Probabilități și Statistică

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GRUPA 251

Temă

1) Fie X și Y 2 v.a discrete independente

$$X: \begin{pmatrix} 2 & 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} \quad Y: \begin{pmatrix} -3 & -2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$$

$$3X = \begin{pmatrix} 3\cdot 2 & 3\cdot 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$X^{-1} = \begin{pmatrix} 2^{-1} & 3^{-1} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$\cos\left(\frac{\pi}{2}\cdot X\right) = ?$

Fie $g: \mathbb{R} \to [-1,1]$, $g(x) = \cos\left(\frac{\pi}{2}\cdot x\right)$

$g(x)$ - compunere de fct elementare

$\Rightarrow g(x)$ continuă

$$\Rightarrow \cos\left(\frac{\pi}{2}X\right) = \begin{pmatrix} \cos\left(\frac{\pi}{2}\cdot 2\right) & \cos\left(\frac{\pi}{2}\cdot 3\right) \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\pi & \cos\frac{3\pi}{2} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$Y^2 = \begin{pmatrix} (-3)^2 & (-2)^2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$Y+3 = \begin{pmatrix} -3+3 & -2+3 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$$

2) Folosind m.a X și Y de la 1) determinați

a) $2X+3Y = \begin{pmatrix} 4 & 6 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} + \begin{pmatrix} -9 & -6 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} 4-9 & 4-6 & 6-9 & 6-6 \\ \frac{1}{5}\cdot\frac{4}{5} & \frac{1}{5}\cdot\frac{1}{5} & \frac{4}{5}\cdot\frac{4}{5} & \frac{4}{5}\cdot\frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} -5 & -2 & -3 & 0 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix} = \begin{pmatrix} -5 & -3 & -2 & 0 \\ \frac{4}{25} & \frac{16}{25} & \frac{1}{25} & \frac{4}{25} \end{pmatrix}$

$3X-Y = \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} - \begin{pmatrix} -3 & -2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} 6+3 & 6+2 & 9+3 & 9+2 \\ \frac{1}{5}\cdot\frac{4}{5} & \frac{1}{5}\cdot\frac{1}{5} & \frac{4}{5}\cdot\frac{4}{5} & \frac{4}{5}\cdot\frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} 9 & 8 & 12 & 11 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix} = \begin{pmatrix} 8 & 9 & 11 & 12 \\ \frac{1}{25} & \frac{4}{25} & \frac{4}{25} & \frac{16}{25} \end{pmatrix}$

$X^2 \cdot Y^3 = \begin{pmatrix} 4 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} \cdot \begin{pmatrix} -27 & -8 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} 4\cdot(-27) & 4\cdot(-8) & 9\cdot(-27) & 9\cdot(-8) \\ \frac{1}{5}\cdot\frac{4}{5} & \frac{1}{5}\cdot\frac{1}{5} & \frac{4}{5}\cdot\frac{4}{5} & \frac{4}{5}\cdot\frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} -108 & -32 & -243 & -72 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix} = \begin{pmatrix} -32 & -72 & -108 & -243 \\ \frac{1}{25} & \frac{4}{25} & \frac{4}{25} & \frac{16}{25} \end{pmatrix}$

3) Determinați parametrii reali p și q știind că

$X: \begin{pmatrix} 1 & 2 \\ p & q \end{pmatrix}$ $Y: \begin{pmatrix} 3 & 9 \\ 0,7 & \frac{p^2+0,02}{0,2} \end{pmatrix}$

Sunt v.a bine definite

X, Y v.a bine definite

$$\Rightarrow \begin{cases} X \Rightarrow \begin{cases} p > 0 \\ q > 0 \\ p+q = 1 \end{cases} \\ Y \Rightarrow \begin{cases} 0,7 > 0 \\ \frac{p^2+0,02}{0,2} > 0 \\ 0,7 + \frac{p^2+0,02}{0,2} = 1 \end{cases} \end{cases}$$

$$0,7 + \frac{p^2+0,02}{0,2} = 1 \quad | \cdot 0,2$$

$$0,02 + p^2 + 0,02 = 0,2$$

$$p^2 + 0,04 = 0,2 \quad | - 0,04$$

$$p^2 = 0,16 \Rightarrow p = \pm\sqrt{0,16} \Rightarrow p = \pm 0,4$$

cum $p > 0 \Rightarrow p = 0,4$

$$p + q = 1 \Rightarrow 0,4 + q = 1 \Rightarrow q = 0,6$$

$$\frac{p^2+0,02}{0,2} = \frac{0,4^2+0,02}{0,2} = \frac{0,16+0,02}{0,2} = \frac{0,18}{0,2} = 0,9$$

$$\Rightarrow X: \begin{pmatrix} 1 & 2 \\ 0,4 & 0,6 \end{pmatrix} \quad Y: \begin{pmatrix} 3 & 9 \\ 0,1 & 0,9 \end{pmatrix}$$

4) Folosind repartiția v.a. de la 1), 2) calculați:

a) $P(2X+3Y>1)$

$2X+3Y: \begin{pmatrix} -5 & -3 & -2 & 0 \\ \frac{4}{25} & \frac{16}{25} & \frac{1}{25} & \frac{4}{25} \end{pmatrix}$

$P(2X+3Y>1) = 1 - P(2X+3Y \leq 1) = 1 - P(2X+3Y=-5)$
$- P(2X+3Y=-3) - P(2X+3Y=-2) - P(2X+3Y=0)$

$P(2X+3Y>1) = 1 - \frac{4}{25} - \frac{16}{25} - \frac{1}{25} - \frac{4}{25} = 0$

$P(2X+3Y>1 \,/\, X>0)$

$X \begin{pmatrix} 2 & 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$

$P(2X+3Y>1 \,/\, X>0) = \dfrac{P(2X+3Y>1 \wedge X>0)}{P(X>0)}$

$2X+3Y$ și $X>0$ nu sunt (reparat) independente $\Rightarrow$ $= 0$

$\Rightarrow P(2X+3Y>1 \,/\, X>0) = \dfrac{P(2X+3Y>1) \cdot P(X>0 \,/\, 2X+3Y>1)}{P(X>0)=1}$

$\Rightarrow P(2X+3Y>1 \,/\, X>0) = 0$

$P(2X+3Y<3 \,/\, Y<-2)$

$+ \begin{pmatrix} -3 & -2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$P(2X+3Y<3 \,/\, Y<-2) = \dfrac{P(2X+3Y<3 \wedge Y<-2)}{P(Y<-2)}$

$2X+3Y<3$ și $Y<-2$ nu sunt (reparat) independente

$$P(2X+3Y<3 \mid Y<-2) = 1 - P(2X+3Y \geq 3 \mid Y<-2)$$

$$= 1 - \frac{P(2X+3Y \geq 3 \cap Y<-2)}{P(Y<-2)}$$

$$= 1 - \frac{P(2X+3Y \geq 3)\, P(Y<-2 \mid 2X+3Y \geq 3)}{P(Y<-2)}$$

$$= 1 - 0 = 1$$

$$\Rightarrow P(2X+3Y<3 \mid Y<-2) = 1$$

$$P(X^2 \cdot Y^3 > 3) =$$

$$X^2 \cdot Y^3 = \begin{pmatrix} -243 & -108 & -72 & -32 \\ \frac{16}{25} & \frac{4}{25} & \frac{4}{25} & \frac{1}{25} \end{pmatrix}$$

$$P(X^2 \cdot Y^3 > 3) = 1 - P(X^2 \cdot Y^3 \leq 3) = 1 - P(X^2 \cdot Y^3 = -243) - P(X^2 \cdot Y^3 = -108) - P(X^2 \cdot Y^3 = -72) - P(X^2 \cdot Y^3 = -32) = 1 - \frac{16}{25} - \frac{4}{25} - \frac{4}{25} - \frac{1}{25} = 0$$

$$\Rightarrow P(X^2 \cdot Y^3 > 3) = 0$$

$$P(X^2 \cdot Y^3 \leq 3)$$

$$P(X^2 \cdot Y^3 \leq 3) = 1 - P(X^2 \cdot Y^3 > 3) = 1 - 0 = 1$$

$$\Rightarrow P(X^2 \cdot Y^3 \leq 3) = 1$$

$P(|2X+3Y| < |3X-Y|) =$

$2X+3Y < 3X-Y \quad | -3X+Y$

$\Rightarrow -X+4Y < 0 \Rightarrow 4Y-X < 0$

$\Rightarrow P(|2X+3Y| < |3X-Y|) = P(4Y-X < 0)$

$4Y: \begin{pmatrix} -12 & -8 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} \quad X: \begin{pmatrix} 2 & 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$

$4Y-X: \begin{pmatrix} -12-2 & -12-3 & -8-2 & -8-3 \\ \frac{4}{25} & \frac{16}{25} & \frac{1}{25} & \frac{4}{25} \end{pmatrix}$

$4Y-X: \begin{pmatrix} -15 & -14 & -11 & -10 \\ \frac{16}{25} & \frac{4}{25} & \frac{4}{25} & \frac{1}{25} \end{pmatrix}$

$\Rightarrow P(4Y-X<0) = P(4Y-X=-15) + P(4Y-X=-14)$
$+ P(4Y-X=-11) + P(4Y-X \in -10)$
$= \frac{16}{25} + \frac{4}{25} + \frac{4}{25} + \frac{1}{25} = 1$

$\Rightarrow P(|2X+3Y| < |3X-Y|) = 1$

Tema1.R - Probabilitati si Statistica

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Incarc pachetele discreteRV si prob, functia suppressMessages este folosita pentru a nu primi output atunci cand pachetul a fost incarcat

```
suppressMessages(library(discreteRV, quietly = T))
suppressMessages(library(prob, quietly = T))
suppressMessages(library(MASS, quietly = T))
```

Problema 1

Problema 1 (scris)

Instantiam cele doua variabile aleatoare prin functia RV(outcomes, probs)

```
(X <-  RV(c(2,3),c(1/5,4/5)))
```

```
## Random variable with 2 outcomes
##
## Outcomes   2   3
## Probs    1/5 4/5
```

```
(Y <-  RV(c(-3,-2),c(4/5,1/5)))
```

```
## Random variable with 2 outcomes
##
## Outcomes  -3  -2
## Probs    4/5 1/5
```

Putem sa aplicam operatiile matematice direct in R, fara sa apelam alte functii

```
(X3 <- 3*X)
```

```
## Random variable with 2 outcomes
##
## Outcomes   6   9
## Probs    1/5 4/5
```

```
(Xem1 <- fractions(X^-1))
```

```
## Random variable with 2 outcomes
##
## Outcomes 1/2 1/3
## Probs    1/5 4/5
```

```
(Xcos <- fractions(cos((pi/2)*X)))
```

```
## Random variable with 2 outcomes
##
## Outcomes  -1   0
## Probs    1/5 4/5
```

```
(Ye2 <- Y^2)
```

```
## Random variable with 2 outcomes
##
## Outcomes   4   9
## Probs    1/5 4/5
```

```
(Yp3 <- Y+3)
```

```
## Random variable with 2 outcomes
##
## Outcomes   0   1
## Probs    4/5 1/5
```

Rezultatele au dat la fel ca pe foaie, cu precizarea ca am folosit functia fractions pentru a transforma numerele decimale in fractii si pentru a scapa de eroarea de calcul a cosinusului, unde `-1.83690953073357e-16` este de fapt 0

Problema 2 (scris)

```
(X2pY3 <- 2*X+3*Y)
```

```
## Random variable with 4 outcomes
##
## Outcomes    -5    -3    -2     0
## Probs     4/25 16/25  1/25  4/25
```

```
(X3mY <- 3*X-Y)
```

```
## Random variable with 4 outcomes
##
## Outcomes     8     9    11    12
## Probs     1/25  4/25  4/25 16/25
```

```
(Xe2Ye3 <- X^2 * Y^3)
```

```
## Random variable with 4 outcomes
##
## Outcomes 4,-27 9,-27  4,-8  9,-8
## Probs     4/25 16/25  1/25  4/25
```

Rezultatele sunt la fel, o limitare a pachetului discreteRV este ca atunci cand inmultim 2 v.a. la afisare o sa avem valorile despartite cu , nu inmultite

```
(Xe2Ye3 <- RV(c(4*-27,9*-27,4*-8,9*-8), c(4/25,16/25,1/25,4/25)))
```

```
## Random variable with 4 outcomes
##
## Outcomes  -243  -108   -72   -32
## Probs    16/25  4/25  4/25  1/25
```

Problema 3 (scris)

