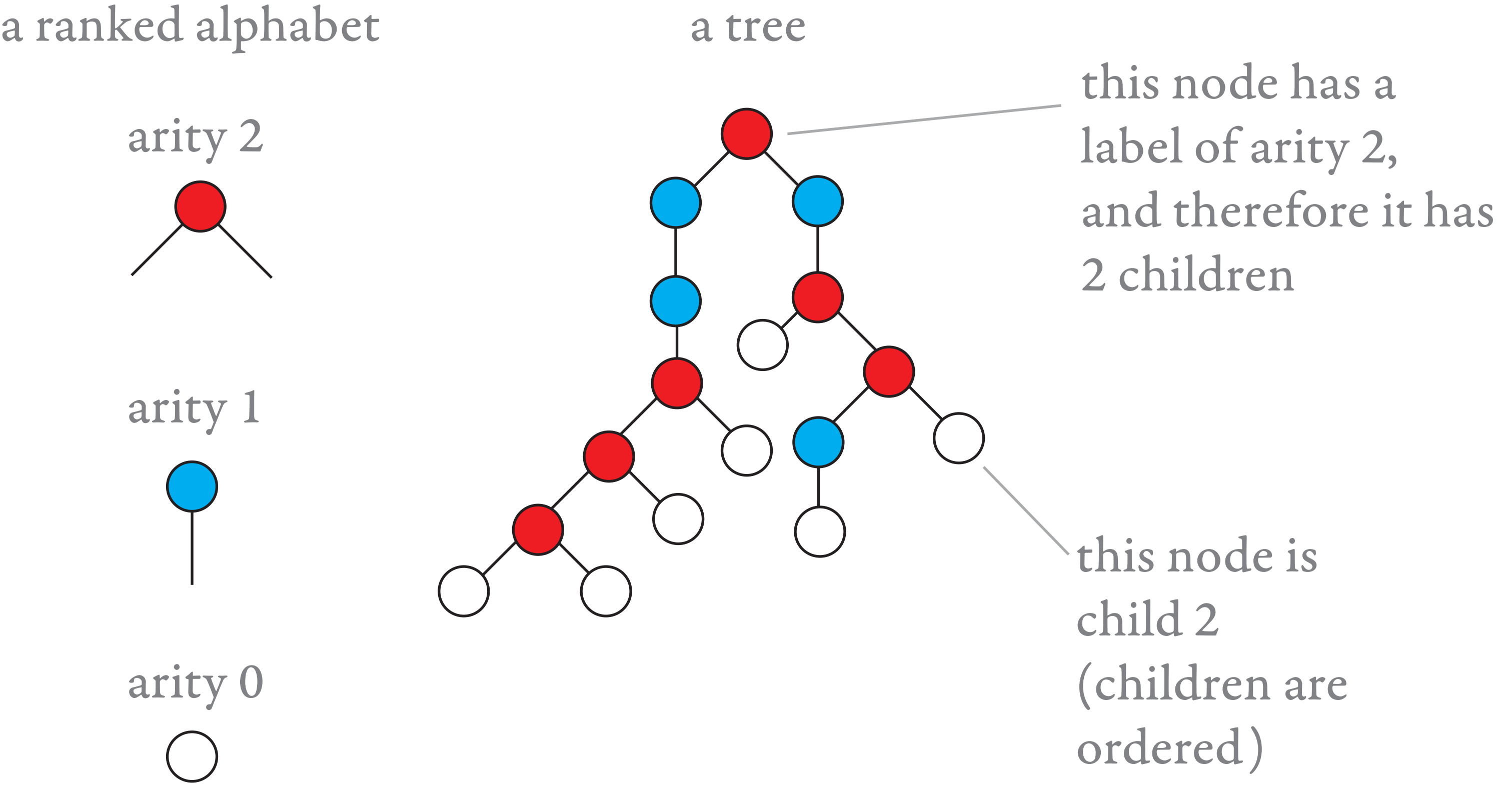
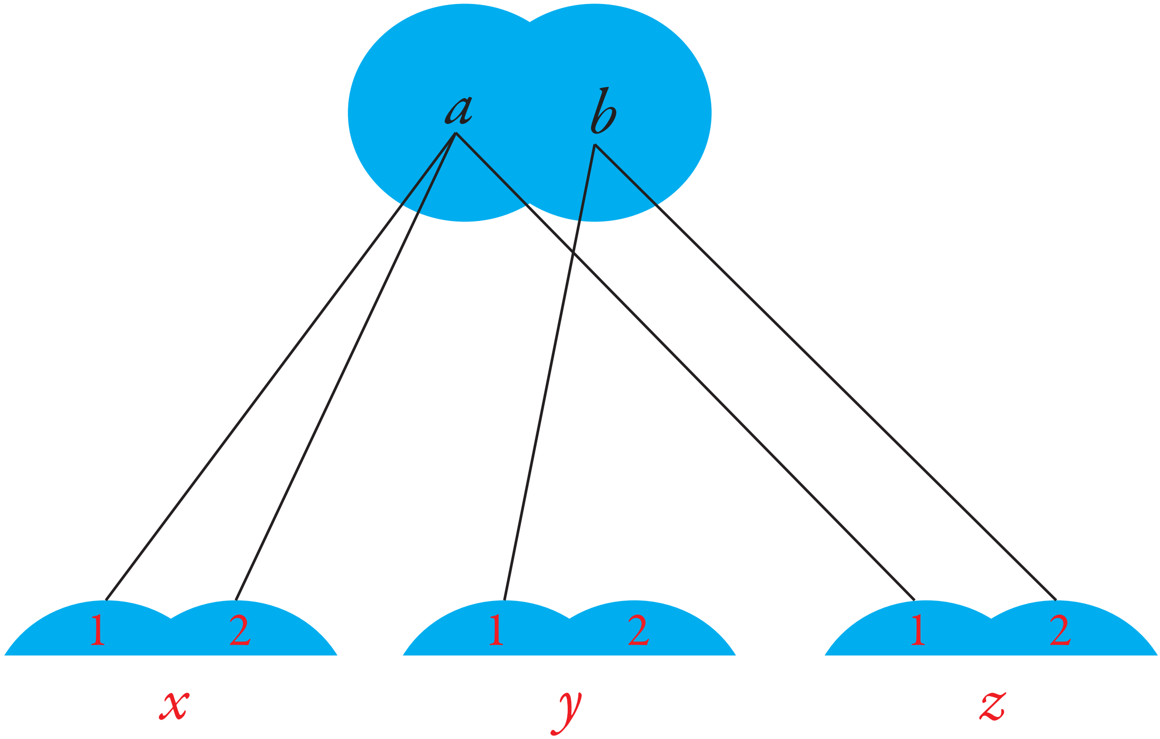
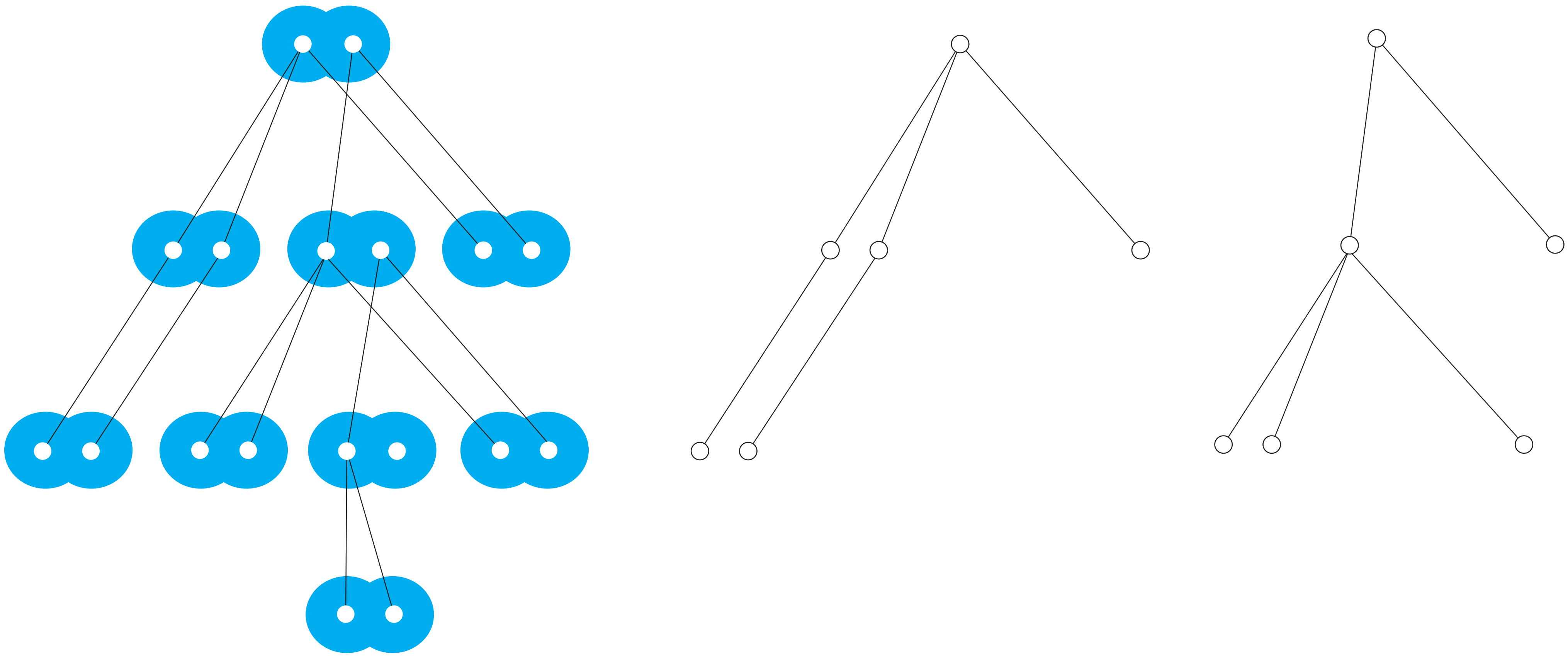
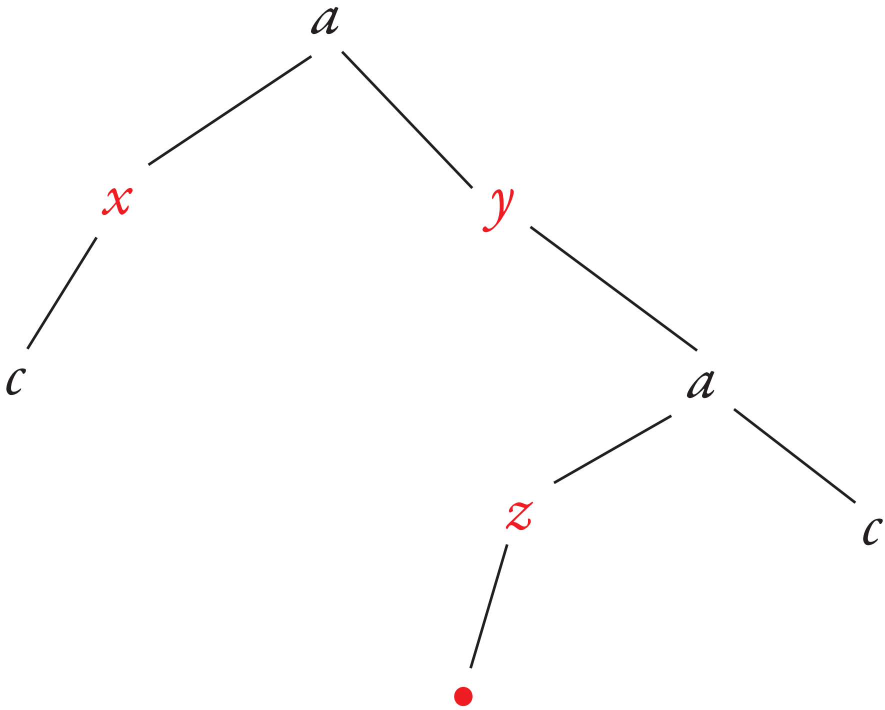

a ranked alphabet
arity 2
arity 1
arity 0
a tree
this node has a label of arity 2, and therefore it has 2 children
this node is child 2 (children are ordered)

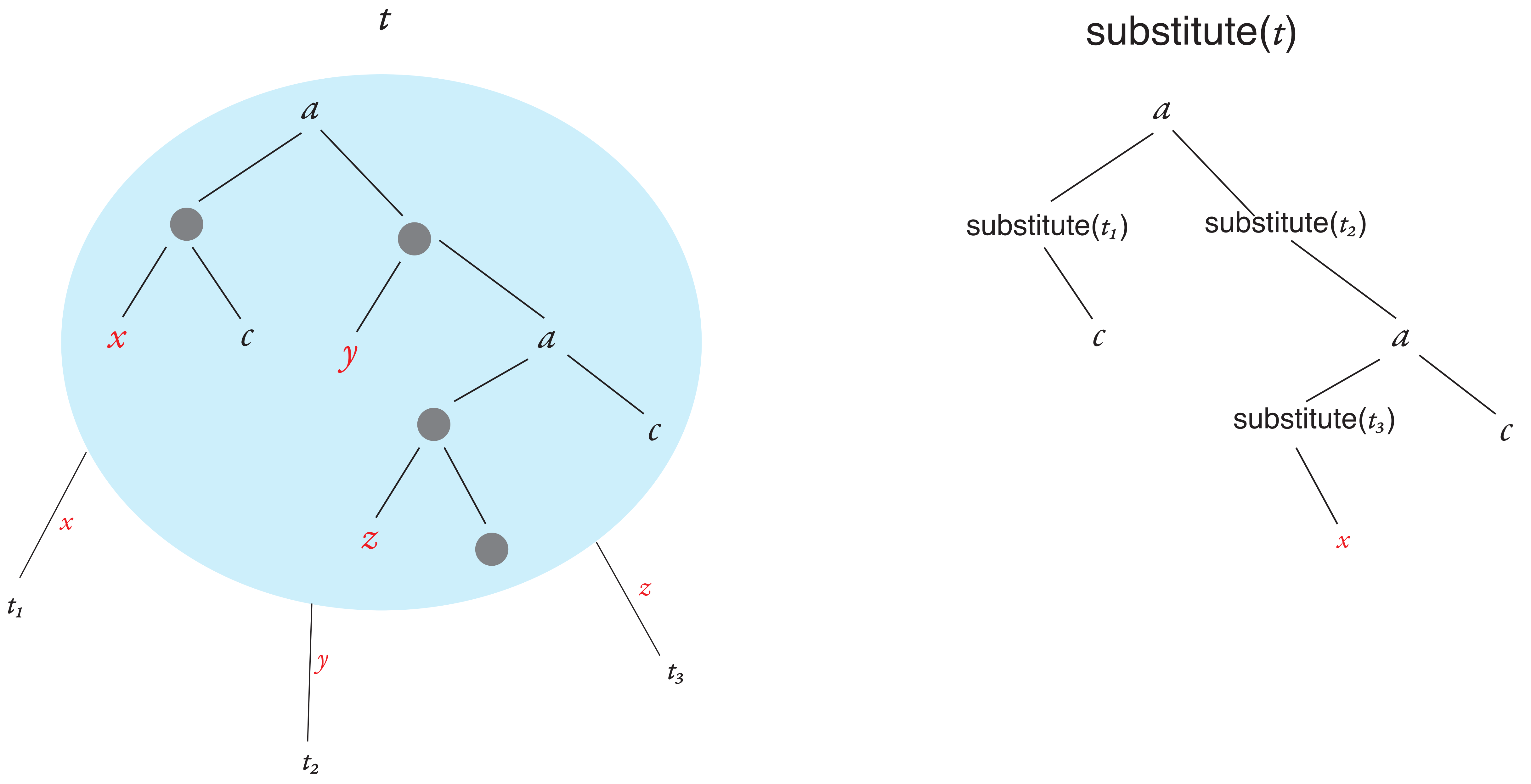


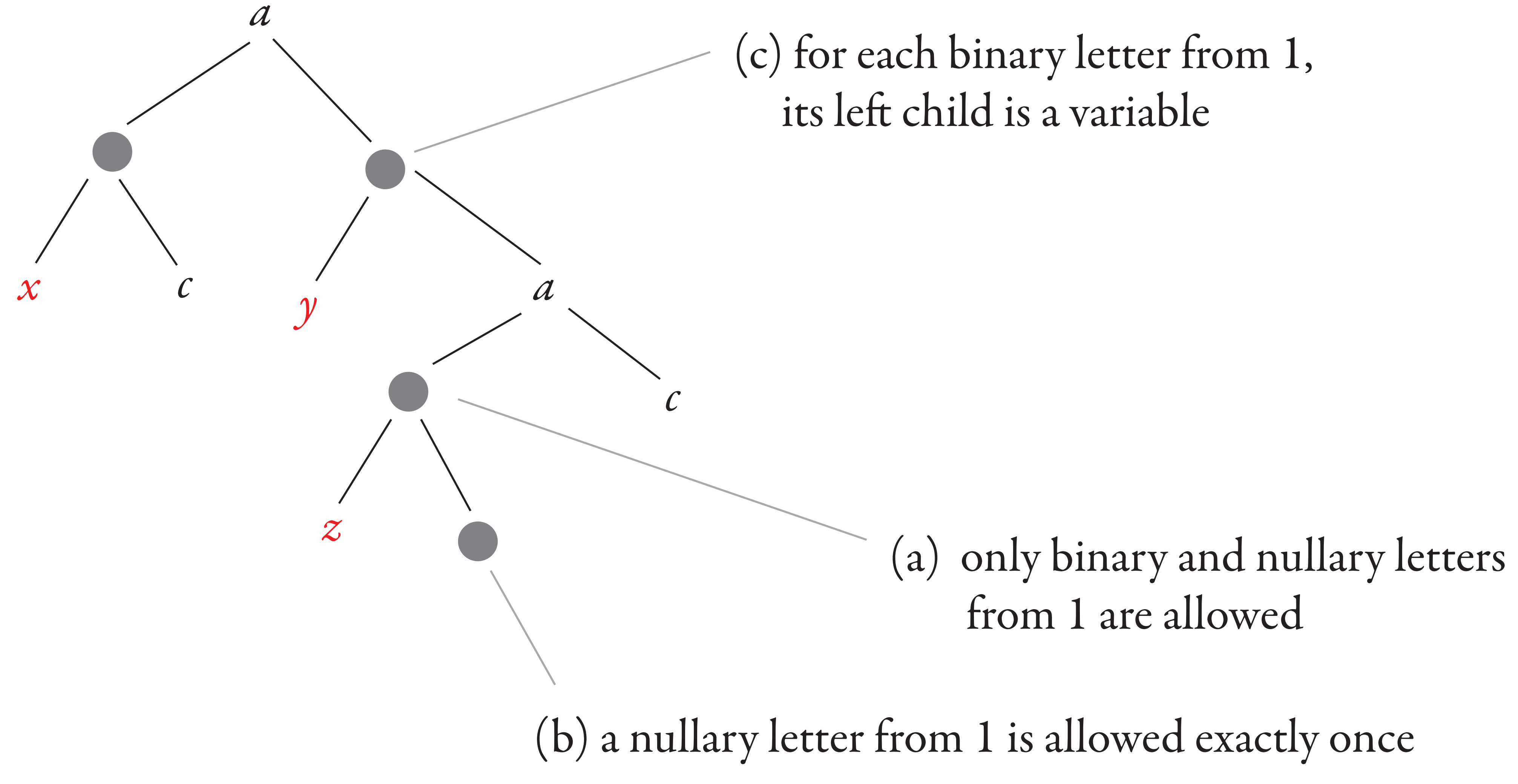
a
b
1
2
1
2
1
2
x
y
z

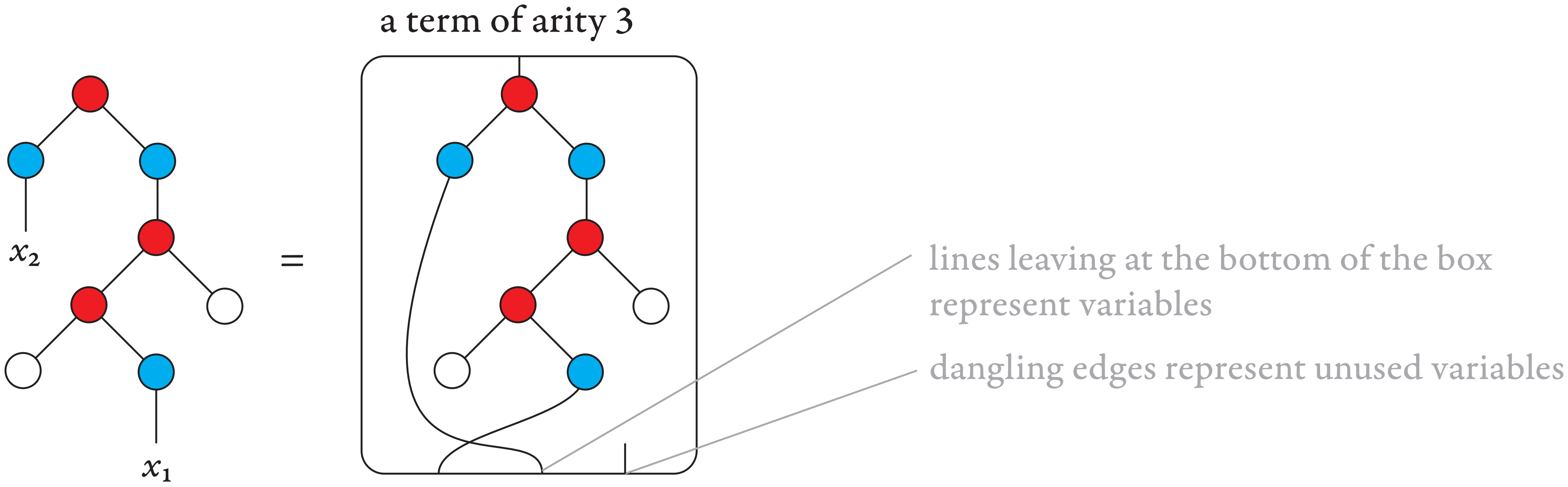
A tree t over $\Sigma^{[2]}$ $\qquad$ $\mathsf{unfold}_1(t)$ $\qquad$ $\mathsf{unfold}_2(t)$

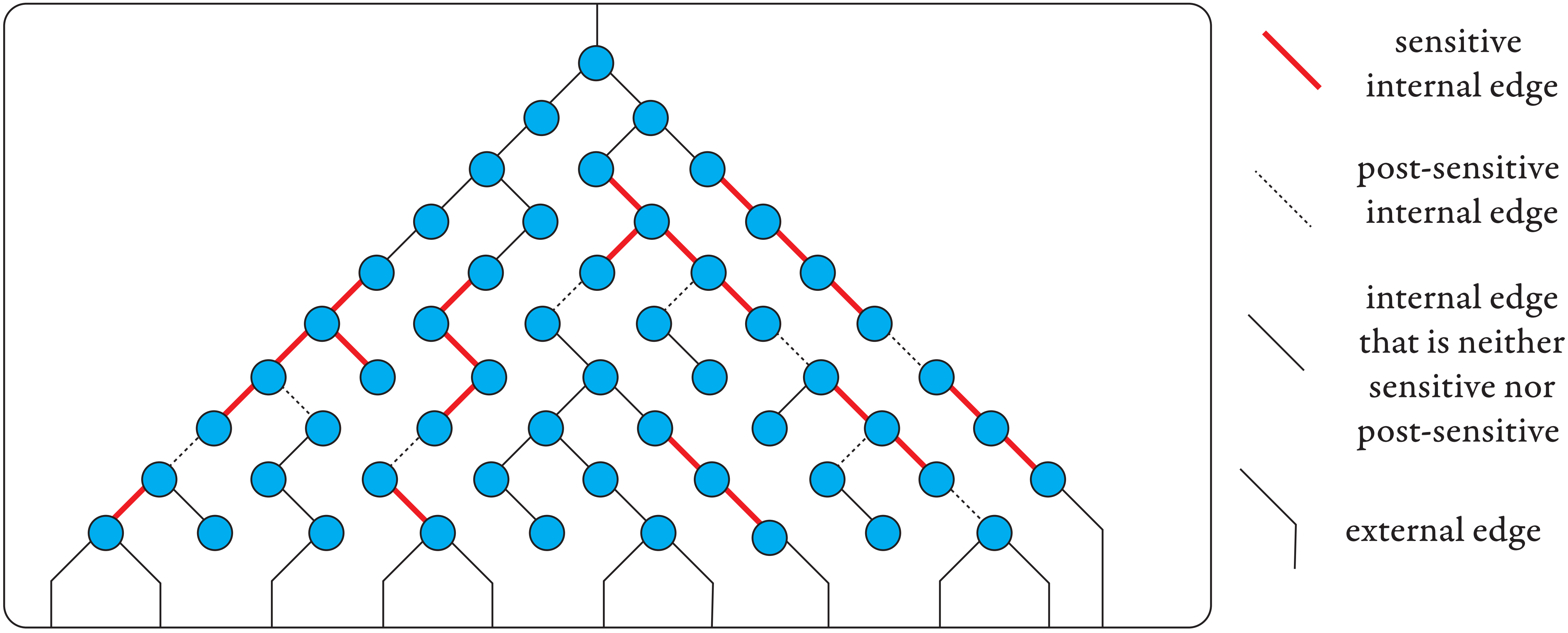


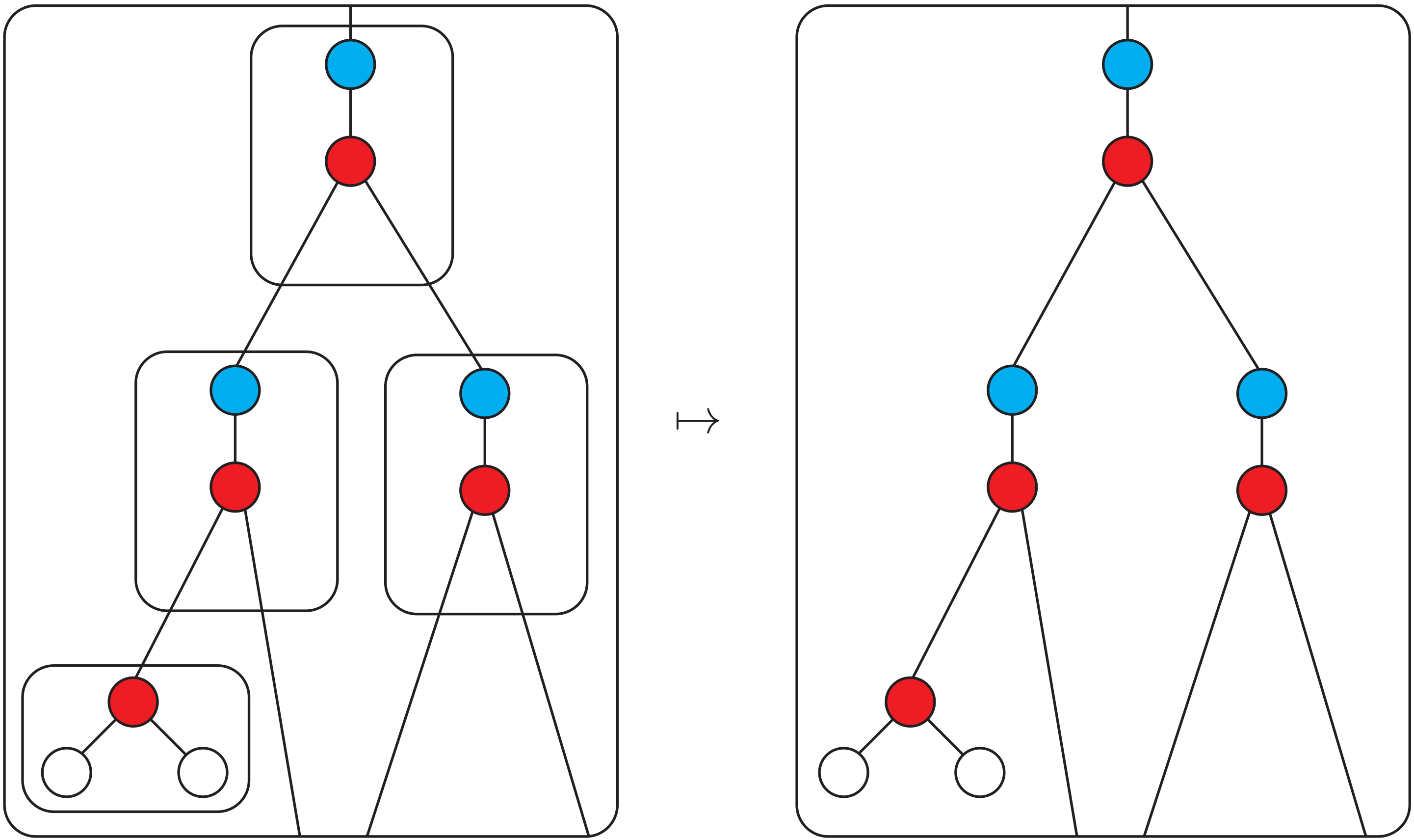

a
x
y
c
a
z
c

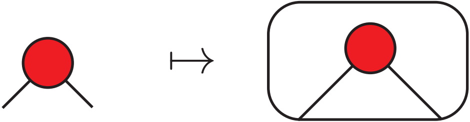

t
a
x
c
y
a
c
z
x
t1
y
t2
z
t3
substitute(t)
a
substitute(t1)
substitute(t2)
c
a
substitute(t3)
c
x

