

Lab 1: Experimental setup and tools

par4111

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1 Introduction

In order to do properly this subject, first, we have to introduce some new concepts and hardware and software environment that we will use during this semester to do all laboratory assignments. The following document contains an introductory approach, step by step introducing those concepts. We will introduce the *Boada* architecture, some of the most important parallelism concepts and several tests to see its effects.

2 Experimental setup

2.1 Node architecture and memory

Boada is a multiprocessor server located at the Computer Architecture Department divided in different nodes, each of them with different architecture and different uses. *Boada* is composed of 8 nodes (from boada-1 to boada-8) and they can be grouped as the following table:

Node name	Processor generation	Interactive	Queue name
boada-1	Intel Xeon E5645	Yes	batch
boada-2 to 4	Intel Xeon E5645	No	execution
boada-5	Intel Xeon E5-2620 v2 + Nvidia K40c	No	cuida
boada-6 to 8	Intel Xeon E5-2609 v4	No	execution2

However in this course we are going to use mainly from boada-1 to boada-4. The easiest way to obtain the information of the hardware used in each node is using the linux commands `lscpu` and `lstopo`(see Figure 1 and 2). This commands can be easily executed in the boada-1 node (because it is interactive), but if we want to use the other nodes we can use the `submit-*.sh` script provided by the PAR professors and use the queue system.

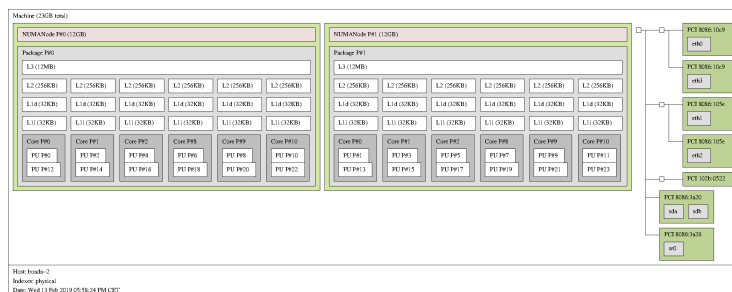


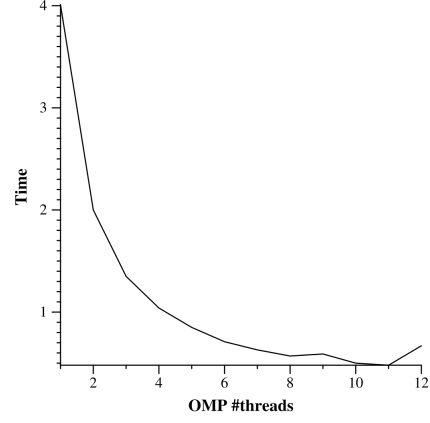
Figure 1: Boada-2 architecture outputed by lstopo.

In the two following sections we are going to see the differences of two different approaches to parallelism, **strong** and **weak**, applied to the *pi_omp.c* program.

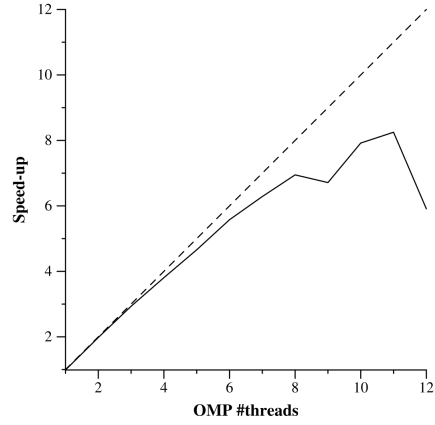
2.2.1 Strong scalability

Strong scalability consists in increasing the number of processors while keeping the problem size the same. This reduces the amount of work each processor has to do, which speeds-up the execution. Nonetheless, the speed-up is bounded by the parallelization of the program and the overhead generated when doing so. Usually a point is reached where adding processors has no further effect on the program or the overhead generated by further parallelizing the program is greater than the added speed-up.

Boada 1: For Boada 1 we can see how execution time is logarithmically reduced in Figure 3. At 11 threads the time seems to slightly increase. To be sure we can look at the speed-up plot, where we can clearly appreciate how speed-up slowly stops increasing and starts to fall down at around 11 threads. This evidence supports the previous statement about time. This is most likely caused by the overhead that parallelizing a program creates: creating new threads, synchronizing the results, etc. The graphic shows how running the program on 12 threads is actually slower than doing so on 11, and even though we cannot see it in the plots, performance would probably keep going down due to the added overhead with each new thread used.



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Min elapsed execution time
Fri Mar 1 16:54:24 CET 2019



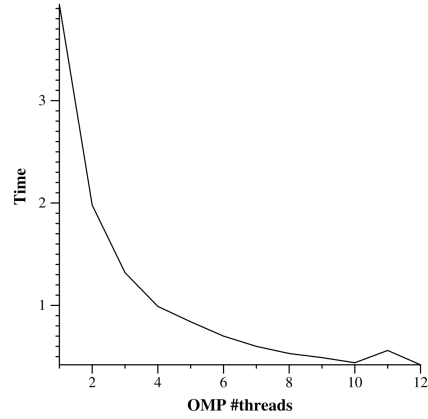
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Speed-up wrt sequential time
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Figure 3: *pi_omp* with strong scalability by Boada-1

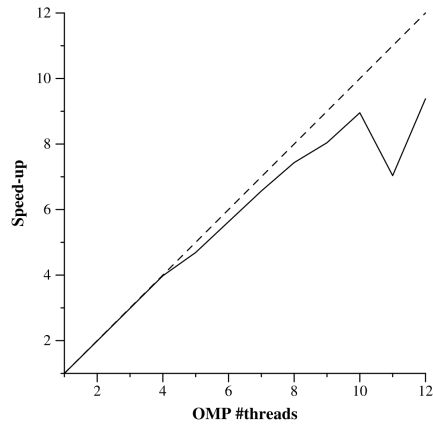
Boada 2 to 4: Since Boada 1 has the same architecture as 2, 3 and 4, it is fair to expect a very similar performance, and in fact that is what Figure 4 shows. However, Boada 4 speed-up does not quite decrease from using 11 threads to 12, although it certainly does from 10 to 11. Again, it would probably keep decreasing from that point on due to the overhead, but there is an important

improvement up until around 10 threads.