R nu are suport nativ pentru expresii simbolice, pentru a nu folosi un alt pachet am ales sa calculez valoarea lui p folosind polyroot, conform conditiilor scrise pe foaie

In cazul nostru polyroot o sa ne intoarca 2 radacini (ecuatie de gradul 2) deci trebuie sa o alegem pe cea pozitiva

```
(radacini <- polyroot(c(0.1*0.2+0.02-1*0.2,0,1)))
```

```
## [1]  0.4+0i -0.4-0i
```

```
(radaciniRe <- Re(radacini))
```

```
## [1]  0.4 -0.4
```

```
(radaciniIm <- Im(radacini))
```

```
## [1]  1.29247e-26 -1.29247e-26
```

Vedem ca radaciniIm primeste valorile foarte apropiate de 0, chiar daca rezultatul initial a dat 0 Avem nevoie de o functie care sa ne zica daca o valoare este foarte apropiata de 0 Am vazut online ca exista functii gen is.zero sau isZero, doar ca eu nu le am in pachetul de baza

```
almostEqual <- function(x, y, tolerance=1e-8) {
  diff <- abs(x - y)
  mag <- pmax(abs(x), abs(y))
  ifelse(mag > tolerance, diff/mag <= tolerance, diff <= tolerance)
}
```

Folosim functia stopifnot pentru a impune conditiile necesare pentru v.a.

```
stopifnot(all(almostEqual(radaciniIm, 0)))
(p <- radaciniRe[radaciniRe>0])
```

```
## [1] 0.4
```

```
stopifnot(length(p) == 1)
(q <- 1-p)
```

```
## [1] 0.6
```

```
(Prob3X <- RV(c(1,2),c(p,q)))
```

```
## Random variable with 2 outcomes
##
## Outcomes   1   2
## Probs    2/5 3/5
```

```
(Prob3Y <- RV(c(3,9),c(0.1, (p^2+0.02)/0.2)))
```

```
## Random variable with 2 outcomes
##
## Outcomes    3    9
## Probs    1/10 9/10
```

Problema 4 (scris)

O sa definesc functia myCondP care calculeaza probabilitatea conditionata pentru 2 v.a. independente

```
myCondP <- function(X, Y)
{
  py <- P(Y)
  if (py==0) return(NaN)
  return((P(X)*P(Y))/P(Y))
}

(P(2*X+3*Y > 1))
```

```
## [1] 0
```

```
(myP <- myCondP(2*X+3*Y > 1, X>0))
```

```
## [1] 0
```

```
(pachetP <- P(2*X+3*Y > 1 | X>0))
```

```
## [1] 0
```

Cele doua valori sunt egale, primul eveniment are p=0 sansa de aparitie deci indiferent de ce ar fi conditionat acesta nu s-ar putea intampla

```
(myP <- myCondP(2*X+3*Y < 3, Y< -2))
```

```
## [1] 1
```

```
(pachetP <- P(2*X+3*Y < 3 | Y < -2))
```

```
## [1] 0.25
```

Observam ca raspunsul a dat diferit in acest caz. Pachetul discreteRV are probleme atunci cand in dreapta semnului este un numar negativ. Putem sa rescriem Y < -2 ca -Y > 2, dar trebuie sa folosim repartitia comuna a celor doua v.a. pentru a obtine valoarea buna

```
negY <- Y*(-1)

produsCartezian <- expand.grid(probs(X2pY3),probs(negY))
jointProbs <- produsCartezian$Var1 * produsCartezian$Var2
(jointNegYsiX2pY3 <- jointRV(list(outcomes(negY), outcomes(X2pY3)), probs=jointProbs))
```

```
## Random variable with 8 outcomes
##
## Outcomes   2,-5   2,-3   2,-2   3,-5    2,0   3,-3   3,-2    3,0
## Probs    16/125 64/125  4/125  4/125 16/125 16/125  1/125  4/125
```

```
(margNegY <- discreteRV::marginal(jointNegYsiX2pY3, 1))
```

```
## Random variable with 2 outcomes
##
## Outcomes   2   3
## Probs    4/5 1/5
```

```
(margX2pY3 <- discreteRV::marginal(jointNegYsiX2pY3, 2))
```

```
## Random variable with 4 outcomes
##
## Outcomes    -5    -3    -2     0
## Probs     4/25 16/25  1/25  4/25
```

```
(myP <- myCondP(2*X+3*Y < 3, Y< -2))
```

```
## [1] 1
```

```
(pachetP <- P(margX2pY3 < 3 | margNegY > 2))
```

```
## [1] 1
```

Rezultatul este acum corect

```
(P(X^2*Y^3 > 3))
```

```
## [1] 1
```

Observam ca P(X^2*Y^3 > 3) a dat gresit deoarece discreteRV nu a inmultit valorile posibile, o sa folosim variabila Xe2Ye3 care contine v.a. in forma buna

```
(P(Xe2Ye3 > 3))
```

```
## [1] 0
```

```
(P(Xe2Ye3 <= 3))
```

```
## [1] 1
```

Pentru ultimul subpunct trebuie sa folosim repartitiile marginale din repartitia comuna a 2X+3Y si 3X-Y Deoarece repartitia comuna are 4 linii si 4 coloane, am decis sa o construiesc prin cod

```
produsCartezian <- expand.grid(probs(X3mY),probs(X2pY3))
jointProbs <- produsCartezian$Var1 * produsCartezian$Var2
(jointX3mYsiX2pY3 <- jointRV(list(outcomes(X2pY3), outcomes(X3mY)), probs=jointProbs))
```

```
## Random variable with 16 outcomes
##
## Outcomes    -5,8    -5,9   -5,11   -5,12    -3,8    -3,9   -3,11   -3,12    -2,8    -2,9   -2,11   -
## Probs      4/625  16/625  16/625  64/625  16/625  64/625  64/625 256/625   1/625   4/625   4/625  16/
##
## Displaying first 12 outcomes
```

```
(margX2pY3 <- discreteRV::marginal(jointX3mYsiX2pY3, 1))
```

```
## Random variable with 4 outcomes
##
## Outcomes    -5    -3    -2     0
## Probs     4/25 16/25  1/25  4/25
```

```
(margX3mY <- discreteRV::marginal(jointX3mYsiX2pY3, 2))
```

```
## Random variable with 4 outcomes
##
## Outcomes     8     9    11    12
## Probs     1/25  4/25  4/25 16/25
```

```
(P(margX2pY3 < margX3mY))
```

```
## [1] 1
```

Cred ca pentru acest subpunct merge si daca le scadem

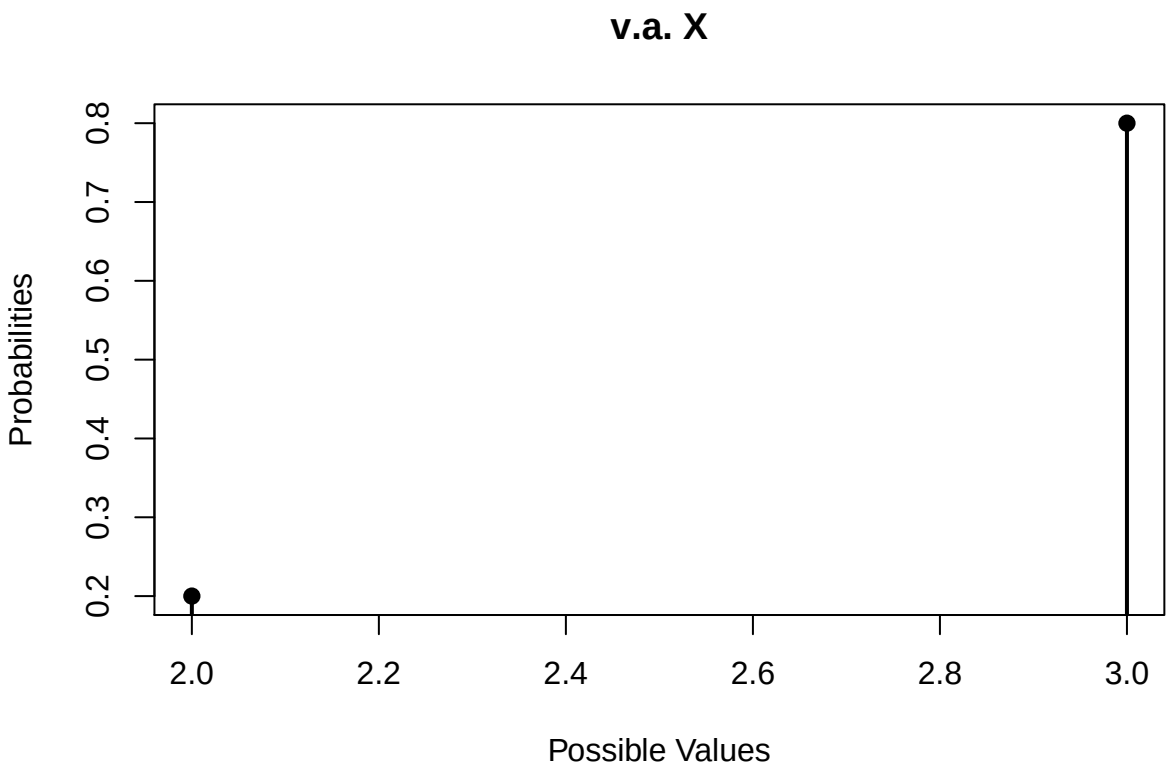
```
(P(X2pY3-X3mY < 0))
```

```
## [1] 1
```

