

a ranked alphabet

arity 2



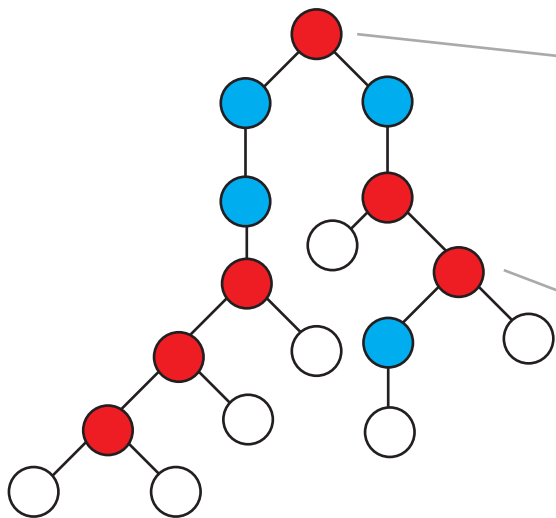
arity 1



arity 0



a tree



this node has a label of arity 2,  
and therefore it has 2 children

this node is child 2  
(children are ordered)



A tree  $t$  over  $\Sigma^{[2]}$



$\text{unfold}_1(t)$



$\text{unfold}_2(t)$





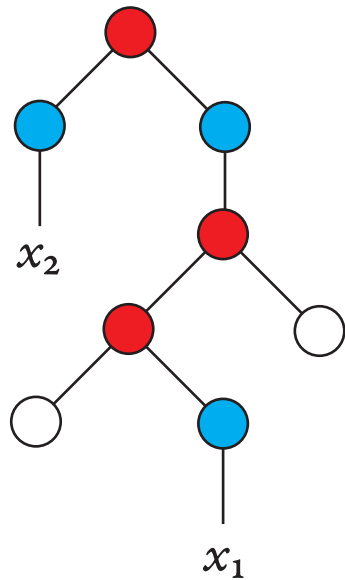
$t$



substitute( $t$ )