It would be fair to say that for the *pi_omp.c* program running Boada 1 to 4, up to 10 threads are beneficial to reduce execution time, but at around 5 threads speed-up starts to increase more slowly at what seems like a logarithmic rate.



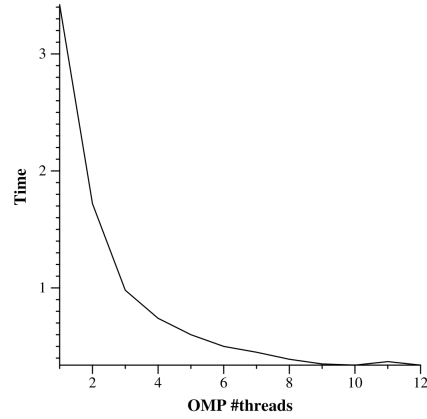
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Min elapsed execution time
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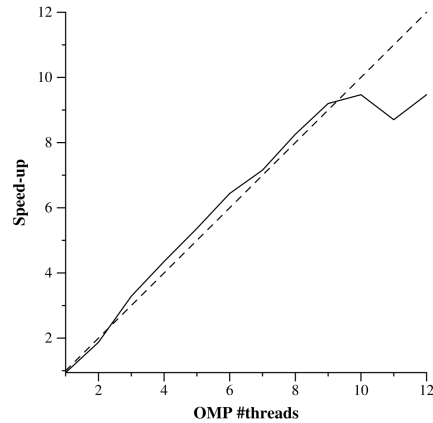
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Speed-up wrt sequential time
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Figure 4: *pi_omp* with strong scalability by Boada-4

Boada 5: Boada 5 uses a different processor, but the overhead principle should still be present because the generated overhead will start to overcome the parallelization speed-up at some point. In fact, looking at the generated graphics, the loss begins at approximately the same number of threads than the previous Boadas (1-4).



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Min elapsed execution time
Fri Mar 1 16:54:17 CET 2019

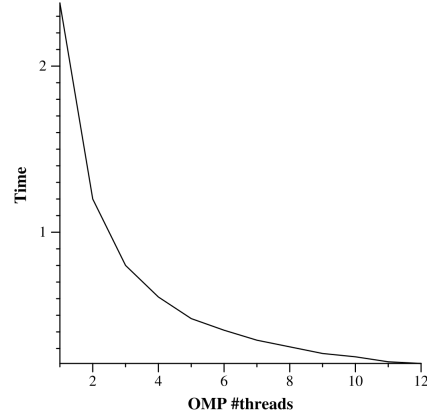


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Speed-up wrt sequential time
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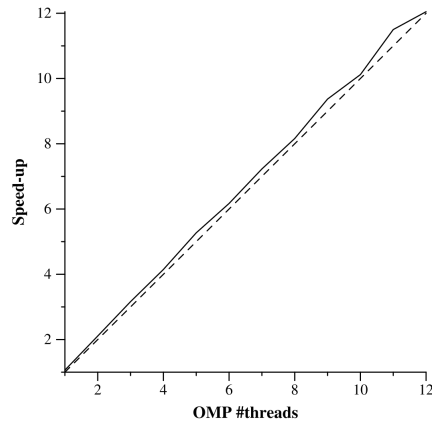
Figure 5: *pi_omp* with strong scalability by Boada-5

Boada 6 to 8: Boada 6 to 8 use again a different processor. This time, however, the speed-up and time plots look quite different from the previous ones. There is no appreciable decrease in speed-up even up to 12 threads. We speculate that this could be of their processors being newer (Boada 6-8 have Intel Xeon E5-2609 v4), which according to Intel's website have a newer instruction set (Intel® AVX2) and Intel® Transactional Synchronization Extensions New Instructions, which make parallel operations more efficient. Again, we are not sure, it is just a possibility we came up with after doing some research about each Boada's processor architecture.

Nevertheless, speed-up should still decay later on if we kept increasing the number of threads, because a point will be reached where the added overhead will surpass the time-reduction gained by parallelism.



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Min elapsed execution time
Wed Feb 13 19:06:28 CET 2019



par4111
Speed-up wrt sequential time
Wed Feb 13 19:06:28 CET 2019

Figure 6: *pi_omp* with strong scalability by Boada-8

2.2.2 Weak scalability

Weak scalability takes a different approach. It takes advantage of the additional power gained by parallelizing the program to increase the problem size proportionally to the number of threads, so that while the speed-up stays more or less the same, the work done increases.

Boada 1: For Boada 1 with weak scalability the graphic clearly shows how speed-up stays mostly the same at first. This is due to increasing the problem size proportionally to the number of threads. At around 9 threads, however, the speed-ups starts to decrease similarly to strong scalability. The reason is the same: excess overhead created by parallelization. To keep speed-up the same from this point on we should start to increase the problem size more slowly while keeping the thread number increment the same.

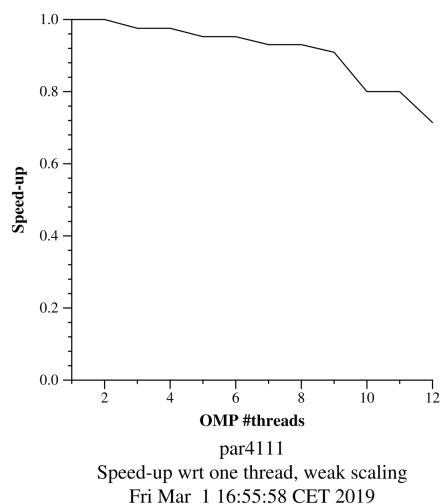


Figure 7: *pi_omp* with weak scalability by Boada-1

Boada 2 to 4: As with strong scalability, Boada 2 to 4 produce a very similar result to Boada 1 due to them having the same processor. There is not much to add with respect to Boada 1.