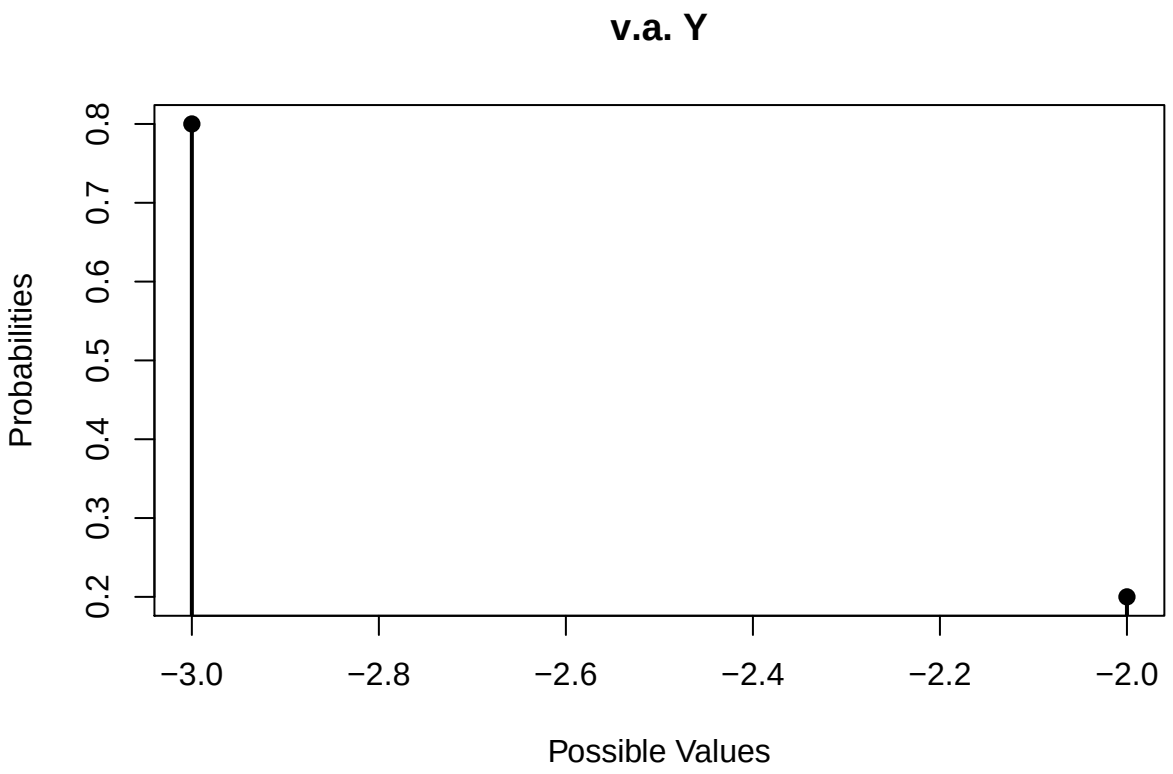
Avem acelasi rezultat

Problema 2

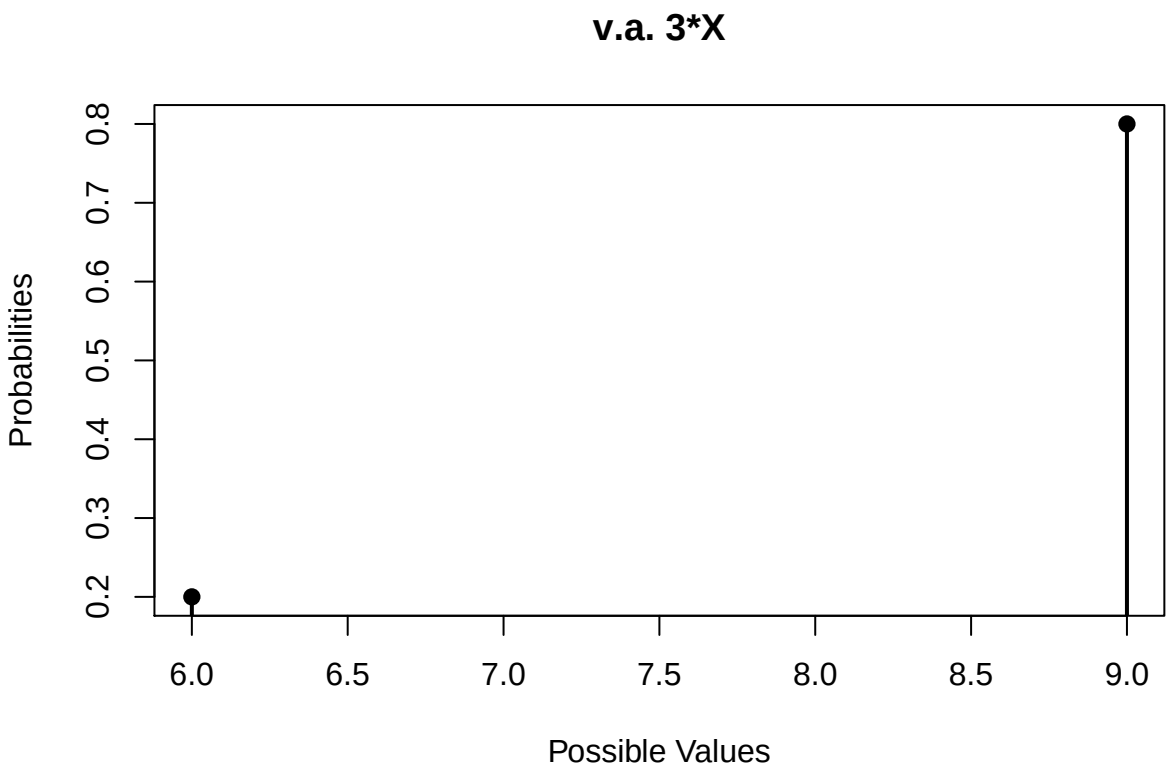
```
plot(X, main='v.a. X')
```



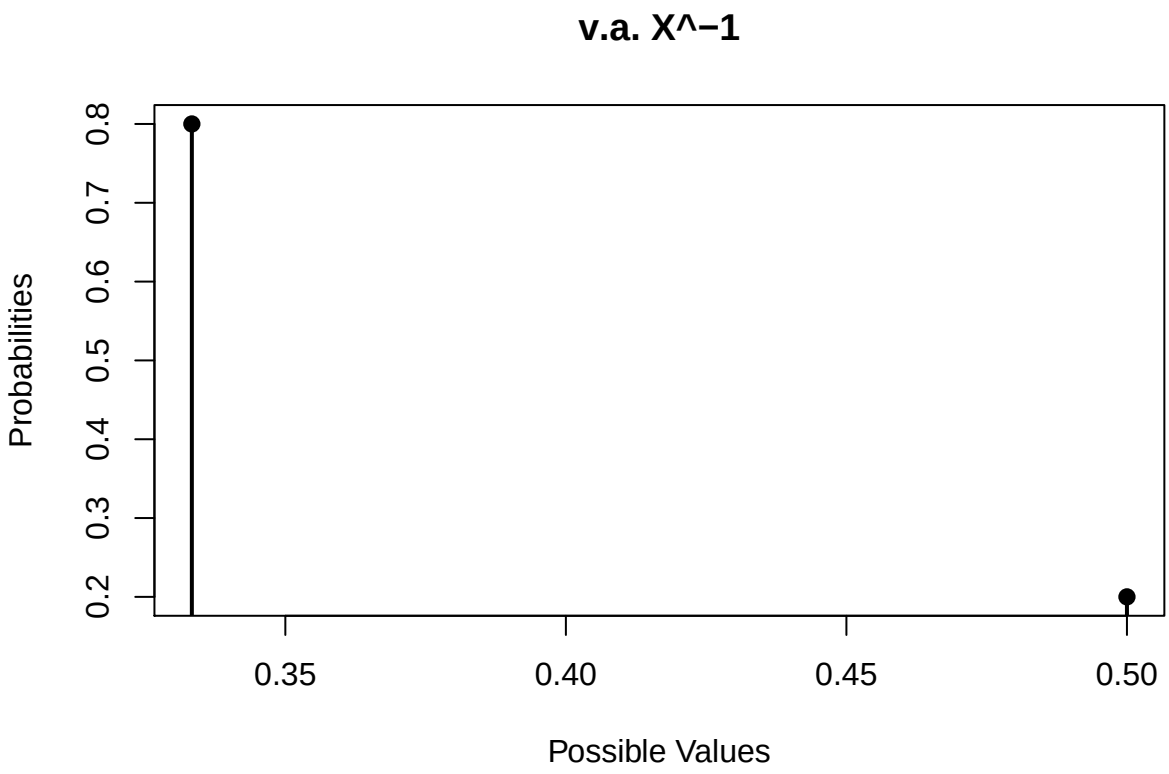
```r
plot(Y, main='v.a. Y')
```



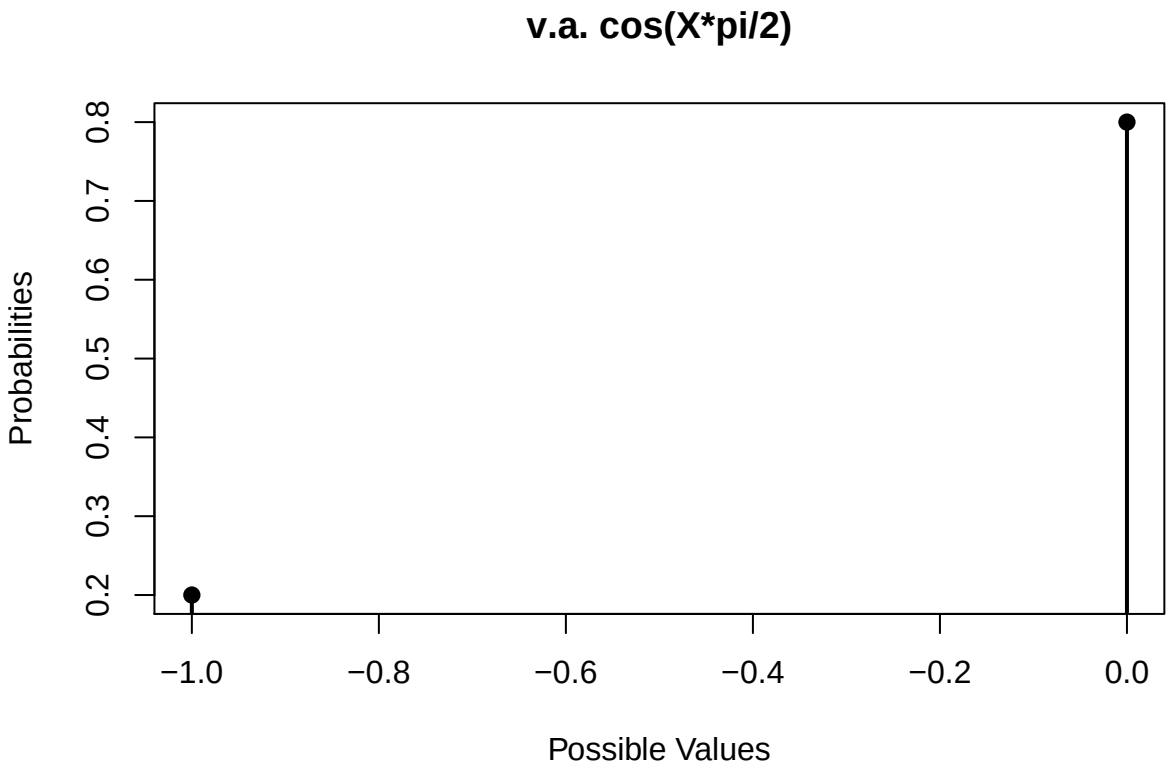
```r
plot(X3, main='v.a. 3*X')
```



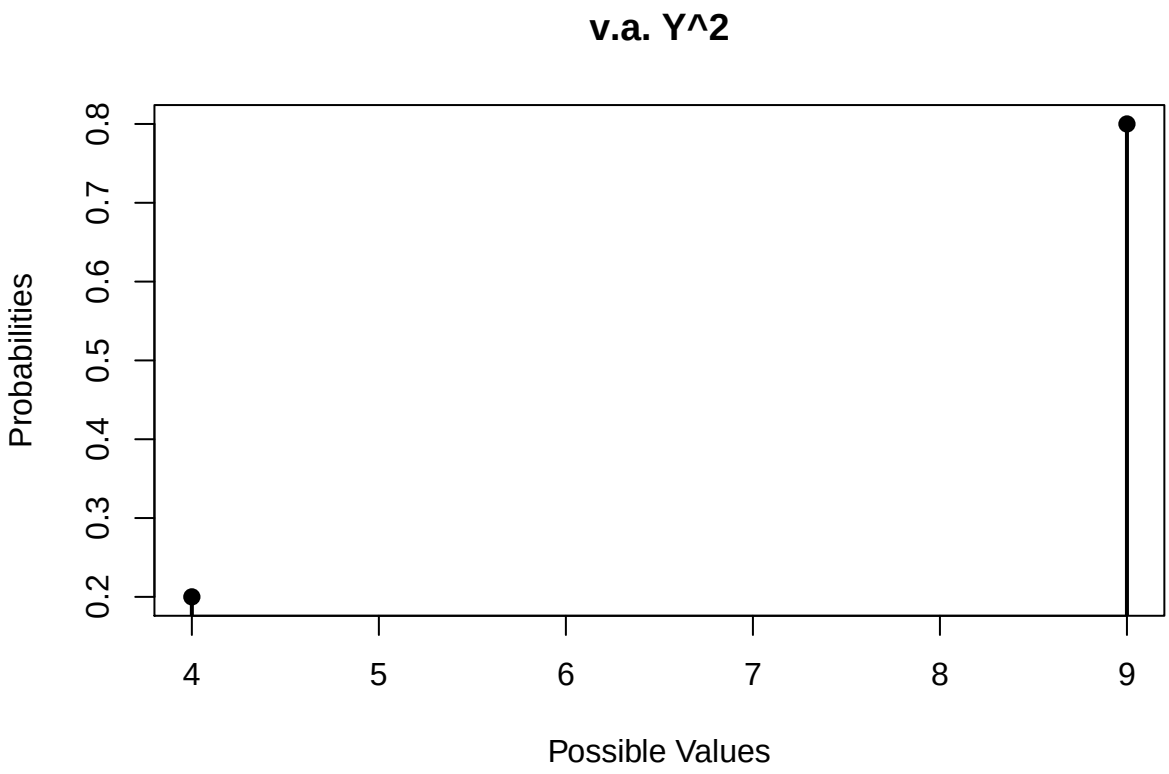
```r
plot(Xem1, main='v.a. X^-1')
```



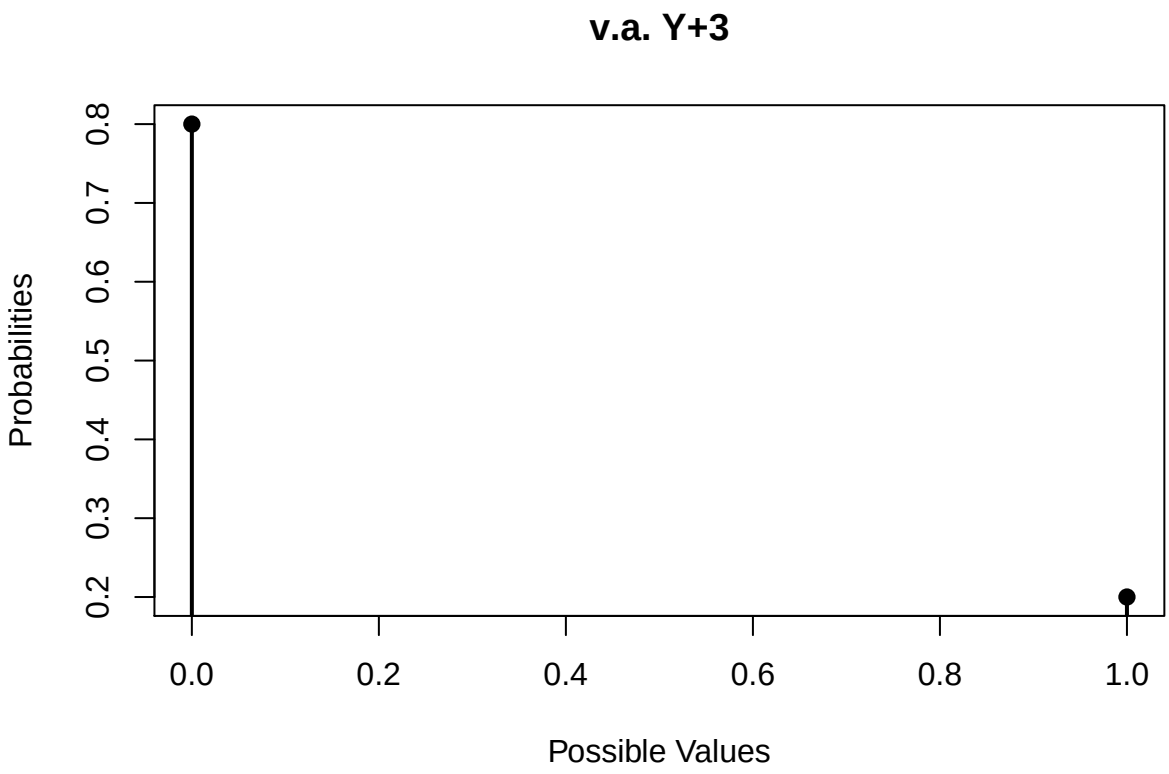
```r
plot(Xcos, main='v.a. cos(X*pi/2)')
```



