Solution exercice 1

The variance, selection coefficient, and allele frequency change of A are, respectively,

$$v = p(1-p) \quad s = w_{\mathrm{A}} - w_{\mathrm{B}} \quad \Delta p = \frac{p(1-p)s}{1+ps}$$

If $p = 1/1000$ and given that $w_{\mathrm{A}} = 1.2$ and $w_{\mathrm{B}} = 1$, we get

$$v = 0.000999 \quad s = 0.2 \quad \Delta p = 0.00019976$$

If $p = 0.5$, we get

$$v = 0.25 \quad s = 0.2 \quad \Delta p = 0.0454545$$

Solution exercice 2

The fitnesses of A and B, and the selection coefficient on A are, respectively

$$w_{\mathrm{A}}(n) = \frac{f_{\mathrm{A}}}{1+\gamma_{\mathrm{A}}\, n} \qquad w_{\mathrm{B}}(n) = \frac{f_{\mathrm{B}}}{1+\gamma_{\mathrm{B}}\, n} \qquad s(n) = w_{\mathrm{A}}(n) - w_{\mathrm{B}}(n)$$

The equilibrium population size in monomorphic populations of each type is, respectively

$$n_{\mathrm{A}}^{*} = \frac{f_{\mathrm{A}}-1}{\gamma_{\mathrm{A}}} \quad \text{and} \quad n_{\mathrm{B}}^{*} = \frac{f_{\mathrm{B}}-1}{\gamma_{\mathrm{B}}}$$

The selection coefficients at the different population sizes are

$$s(10000) = 0.083 \qquad s(10250) = 0.073 \qquad s(10750) = 0.056$$

Allele A is favored at all population sizes and thus goes to fixation. Hence, the long term population size is n_{A}^{*}.

Solution exercice 3

The fitnesses of A and B for the coordination game are

$$w_{\mathrm{A}} = 1 + p\,(H + B) \qquad w_{\mathrm{B}} = 1 + pH + (1 - p)H = 1 + H$$

The selection coefficient is

$$s(p) = w_{\mathrm{A}} - w_{\mathrm{B}} = pB - (1 - p)H$$

Solution exercice 3

From the selection coefficient when $H = 1$ and $B = 1$ we have

$$s(p) = pB - (1-p)H = 2p - 1,$$

which is downward sloping in p, we see that there is only one interior equilibrium (satisfying $s(p^*) = 0$)

$$p = \frac{H}{B+H} = \frac{1}{2}$$

- Near $p \approx 0$, the selection coefficient is negative, which implies $\Delta p < 0$ (hunting hare is favored).
- Near $p \approx 1$, the selection coefficient is positive, which implies $\Delta p > 0$ (Hunting stag is favored).

Hence, the interior point $p^* = 1/2$ is unstable and the two boundary points are stable.