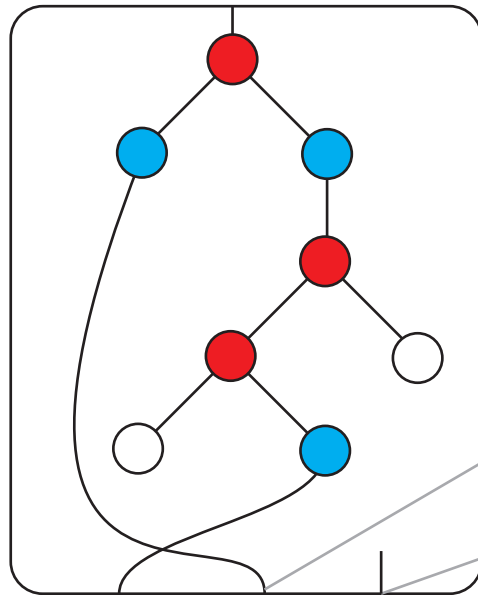






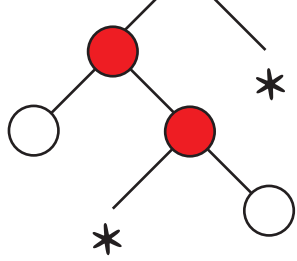
=

a term of arity 3

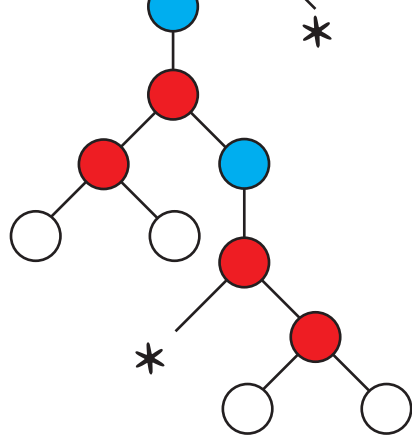


lines leaving at the bottom of the box  
represent variables

dangling edges represent unused variables



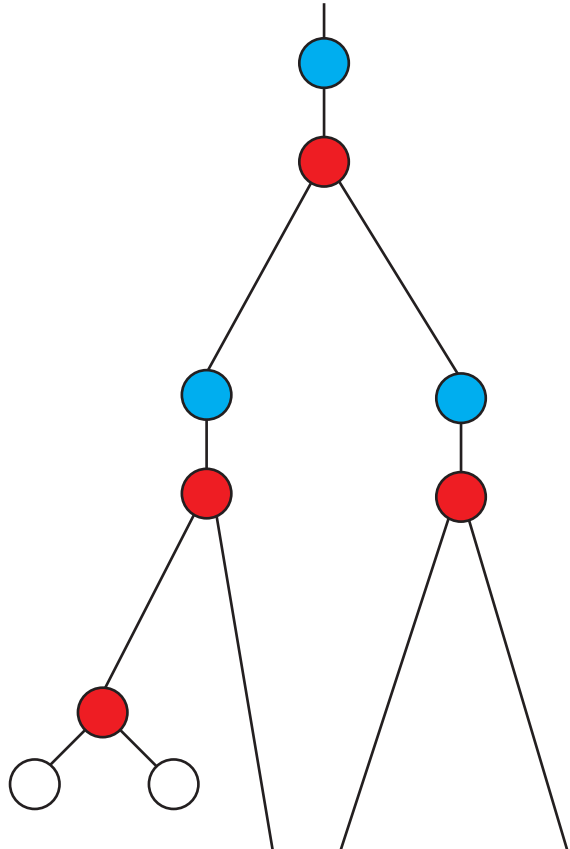
$\mathsf{T}f$   
 $\mapsto$







$\mapsto$





a term



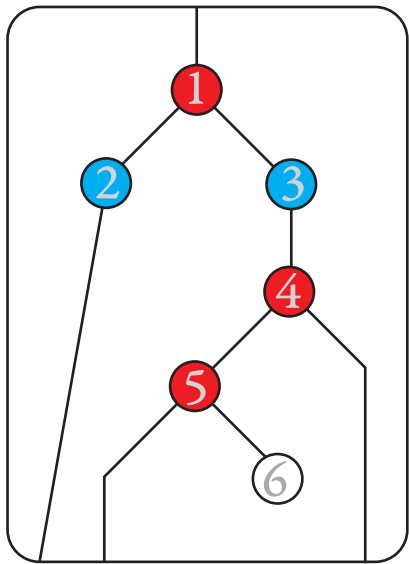
ancestor equivalence



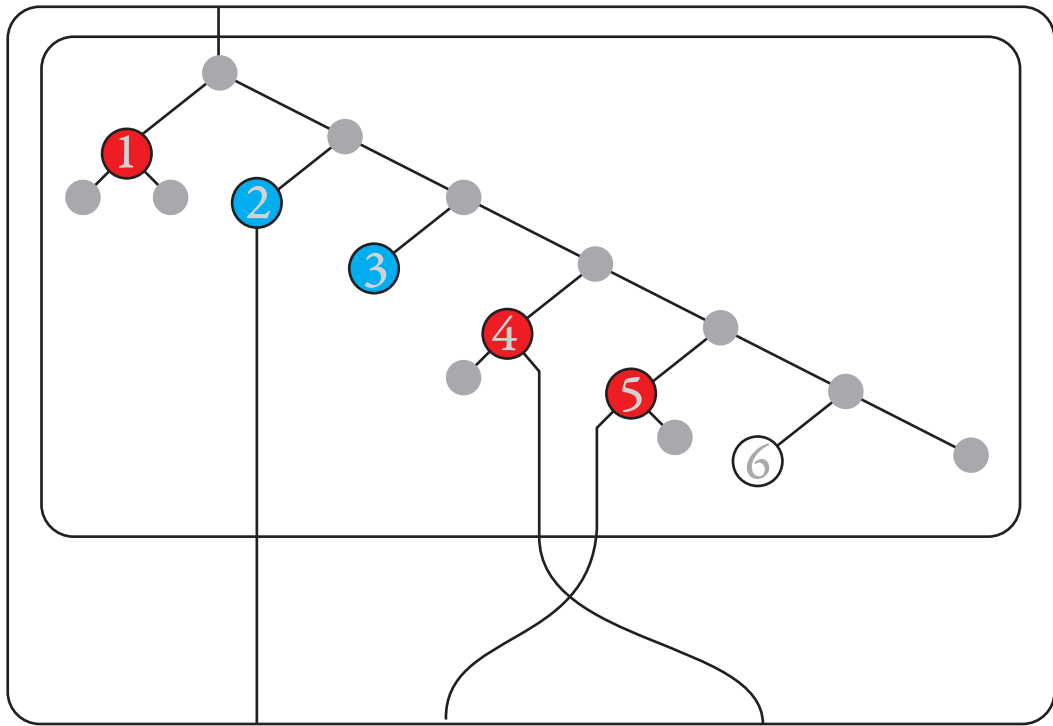
descendant equivalence



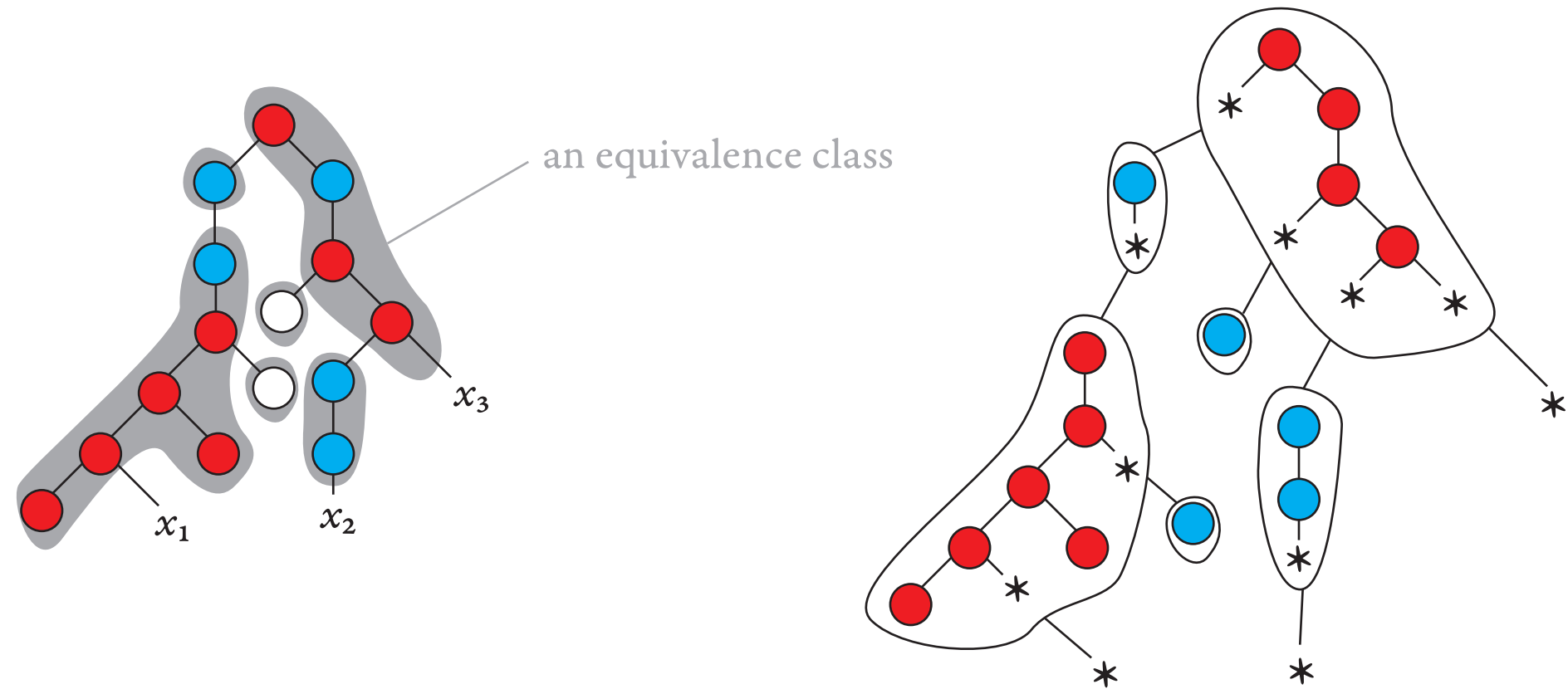




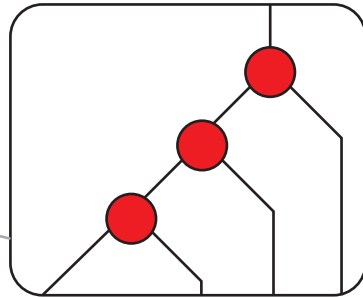
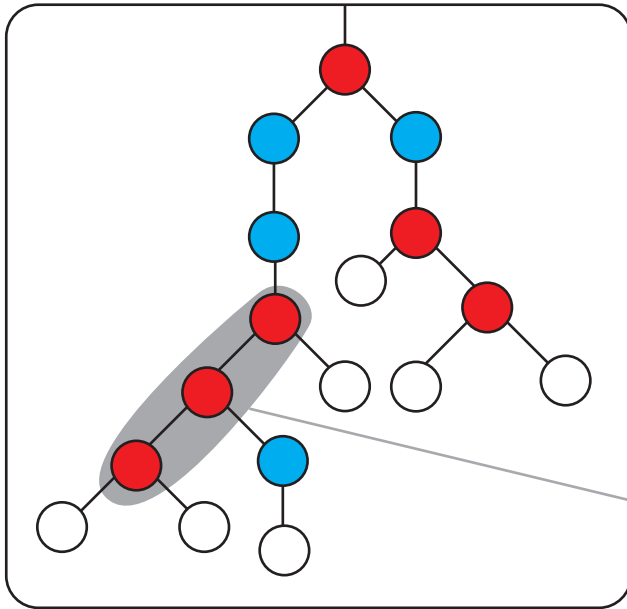
$\mapsto$



a factorisation equivalence



