

Import relevant packages here.

```
In [21]: import matplotlib.pyplot as plt
import pandas as pd
import numpy as numpy
```

Load the data and verify it is loaded correctly.

- Print it (head, tail, or specific rows, choose a sensible number of rows).
- Compare it to the source file.

```
In [22]: data = pd.read_csv('cf_data.csv')
```

```
display(data.head(10))
display(data.tail(10))
```

	dv	s	a
0	-0.743240	53.5427	1.242570
1	-0.557230	53.6120	1.777920
2	-0.454769	53.6541	0.544107
3	-0.525396	53.7030	-0.294755
4	-0.601285	53.7592	-0.290961
5	-0.682448	53.8232	-0.283414
6	-0.768859	53.8957	-0.271604
7	-0.860452	53.9770	-0.133532
8	-0.832777	54.0678	0.243356
9	-0.576125	54.1436	0.406759

	dv	s	a
73898	0.141326	16.3446	0.594131
73899	0.201311	16.3098	-0.332118
73900	0.157088	16.3044	-0.215962
73901	5.199570	117.1910	0.820016
73902	5.262130	116.8660	-0.008254
73903	5.198740	116.1390	-0.795081
73904	5.104280	115.6270	-0.314263
73905	5.137640	115.1180	0.232283
73906	5.153480	114.5990	0.262078
73907	5.258880	113.1120	-0.612440

In the ensuing, you will use `numpy`.

Let's create a grid for the values to plot. But first create **two arrays named `dv` and `s`** using `numpy.linspace` that hold the grid values at the relevant indices in their respective dimension of the grid.

Create a **grid named `a`** with zeros using `numpy.zeros` in to which calculated acceleration values can be stored.

Let the grid span:

- Speed difference `dv` [m/s]
 - From -10 till 10
 - With 41 evenly spaced values
- Headway `s` [m]
 - From 0 till 200
 - With 21 evenly spaced values

```
In [23]: dv = numpy.linspace(-10, 10, 41)
s = numpy.linspace(0, 200, 21)
a = numpy.zeros((len(s), len(dv)))
```

Create from the imported data 3 separate `numpy` arrays for each column `dv`, `s` and `a`. (We do this for speed reasons later.)

- Make sure to name them differently from the arrays that belong to the grid as above.
- You can access the data of each column in a `DataFrame` using `data.xxx` where `xxx` is the column name (not as a string).
- Use the method `to_numpy()` to convert a column to a `numpy` array.

```
In [24]: DV = (data.dv).to_numpy()
S = (data.s).to_numpy()
A = (data.a).to_numpy()
```

Create an algorithm that calculates all the acceleration values and stores them in the grid. The algorithm is described visually in the last part of the lecture. At each grid point, it calculates a weighted mean of all measurements. The weights are given by an exponential function, based on the 'distance' between the grid point, and the measurement values of `dv` and `s`. To get you started, how many `for`-loops do you need?

For this you will need `math`.

Use an *upsilon* of 1.5m/s and a *sigma* of 30m.

Warning: This calculation may take some time. So:

- Print a line for each iteration of the outer-most `for`-loop that shows you the progress.
- Test you code by running it only on the first 50 measurements of the data.

```
In [26]: # Define Constants
UPSILON = 1.5 # [ m / s ] meters per second
SIGMA = 30 # [ m ] meters

# Initialize
for i in range(len(s)):
    for j in range(len(dv)):
        sum_omega_A = 0
        sum_omega = 0
        for k in range(len(data)):
            omega = numpy.exp((-abs(DV[k] - dv[j]) / UPSILON) - (abs(S[k] - s[i]) / SIGMA))

            sum_omega_A += (omega * A[k])
            sum_omega += omega

            A_weight = sum_omega_A / sum_omega
            a[i][j] = A_weight

print(a)

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