Exercises: Natural selection

(1) Consider an evolving population in which there are two alleles, A and B, that are subject to natural selection. Suppose the population consists of 10'000 individuals, out of which 10 carry allele A and all other individuals carry allele B. Assume that the fitness of an A individual is 1.2 and the fitness of a B individual is 1. For this model:

(i) Calculate the variance in allele frequency in the population.

(ii) Calculate the selection coefficient on allele A.

(iii) Calculate the change in frequency of allele A over one demographic time period.

(iv) Repeat steps (i)–(iii), assuming that the number of A individuals is now 5'000.

(v) Considering selection to be constant in this population, what will be the equilibrium frequency of A in the long run?

(2) Consider an evolving population in which there are two alleles, A and B, subject to density-dependent natural selection. The fitness of individuals is given by a Beverton-Holt model, with parameters $f_{\mathrm{A}} = 6.5$, $\gamma_{\mathrm{A}} = 0.0005$, $f_{\mathrm{B}} = 2$, $\gamma_{\mathrm{B}} = 0.0001$. For this model

(i) Calculate the equilibrium population size when the population is monomorphic for the B allele.

(ii) What is the selection coefficient on the A allele if the population size is $n =$10'000? $n =$10'250? $n =$10'750?

(iii) What will be the long run equilibrium frequency of the A allele and what will be the corresponding equilibrium population size?

(3) Suppose now that we have a population in which there are two alleles, A and B, that are subject to frequency-dependent selection. Each allele is assumed to code for a discrete strategy to be adopted in a pairwise interaction game. The following payoff matrix shows the payoff obtained by a player 1 (who can carry either A or B allele) when matched with a player 2 (who can also carry either A or B allele):

$$\begin{array}{ccc} & & \text{Player 2:} \\ & & \begin{array}{cc} \text{A "Stag"} & \text{B "Hare"} \end{array} \\ \text{Player 1:} & \begin{array}{c} \text{A "Stag"} \\ \text{B "Hare"} \end{array} & \begin{pmatrix} H+B & 0 \\ H & H \end{pmatrix} \end{array} \tag{1}$$

These are the payoffs of player 1 for the so-called *stag hunt* game[1]. It is a coordination game in which there are two possible actions: hunt a stag (strategy Stag) or a hare (strategy Hare). Getting a stag is more valuable in terms of energetic value and thus results in a higher payoff than getting a hare. However, a successful stag hunt requires that the stag-hunting individual is teamed up with another stag-hunter. For this model:

(i) Determine the average fitness of a carrier of each allele under random matching of individuals, given that the frequency of A allele is p.

(ii) Determine the expression for the selection coefficient on allele A.

(iii) Determine the equilibrium point(s) of the allele frequency dynamics.

(iv) Determine if the equilibrium point(s) is(are) stable or unstable when $H = 1$ and $B = 1$.

[1] stag = male deer