

Leverging Sparsity for Fast Jacobians and Hessians

An Illustrated Guide to Automatic Sparse Differentiation

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Matrices vs. Operators

Foo

$$\mathbf{J}_f(\mathbf{x}) = \mathbf{J}_{h(g(\mathbf{x}))} \cdot \mathbf{J}_g(\mathbf{x})$$

$$\mathbf{D}f(\mathbf{x}) = \mathbf{D}h(g(\mathbf{x})) \cdot \mathbf{D}g(\mathbf{x})$$

Forward and reverse mode

Foo

$$\mathbf{D}f(\mathbf{x}) = \mathbf{D}h(g(\mathbf{x})) \cdot \mathbf{D}g(\mathbf{x})$$
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Stretching

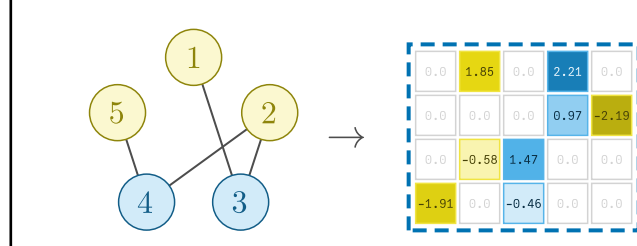
Foo

Automatic Sparse Differentiation

Foo

$$\mathbf{J}_f(\mathbf{x}) = \mathbf{J}_{h(g(\mathbf{x}))} \cdot \mathbf{J}_g(\mathbf{x})$$

$$\mathbf{D}f(\mathbf{x}) = \mathbf{D}h(g(\mathbf{x})) \cdot \mathbf{D}g(\mathbf{x})$$



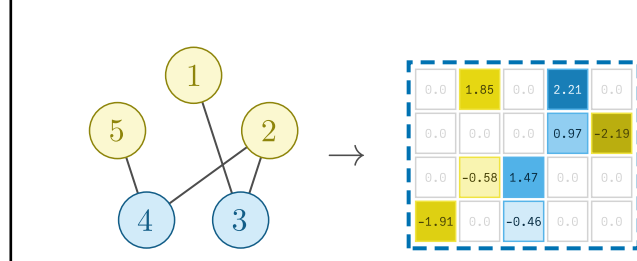
Pattern detection

Foo

$$\mathbf{J}_f(\mathbf{x}) = \mathbf{J}_{h(g(\mathbf{x}))} \cdot \mathbf{J}_g(\mathbf{x})$$

Coloring

Foo



Bicoloring

Foo

$$\mathbf{J}_f(\mathbf{x}) = \mathbf{J}_{h(g(\mathbf{x}))} \cdot \mathbf{J}_g(\mathbf{x})$$

Demonstration

```
using DifferentiationInterface
using SparseConnectivityTracer: TracerSparsityDetector
using SparseMatrixColorings: GreedyColoringAlgorithm
import ForwardDiff

ad_backend = AutoForwardDiff()

asd_backend = AutoSparse(ad;
    TracerSparsityDetector(),
    GreedyColoringAlgorithm()
)

jacobian(f, ad_backend, x)
jacobian(f, asd_backend, x)
```

Align them to the bottom.