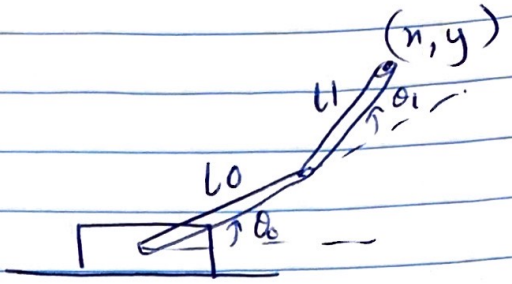
Part 1



A) Forward Model

$L_0 = 0.1$, $L_1 = 0.11$

$$x = L_0 \cos(\theta_0) + L_1 \cos(\theta_0 + \theta_1)$$
$$y = L_0 \sin(\theta_0) + L_1 \sin(\theta_0 + \theta_1)$$

} End effector positions

B) Jacobian Formula

$$J = \frac{\partial f(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial x}{\partial \theta_0} & \frac{\partial x}{\partial \theta_1} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} \end{bmatrix}$$

$$J(\theta) = \begin{bmatrix} -L_0 \sin(\theta_0) - L_1 \sin(\theta_0 + \theta_1) & -L_1 \sin(\theta_0 + \theta_1) \\ L_0 \cos(\theta_0) + L_1 \cos(\theta_0 + \theta_1) & L_0 \cos(\theta_0 + \theta_1) \end{bmatrix}$$

1000, 2

c) Algorithm for Inverse

$$dx = J(\theta)\, d\theta$$

$$\Rightarrow \Delta x = J(\theta)\, \Delta\theta$$

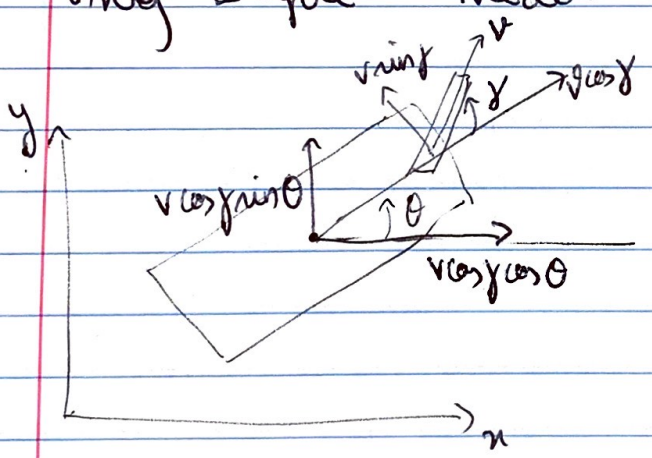
$$\Rightarrow \Delta\theta = J^{-1}(\theta)\, \Delta x$$

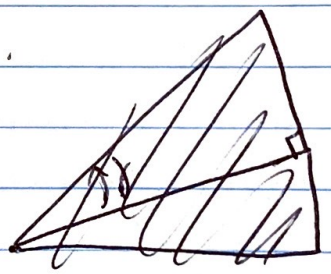
To avoid singularities with $J^{-1}(\theta)$, use pseudoinverse

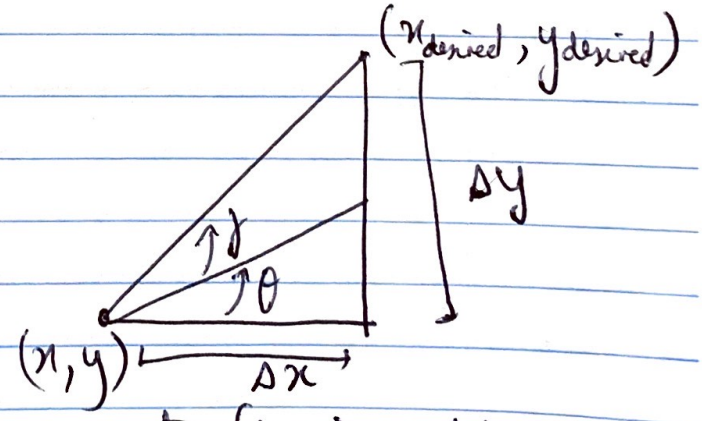
$$\Rightarrow \Delta\theta = J^{+}(\theta)\, \Delta x$$

