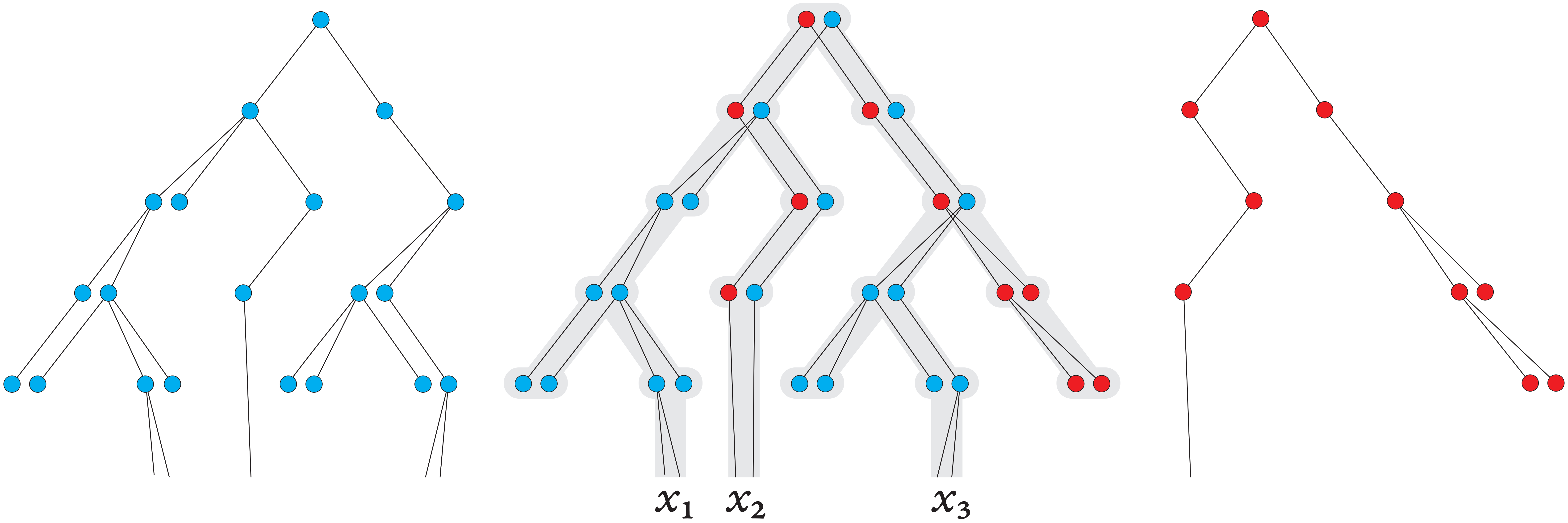

input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 0


1
2
3
4
5
6
7
8
9
10
11
↦
1
2
3
4
5
6
7
8
9
10
11

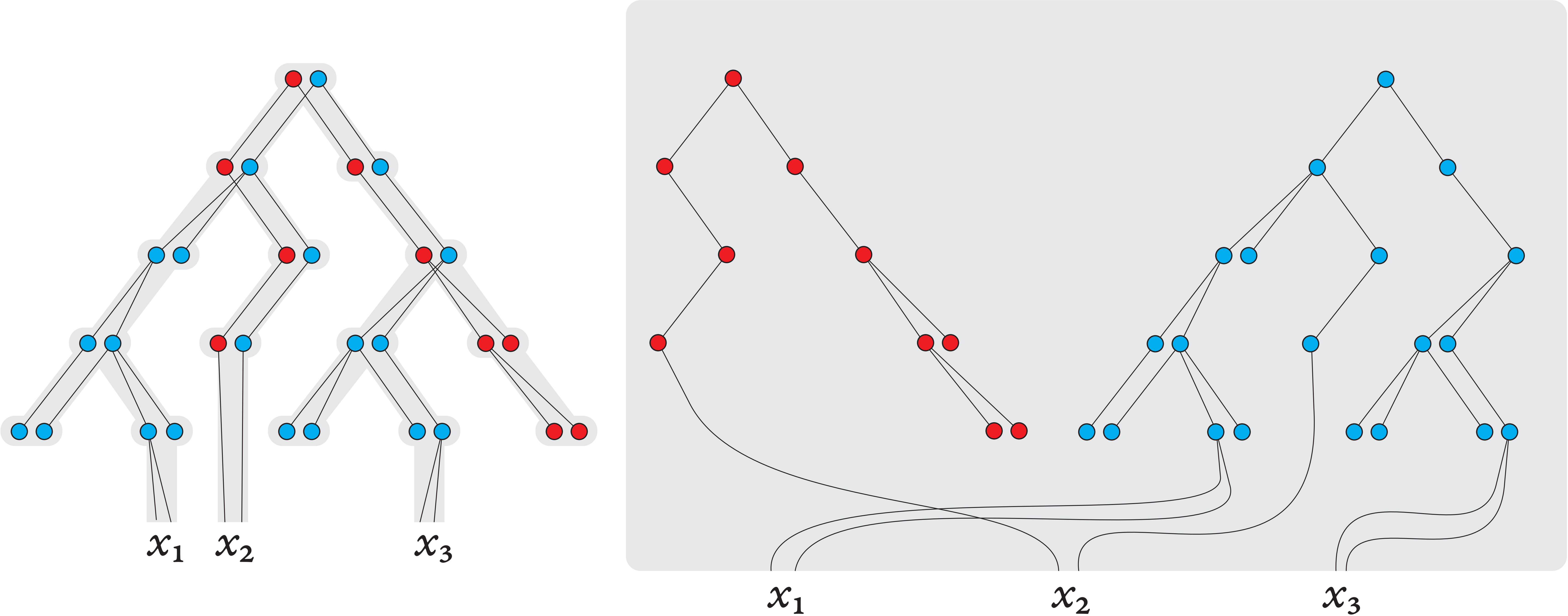
a term of arity 4

