Семинар 1

2024 курс: Генеративные модели

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$X_1, \ldots X_n \sim U[0;\theta]$ не знаем

$\hat{\theta}_{ML} = ?$

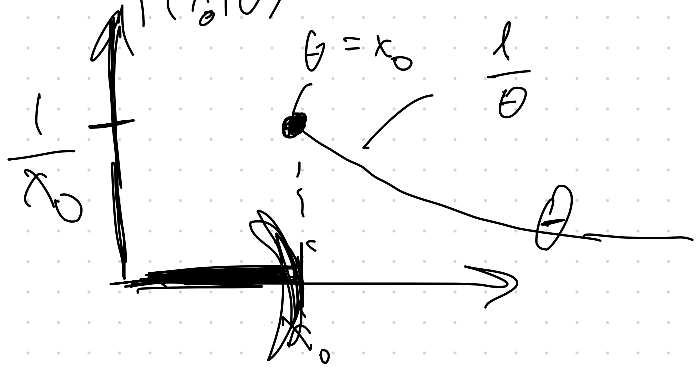
$\frac{X_1 + \ldots + X_n}{n} = \mathbb{E}X = \frac{\theta}{2}$

$$P(\cancel{X}|\theta) = \begin{cases} \left(\frac{1}{\theta}\right)^n & \cancel{X} \in [0,\theta]^n \\ 0 & \cancel{X} \notin [0;\theta]^n \end{cases}$$

$\max(X_1 \ldots X_n) > \theta$

$\left(\frac{1}{\theta}\right)^n, \quad \cancel{X} \in [0;\theta]^n$

$P(x_0|\theta) = f(\theta)$



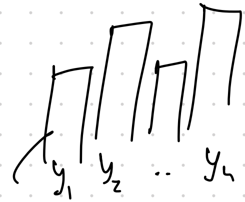
$X_0 \sim U[0;\theta]$

$$\hat{\theta}_{ML} = \max(x_1 \dots x_n)$$

$$P(x_1, \cancel{x_2} \mid \theta) = P(x_1 \mid \theta) \cdot P(x_2 \mid \theta)$$

$$P(X \mid \theta) = \begin{cases} \frac{1}{\theta^n}, & X \in [0;\theta]^n \Rightarrow \begin{cases} \max(x_1 \dots x_n) \leq \theta \\ \min(x_1 \dots x_n) \geq 0 \end{cases} \\ 0, & X \notin [0;\theta]^n \end{cases}$$

$$X_1 \dots X_N = X \sim Cat(\theta)$$



$$\theta_1 \dots \theta_k = \theta$$

$$\theta_1 = p(x \in y_1 | \theta)$$

$$n_1 = \sum_x I[x \in y_1]$$

$$\log p(X|\theta) = \sum_{i=1}^{k} n_i \log \theta_i$$

$$\sum_{i=1}^{k} \theta_i = 1, \quad \theta_i \geq 0$$

$$L = \sum_{i=1}^{k} n_i \log \theta_i + \lambda \left(\sum_{i=1}^{k} \theta_i - 1 \right)$$

$$\frac{\partial L}{\partial \theta_i} = \frac{n_i}{\theta_i} + \lambda = 0 \Rightarrow \boxed{\theta_i = \frac{n_i}{N}}$$