a tree



a term that  
represents a  
part of the tree





input alphabet

arity 2



arity 1



arity 0



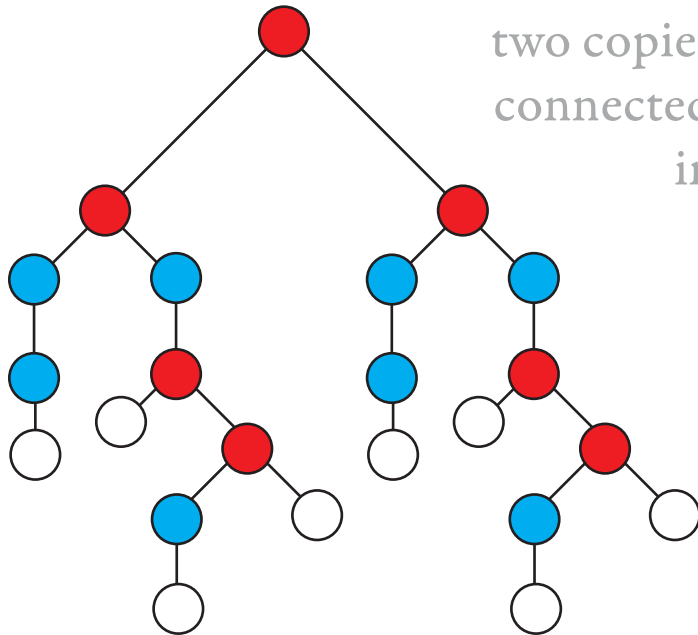
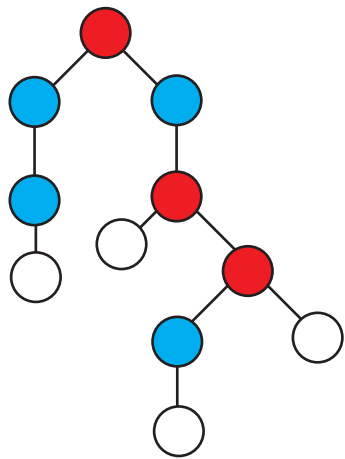
output alphabet

arity 2



arity 0





two copies of the input tree,  
connected by a binary node  
in the root





input alphabet

arity 2



arity 1



arity 0



output alphabet

arity 2



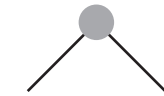
arity 1



arity 0

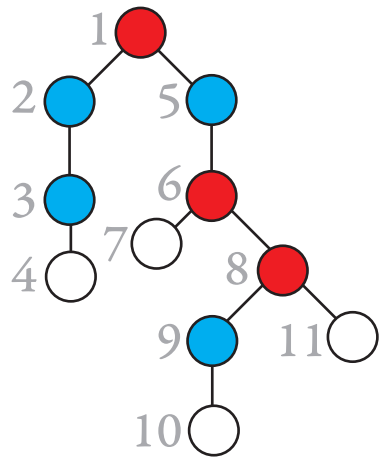


arity 2

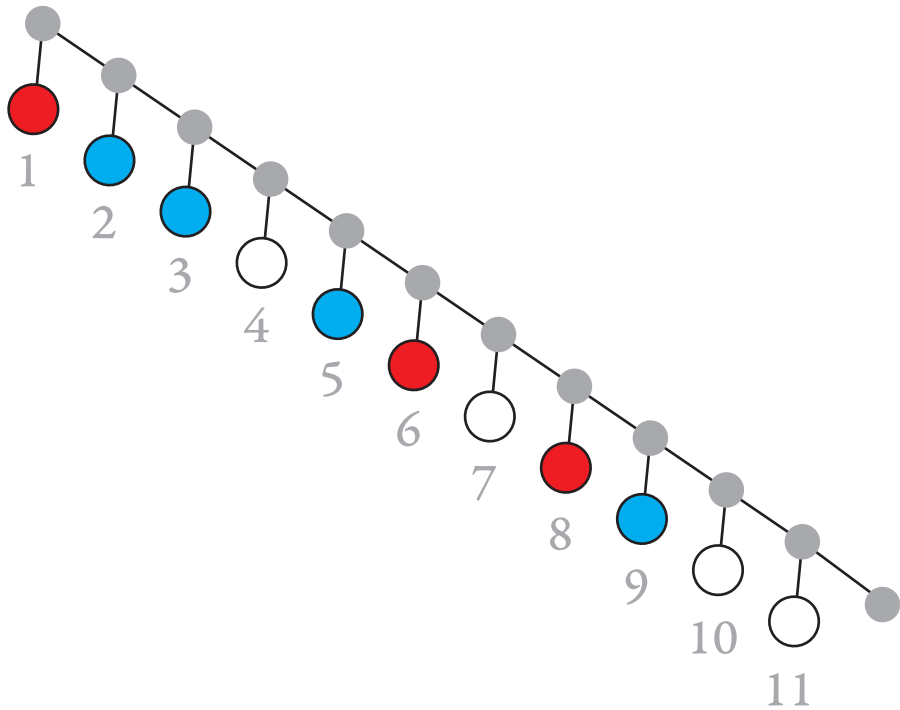


arity 0





$\mapsto$







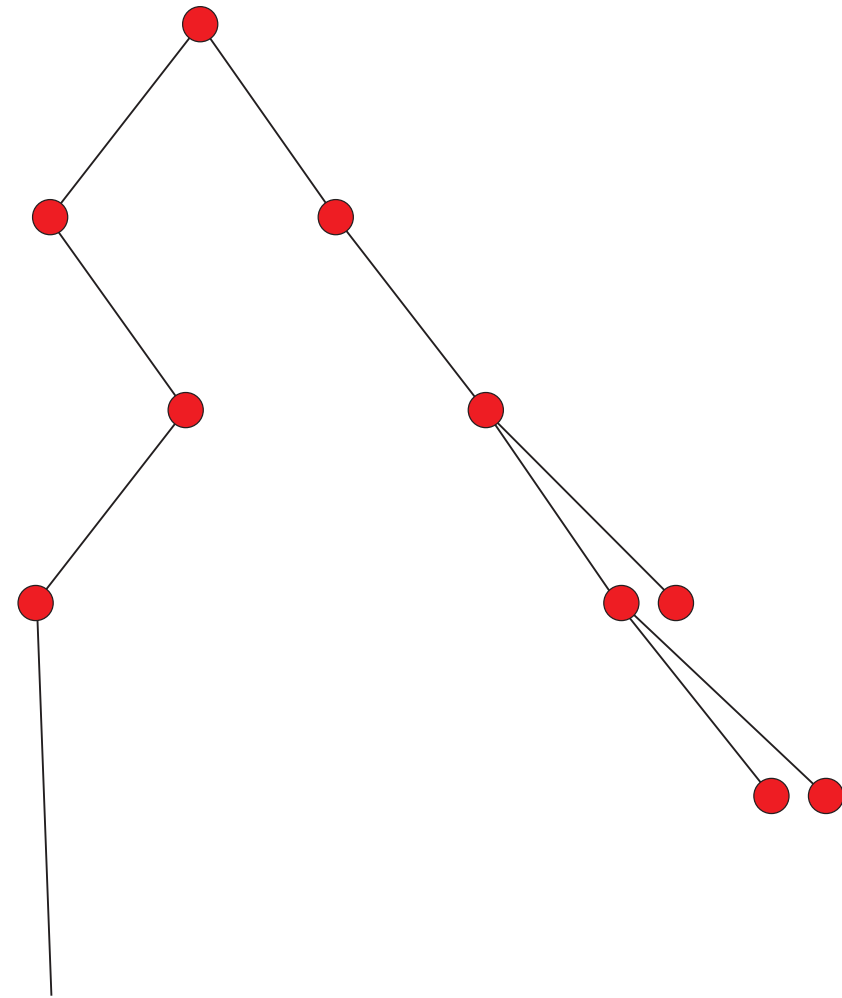
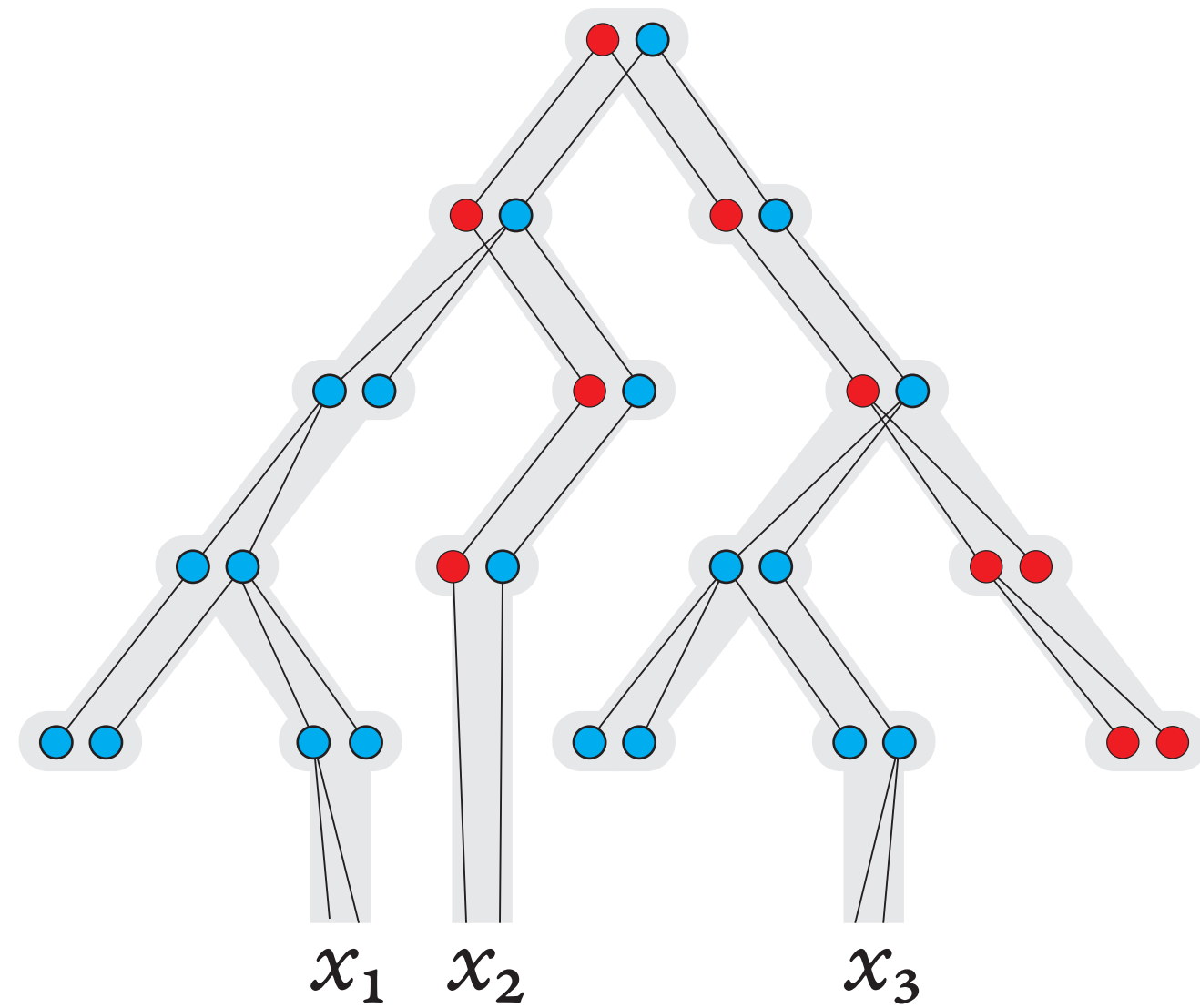