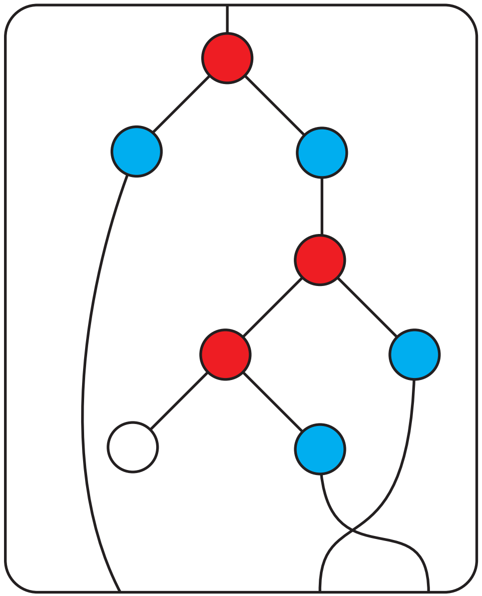


a term of arity 0



x_1
x_2
x_3

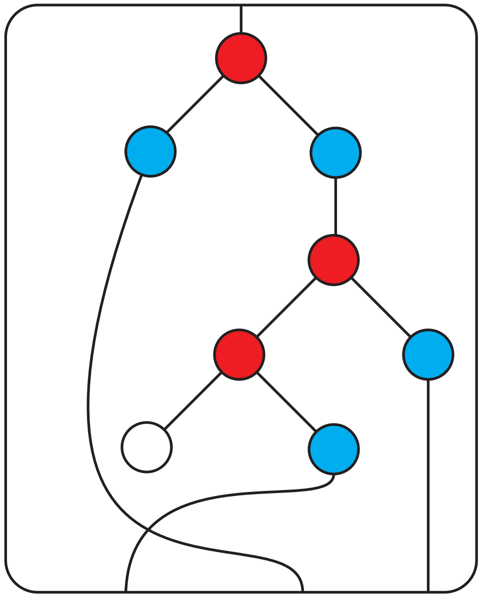

x_1
x_2
x_3
x_1
x_2
x_3



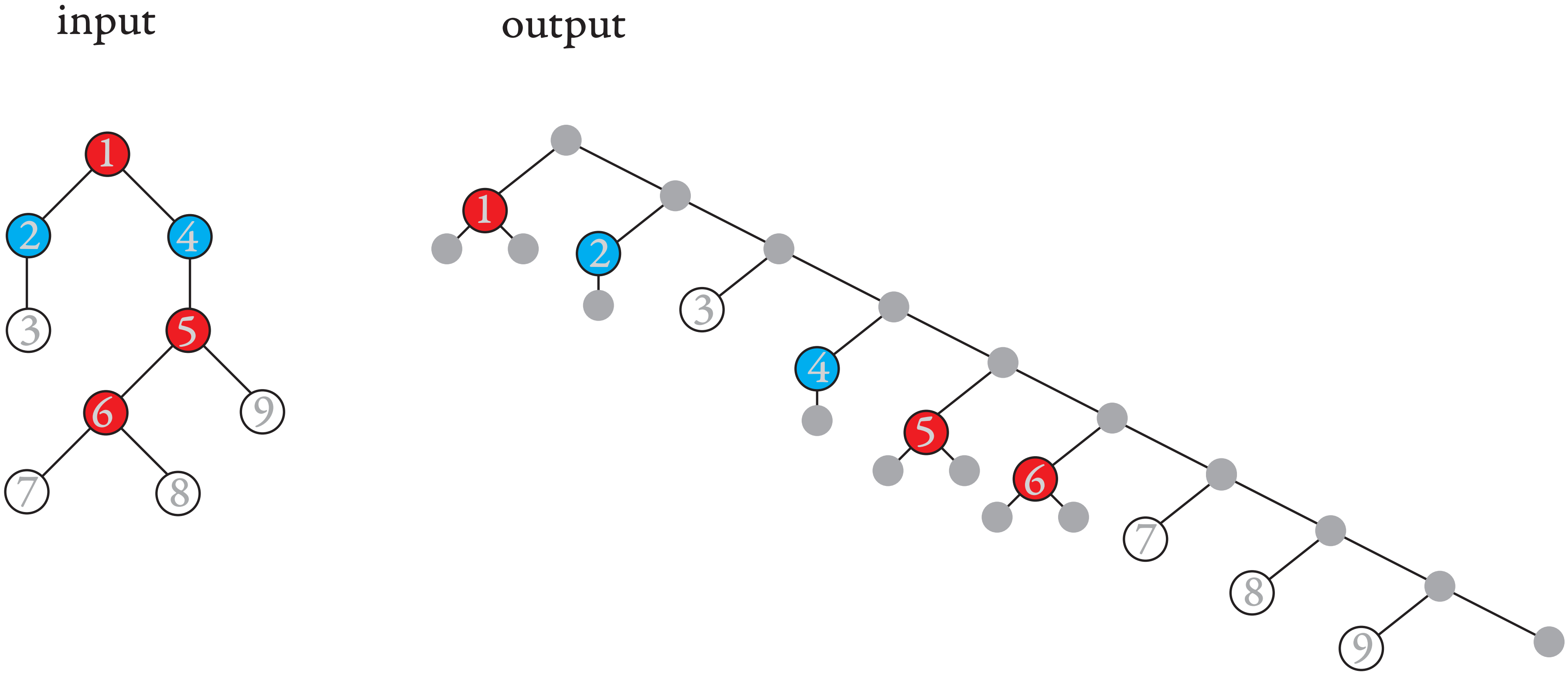
satisfies (*)

(*)

If the root has arity n,
and $1 \leq i < j \leq n$, then
all ports of the j-th
subterm of the root are
after all ports of the
i-th subterm of the root



violates (*)


input
output
1
2
3
4
5
6
7
8
9
1
2
3
4
5
6
7
8
9