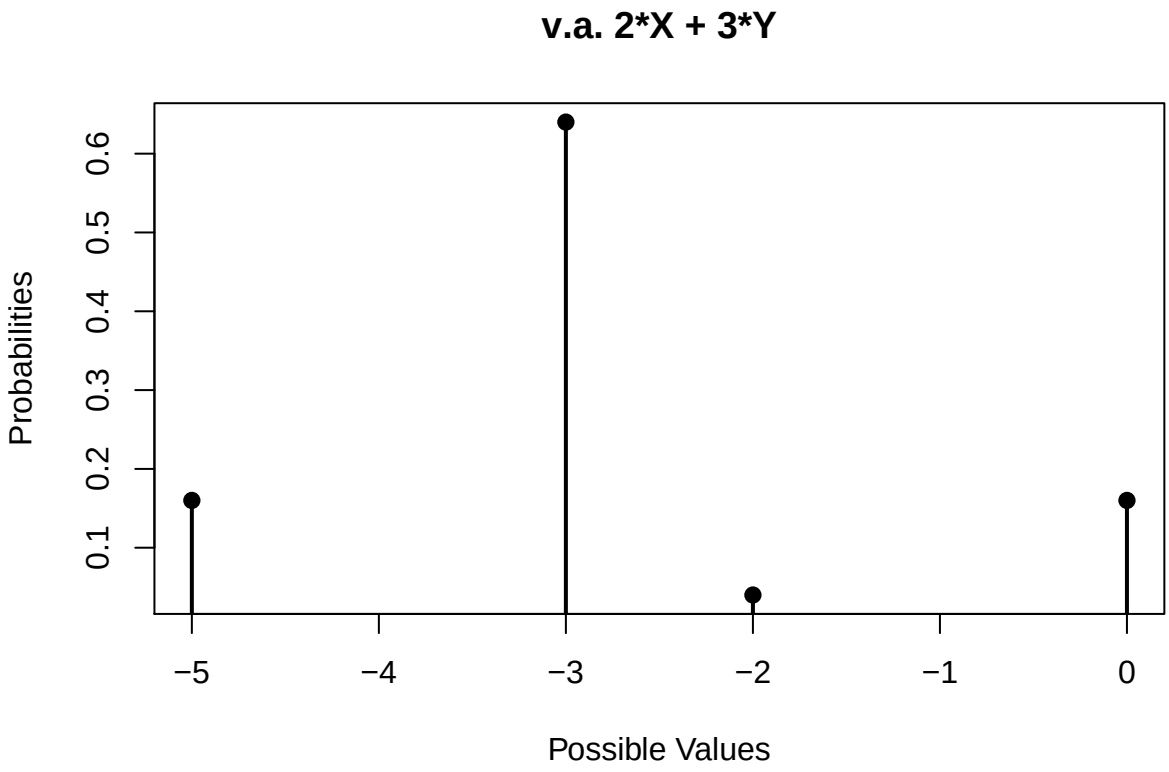
```r
plot(Ye2, main='v.a. Y^2')
```



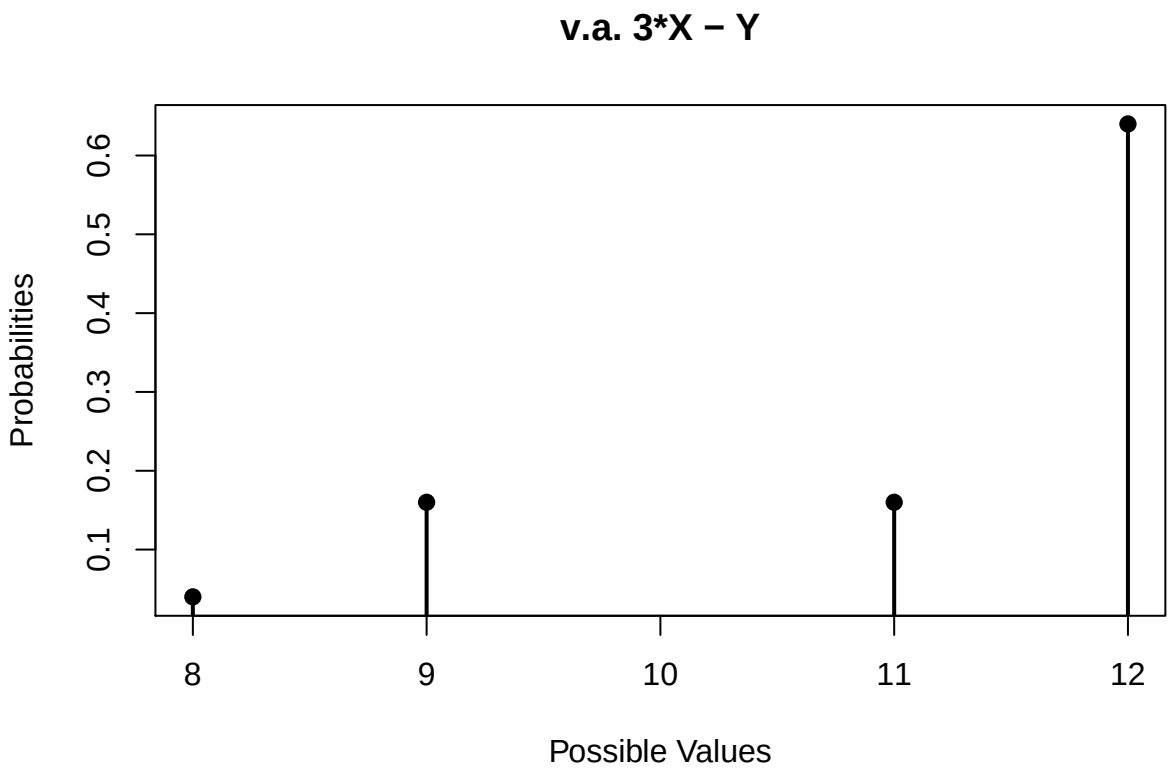
```r
plot(Yp3, main='v.a. Y+3')
```



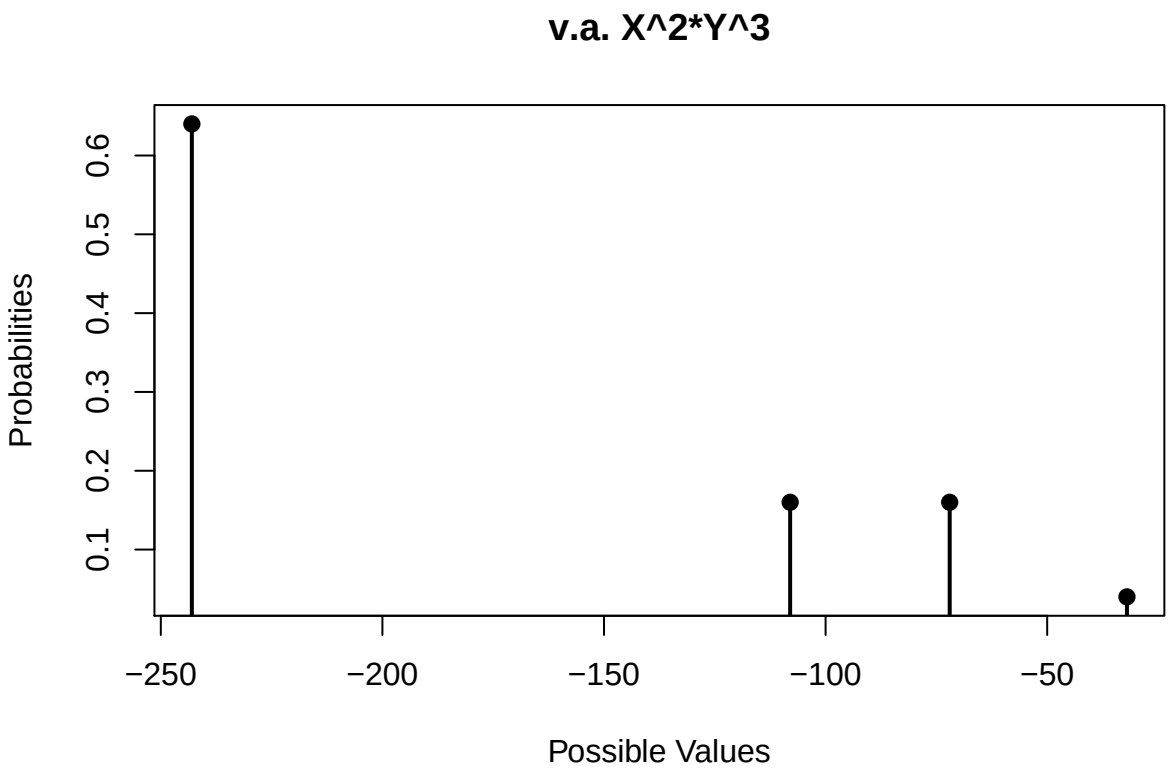
```r
plot(X2pY3, main='v.a. 2*X + 3*Y')
```



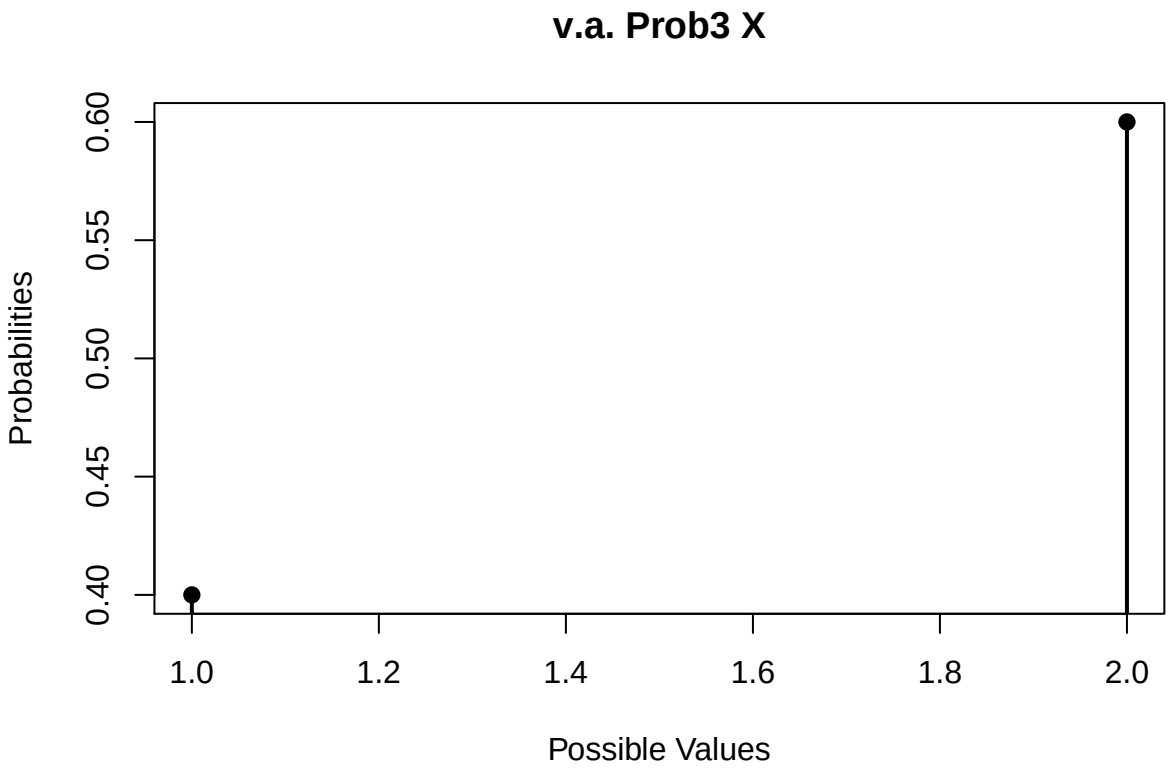
```r
plot(X3mY, main='v.a. 3*X - Y')
```