a term of arity 4



a term of arity 0





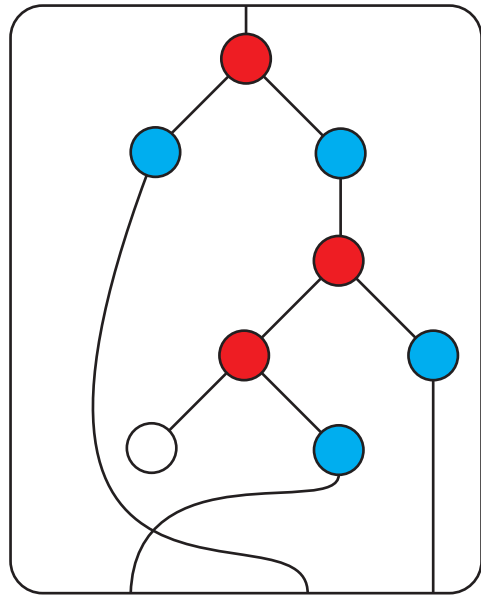




satisfies (\*)

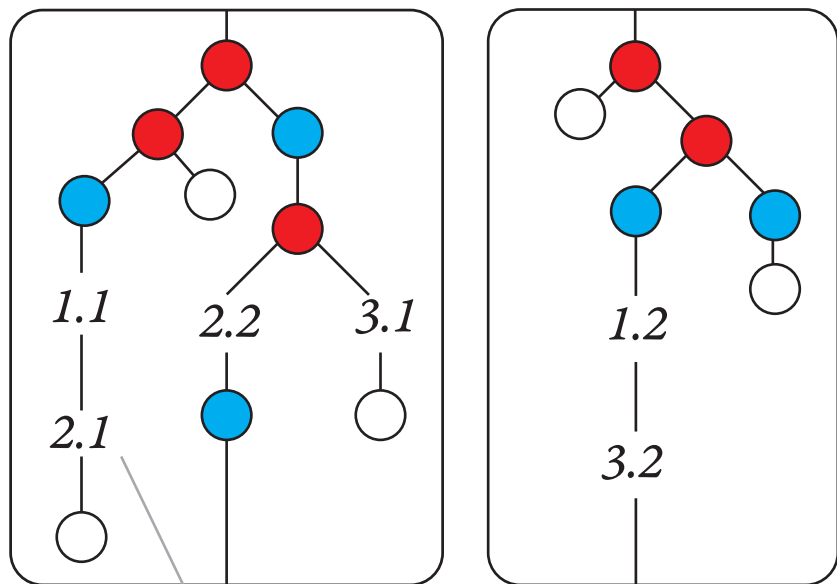
(\*)

If the root has arity  $n$ ,  
and  $1 \leq i < j \leq n$ , then  
all ports of the  $j$ -th  
subterm of the root are  
after all ports of the  
 $i$ -th subterm of the root



violates (\*)

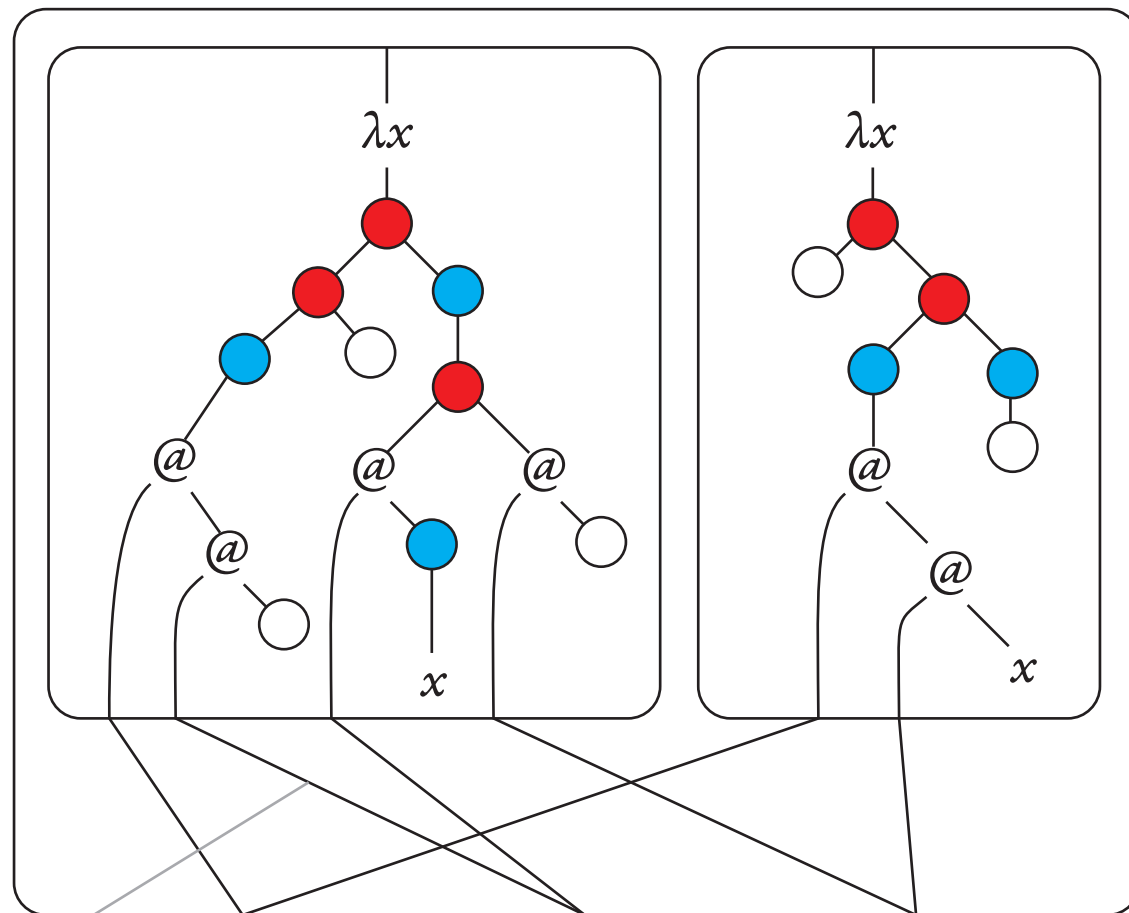
a register update



Variable  $i.j$  represents register  $i$  in the  $j$ -th argument of the register update.

In the dual, this variable is mapped to the  $i$ -th edge which enters the  $j$ -th port of the reducer.

