

a ranked alphabet

arity 2



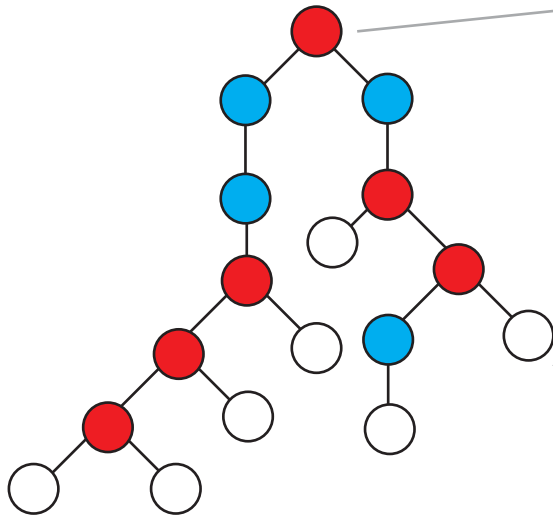
arity 1



arity 0



a tree



this node has a
label of arity 2,
and therefore it has
2 children

this node is
child 2
(children are
ordered)



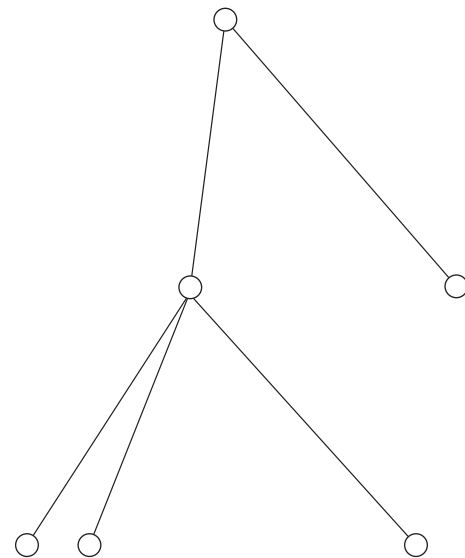
A tree t over $\Sigma^{[2]}$

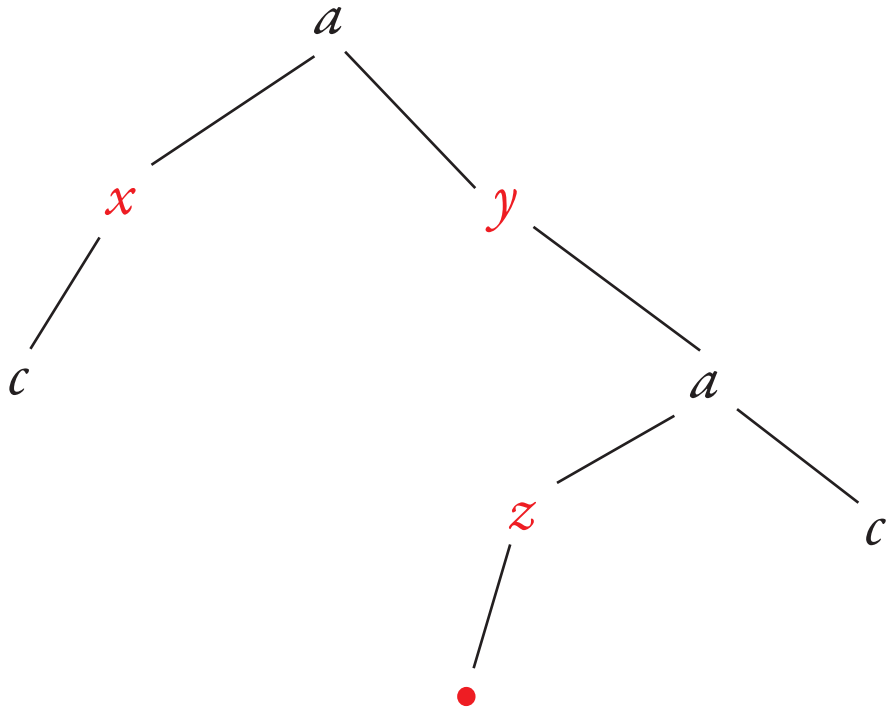


$\text{unfold}_1(t)$



$\text{unfold}_2(t)$

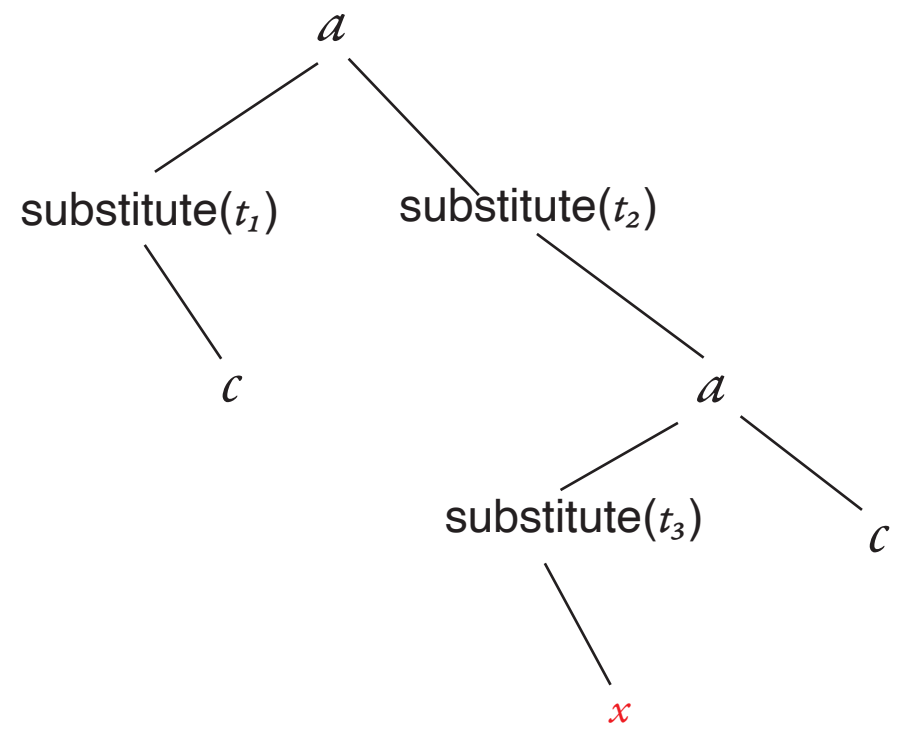




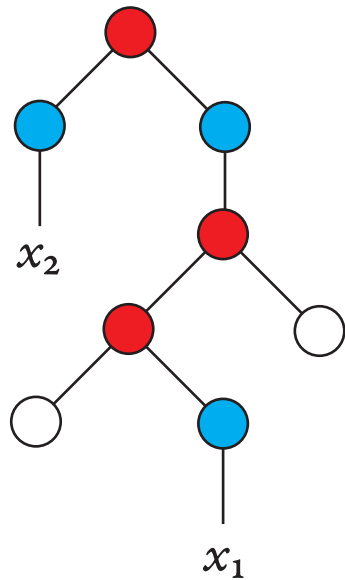
t



substitute(t)

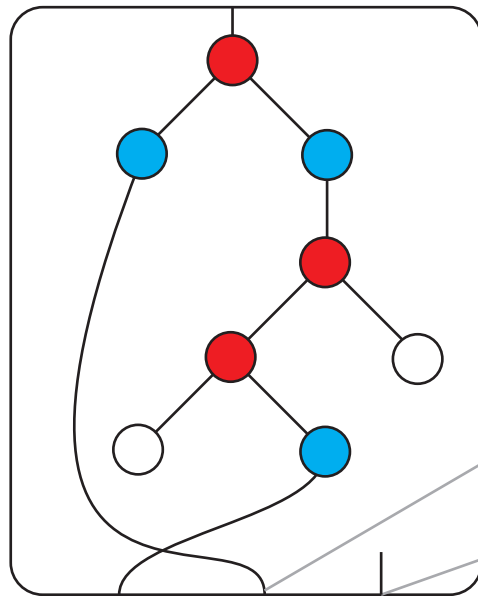






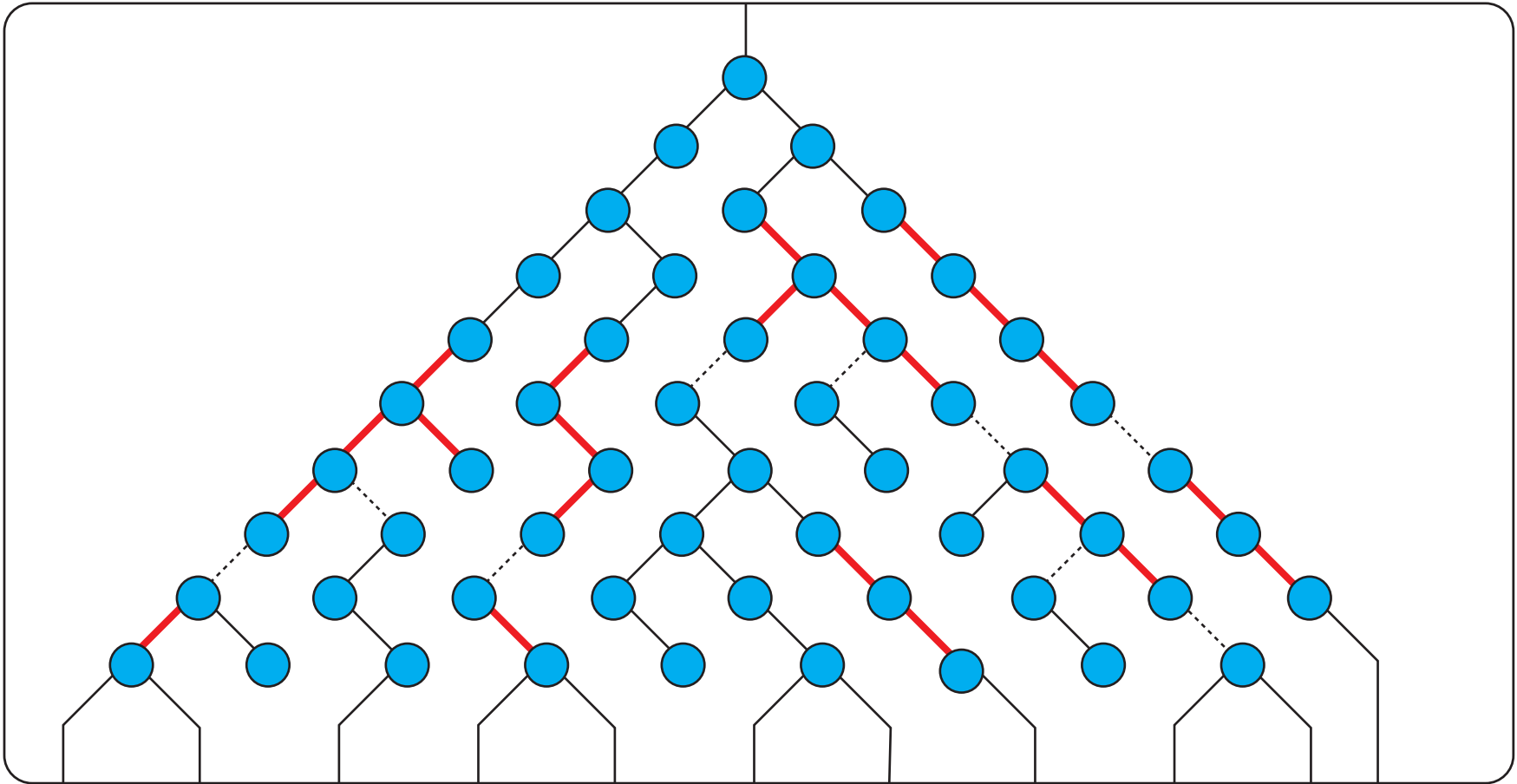
=




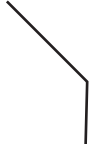
a term of arity 3



lines leaving at the bottom of the box
represent variables

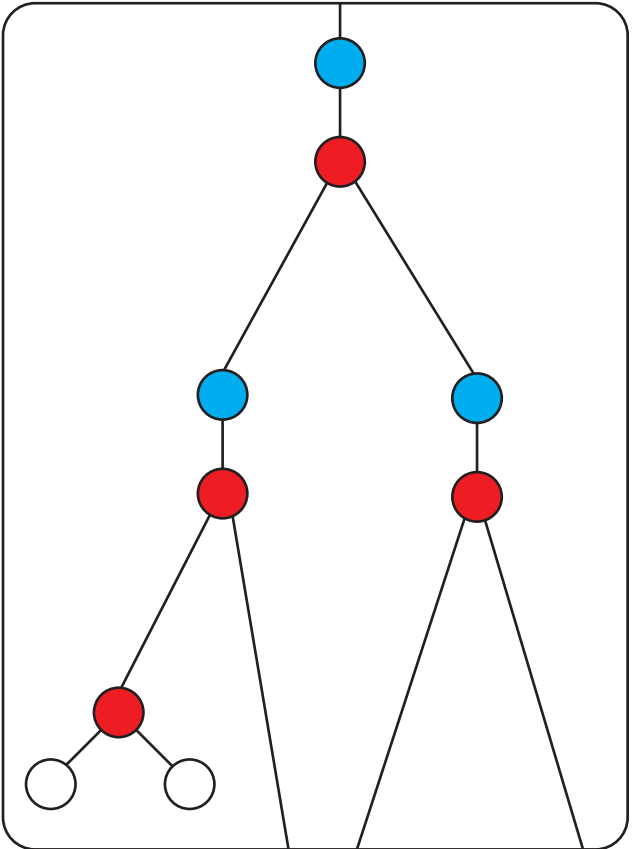
dangling edges represent unused variables



-  sensitive internal edge
-  post-sensitive internal edge
-  internal edge that is neither sensitive nor post-sensitive
-  external edge