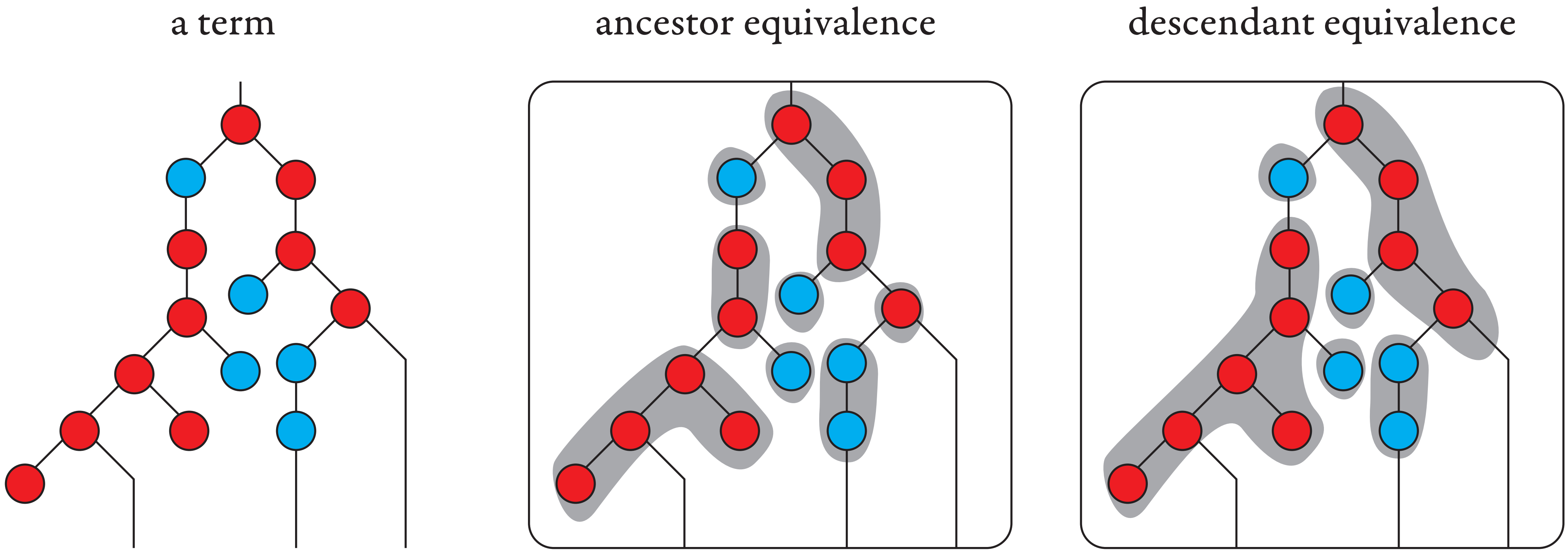

a
x
c
y
a
c
z
(c) for each binary letter from 1,
its left child is a variable
(a) only binary and nullary letters
from 1 are allowed
(b) a nullary letter from 1 is allowed exactly once

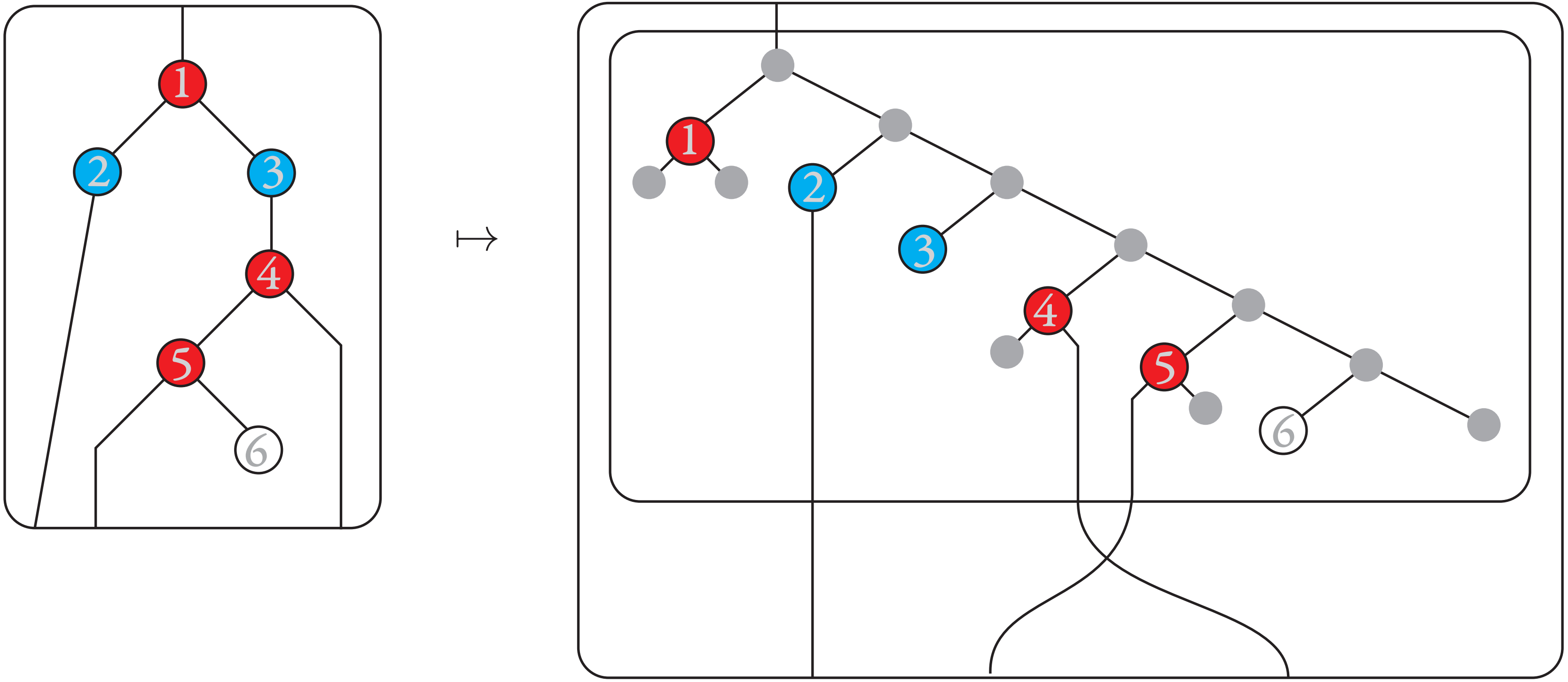

a term of arity 3
x_2
x_1
=
lines leaving at the bottom of the box represent variables
dangling edges represent unused variables


sensitive internal edge
post-sensitive internal edge
internal edge that is neither sensitive nor post-sensitive
external edge

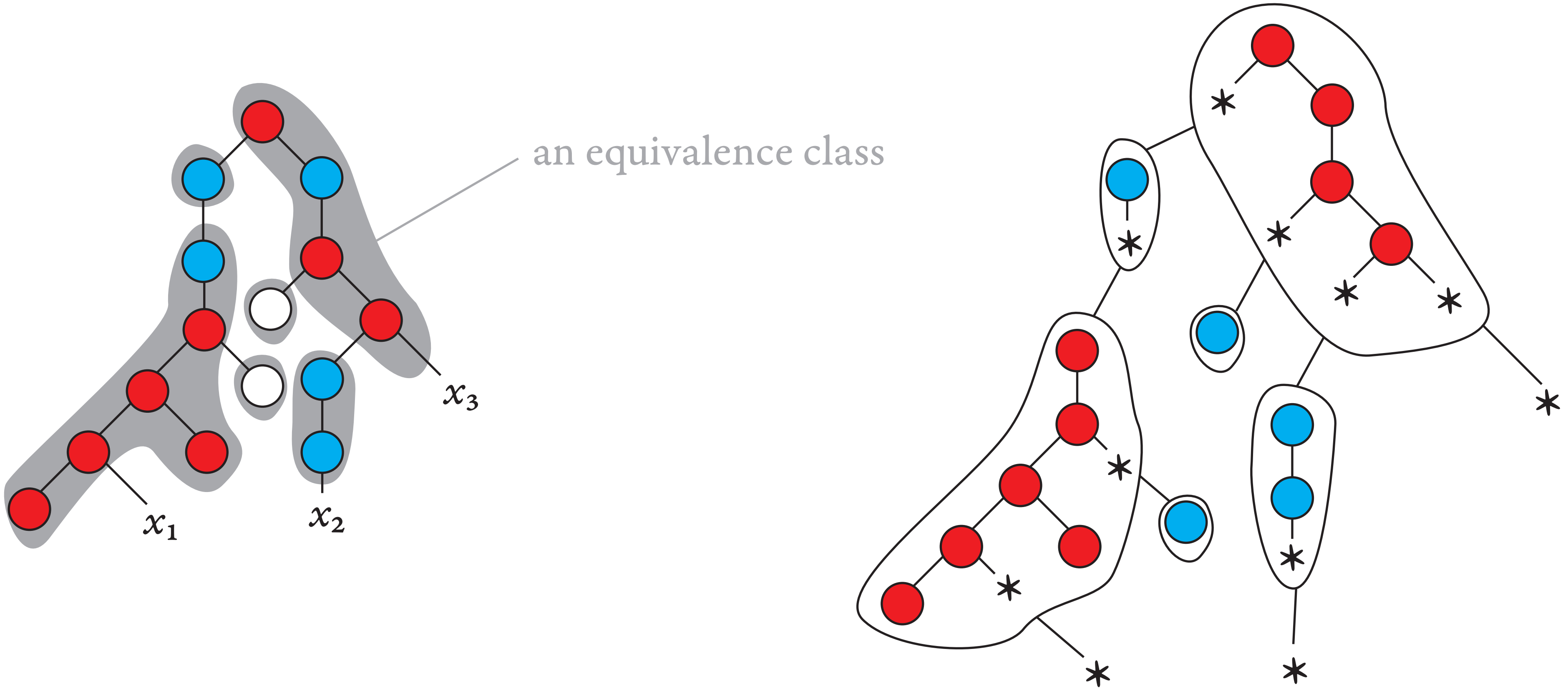




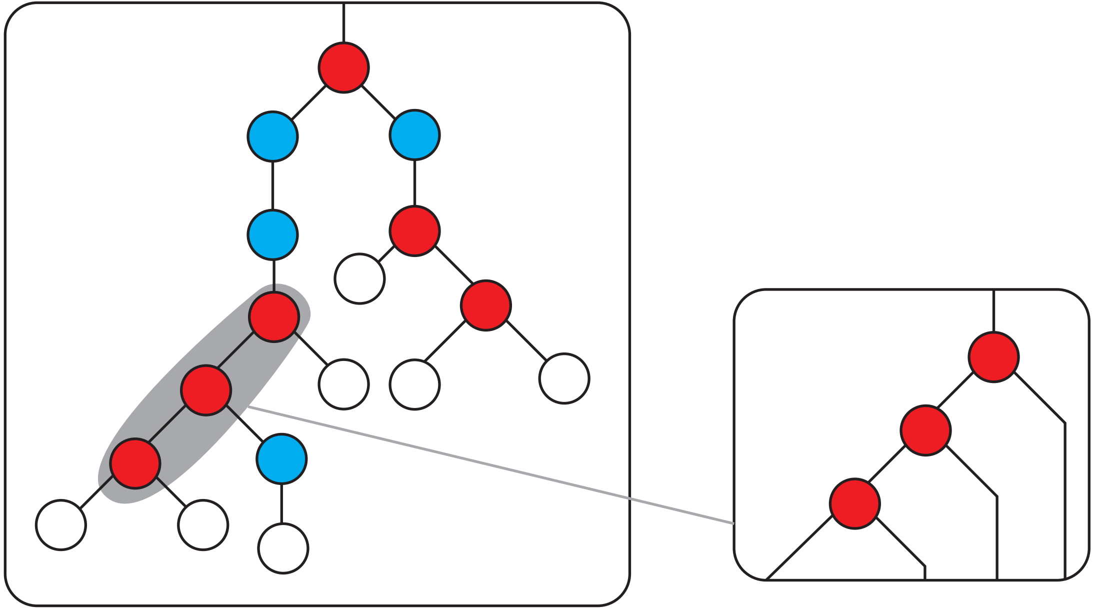

a term
ancestor equivalence
descendant equivalence


1
2
3
4
5
6
↦
1
2
3
4
5
6

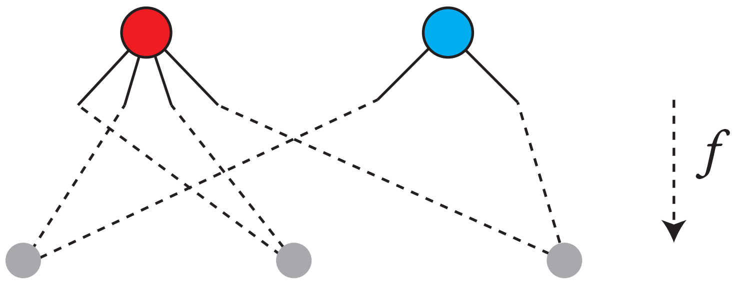
a factorisation equivalence



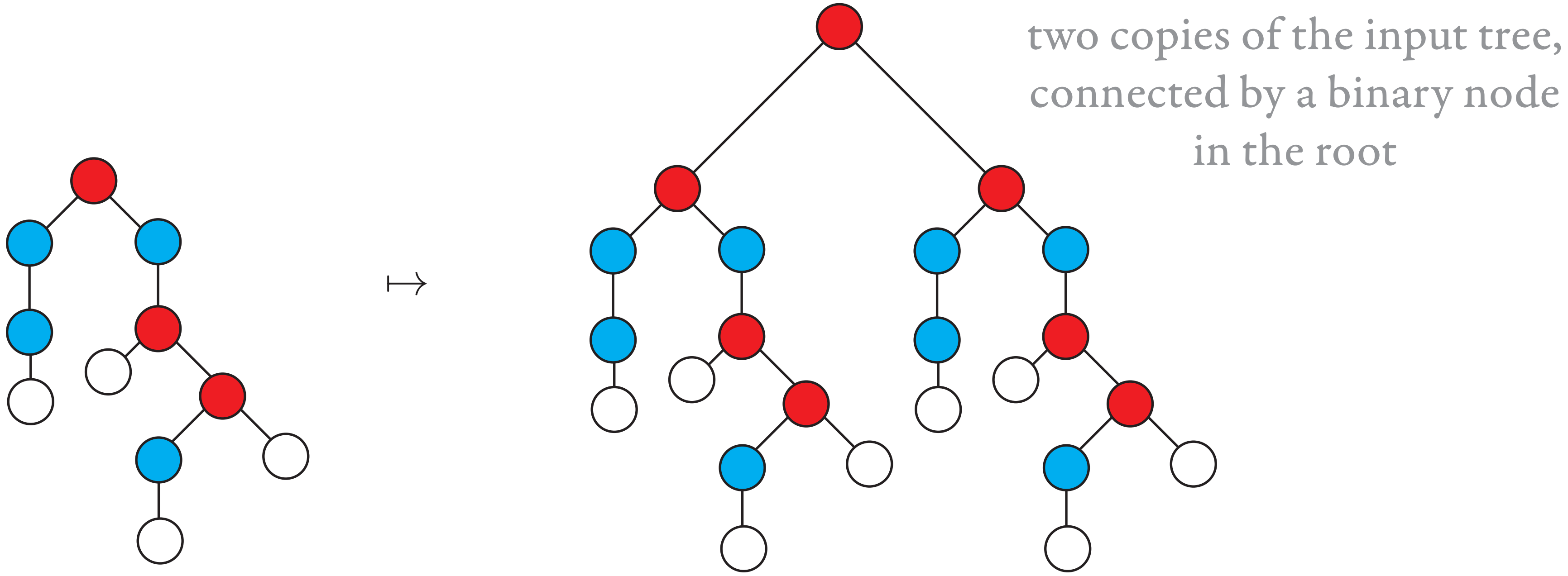
a tree

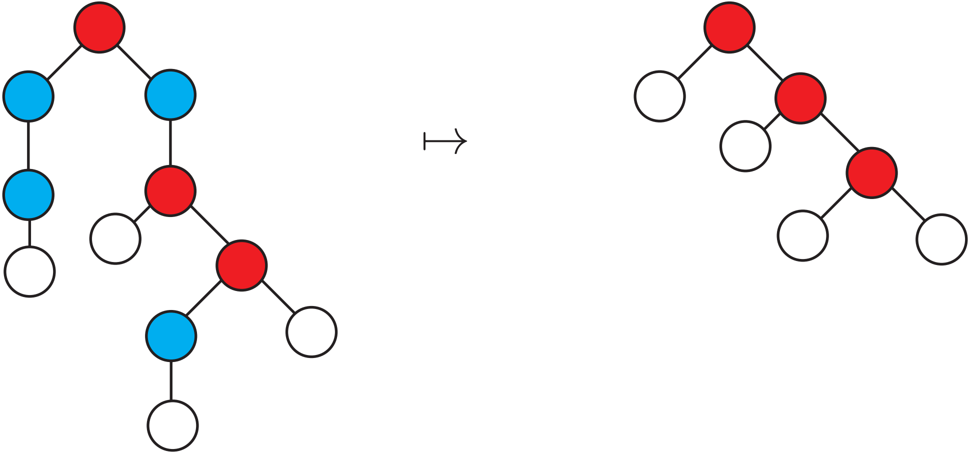


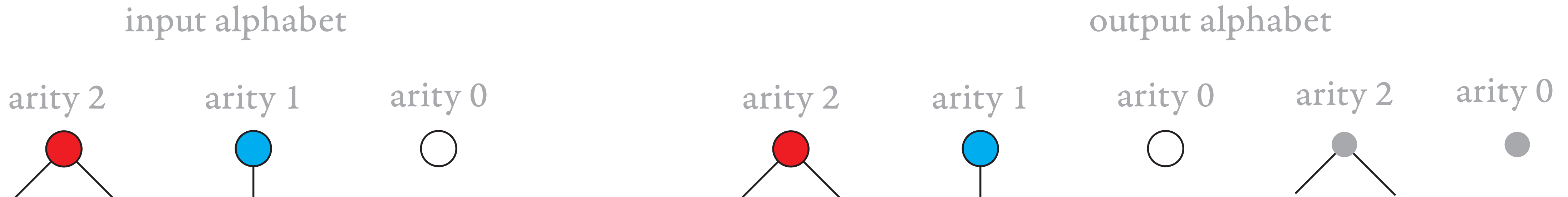
a term with 4 ports that represents part of the tree

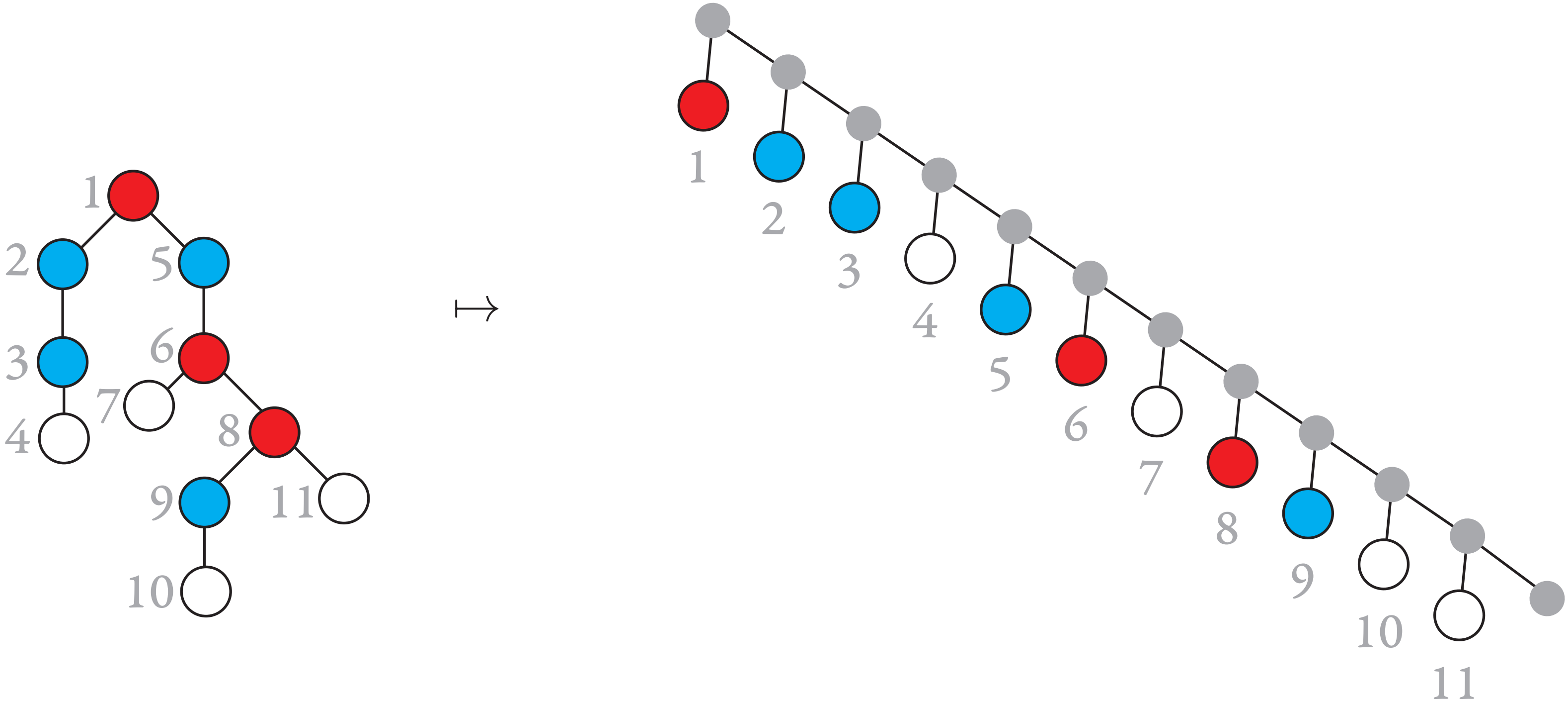

f


input alphabet
arity 2
arity 1
arity 0
output alphabet
arity 2
arity 0

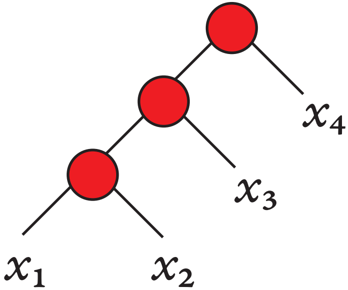

two copies of the input tree,
connected by a binary node
in the root



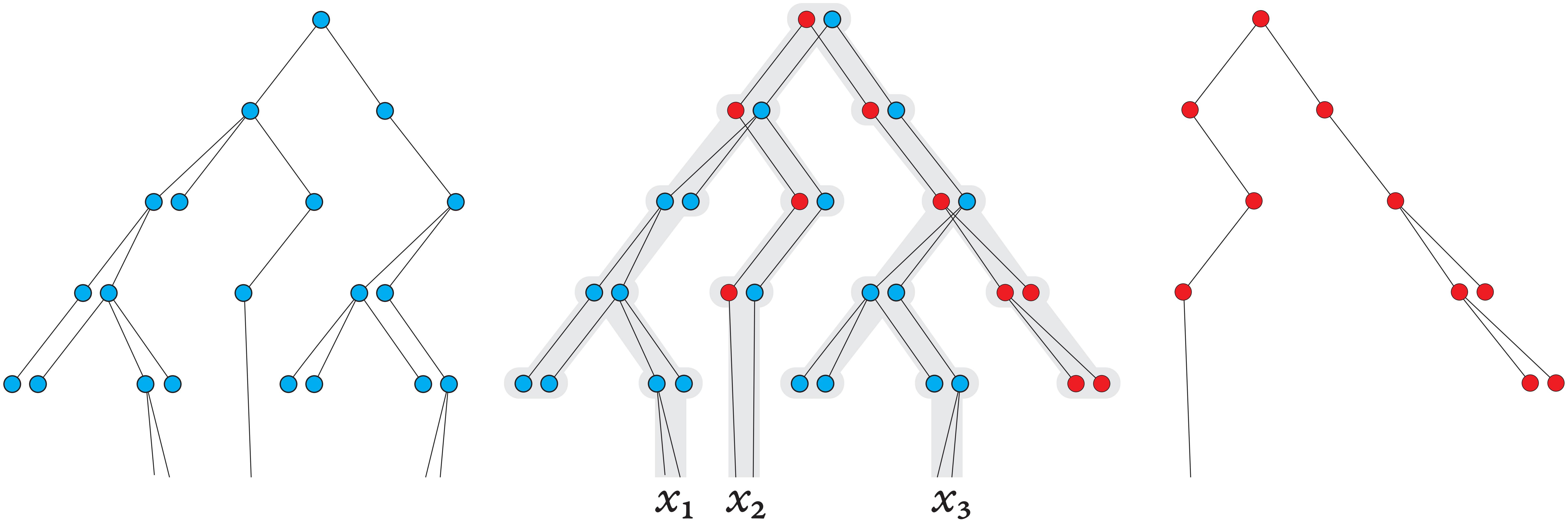

input alphabet
output alphabet
arity 2
arity 1
arity 0
arity 2
arity 1
arity 0
arity 2
arity 0

