Solution exercice 1

The equilibrium frequency (i.e. $\Delta p = 0$) of A under only mutation is

$$p^* = \frac{\mu_\mathrm{B}}{\mu_\mathrm{A} + \mu_\mathrm{B}}$$

with $\mu_\mathrm{A} = 0.01$ and $\mu_\mathrm{B} = 0.02$, the equilibrium frequency of B is

$$(1 - p^*) = \frac{\mu_\mathrm{A}}{\mu_\mathrm{A} + \mu_\mathrm{B}} = 0.333333$$

Solution exercice 2

The change in the frequency of allele A is

$$\Delta p = \Delta p_{\mathrm{s}} + \Delta p_{\mu} = \frac{p(1-p)s}{\underbrace{1-(1-p)s}_{\bar{w}}} + \frac{(1-p)\mu_{\mathrm{B}} w_{\mathrm{B}} - p\mu_{\mathrm{A}} w_{\mathrm{A}}}{\underbrace{1-(1-p)s}_{\bar{w}}}$$

where $s = w_{\mathrm{A}} - w_{\mathrm{B}}$. With $w_{\mathrm{A}} = 1$, $w_{\mathrm{B}} = 0.99$, $\mu_{\mathrm{A}} = 0.01$, $\mu_{\mathrm{B}} = 0.02$, we get:

$$\Delta p_{\mathrm{s}} \approx 2.41 \times 10^{-3} \text{ and } \Delta p_{\mu} \approx 7.93 \times 10^{-3} \text{ when } p = 0.4 \quad (1)$$

$$\Delta p_{\mathrm{s}} \approx 1.60 \times 10^{-3} \text{ and } \Delta p_{\mu} \approx -4.05 \times 10^{-3} \text{ when } p = 0.8 \quad (2)$$

From figure 1, $p^* \approx 0.62$ is a stable equilibrium.

Solution exercice 3

(i) The change in the frequency of allele A when $\mu_\mathrm{B} = 0$ is

$$\Delta p = \frac{p(1-p)s}{1-(1-p)s} - \frac{\mu_\mathrm{A} w_\mathrm{A} p}{1-(1-p)s}$$

where $s = w_\mathrm{A} - w_\mathrm{B}$. Solving for p, we get $p^* = 0$ (trivial solution) or

$$p^* = \frac{w_\mathrm{A}(1-\mu_\mathrm{A}) - w_\mathrm{B}}{w_\mathrm{A} - w_\mathrm{B}} \tag{3}$$

which equates to 0 when $w_\mathrm{A} = 1, w_\mathrm{B} = 0.99, \mu_\mathrm{A} = 0.01, \mu_\mathrm{B} = 0$.

(ii) The effect of the mutation is always larger than the effect of selection, and the pressure is in the opposite direction as selection.

Solution exercice 4

(i) We have $s_{\mathrm{nat}} = w_{\mathrm{A}} - w_{\mathrm{Bnat}} = 1 - 0.8 = 0.2$ in the natural environment while

$$s_{\mathrm{mod}} = w_{\mathrm{A}} - \underbrace{(kw_{\mathrm{A}} + (1-k)w_{\mathrm{Bnat}})}_{w_{\mathrm{Bmod}}} = s_{\mathrm{nat}}(1-k) \quad \text{“modern environment”}$$

Thus k reduces the selection coefficient.

(ii) Under a mutation-selection balance, the frequency of A is

$$p_e^* = 1 - \frac{\mu_{\mathrm{A}}}{s_e}, \text{ where } e \in \{\mathrm{nat}, \mathrm{mod}\}$$

Letting $\mu_{\mathrm{A}} = 0.0001$ gives:
$p^* = 0.9995$ in a “natural environment” (k=0)
$p^* = 0.5$ in a “modern environment”(k=0.999)

(iii) $L = \frac{w_{\max} - \bar{w}}{w_{\max}} = \mu_{\mathrm{A}}/(1-k)$
with $w_{\max} = w_{\mathrm{A}} = 1$ and $\bar{w} = p_m^* + (1 - p_m^*)w_{\mathrm{B}}$.