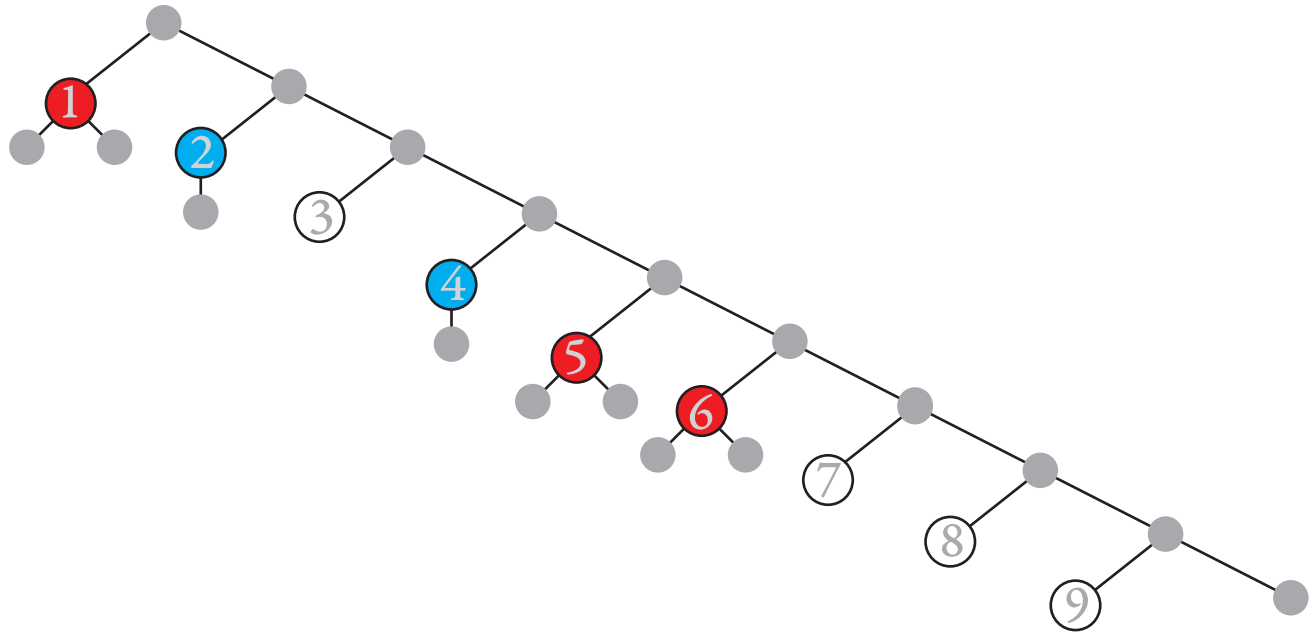
its dual



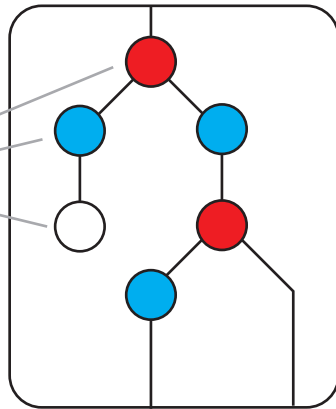
input



output

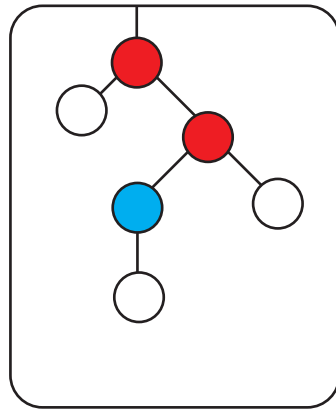


register  $r$

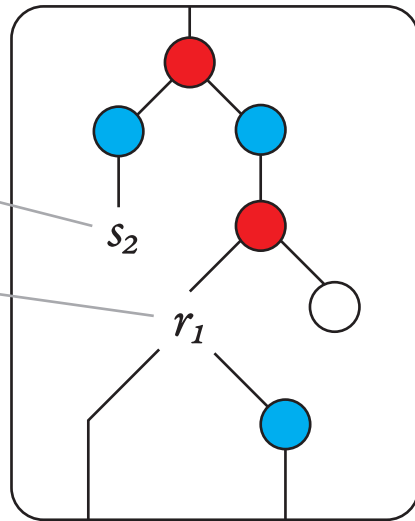


letters of the output alphabet

register  $s$



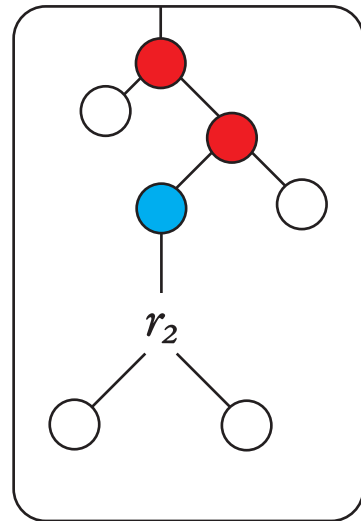
register  $r$



copy 2 of register  $s$

copy 1 of register  $r$

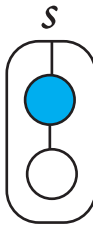
register  $s$











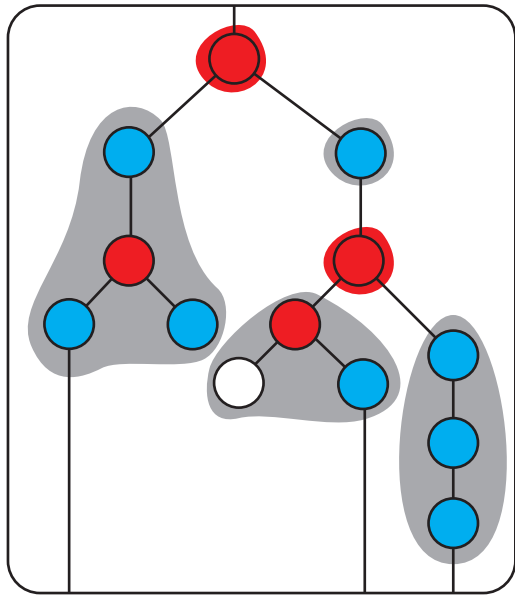




factors without  
branching nodes

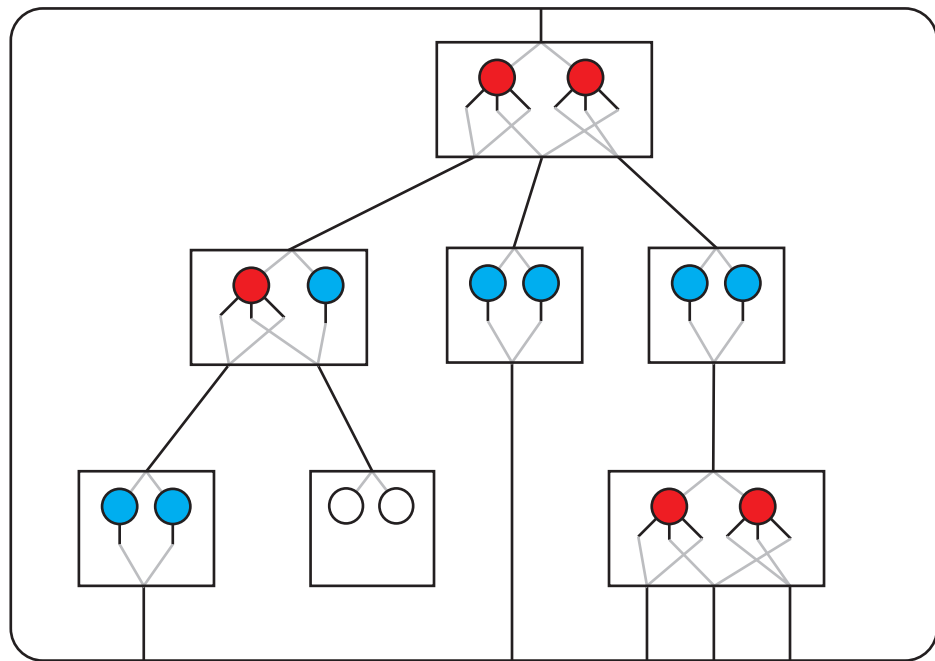


factors with  
branching nodes

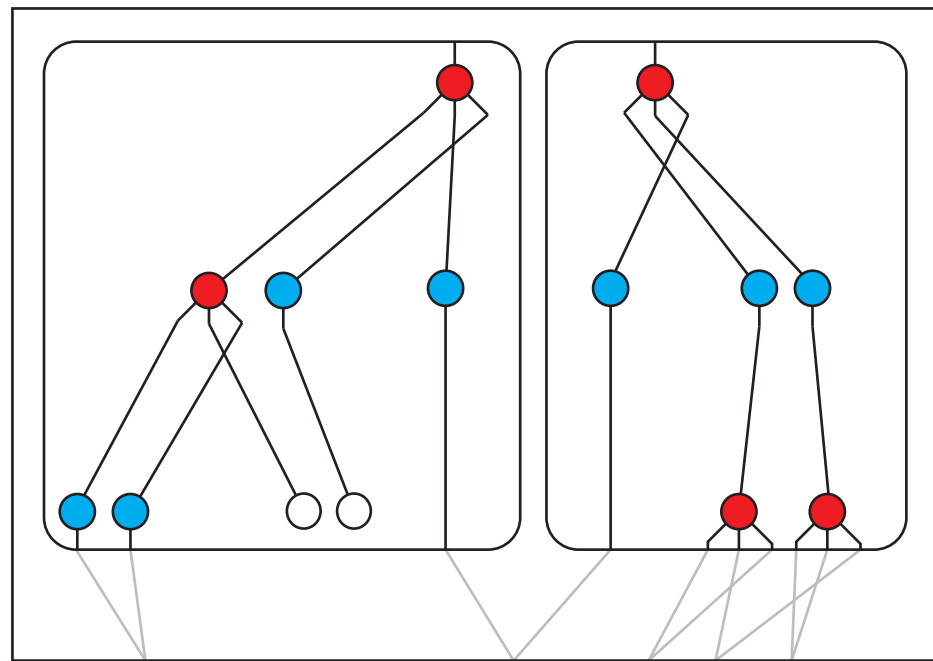




input

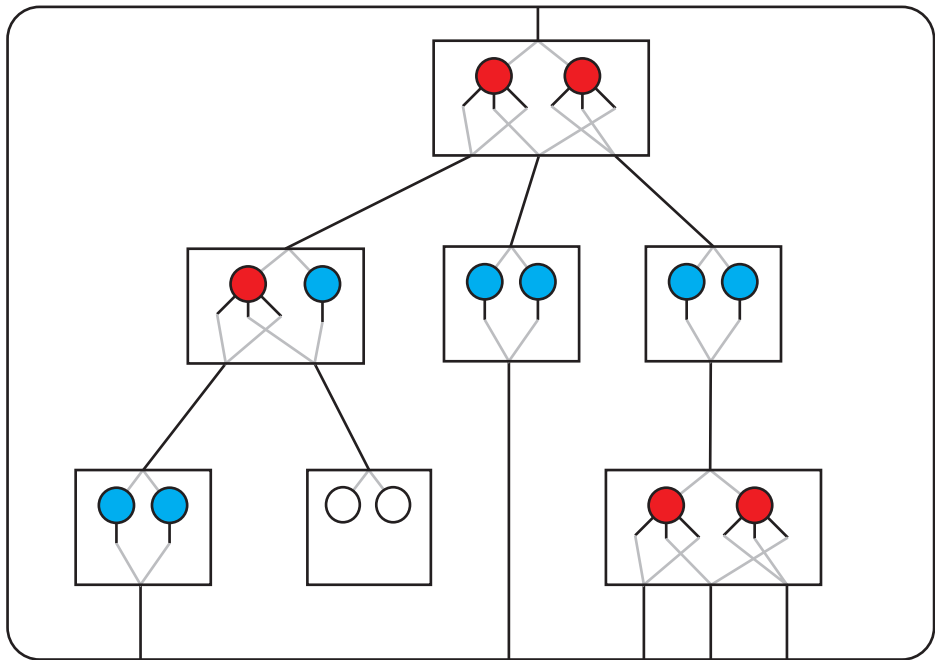


output









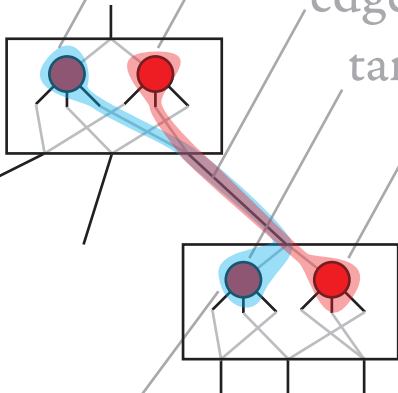
source 1 of  $e$

source 2 of  $e$

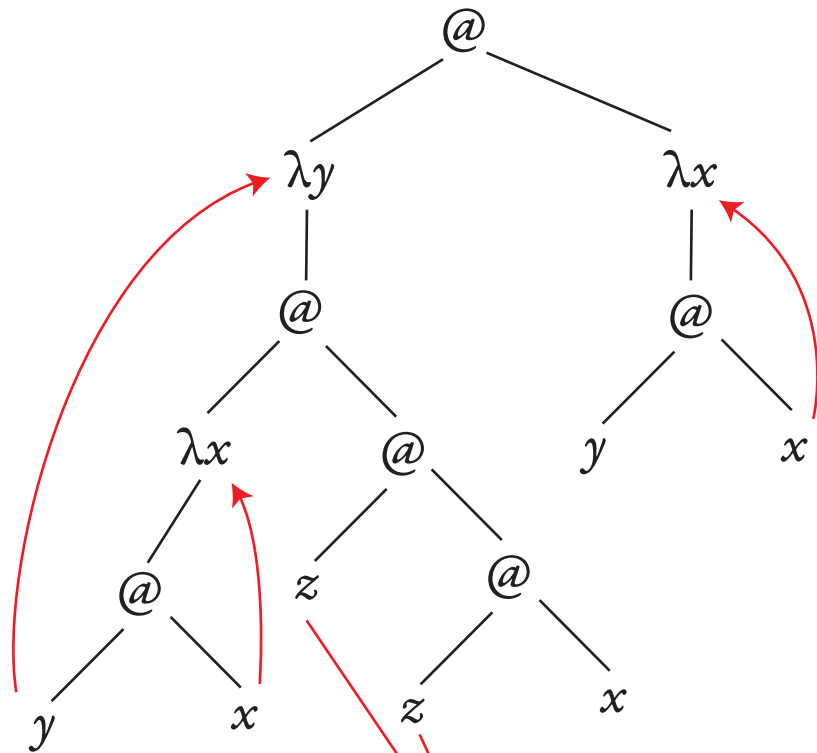
edge  $e$

target 1 of  $e$

target 2 of  $e$



linear

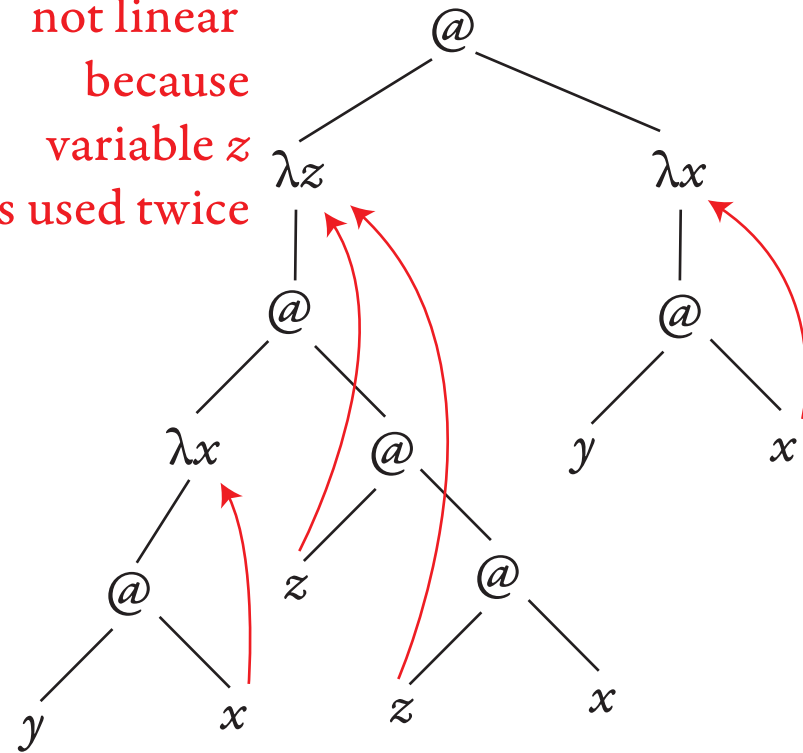


we only count  
variables used  
in their scope

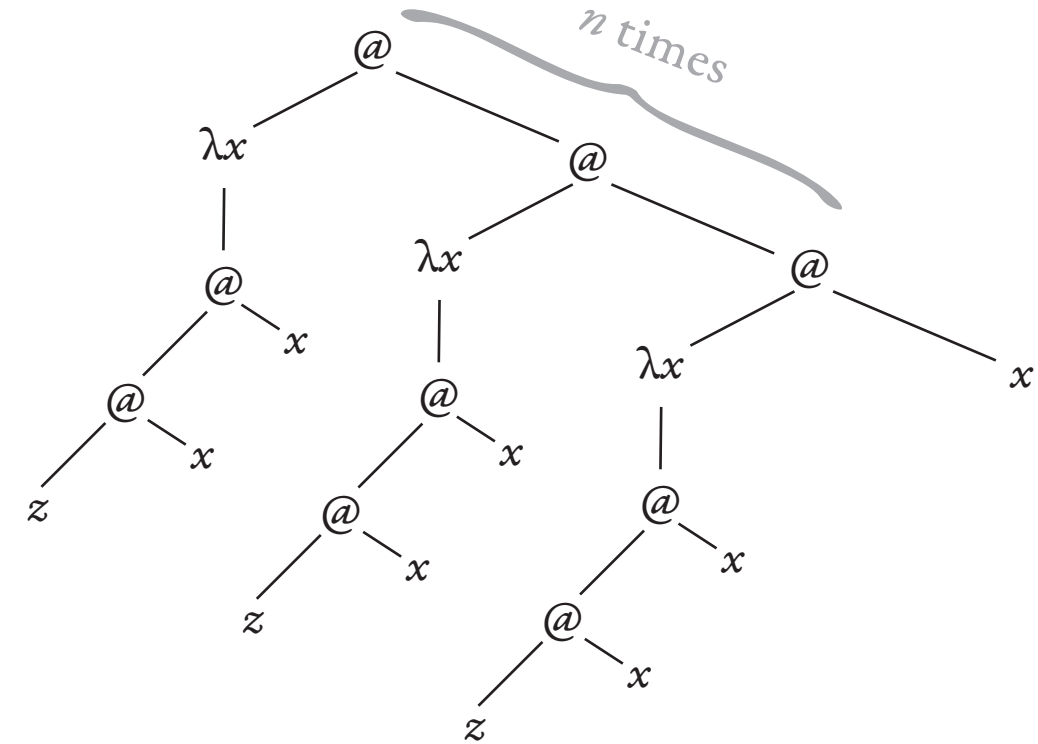
variable  $z$  can be used twice because it is free

not linear

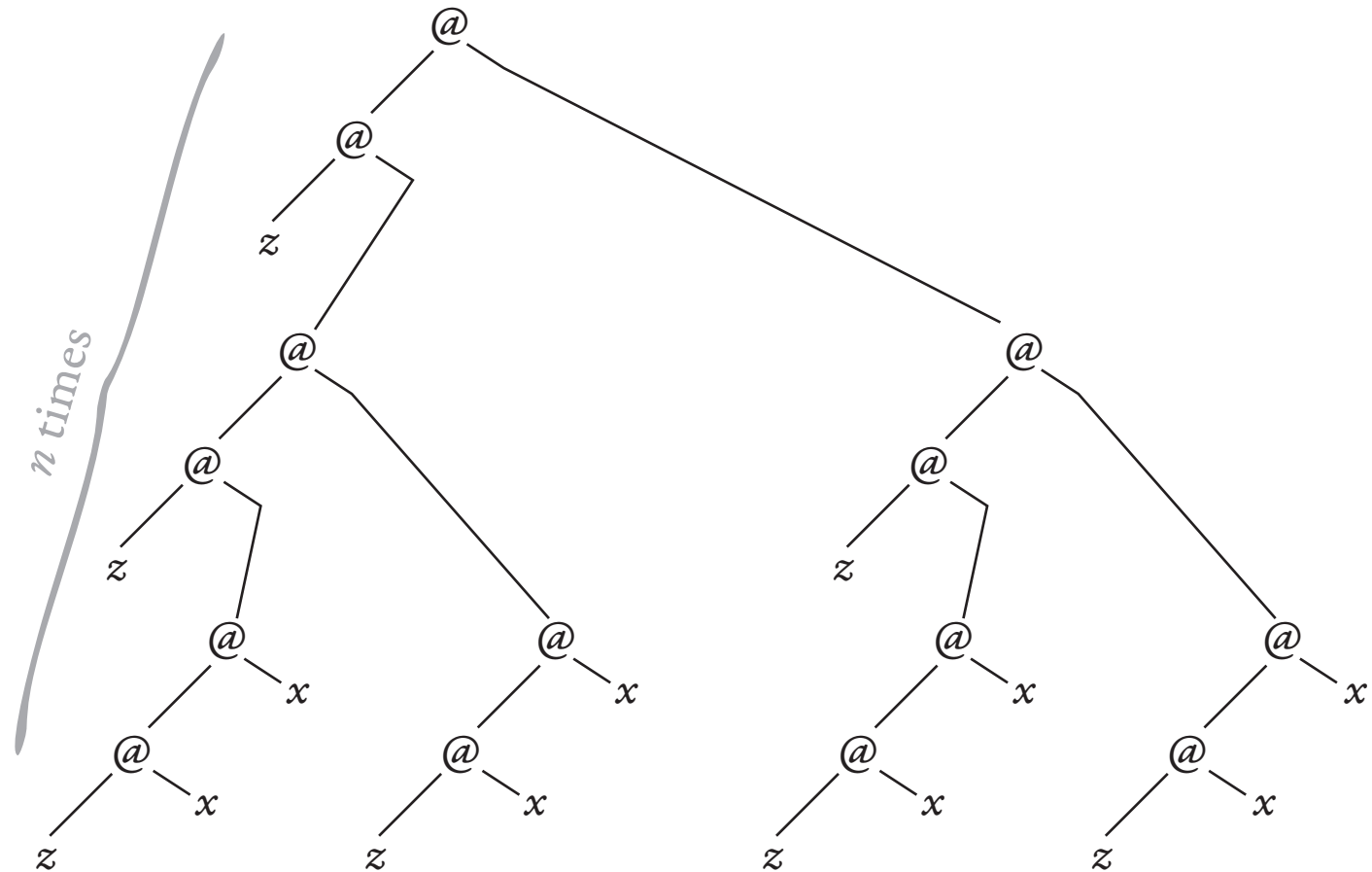
not linear  
because  
variable  $z$   
is used twice