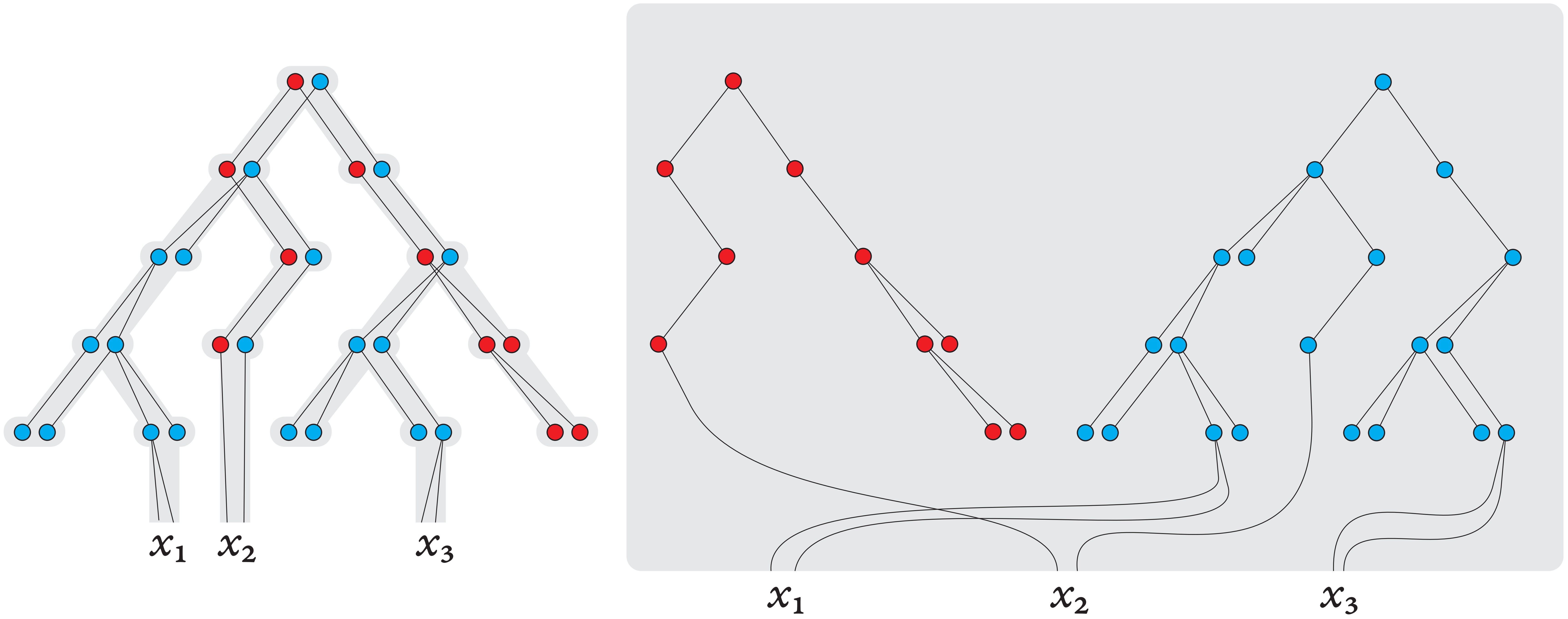

1
2
5
3
6
7
4
8
9
11
10
↦
1
2
3
4
5
6
7
8
9
10
11

a term of arity 4

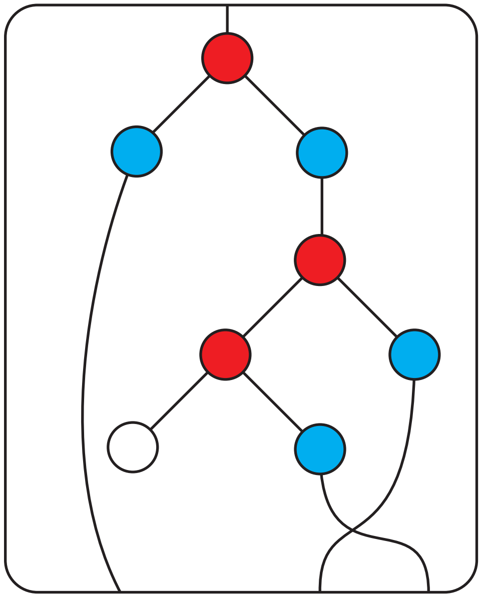


a term of arity 0


x_1 x_2 x_3



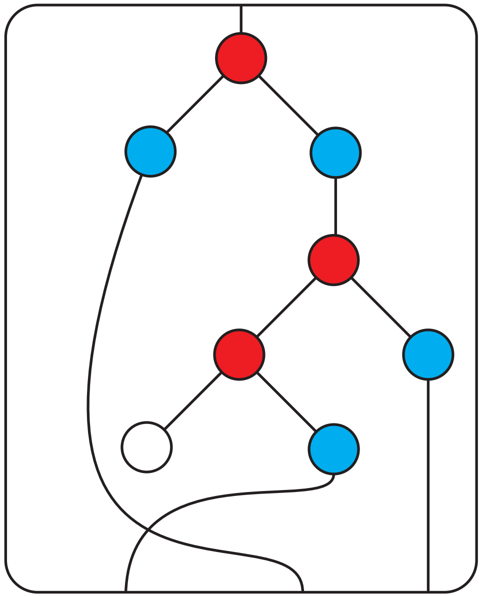
x_1 x_2 x_3
x_1 x_2 x_3



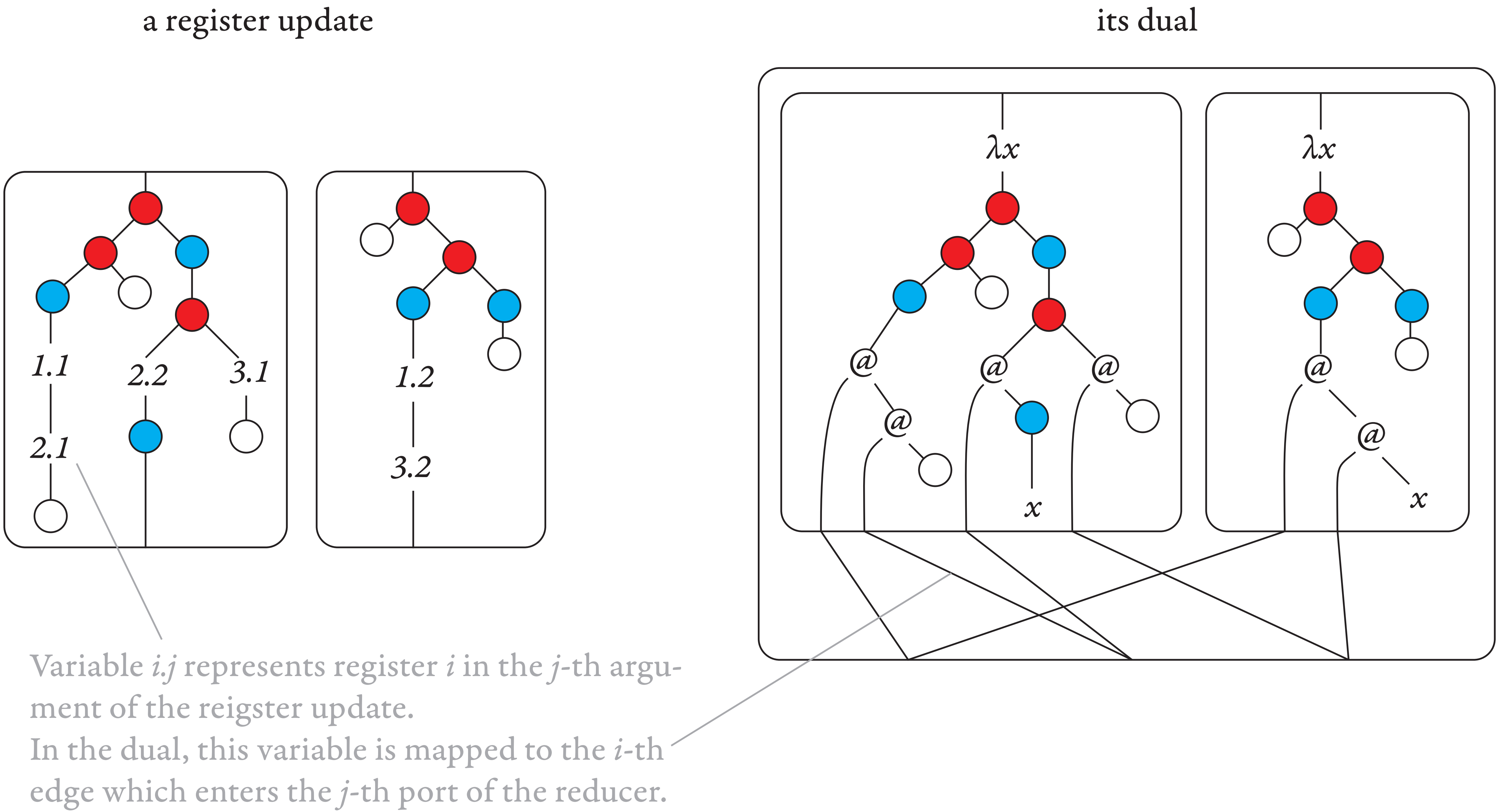
satisfies (*)

(*)

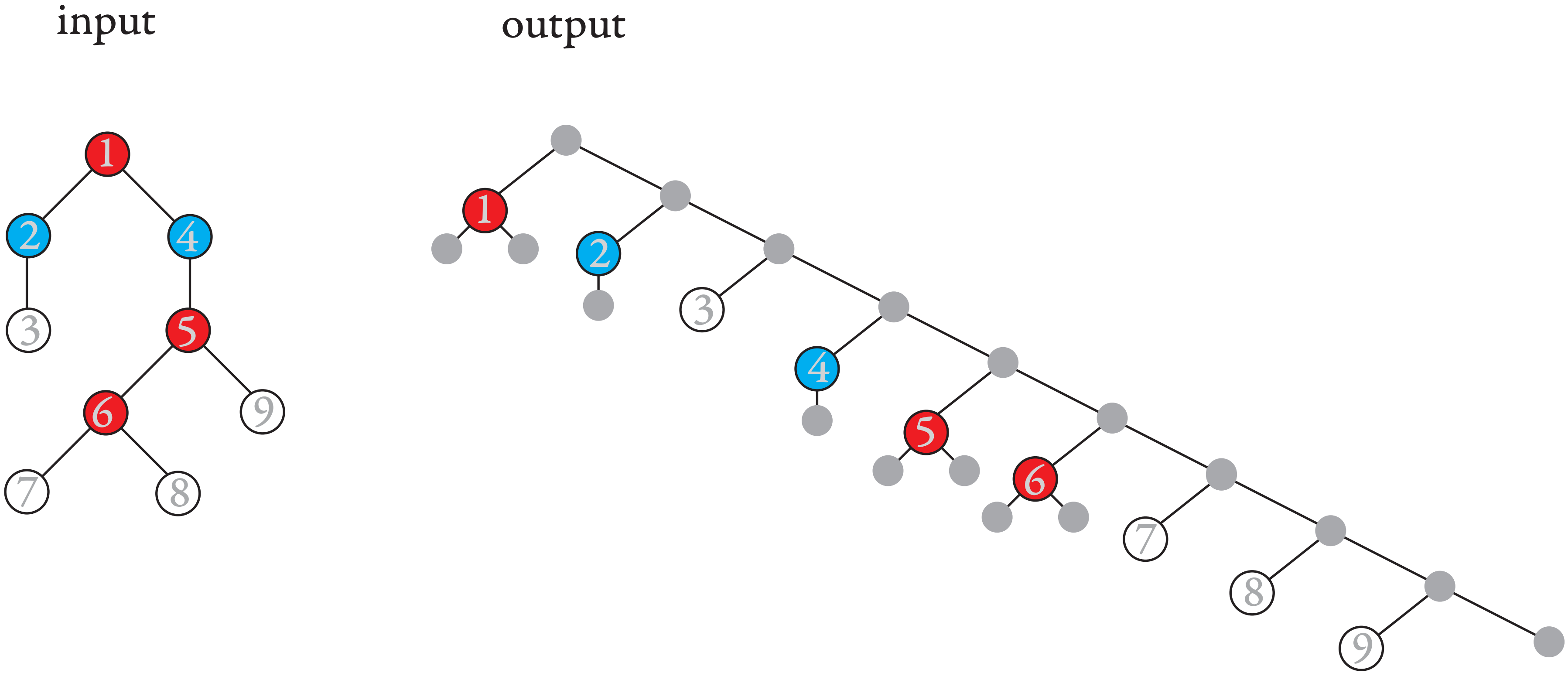
If the root has arity n, and $1 \leq i < j \leq n$, then all ports of the j-th subterm of the root are after all ports of the i-th subterm of the root



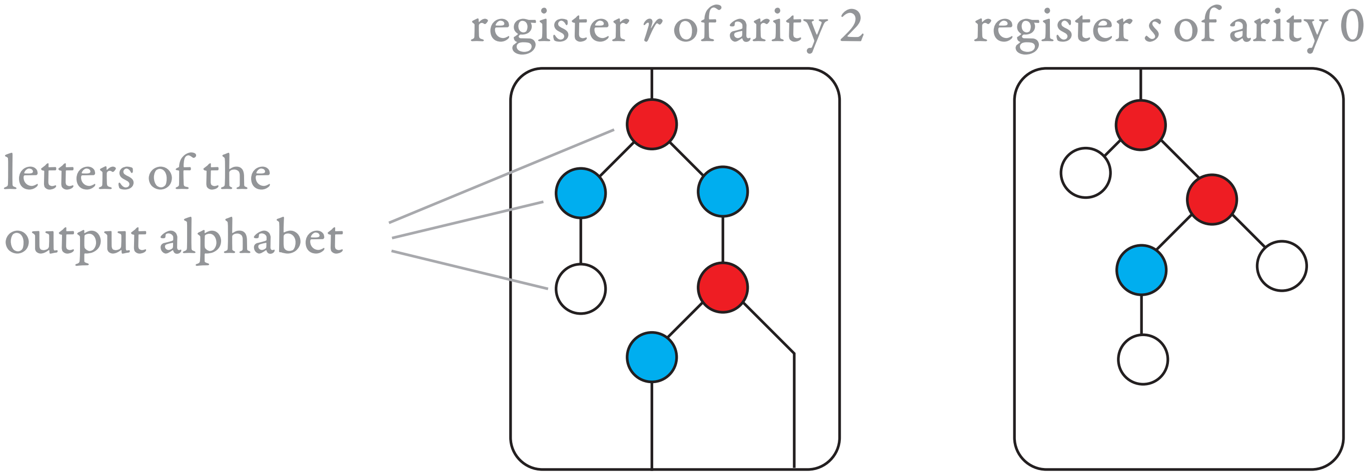
violates (*)

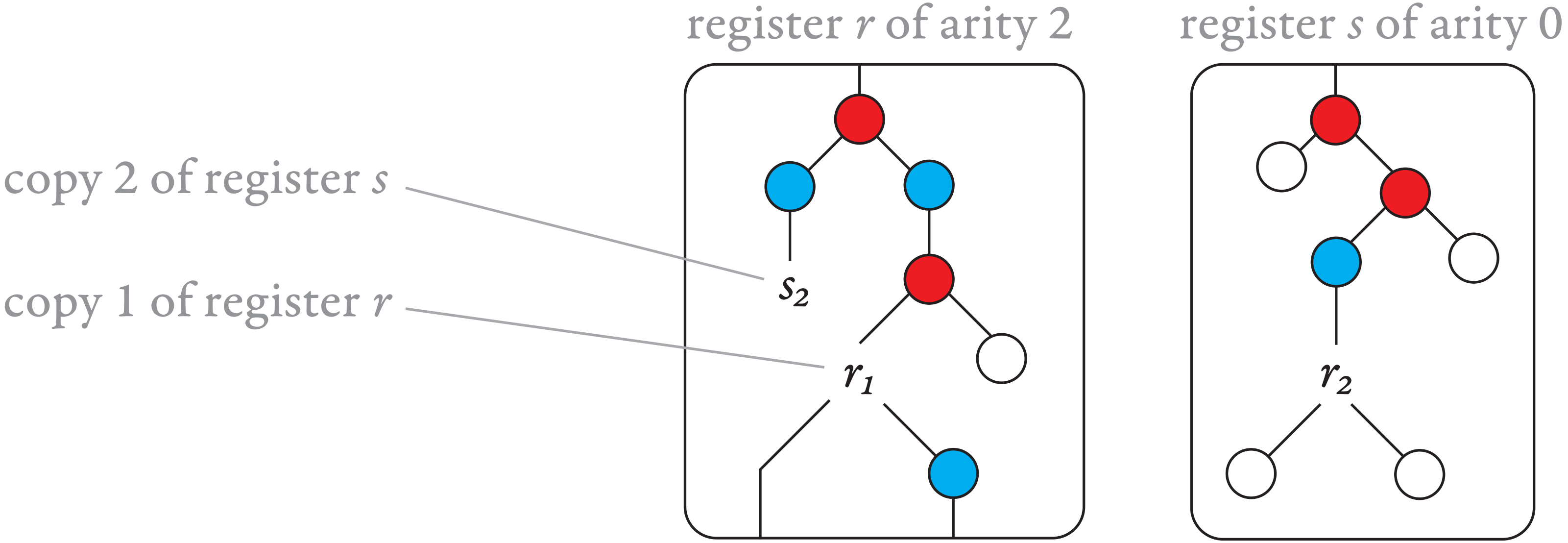


a register update
its dual
1.1
2.2
3.1
2.1
1.2
3.2
λx
λx
@
@
@
@
@
@
x
x
Variable i.j represents register i in the j-th argument of the reigster update.
In the dual, this variable is mapped to the i-th edge which enters the j-th port of the reducer.



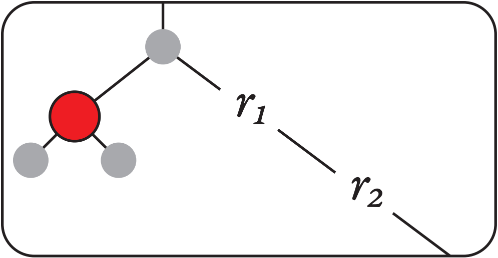
input
output
1
2
4
3
5
6
9
7
8
1
2
3
4
5
6
7
8
9


register r of arity 2
register s of arity 0
letters of the output alphabet

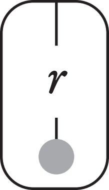


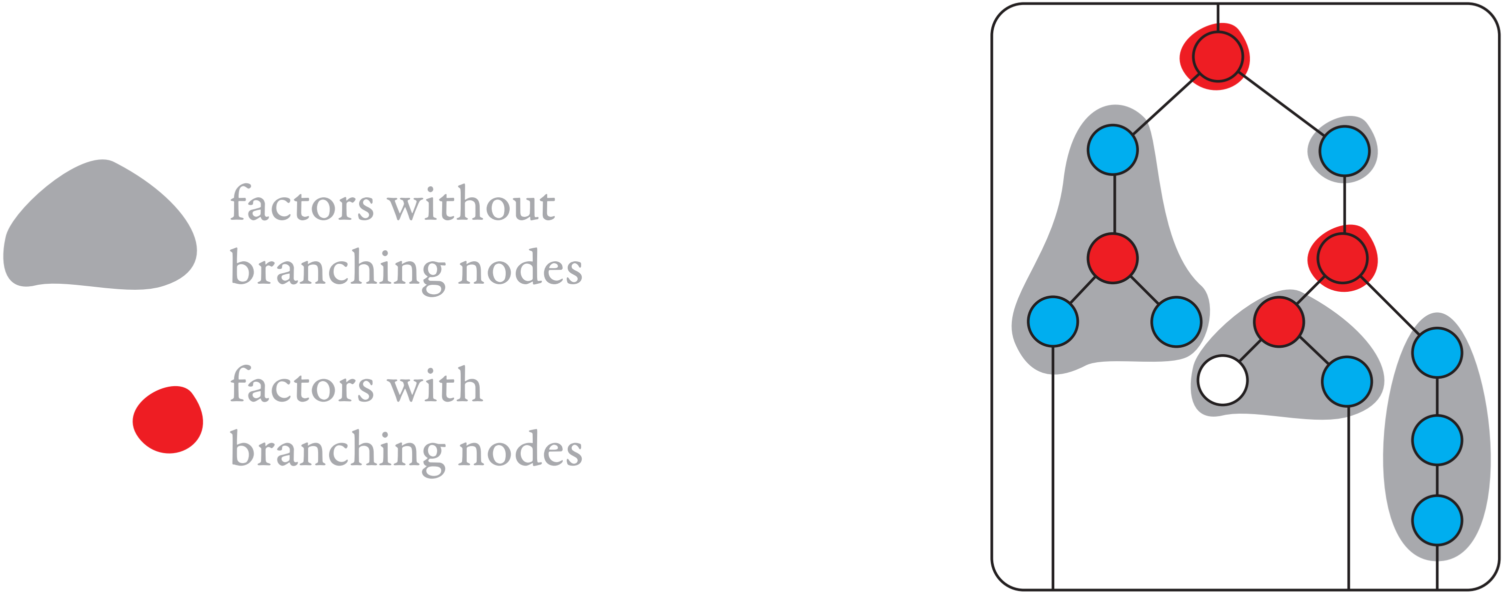
register r of arity 2
register s of arity 0
copy 2 of register s
copy 1 of register r
s_2
r_1
r_2