PART 2

Using bicycle model:



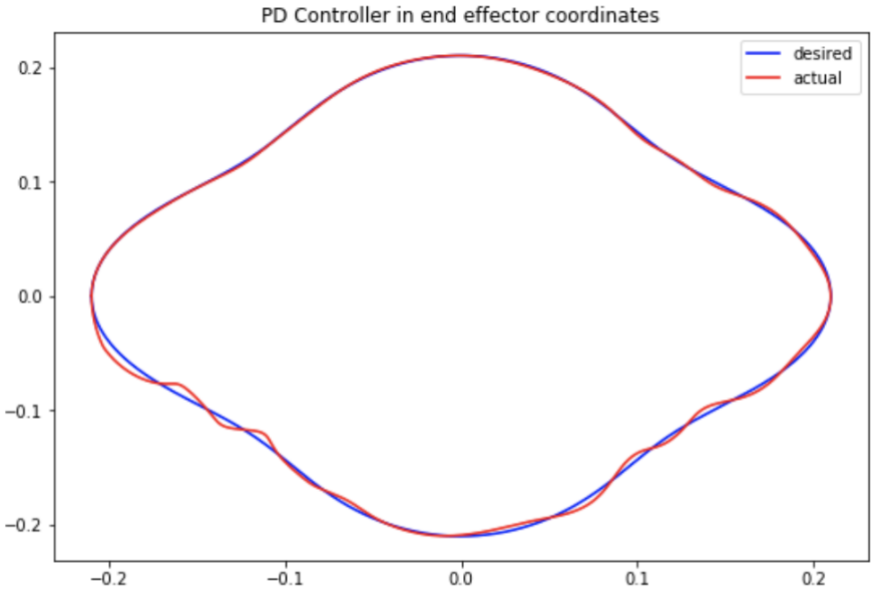




$$\tan(\gamma + \theta) = \frac{\Delta y}{\Delta x}$$

Part 1

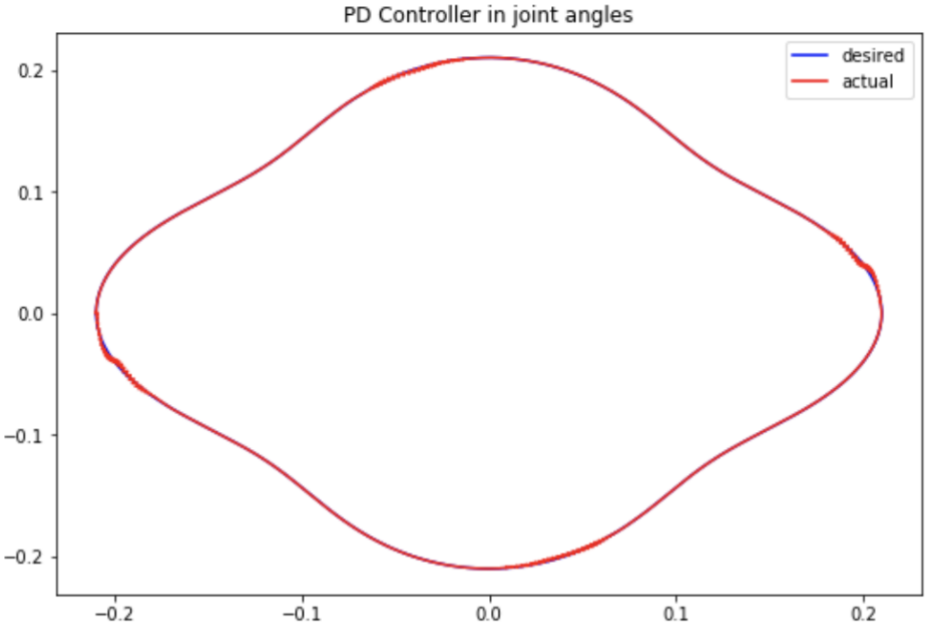
D) Graph of actual and desired robot trajectory with using error in end effector as input




```
MSE =  8.468949988734487e-06
Gains used --> Kp = 2.4, Kd = 0.15
```

Gains used: Kp = 2.4, Kd = 0.15
MSE: 8.47×10^{-6}

E) Graph of actual and desired robot trajectory with using error in joint angles as input




```
MSE =  4.29363859003678e-06
Gains used --> Kp = 8.0, Kd = 0.18
```

Gains used: Kp = 8.0, Kd = 0.18
MSE: 4.29×10^{-6}

Part 2

A) Controller

A PD controller similar to part 1 was used to control the thrust. No controller was used to control the wheel angle given by γ. That is calculated definitively and fed in as an input.