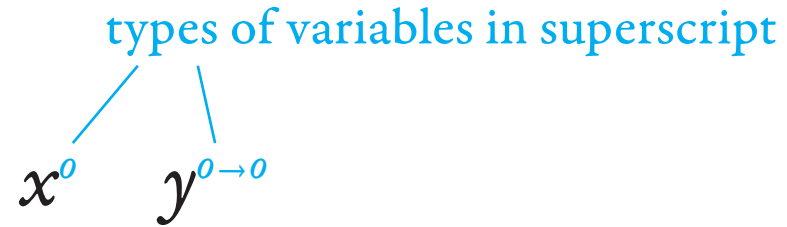
a  $\lambda$ -term of size  $O(n)$



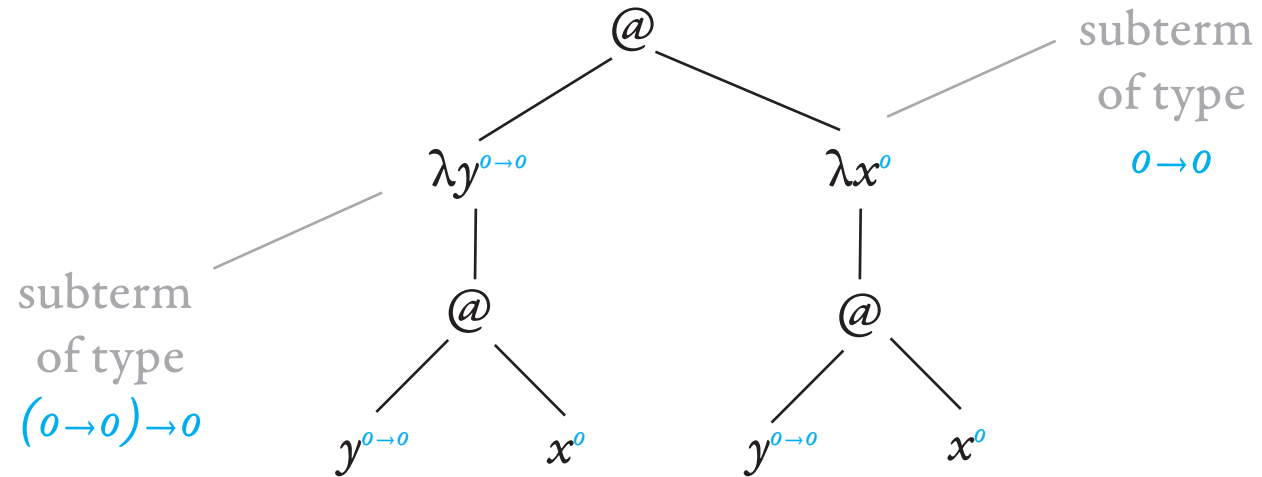
its normal form of size  $O(2^n)$



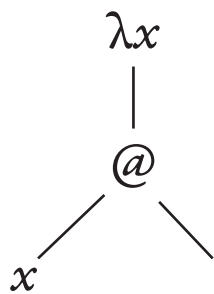
variables



$\lambda$ -term of type  $o$



@



$\lambda x.$



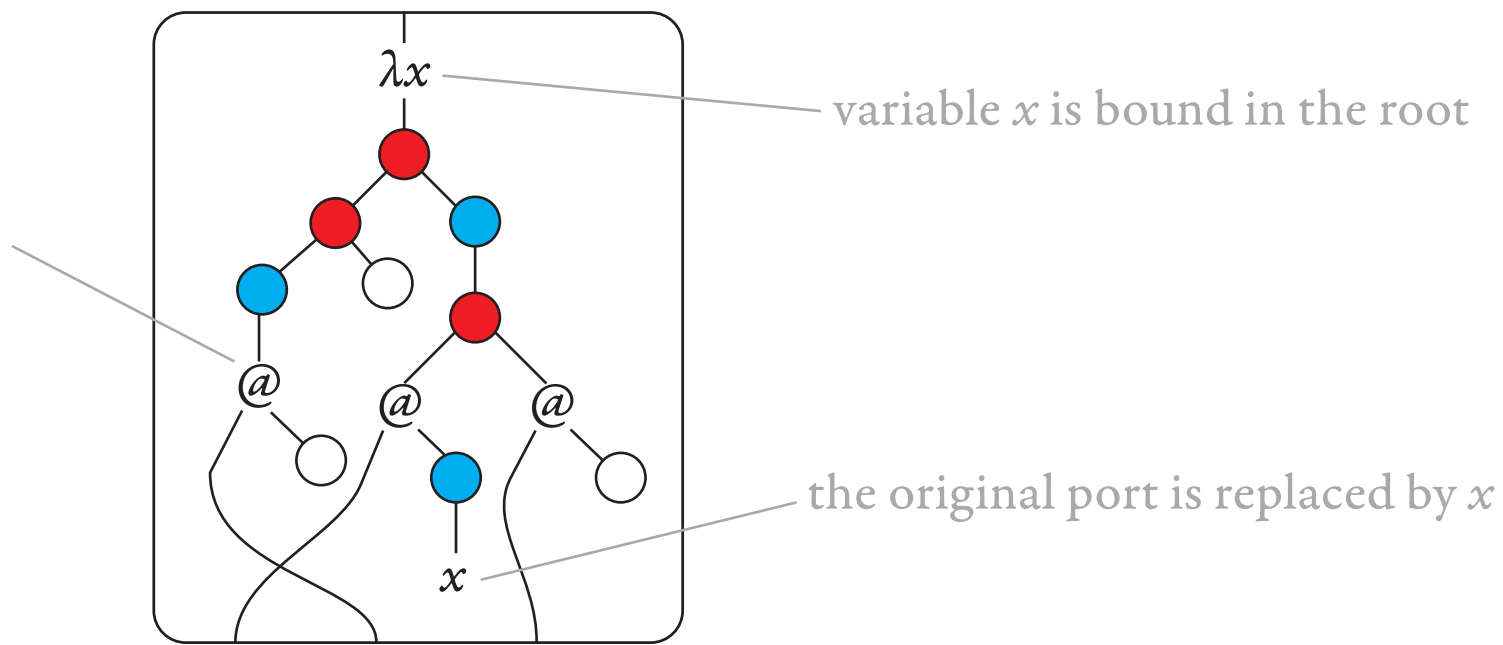
*r*

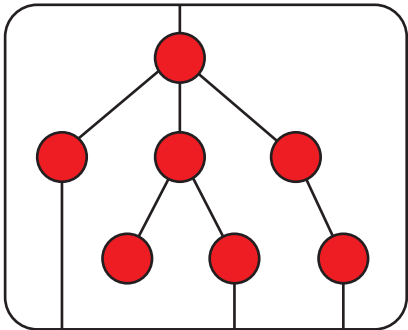


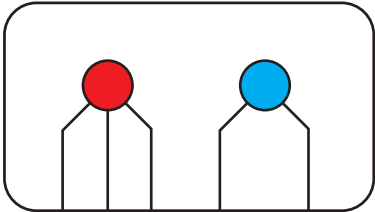


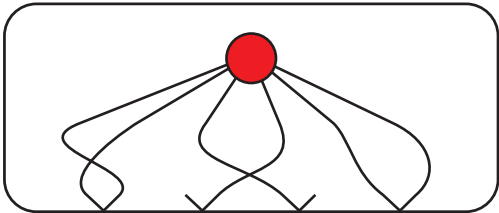
placeholder for the term  
stored in the unique register  
of the 2nd child