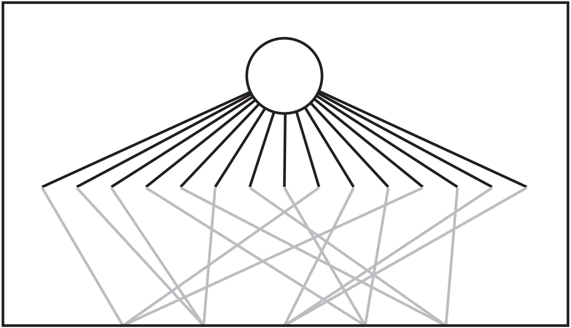

r
r
r
s


r_1
r_2

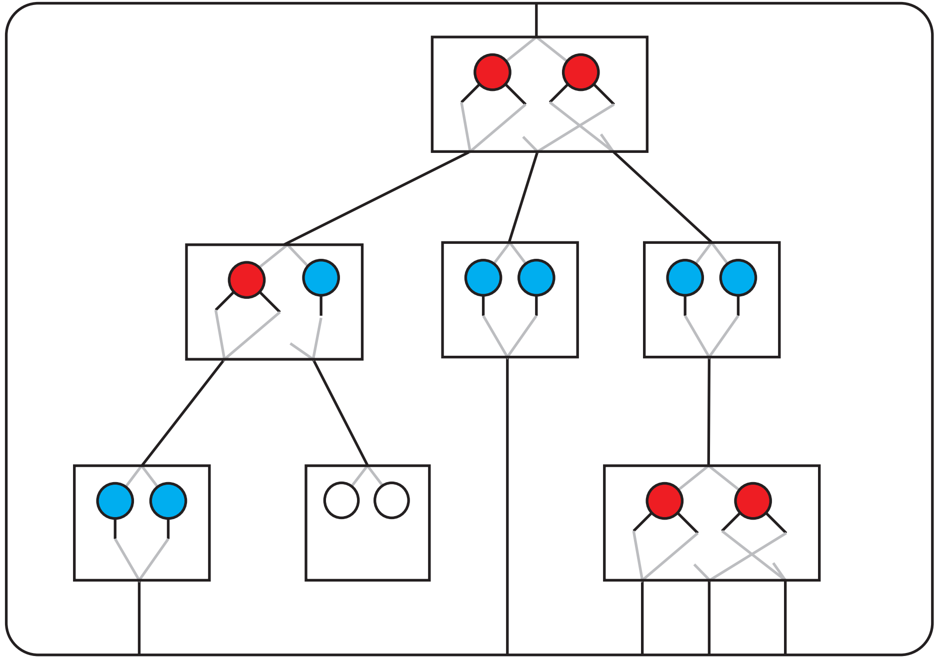

r
s
s
r
r
s


r

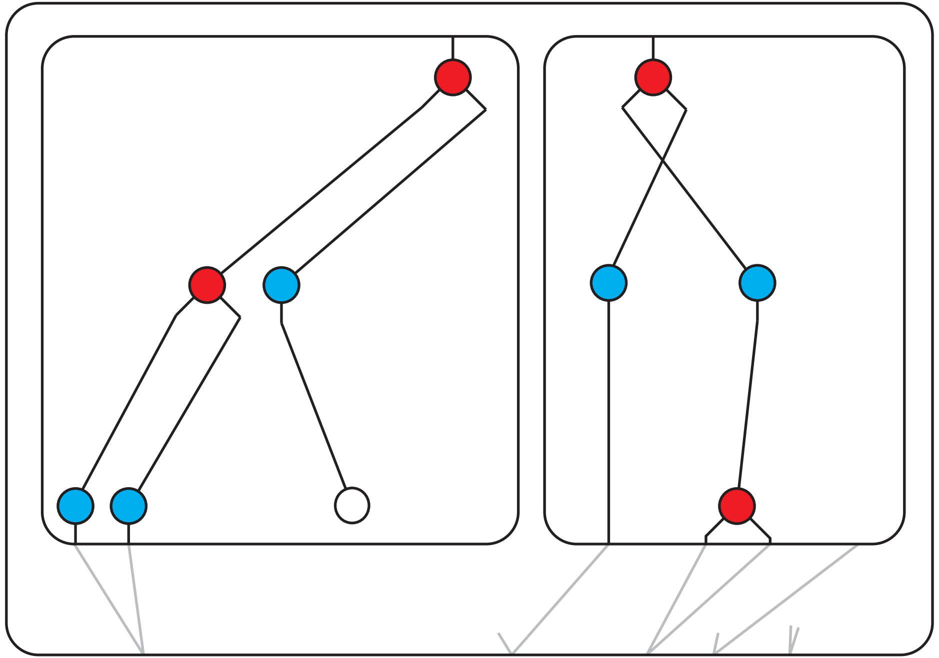

factors without branching nodes
factors with branching nodes

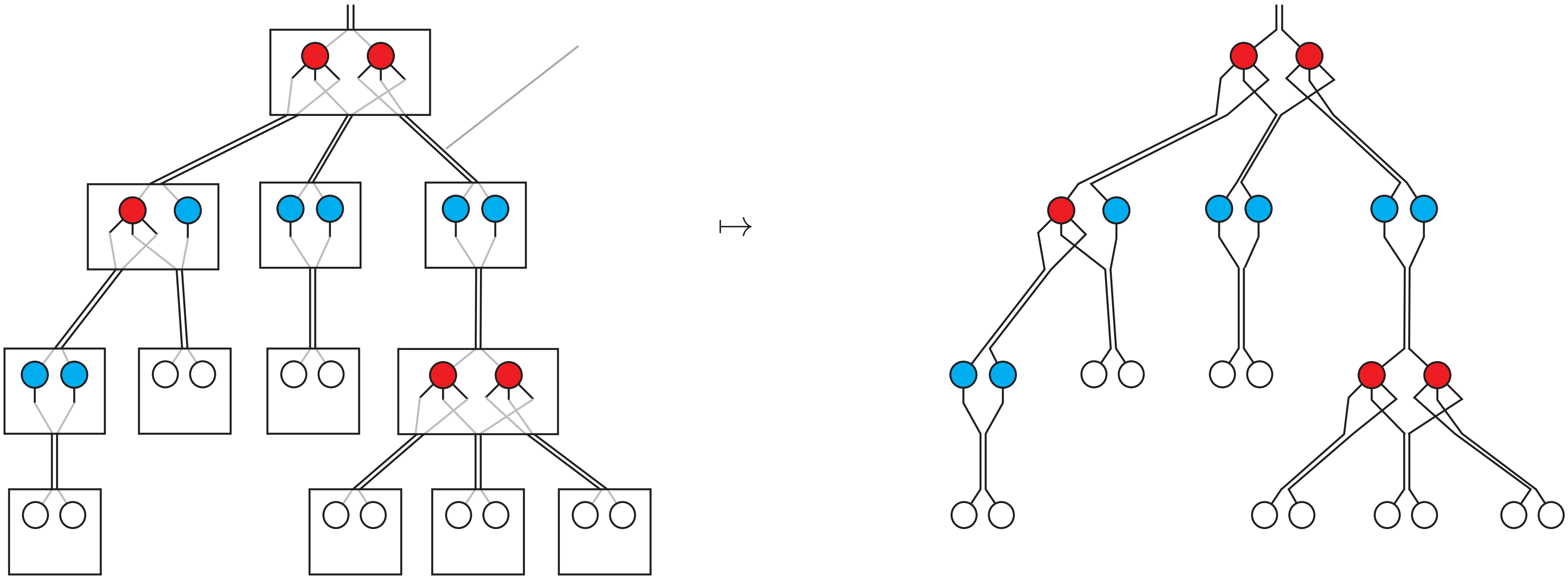


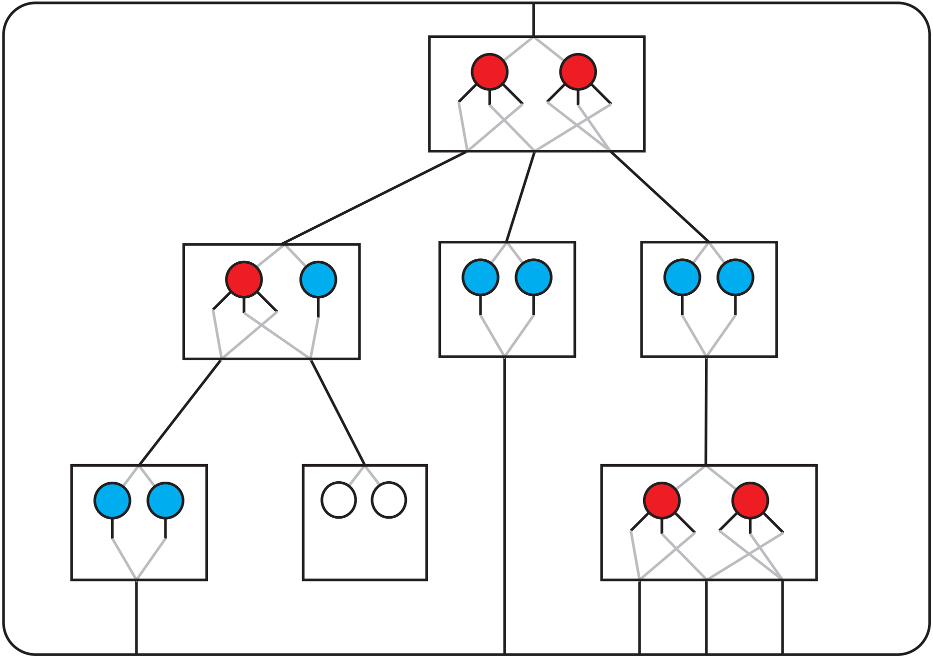
a term of matrix powers

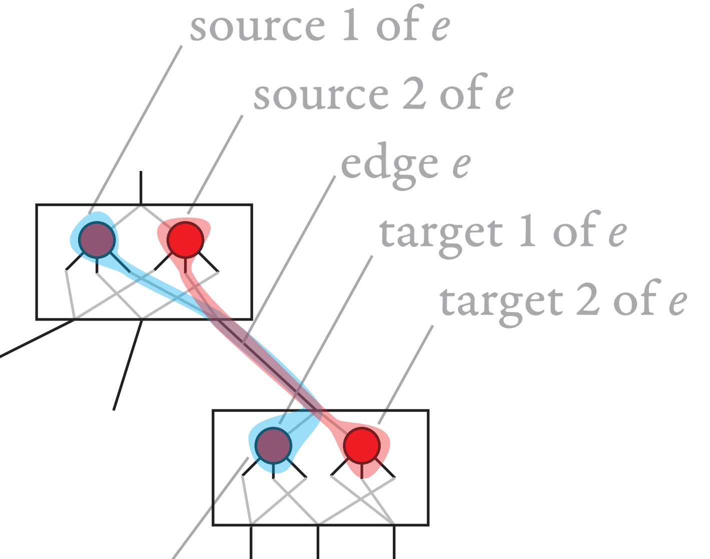


its term unfolding

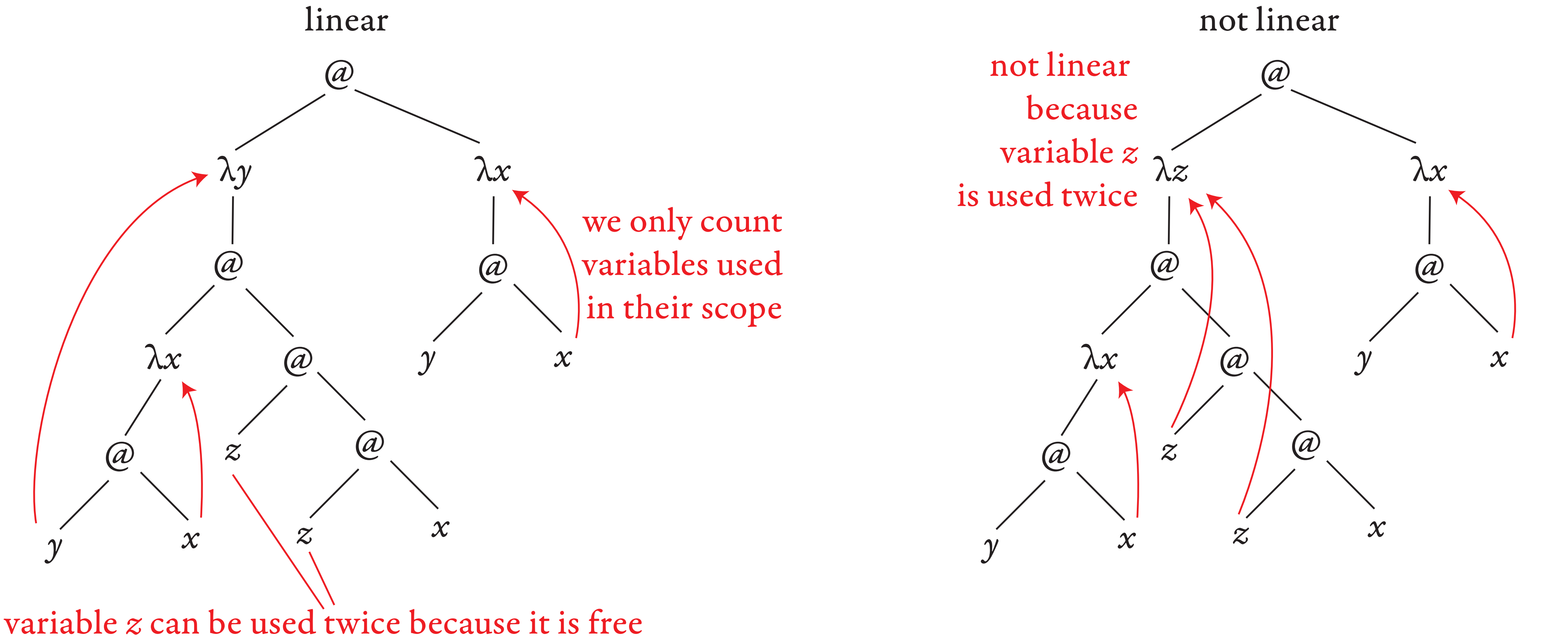






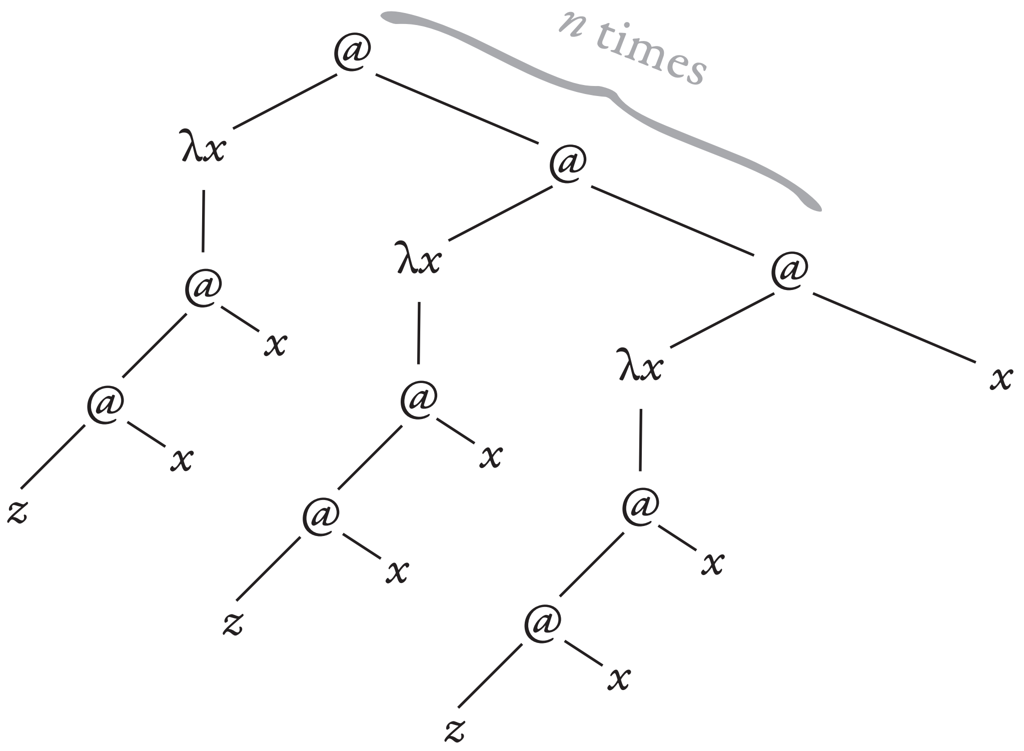


source 1 of e
source 2 of e
edge e
target 1 of e
target 2 of e

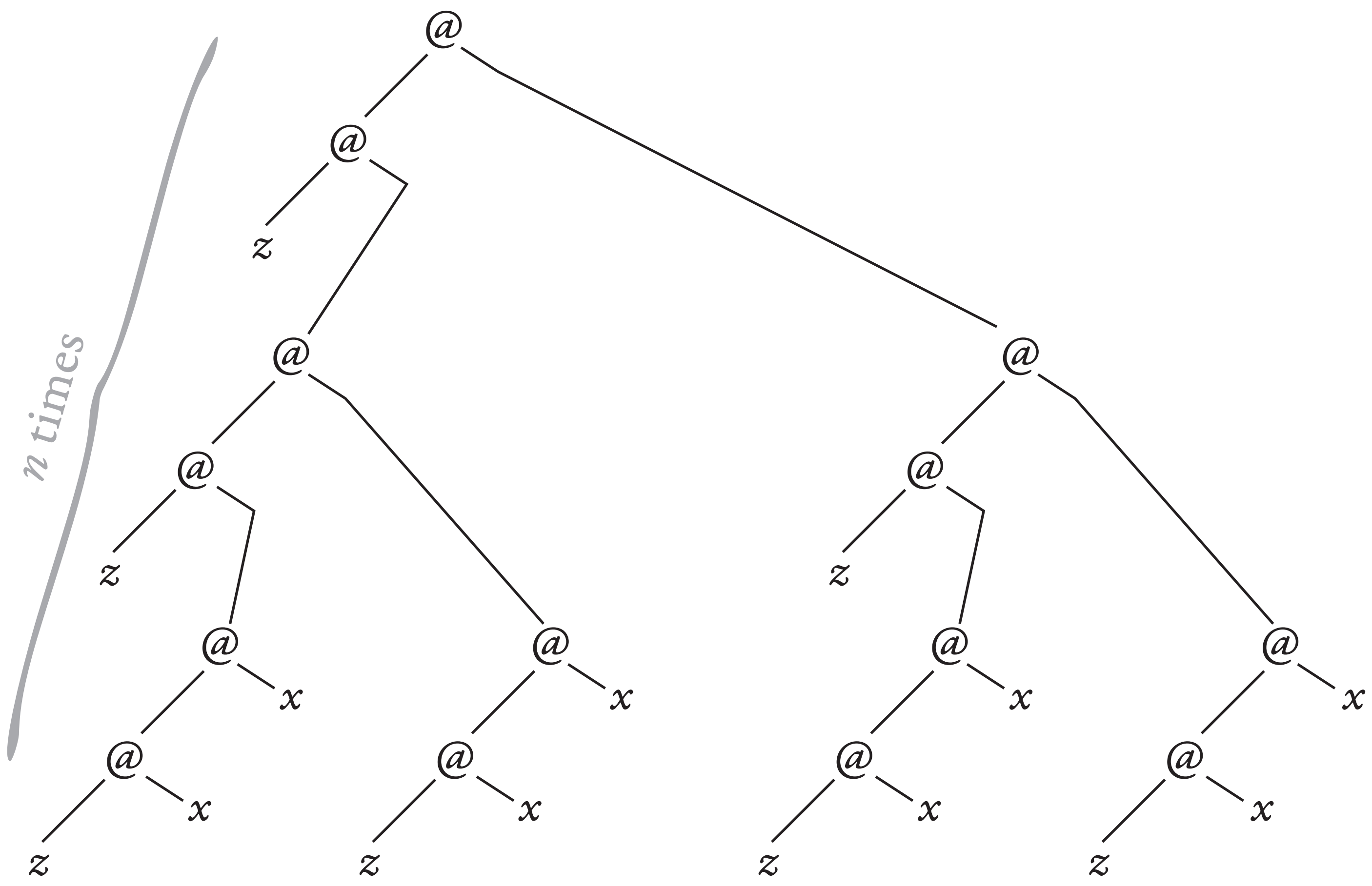


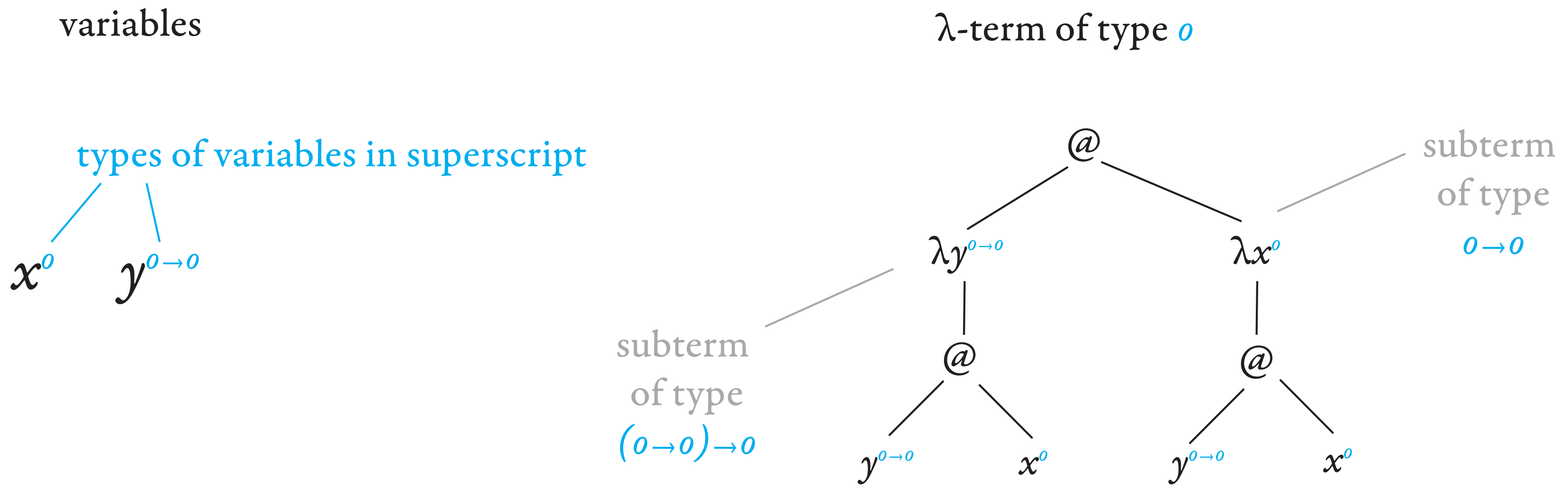
linear
not linear
we only count
variables used
in their scope
variable z can be used twice because it is free
not linear
because
variable z
is used twice

a λ-term of size $O(n)$



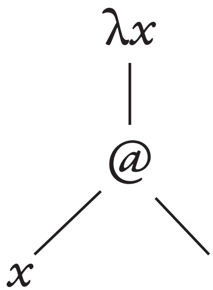
its normal form of size $O(2^n)$



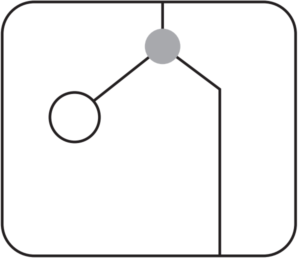


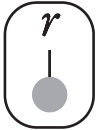
variables
λ-term of type o
types of variables in superscript
x^{o}
$y^{o\to o}$
@
$\lambda y^{o\to o}$
λx^{o}
@
@
$y^{o\to o}$
x^{o}
$y^{o\to o}$
x^{o}
subterm of type $o\to o$
subterm of type $(o\to o)\to o$

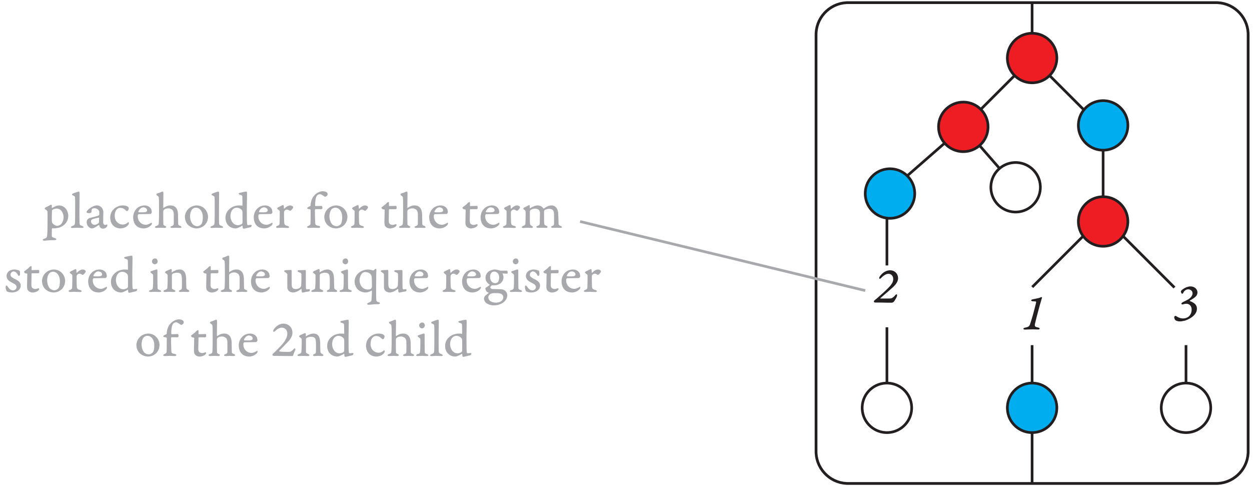
@

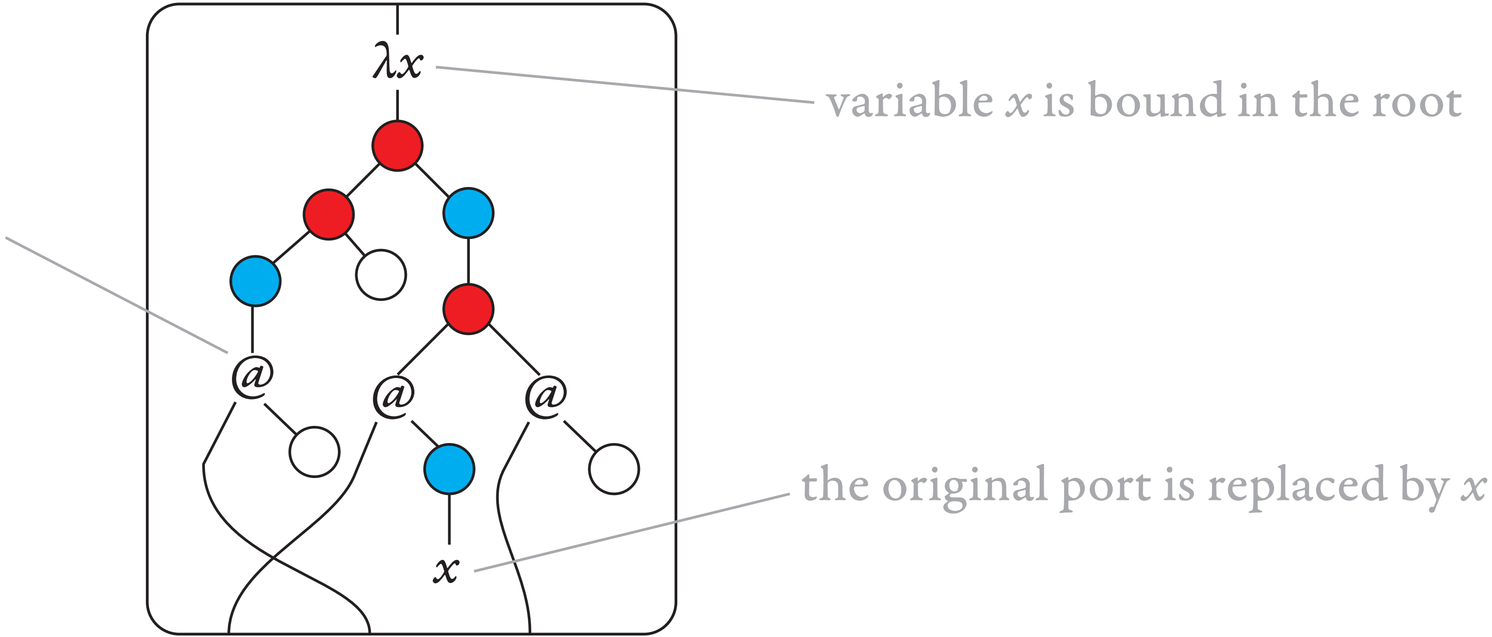


$\lambda x.$

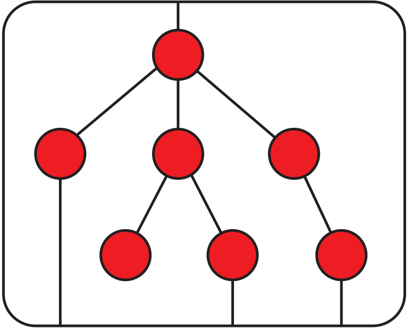



r

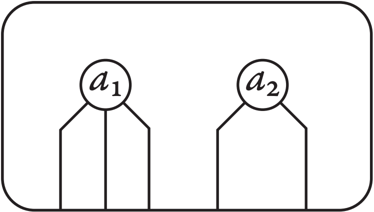

placeholder for the term
stored in the unique register
of the 2nd child
2
1
3



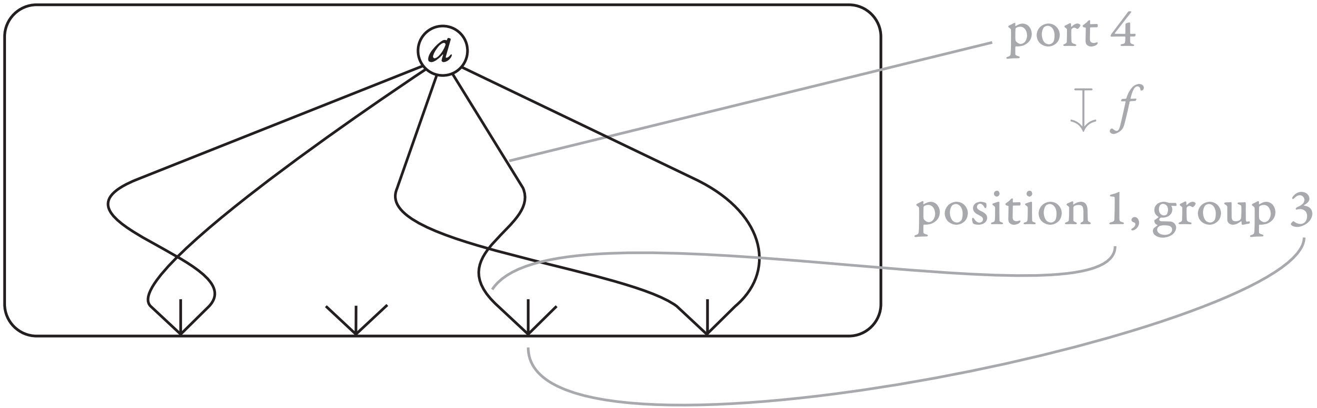
λx
@
@
@
x
variable x is bound in the root
the original port is replaced by x