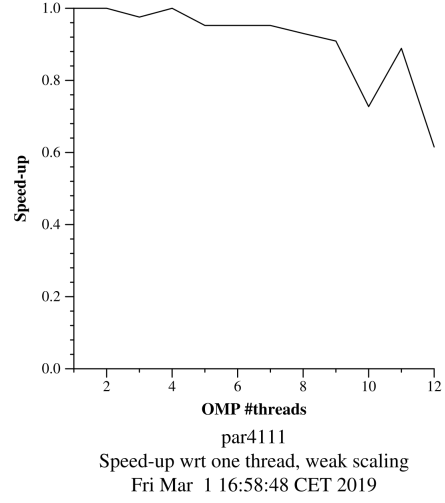


Figure 8: *pi_omp* with weak scalability by Boada-3

Boada 5: For Boada 5 we observe a similar behaviour, but this time speed-up goes over 1, meaning the problem size could be increased faster if we wanted to keep speed-up steady. In the end, however, overhead still counters the extra processors and speed-up goes down at around 10-12 threads.

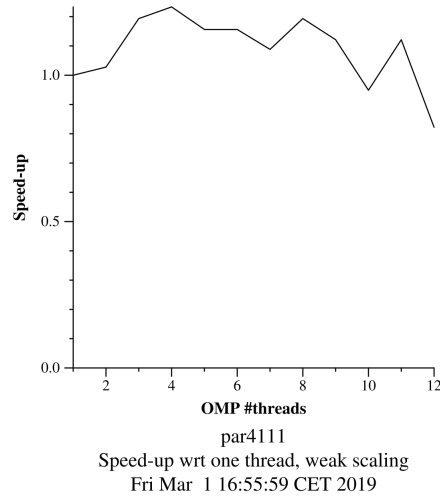


Figure 9: *pi_omp* with weak scalability by Boada-5

Boada 6 to 8: With Boada 6 to 8 we encounter the same situation as in strong scalability. There is no clear sign of overhead occurring in Figure 10. Again, it would happen if we kept increasing the number of threads assuming the program could use them all, but up to 12 threads there is not a real downside of parallelizing the program too much.

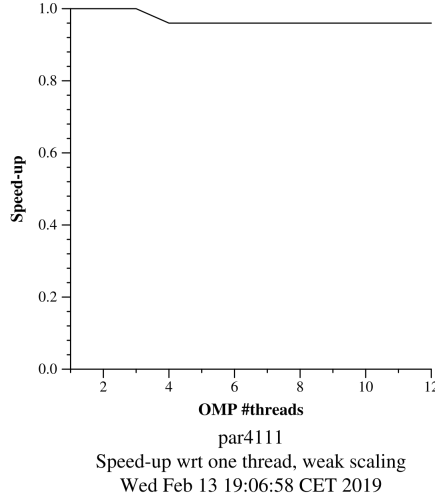


Figure 10: *pi_omp* with weak scalability by Boada-6

3 Systematically analysing task decompositions with Tareador

3.1 Introduction

The objective of this laboratory is learn how to use *Tareador*, an environment to analyse the potential parallelism that can be obtained when a certain task decomposition is applied to a code. We will introduce how it works and we will experiment and analyse decomposition with a sequential code called 3DFFT.

3.2 Analysis of task decompositions for 3DFFT

Once we have seen the basic features in *Tareador* we can now proceed to explore new tasks decompositions for a piece of code. Down below we will incrementally generate five new task decompositions and the potential parallelism (T_1/T_∞) from the task dependence graph generated by *Tareador*.

To obtain T_∞ we will assume that each instruction takes one time unit to execute and simulate the execution of the graph with a large number of processors.

Once we have created those tasks, we can visualize the dependency graph using *Tareador*. Each node of the graph represents a task: diferents shapes and colours are used to identify task instances and each one is labeled with a task instance number and some important information like the number of instructions. Also the size of the shape reflects in some way its granularity.

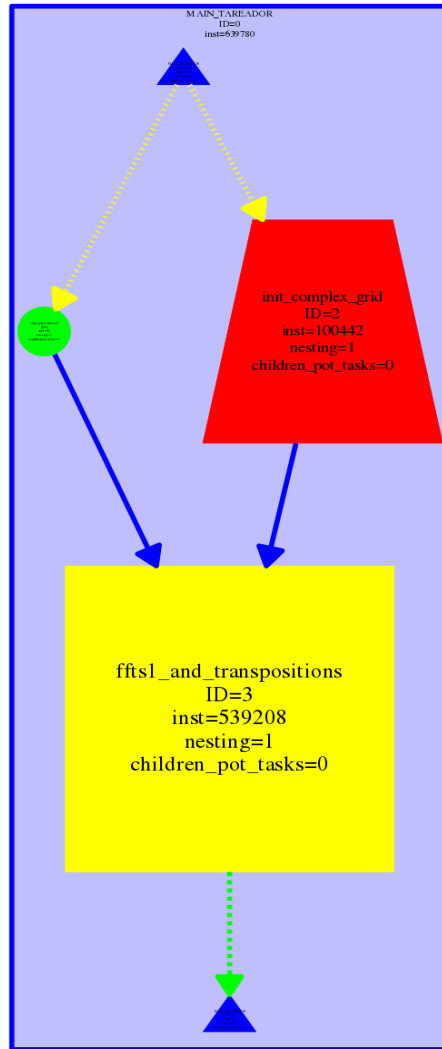


Figure 11: Dependency graph for the original version.

3.2.1 Version 1

The first version consists in replacing the task named `ffts1_and_transpositions` with a sequence of finer grained tasks, one for each function invocation inside it. The modified code is the following:

```
...
tareador_start_task("0");
ffts1_planes(p1d, in_fftw);
```

```

tareador_end_task("0");

tareador_start_task("1");
transpose_xy_planes(tmp_fftw, in_fftw);
tareador_end_task("1");

tareador_start_task("2");
ffts1_planes(p1d, tmp_fftw);
tareador_end_task("2");

tareador_start_task("3");
transpose_zx_planes(in_fftw, tmp_fftw);
tareador_end_task("3");

tareador_start_task("4");
ffts1_planes(p1d, in_fftw);
tareador_end_task("4");

tareador_start_task("5");
transpose_zx_planes(tmp_fftw, in_fftw);
tareador_end_task("5");

tareador_start_task("6");
transpose_xy_planes(in_fftw, tmp_fftw);
tareador_end_task("6");

```

Once we have created all these tasks, we have to execute the script `./run-tareador.sh VERSION1` and visualize the task dependence graph, see Figure 12. As we can see, comparing the original graph, see Figure ??, now the shape that was associated to `ffts1.and.transpositions` has now been divided into several other shapes which represents more granularity.