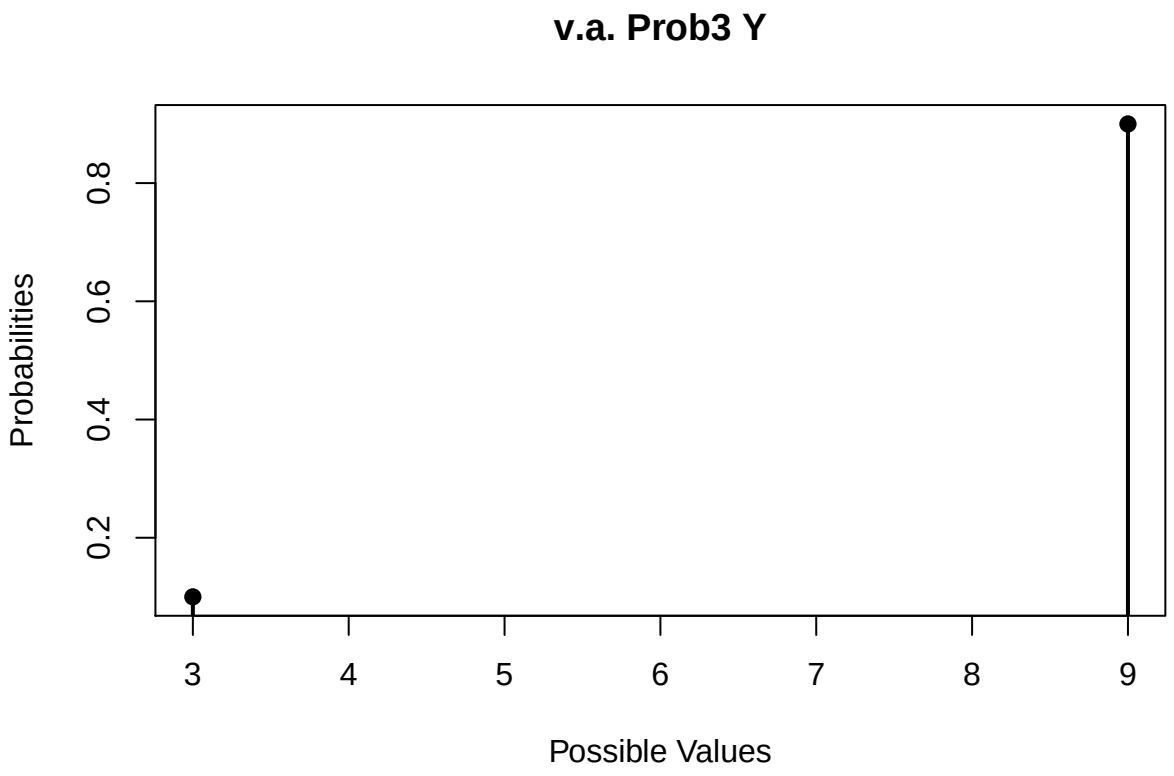
```r
plot(Xe2Ye3, main='v.a. X^2*Y^3')
```



```r
plot(Prob3X, main='v.a. Prob3 X')
```



```r
plot(Prob3Y, main='v.a. Prob3 Y')
```

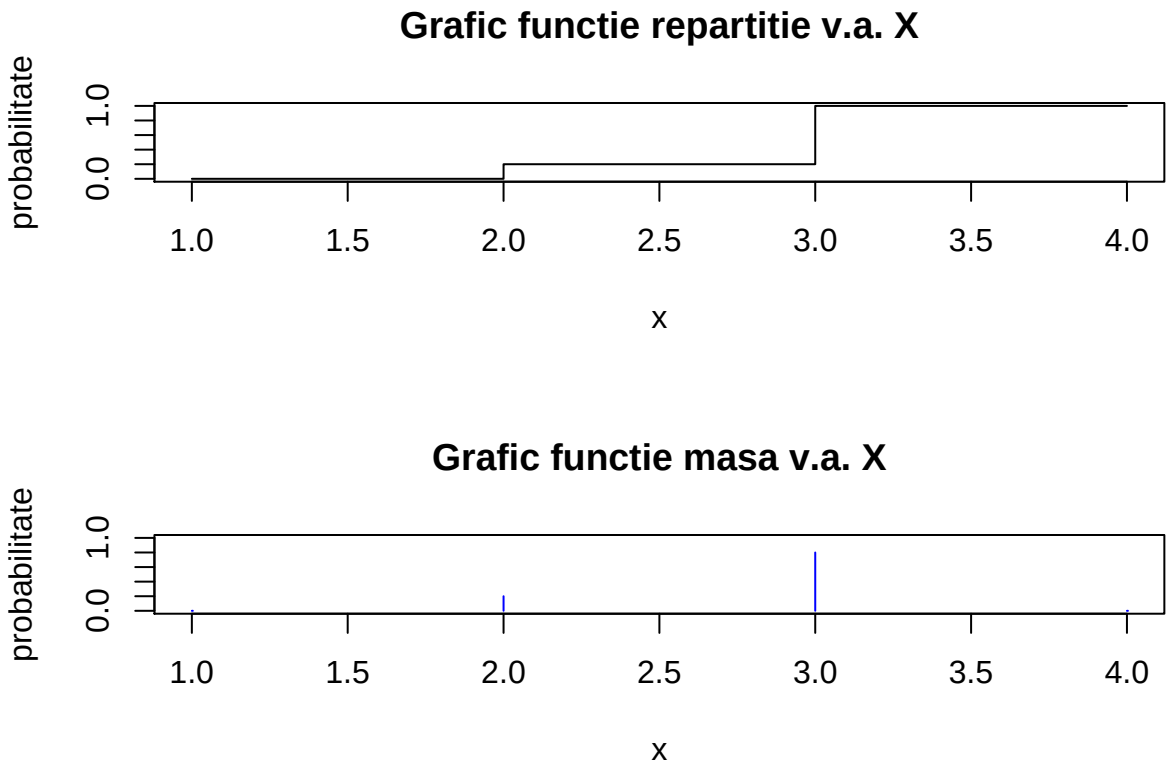


Problema 3

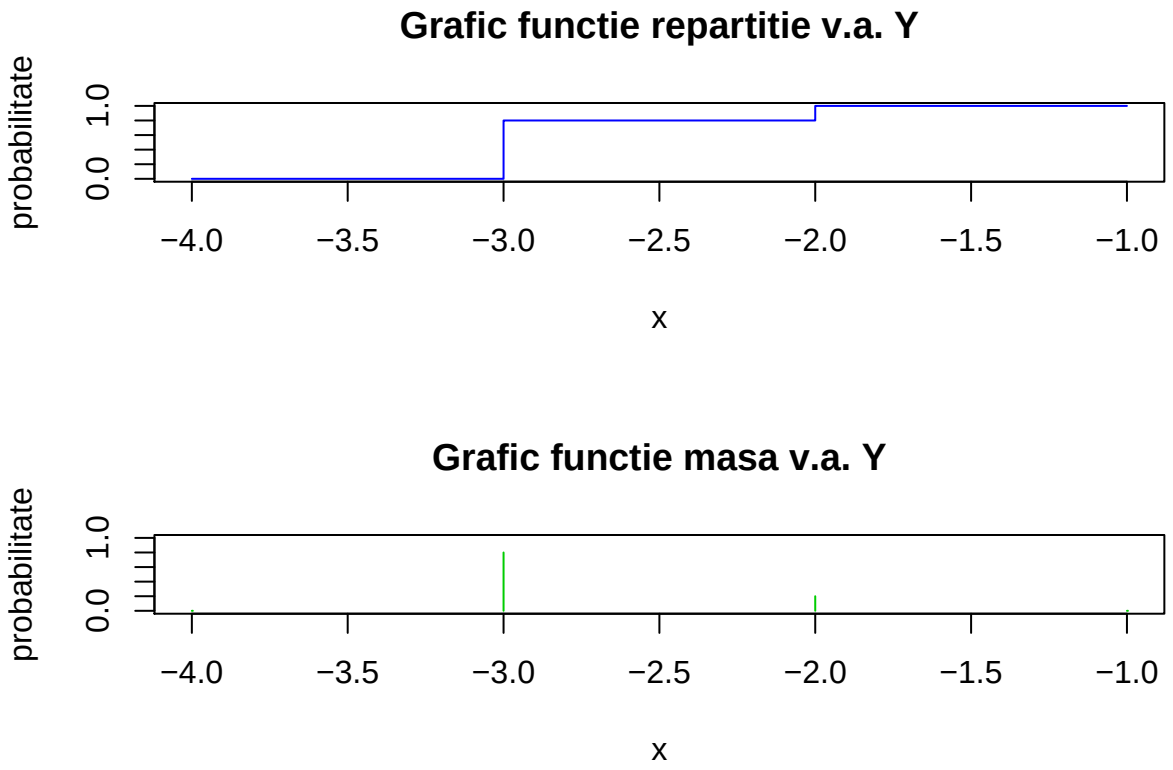
```r
fMasa <- function(X, x)
{
  xi <- outcomes(X)
  idx <- match(x, xi)

  el <- probs(X)[idx]
  el[is.na(el)] <- 0
  return(unname(el))
}

fRepartitie <- function(X, x)
{
  ps = c()
  for (val in x)
  {
    p <- sum(probs(X)[outcomes(X) <= val])

    ps <- c(ps,p)
  }
```

```
  return(ps)
}

fAfisare <- function(X, nume='')
{
  culori <- sample(c(1:4,6), 2)

  LB <- min(outcomes(X))
  LB <- LB - floor(abs(LB/10)) - 1
  UB <- max(outcomes(X))
  UB <- UB + floor(abs(UB/10)) + 1
  x_axis <- LB:UB
  x_axis <- sort(union(x_axis, outcomes(X)))

  par(mfrow=c(2,1))
  plot(x_axis, fRepartitie(X, x_axis), main=paste('Grafic functie repartitie',nume)
       , col=culori[1], 's', xlab='x', ylab='probabilitate', ylim=c(0,1))
  plot(x_axis, fMasa(X, x_axis), main=paste('Grafic functie masa',nume)
       , col=culori[2],type="h", xlab='x', ylab='probabilitate', ylim=c(0,1))
}
```