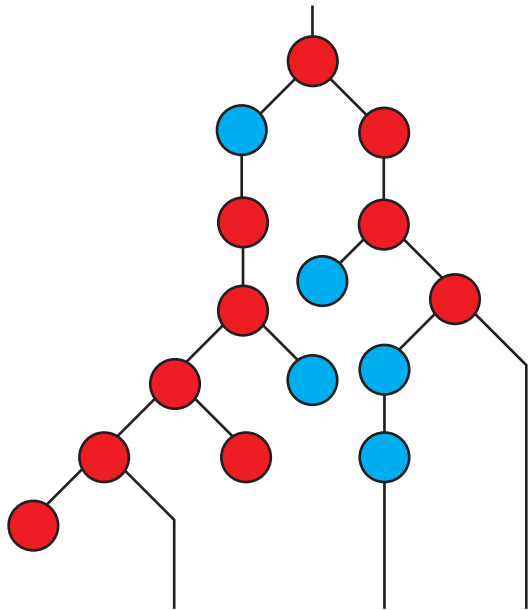


\mapsto

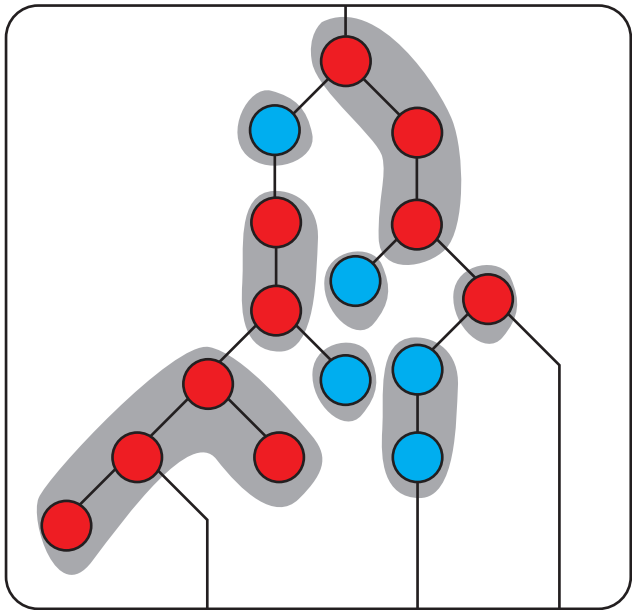




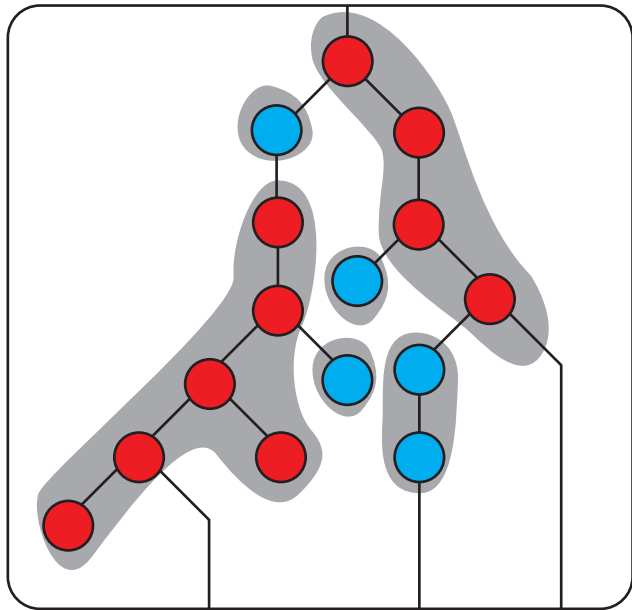
a term



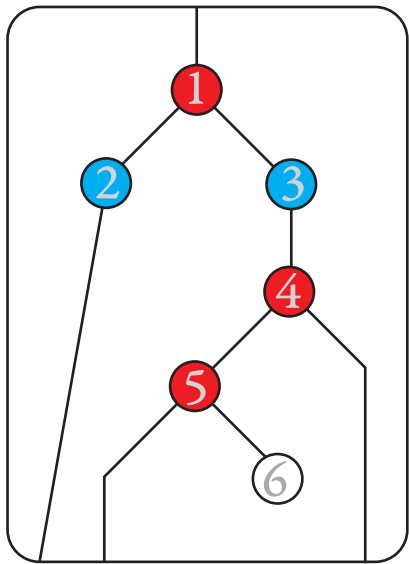
ancestor equivalence



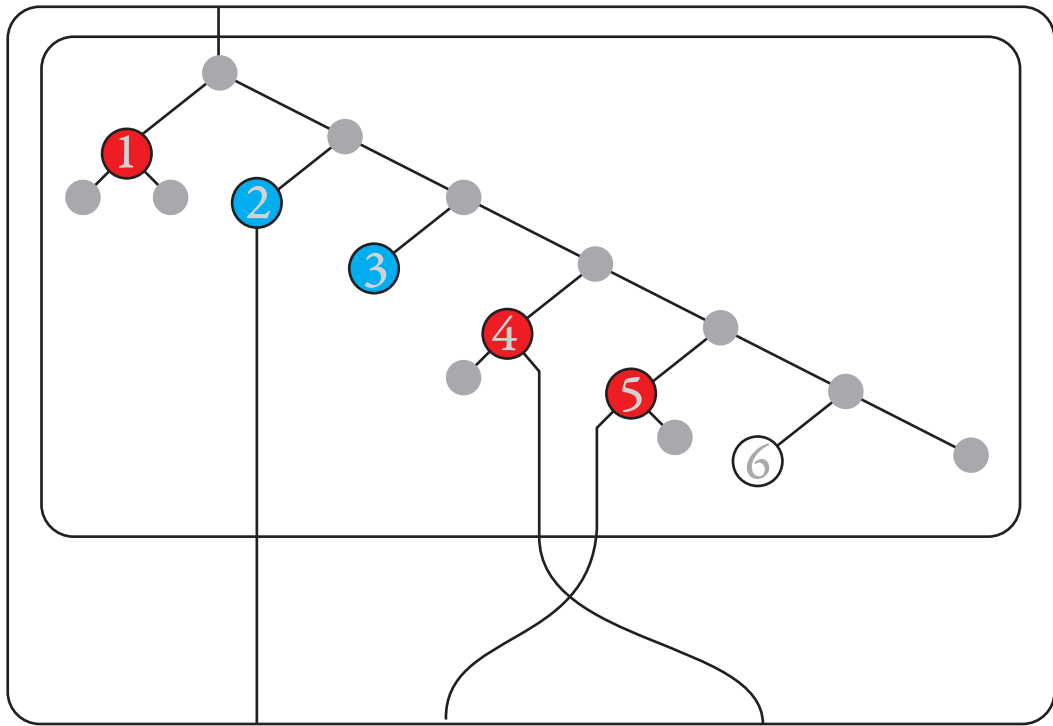
descendant equivalence



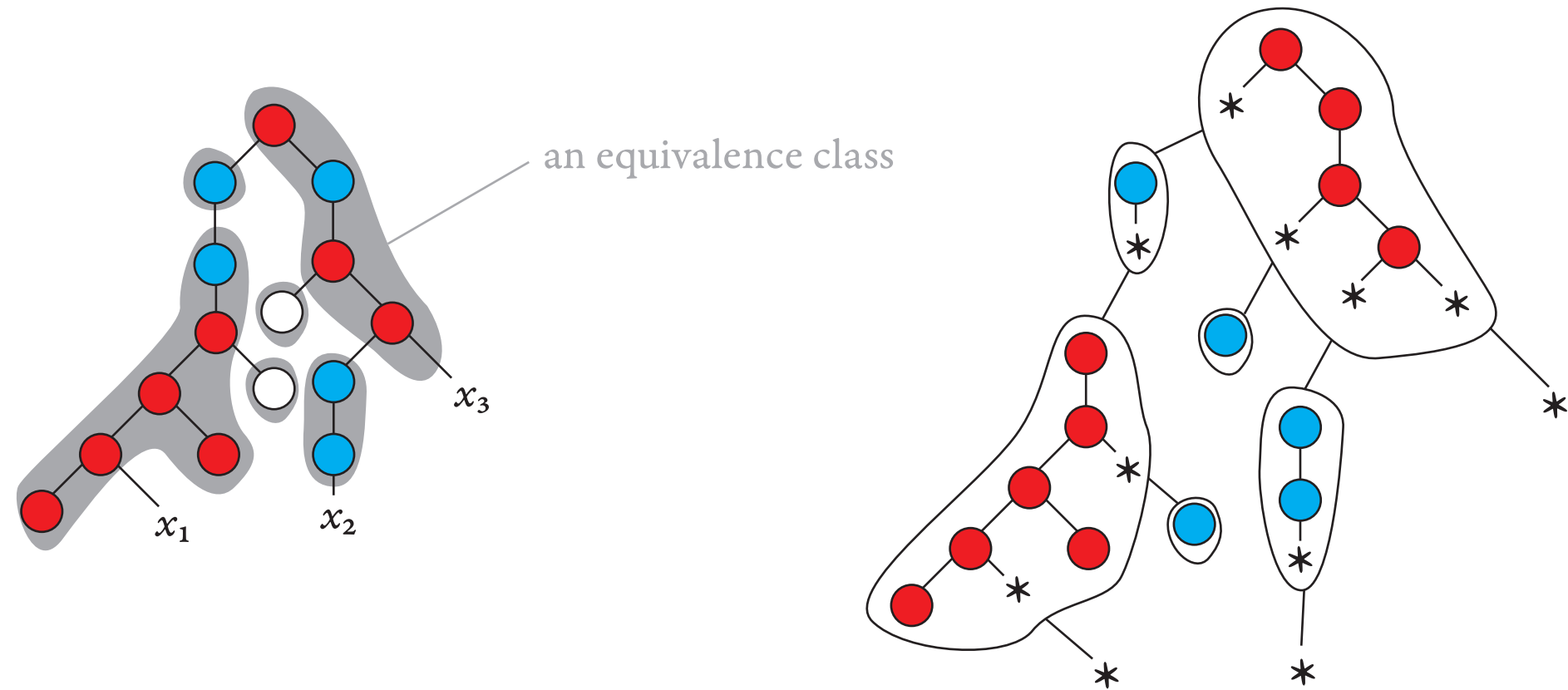




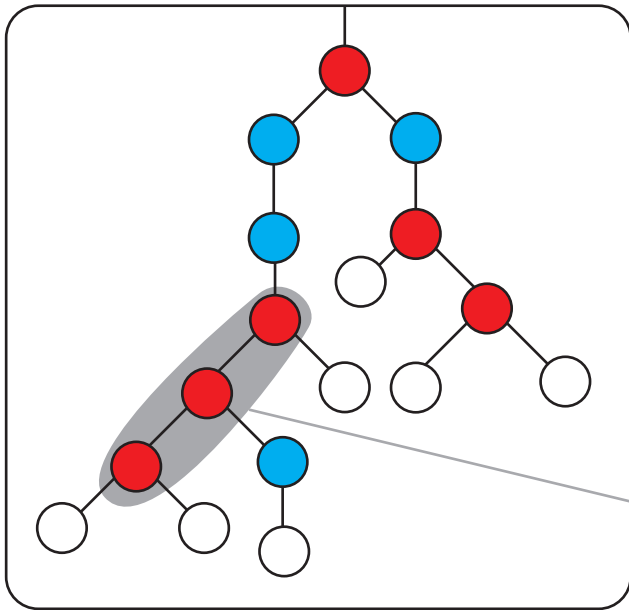
\mapsto



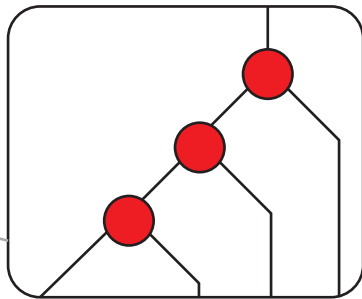
a factorisation equivalence



a tree



a term with
4 ports that
represents
part of the
tree





input alphabet

arity 2



arity 1



arity 0



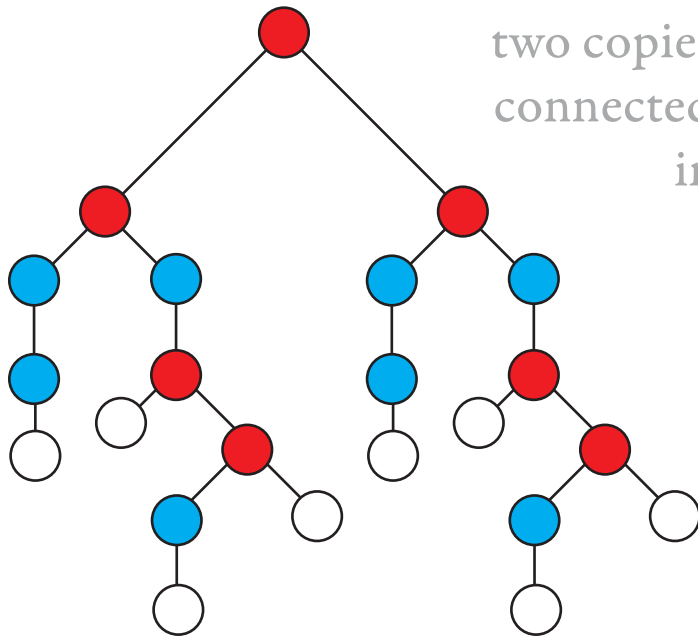
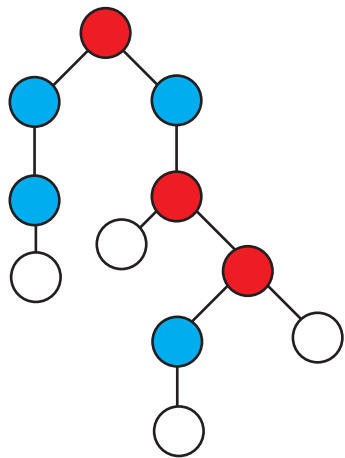
output alphabet

arity 2



arity 0





two copies of the input tree,
connected by a binary node
in the root





input alphabet

arity 2



arity 1



arity 0



output alphabet

arity 2



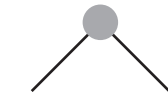
arity 1



arity 0

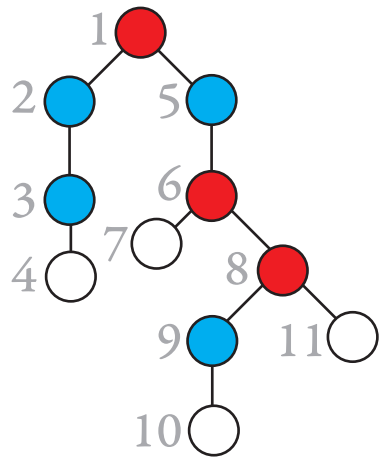


arity 2

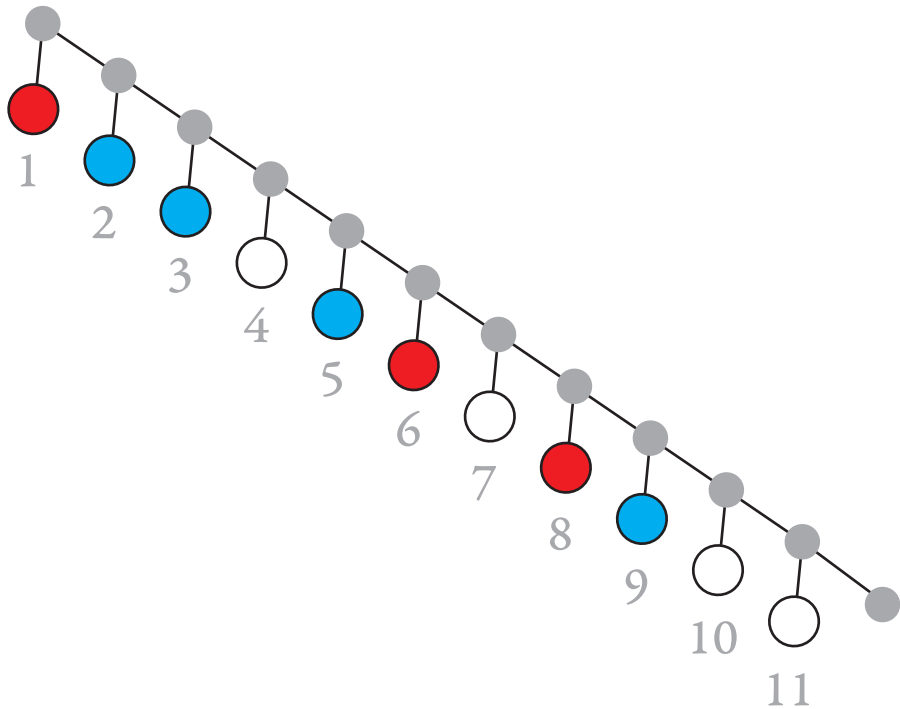


arity 0





\mapsto





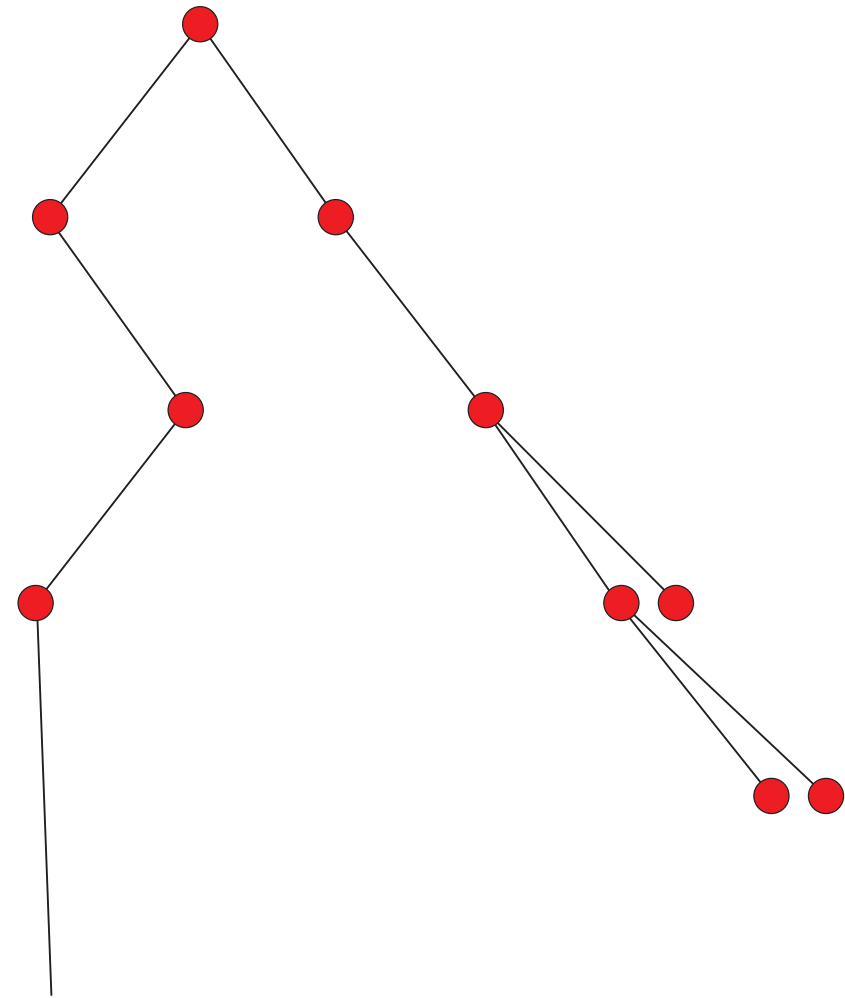
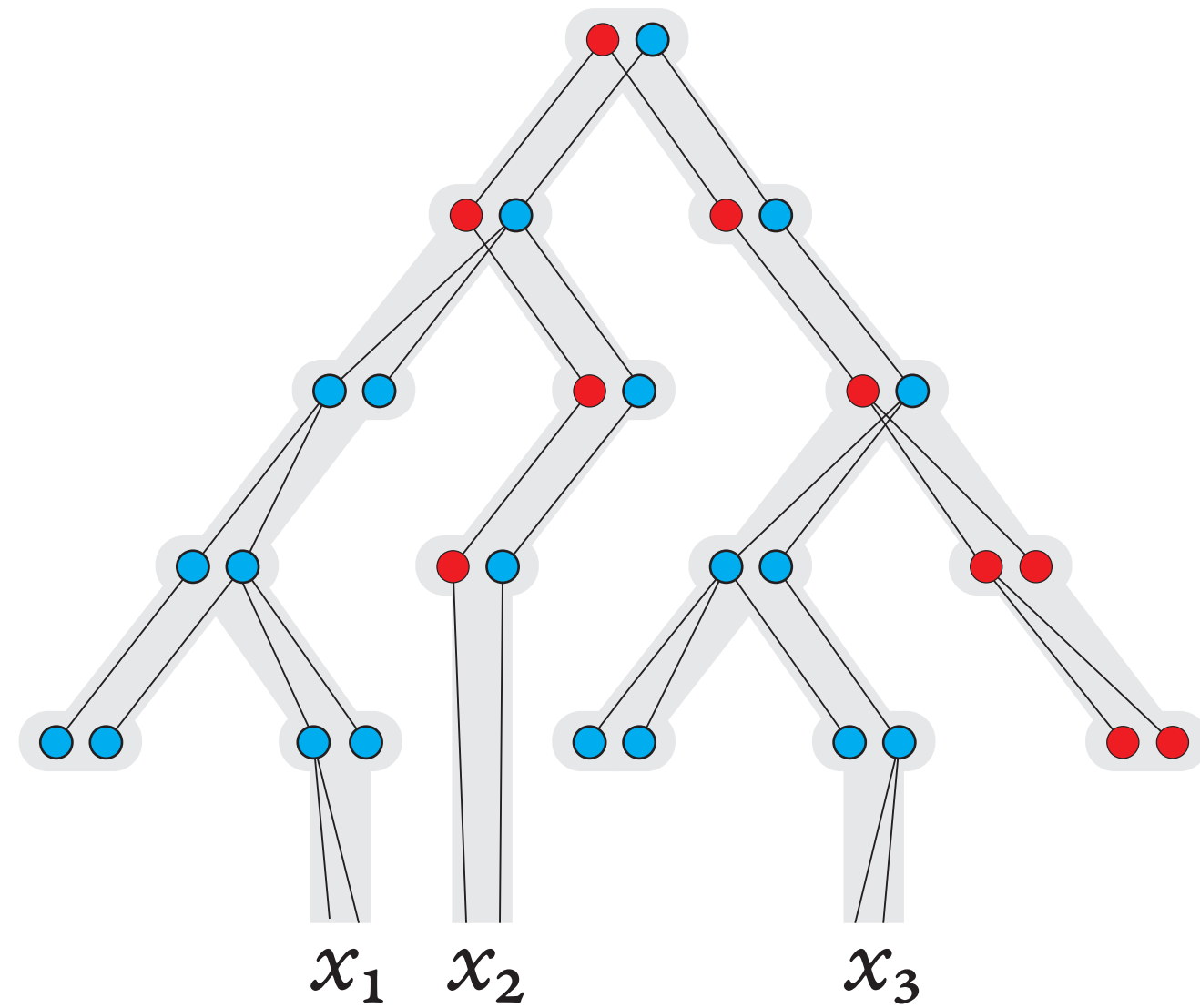


a term of arity 4



a term of arity 0



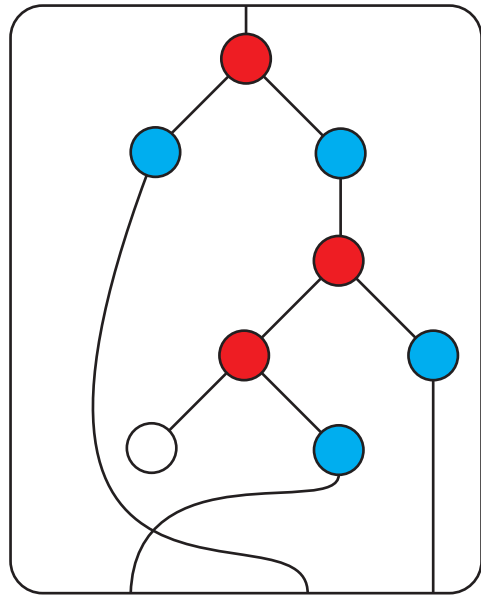




satisfies (*)

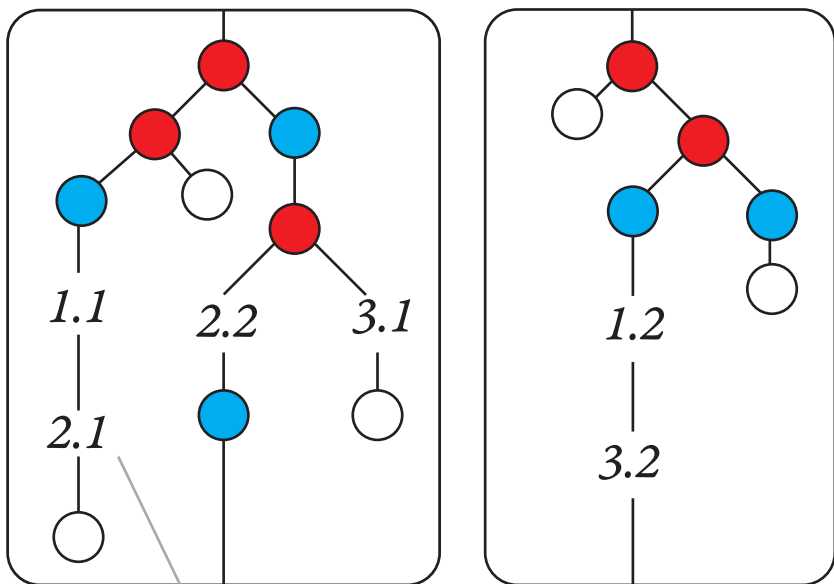
(*)

If the root has arity n ,
and $1 \leq i < j \leq n$, then
all ports of the j -th
subterm of the root are
after all ports of the
 i -th subterm of the root



violates (*)

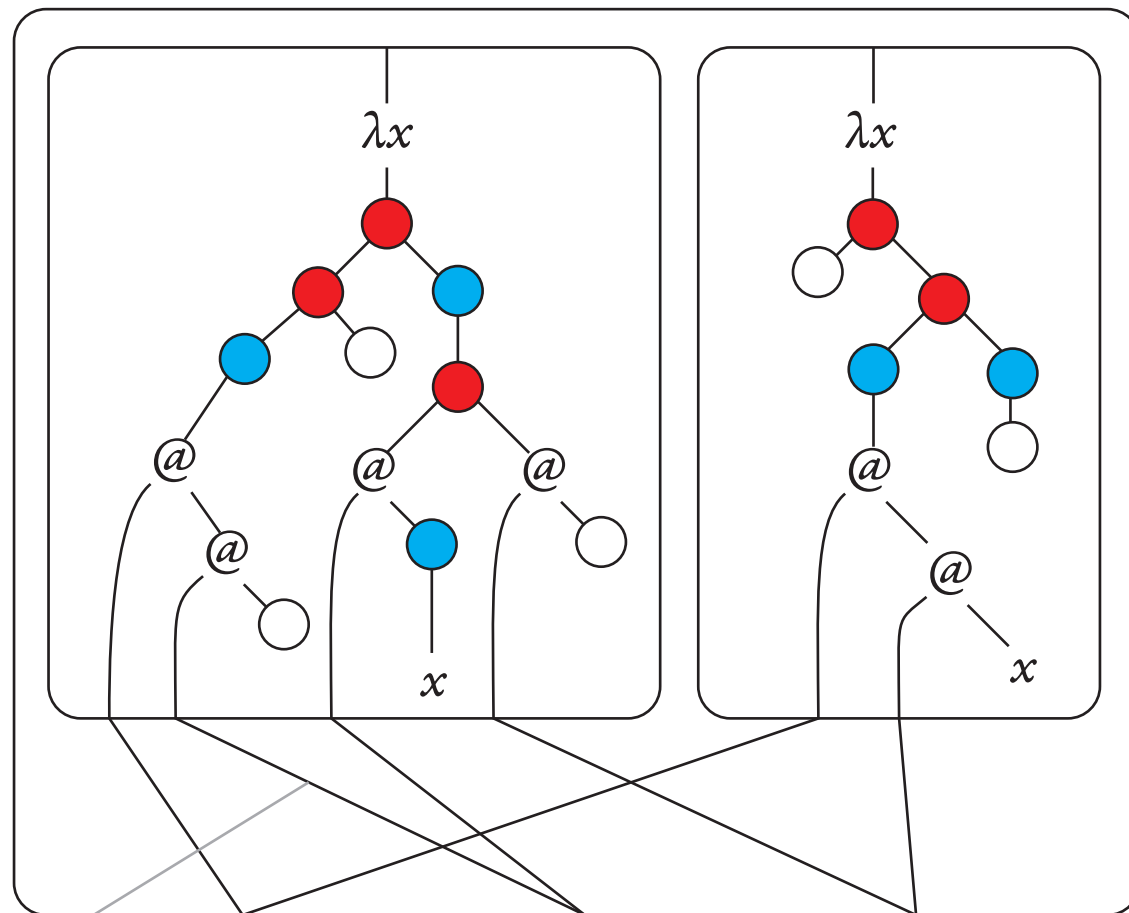
a register update



Variable $i.j$ represents register i in the j -th argument of the register update.