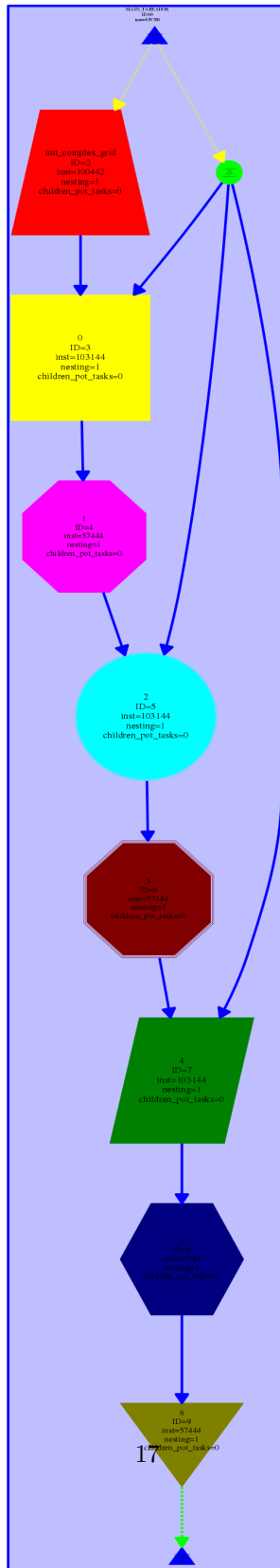


Figure 12: Dependency graph for the first version.

3.2.2 Version 2

The second version, starting from the first one, consists in replacing the definition of tasks associated to function invocations `ffts1_planes` with fine-grained tasks defined inside the function body and associated to individual iterations of the `k` loop. The changes that have been made in the code for this version are the following ones:

```
void ffts1_planes(fftwf_plan p1d, fftwf_complex in_fftw [[ N][N]] {
    int k,j;

    for (k=0; k<N; k++) {
        tareador_start_task("ffts1_planes_loop_k");
        for (j=0; j<N; j++) {
            fftwf_execute_dft ( p1d, (fftwf_complex *)in_fftw [k][j ][0], (
                fftwf_complex *)in_fftw [k][j ][0]) ;
        }
        tareador_end_task("ffts1_planes_loop_k");
    }
}
```

```
int main(){
    ...
    tareador_start_task("1");
    transpose_xy_planes(tmp_fftw, in_fftw );
    tareador_end_task("1");

    ffts1_planes (p1d, tmp_fftw);

    tareador_start_task("3");
    transpose_zx_planes(in_fftw , tmp_fftw);
    tareador_end_task("3");

    ffts1_planes (p1d, in_fftw );

    tareador_start_task("5");
    transpose_zx_planes(tmp_fftw, in_fftw );
    tareador_end_task("5");

    tareador_start_task("6");
    transpose_xy_planes(in_fftw , tmp_fftw);
    tareador_end_task("6");
    ...
}
```

}

As we can see in the outputted dependency graph given by the *Tareador* our dependency graph has now changed and there are several more shapes. The task that previously was `ffts1_planes` has been divided into several more.

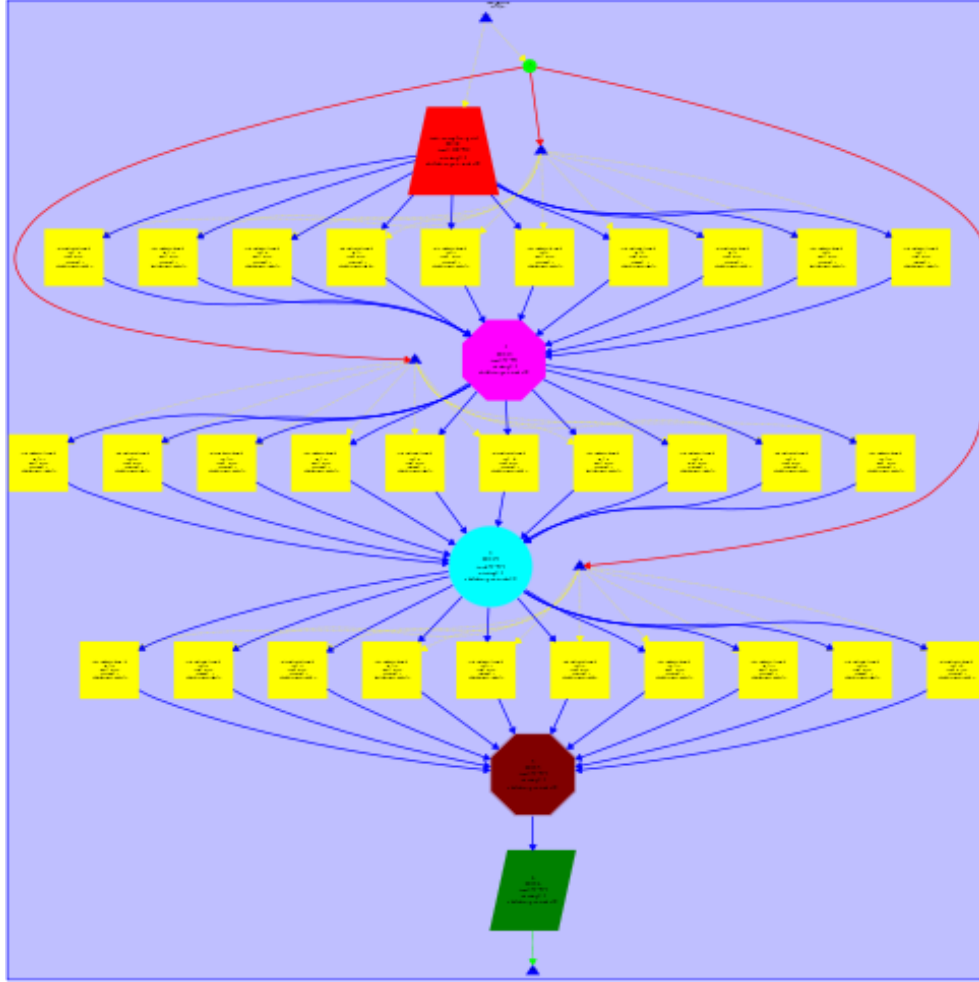


Figure 13: Dependency graph for the second version.