$Thrust = \left[K_p \times \sqrt{(\Delta x^2 + \Delta y^2)}\right] + \left[K_d \times (v_{desired} - v_{current})\right]$, where

$v_{current} = \sqrt{\left({v_x}^2 + {v_y}^2\right)}$ $\quad v_{desired} = \sqrt{\frac{(\Delta x^2 + \Delta y^2)}{\cos^2 \gamma}}$ $\quad \gamma = tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) - \theta$

$\Delta y = y_{desired} - y_{current}$ $\quad \Delta x = x_{desired} - x_{current}$

$K_p = Proportional\ gain,\ K_d = Differential\ gain$

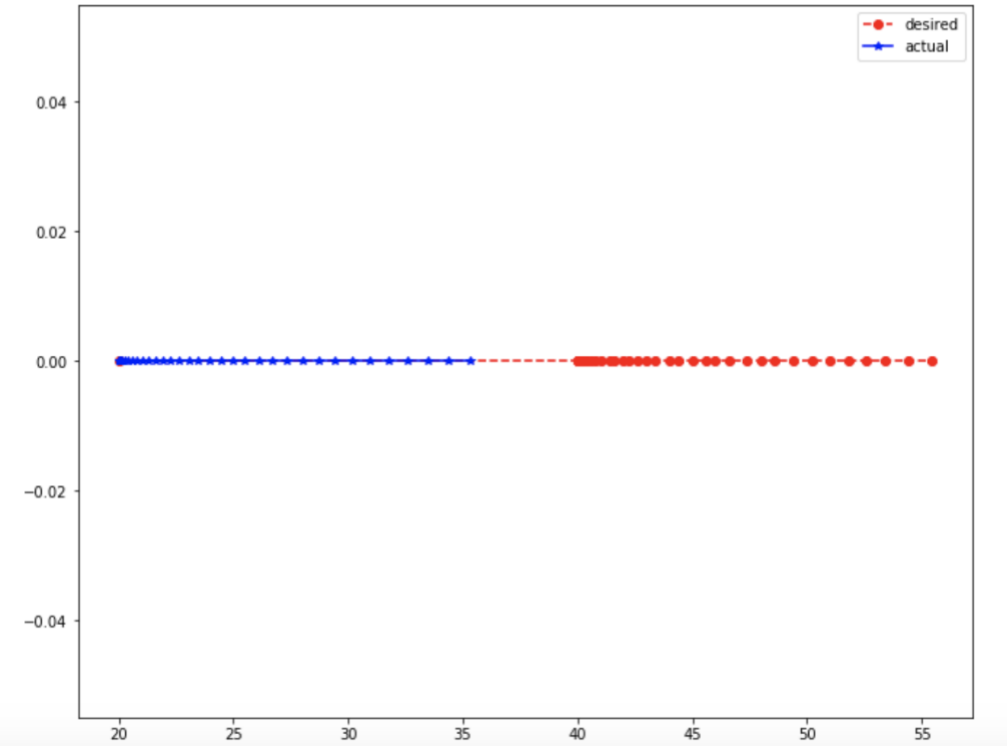
$x_{desired}$, $y_{desired}$, $x_{current}$, $y_{current}$, v_x, v_y, θ are read from the observation.

B) Results

For gain of $K_p = 1.05,\ K_d = 0.33$, the plots are:

1) Linear Track

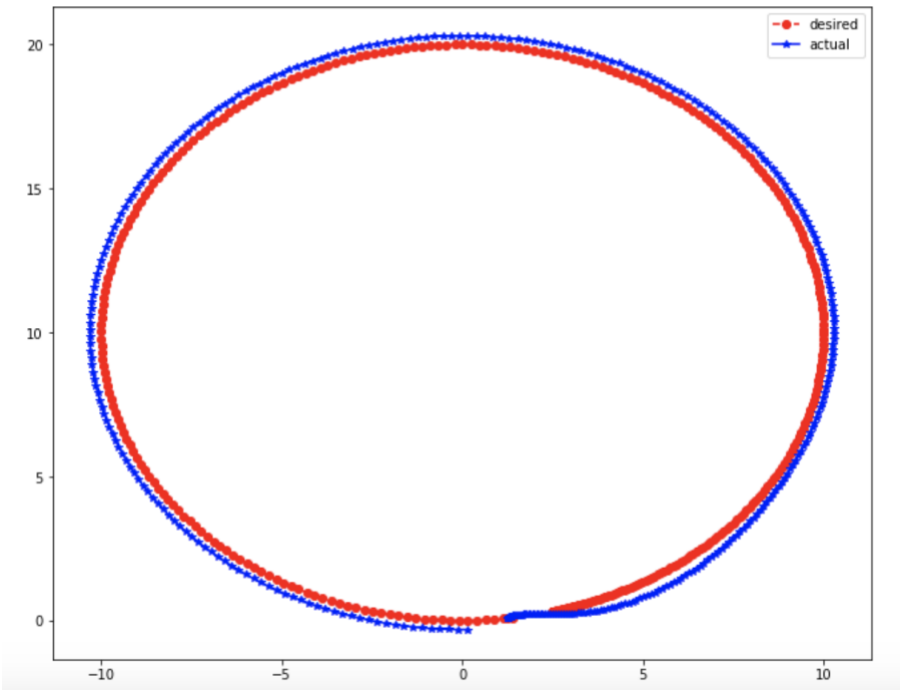
```
Steps taken = 34
MSE between actual and desired trajectories =  388.36986321483533
```



Steps taken = 34, MSE = 388.369

2) Circle Track

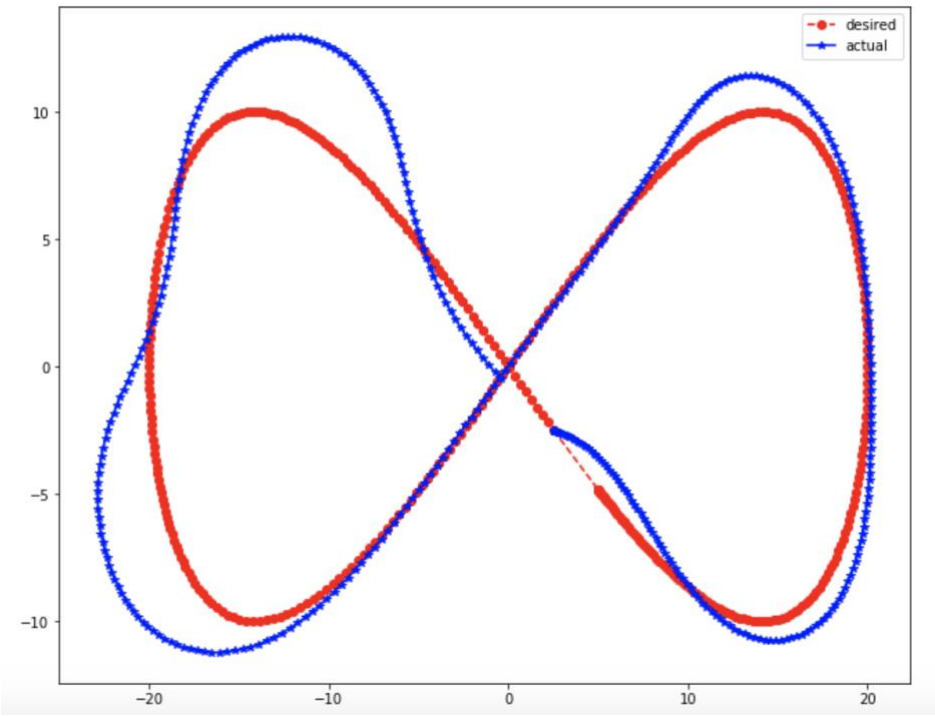
```
Steps taken = 342
MSE between actual and desired trajectories =  1.7063010753262253
```



Steps taken = 342, MSE = 1.70

3) FigureEight Track

```
Steps taken = 403
MSE between actual and desired trajectories =  9.361375979211653
```



Steps taken = 403, MSE = 9.36

Although the controller is able to make the car stay almost on track, the error can be completely eliminated by using an integral controller too. This is reserved for future work.