In the dual, this variable is mapped to the i -th edge which enters the j -th port of the reducer.

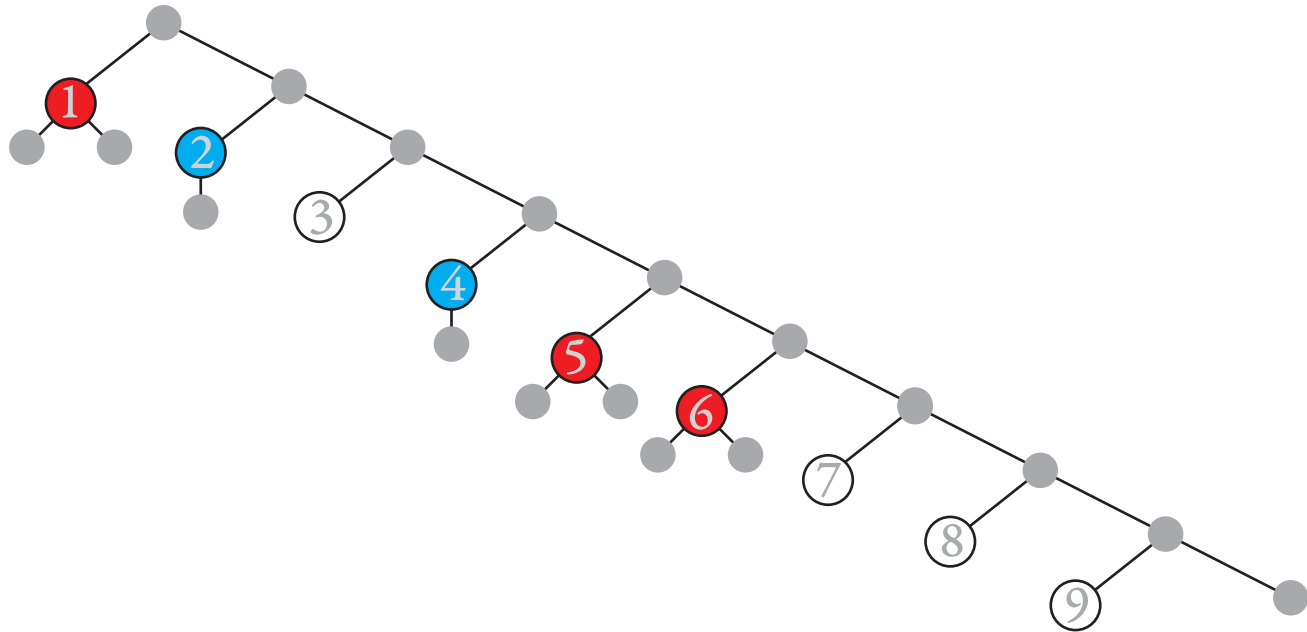
its dual



input

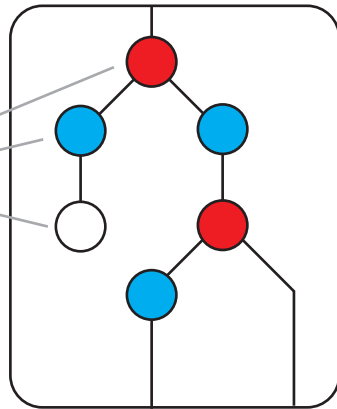


output

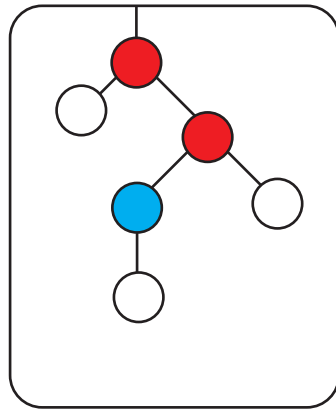


letters of the
output alphabet

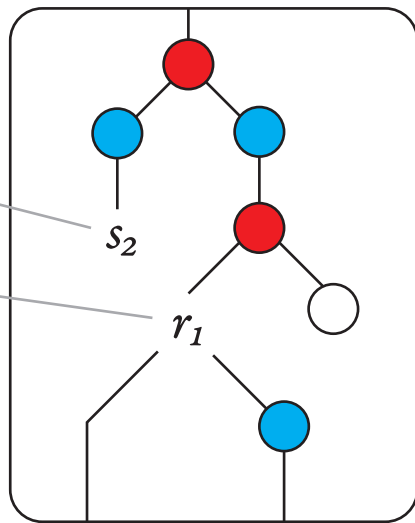
register r of arity 2



register s of arity 0



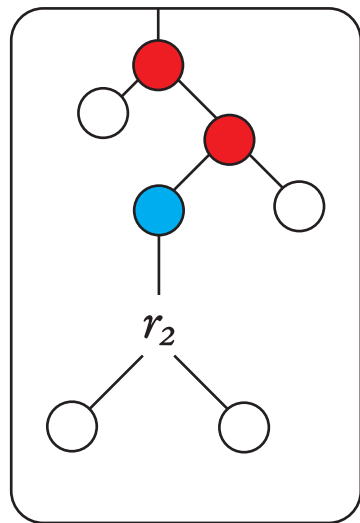
register r



copy 2 of register s

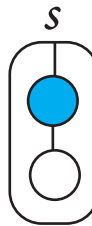
copy 1 of register r

register s









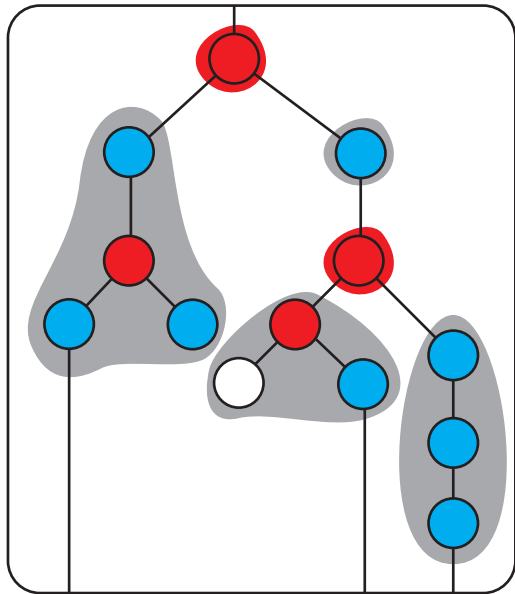




factors without
branching nodes

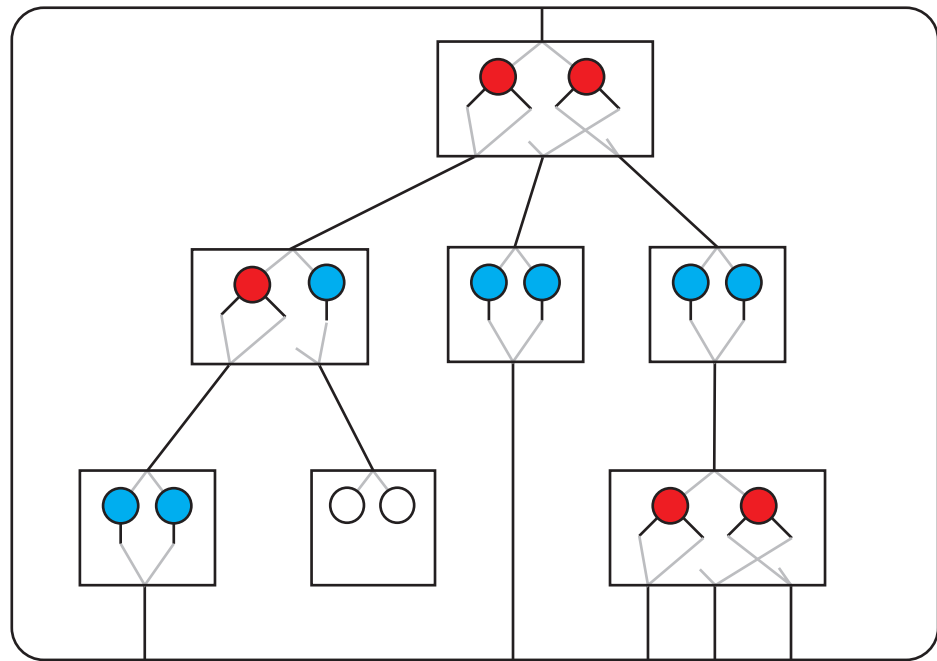


factors with
branching nodes

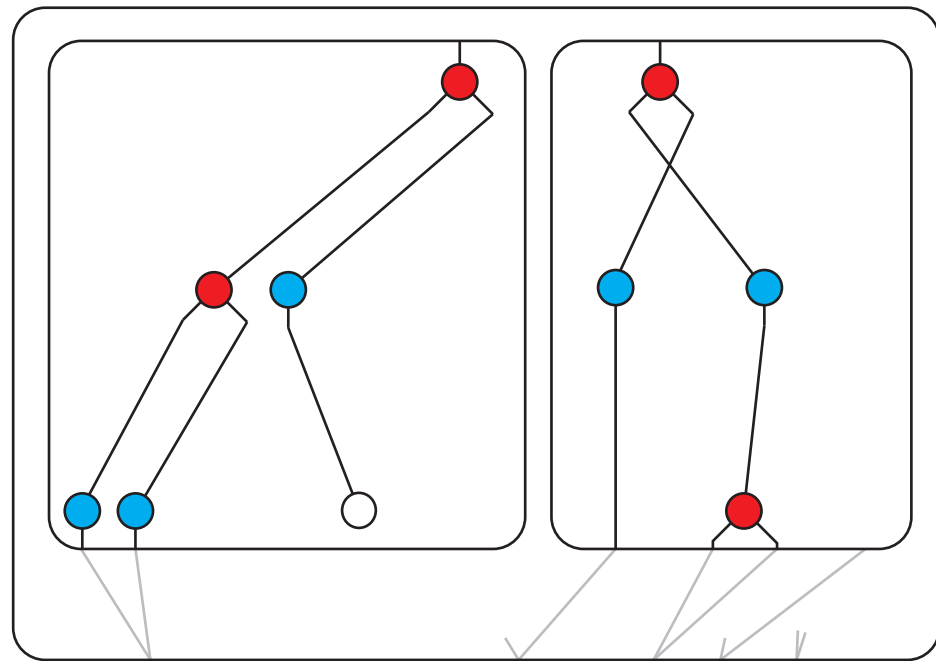


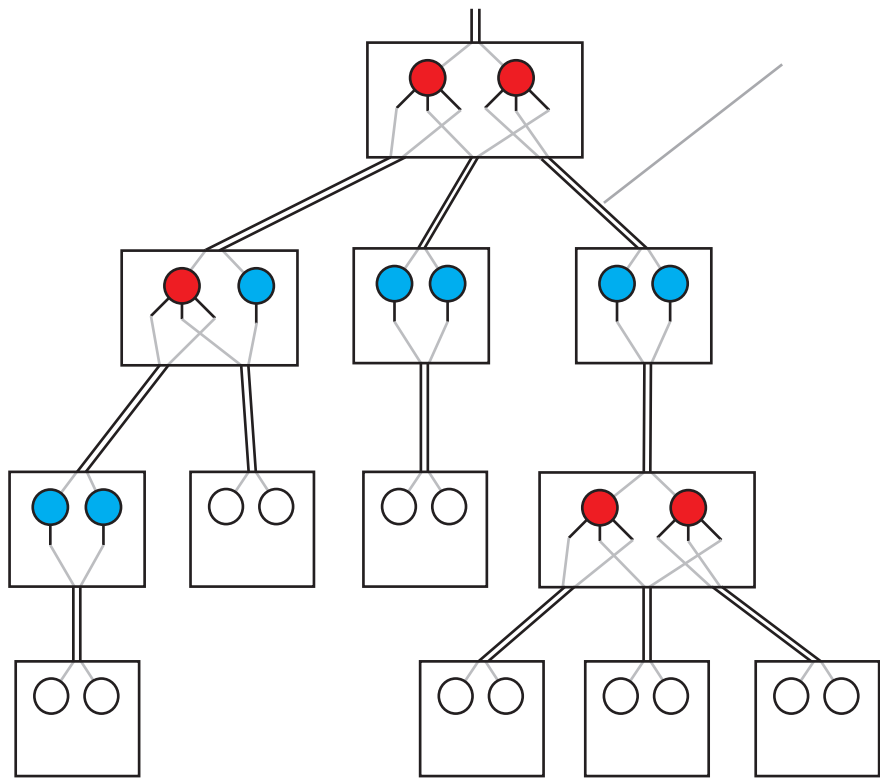


a term of matrix powers

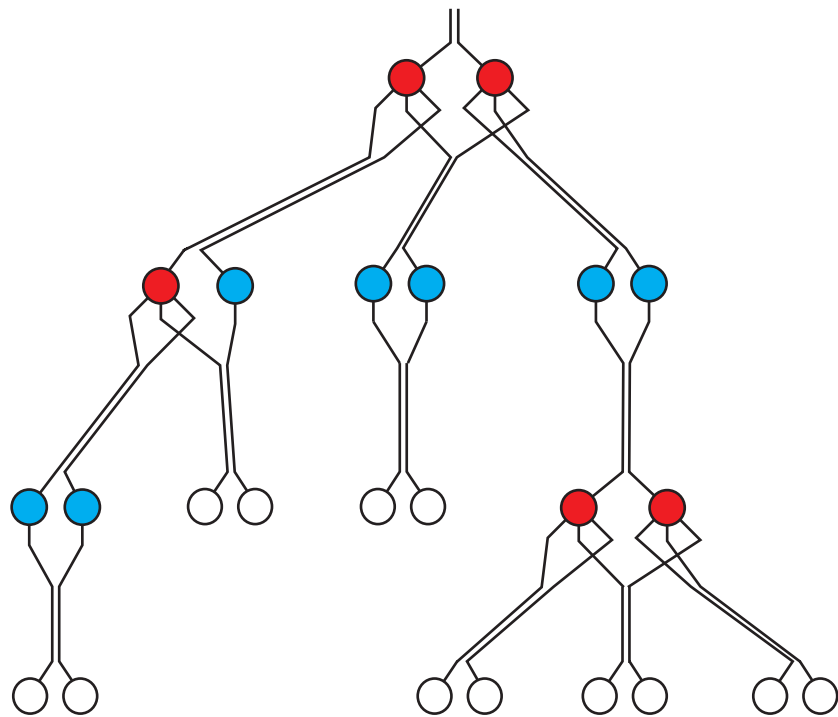


its term unfolding

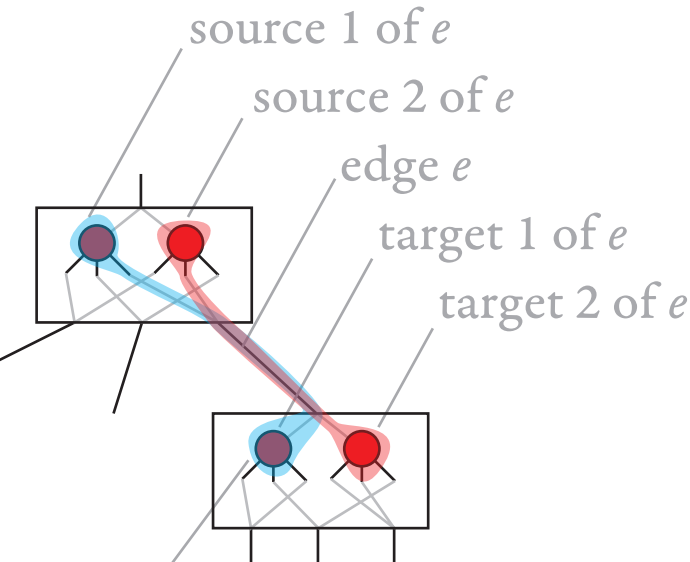




\mapsto







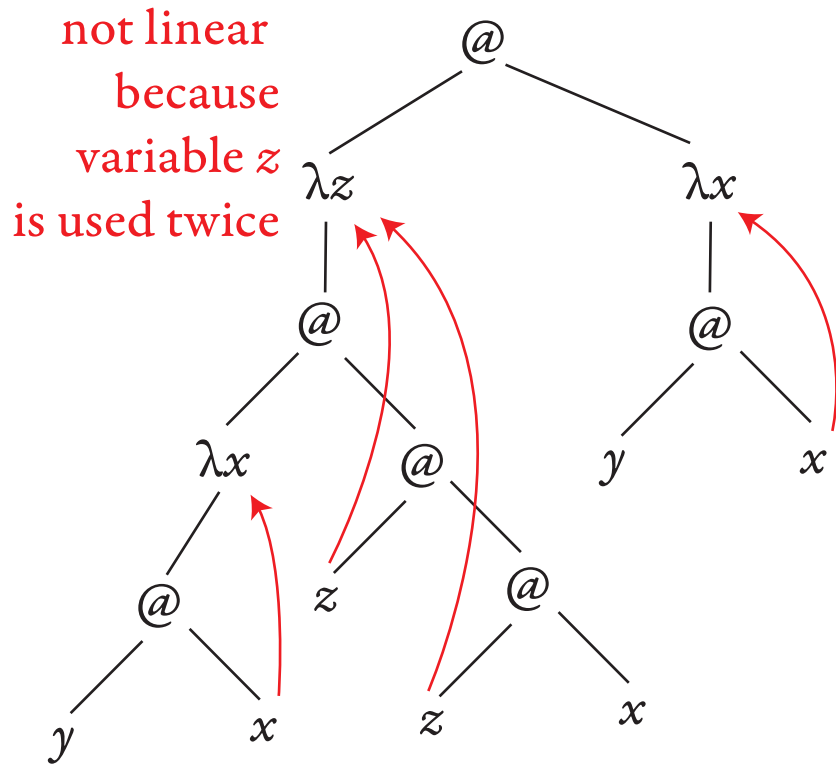
linear



we only count
variables used
in their scope

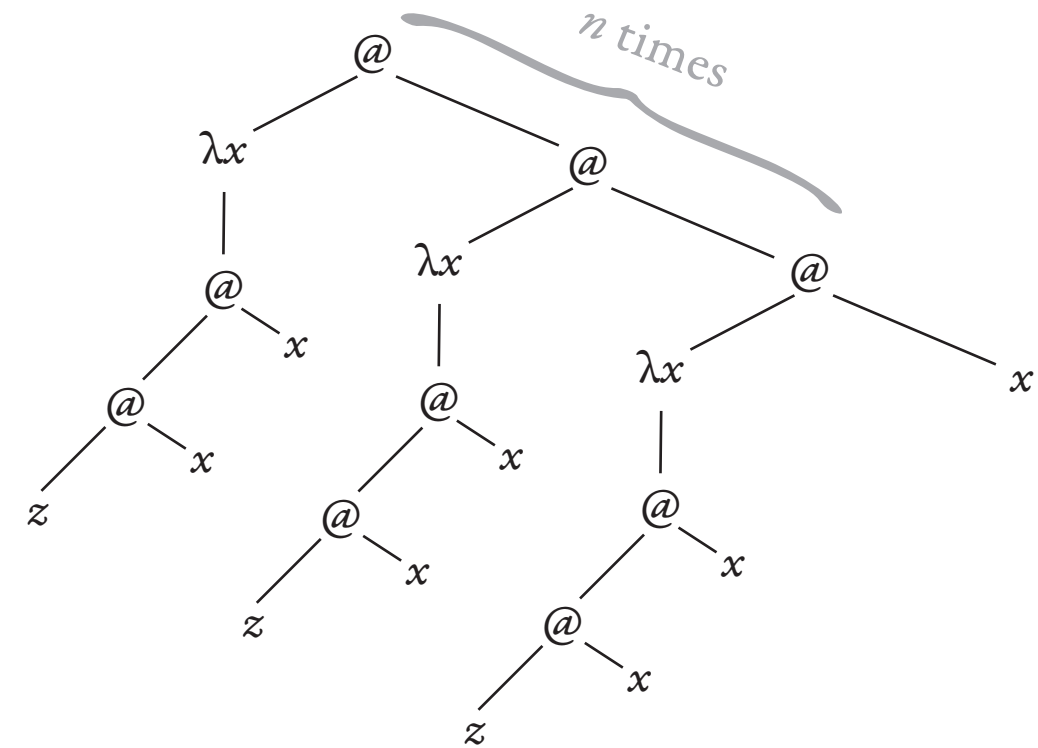
variable z can be used twice because it is free