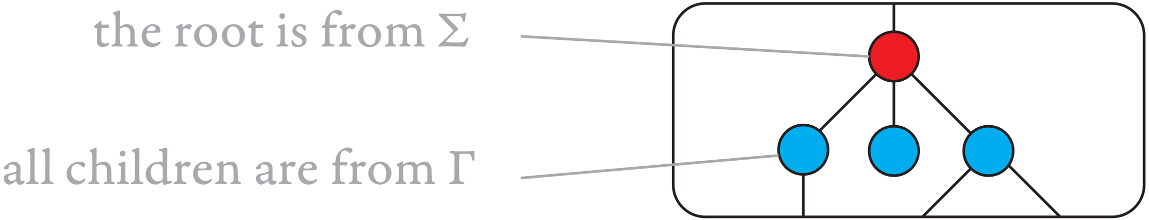
dangling edges
represent ports

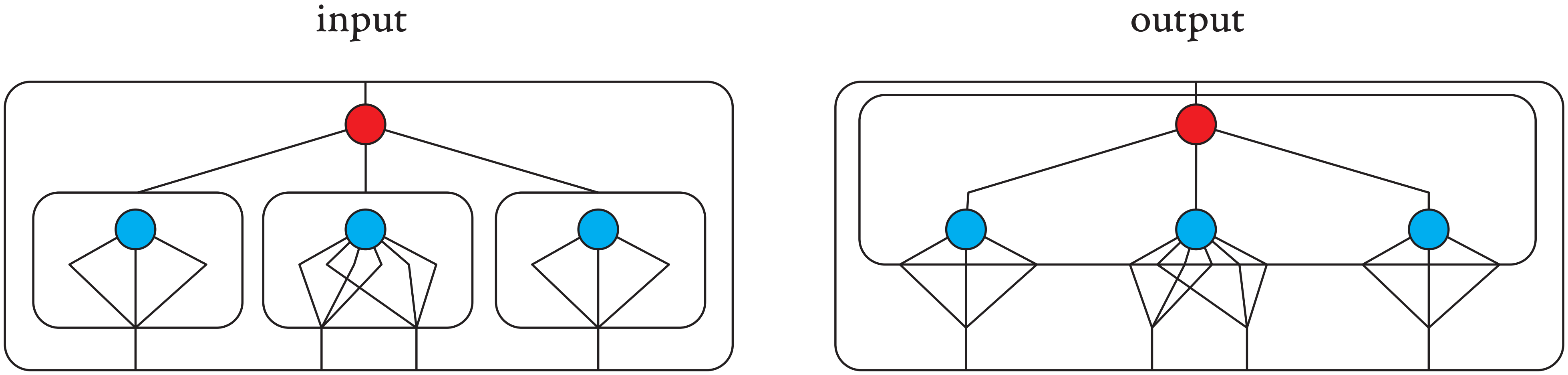


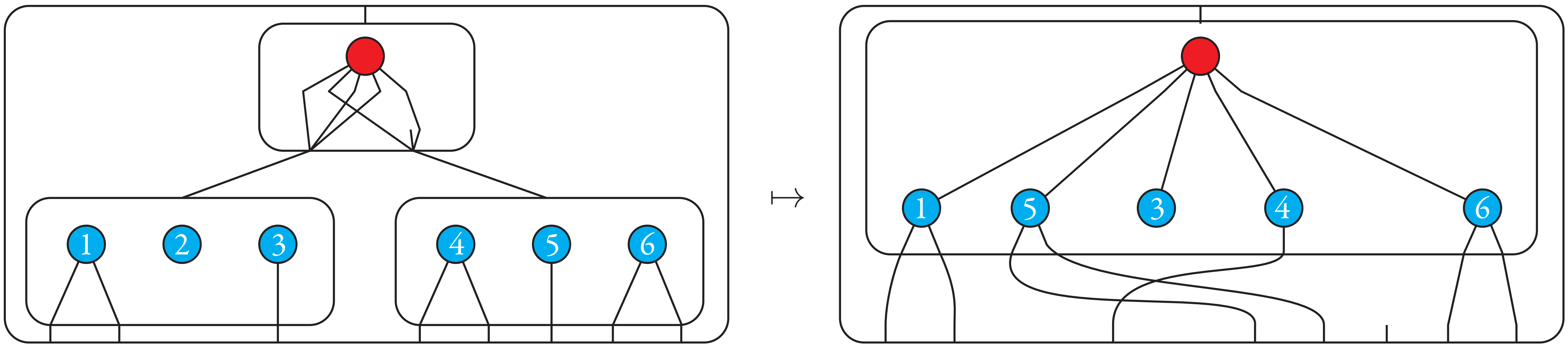
a_1
a_2

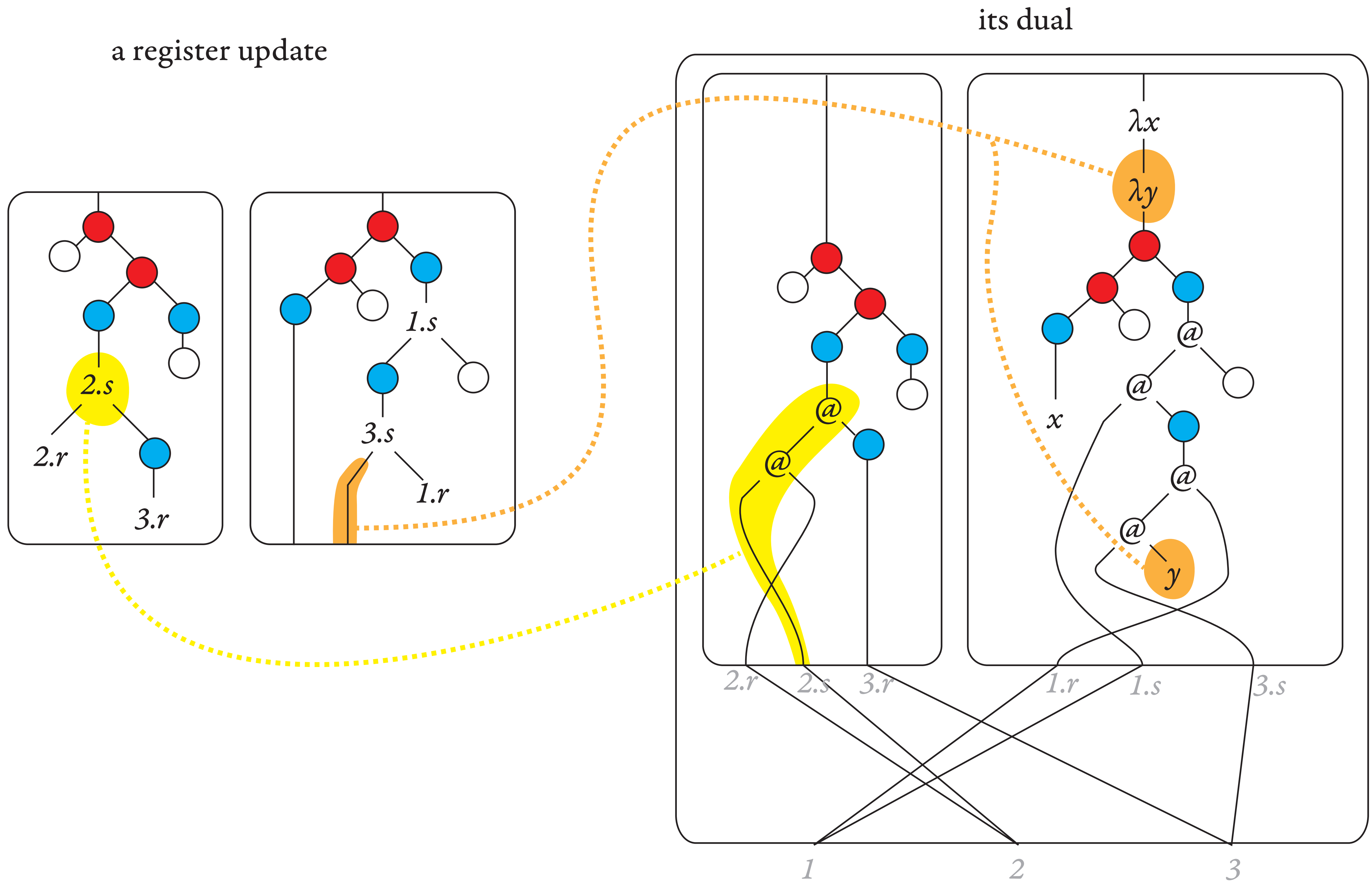


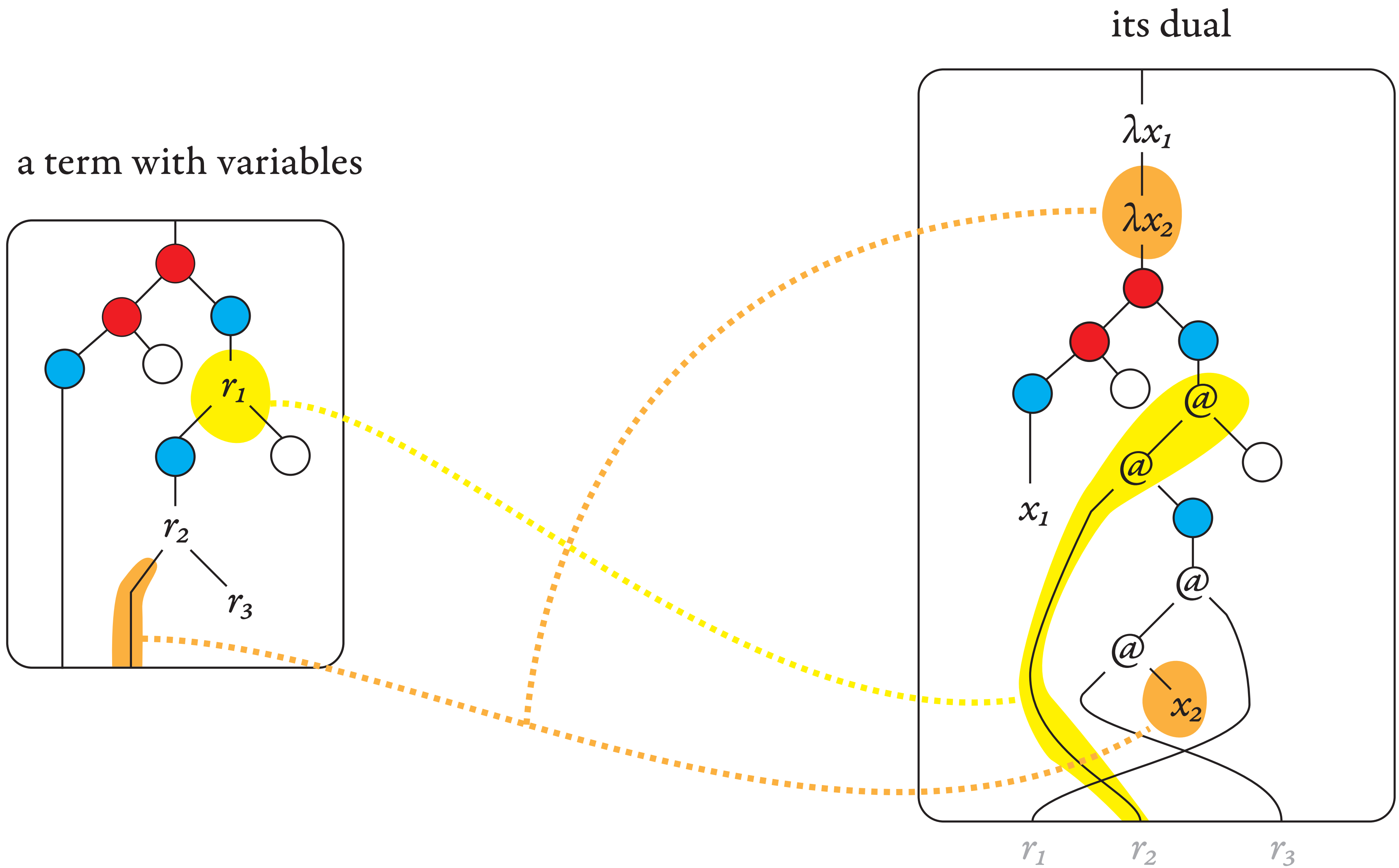
a
port 4
↧ f
position 1, group 3


the root is from Σ
all children are from Γ

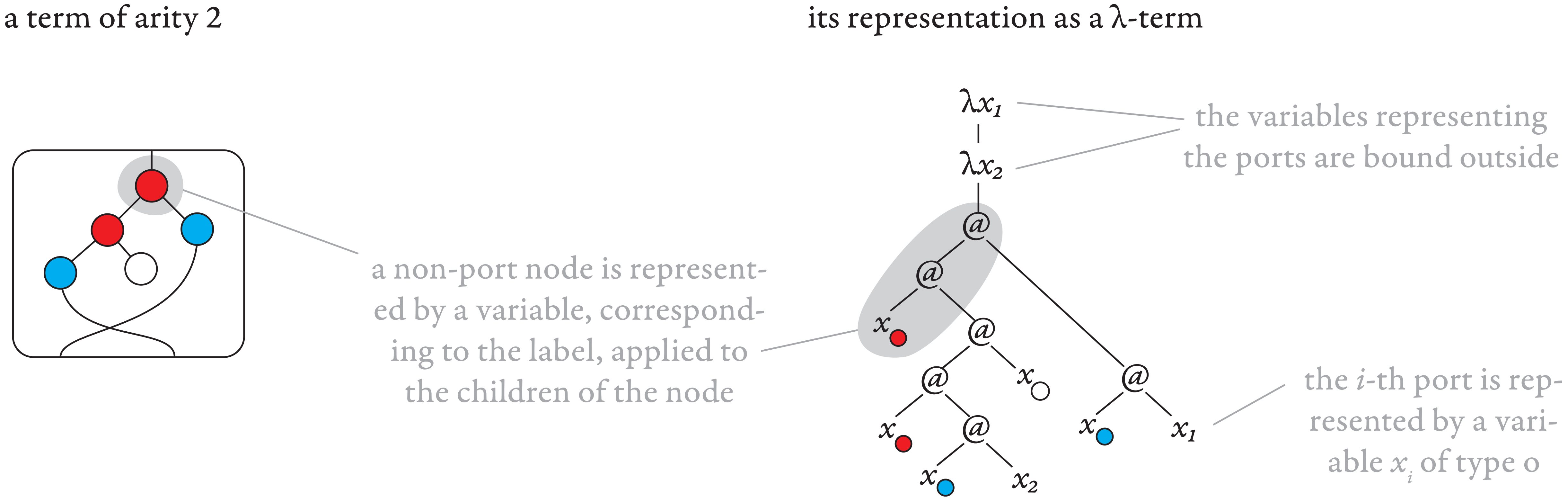

input
output

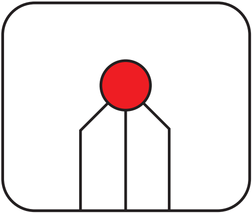

1
2
3
4
5
6
$\mapsto$
1
5
3
4
6


a register update
its dual
2.s
2.r
3.r
1.s
3.s
1.r
λx
λy
@
x
y
2.r
2.s
3.r
1.r
1.s
3.s
1
2
3

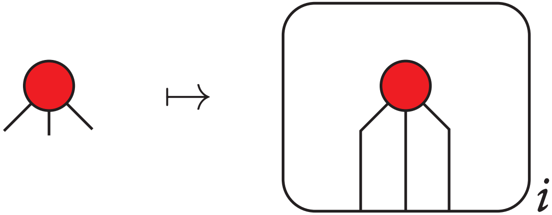


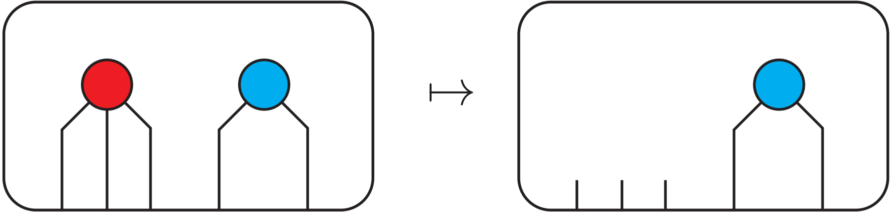
a term with variables
its dual
r1
r2
r3
λx1
λx2
@
x1
x2
r1
r2
r3

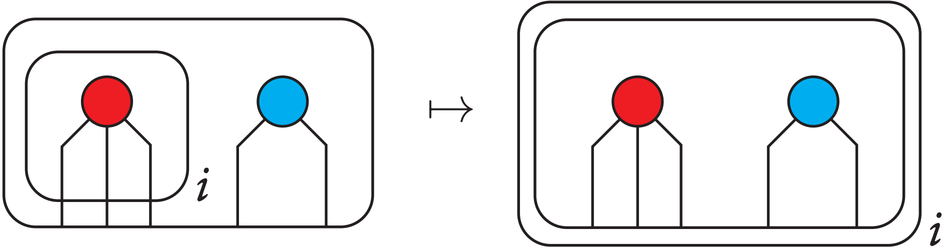

a term of arity 2
its representation as a λ-term
λx_1
λx_2
the variables representing the ports are bound outside
@
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node
x_1
x_2
the i-th port is represented by a variable x_i of type o