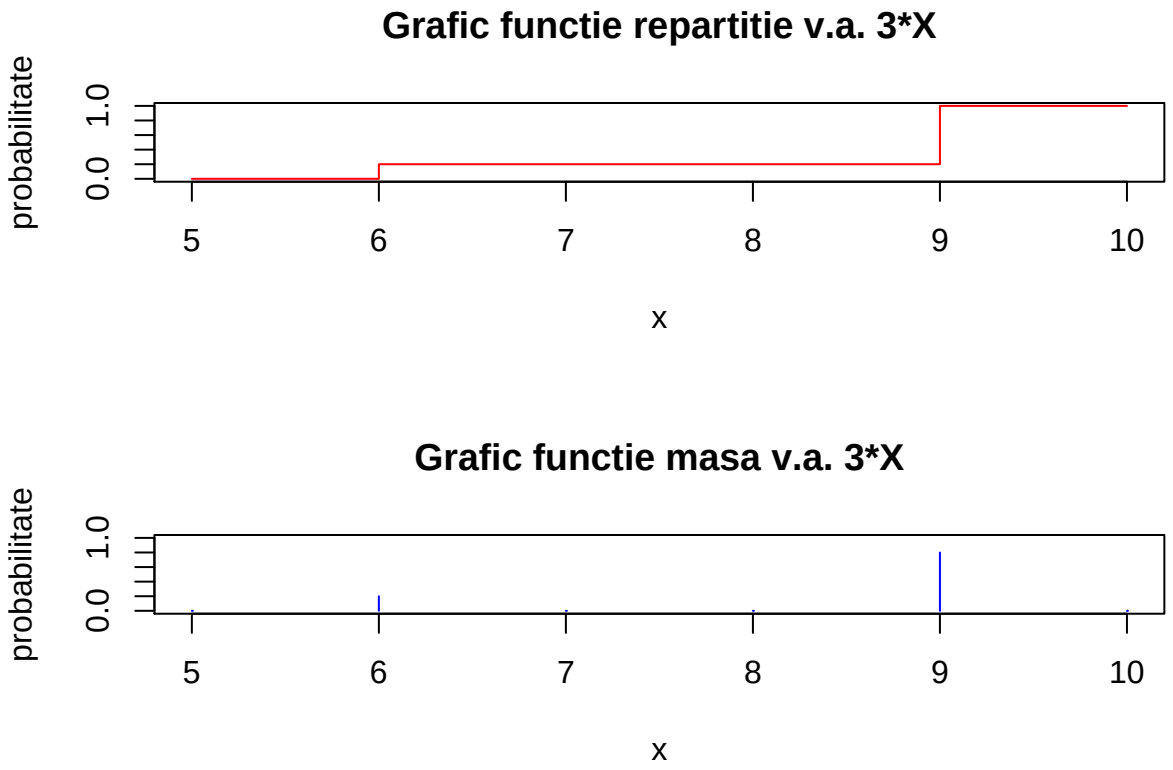
```
fAfisare(X, 'v.a. X')
```



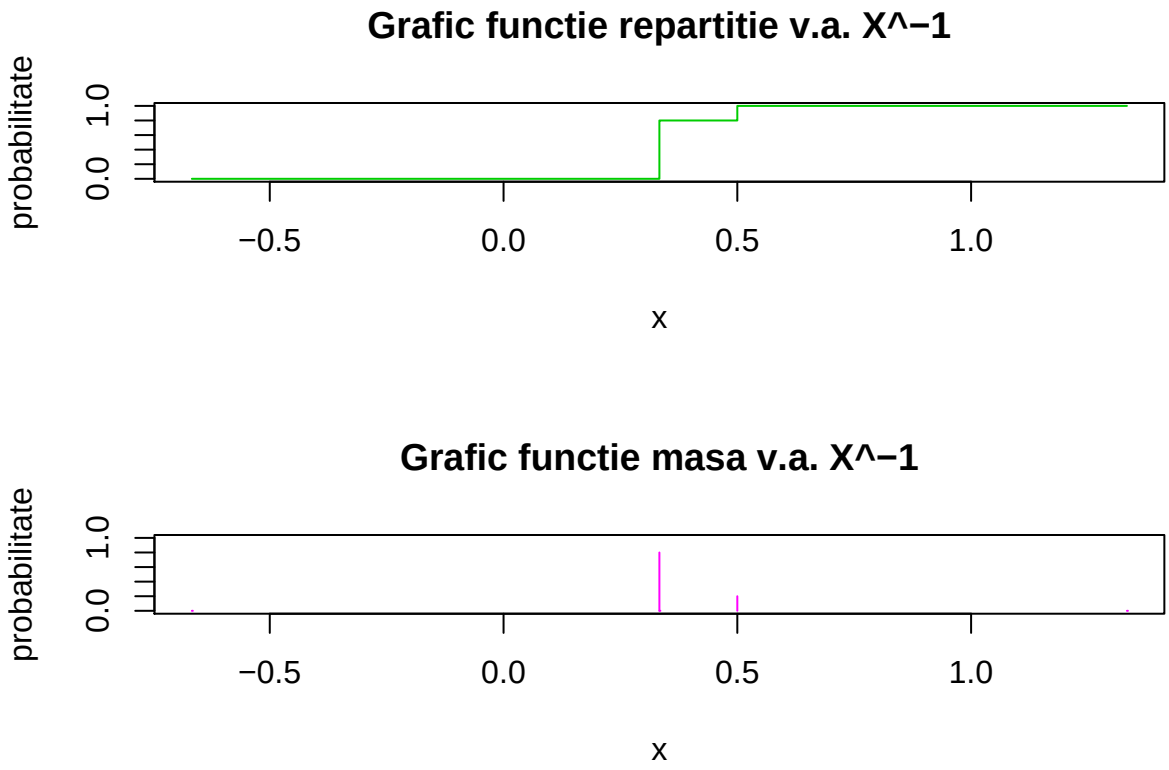
```
fAfisare(Y, 'v.a. Y')
```



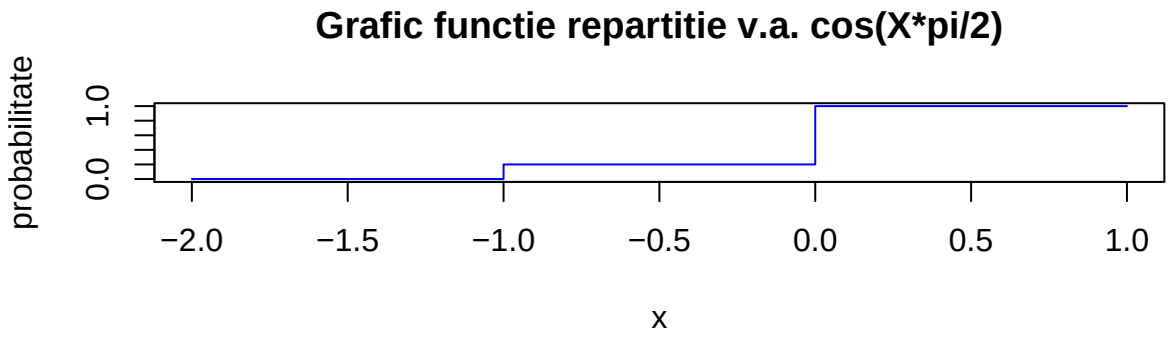
```
fAfisare(X3, 'v.a. 3*X')
```

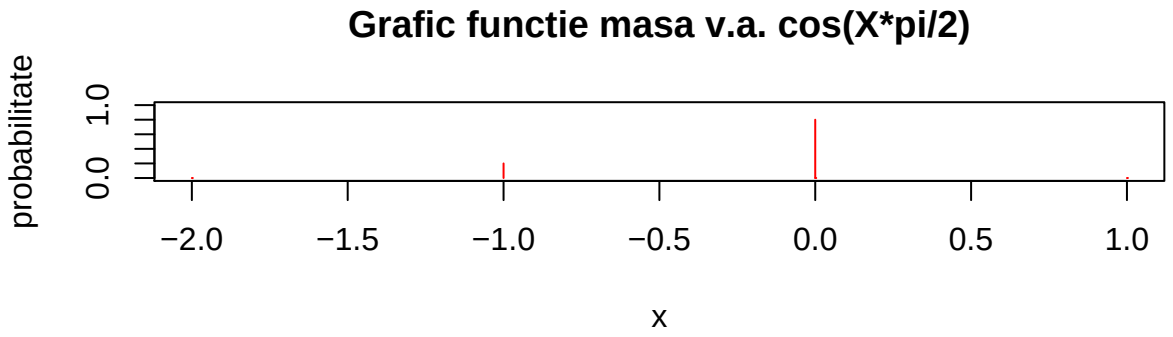


```
fAfisare(Xem1, 'v.a. X^-1')
```

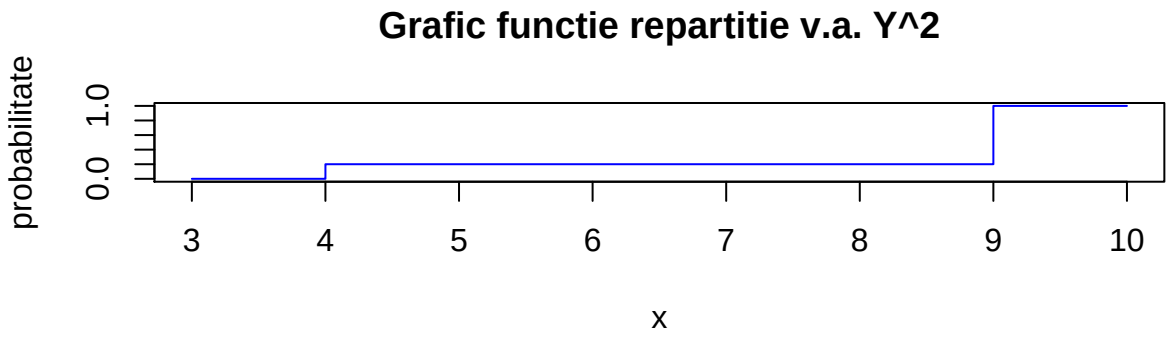


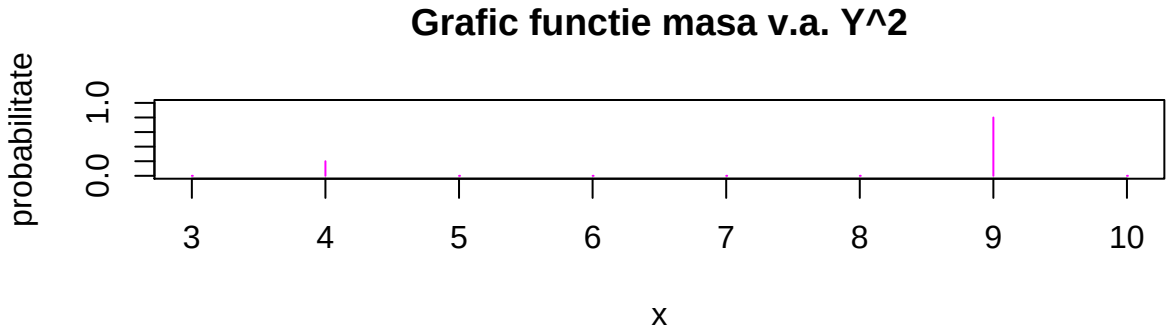
```
fAfisare(Xcos, 'v.a. cos(X*pi/2)')
```



