In a standard linear model, the test statistics F = (RSS_-RSS,) (df, -df) RSS_-RSS,) (df, -df) RSS_-RSS,) (df, -df) RSS_-RSS,) (df, -df) RSS_-RSS,) (df, -df) (also equivalent to comparing fitted values) (who A fields out the? teams of intood.) A DOMAN See Schet pp. 96-97. Responsible Police Poli The second form is convenient because it doesn't regive relitting the model. the motivation is that $A\hat{\theta}$ has various $(A\hat{x}^Tx)^T\hat{x}^Ty) = \tau^TA(x^Tx)^T\hat{x}^TX(x^Tx)^TA^T$ $= \sigma^TA(x^Tx)^TA^T$ For policies, A & has voraice of A BI BT B BIT AT So the test statistic is 1 2 × × 4 5. [A BI. (8TB) 81 TAT] . A X Somice BTB may not be of full work the mating being most be of this expersion may not be of full work. An approximation may therefore be seggived.

No: A & = 2.

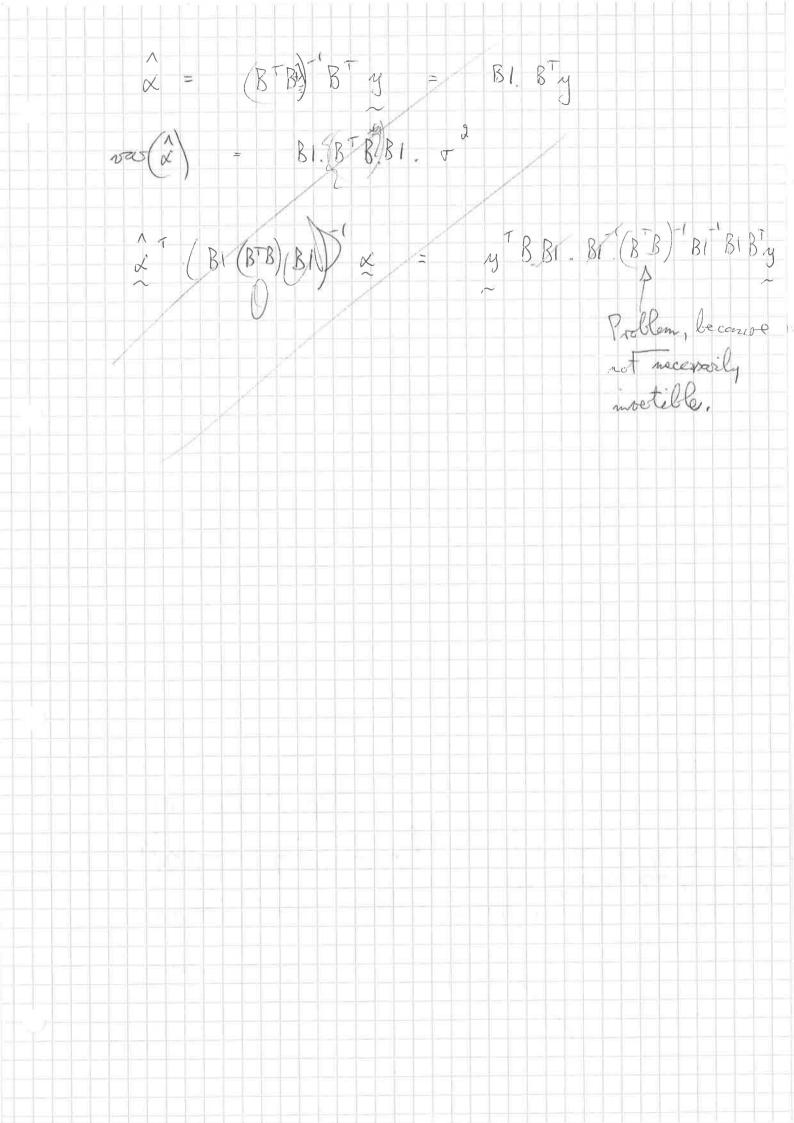
Numerator: y BB, AT A. A B, BTy

£ = B, B T y

A x = A B, B y = A B, B T B x + A B, B T B * E

The terms involved in AX are all 0, or the fenalty term winde 8, should be ormall for the term. For the fact of B1.858 fulled out by A, we should therefre expect 38, to De close to 8TB.