not linear

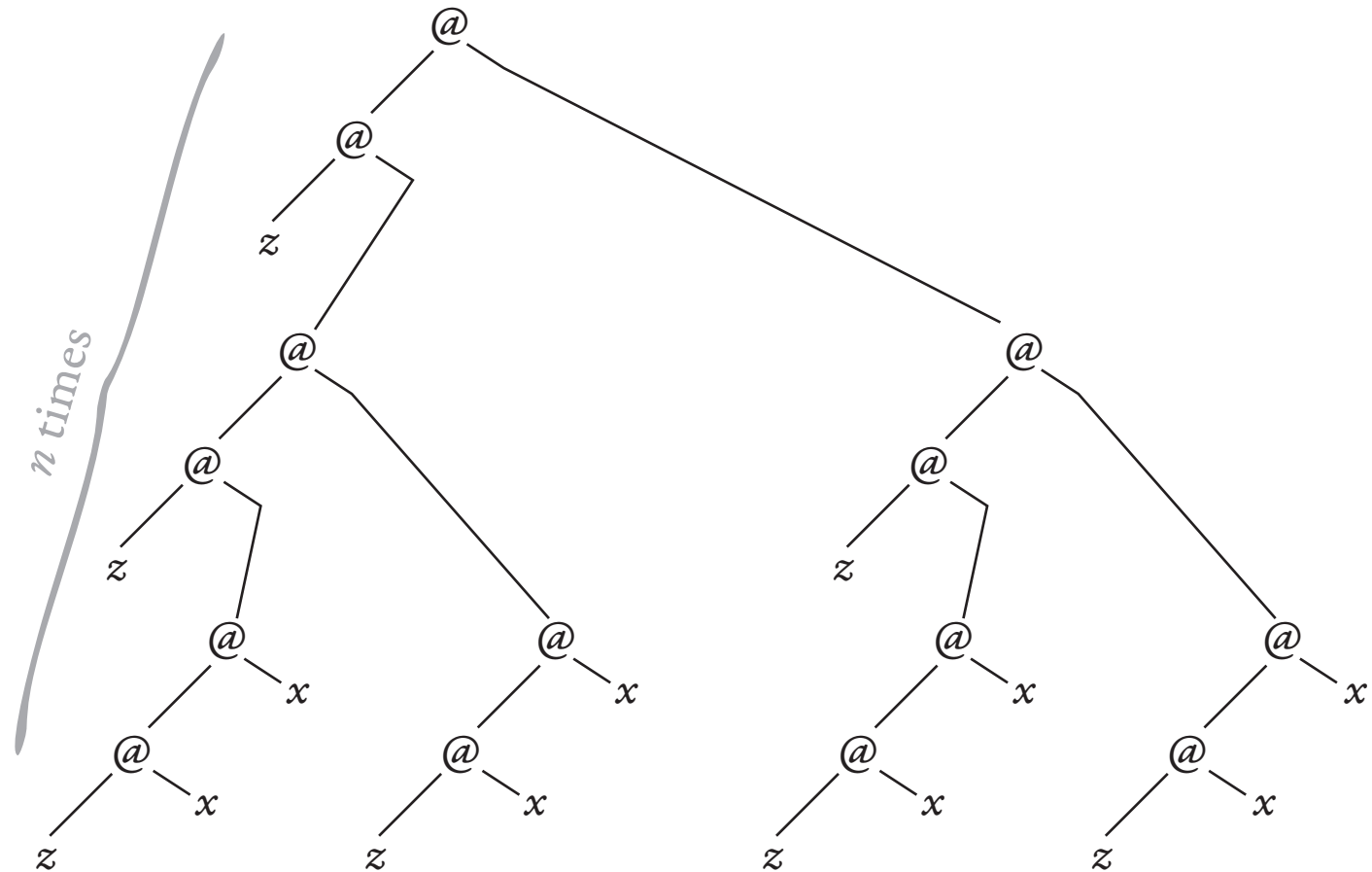


not linear
because
variable z
is used twice

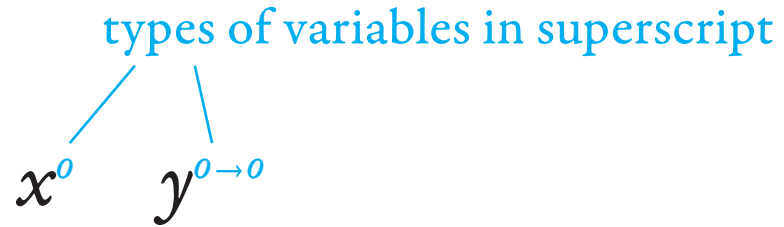
a λ -term of size $O(n)$



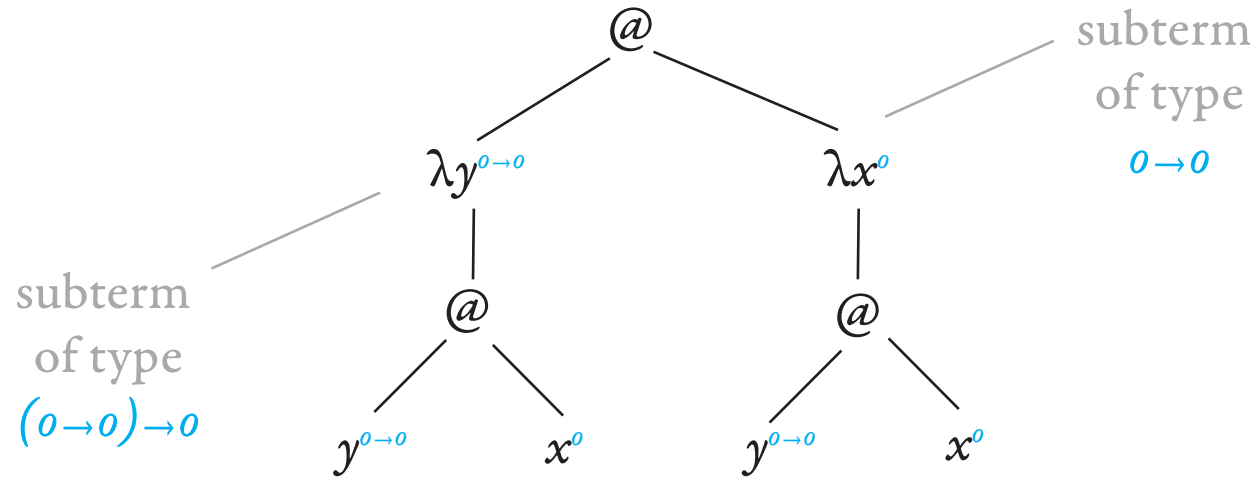
its normal form of size $O(2^n)$



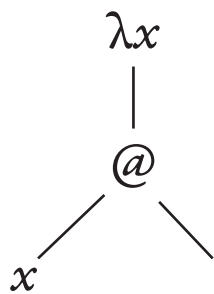
variables



λ -term of type o



@



$\lambda x.$

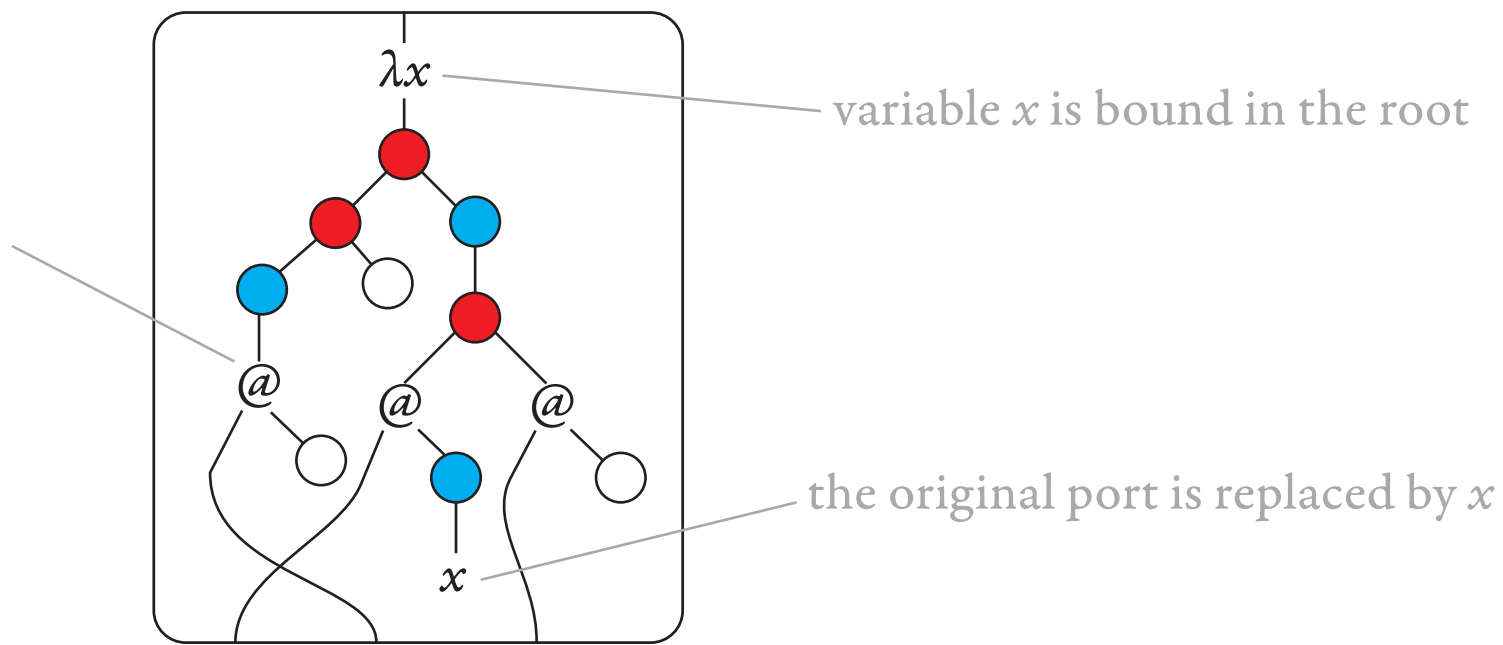


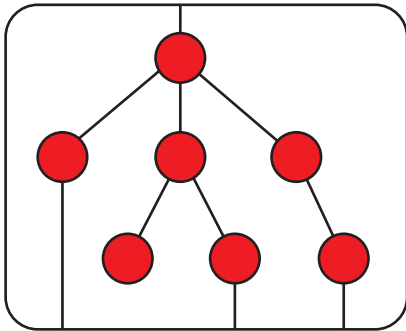
r



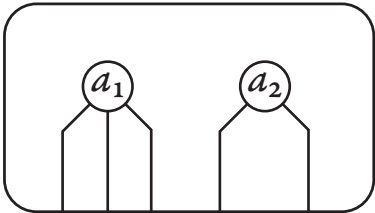
placeholder for the term
stored in the unique register
of the 2nd child