3.2.3 Version 3

The third version, starting from the second one, consists in replacing the definition of tasks associated to function invocations `transpose_xy_planes` and `transpose_zx_planes` with fine-grained tasks inside the corresponding body functions and associated to individual iterations of the `k` loop, similarly it was

made in the second version.

```
void transpose_xy_planes(fftwf_complex tmp_fftw [[N][N], fftwf_complex
    in_fftw [[N][N]) {
    int k,j,i;

    for (k=0; k<N; k++) {
        tareador_start_task("transpose_xy_planes_loop_k");
        for (j=0; j<N; j++) {
            for (i=0; i<N; i++)
            {
                tmp_fftw[k][i][j][0] = in_fftw[k][j][i][0];
                tmp_fftw[k][i][j][1] = in_fftw[k][j][i][1];
            }
        }
        tareador_end_task("transpose_xy_planes_loop_k");
    }
}

void transpose_zx_planes(fftwf_complex in_fftw [[N][N], fftwf_complex
    tmp_fftw [[N][N]) {
    int k, j, i;

    for (k=0; k<N; k++) {
        tareador_start_task("transpose_zx_planes_loop_k");
        for (j=0; j<N; j++) {
            for (i=0; i<N; i++)
            {
                in_fftw[i][j][k][0] = tmp_fftw[k][j][i][0];
                in_fftw[i][j][k][1] = tmp_fftw[k][j][i][1];
            }
        }
        tareador_end_task("transpose_zx_planes_loop_k");
    }
}

int main(){
    ...
    tareador_start_task("init_complex_grid");
    init_complex_grid(in_fftw);
    tareador_end_task("init_complex_grid");

    STOP_COUNT_TIME("Init Complex Grid FFT3D");

    START_COUNT_TIME;
```

```

ffts1_planes (p1d, in_fftw );
transpose_xy_planes(tmp_fftw, in_fftw );
ffts1_planes (p1d, tmp_fftw);
transpose_zx_planes(in_fftw , tmp_fftw);
ffts1_planes (p1d, in_fftw );
transpose_zx_planes(tmp_fftw, in_fftw );
transpose_xy_planes(in_fftw , tmp_fftw);
...
}

```

Like in the previous cases, we can now see in the dependency graph our results and the level of granularity we are getting.

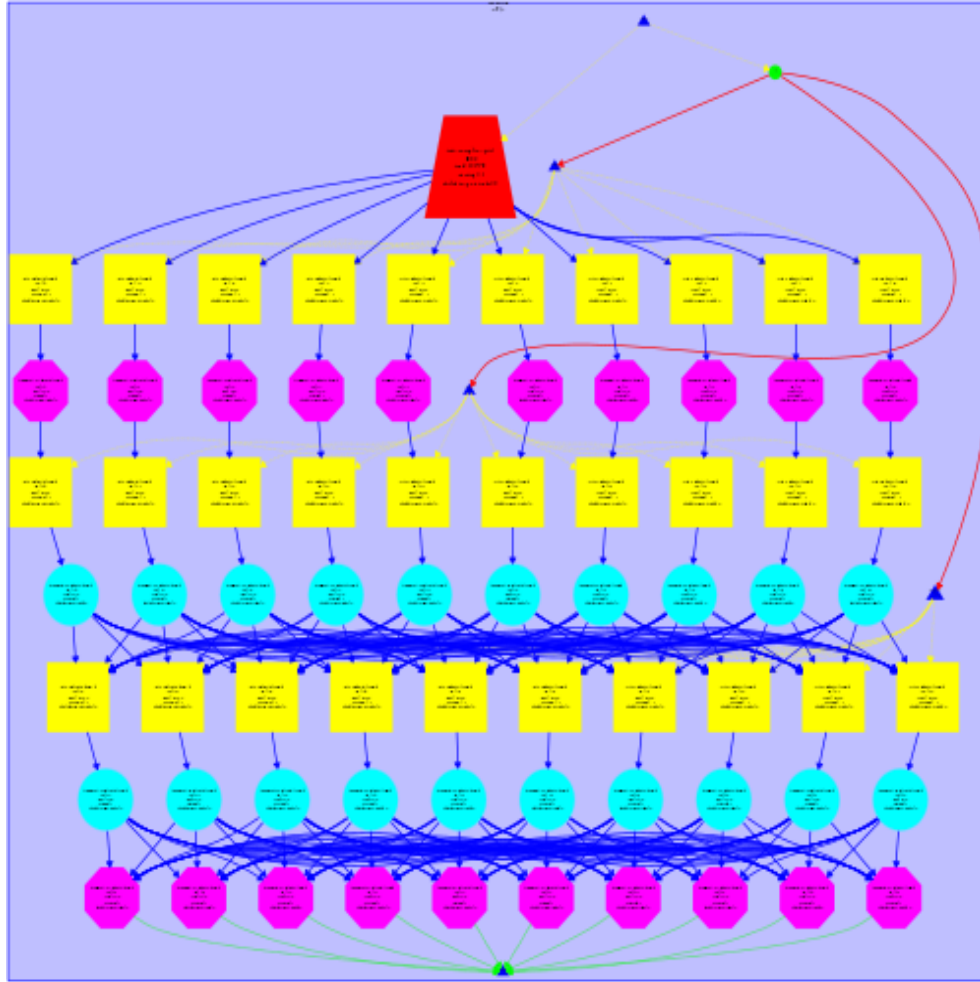


Figure 14: Dependency graph for the third version.

