Exercice 1

$$s_{\mathrm{J}}(x, x_{\mathrm{r}}) = \left(\frac{x}{x + x_{\mathrm{r}}}\right)(1 - x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{f(x)}{g(x)} = \frac{(\mathrm{d}f(x)/\,\mathrm{d}x)}{g(x)} - \frac{f(x)\,(\mathrm{d}g(x)/\,\mathrm{d}x)}{g(x)^2}$$

Exercice 1

Fitness in our model is

$$w(x, x_{\mathrm{r}}, y) = k s_{\mathrm{J}}(x, x_{\mathrm{r}}) \ \text{ where } \ s_{\mathrm{J}}(x, x_{\mathrm{r}}) = \frac{x - x^2}{x + x_{\mathrm{r}}}.$$

So, the direct and indirect fitness effects are

$$k \left. \frac{\partial s_{\mathrm{J}}(x, x_{\mathrm{r}})}{\partial x} \right|_{x = x_{\mathrm{r}} = y} = \frac{k}{4} \left(\frac{1}{y} - 3 \right)$$

$$k \left. \frac{\partial s_{\mathrm{J}}(x, x_{\mathrm{r}})}{\partial x_{\mathrm{r}}} \right|_{x = x_{\mathrm{r}} = y} = -\frac{k}{4} \left(\frac{1}{y} - 1 \right).$$

Hence, the behaviour is selfish when $y < 1/3$ and spiteful when $y > 1/3$.

Exercice 1

The selection gradient is

$$
\begin{aligned}
S(y) &= k\left[\left.\frac{\partial s_{\mathrm{J}}(x,x_{\mathrm{r}})}{\partial x}\right|_{x=x_{\mathrm{r}}=y} + \left.\frac{\partial s_{\mathrm{J}}(x,x_{\mathrm{r}})}{\partial x_{\mathrm{r}}}\right|_{x=x_{\mathrm{r}}=y} r\right] \\
&= \frac{k}{4}\left(\frac{1-r}{y} - 3 + r\right)
\end{aligned}
$$

Solving $S(y^*) = 0$, the unique singular strategy is $y^* = \dfrac{1-r}{3-r}$. It is convergence stable as:

- $S(0) = \infty > 0$, so selection favour signalling when absent.
- $S(1) = -k/2 < 0$, so selection favour signalling when full.

We then have $y^* = 0$ ($r = 1$), $y^* = 1/5$ ($r = 1/2$), and $y^* = 1/3$ ($r = 0$).

Exercice 1

In a population at the convergence stable strategy y^*, survival is

$$s_{\mathrm{J}}(y^*, y^*) = \left(\frac{1}{2}\right)(1 - y^*) \text{ where } \quad y^* = \frac{1-r}{3-r}$$

Substituting and simplifying we get that survival is

$$s_{\mathrm{J}}(y^*, y^*) = \frac{1}{3-r}$$

which increases with relatedness because relatedness mediates competition.

Exercice 1

The convergence stable equilibrium is

$$y^* = \frac{1-r}{3-r}$$

which means that signalling will

$$\text{decrease when } y > \frac{1-r}{3-r}$$
$$\text{increase when } y < \frac{1-r}{3-r}$$

So if the population expresses $y = 0.1$, signalling will decrease when

$$0.1 > \frac{1-r}{3-r}, \text{ which holds for } r \in (0.78, 1]$$