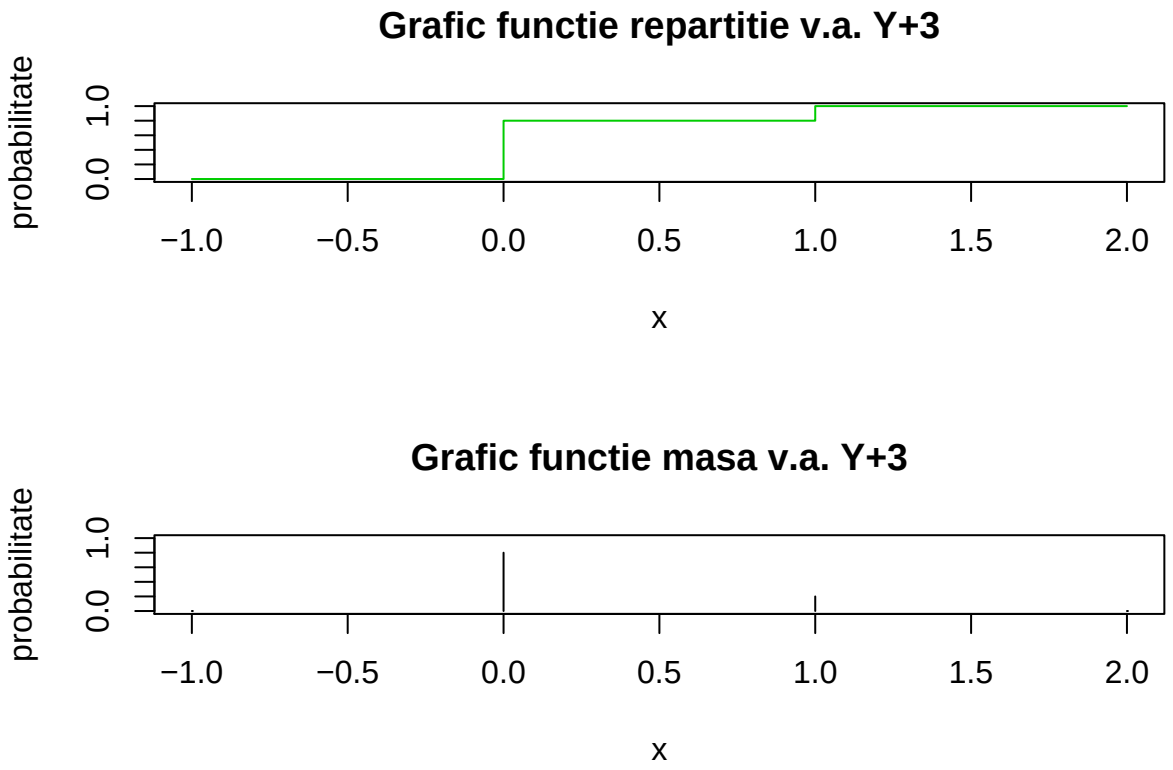


```
fAfisare(Ye2, 'v.a. Y^2')
```

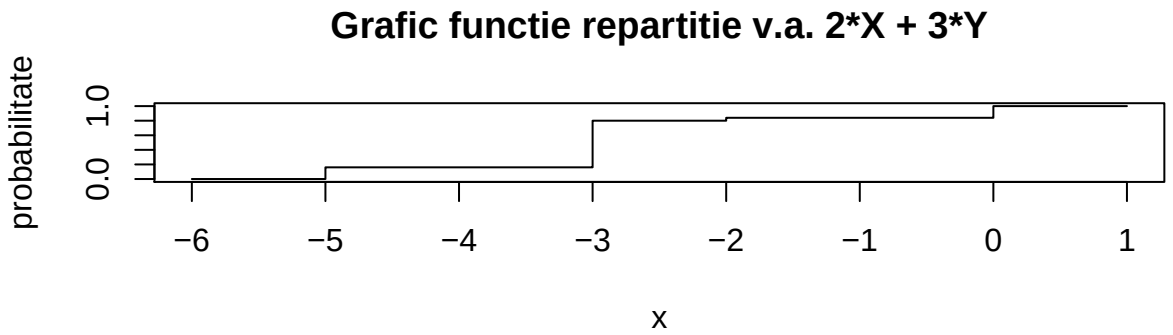


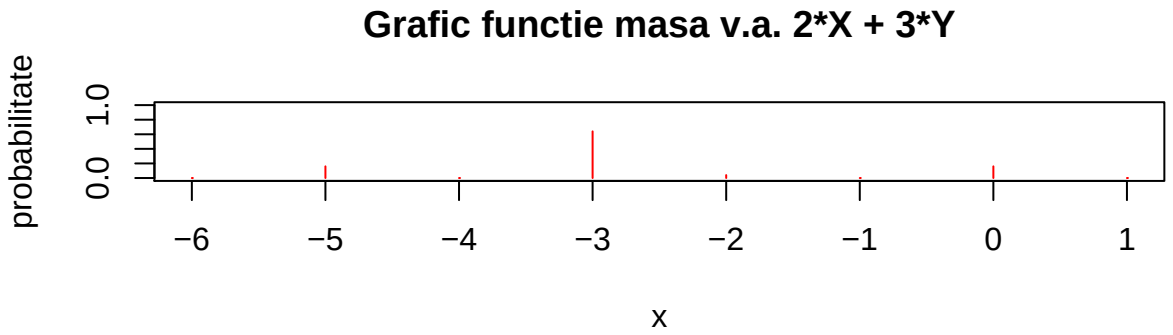


```
fAfisare(Yp3, 'v.a. Y+3')
```

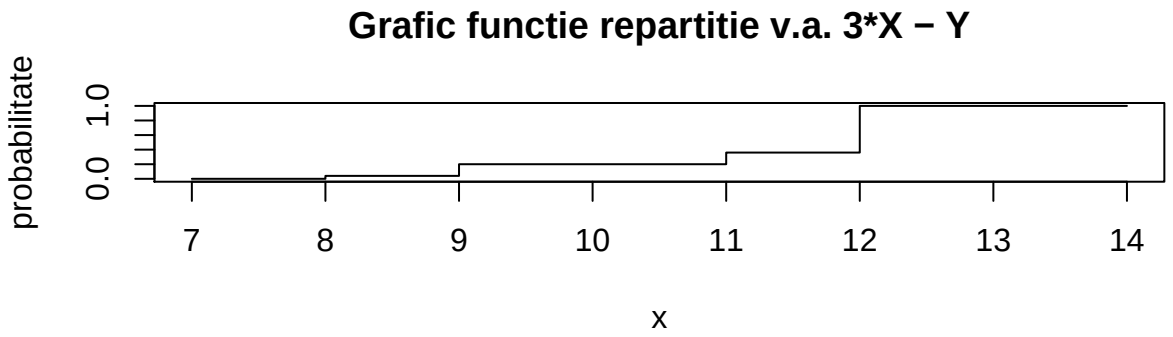


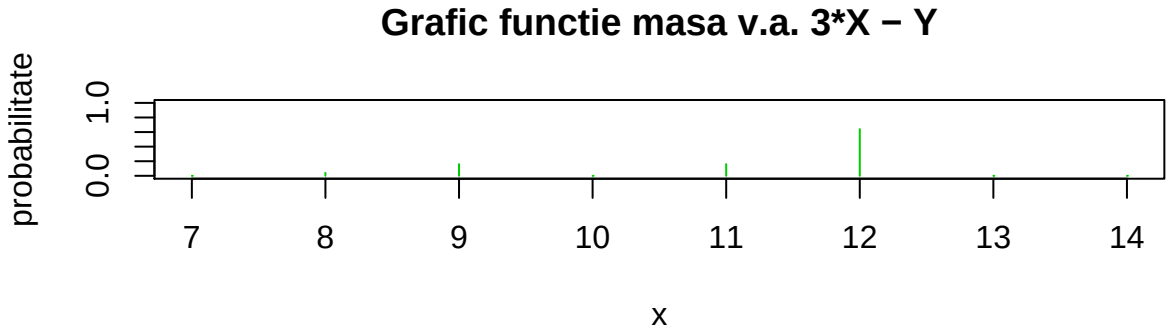
```
fAfisare(X2pY3, 'v.a. 2*X + 3*Y')
```



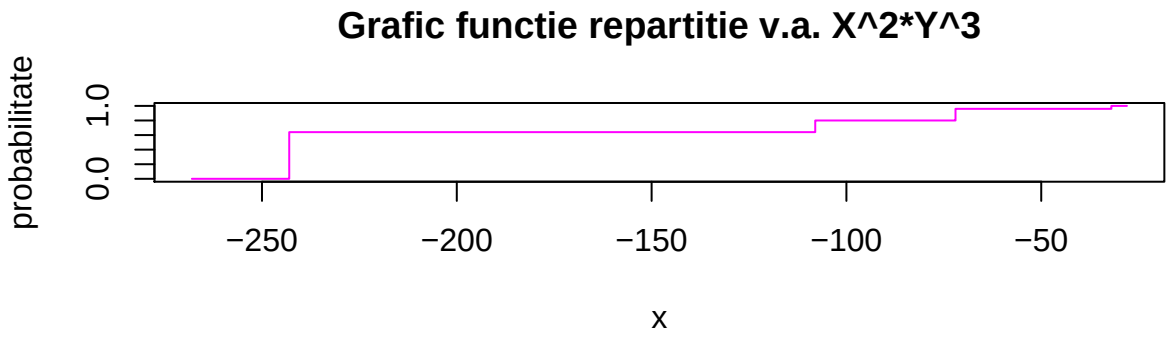


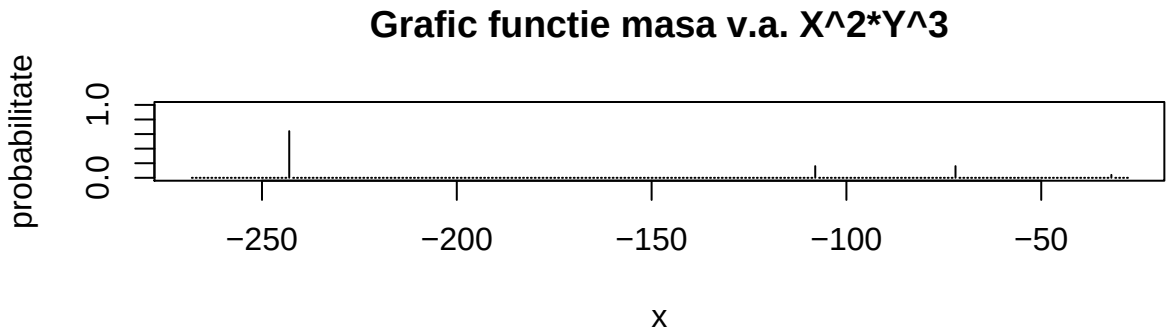
```
fAfisare(X3mY, 'v.a. 3*X - Y')
```



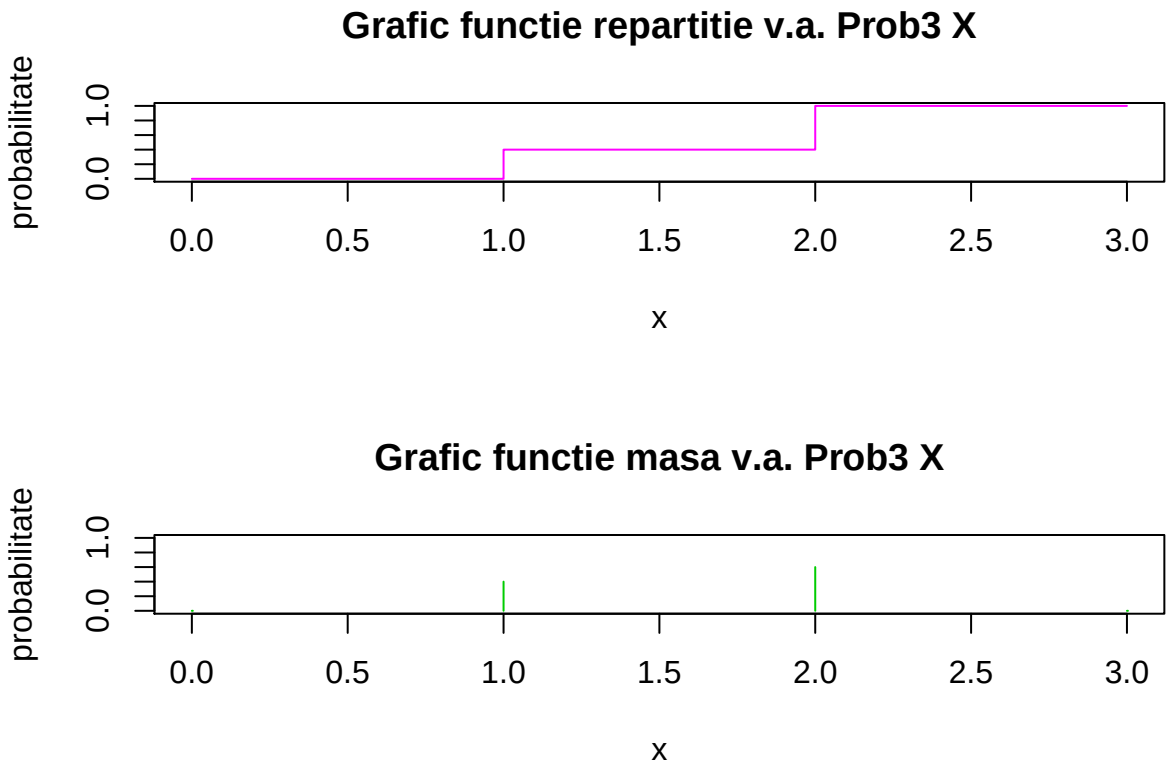


```
fAfisare(Xe2Ye3, 'v.a. X^2*Y^3')
```