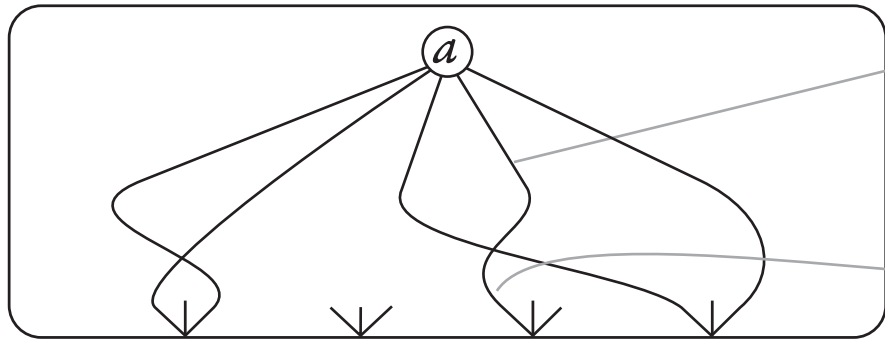






dangling edges
represent ports





port 4

$\Downarrow f$

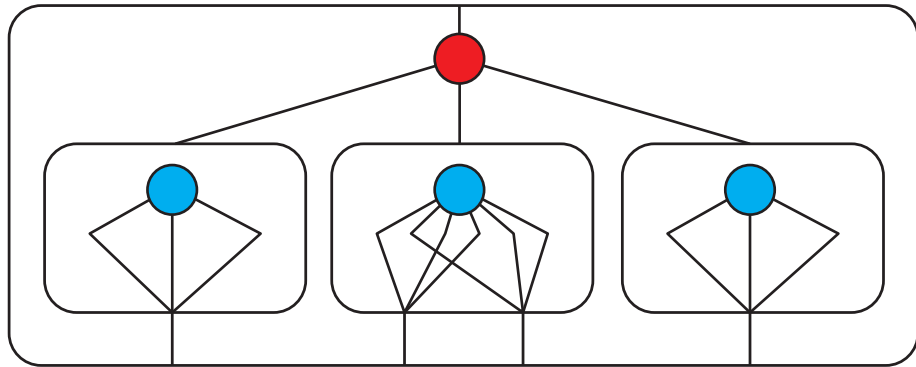
position 1, group 3

the root is from Σ

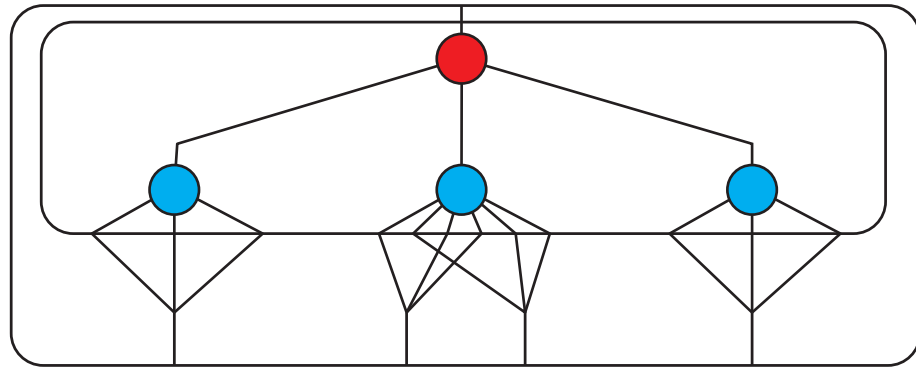
all children are from Γ

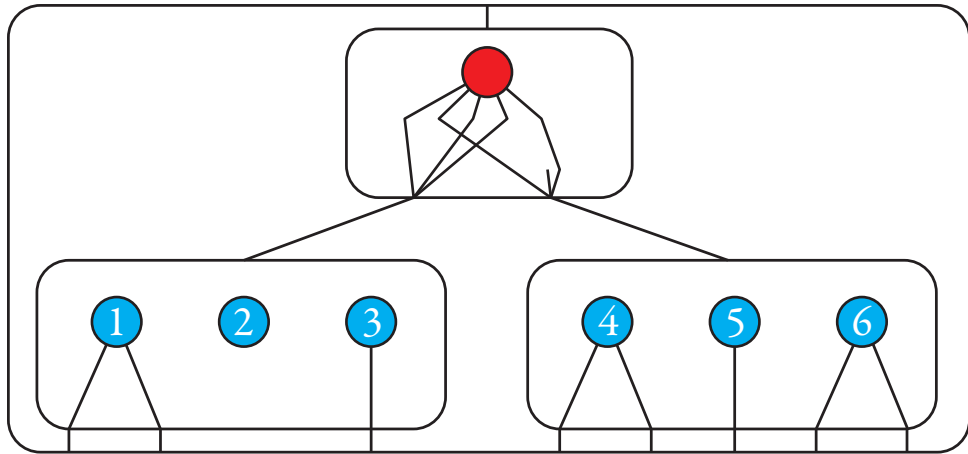


input

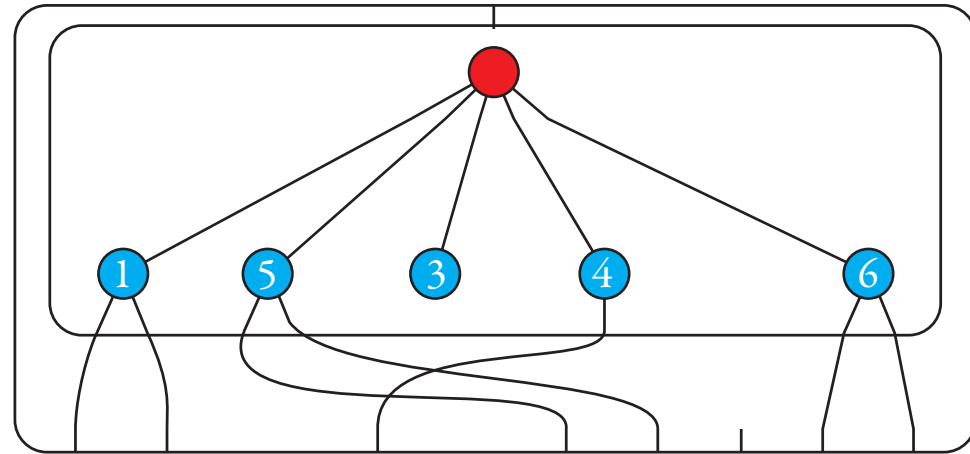


output

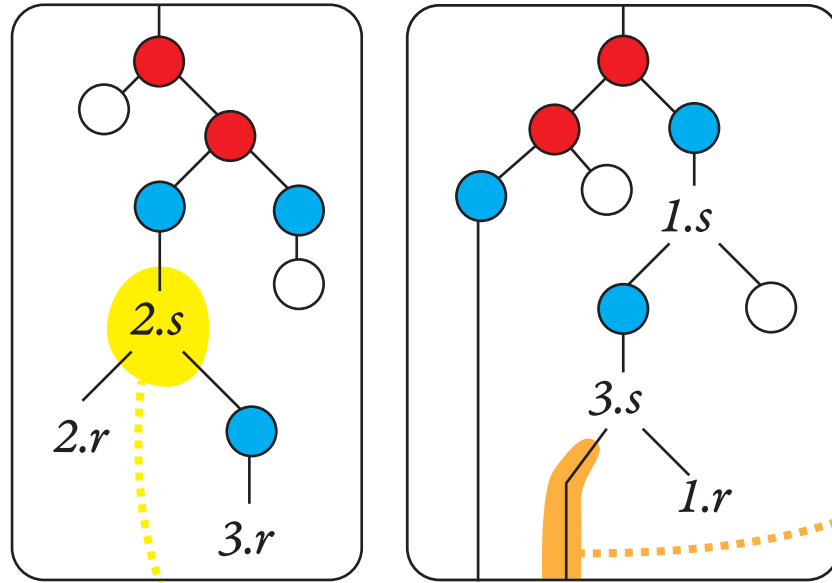




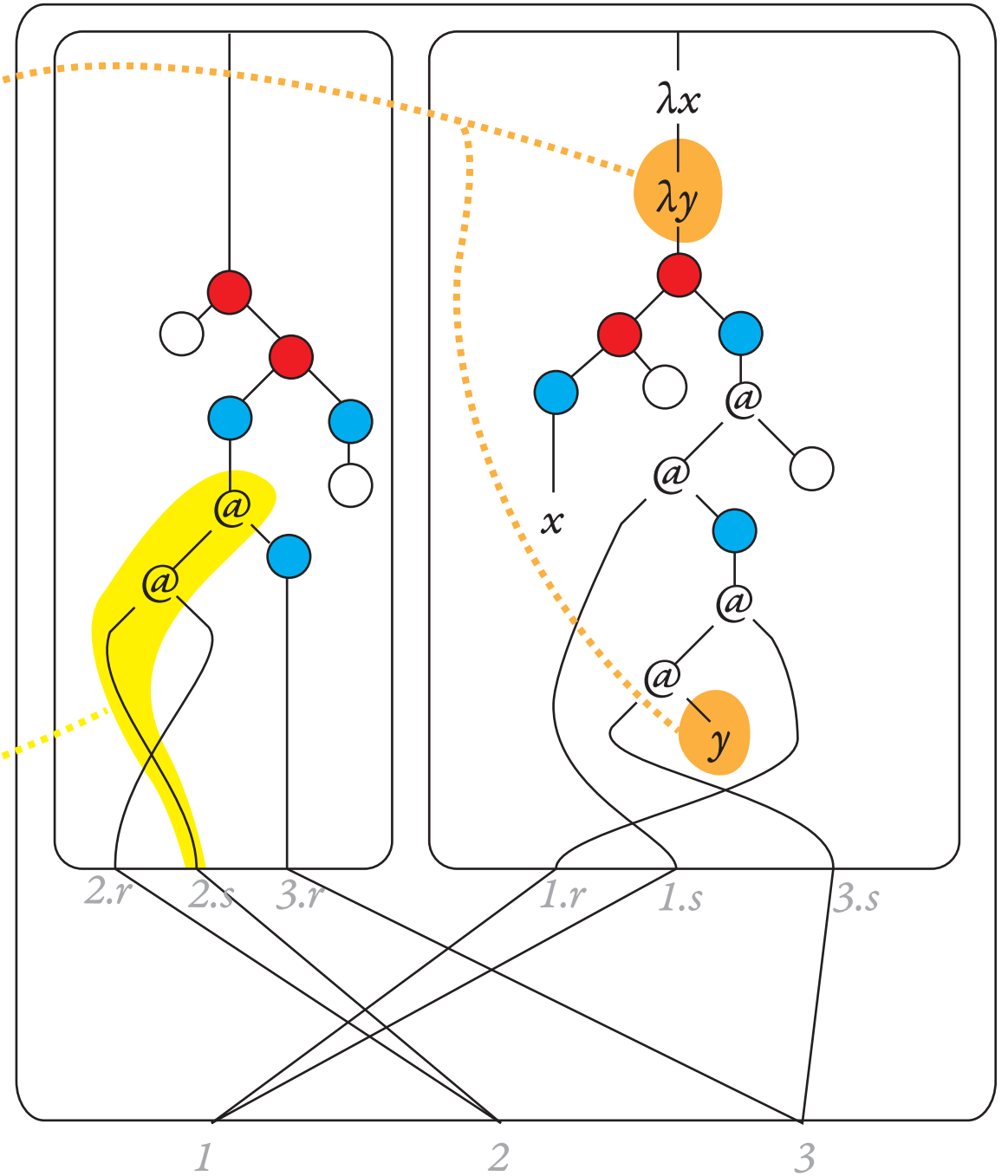
\mapsto



a register update



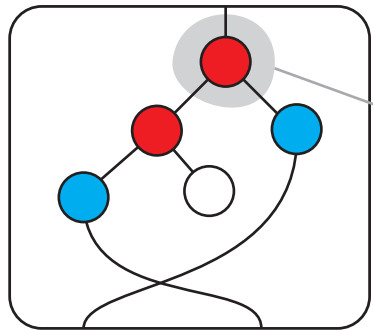
its dual



The diagram shows a tree structure with nodes colored red, blue, and white. A yellow circle labeled r_1 is highlighted, and an orange shape labeled r_2 is highlighted. A dashed line connects r_1 and r_2 .

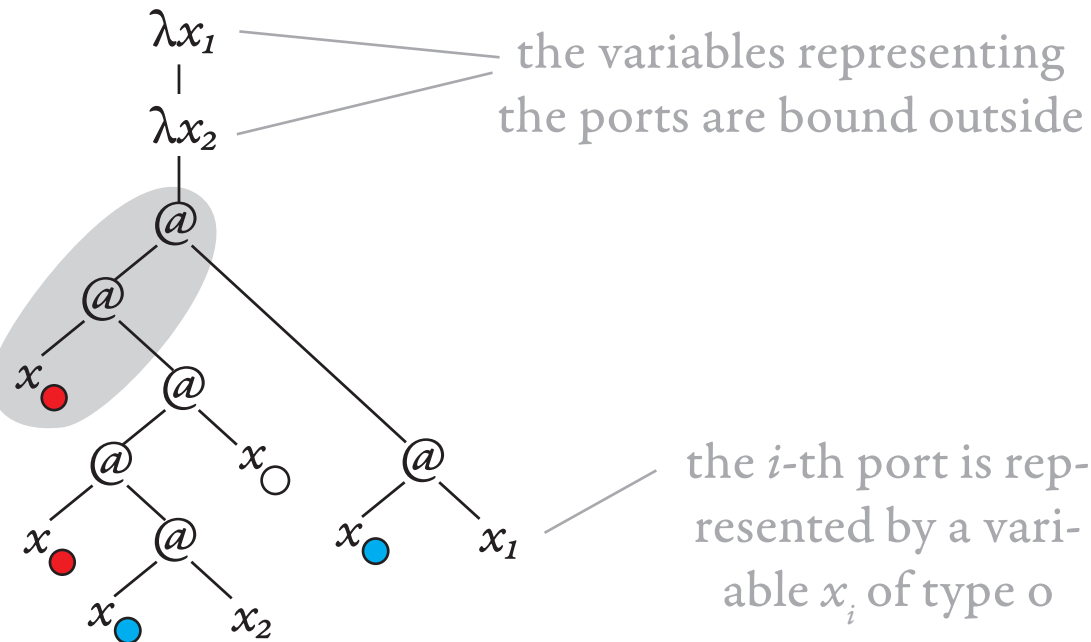
[illegible]

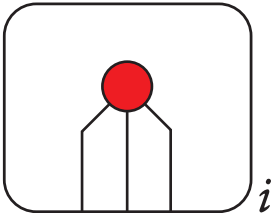
a term of arity 2

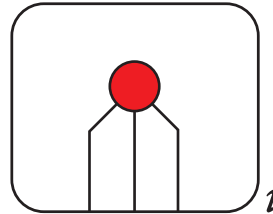


a non-port node is represented by a variable, corresponding to the label, applied to the children of the node

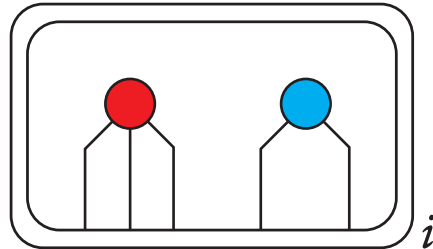
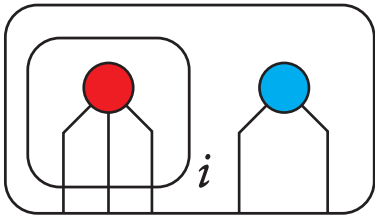
its representation as a λ -term

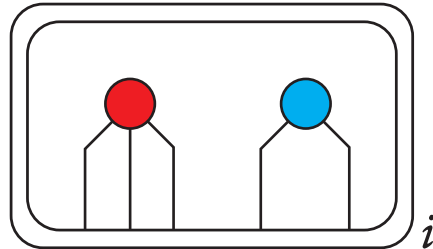
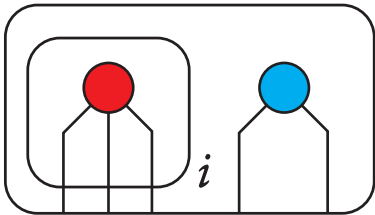


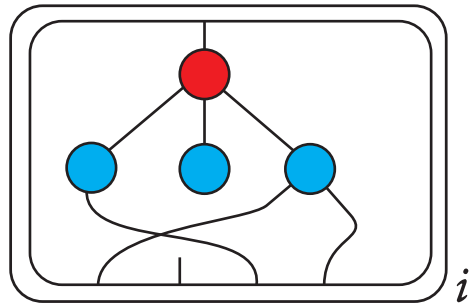
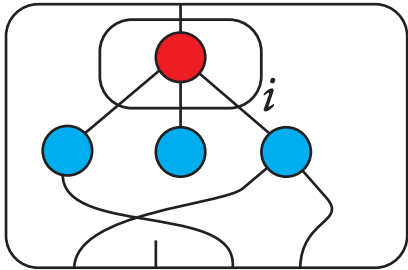


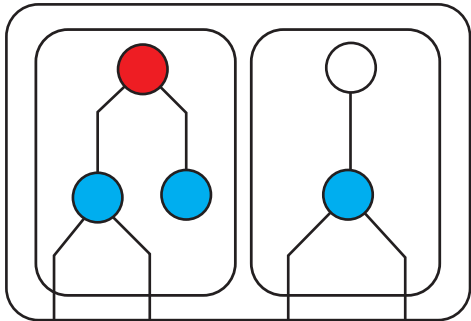
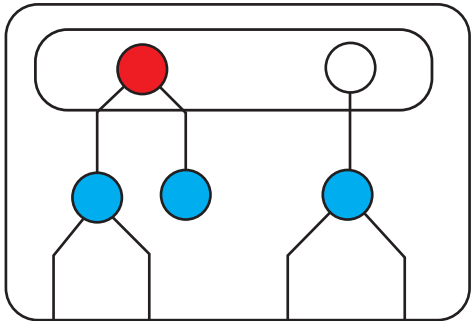


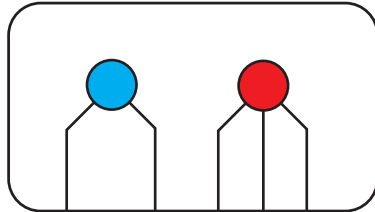




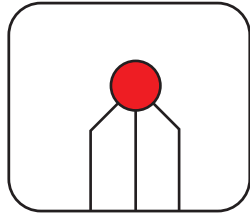


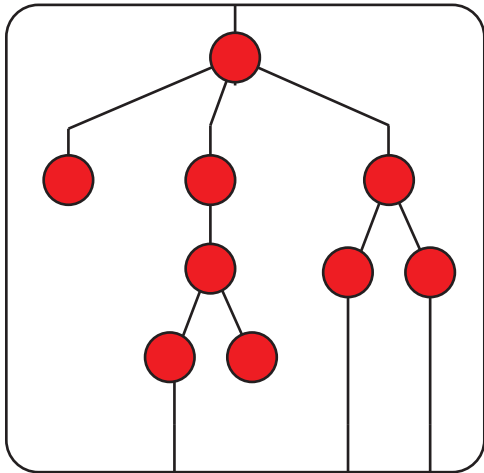
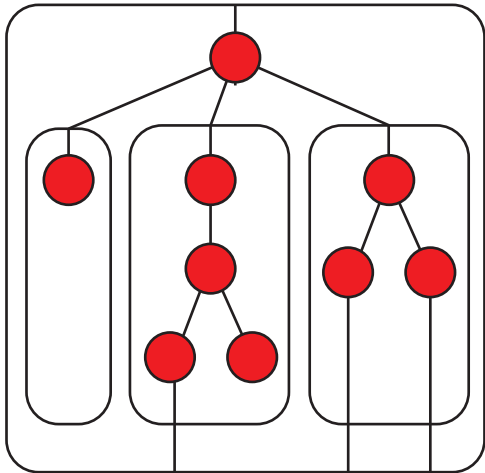


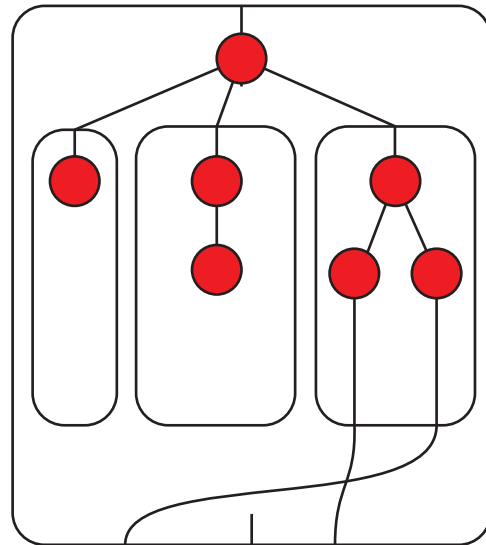
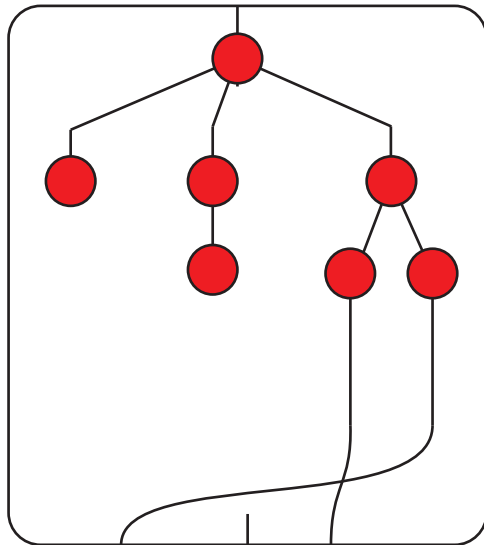


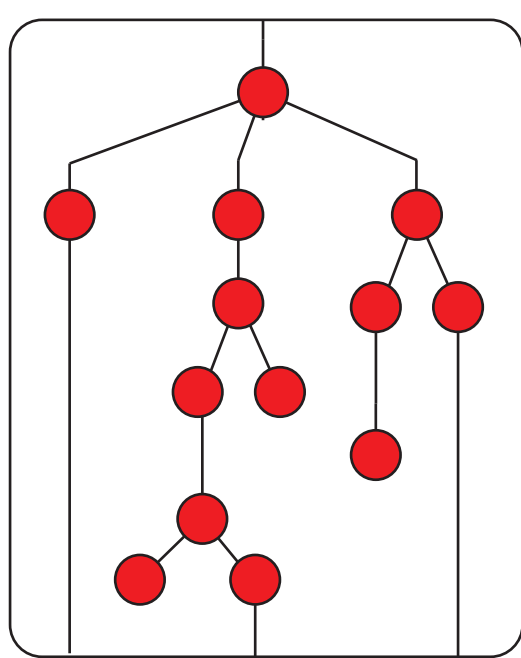
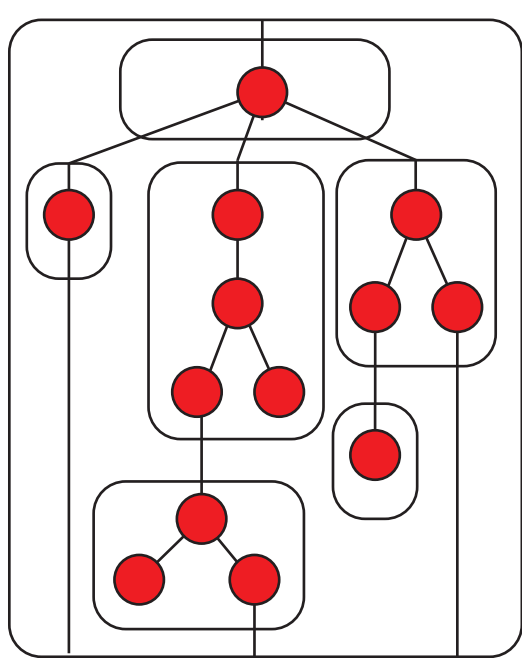


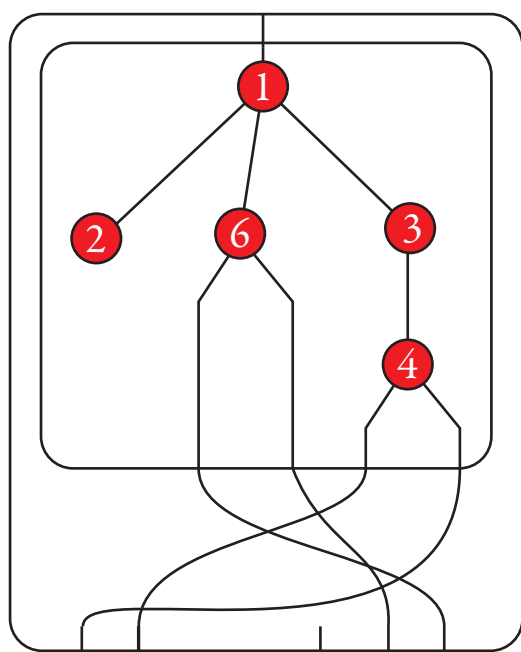


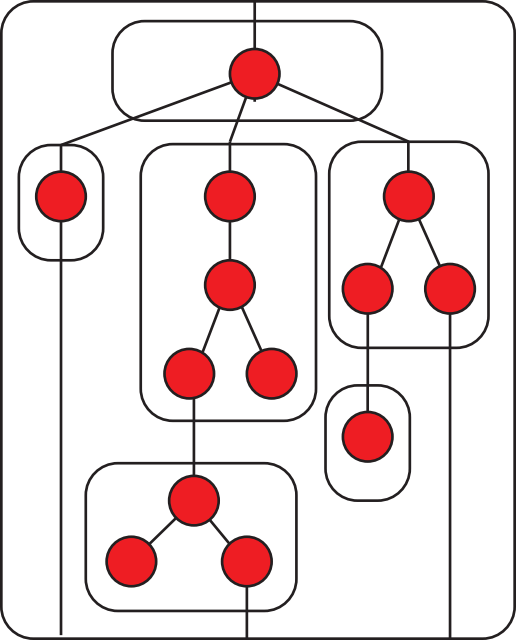




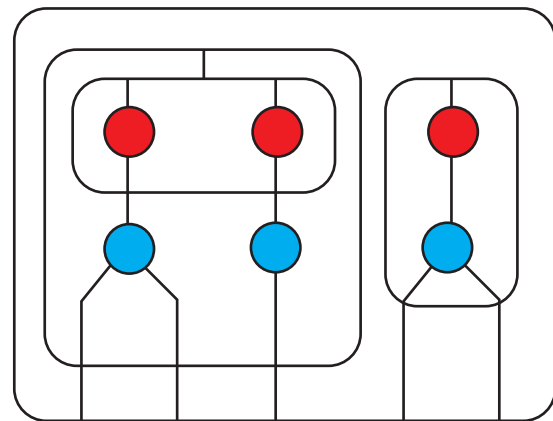
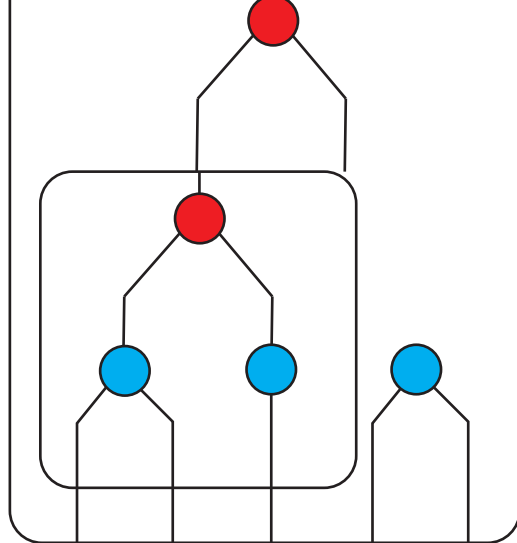
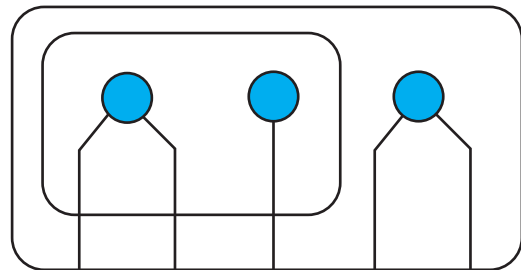


