Solutions to Exercise Set 3

Problem 1

With Bland's rule, we get the pivots given below.

	x_1	x_2	x_3	x_4	x_5	x_6	z		ratio	
	1	4	1	0	3	0	0	5	5	
$T_1 =$	0	-2	**2**	1	5	0	0	4	2	$\leftarrow$
	0	7	2/3	0	8	1	0	2	3	
	0	10	-5	0	3	0	1	23		
			$\uparrow$							

Tableau T_2 is feasible and unbounded. The optimization process is stopped and there is no next pivot.

	x_1	x_2	x_3	x_4	x_5	x_6	z		ratio	
	4	7	0	0	1	4	0	4	4/7	
$T_3 =$	2	8	0	1	0	3	0	0	0	
	-2	**9**	1	0	0	2	0	0	0	$\leftarrow$
	5	-2	0	0	0	-5	1	3		
		$\uparrow$								

	x_1	x_2	x_3	x_4	x_5	x_6	z		ratio	
	5	0	0	1	4	1	0	1	1	
$T_4 =$	-3	0	1	-7	5	0	0	14	-	
	2	1	0	**8**	2	0	0	8	1	$\leftarrow$
	8	0	1	-3	-2	0	1	-4		
				$\uparrow$						

Tableau T_5 is optimal.

	x_1	x_2	x_3	x_4	x_5	x_6	z		ratio	
	3	3	1	0	4	0	0	0	0	$\leftarrow$
$T_6 =$	2	2	0	0	3	1	0	1	1/2	
	0	5	0	1	2	0	0	4	-	
	-8	4	0	0	-1	0	1	2		
	$\uparrow$									

Problem 2

$T_0 =$

x_1	x_2	x_3	x_4	z		ratio	
1	-1	1	0	0	1	1	$\leftarrow$
-3	1	0	1	0	0	-	
-1	-4	0	0	1	0		
$\uparrow$							

$T_1 =$

x_1	x_2	x_3	x_4	z	
1	-1	1	0	0	1
0	-2	3	1	0	3
0	-5	1	0	1	1

This problem has no finite optimum.

Problem 3

The initial tableau is given by:

$T_0 =$

x_1	x_2	x_3	x_4	x_5	z	
1	-1	1	0	0	0	2
-1	-2	0	1	0	0	-1
-2	3	0	0	1	0	-6
-2	-3	0	0	0	1	0

This tableau is not feasible, we apply phase I:

$T_0^{\text{aux}} =$

x_0	x_1	x_2	x_3	x_4	x_5	z	z'	
0	1	-1	1	0	0	0	0	2
-1	-1	-2	0	1	0	0	0	-1
-1	-2	3	0	0	1	0	0	-6
0	-2	-3	0	0	0	1	0	0
1	0	0	0	0	0	0	1	0

$T_1^{\text{aux}} =$

x_0	x_1	x_2	x_3	x_4	x_5	z	z'	
0	**1**	-1	1	0	0	0	0	2
0	1	-5	0	1	-1	0	0	5
1	2	-3	0	0	-1	0	0	6
0	-2	-3	0	0	0	1	0	0
0	-2	3	0	0	1	0	1	-6

$T_2^{\text{aux}} =$

x_0	x_1	x_2	x_3	x_4	x_5	z	z'	
0	1	-1	1	0	0	0	0	2
0	0	-4	-1	1	-1	0	0	3
1	0	-1	-2	0	-1	0	0	2
0	0	-5	2	0	0	1	0	4
0	0	1	2	0	1	0	1	-2

Phase I has completed but the optimal value is not null. The initial problem has no feasible solution.

Problem 4

We number the different schedules from 1 to 7 in the order given by the table and we define the following decision variables:

$$x_i = \begin{cases} 1 & \text{if schedule } i \text{ is kept} \\ 0 & \text{otherwise} \end{cases} \qquad i = 1,\ldots,7.$$

The period from 9 am to 5 pm is split into 1-hour periods $[j,j+1], j = 9, \cdots ,16$. Each of these periods must be covered at least by one schedule (at least one driver) meaning that for period $[j,j+1]$, we must have :

$$\sum_i x_i \geq 1,$$

where the sum is taken over all the schedules i that cover the period $[j,j+1]$.

The problem to solve is:

$$\begin{array}{llccccccccccccccc}
\text{Min} & z = & 18x_1 & + & 30x_2 & + & 38x_3 & + & 14x_4 & + & 22x_5 & + & 16x_6 & + & 9x_7 & & \\
\text{s.t.} & & x_1 & + & x_2 & & & & & & & & & & & \geq & 1 \\
& & & & x_2 & + & x_3 & & & & & & & & & \geq & 1 \\
& & & & x_2 & + & x_3 & + & x_4 & & & & & & & \geq & 1 \\
& & & & & & x_3 & + & x_4 & + & x_5 & & & & & \geq & 1 \\
& & & & & & x_3 & + & x_4 & + & x_5 & + & x_6 & & & \geq & 1 \\
& & & & & & x_3 & & & + & x_5 & + & x_6 & & & \geq & 1 \\
& & & & & & & & & & & & x_6 & + & x_7 & \geq & 1 \\
& & x_1 & , & x_2 & , & x_3 & , & x_4 & , & x_5 & , & x_6 & , & x_7 & \in & \{0,1\}
\end{array}$$

Remark: as the constraints for periods [9,10] and [10,11] are the same, we only keep one in the formulation.