Exercises: kin selection, part II (parental care)

Consider a population in which females produce two offspring. A mother monitors her offspring's needs in terms of resources. Offspring express their needs by signalling and the mother provides a greater amount of care to the offspring with the highest level of signalling (e.g. the "noisiest" offspring). Without loss of generality, we can assume that the level of signalling (e.g. "noise") varies between zero and one. Let us then assume that a focal offspring receives

$$\frac{x}{x+x_{\rm r}}$$

units of care (or units of resources), where x is the level of signalling of the focal offspring and x_r is the level of signalling of its sibling. In turn, such maternal care increases the survival probability of offspring. However, signalling is costly in time and energy, therefore it also decreases the probability of survival when expressed. Overall, we assume that the expected survival probability of a focal offspring with level x of signalling when its sibling expresses level x_r is

$$s_{\mathrm{J}}(x, x_{\mathrm{r}}) = \left(\frac{x}{x + x_{\mathrm{r}}}\right) (1 - x),$$

where (1-x) accounts for the cost of signalling.

We assume that the fitness of an individual is proportional to its survival. So an individual expressing a level x of signalling while interacting with a relative with level x_r of signalling has fitness $w(x, x_r) = ks_J(x, x_r)$ for some constant k. For this model:

- (i) Calculate the direct and indirect fitness effects of signalling (i.e., the marginal changes in fitness).
- (ii) What category of social behaviour signalling belongs to?
- (iii) Find the singular level of signalling (the singular strategy) and determine whether this strategy is convergence stable.
- (iv) How does the singular strategy vary when offspring are: monozygotic twin, full siblings, and unrelated?
- (v) Calculate the survival of an individual in a monomorphic population expressing the convergence stable strategy of signalling.
- (vi) Suppose that the signalling level in the population is y = 0.1. For what range of relatedness values will this level decrease due to selection?