The confount (+) should thought be close to 0. We might than reasonably exfect that to behave like A B, B & E.

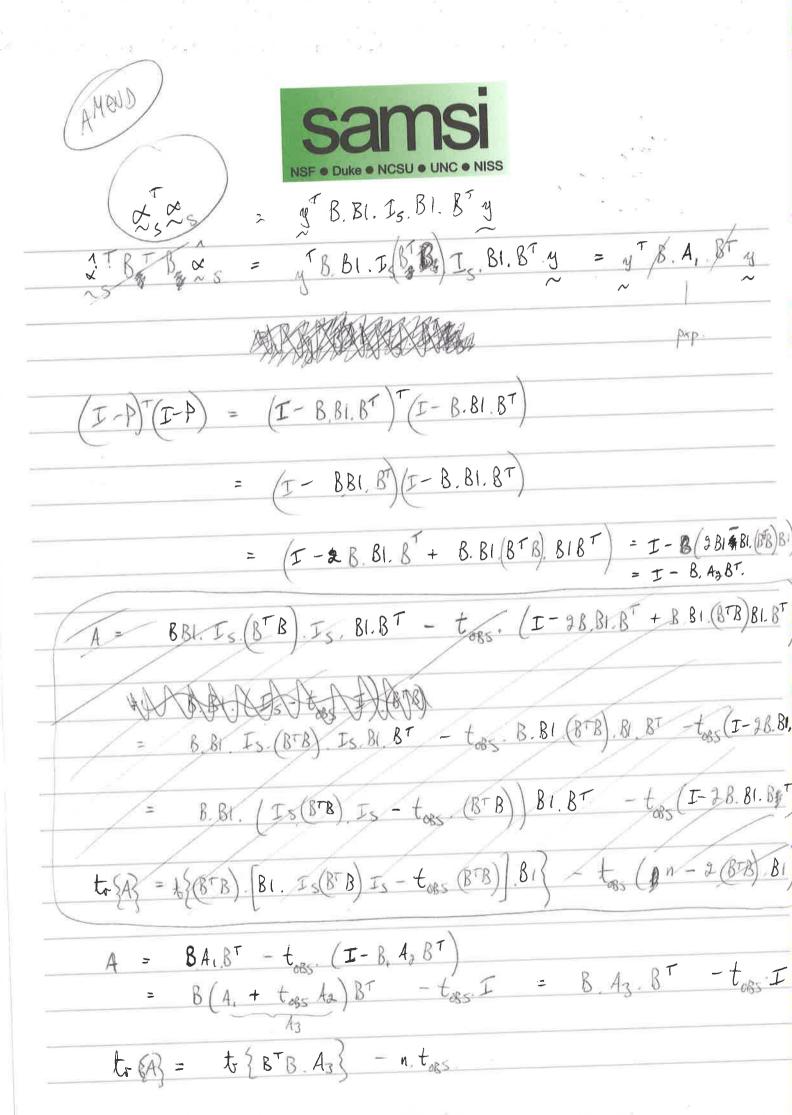


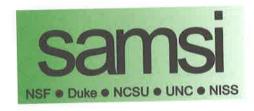
Js: MxP

benominator is (I-BA2 BT).

$$A = BA, B^{T} - t_{abs} \cdot (I - BA, B^{T})$$

Results derived entier then follows





$$A^{3} = DA^{3}B^{T} - t_{obs} I) (B A_{3}B^{T} - t_{obs} I)$$

$$= B.A_{3}B^{T}B A_{3}B^{T} - 2t_{obs} B A_{3}B^{T} + t_{obs}^{2} I$$

$$= B.A_{3}(B^{T}B) A_{3}B^{T} - 2t_{obs} B A_{3}B^{T} + t_{obs}^{2} I$$

$$= B.A_{4}B^{T} + t_{obs}^{2} I$$

$$= B.A_{4}B^{T} + t_{obs}^{2} I$$

$$= B.A_{4}(B^{T}B) A_{3}B^{T} - t_{obs}^{2} I$$

$$= B.A_{4}(B^{T}B) A_{3}B^{T} + t_{obs}^{2} B A_{3}B^{T} - t_{obs} B A_{4}B^{T} - t_{obs}^{2} I$$