3.2.4 Version 4

The forth version, starting from the third one, consists in replacing the definition of tasks associated to function invocations `init_complex_grid` with fine-grained tasks inside the corresponding body functions and associated to individual iterations of the `k` loop, similarly it was made in the previous version.

```
void init_complex_grid(fftwf_complex in_fftw [] [N][N]) {
    int k,j,i;

    for (k = 0; k < N; k++) {
        tareador_start_task("transpose.init_complex_grid.loop_k");
```

```

    for (j = 0; j < N; j++) {
        for (i = 0; i < N; i++)
        {
            in_fftw[k][j][i][0] = (float) (sin(M_PI*((float)i)/64.0)+sin(M_PI
                *((float)i)/32.0)+sin(M_PI*((float)i/16.0)));
            in_fftw[k][j][i][1] = 0;
        }
    }
    #if TEST
        out_fftw[k][j][i][0] = in_fftw[k][j][i][0];
        out_fftw[k][j][i][1] = in_fftw[k][j][i][1];
    #endif
}
}

tareador_end_task("transpose_init_complex_grid_loop_k");
}
}
int main(){
...

    init_complex_grid(in_fftw);
    STOP_COUNT_TIME("Init Complex Grid FFT3D");

    START_COUNT_TIME;

    ffts1_planes(p1d, in_fftw);
    transpose_xy_planes(tmp_fftw, in_fftw);
    ffts1_planes(p1d, tmp_fftw);
    transpose_zx_planes(in_fftw, tmp_fftw);
    ffts1_planes(p1d, in_fftw);
    transpose_zx_planes(tmp_fftw, in_fftw);
    transpose_xy_planes(in_fftw, tmp_fftw);
    ...
}

```

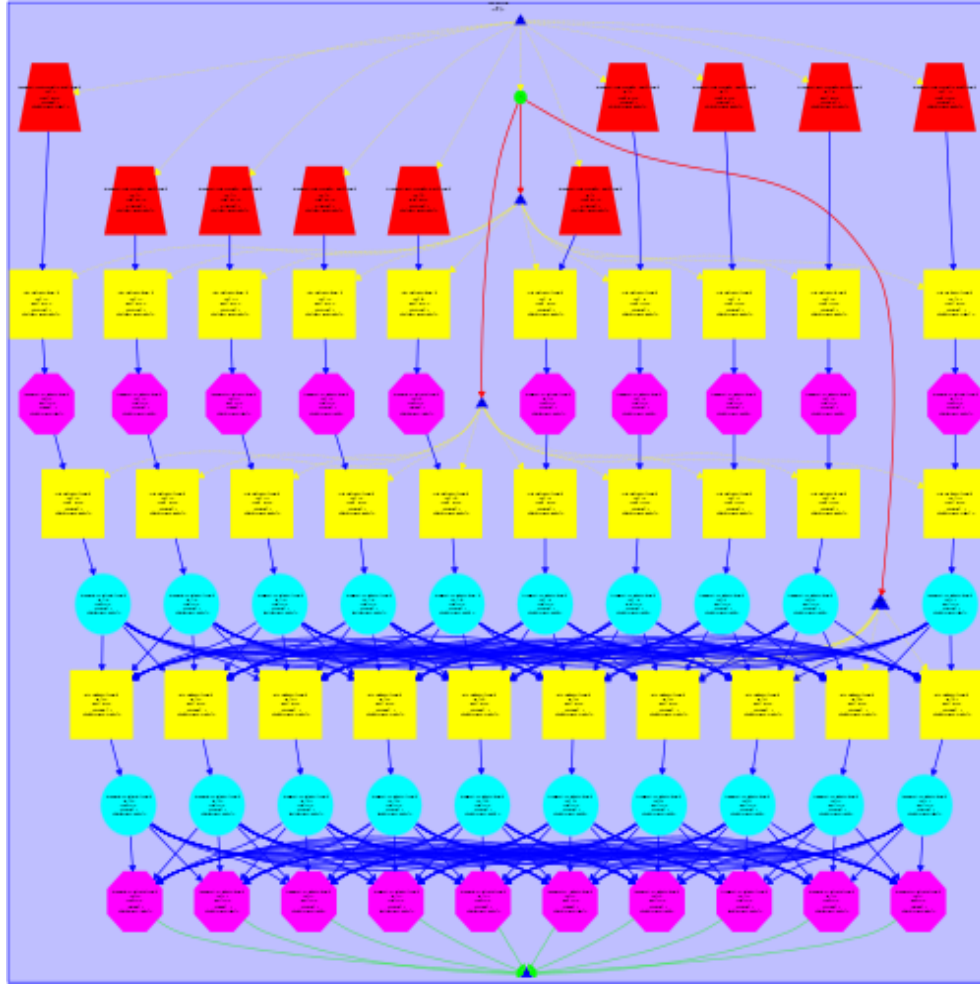


Figure 15: Dependency graph for the forth version.

3.2.5 Version 5

This is the final version; in this version we will explore even more finer-grained tasks. In order to continue this task we observed the forth figure given by *Tareador* which corresponds to the forth version of the code.

As we can see in the Figure ?? the task granularity that has less granularity is the *ffts1_planes_loop_k* with 10305 instructions. So we deepen in the code of the corresponding function and we created tasks in a loop deeper than our first approach.

```
void ffts1_planes( fftwf_plan p1d, fftwf_complex in_fftw [[ N][N]] {
    int k,j;
```



```

for (k=0; k<N; k++) {
  for (j=0; j<N; j++) {
    tareador_start_task(" ffts1_planes_loop_j ");

    fftwf_execute_dft ( p1d, (fftwf_complex *)in_fftw [k][j ][0], (
      fftwf_complex *)in_fftw [k][j ][0]) ;
    tareador_end_task(" ffts1_planes_loop_j ");
  }
}

```

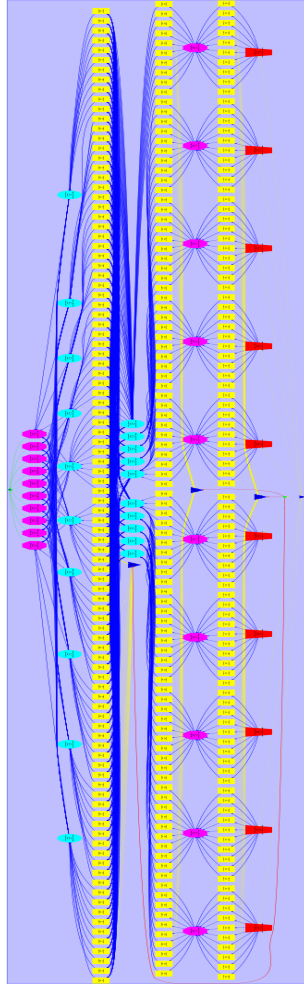


Figure 16: Dependency graph for the final version.

