Exercises: adaptive dynamics, part II

(1) Suppose that individuals engage in pairwise interactions (i.e., between two individuals) to compete for a resource with value V. The outcome of a contest impacts the fitness of an individual. During the pairwise interaction, the two individuals each play one of either two possible strategies: "Hawk" or "Dove". The phenotype $x \in [0, 1]$ of a focal mutant individual determines its probability of playing Hawk in an otherwise monomorphic population with phenotype y for this trait. We assume individuals are randomly matched, and we use the Hawk-Dove game pay-off matrix introduced in Chapter 3. We can then write the fitness of the focal mutant individual with phenotype x in the otherwise monomorphic population as:

$$w(x,y) = \underbrace{k}_{\text{A}} \left(\underbrace{1}_{\text{B}} + \overbrace{\underbrace{xy\left(\frac{v}{2} - c\right)}_{\text{C}} + \underbrace{x(1-y)v}_{\text{D}} + \underbrace{(1-x)(y) \times 0}_{\text{E}} + \underbrace{(1-x)(1-y)\frac{v}{2}}_{\text{F}}}^{\text{average payoff of the game}} \right).$$

A. Constant accounting for density-dependent competition. Can be held fixed.

B. Baseline fecundity.

C. With a probability xy, the focal individual and its opponent both play Hawk and fight over the resource. Then on average, each individual is assumed to get half of the resource, but pays a cost c for fighting.

D. With a probability $x(1-y)$, the focal individual plays Hawk, but its opponent plays Dove. The focal individual gets the resource in its entirety, without paying the cost of a fight.

E. With a probability $(1-x)y$, the focal individual plays Dove while its opponent plays Hawk, thus the focal individual gets nothing.

F. With a probability $(1-x)(1-y)$, the focal individual and its opponent both play Dove and share the resource equally without a fight.

(i) Calculate the singular probability of playing Hawk.

(ii) Is this strategy convergence stable? Is it uninvadable?

Note that here, strategy x defines a mixed strategy. This contrasts with the Hawk-Dove game analysed in Chapter 2, where individuals expressed pure strategies (i.e. fixed rather than probabilistic): playing Hawk or playing Dove.

(2) Consider again a population where there are pairwise interactions between individuals like in exercise (1). But suppose now that the fitness of a focal mutant with phenotype x when interacting in a pairwise manner with an individual with phenotype y is given by the following fitness function

$$w(x,y) = \underbrace{k}_{\text{A}} \Big(\underbrace{1}_{\text{B}} + \overbrace{\underbrace{[1 - b(x+y)]}_{\text{C}} x}^{\text{average game payoff}} - \underbrace{c}_{\text{D}} x \Big).$$

This can be thought of as describing the *Cournot duopoly* competition model, where the evolving phenotype x describes the quantity of some good an individual puts on the market, with:

A. Constant accounting for density-dependent competition. Can be held fixed.

B. Baseline fecundity;

C. The return from "selling" a quantity x of this good;

D. The cost of producing x of this good.

(i) Calculate the singular strategy for this model.

(ii) Determine the condition (in terms of parameters b and c) under which this strategy is convergence stable and uninvadable.