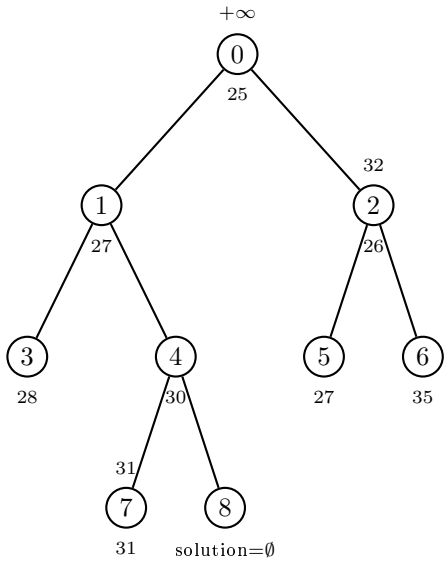
Exercise Set 7

Problem 1

We consider below the enumeration tree of a minimization problem.



a) Give the best upper and lower bound of the optimal value z.

b) Which nodes can be discarded and which ones still need to be explored?

Problem 2

Using the B&B algorithm, solve the following knapsack problem where we can select several times items 1 and 2.

$$\begin{array}{lll} \max & 10x_1 + 12x_2 + 7x_3 + \frac{3}{2}x_4 & \\ & & \\ \text{s.t.} & 4x_1 + 5x_2 + 3x_3 + x_4 & \leq \quad 10 \\ & x_1,x_2 & \in \quad \mathbb{Z}_+ \\ & x_3,x_4 & \in \quad \{0,1\} \end{array}$$

Hint: It is quite obvious that $x_1,x_2 \in \{0,1,2\}$. When branching on x_i, then consider the cases $x_i = 0$, $x_i = 1$, $x_i = 2$ for $i = 1,2$.

Problem 3

A person has 10'000 francs that he would like to invest over a 1-year horizon. Current purchase prices and sales prices in 1 year can be found in the table below:

Item	Purchase price	Sales price (in 1 year)	Return %
1	4 000	8 000	100 %
2	5 000	6 500	30 %
3	2 500	3 500	40 %
4	2 000	2 500	25 %
5	3 500	4 500	$\sim$ 29 %

The investor can buy only one copy of each item. To solve this problem, we introduce the binary variables x_i $(1 \leq i \leq 5)$:

$$x_i = \begin{cases} 1 & \text{investor purchases item } i \\ 0 & \text{otherwise} \end{cases}$$

and we build the enumeration tree as follows: within each node of the tree, we put the remaining amount of money to purchase other items. Beside each node, we put the pair defined by

1) the realized profit after the sales of the purchased items;
2) an upper bound for the potential profit for the items that we could still purchase.

Computation of an upper bound. If we haven't yet decided about the purchase of the last four items and if we can still invest x francs, then an upper bound is given by the maximal return for the items that are still available multiplied by the amount to invest. In our case, $\frac{40}{100}x$ francs.

Branching. We start branching with x_1, then x_2, x_3, x_4, and x_5.

Selection of the active node. We select the node having the best potential. Concretely, we sum each figure in the pair (profit, upper bound) at each node and we explore the node with the highest sum.

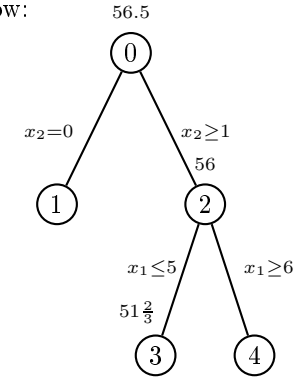
Using the B&B algorithm and based on the approach described above, determine which items should the investor purchase in order to maximize his profit. Give also his profit.

Problem 4

We consider the following problem:

$$\begin{array}{lrcl} \max & 9x_1 + 5x_2 & & \\ \text{s.t.} & 4x_1 + 9x_2 & \leq & 35 \\ & x_1 & \leq & 6 \\ & x_1 - 3x_2 & \geq & 1 \\ & 3x_1 + 2x_2 & \leq & 19 \\ & x_1, x_2 & \in & \mathbb{Z}_+ \end{array}$$

We first branch on x_2 ($x_2 = 0$ and $x_2 \geq 1$) and then on x_1 ($x_1 \leq 5$ and $x_1 \geq 6$). The optimal values of some relaxation problems are given below:



Solve this problem by completing the enumeration tree given above. What is the optimal solution? What is its value?