3.2.6 Comparison between version 4 and version 5

Time comparison (in ns) between version 4 and version 5

Number of processors

	1	2	4	8	16	32
v4	639.780.001	320.310.001	165.389.001	91.496.001	64.018.001	64.018.001
v5	639.780.001	321.493.001	172.584.001	99.126.001	53.554.001	44.356.001

Time comparison between v4 & v5

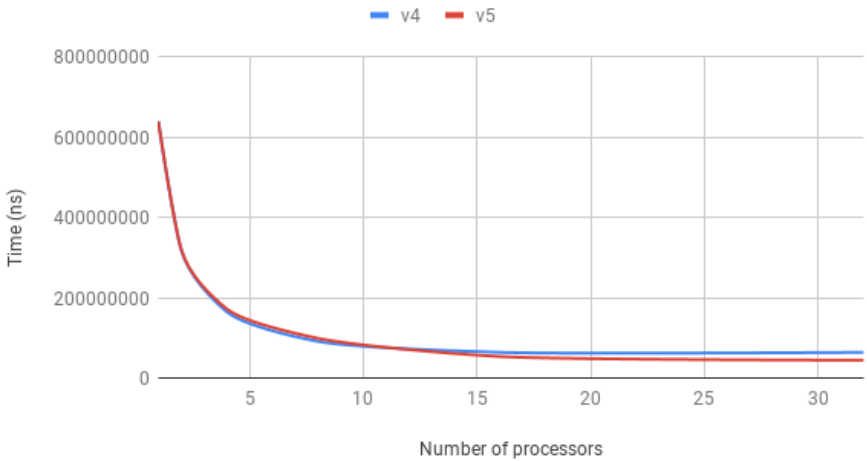


Figure 17: Time comparison between v4 and v5.

Speedup between version 4 and version 5

Number of processors						
	1	2	4	8	16	32
Speedup	1	0.99632029	0.95831015	0.92302725	1.1953915	1.4432771

Speedup between v4 and v5

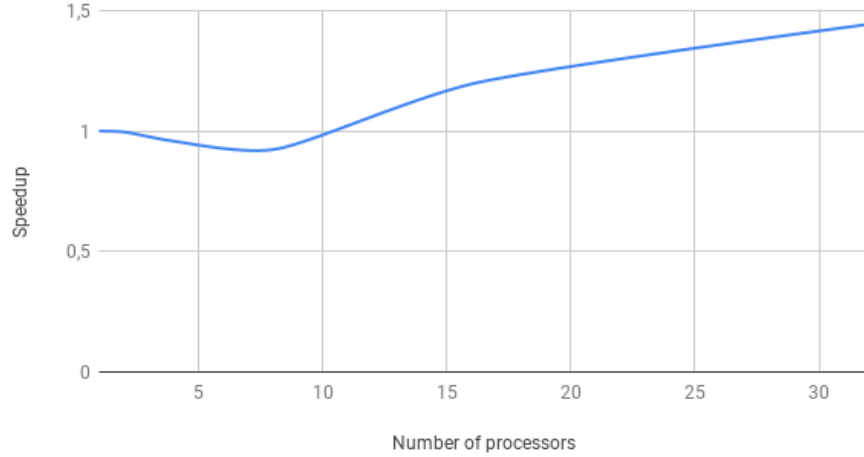


Figure 18: Speedup between v4 and v5.

From this two plots we can observe that there's a moment when it does not matter how many processors you use, you cannot improve the execution time we can get. In the case of the fourth version we can see it clearly comparing the time we get with 16 processors and the time we get with 32.

Moreover, there is still another interesting point we can still analyse. Even though the fifth version had more finer-grained tasks (check Figure 16 when we use less than 16 processors v4 gets a better time execution. That could be justified with the time is focused for making the finer-grained tasks work together because of the overhead.

3.2.7 Summary

To sum up, we show a table with all the data obtained with our experiments. We can see that in all the versions excepting for the first ones, we increase the parallelism.

Version	T_1	T_∞	Parallelism
seq	639,780,001	639,707,001	1.00011411474
v1	639,780,001	639,707,001	1.00011411474
v2	639,780,001	361,190,001	1.77131149597
v3	639,780,001	154,354,001	4.14488770524
v4	639,780,001	64,018,001	9.99375161683
v5	639,780,001	38,224,001	16.7376513254

4 Understanding the parallel execution

In this final section we have used several *Paraver* configurations to be able to understand better the parallel executions. For example there is the Statistics Configuration File, see Figures 20 and 19, where we can see the % of each part of the execution time.

	Running	Synchronization	Scheduling and Fork/Join	I/O	Others
THREAD 1.1.1	99.93 %	0.05 %	0.01 %	0.01 %	0.00 %
Total	99.93 %	0.05 %	0.01 %	0.01 %	0.00 %
Average	99.93 %	0.05 %	0.01 %	0.01 %	0.00 %
Maximum	99.93 %	0.05 %	0.01 %	0.01 %	0.00 %
Minimum	99.93 %	0.05 %	0.01 %	0.01 %	0.00 %
StDev	0 %	0 %	0 %	0 %	0 %
Avg/Max	1	1	1	1	1

Figure 19: Statistics from the original 3dfftomp version with 1 processor. State_profile configuration was used.

	Running	Not created	Synchronization	Scheduling and Fork/Join	I/O	Others
THREAD 1.1.1	95.10 %	-	4.75 %	0.12 %	0.02 %	0.00 %
THREAD 1.1.2	39.33 %	58.86 %	1.78 %	-	0.02 %	-
THREAD 1.1.3	39.17 %	58.86 %	1.95 %	-	0.02 %	-
THREAD 1.1.4	38.91 %	58.86 %	2.21 %	-	0.02 %	-
THREAD 1.1.5	36.92 %	58.86 %	4.20 %	-	0.02 %	-
THREAD 1.1.6	36.49 %	58.86 %	4.63 %	-	0.02 %	-
THREAD 1.1.7	40.06 %	58.87 %	1.05 %	-	0.02 %	-
THREAD 1.1.8	36.57 %	58.87 %	4.54 %	-	0.02 %	-
Total	362.56 %	412.05 %	25.12 %	0.12 %	0.14 %	0.00 %
Average	45.32 %	58.86 %	3.14 %	0.12 %	0.02 %	0.00 %
Maximum	95.10 %	58.87 %	4.75 %	0.12 %	0.02 %	0.00 %
Minimum	36.49 %	58.86 %	1.05 %	0.12 %	0.02 %	0.00 %
StDev	18.86 %	0.00 %	1.43 %	0 %	0.00 %	0 %
Avg/Max	0.48	1.00	0.66	1	0.89	1

Figure 20: Statistics from the original 3dfftomp version with 8 processors. State_profile configuration was used.