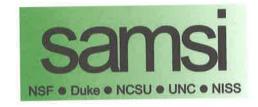
$$= B.A_{4}(B^{T}B) A_{3}B^{T} + t_{obs}^{2} A_{3}B^{T} - t_{obs} B A_{4}B^{T} - t_{obs}^{3} I$$

$$= B.A_{4}(B^{T}B) A_{3} + t_{obs}^{2} A_{3}B^{T} - t_{obs} A_{4}B^{T} - t_{obs}^{3} I$$

$$= B.A_{4}(B^{T}B) A_{3} + t_{obs}^{2} A_{3}B^{T} - t_{obs}^{3} A_{4}B^{T} - t_{obs}^{3} I$$

$$= B.A_{5}(B^{T}B) A_{5} - t_{obs}^{3} I$$

Pert this into an anova son fraction



$$t_{A\Sigma} = t_{A\Sigma} = t_{AS} = t$$

etc.

Can we efficiently calculate BTZB?

INFORENCE FOR P-SPLINE ASSITIVE MOBERS

TEST OF NO EFFECT

Porable test statistics are

(RSSo- RSS,) / df, -dfo)

RSS, (n-df,)

AT. To B mondored (df-dfo)

T. (mi, -mio) 2 / df, -dfo)

ESS, (n-df,)

There statistics are all equivalent for linear models (scools theorem 4.1) may not be for additive models.

In other words, RSSO-RSS, can be refresented as $\hat{R}^{T} \cdot \hat{R}^{D}_{B} \hat{R}^{D}_{A}$ of as $\sum_{i=1}^{n} (\hat{M}_{i} - \hat{M}_{i})^{2}$.

The $\hat{R}^{T} \cdot \hat{R}^{D}_{B} \hat{R}^{D}_{A}$ form is attentive because of its lover dimensionalty.

$$E = \begin{cases} e & e \\ e & n \end{cases}$$

$$E^{T} = \begin{cases} e \\ e & n \end{cases}$$

$$E^{T} = \begin{cases} e \\ e & n \end{cases}$$

$$E^{T} = \begin{cases} e \\ e & n \end{cases}$$

$$\tilde{\Sigma} = \varphi \wedge \varphi^{T}$$

$$\tilde{\Sigma}' = \varphi \wedge \varphi^{T}$$

$$\tilde{\Sigma}'' = \varphi \wedge \varphi^{T} \varphi \wedge \varphi^{T}$$

$$= \varphi \wedge \varphi^{T}$$

 $B(A) = m(x) + \varepsilon;$ $\hat{M} = B(BTB + \lambda DTD)^{-1}BTy$ $\mathcal{L}(\hat{m}) = B(BTB + \lambda DTB)^{-1}B^{\dagger}. m = B(BTB[I + \lambda(BTB)^{-1}(DTD)]^{-1}B^{\dagger}. m$ = B [I + 2(BTB)-(DTA)]-(BTB)-(BTM) The has a referentation as & the this bedies to and mile som of and there are two adjustments here. One is the representation of m by Bx. of m by Bx. the other is the adjustment in the (I tem. This is a form which has an alterative analytic expression. Chack this out. Of the give anymatotic enternies. Kausemann et al. TESTING Testing for m(u) = 0. This is a standard segressian model, so compare RSS'O.

RSS = y (I - B(BTB + 4 DTD) BT) (I - B(BTB + 4 DTD) BT) y y T[I - 2 B(BTB+4 DTD) - BT + B(B) - BTB() - BTJ y RSS0 = yt [- . - Bo] y (RSS0 - RSS1) = yT (I-Po)(I-Po) - (I-Pi)(I-Pi) y $= y^{T} \left[P_{0}^{2} - g P_{0} - P_{1}^{2} - g P_{1} \right] y = g^{T} A y$ The quadratic form argument is based on to {Ai}. to [A] & is trace of a pxp matri. to 40) moroloes PoP, of P, Po. Pop = Bo (Bot 8+1 bob) Bot B. (B, TB, +1 D, TO) BT This has lower dimension, so ox. who hock at f