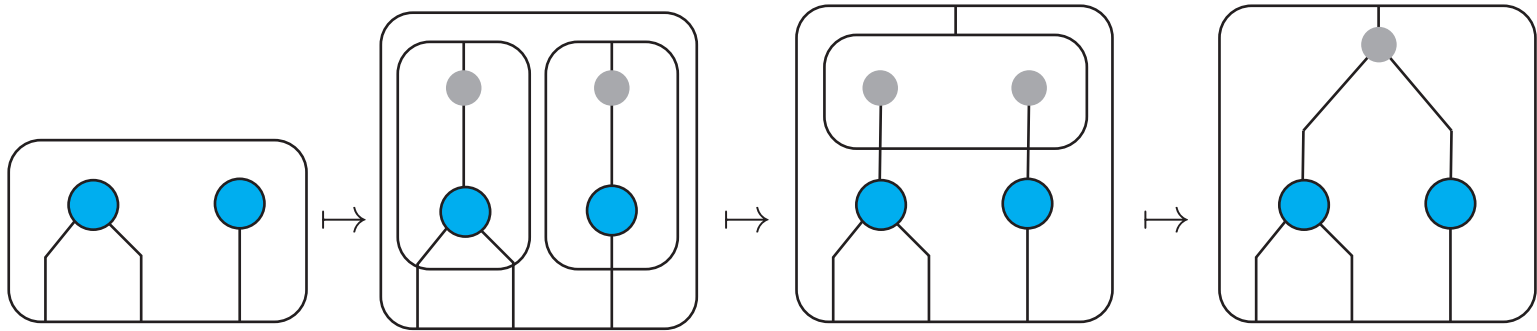






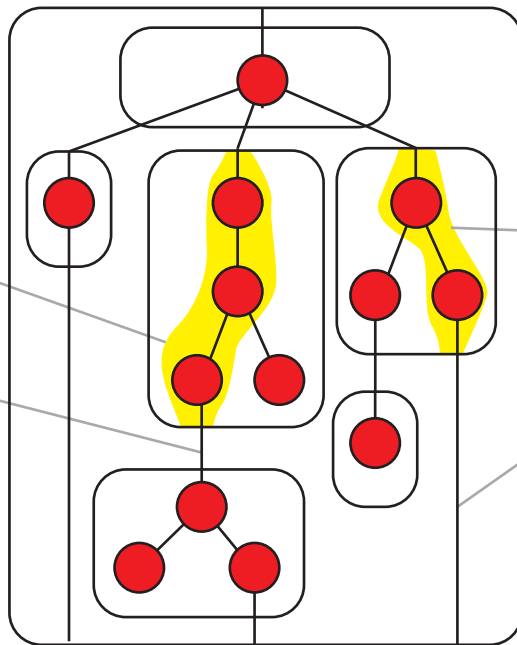




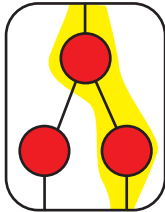




the subbranch
corresponding to
an internal edge

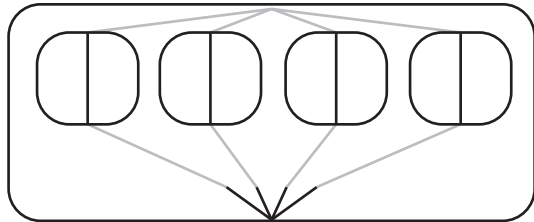


the subbranch
corresponding to
an external edge



a branch can be visualised as
a term with a distinguished
root-to-port path

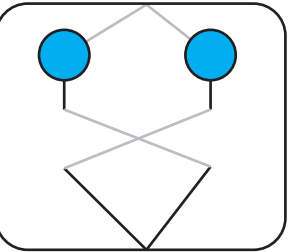




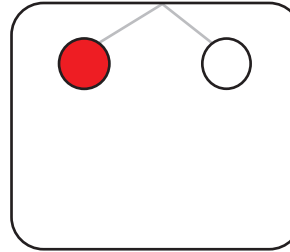
a tuple of k identity terms
with all their ports folded
into one

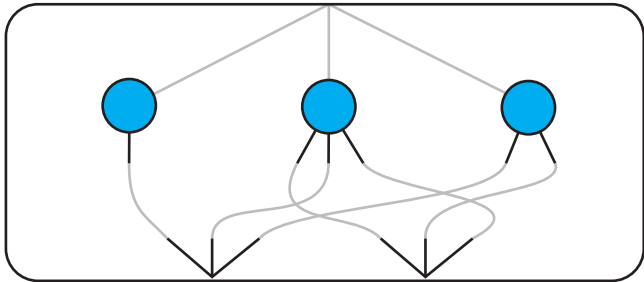
$$\Sigma = \{ \text{blue circle with stem}, \text{red circle}, \text{white circle} \}$$