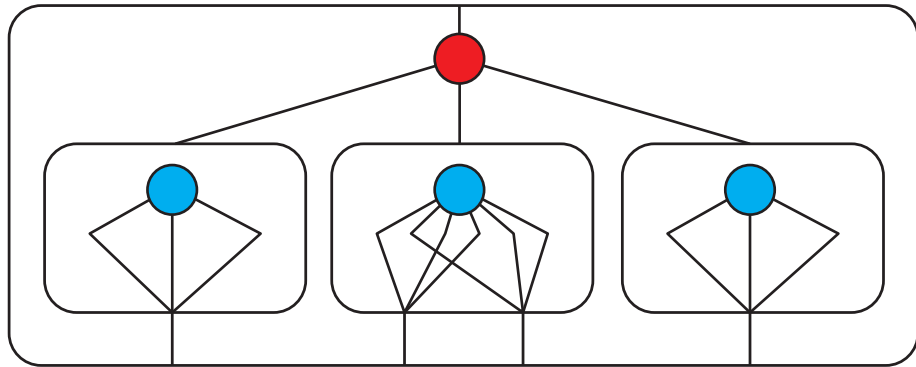


the root is from  $\Sigma$

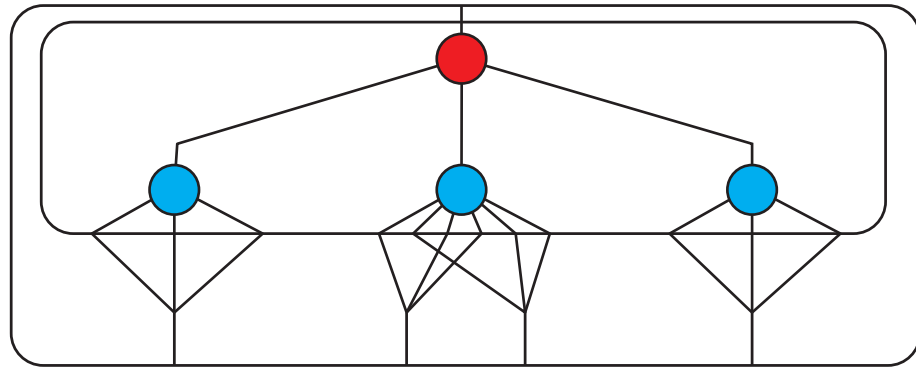
all children are from  $\Gamma$

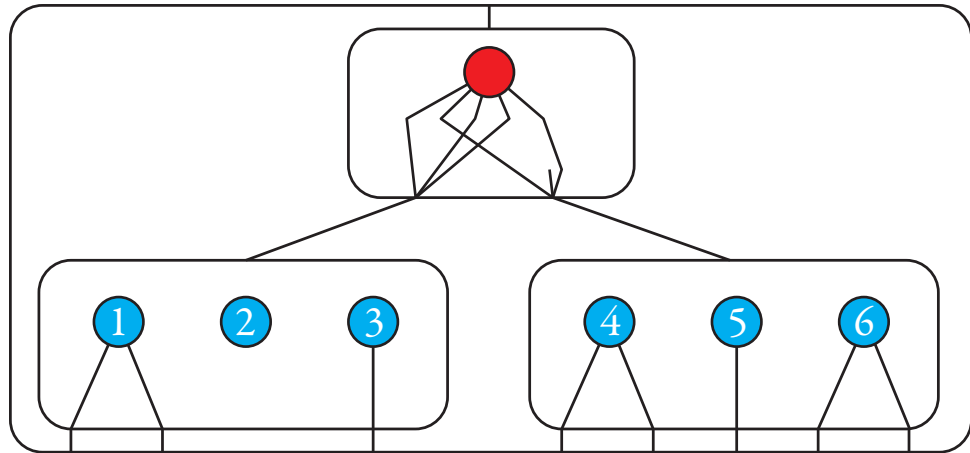


input

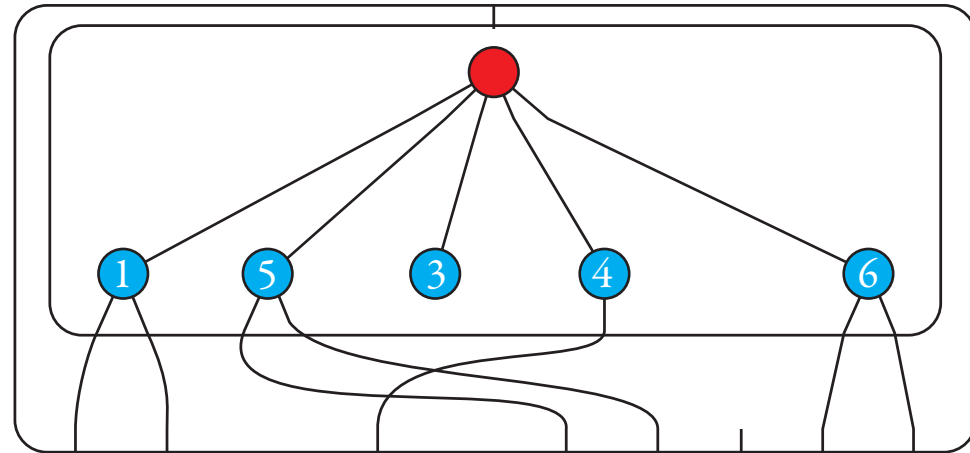


output



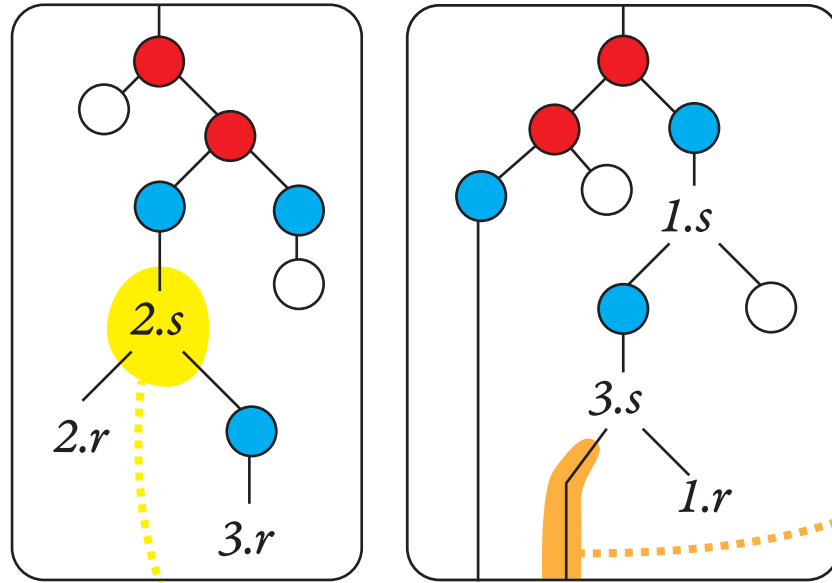


$\mapsto$

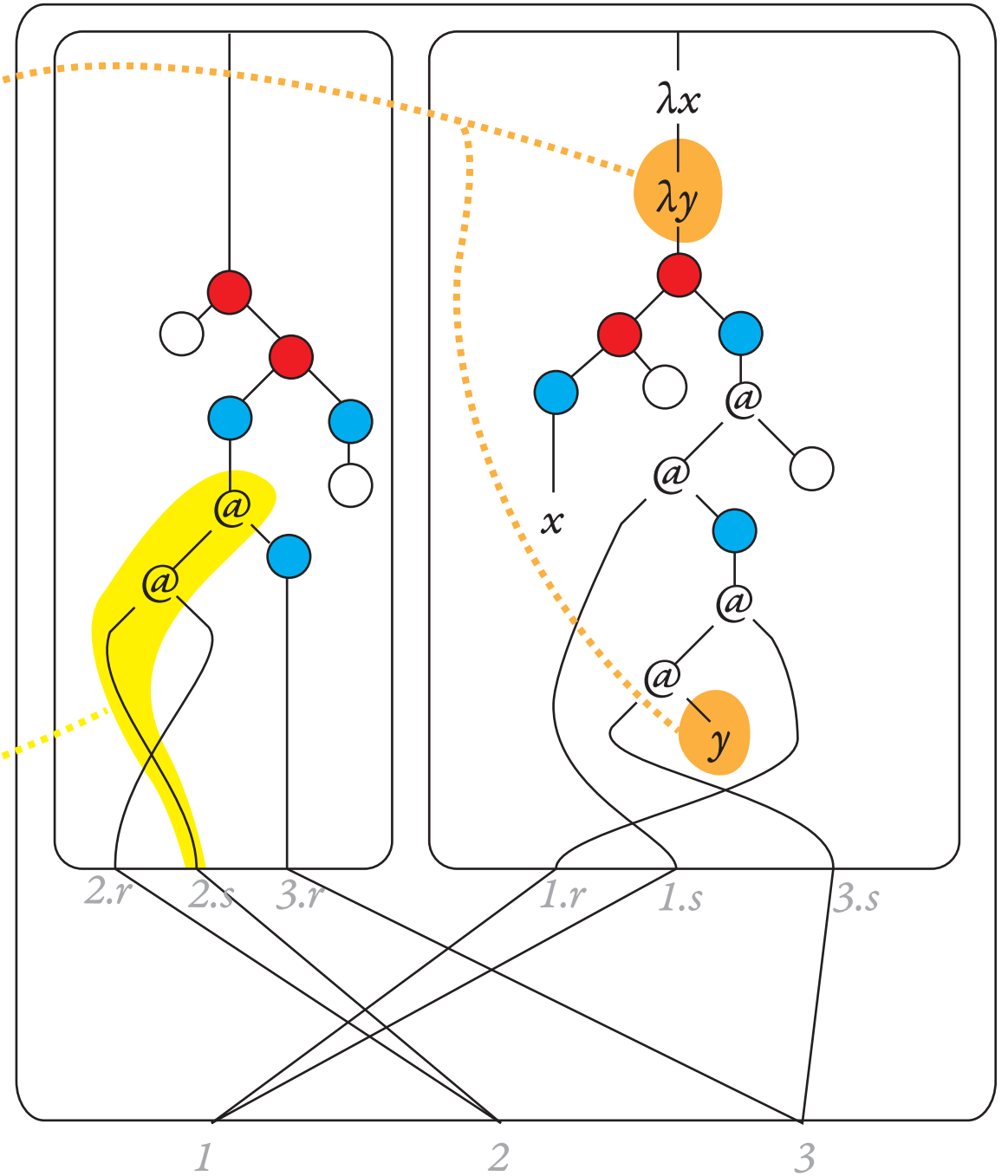




a register update



its dual



The diagram shows a binary tree structure. The root node is red. Its left child is red, and its right child is blue. The red node's left child is blue, and its right child is white. The blue node's left child is blue, and its right child is white. A yellow circle labeled  $r_1$  highlights the blue node that is the right child of the root. An orange shape labeled  $r_2$  highlights the subtree rooted at the blue node that is the left child of the root. A dashed orange line labeled  $r_3$  indicates a path from the root to the right child of the root, and then to the right child of that node.

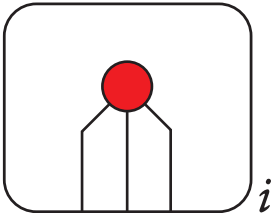
[illegible]

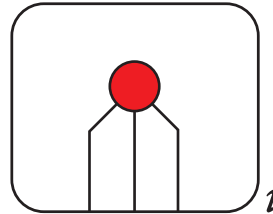
a non-port node is represented by a variable, corresponding to the label, applied to the children of the node

- the variables representing the ports are bound outside

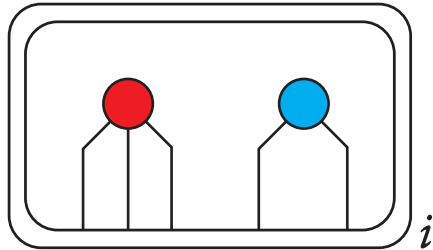
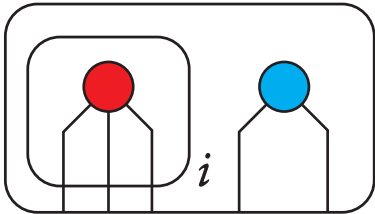
the  $i$ -th port is represented by a variable  $x_i$  of type  $\mathbf{o}$



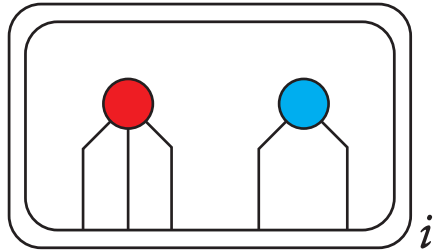
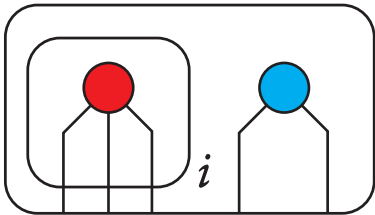


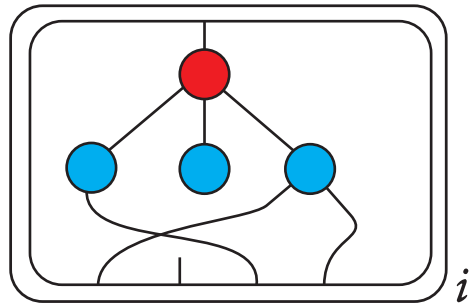
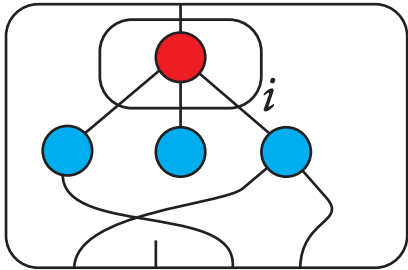


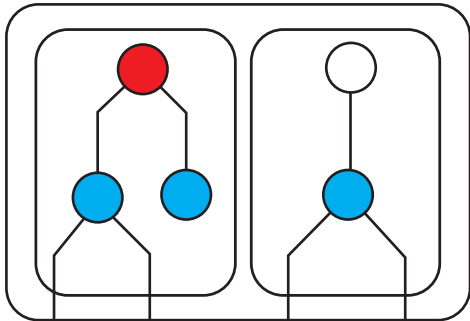
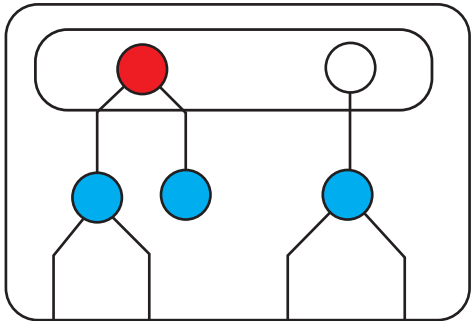


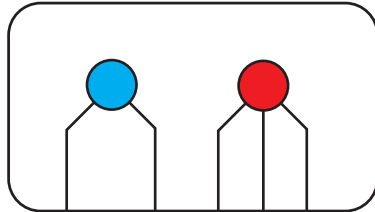




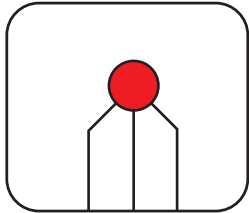


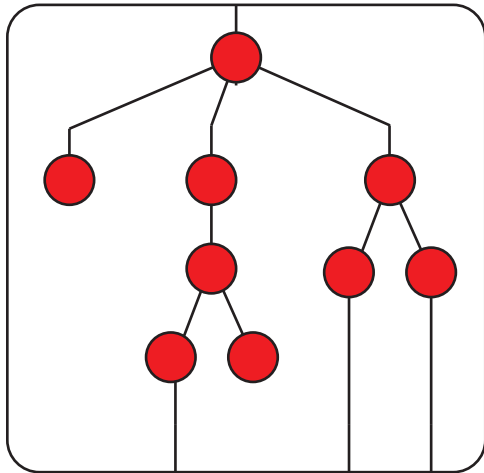
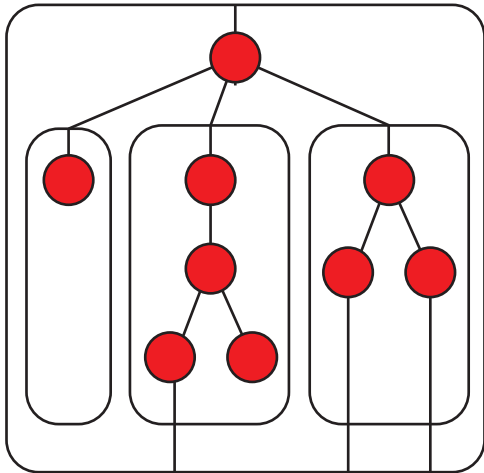


