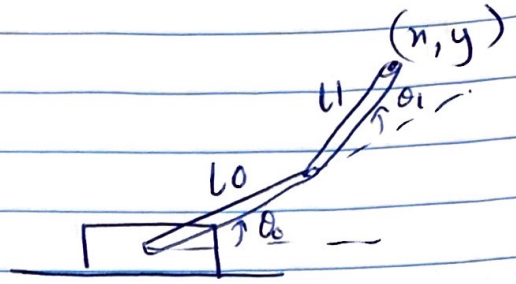


Part 1

A) Forward Model



$$l_0 = 0.1 \rightarrow l_1 = 0.11$$

$$\begin{aligned} n &= l_0 \cos(\theta_0) + l_1 \cos(\theta_0 + \theta_1) \\ y &= l_0 \sin(\theta_0) + l_1 \sin(\theta_0 + \theta_1) \end{aligned} \quad \left. \vphantom{\begin{aligned} n &= l_0 \cos(\theta_0) + l_1 \cos(\theta_0 + \theta_1) \\ y &= l_0 \sin(\theta_0) + l_1 \sin(\theta_0 + \theta_1) \end{aligned}} \right\} \begin{array}{l} \text{End effector} \\ \text{position} \end{array}$$

B) Jacobian Formula

~~$$J = \frac{\partial d(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial n}{\partial \theta_0} & \frac{\partial n}{\partial \theta_1} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} \end{bmatrix}$$~~

$$J = \frac{\partial d(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial n}{\partial \theta_0} & \frac{\partial n}{\partial \theta_1} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} \end{bmatrix}$$

$$J(\theta) = \begin{bmatrix} -l_0 \sin(\theta_0) - l_1 \sin(\theta_0 + \theta_1) & -l_1 \sin(\theta_0 + \theta_1) \\ l_0 \cos(\theta_0) + l_1 \cos(\theta_0 + \theta_1) & l_1 \cos(\theta_0 + \theta_1) \end{bmatrix}$$

c) Algorithm for Inverse

$$dx = J(\theta) d\theta$$

$$\Rightarrow \Delta x = J(\theta) \Delta \theta$$

$$\Rightarrow \Delta \theta = J^{-1}(\theta) \Delta x$$

To avoid singularities with $J^{-1}(\theta)$, use pseudoinverse

$$\Rightarrow \Delta \theta = J^{\dagger}(\theta) \Delta x$$

PART 2

Using bicycle model:

