Exercise Set 10

Problem 1

Suppose we can buy a chemical for 10$ per ounce (oz). There are only 17.25 oz available. We can transform this chemical into two products: A and B. Transforming to A costs 3$ per oz, while transforming to B costs 5$ per oz. If x_1 oz of A are produced, the price we get for A is $30 - x_1$ and $50 - 2x_2$ for B if x_2 oz of B are produced. How much chemical should we buy to maximize our profit, and what should we transform it to?

Problem 2

Let's consider the following logarithmic barrier problem:

$$\begin{array}{ll} min & 5x_1 + 7x_2 - 4x_3 - \sum_{j=1}^{3} ln(x_j) \\ \text{s.t.} & x_1 + 3x_2 + 12x_3 = 37 \\ & x_1 > 0, x_2 > 0, x_3 > 0 \end{array}$$

Give its dual problem.

Problem 3

We consider the following function $f(x,y) = xy$ with the constraint that $3x^2 + y^2 = 6$.

a) Compute the points satisfying the KKT conditions.

b) For each of these points, determine if it is a maximum, a minimum, or none of these.

Problem 4

Suppose we have a refinery that must ship finished goods to some storage tanks. Suppose further that there are two pipelines, A and B, to do the shipping. The cost of shipping x units on A is ax^2; the cost of shipping y units on B is by^2, where $a > 0$ and $b > 0$ are given. How can we ship Q units while minimizing cost? Determine the optimal quantities x^* and y^*.

Problem 5

How should one divide his/her savings between three mutual funds with expected returns 10%, 10% and 15% respectively, so as to minimize risk while achieving an expected return of 12%. We measure risk as the variance of the return on the investment. We assume that the variance of the portfolio is given by

$$0.04x^2 + 0.08y^2 + 0.02xy + 0.16z^2 + 0.04yz,$$

where x is the fraction invested in Fund 1, y in Fund 2 and z in Fund 3. We don't impose non-negativity constraints on the variables x,y,z.