FER GRAFICA A PARTIR D'AQUEST VALORS; AIXÒ ES LA PRIMERA VERSIÓ

Temps paral·lelitzat en un processador(per parts) 343,032.74 us 330,705.06 us 105,560.45 us 146,440.56 us 199,973.45 us 401,774.42 us Total paralelitzat 1.527.486,68 us Temps total un processador 2894642449 ns

Temps paral·lelitzat en el 8 processadors (per parts) 1,284,479.03 us 1,145,823.44 us 709,704.96 us 496,762.32 us 321,938.67 us 579,582.94 us Total paralelitzat 4538291,36 us Temps total 1.400.960.654 ns

	[105,560..106,807]	[145,456..146,703]	[199,065..200,312]	[200,312..201,559]	[329,972..331,218]	[342,439..343,686]
THREAD 1.1.1	105,560.45 us	146,440.56 us	199,973.45 us	401,774.42 us	330,705.06 us	343,032.74 us
Total	105,560.45 us	146,440.56 us	199,973.45 us	401,774.42 us	330,705.06 us	343,032.74 us
Average	105,560.45 us	146,440.56 us	199,973.45 us	401,774.42 us	330,705.06 us	343,032.74 us
Maximum	105,560.45 us	146,440.56 us	199,973.45 us	401,774.42 us	330,705.06 us	343,032.74 us
Minimum	105,560.45 us	146,440.56 us	199,973.45 us	401,774.42 us	330,705.06 us	343,032.74 us
StDev	0 us	0 us	0 us	0 us	0 us	0 us
Avg/Max	1	1	1	1	1	1

Figure 21: blablabla

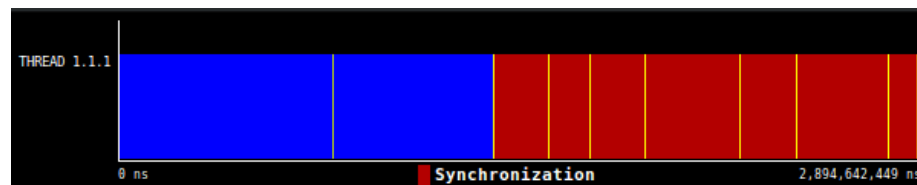


Figure 22: blablabla

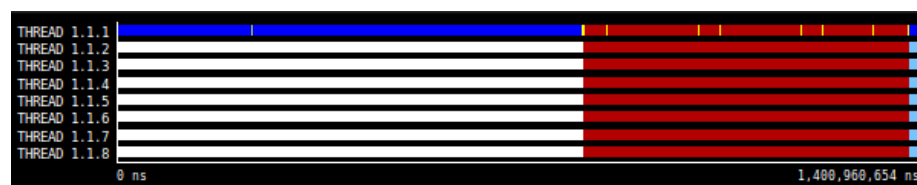


Figure 23: blablabla

	[36,206.3..36,859.2]	[40,123.5..40,776.3]	[61,667.9..62,320.7]	[88,435.2..89,088.1]	[142,623..143,276]	[160,250..160,903]
THREAD 1.1.1	72,447.61 us	40,329.82 us	62,095.12 us	88,713.71 us	143,228.32 us	160,560.12 us
THREAD 1.1.2	72,447.99 us	40,235.57 us	62,096.42 us	88,712.71 us	143,227.68 us	160,560.16 us
THREAD 1.1.3	72,448.37 us	40,235.61 us	62,095.04 us	88,712.80 us	143,227.55 us	160,559.33 us
THREAD 1.1.4	72,448.57 us	40,235.62 us	62,094.92 us	88,712.69 us	143,227.62 us	160,559.26 us
THREAD 1.1.5	72,447.44 us	40,251.83 us	62,095.08 us	88,713.86 us	143,228.28 us	160,559.87 us
THREAD 1.1.6	72,447.32 us	40,246.31 us	62,095.31 us	88,713.38 us	143,228.23 us	160,560.81 us
THREAD 1.1.7	72,448.24 us	40,216.37 us	62,095.34 us	88,712.57 us	143,227.58 us	160,559.38 us
THREAD 1.1.8	72,447.40 us	40,187.54 us	62,095.08 us	88,713.25 us	143,228.17 us	160,560.09 us
Total	579,582.94 us	321,938.67 us	496,762.32 us	709,704.96 us	1,145,823.44 us	1,284,479.03 us
Average	72,447.87 us	40,242.33 us	62,095.29 us	88,713.12 us	143,227.93 us	160,559.88 us
Maximum	72,448.57 us	40,329.82 us	62,096.42 us	88,713.86 us	143,228.32 us	160,560.81 us
Minimum	72,447.32 us	40,187.54 us	62,094.92 us	88,712.57 us	143,227.55 us	160,559.26 us
StDev	0.46 us	38.03 us	0.45 us	0.47 us	0.33 us	0.50 us
Avg/Max	1.00	1.00	1.00	1.00	1.00	1.00

Figure 24: blablalabla

The results obtained are in the following table.

3dfft: Tpar = 1523 ms Tseq = 1372 ms

3dfft_improved: Tpar = 2087 ms Tseq = 361 ms

3dfft_megaImproved: Tpar = 2258 ms Tseq = 500 ms

Version	ϕ	S_∞	T_1	T_8	S_8
initial version in <i>3dfft_omp.c</i>	0.53	2.12	2895 ms	1400 ms	2.07
new version with improved ϕ	0.85	6.67	2448 ms	960 ms	2.55
final version with reduced parallelisation overhead	0.82	5.56	2758 ms	1015 ms	2.72

$$\phi = T_{par} / (T_{seq} + T_{par})$$

$$S_\infty = 1 / (1 - \phi)$$