Exercise Set 3

Problem 1

For each of the following tableaus, determine the next pivot using the simplex algorithm with Bland's rule.

	x_1	x_2	x_3	x_4	x_5	x_6	z	
[1	4	1	0	3	0	0	5
$T_1 =$	0	-2	2	1	5	0	0	4
-	0	7		0	8	1	0	2
	0	10	2/3 -5	0	3	0	1	23
	L							
	x_1	x_2	x_3	x_4	x_5	x_6	z	
	0	-6	0	$\frac{x_4}{-3/2}$	-1	1	0	2
$T_2 =$	1 0	4	0	-1	1	0	0	23
	0	2	1	-3	-2	0	0	1
	0	0	0	-2	3/2	0	1	12
'							-	
	x_1	x_2	x_3	x_4	x_5	x_6	z	
	4	7	0	0	1	4	0	4
$T_3 =$	2 -2 5	8	0	1	0	3	0	0
	-2	9	1	0	0	2	0	0
	5	-2	0	0	0	-5	1	3
	x_1	x_2	x_3	x_4	x_5	x_6	z	
	5	$\frac{x_2}{0}$	$\frac{x_3}{0}$	$\frac{x_4}{1}$	4	$\frac{x_6}{1}$	0	1
$T_4 =$	5	0	0 1	1 -7	4 5		0	14
$T_4 =$	5 -3 2	0	0 1 0	1 -7 8	4 5 2	1	0 0 0	
$T_4 =$	5	0	0 1	1 -7	4 5	1 0	0	14
$T_4 =$	5 -3 2	0 0 1	0 1 0	1 -7 8	4 5 2	1 0 0	0 0 0	14 8
$T_4 =$	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $	0 0 1 0	$0 \\ 1 \\ 0 \\ 1$ x_3	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $	4 5 2	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} $	0 0 0 1	14 8 -4
	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $	$0 \\ 0 \\ 1 \\ 0$ x_2	0 1 0 1	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{r} x_4 \\ 5 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} $	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$	14 8 -4
$T_4 =$ $T_5 =$	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $	$0 \\ 0 \\ 1 \\ 0$ x_2 x_2 x_2	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} $ $ \begin{array}{c} x_3 \\ 0 \\ 0 \end{array} $	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \\ x_5 \\ 0 \\ 1 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \end{array} $	0 0 0 1 z 0 0	14 8 -4
	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $ $ \begin{array}{c c} x_1 \\ 0 \\ 0 \end{array} $	$0 \\ 0 \\ 1 \\ 0$ x_2 2 -4 5	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} $ $ \begin{array}{c} x_3 \\ 0 \\ 0 \\ 1 \end{array} $	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{r} x_4 \\ 5 \\ 3 \\ 4 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \\ 1 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \\ 5 \end{array} $	0 0 0 1 2 0 0 0	14 8 -4 10 2 3
	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $	$0 \\ 0 \\ 1 \\ 0$ x_2 x_2 x_2	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} $ $ \begin{array}{c} x_3 \\ 0 \\ 0 \end{array} $	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{r} x_4 \\ 5 \\ 3 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \\ x_5 \\ 0 \\ 1 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \end{array} $	0 0 0 1 z 0 0	14 8 -4
	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $ $ \begin{array}{c c} x_1 \\ 0 \\ 0 \end{array} $	$0 \\ 0 \\ 1 \\ 0$ x_2 2 -4 5	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} $ $ \begin{array}{c} x_3 \\ 0 \\ 0 \\ 1 \end{array} $	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{r} x_4 \\ 5 \\ 3 \\ 4 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \\ 1 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \\ 5 \end{array} $	0 0 0 1 2 0 0 0	14 8 -4 10 2 3
	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $ $ \begin{array}{c c} x_1 \\ 1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} $ $ \begin{array}{c} x_2 \\ 2 \\ -4 \\ 5 \\ 4 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} $ $ \begin{array}{c} x_3 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} $	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{r} x_4 \\ 5 \\ 3 \\ 4 \\ 2 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \\ 5 \\ 0 \end{array} $	0 0 0 1 2 0 0 0 0	14 8 -4 10 2 3 20
$T_5 =$	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $ $ \begin{array}{c c} x_1 \\ 0 \\ 0 \\ \end{array} $ $ \begin{array}{c c} x_1 \\ 3 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} $ $ \begin{array}{c} x_2 \\ 2 \\ -4 \\ 5 \\ 4 \end{array} $	$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x_3 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} x_3 \\ 1 \end{bmatrix}$	$ \begin{array}{r} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{r} x_4 \\ 5 \\ 3 \\ 4 \\ 2 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \\ 5 \\ 0 \end{array} $	0 0 0 1 2 0 0 0 0 1	14 8 -4 10 2 3 20
	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $ $ \begin{array}{c c} x_1 \\ 0 \\ 0 \\ \end{array} $ $ \begin{array}{c c} x_1 \\ 3 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} $ $ \begin{array}{c} x_2 \\ 2 \\ -4 \\ 5 \\ 4 \end{array} $ $ \begin{array}{c} x_2 \\ 3 \\ 2 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} $ $ \begin{array}{c} x_3 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{c} x_4 \\ 5 \\ 3 \\ 4 \\ 2 \end{array} $ $ \begin{array}{c} x_4 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_5 \\ 4 \\ 3 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 5 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 0 \\ 1 \end{array} $	0 0 0 1 2 0 0 0 0 1	14 8 -4 10 2 3 20
$T_5 =$	$ \begin{array}{c c} 5 \\ -3 \\ 2 \\ 8 \end{array} $ $ \begin{array}{c c} x_1 \\ 1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} $ $ \begin{array}{c} x_2 \\ 2 \\ -4 \\ 5 \\ 4 \end{array} $	$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x_3 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} x_3 \\ 1 \end{bmatrix}$	$ \begin{array}{c} 1 \\ -7 \\ 8 \\ -3 \end{array} $ $ \begin{array}{c} x_4 \\ 5 \\ 3 \\ 4 \\ 2 \end{array} $ $ \begin{array}{c} x_4 \\ 0 \end{array} $	$ \begin{array}{c} 4 \\ 5 \\ 2 \\ -2 \end{array} $ $ \begin{array}{c} x_5 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} x_6 \\ 3 \\ 6 \\ 5 \\ 0 \end{array} $	0 0 0 1 2 0 0 0 0 1	14 8 -4 10 2 3 20

Problem 2

Solve the following LP with the simplex algorithm:

$$\begin{array}{rclrcl}
\text{Max } z & = & x_1 & + & 4x_2 \\
s.t. & & x_1 & - & x_2 & \leq & 1 \\
& & -3x_1 & + & x_2 & \leq & 0 \\
& & x_1 & , & x_2 & \geq & 0
\end{array}$$

Problem 3

Solve the following LP with the simplex algorithm:

Problem 4

The table below contains possible schedules for drivers of a bus company. The latter would like to determine the schedules at the lowest cost which guarantee that at least one driver is present from 9 am to 5 pm.

Schedule	9 am to	9 am to	11 am to	12 pm to	1 pm to	2 pm to	4 pm to
	11 am	1 pm	$4 \mathrm{\ pm}$	3 pm	$4 \mathrm{pm}$	5 pm	$5~\mathrm{pm}$
Cost	18	30	38	14	22	16	9

Formulate this problem as an **integer** LP