P-GAMs

For each component, compute the model which is restricted to be levier using the adjustment formula in Selver. (Check this remains valid in the fenalty wold: "fined" argument too.)

$$\begin{split} \left\| B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right) \right\| &= B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \stackrel{\mathsf{T}}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} + \stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right) + \left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right) + \left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right) + \left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right) + \left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right) + \left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right) + \left(\stackrel{\circ}{\partial}_{\mathsf{N}} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right) \\ &= \left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial}_{\mathsf{N}} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} \right)^{\mathsf{T}} B^{\mathsf{T}} B\left(\stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} - \stackrel{\circ}{\partial} -$$

$$\xi' \xi = (y - x) (y - x)$$

$$= (y - x) (y - x)$$

P-GAMS

Immoration:

very small boundaridth. This can go in the denounates of F-statistics.

Is there a natural largest of (smallest 1) which can be used? That anther bounds to of min (M, nboses).

More exactly, the infer limit is the imber of boxis frections which have non-zero evaluations of frections has evaluate so which we all per than this affer limit cannot be achieved.

Anova by gradatic forms.

Enfloration of profesties by matrix mierria, when all bains functions have at least one na-get avaluation.

Graphics for no-effect of linear terms, for 1-d + 2-d covariates.

Asimation for 3-d covariates.

Addition of covalited evers? Similtaneous est?

An alterative representation, which is equivalent because this is a regression model, is something like lass of a corresponding to the component of interest. $= \left| \left(81 \right), \left(8^{T} \right) \right|_{S} = \left(81 \right)_{S} \, 8^{T} y$ $\hat{\lambda}_{S} = \frac{1}{8} \left(8^{T}B + 4 \delta^{T} \delta \right)^{-1} 8^{T} y$ || 25 || = y+B (BI & BIs) BT my, where BIs has releasent sons get to o. There is pobably a meater decomposition. (bone this before?) toole of Seale for details. How does the bies argument work when a component is linear or o? SM code: infrastrictere of regression should be usable a bord linear specific code is limited. I separate code to the code of the code of parties out platting, Graffies for additive model conforats, including onfaces, could be added to the paper. Possibly better in the lang-run to do everything in terms of a general run for for addition models? Always fort in an 'x' column for each composant to that the additional terms can be assessed for man-linearity? There may be an easier way to do this by assessing the 2's terms afformately: 12. !! Is there a case for moving a come large of denominator? (2=0). - things dane better faster (3 coverates flow) 1 3 covariates - modertand - do things in special ways (amora). - GLAM - link with som infraction tope; oftening graffing, - formla 41 - nottoods for flot, summony, amount, predict
- on over away but good.
- flot of 3-d conversates.

Problem at x1,x2 section of additive models)
in sm-pam-test.r.

sm. hgam

B

Choose I though of woning unwovate tems

(4 efficient was of BB). That's what happens in the word gan.

Fitted fram has it a projection matrix decomposition just by taking the afforbrate columns of BX. Calculate of o from this.

Extende ever of A .

Put all this in a runnary f.

anova of which looks at highest oder tems,
Integrate north on tests.

Allow linear tems.

Horology

DEGREES OF FREEDOM

All

B
$$\beta^{-1} = B (B^{T}B^{+}P)^{-1}B^{T}y$$

If $= t \{B^{T}B B_{i}\}$

Composed

 $\hat{A}_{i} = B \hat{\alpha}_{i}$
 $= [aB_{i}]^{\frac{1}{2}}$, where B_{i} has a_{i} in colp. It associates with coup. i.

 $= [aB_{i}]^{\frac{1}{2}}(B^{T}B^{+}P)^{-1}B^{T}y$
 $P_{i} = [aB_{i}]^{\frac{1}{2}}(B^{T}B^{+}P)^{-1}B^{T}y$
 $P_{i} = [aB_{i}]^{\frac{1}{2}}(B^{T}B^{+}P)^{-1}B^{T}y$
 $= t \{B_{i}]^{\frac{1}{2}}[aB_{i}]^{\frac{1}{2}}[aB_{i}]^{\frac{1}{2}}$
 $= t \{B_{i}]^{\frac{1}{2}}[aB_{i}]^{\frac{1}{2}}$
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