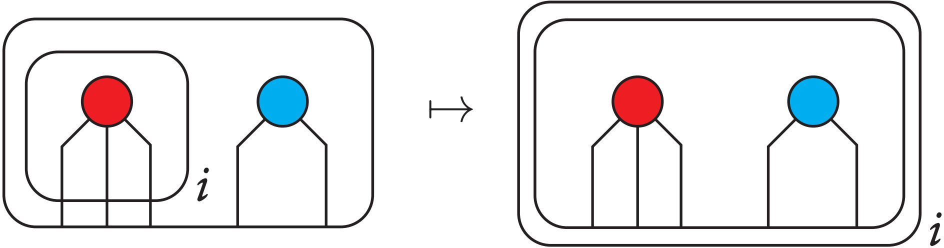


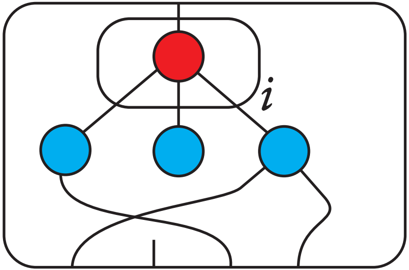
i


i

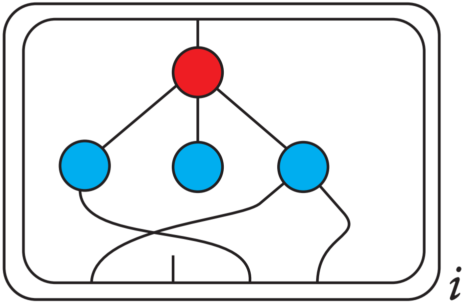


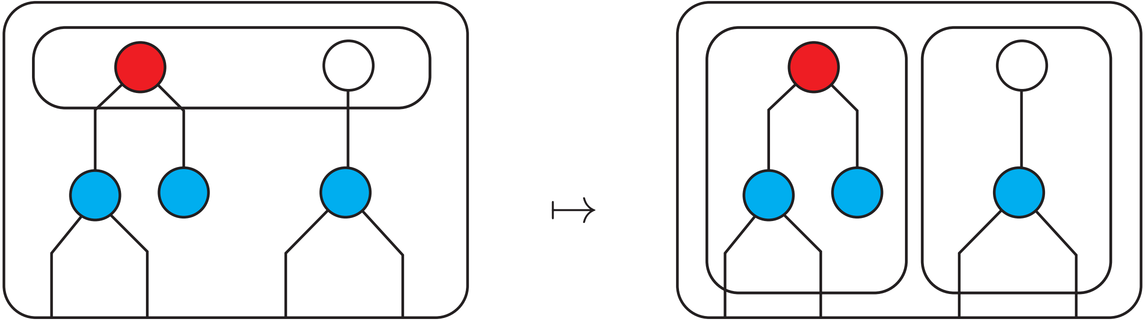

i
i

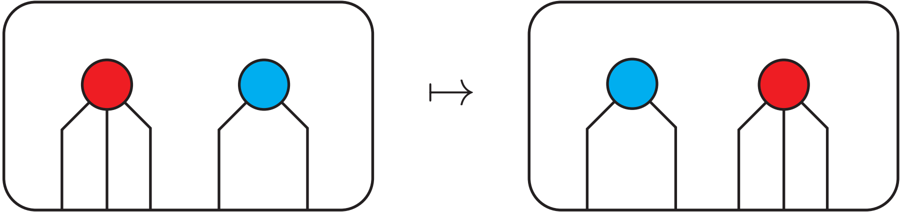

i
i

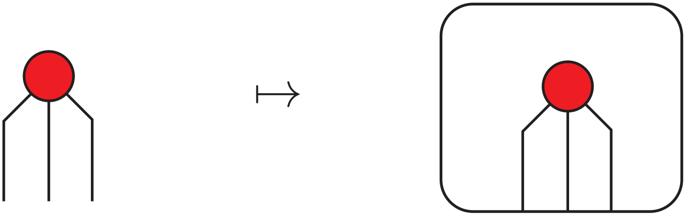


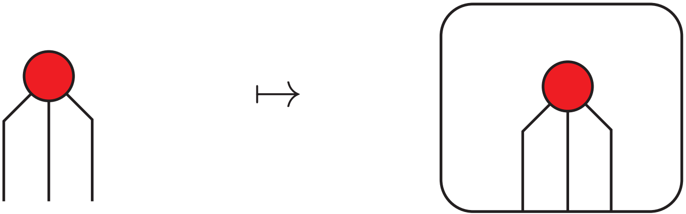
$\mapsto$

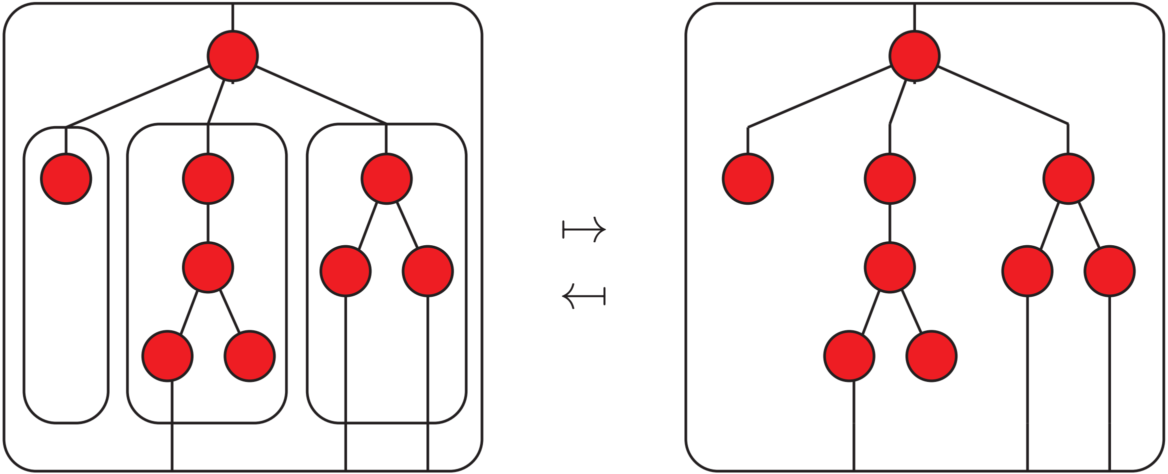


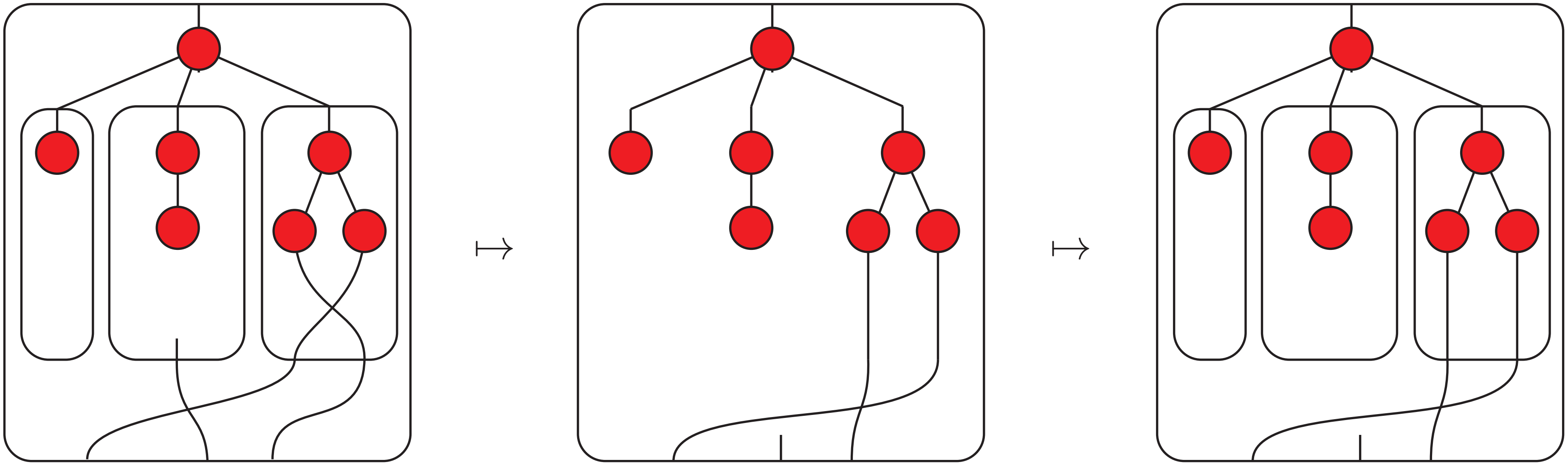


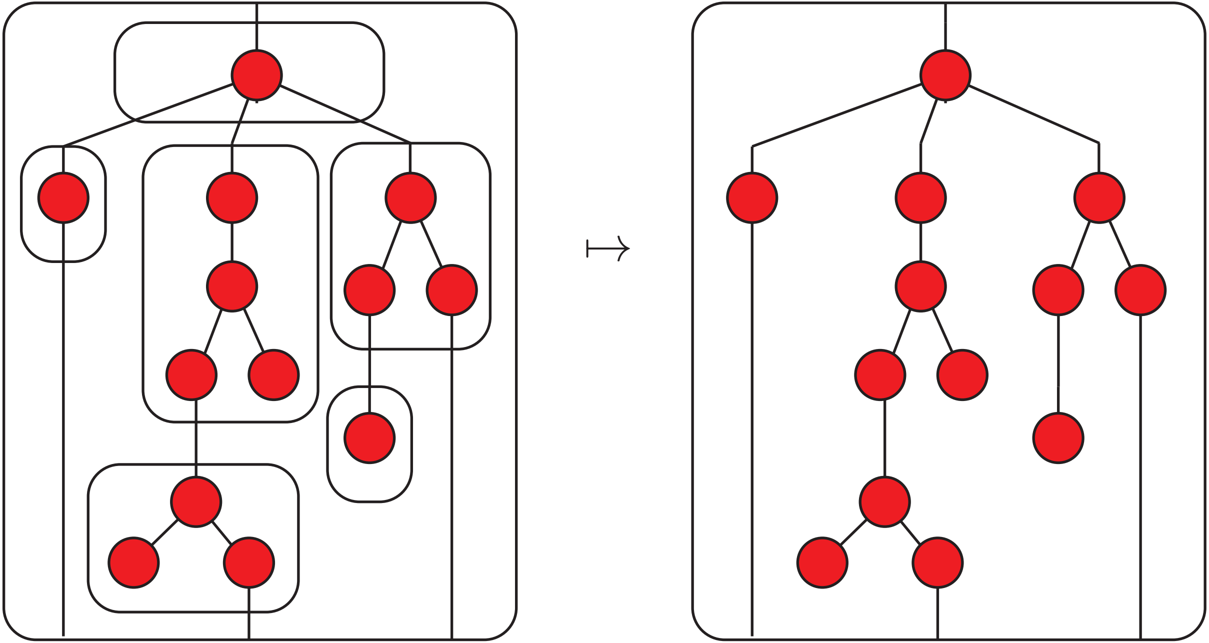


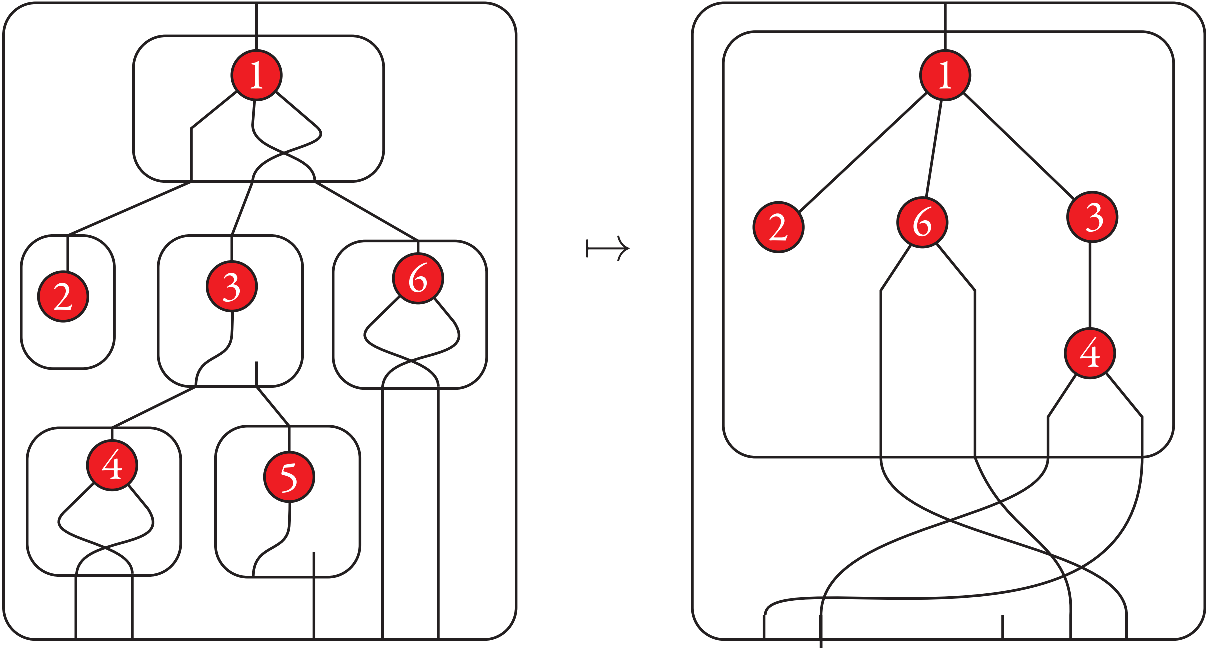




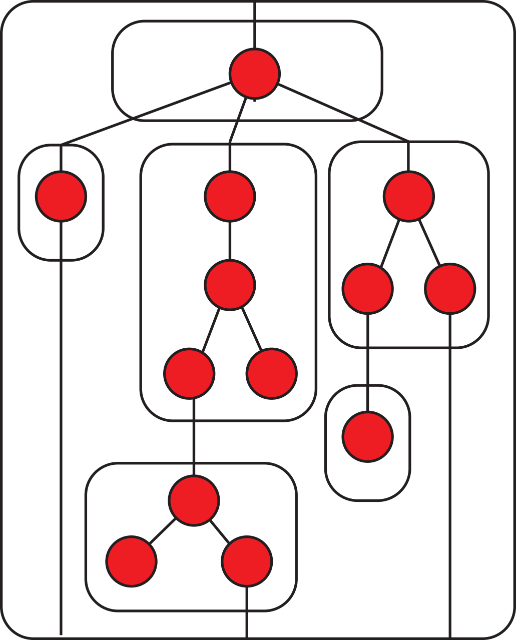


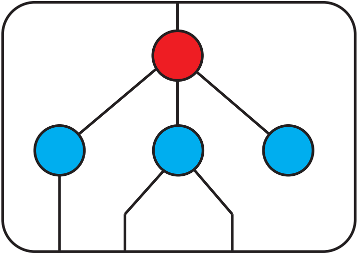


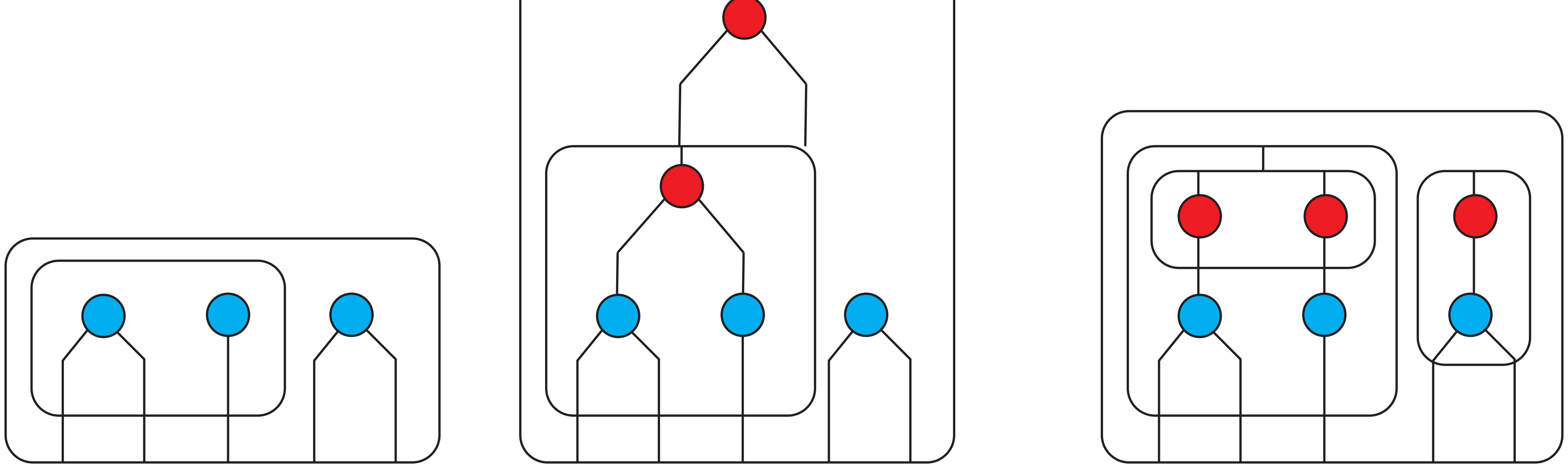


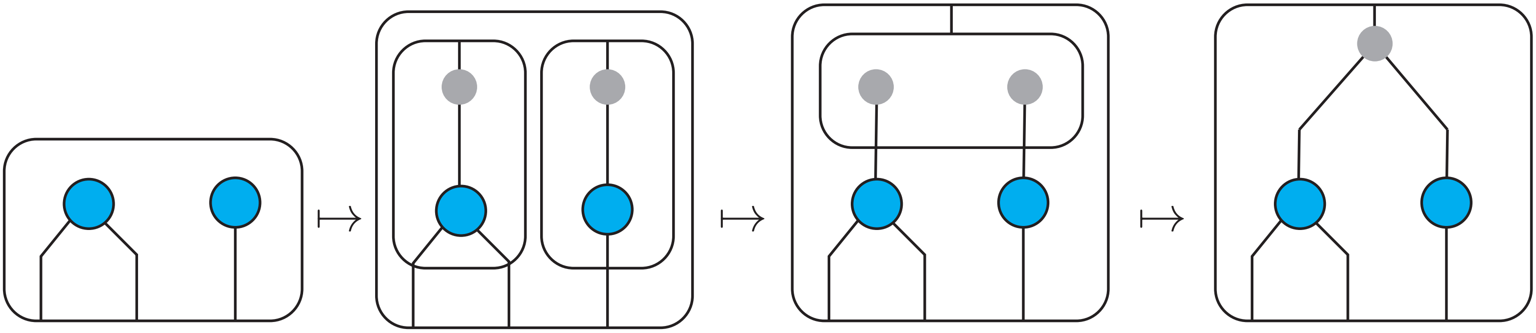


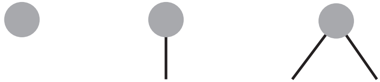
1
2
3
6
4
5
1
2
6
3
4

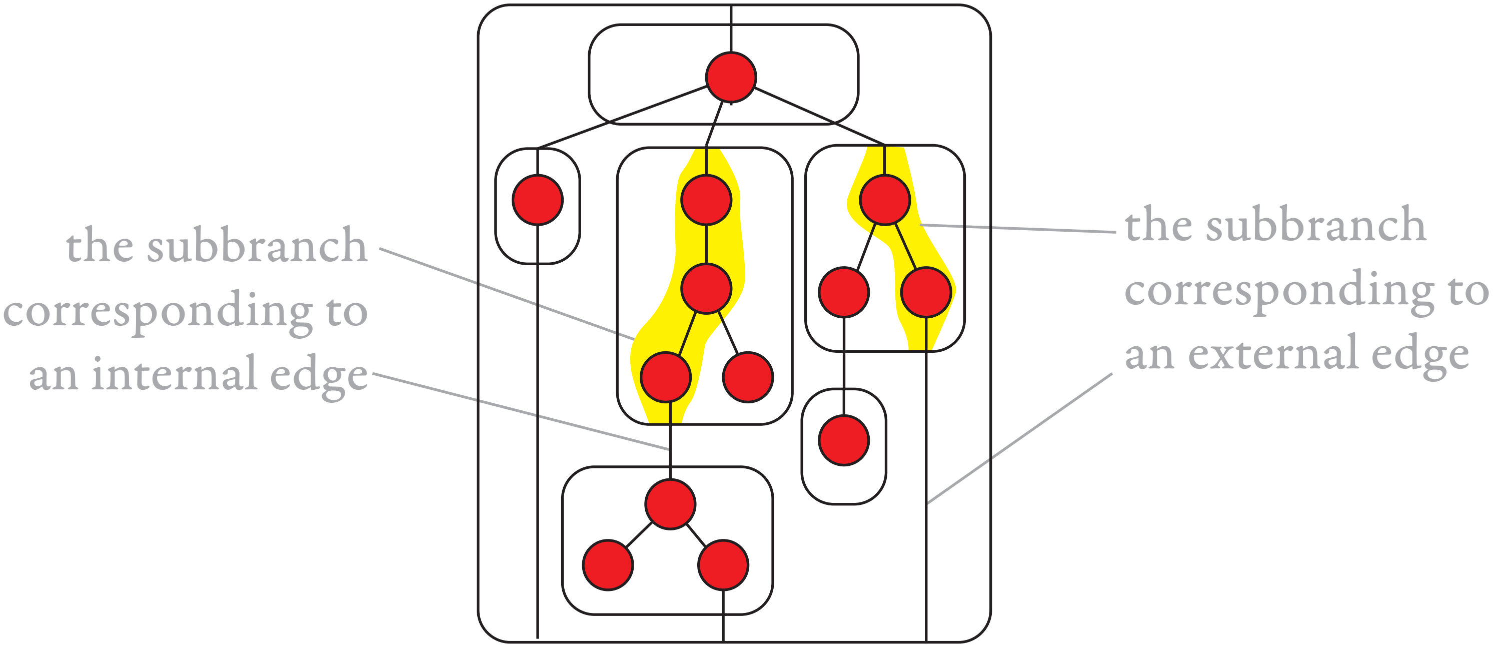


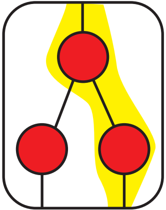




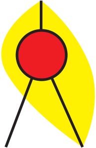


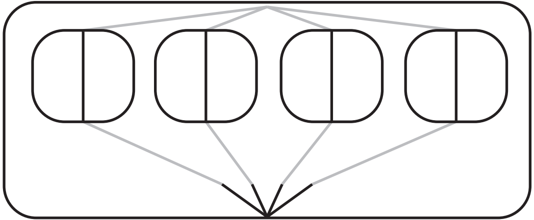



the subbranch corresponding to an internal edge
the subbranch corresponding to an external edge

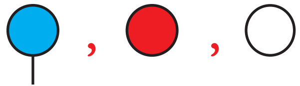


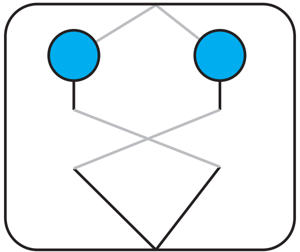
a branch can be visualised as
a term with a distinguished
root-to-port path

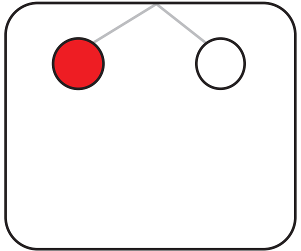


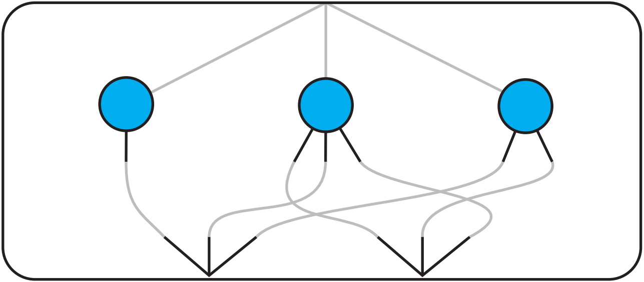


a tuple of *k* identity terms with all their ports folded into one

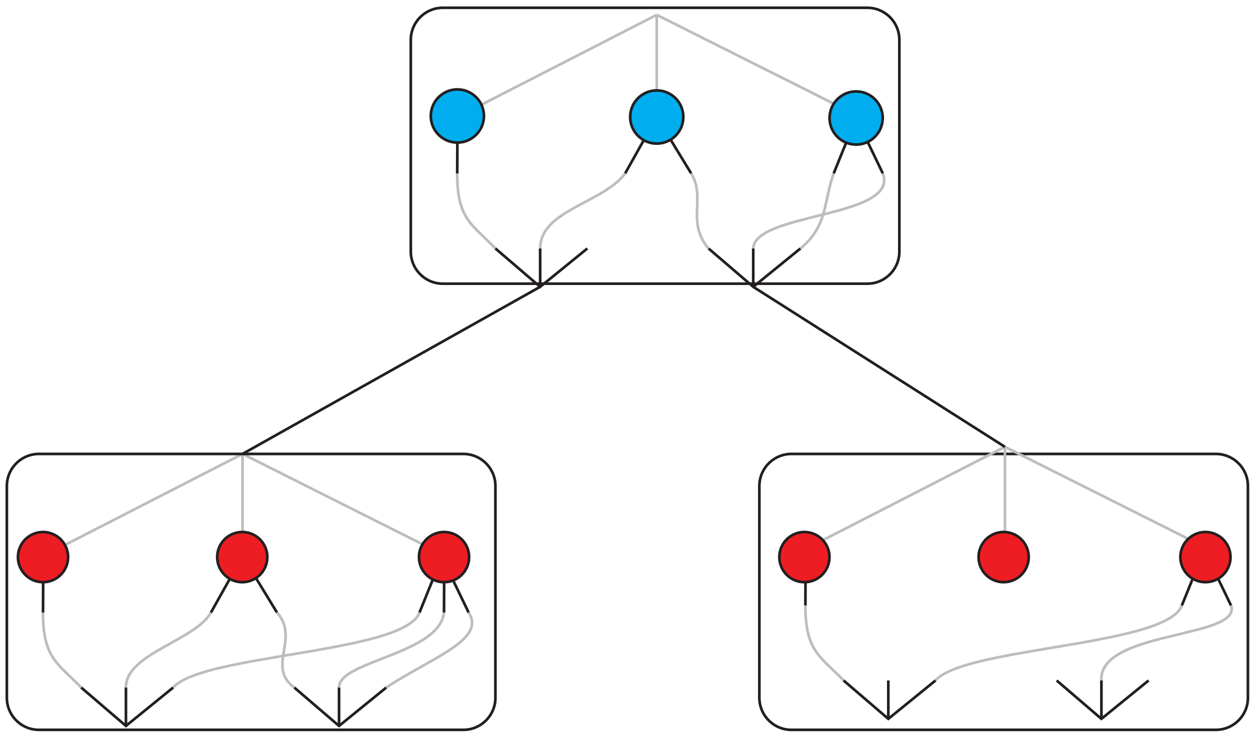
$\Sigma = \{$  $\}$

$a \in \Sigma^{[2]}$ 

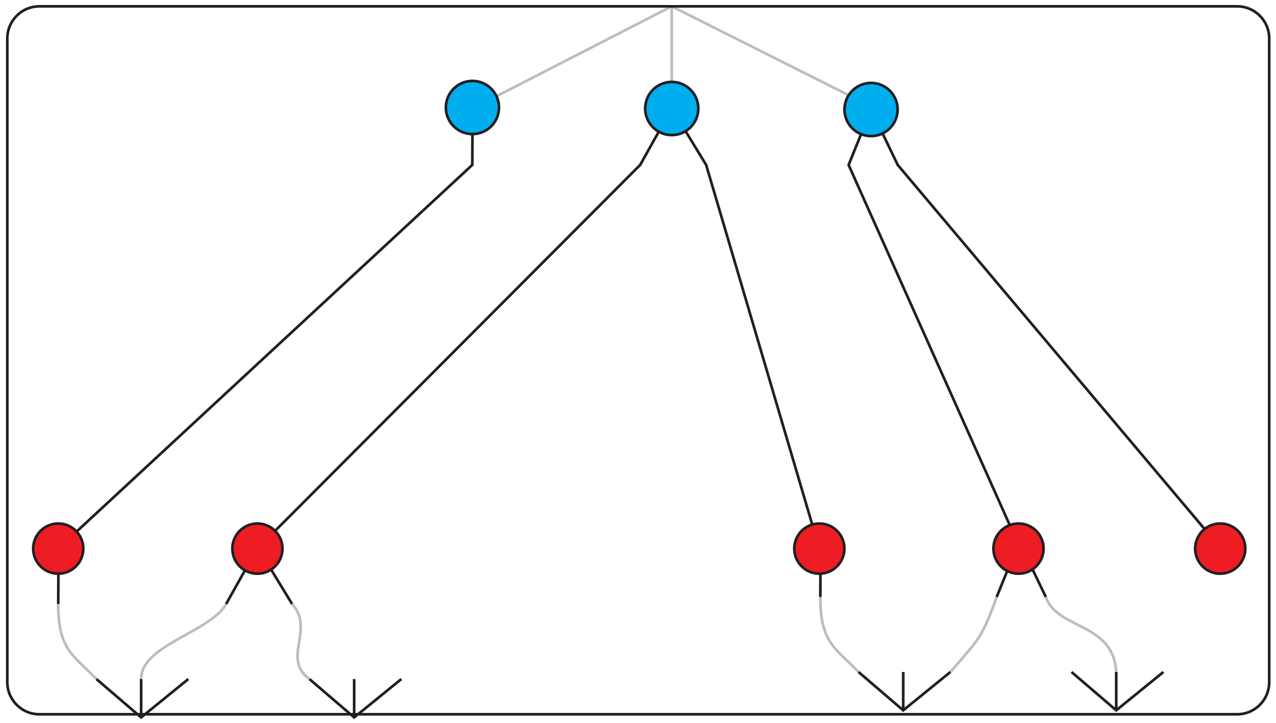
$b \in \Sigma^{[2]}$ 

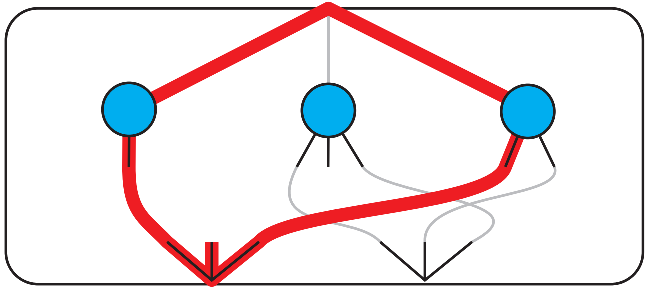


a shallow term of matrix powers



its shallow unfolding

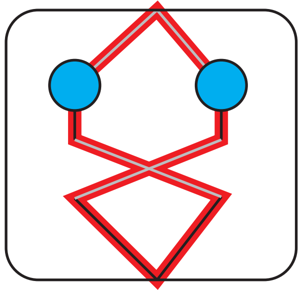




twist of port 1

1 2 3

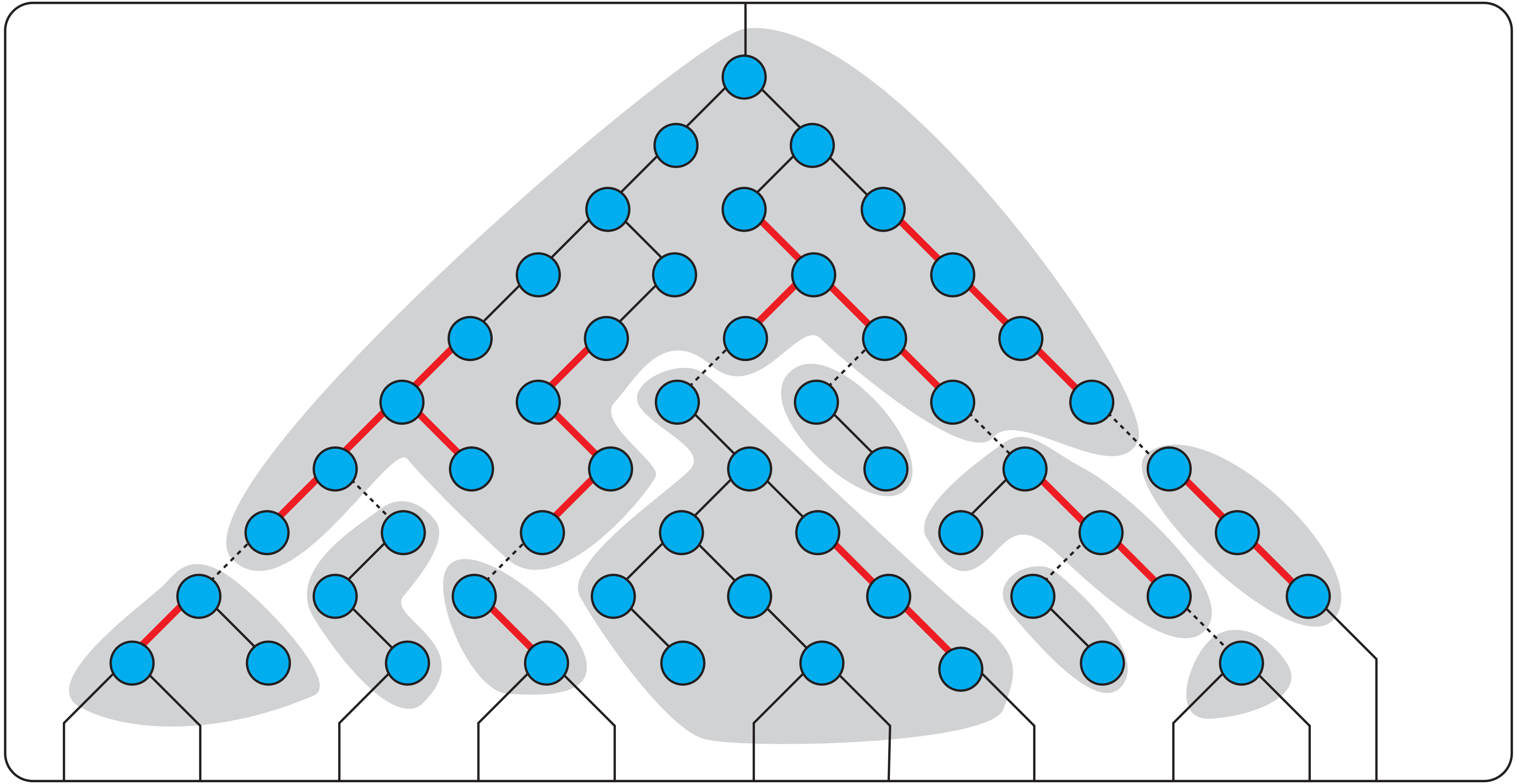
1 2 3

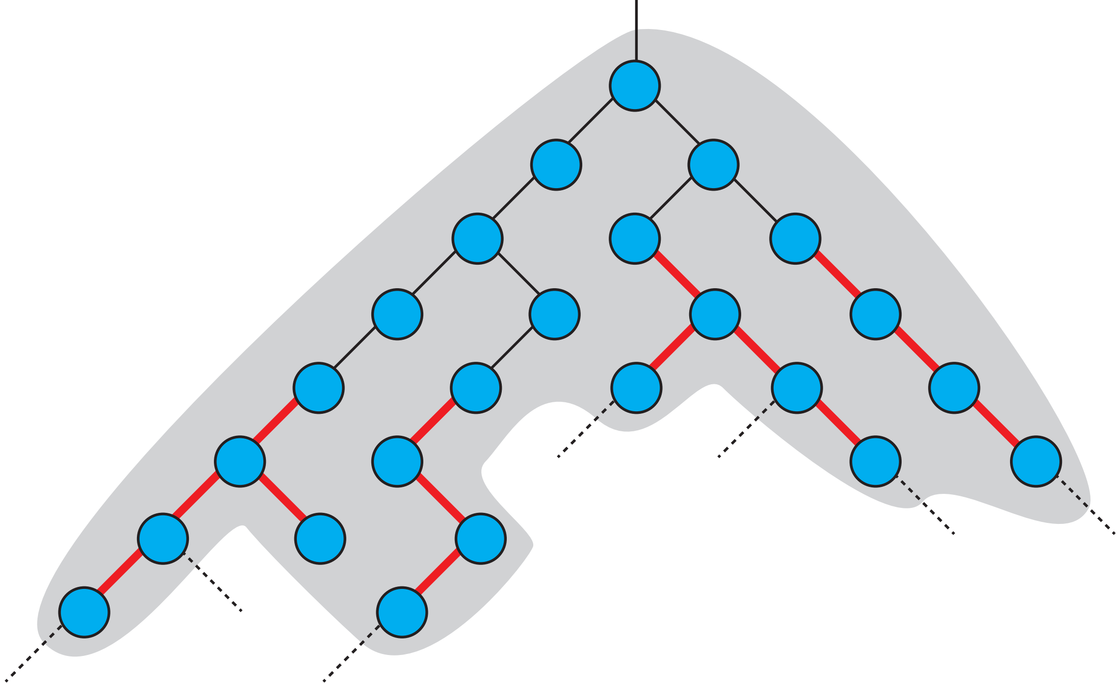


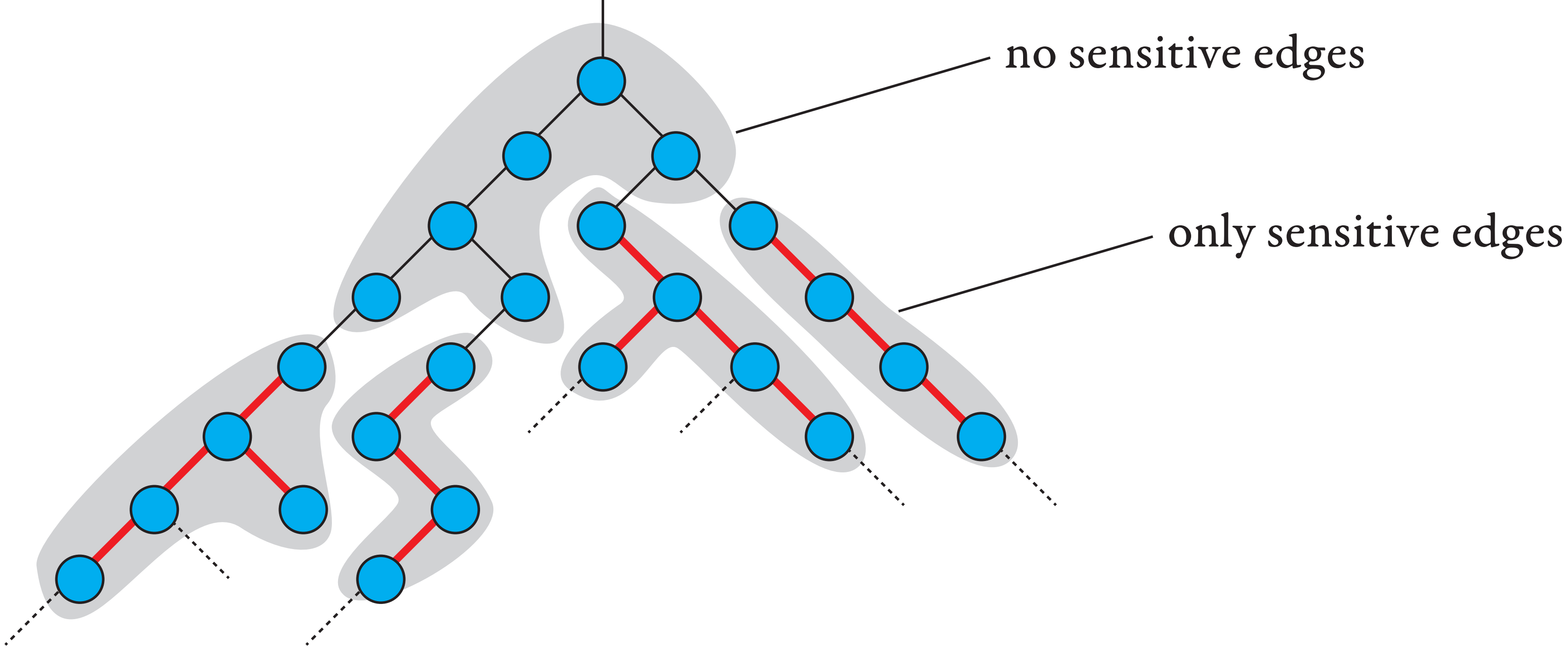
twist of port 1

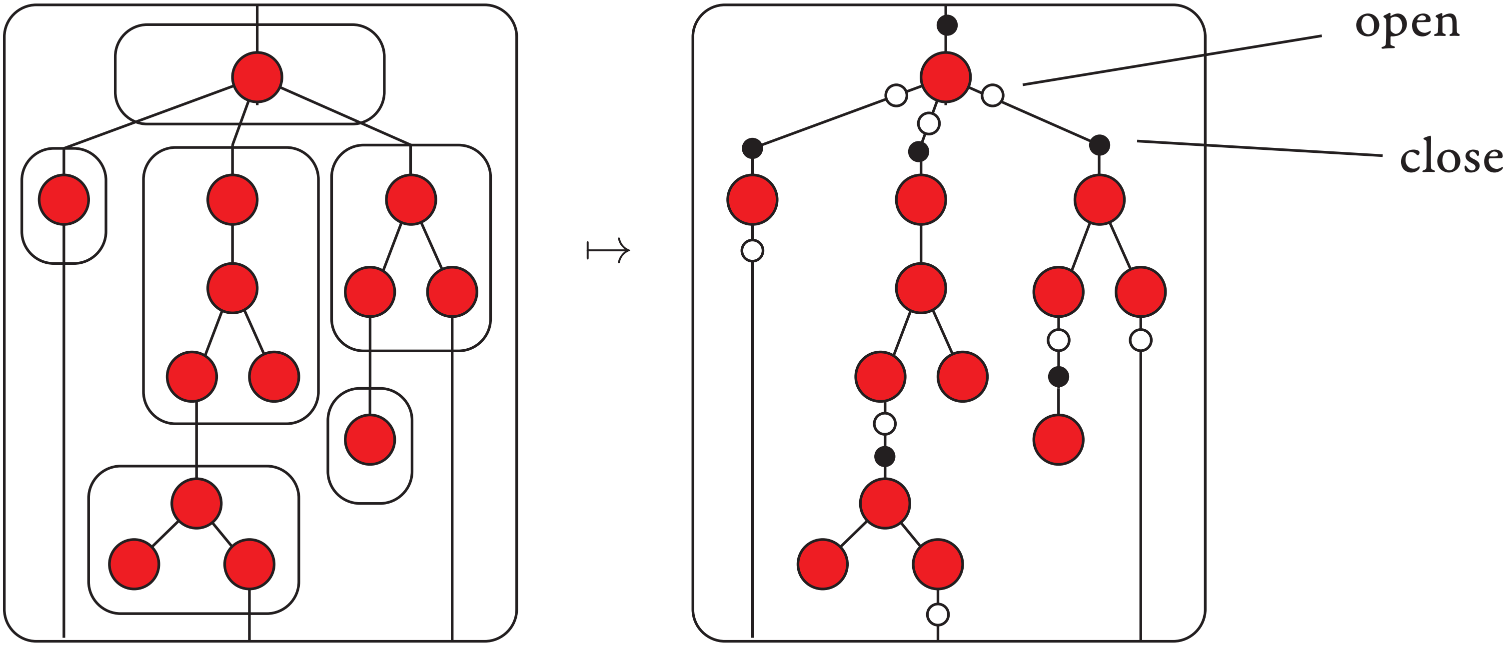
1 2

1 2

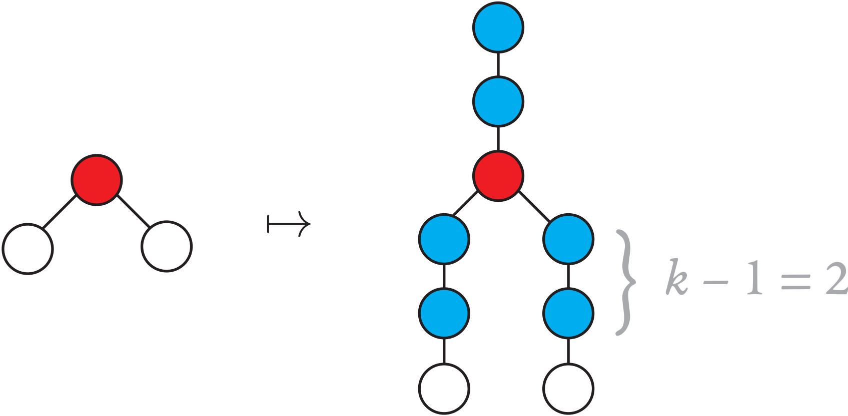


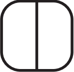


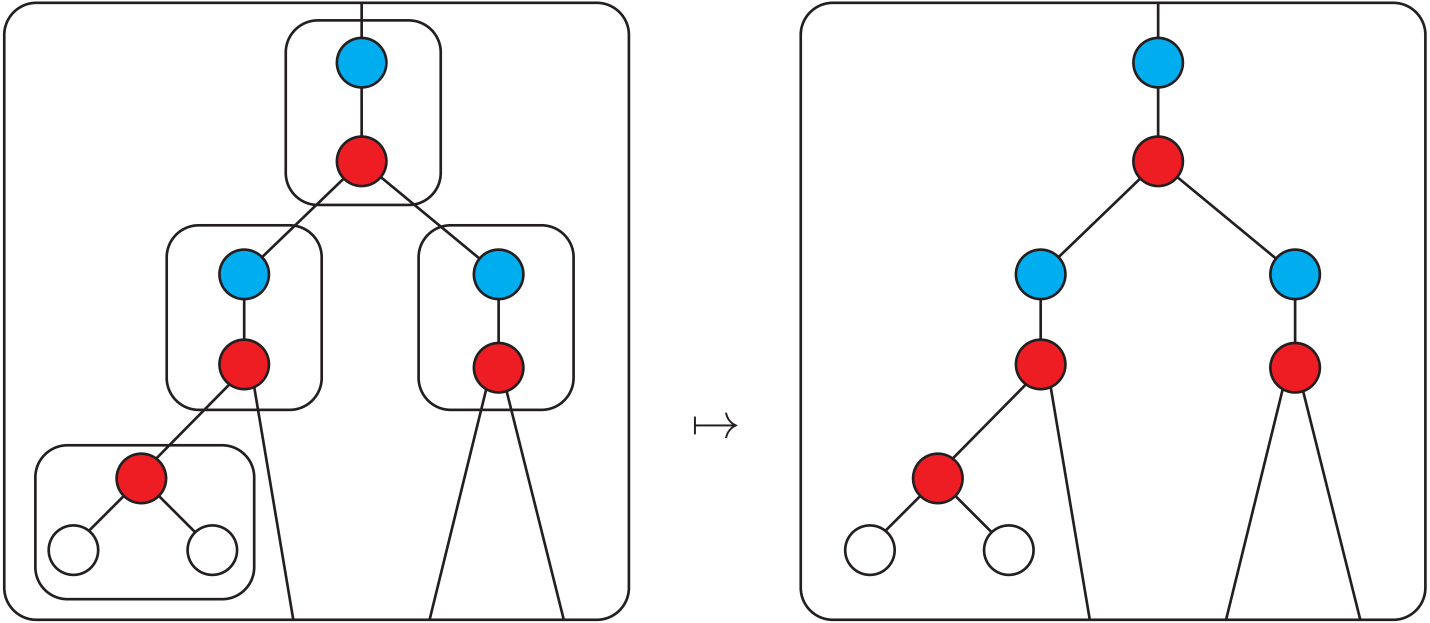

no sensitive edges
only sensitive edges

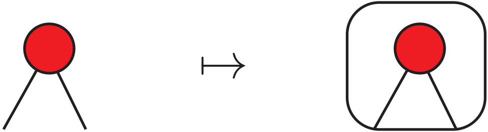


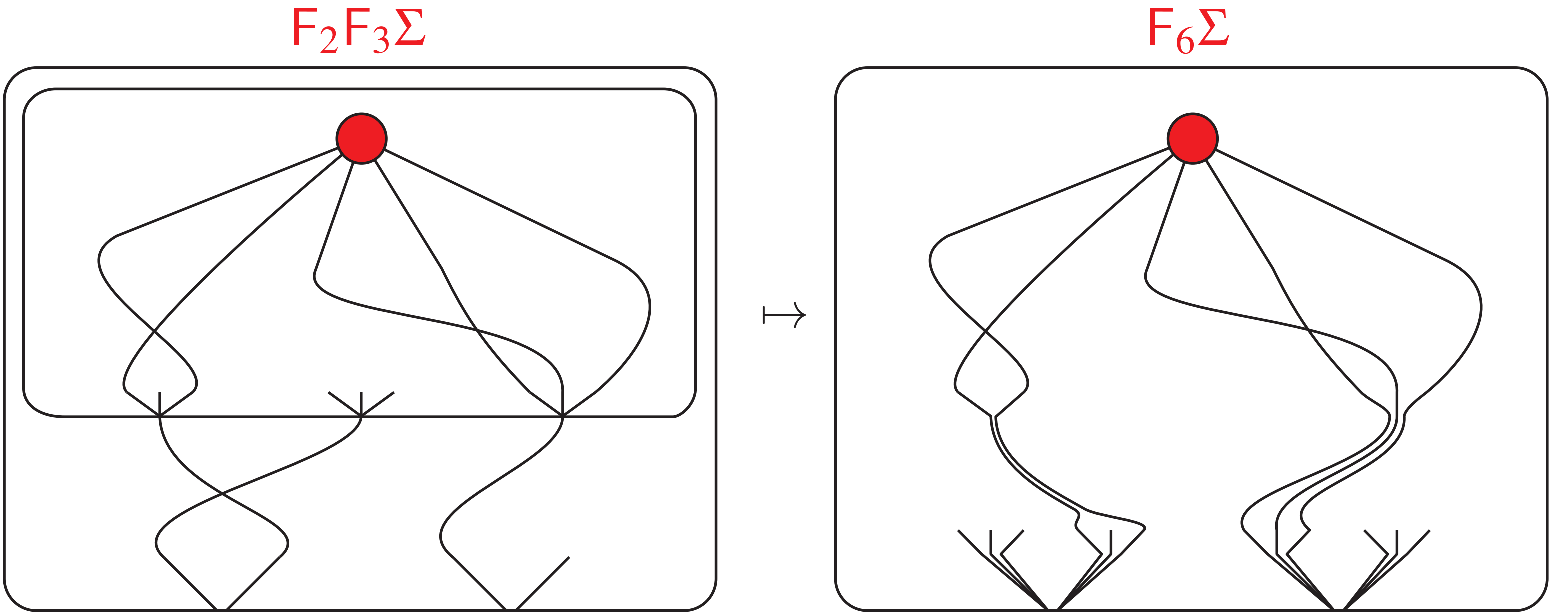
open
close

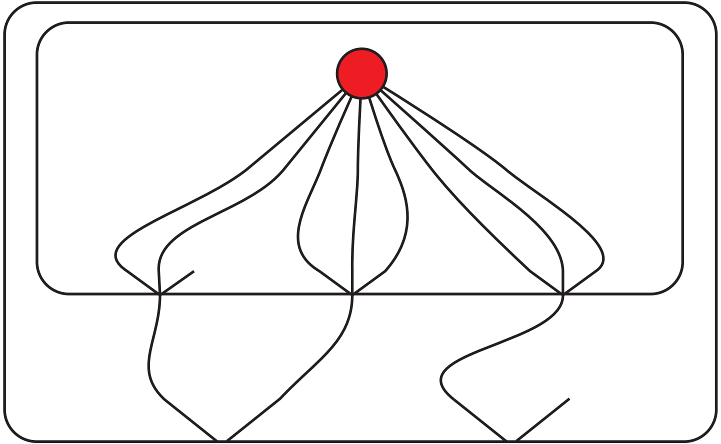

$k - 1 = 2$



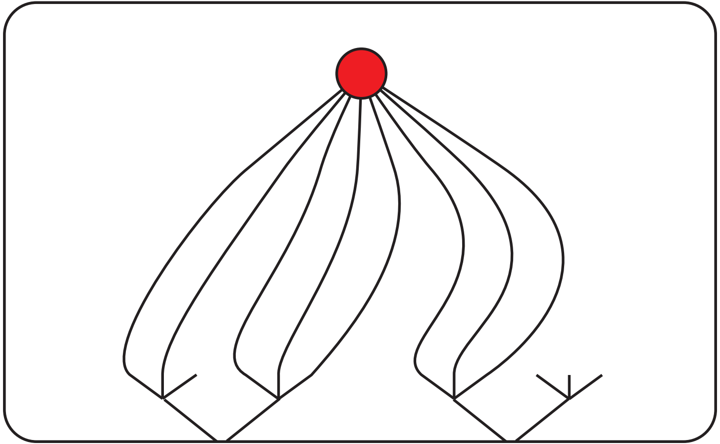


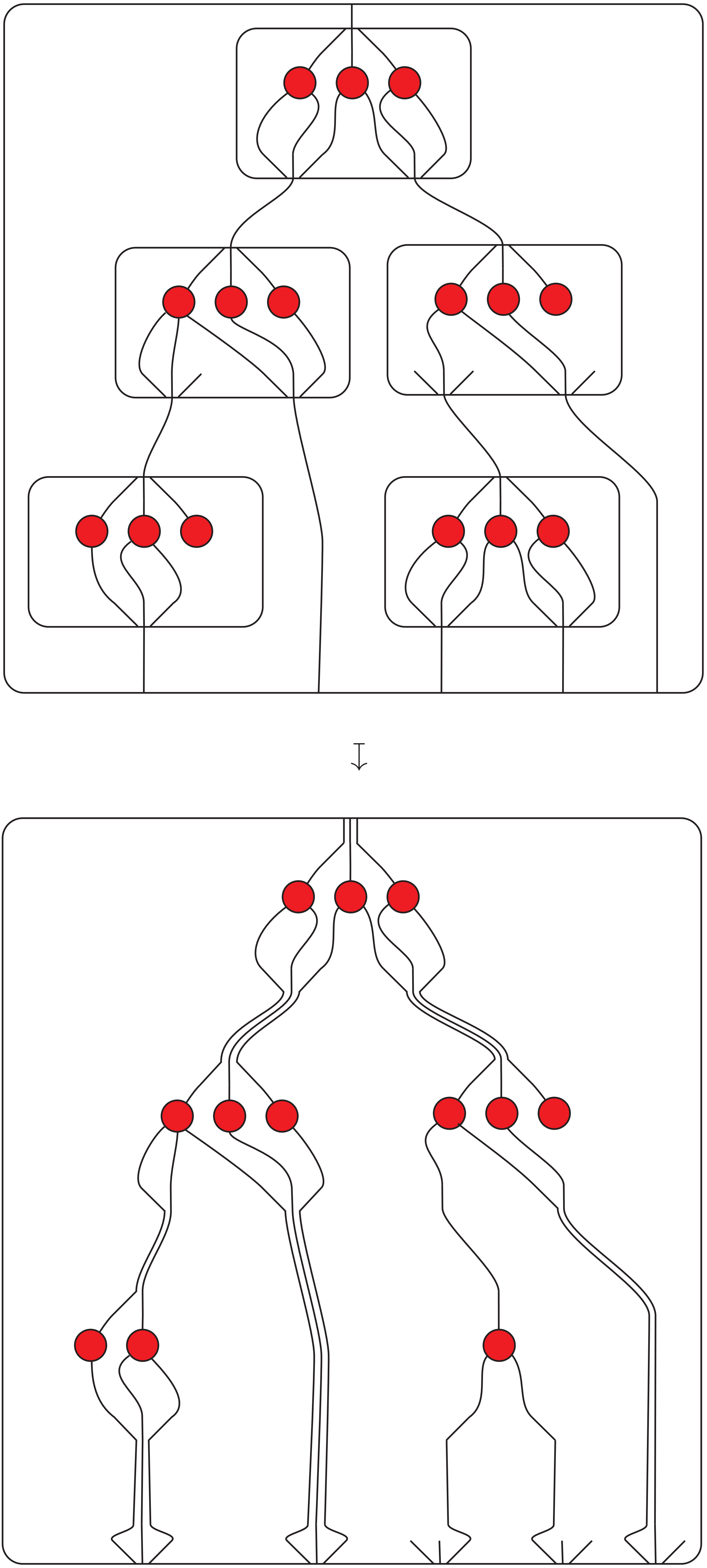



$F_2F_3\Sigma$
$\mapsto$
$F_6\Sigma$

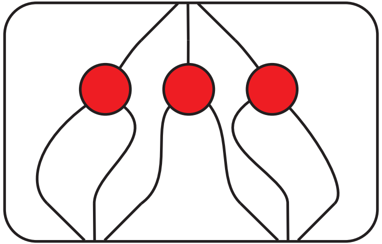


$\mapsto$

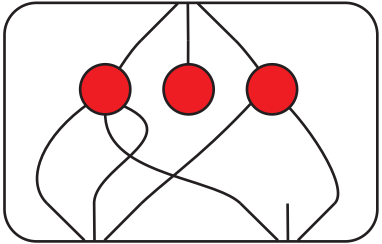




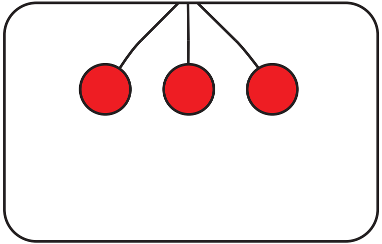
arity 2

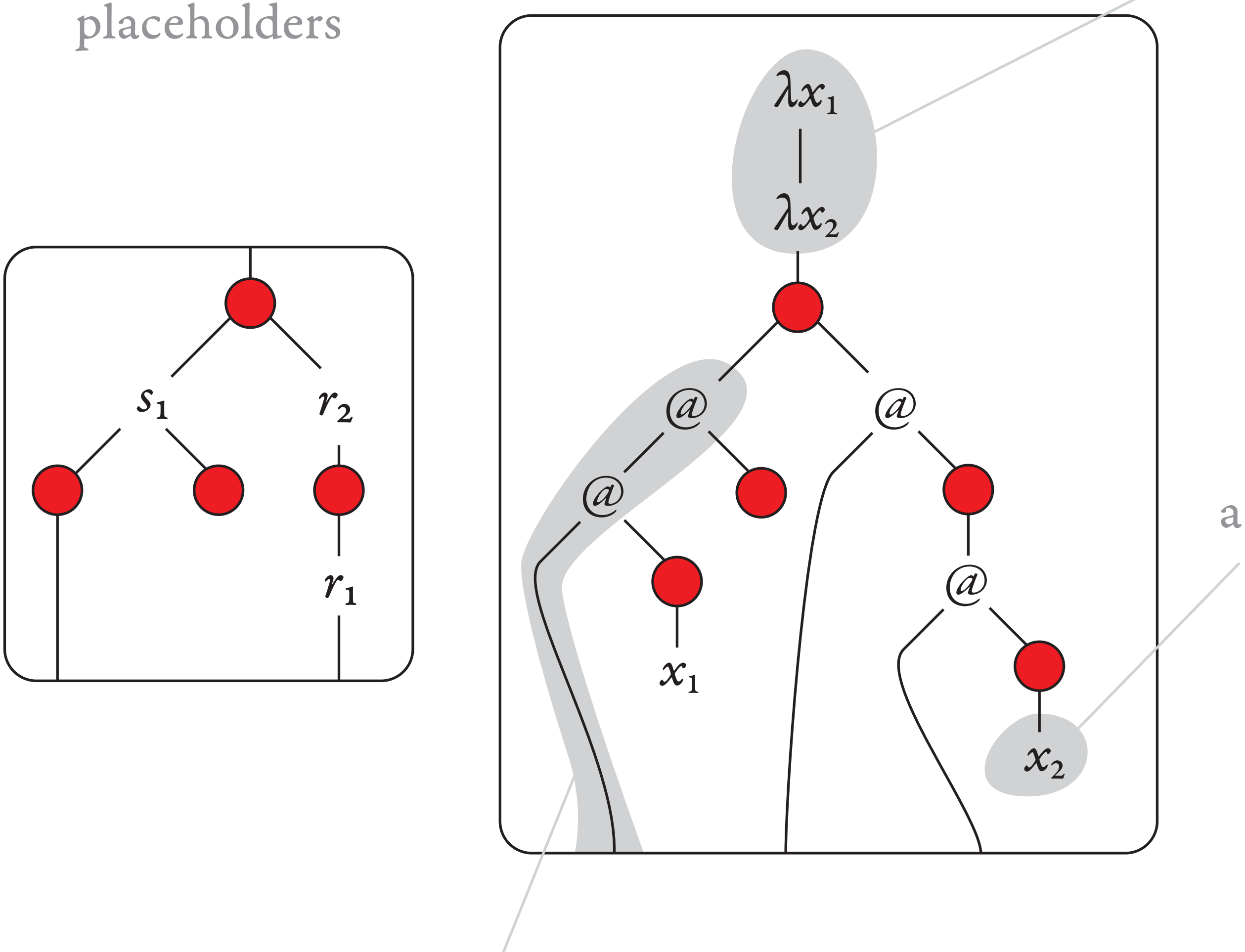


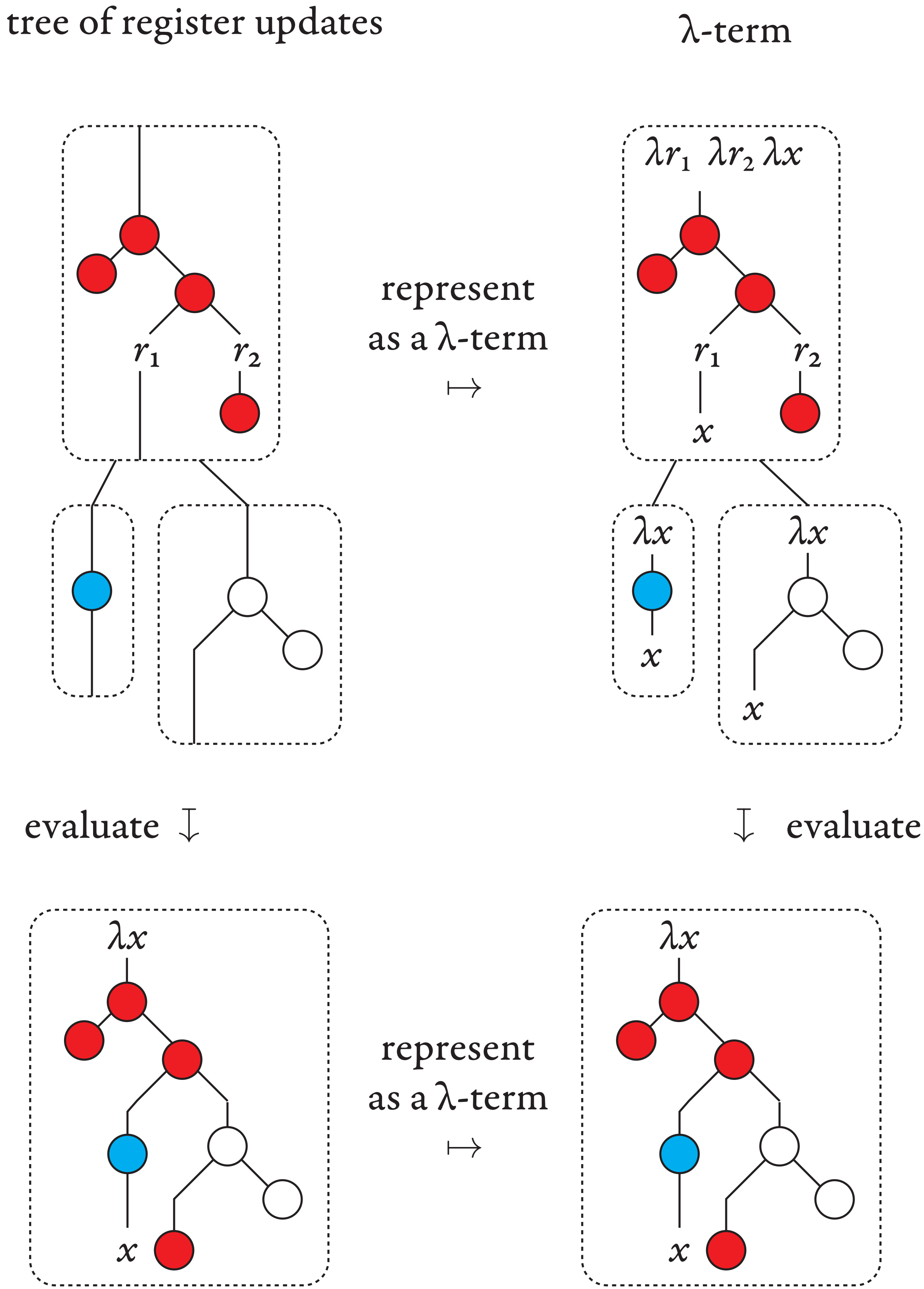
arity 2



arity 0

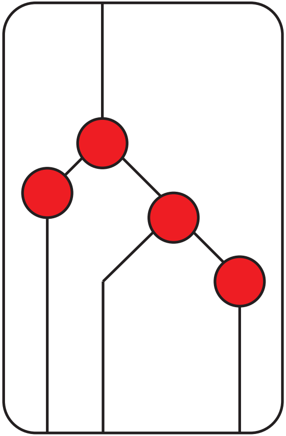



ipnput:
a term with
placeholders
output:
its λ-representation
one bound
variable
for each port
in the input
each port of
the input
is replaced by
a corresponding
variable
each placeholder in the input
is replaced by a port applied to its children using @
s_1
r_2
r_1
λx_1
λx_2
@
@
@
@
x_1
x_2