$$a \in \Sigma^{[2]}$$

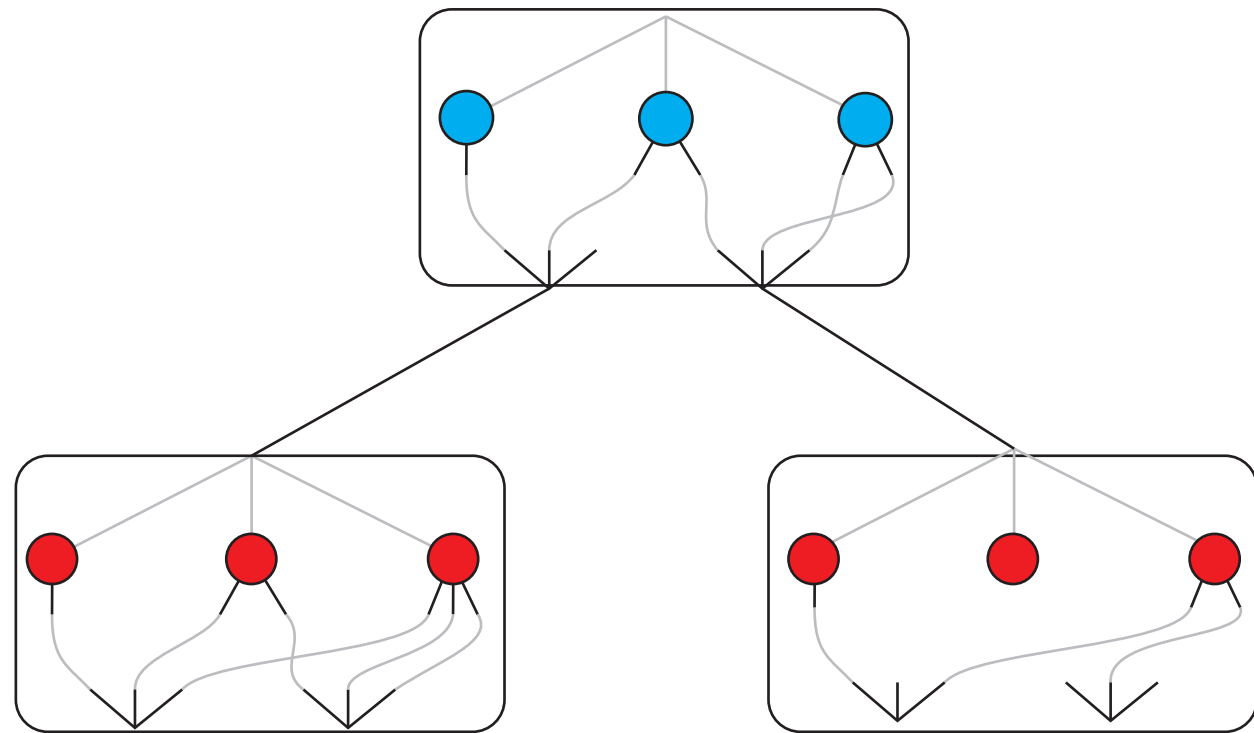


$$b \in \Sigma^{[2]}$$

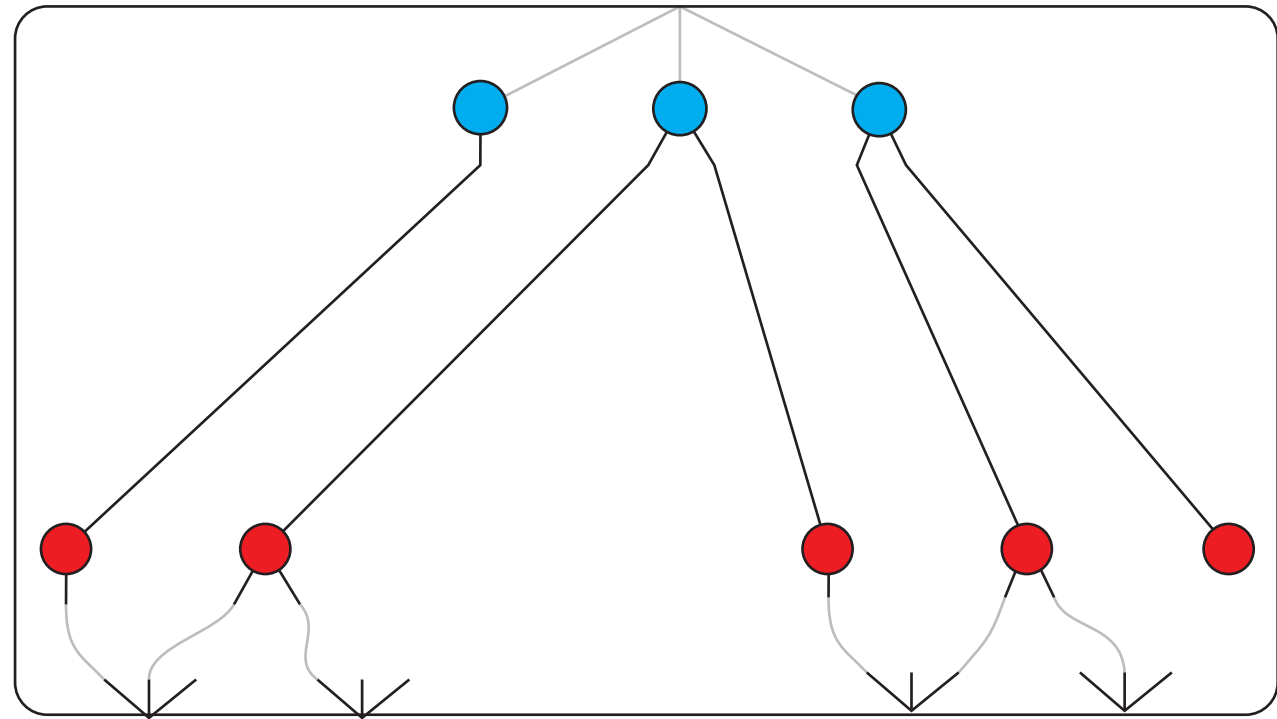


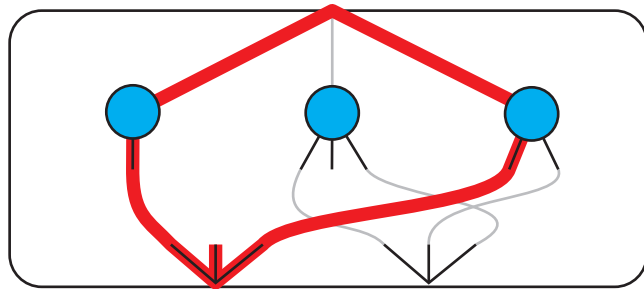


a shallow term of matrix powers



its shallow unfolding





twist of port 1

1

2

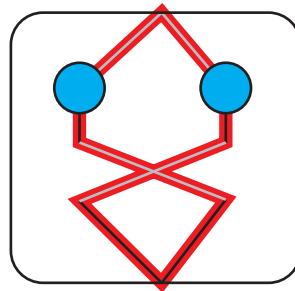
3



1

2

3



twist of port 1

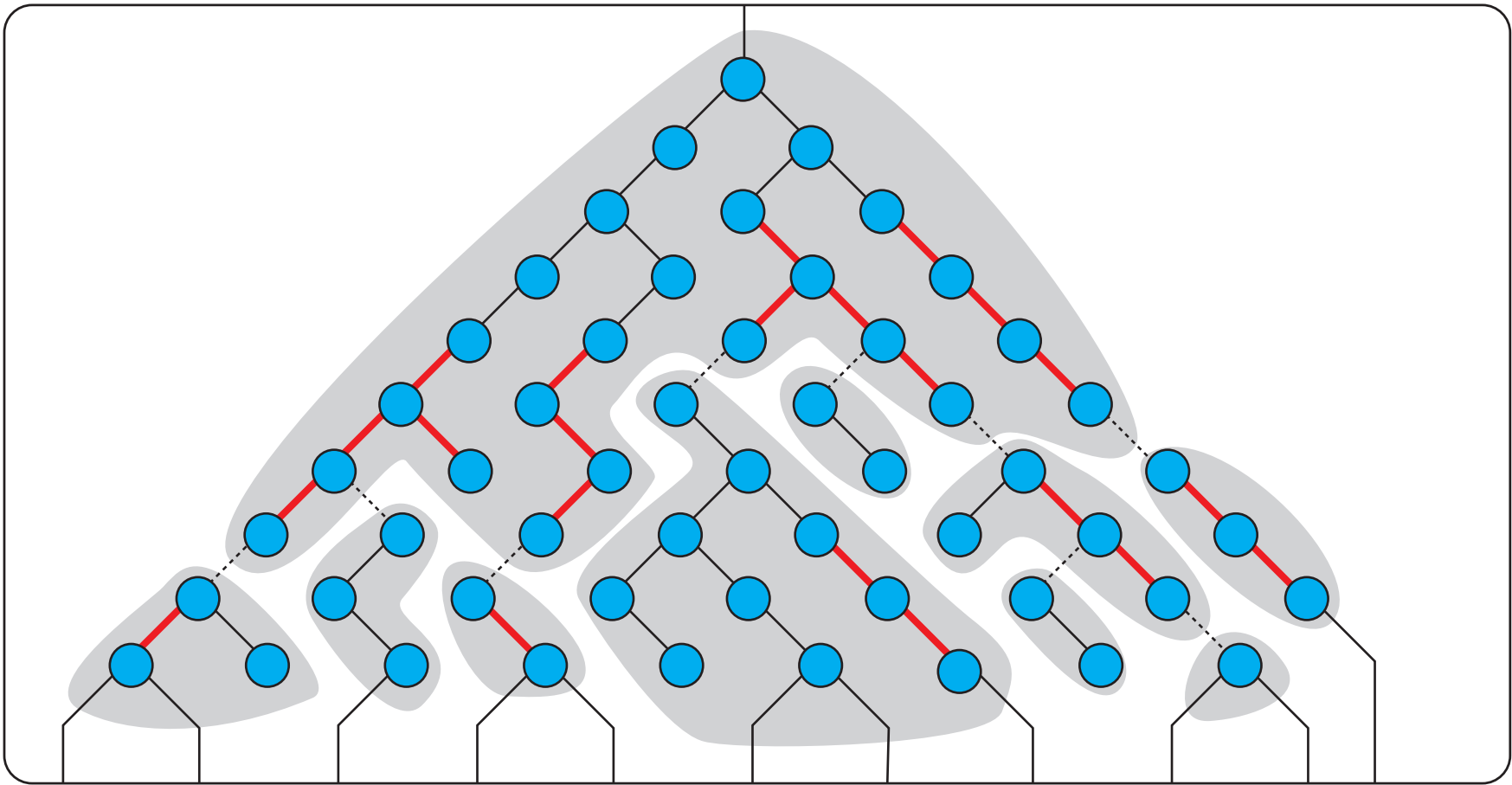
1

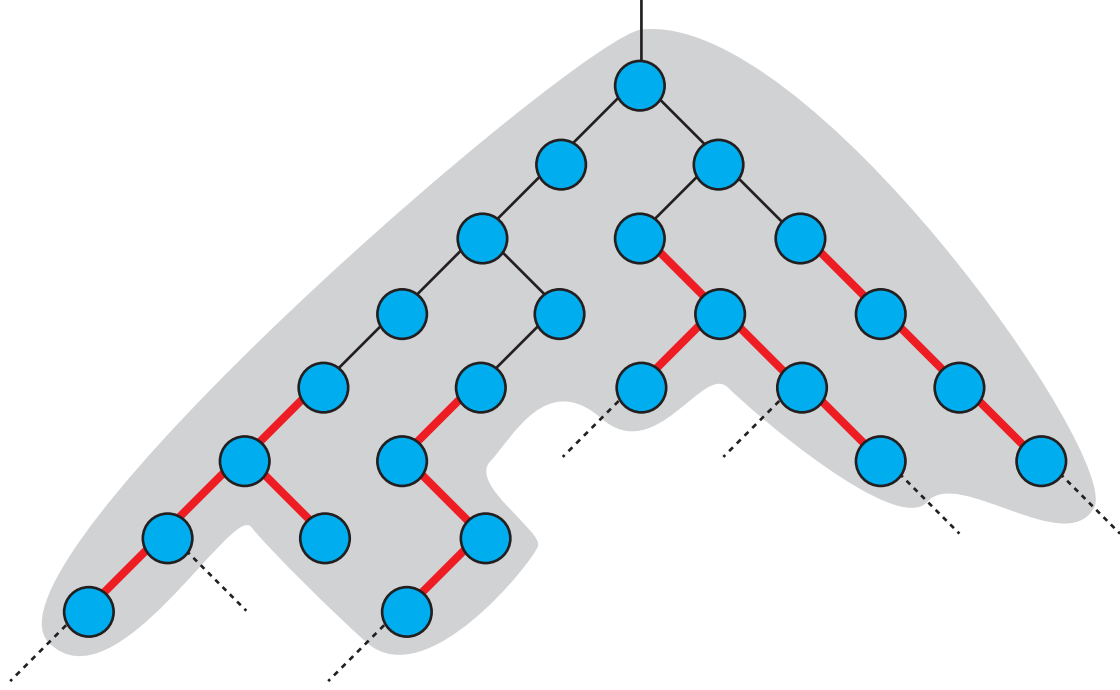
2

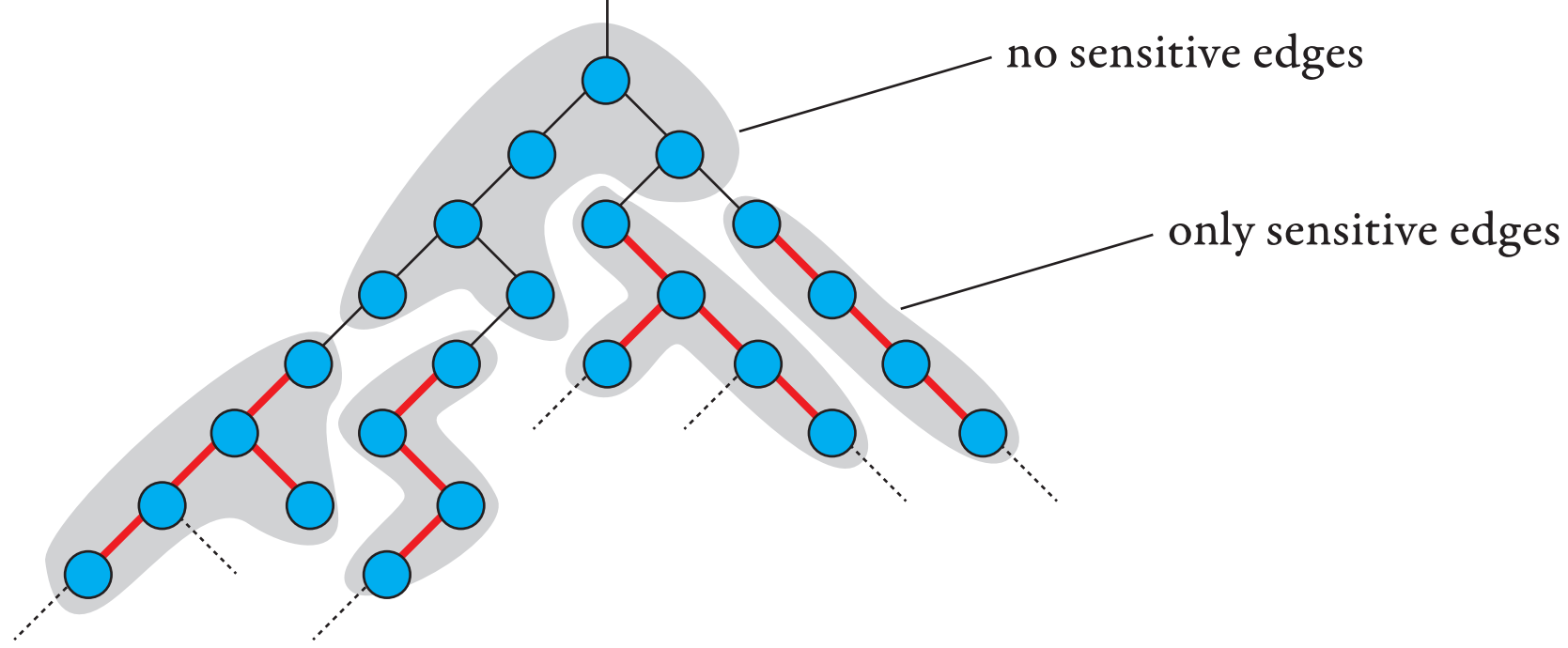


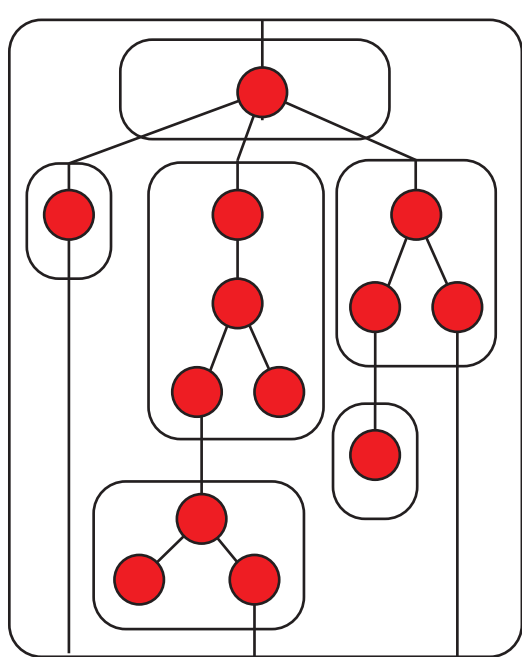
1

2

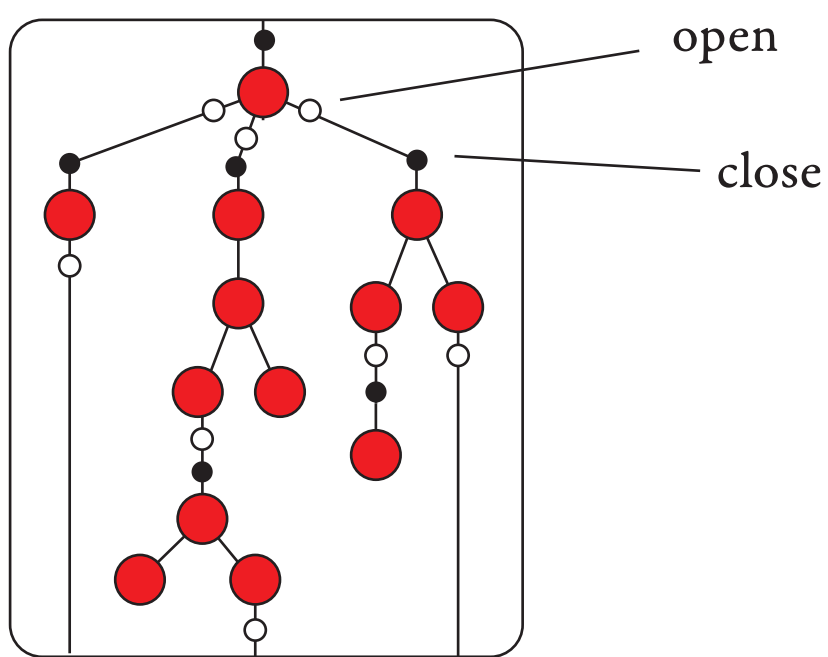






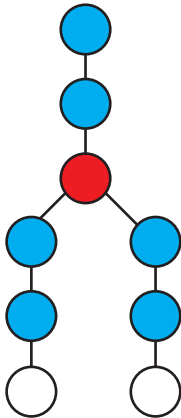
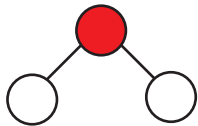


\mapsto



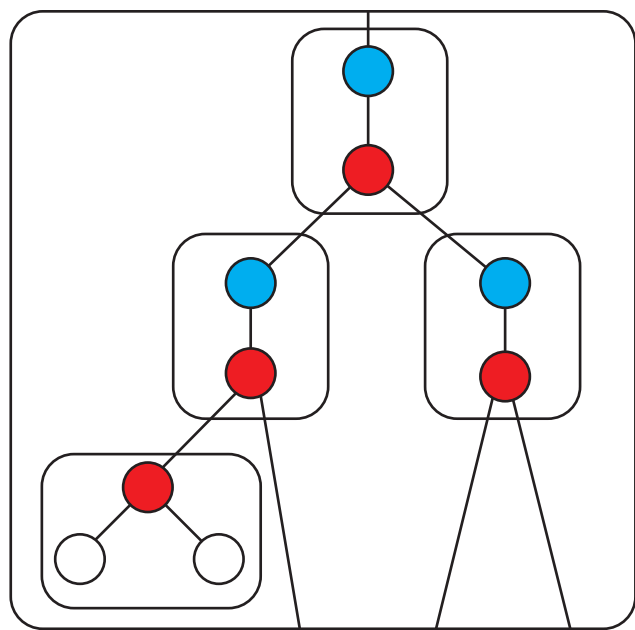




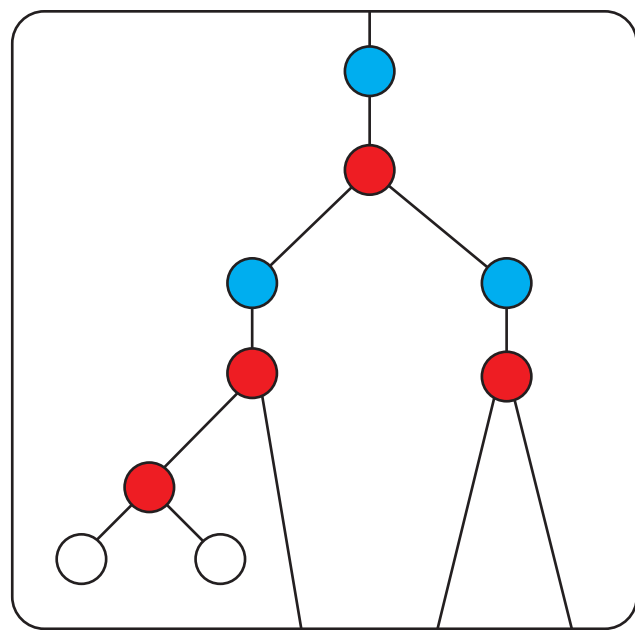


$$0_k - 1 = 2$$



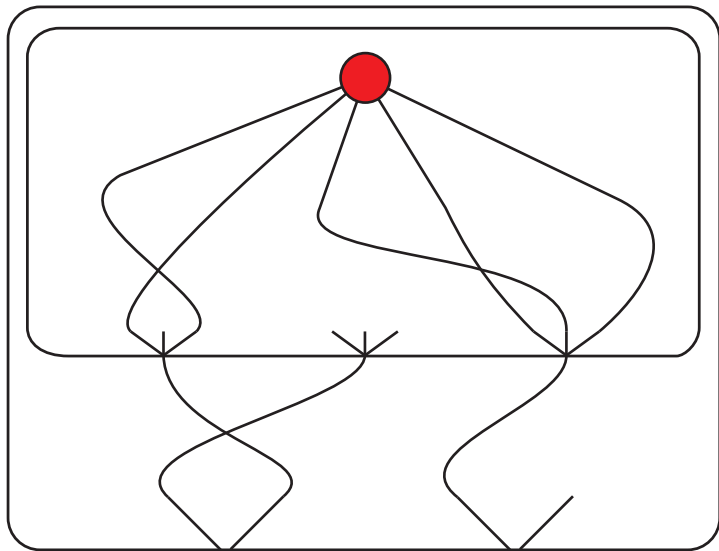


\mapsto

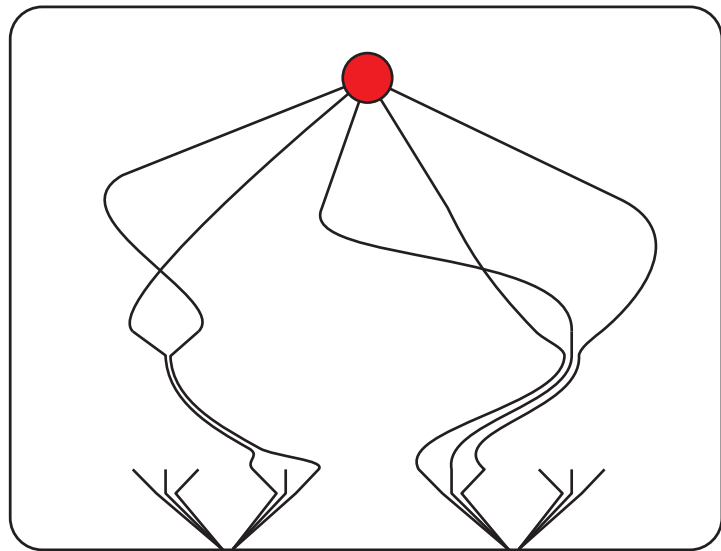


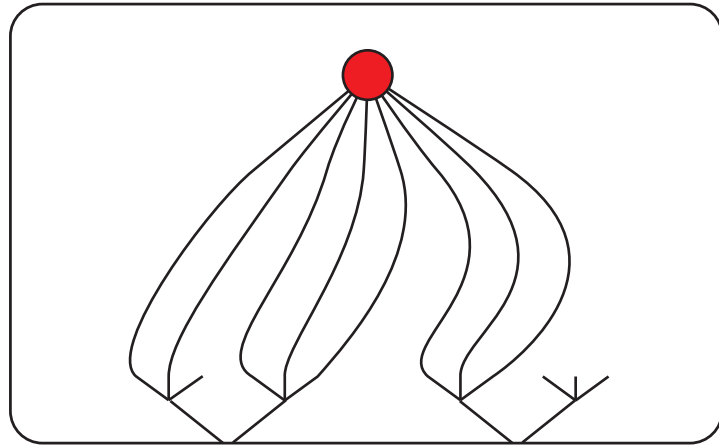
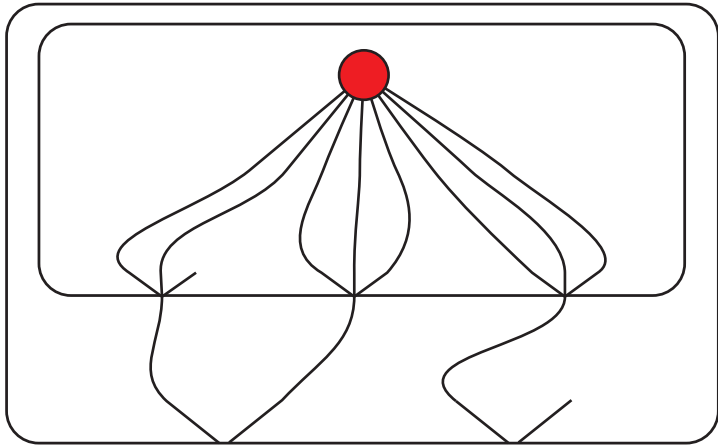


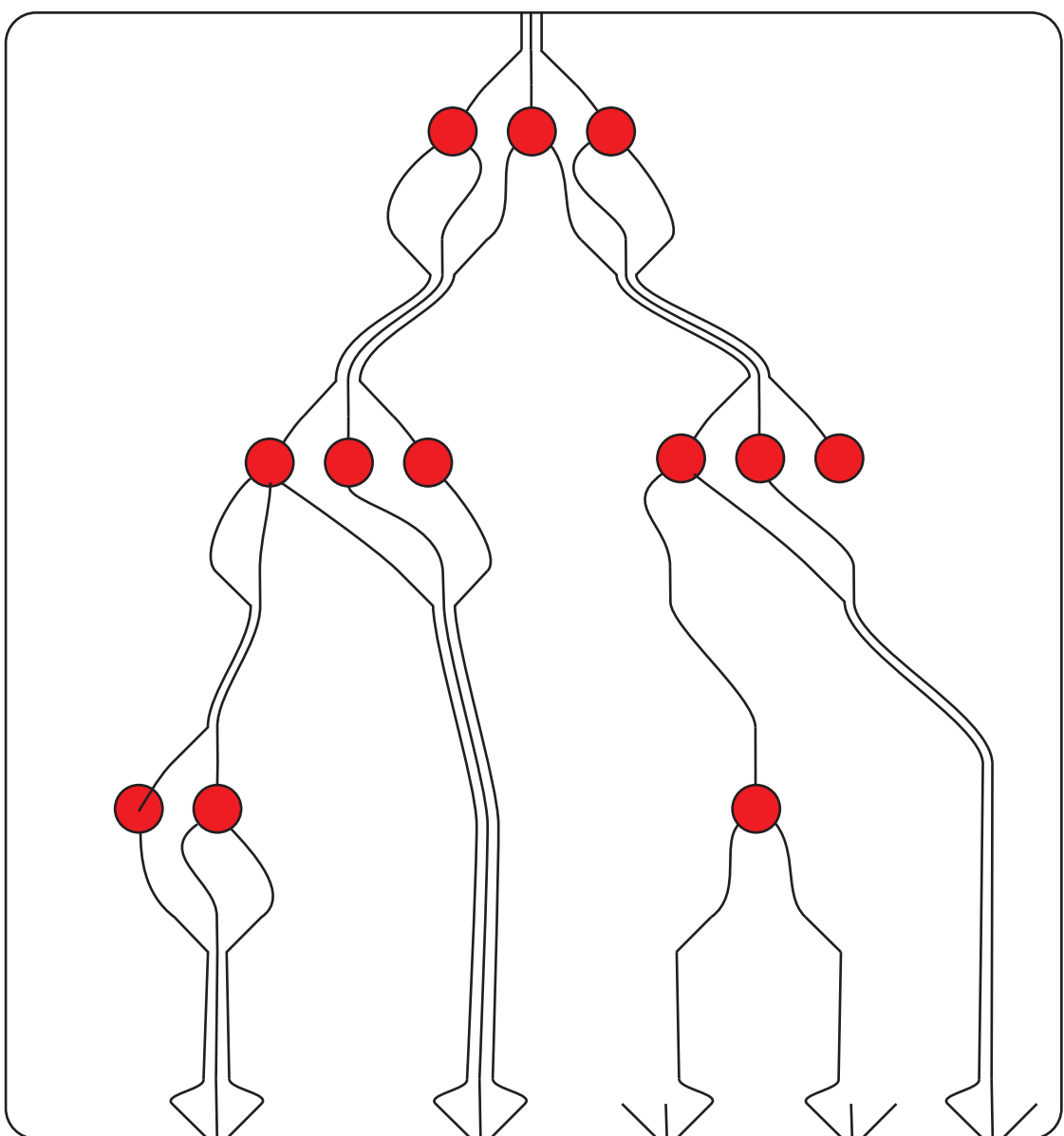
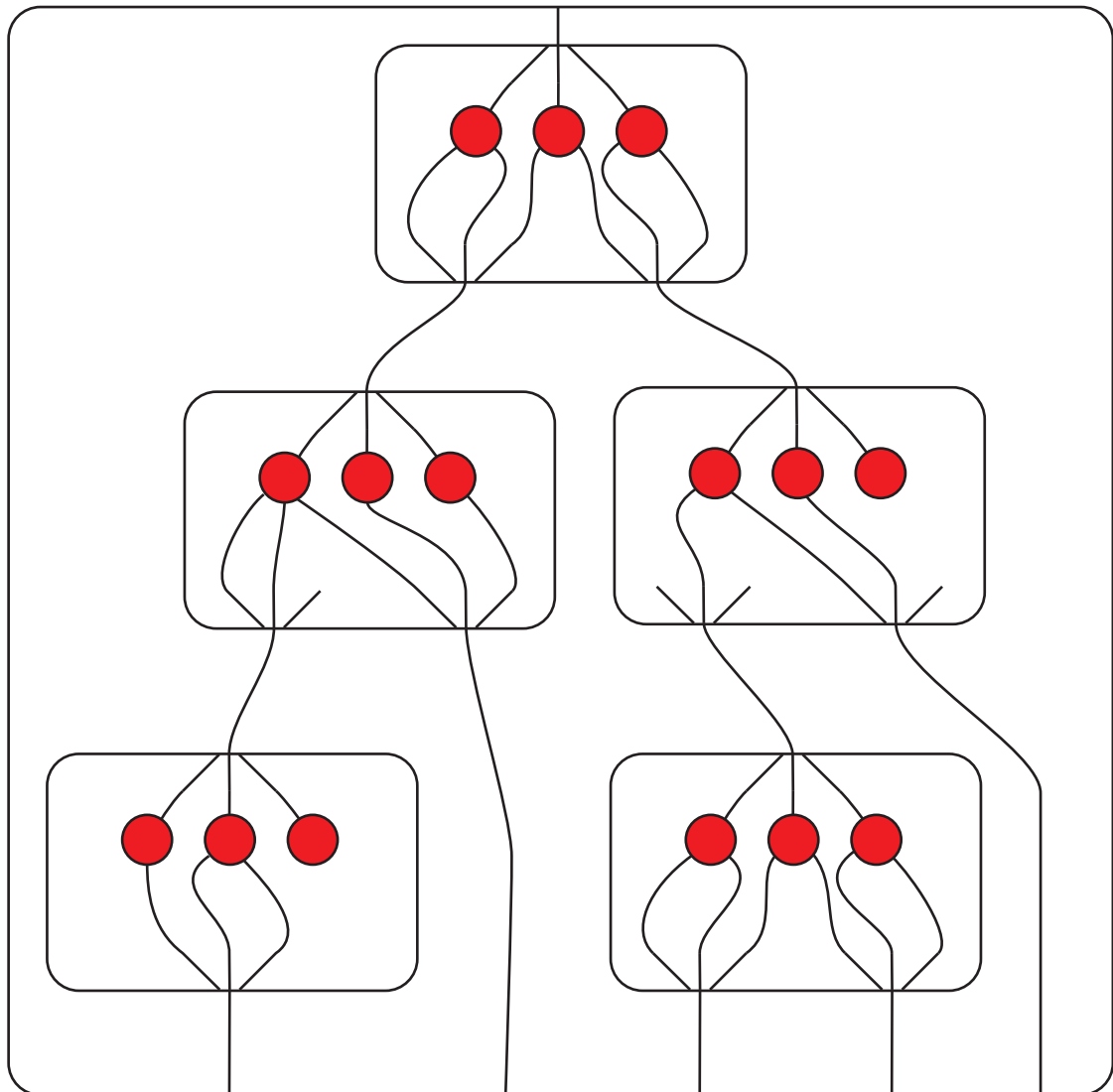
$F_2 F_3 \Sigma$



