Exercice 1: Hawk-Dove game

Fitness is

$$w(x,y) = k\left(1 + xy\left(\frac{V}{2} - C\right) + (1-x)(1-y)\frac{V}{2} + x(1-y)V\right)$$

Hence, the selection gradient is

$$\begin{aligned} S(y) &= \frac{\partial w(x,y)}{\partial x}|_{x=y} \\ &= k\left(y\left(\frac{V}{2} - C\right) + (1-y)\frac{V}{2}\right) = k\left(\frac{V}{2} - Cy\right) \end{aligned}$$

The singular strategy satisfies $S(y^*) = 0$ and is given by

$$y^* = \frac{V}{2C}$$

Exercice 1: Hawk-Dove game

The singular strategy is convergence stable at $y^* = V/(2C)$ if

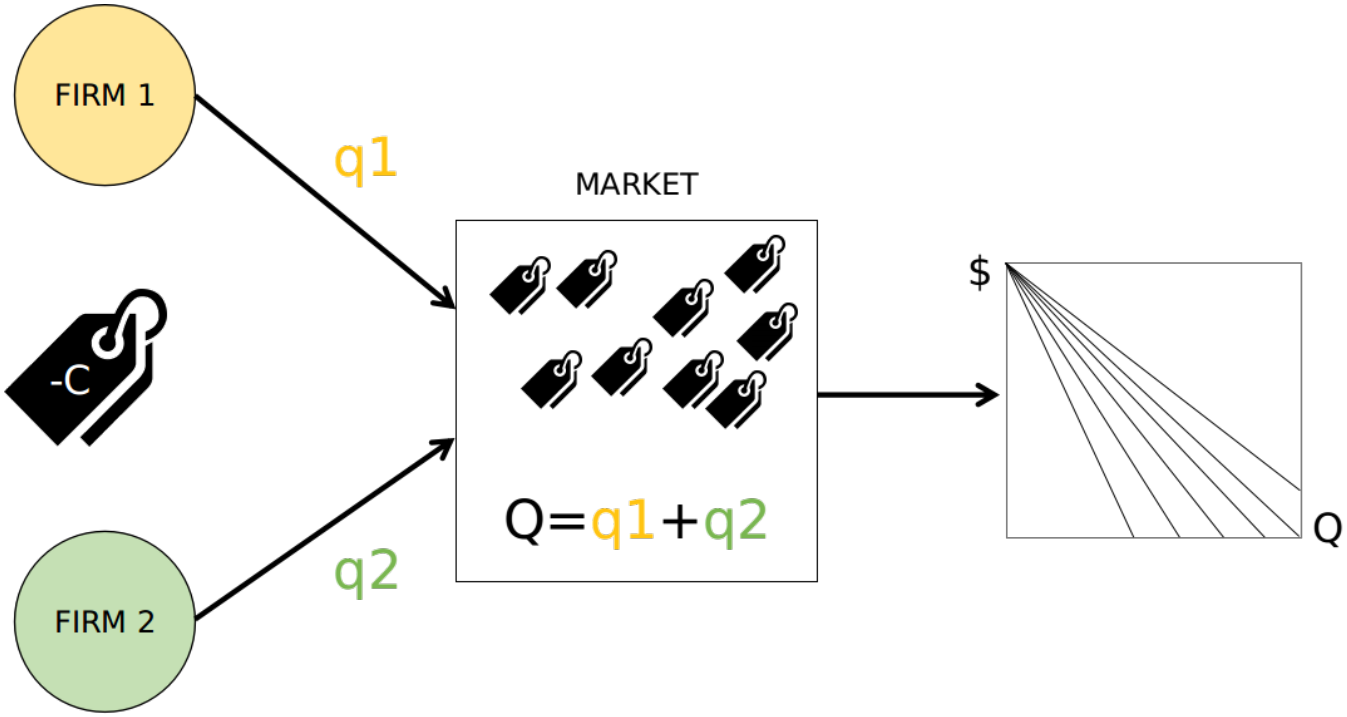
$$\frac{\mathrm{d}S(y)}{\mathrm{d}y}|_{y=y^*} = \frac{\mathrm{d}}{\mathrm{d}y} k \left(\frac{V}{2} - Cy \right) = -kC < 0$$

We also have

$$\frac{\partial^2 w(x,y)}{\partial x^2}|_{x=y=y^*} = 0$$

Hence, the singular strategy is both convergence stable and uninvadable.

Cournot duopoly model



Profit π_i for company or "individual" $i \in \{1, 2\}$ is $\pi_i = pq_i - Cq_i$ where q_i is the quantity provided by individual i, C is the cost of production and p is the price on the market

$$p = 1 - B(q_1 + q_2),$$

where the parameter B tunes the impact of quantity on price.

Exercice 2: Cournot game

Fitness is

$$w(x,y) = k\Big(1 + [1 - B(x+y)]\,x - Cx\Big)$$

Hence, the selection gradient is

$$S(y) = \left.\frac{\partial w(x,y)}{\partial x}\right|_{x=y} = k\,(1 - 2xB - yB - C)|_{x=y} = k\,(1 - 3yB - C)$$

The singular strategy satisfies $S(y^*) = 0$ and is given by

$$y^* = \frac{1 - C}{3B}$$

Exercice 2

The singular strategy is convergence stable at $y^* = (1 - C)/(3B)$ if

$$\frac{\mathrm{d}S(y)}{\mathrm{d}y}|_{y=y^*} = \frac{\mathrm{d}}{\mathrm{d}y} k\,(1 - 3yB - C) = -3kB < 0$$

The singular strategy is uninvadable if

$$\frac{\partial^2 w(x,y)}{\partial x^2}|_{x=y=y^*} = -2kB \leq 0$$

Hence, if $B > 0$ the singular strategy is both convergence stable and uninvadable.

In the Cournot model, $B < 0$ means that increasing supply increases the price on the market, which is unrealistic.