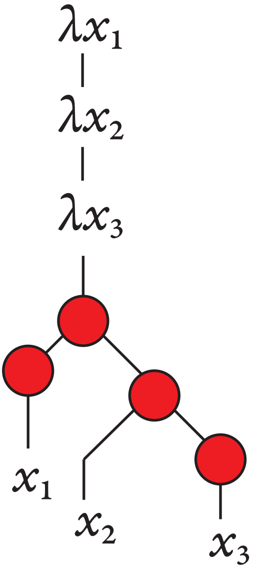


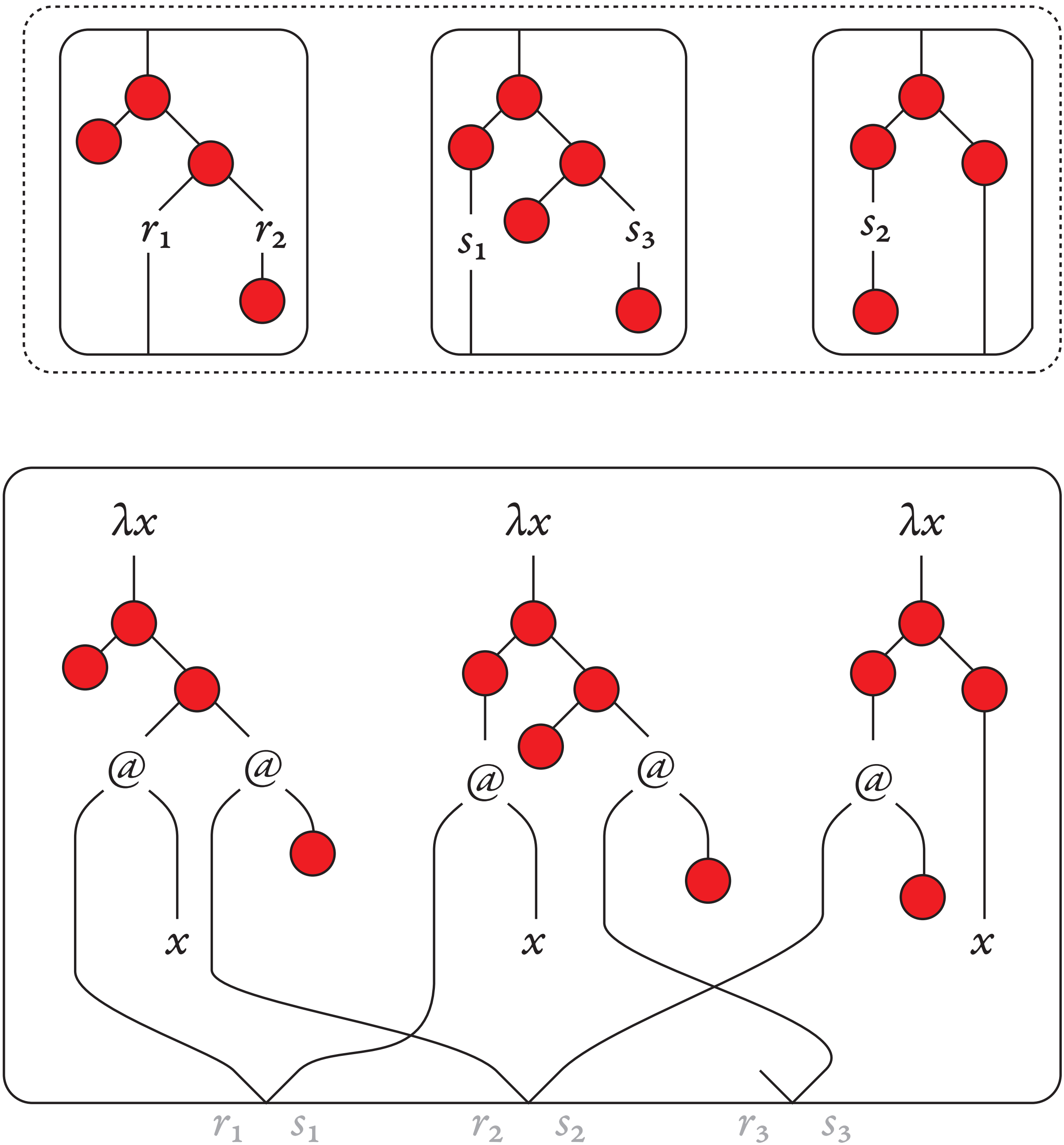
tree of register updates
λ-term
λr₁ λr₂ λx
r₁
r₂
x
represent
as a λ-term
↦
λx
x
evaluate ↧
↧ evaluate
represent
as a λ-term
↦

a term

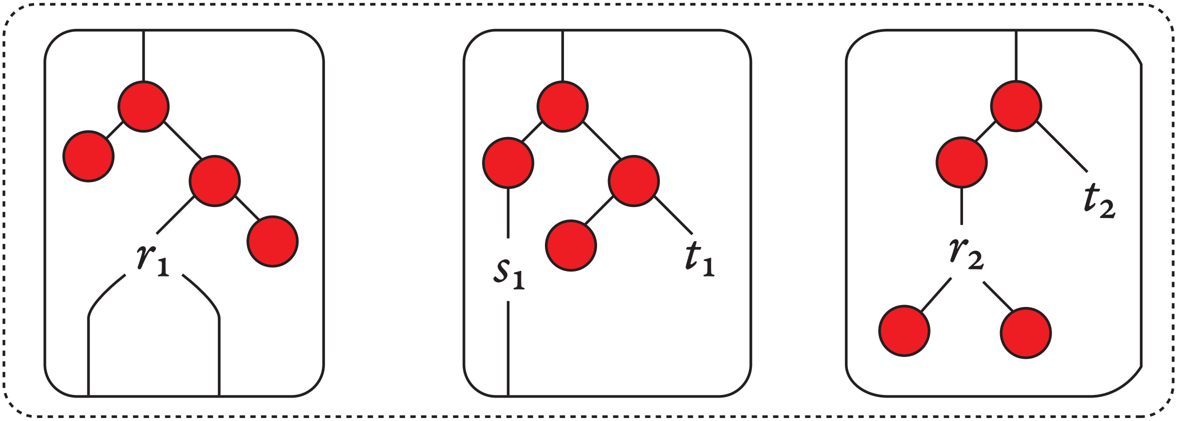


its λ-representation

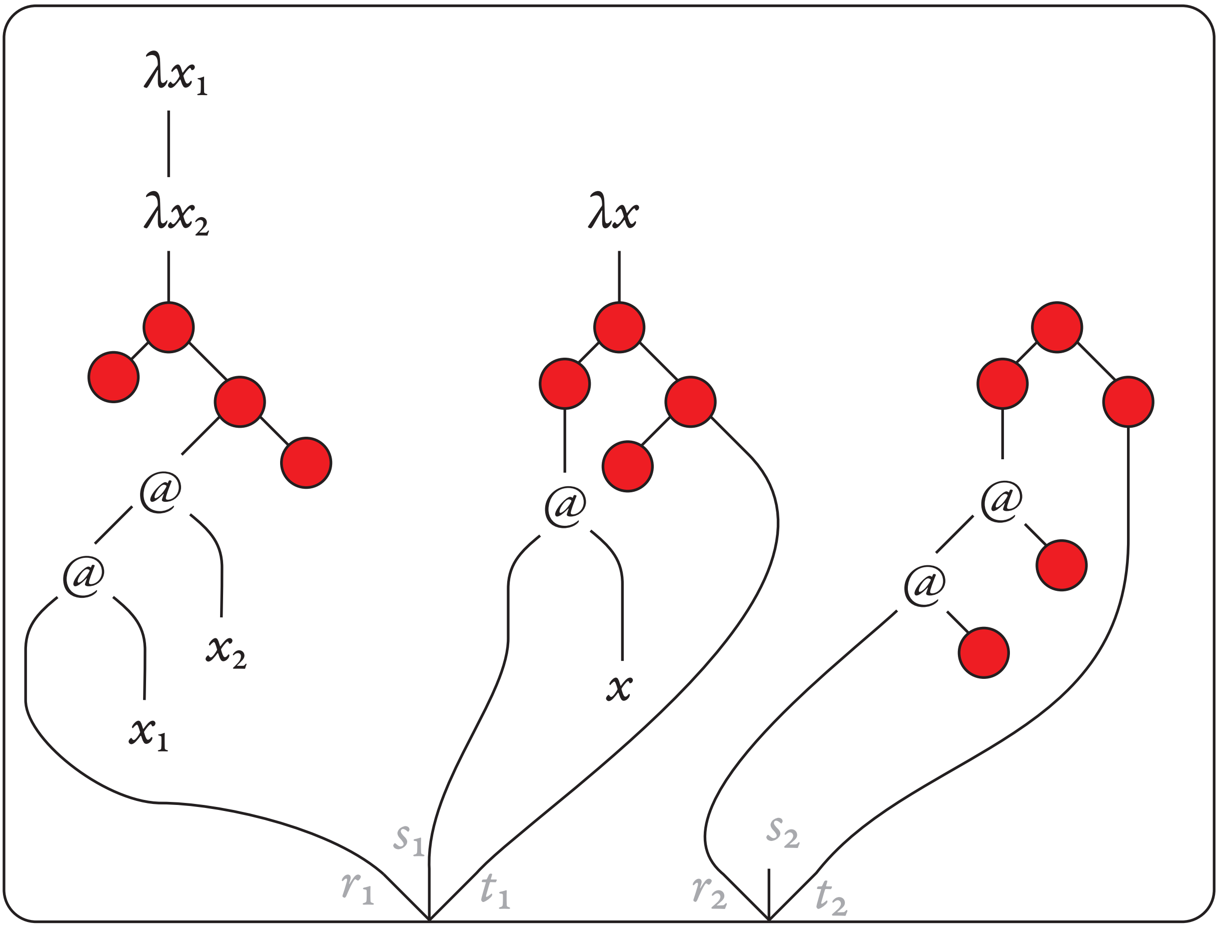


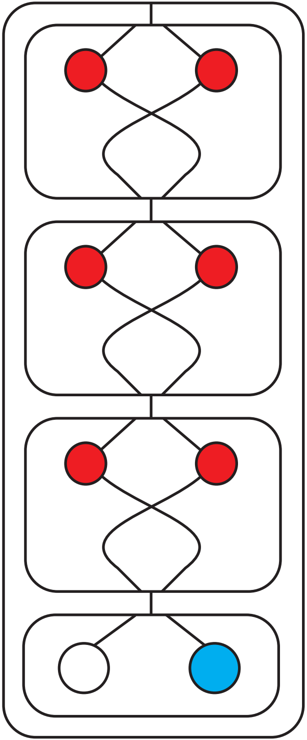

r_1
r_2
s_1
s_3
s_2
λx
λx
λx
@
@
@
@
@
x
x
x
r_1
s_1
r_2
s_2
r_3
s_3

a register update

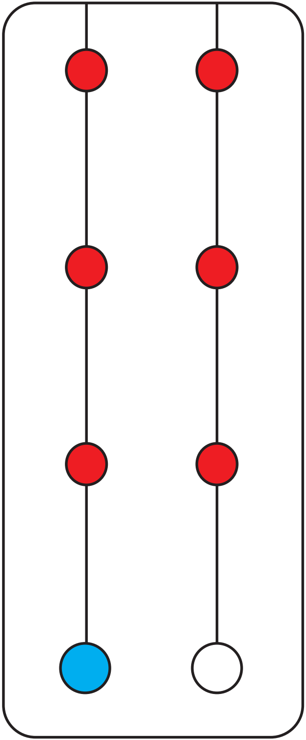


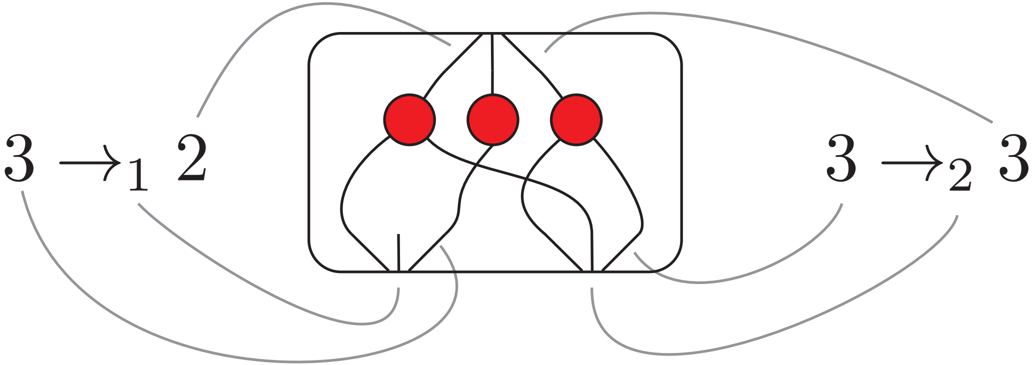
its λ-representation

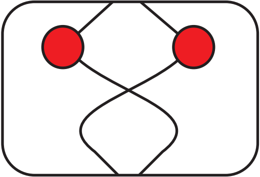




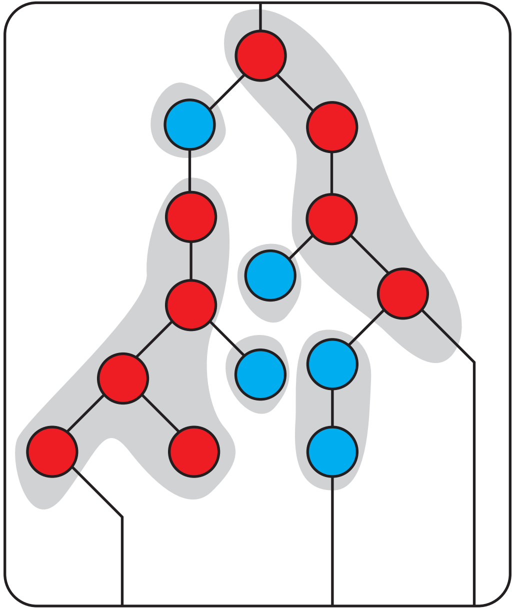
$\mapsto$



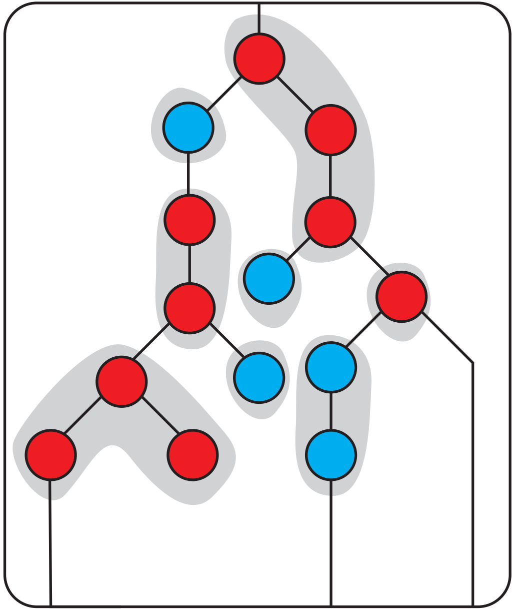

$3 \to_1 2$
$3 \to_2 3$

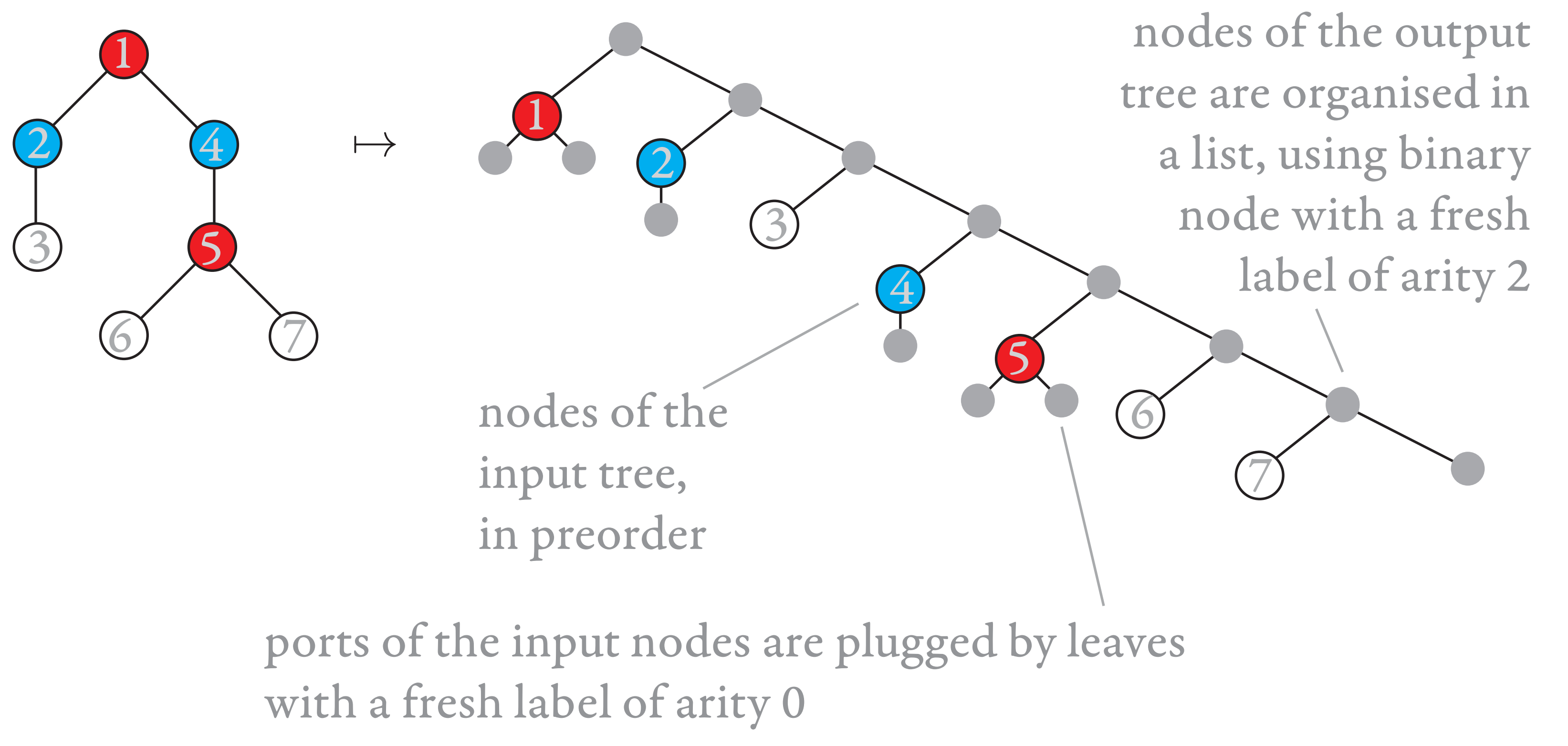


$\uparrow$-equivalence

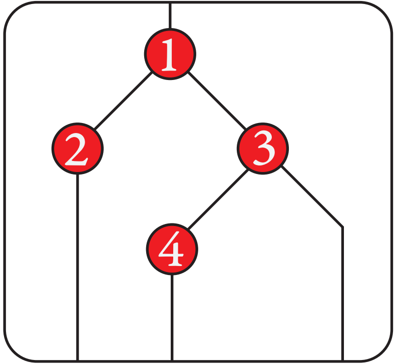


$\downarrow$-equivalence

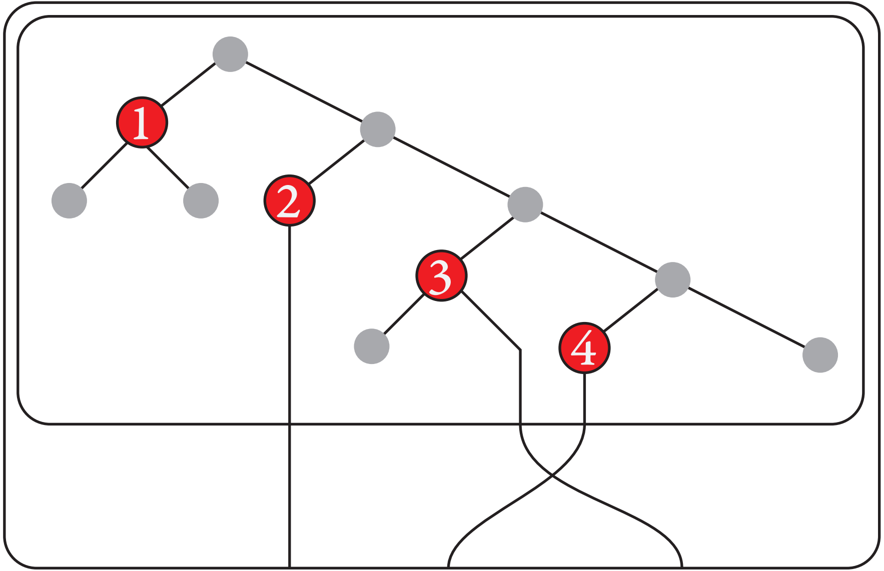




1
2
3
4
5
6
7
↦
nodes of the output tree are organised in a list, using binary node with a fresh label of arity 2
nodes of the input tree, in preorder
ports of the input nodes are plugged by leaves with a fresh label of arity 0

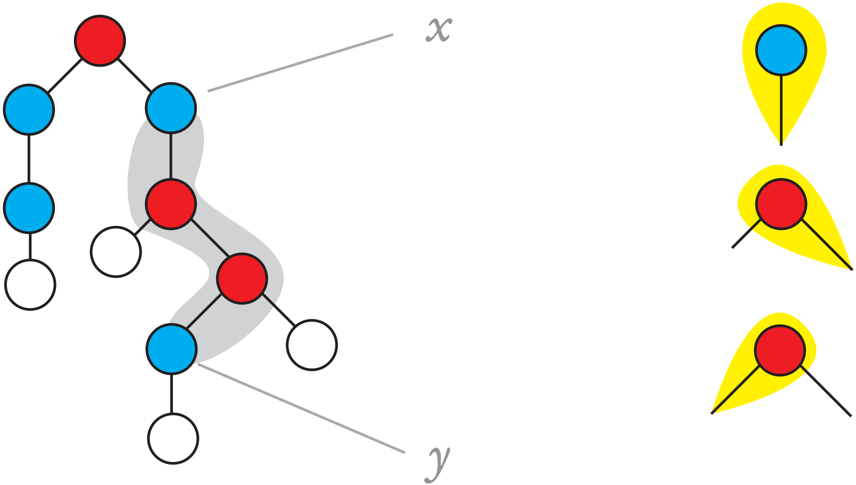


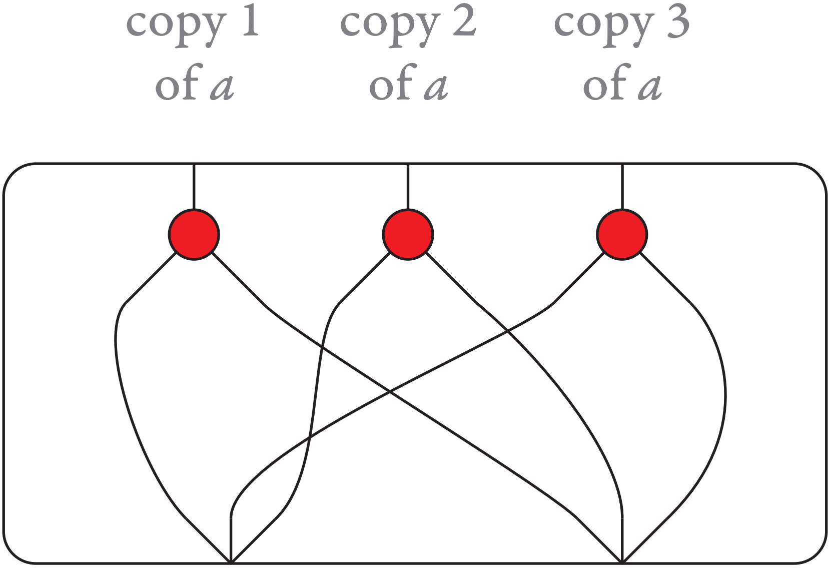
$\mapsto$

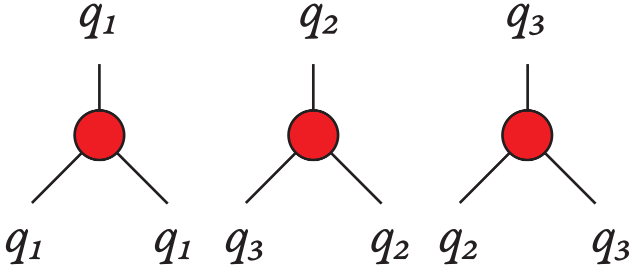


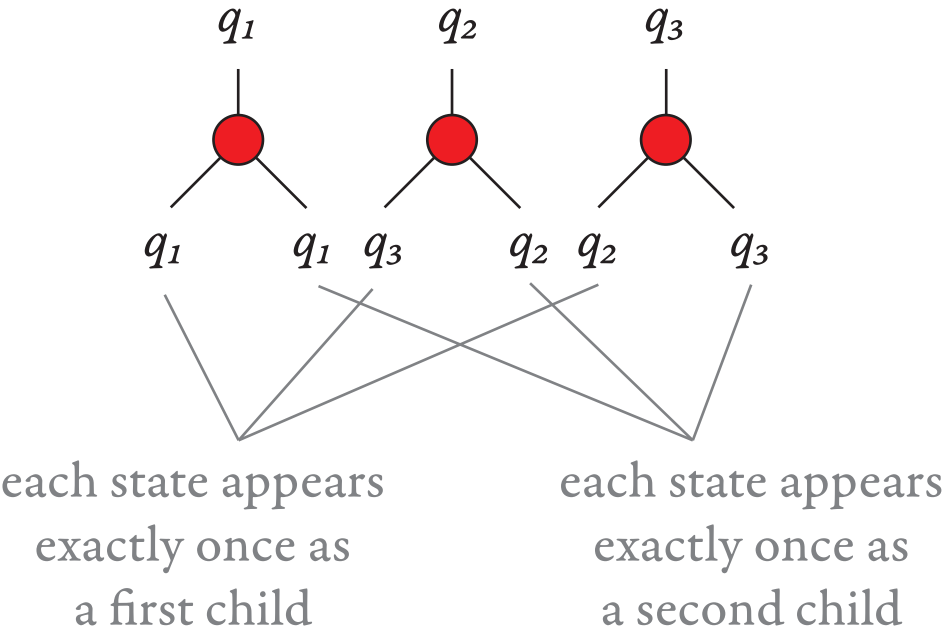
tree with a path from x to y

word corresponding to the path

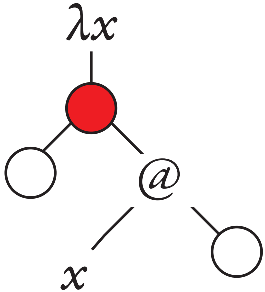



copy 1 of a
copy 2 of a
copy 3 of a

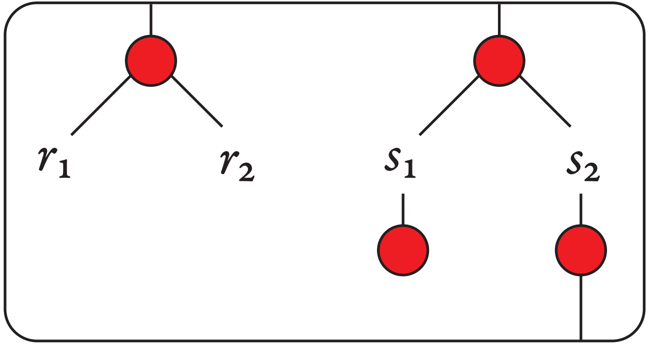

q_1
q_1
q_1
q_2
q_3
q_2
q_3
q_2
q_3



q_1
q_2
q_3
q_1
q_1
q_3
q_2
q_2
q_3
each state appears exactly once as a first child
each state appears exactly once as a second child


λx
@
x

a register valuation



its λ-representation