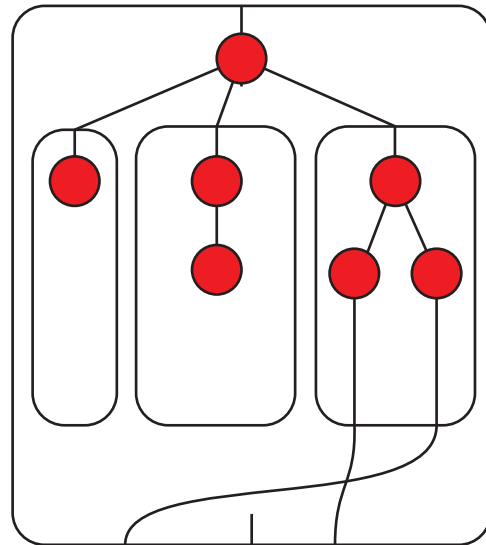
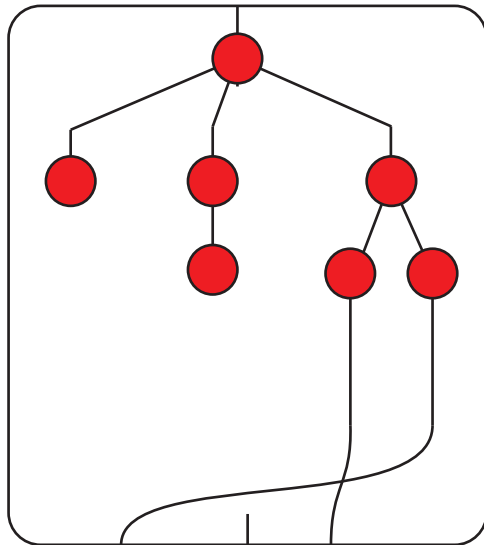




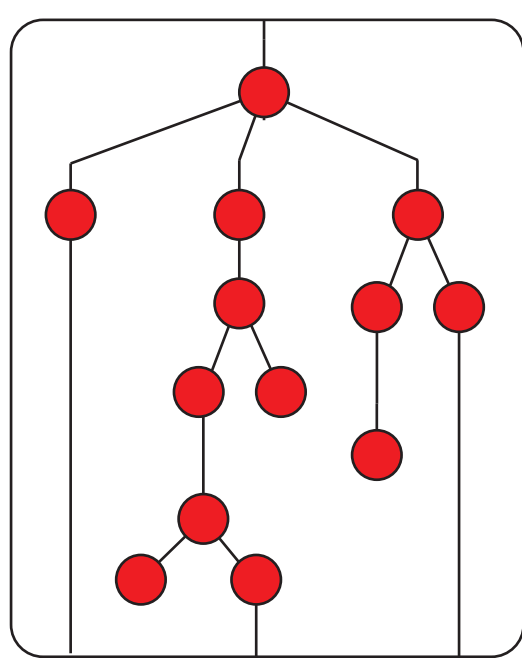


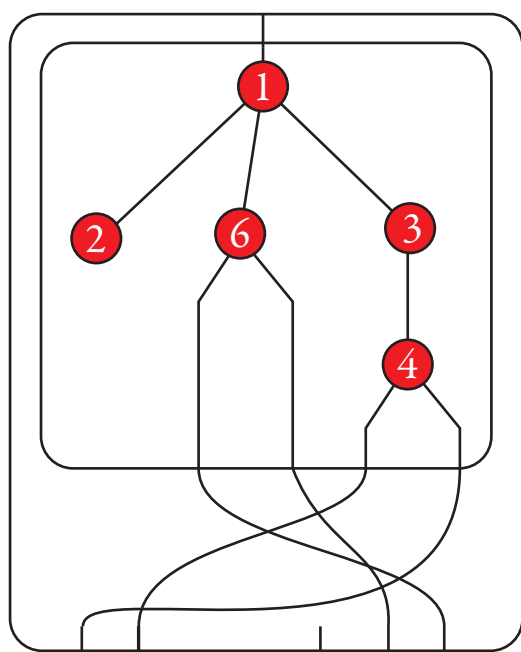


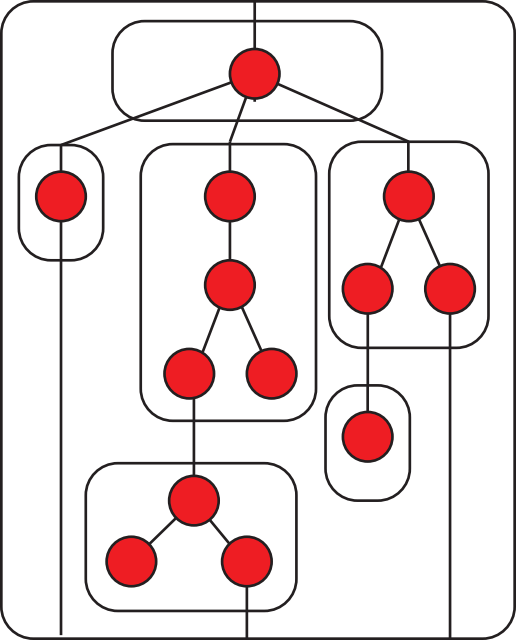




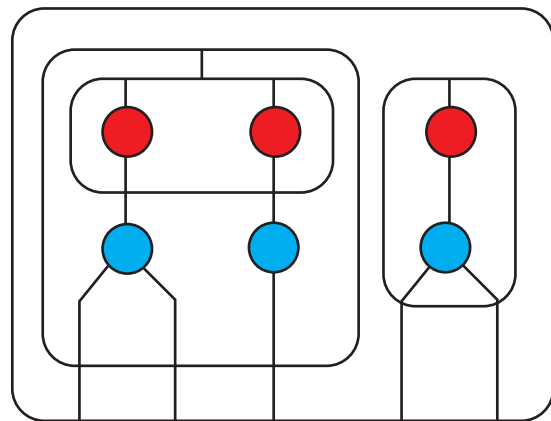
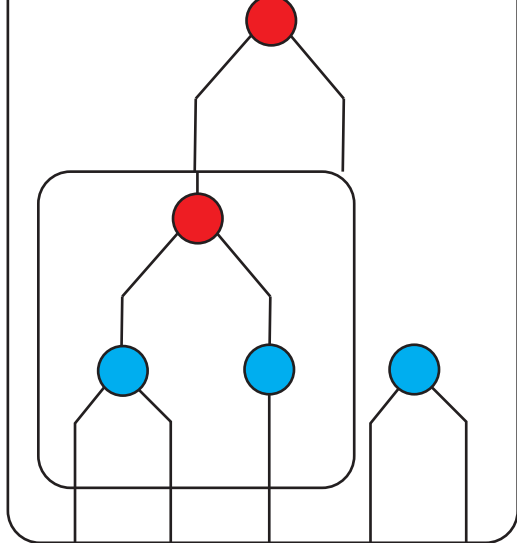
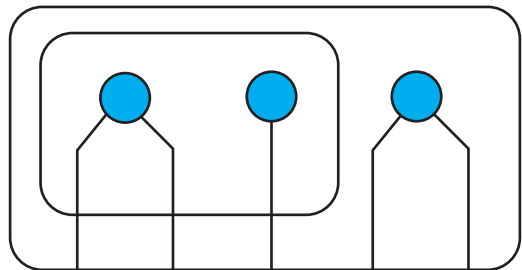


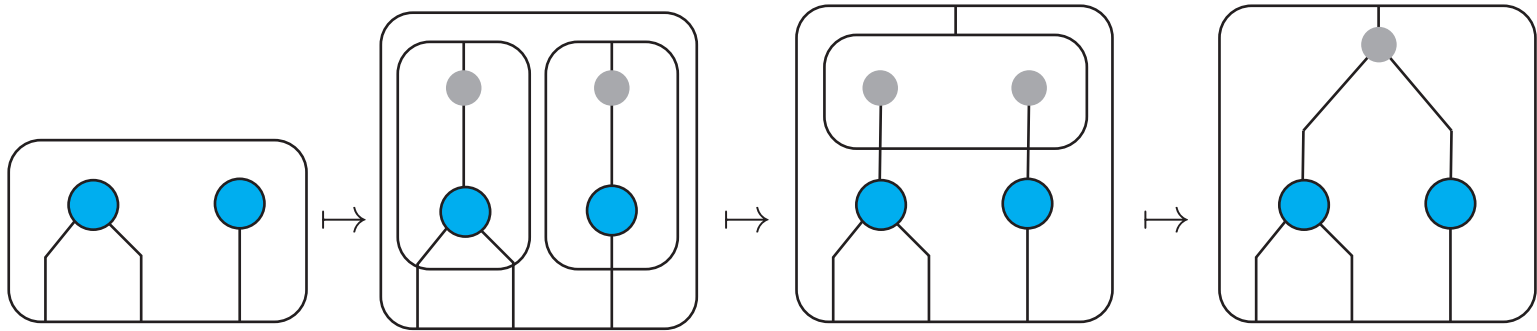






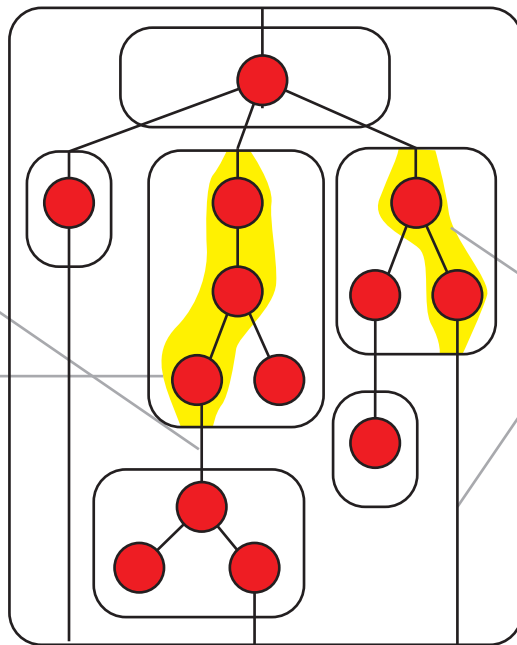






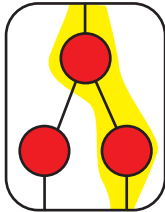


internal edge  
and its  
corresponding subbranch



external edge  
and its  
corresponding subbranch





a branch can be visualised as  
a term with a distinguished  
root-to-port path