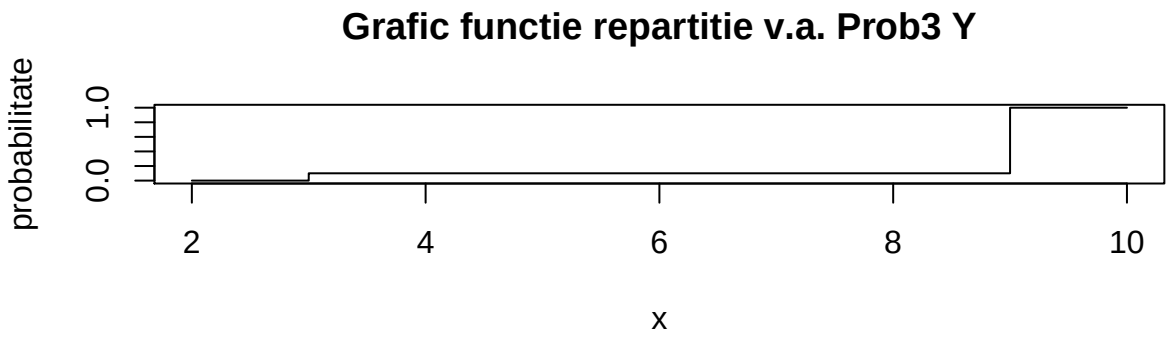


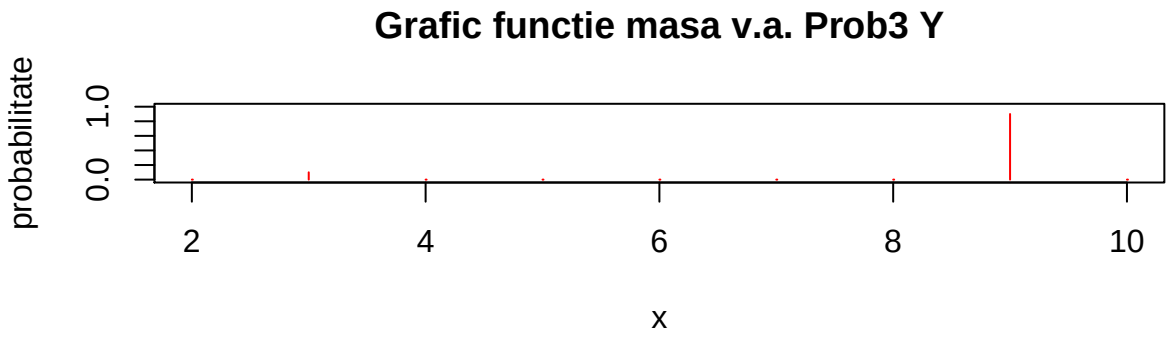


```
fAfisare(Prob3X, 'v.a. Prob3 X')
```



```
fAfisare(Prob3Y, 'v.a. Prob3 Y')
```





Problema 4

```
Z <- function(X, Y)
{
  moneda <- sample(c('H','T'),1)

  if (moneda == 'H') return(X)
  else return(Y)
}
```

generateVA este functia care genereaza n v.a. din Z

```
generateVA <- function(n, Z, X, Y)
{
  vas <- list()

  for (i in 1:n)
  {
    vas[[i]] <- Z(X,Y)
  }
```

```
  return(vas)
}


n <- 1000
Zs <- generateVA(n, Z, X, Y)
nr_X <- 0
```

Calculam de cate ori apar X si Y in generateVA

```
LB <- Inf
UB <- -Inf
for (i in 1:n)
{
  if (all(outcomes(Zs[[i]]) == outcomes(X)))
  {
    nr_X <- nr_X + 1
  }
  LB <- min(c(outcomes(Zs[[i]]), LB))
  UB <- max(c(outcomes(Zs[[i]]), UB))
}
nr_Y <- n-nr_X

(nr_X)
```

```
## [1] 491
```

```
(nr_Y)
```

```
## [1] 509
```

```
(probX <- nr_X/n)
```

```
## [1] 0.491
```

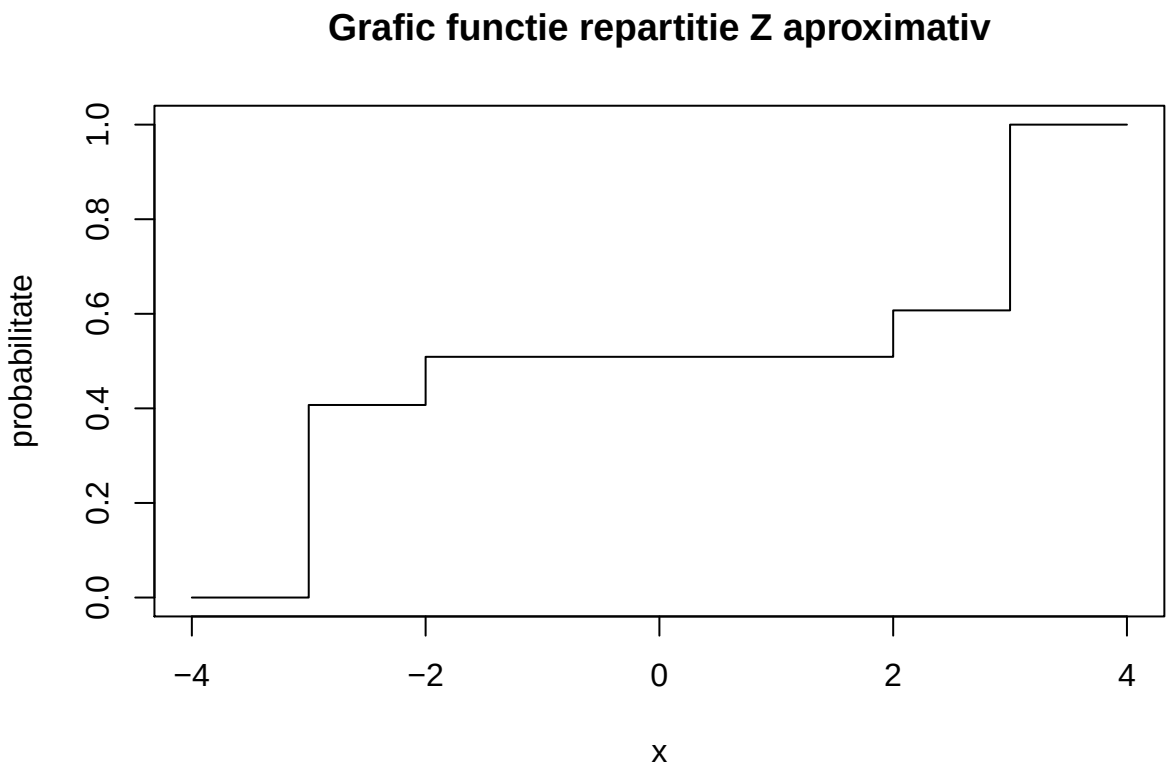
```
(probY <- nr_Y/n)
```

```
## [1] 0.509
```

Calculam o repartitie “ponderata”

```
LB <- LB - floor(abs(LB/10)) - 1
UB <- UB + floor(abs(UB/10)) + 1
x_axis <- LB:UB
x_axis <- sort(union(x_axis, outcomes(X), outcomes(Y)))

y_axis <- c()

for (i in x_axis)
{
```

```
  y_axis <- c(y_axis, probX * fRepartitie(X, i) + probY * fRepartitie(Y, i))
}

plot(x_axis, y_axis, main=paste('Grafic functie repartitie','Z aproximativ'),
     col=1, 's', xlab='x', ylab='probabilitate', ylim=c(0,1))
```



Din grafic putem deduce punctele in care repartitia se schimba (creste), acelea vor fi valorile posibile din v.a. Z

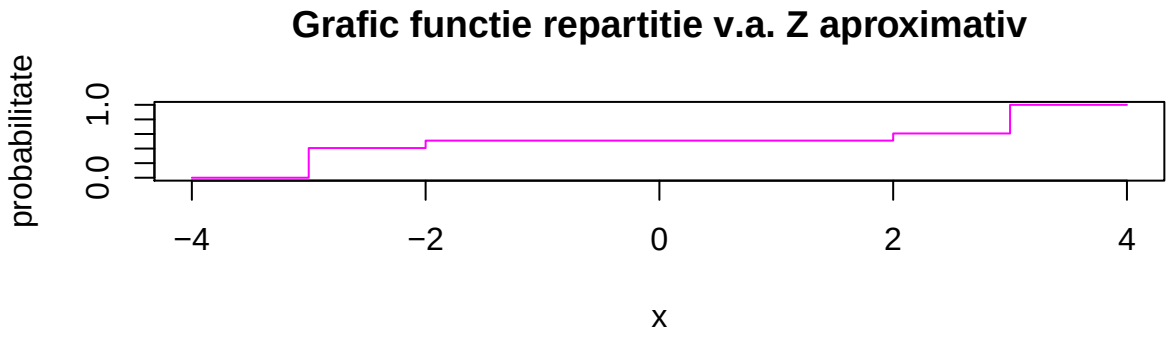
```
val_va <- c()
prob_va <- c()
lastVal <- numeric(0)
for (i in 1:length(x_axis))
{
  if (isTRUE(y_axis[i] != lastVal))
  {
    val_va <- c(val_va, x_axis[i])
    prob_va <- c(prob_va, y_axis[i]-lastVal)
  }
  lastVal <- y_axis[i]
}
```

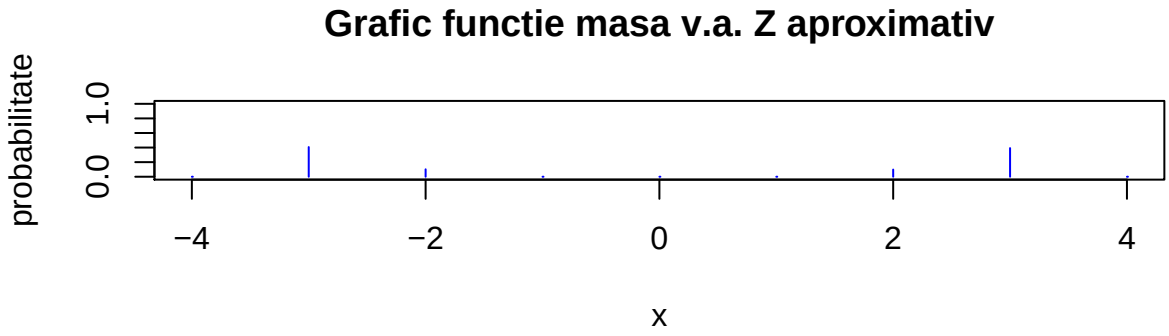
Construim o v.a. aproximativa

```
Zaprox <- RV(val_va,probs = prob_va)
(Zaprox)
```

```
## Random variable with 4 outcomes
##
## Outcomes       -3       -2        2        3
## Probs    509/1250 509/5000 491/5000 491/1250
```

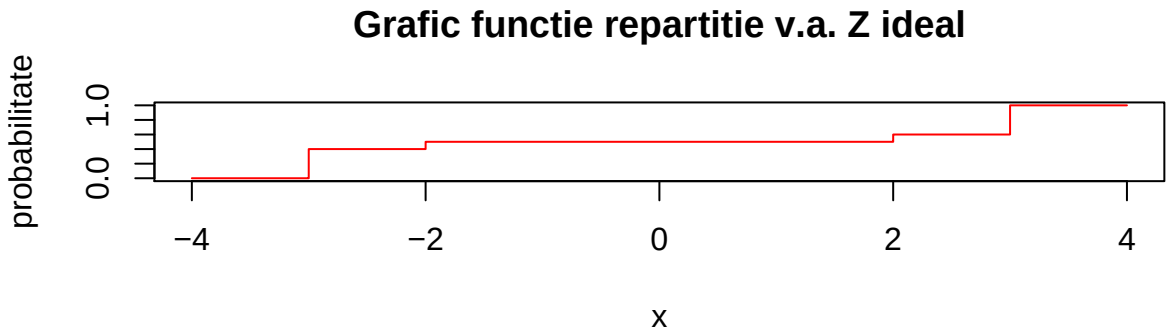
```
fAfisare(Zaprox, 'v.a. Z aproximativ')
```

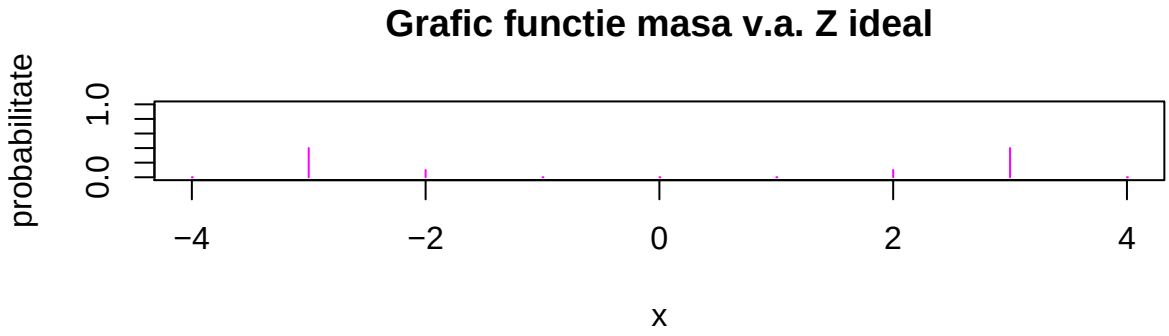




Din valori ne putem da seama cum ar arata Z ideal, atunci cand n -> Inf

```
Zideal <- RV(c(-3,-2,2,3),c(2/5,1/10,1/10,2/5))

fAfisare(Zideal, 'v.a. Z ideal')
```





In concluzie, este posibil sa aflam repartitia si functia de masa pentru Z, indiferent de probabilitatea de aparitie a lui X sau Y sau a numarului de v.a. din Z