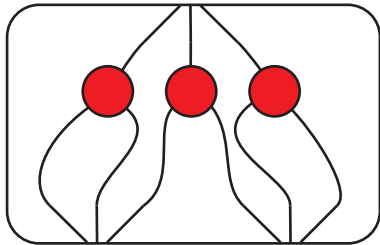
$F_6 \Sigma$



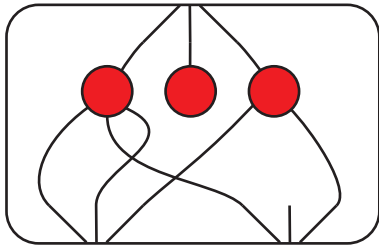




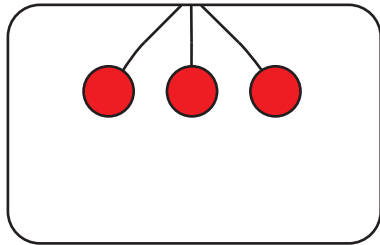
arity 2

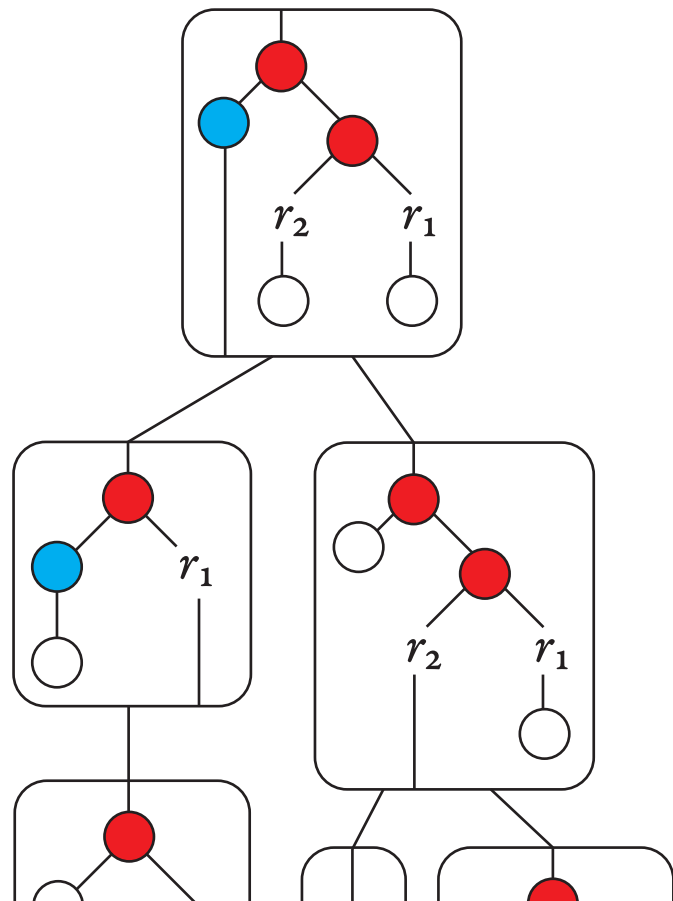


arity 2



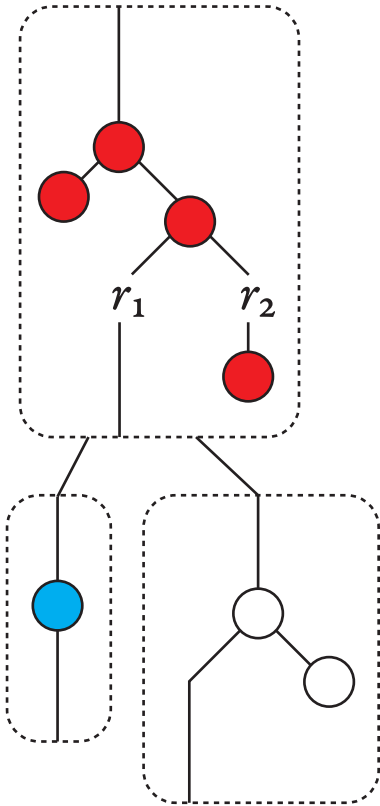
arity 0





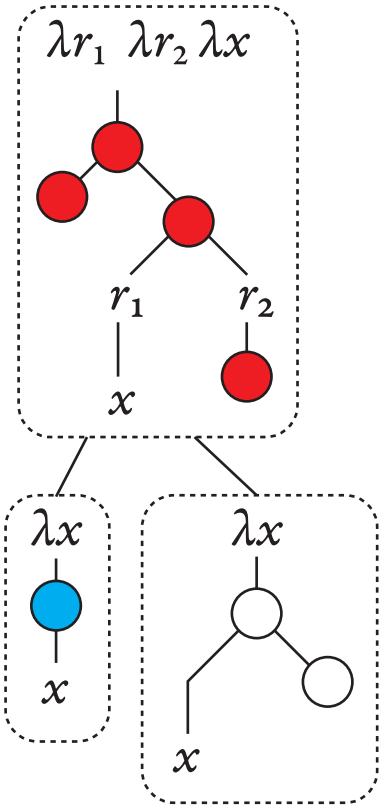
tree of register updates

λ -term



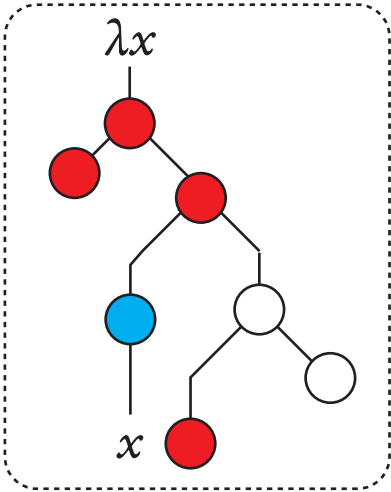
represent
as a λ -term

\mapsto



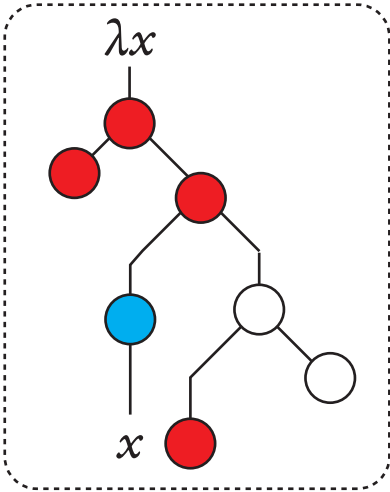
evaluate \Downarrow

\Downarrow evaluate

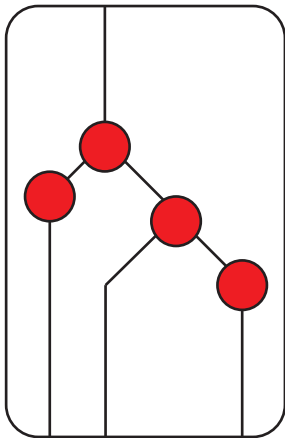


represent
as a λ -term

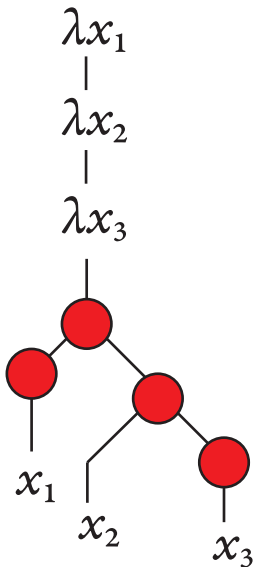
\mapsto

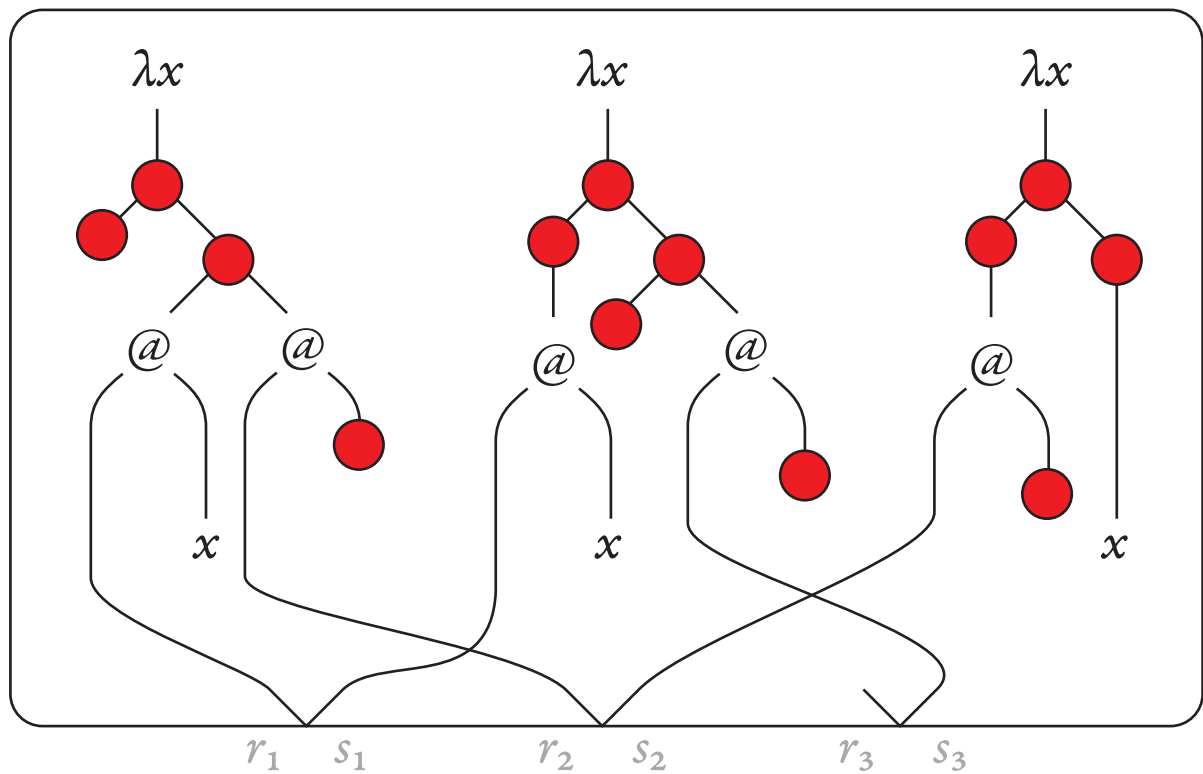
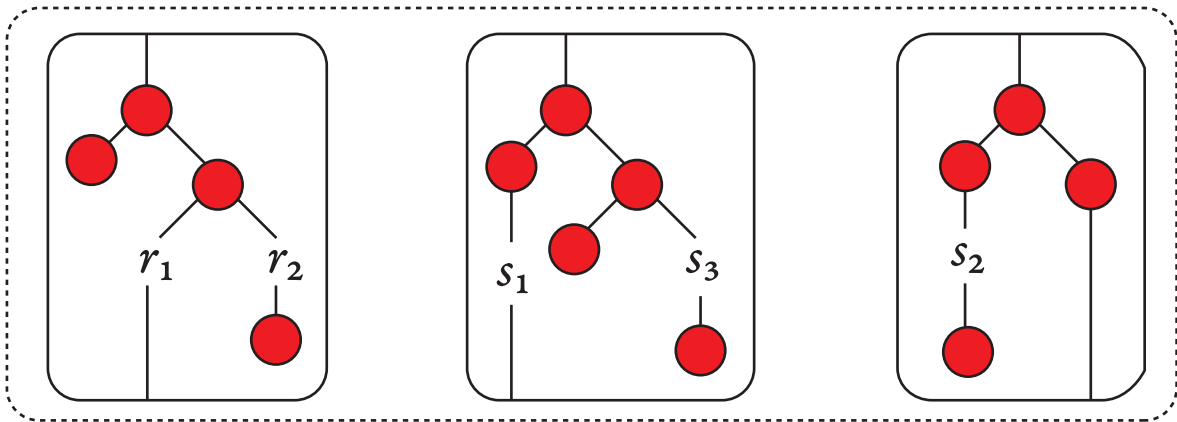


a term

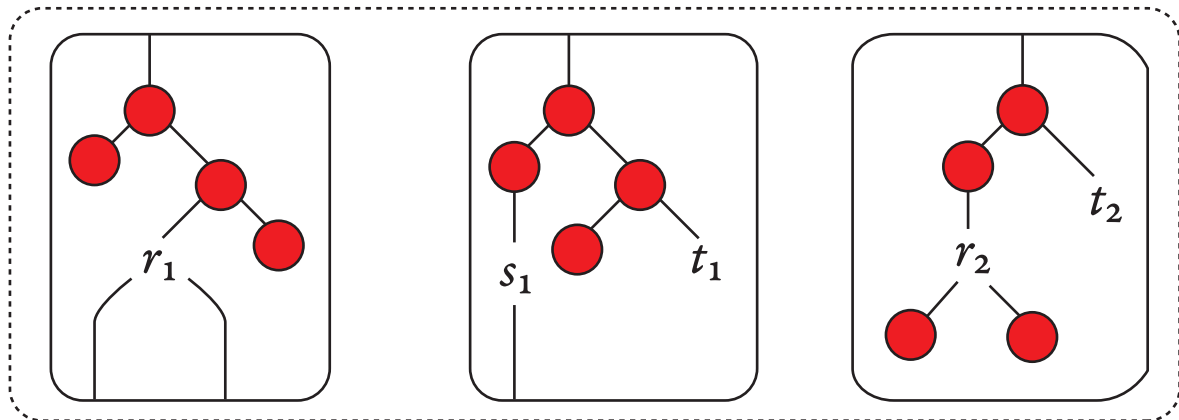


its λ -representation

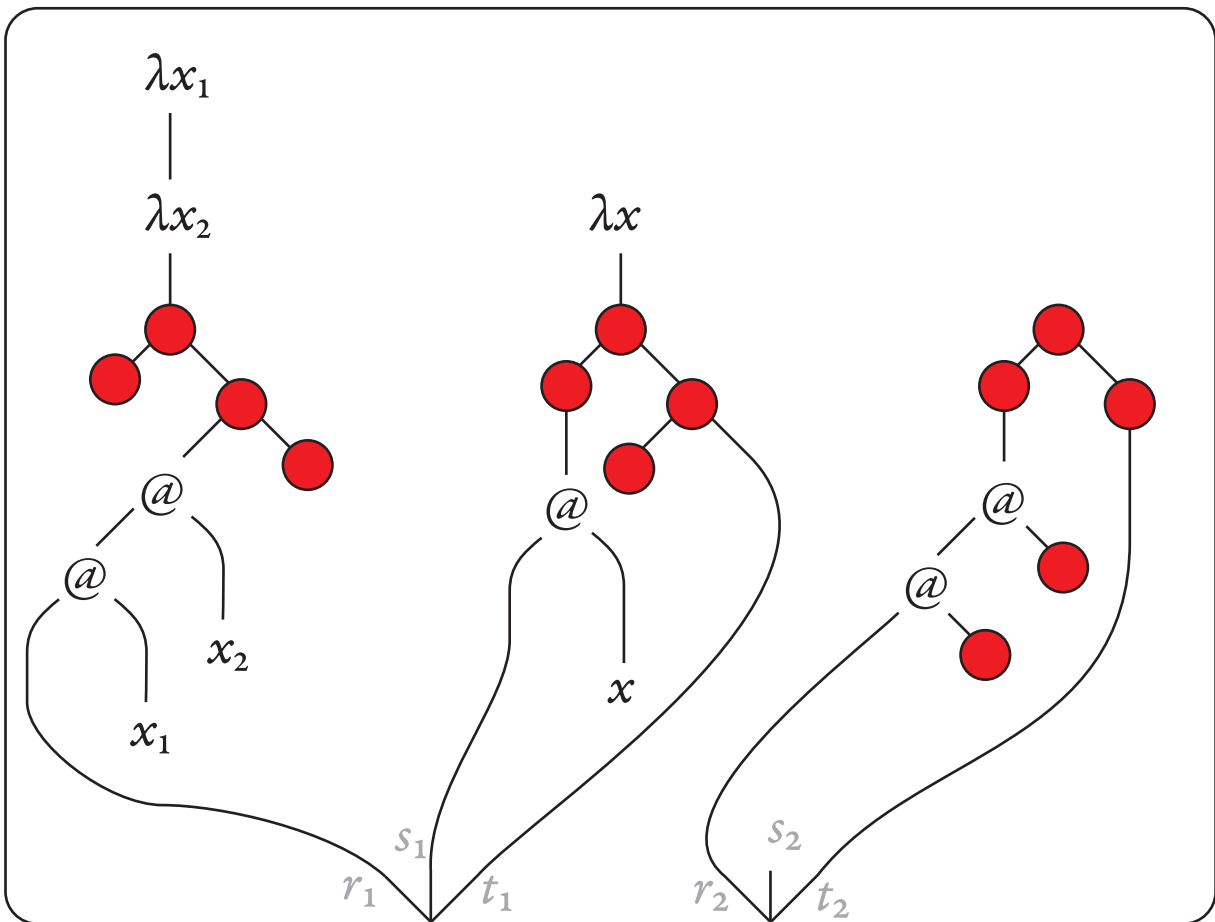


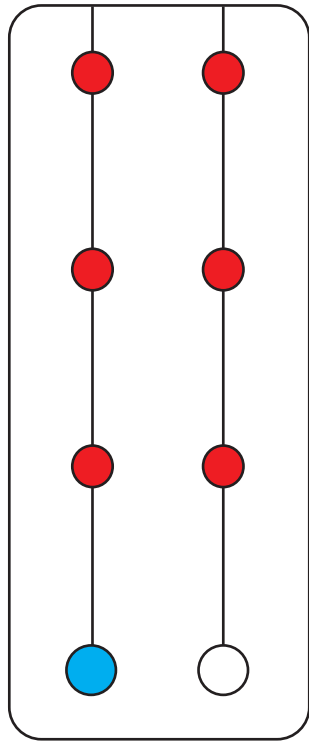
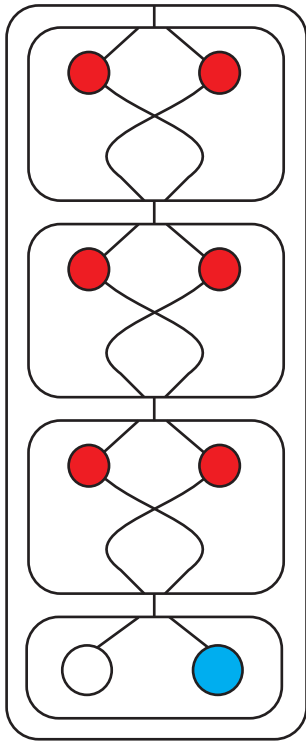


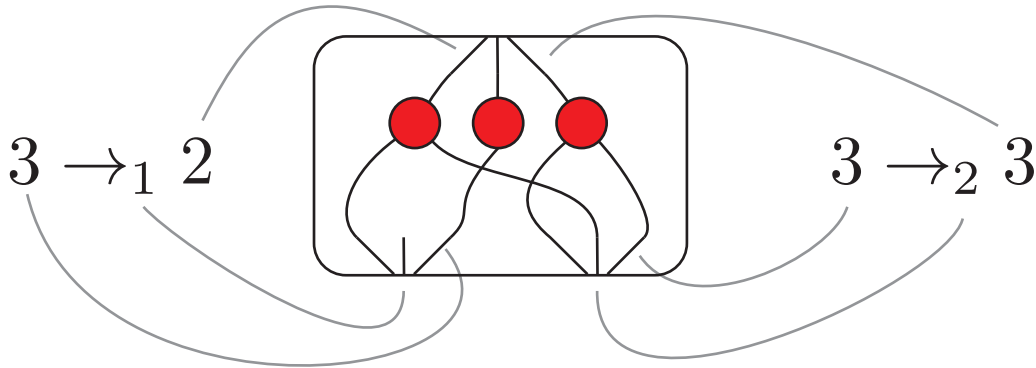
a register update

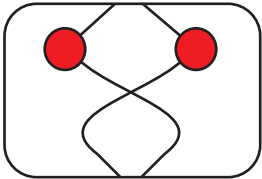


its λ -representation

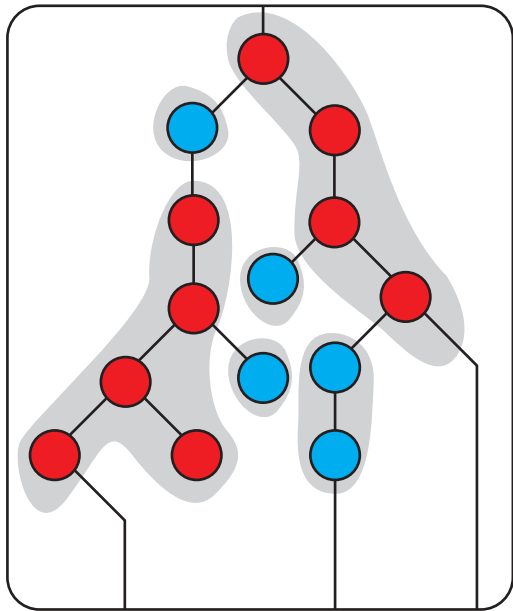








↑-equivalence



↓-equivalence

