# Exercise Set 4

## Problem 1

For each of the following tableaus:

$$T_1 = \begin{array}{c} \begin{array}{cccccc|c|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & z & \end{array} \\ \begin{array}{|cccccc|c|c|} \hline 1 & 0 & -3 & -1 & 0 & 1 & 0 & -1 \\ -1 & 0 & -2 & 2 & 1 & 0 & 0 & -1/2 \\ 4 & 1 & 1 & -4 & 0 & 0 & 0 & 0 \\ \hline 3 & 0 & 6 & 0 & 0 & 0 & 1 & 9 \\ \hline \end{array} \end{array}$$

$$T_2 = \begin{array}{c} \begin{array}{cccccc|c|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & z & \end{array} \\ \begin{array}{|cccccc|c|c|} \hline 0 & 0 & -6 & 3/2 & -1 & 1 & 0 & -2 \\ 1 & 0 & 4 & -1 & 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & -3 & -2 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & -2 & 3/2 & 0 & 1 & 12 \\ \hline \end{array} \end{array}$$

$$T_3 = \begin{array}{c} \begin{array}{cccccc|c|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & z & \end{array} \\ \begin{array}{|cccccc|c|c|} \hline 1 & 3 & 2 & 0 & 1 & 0 & 0 & 2 \\ 0 & 4 & 1/3 & 1 & 1 & 0 & 0 & 3 \\ 0 & 2 & 2 & 0 & 2 & 1 & 0 & 1 \\ \hline 0 & 1 & 2/3 & 0 & 3/2 & 0 & 1 & -8 \\ \hline \end{array} \end{array}$$

$$T_4 = \begin{array}{c} \begin{array}{cccccc|c|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & z & \end{array} \\ \begin{array}{|cccccc|c|c|} \hline 1 & 1 & 3/2 & 3 & 0 & 0 & 0 & -2 \\ 1/2 & 0 & 2 & 1 & 1 & 0 & 0 & 3 \\ 3/2 & 0 & 3 & 3/2 & 0 & 1 & 0 & -1 \\ \hline 1 & 0 & 2 & 1 & 0 & 0 & 1 & 5 \\ \hline \end{array} \end{array}$$

determine if it is:

| | | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
|---|---|---|---|---|---|
| i) | primal-feasible | ☐ | ☐ | ☐ | ☐ |
| ii) | dual-feasible | ☐ | ☐ | ☐ | ☐ |
| iii) | optimal | ☐ | ☐ | ☐ | ☐ |
| iv) | primal-unbounded | ☐ | ☐ | ☐ | ☐ |
| v) | dual-unbounded | ☐ | ☐ | ☐ | ☐ |
| vi) | primal-degenerated | ☐ | ☐ | ☐ | ☐ |
| vii) | dual-degenerated | ☐ | ☐ | ☐ | ☐ |

# Problem 2

Give the dual problem of each of the following LPs:

a)

$$
\begin{array}{llrcrcrcr}
\text{Max} & z = & & & 7x_2 & + & 2x_3 & & \\
\text{s.t.} & & x_1 & + & x_2 & + & 2x_3 & = & -1 \\
& & -2x_1 & + & x_2 & + & x_3 & \leq & 7 \\
& & x_1 & & & & & \in & \mathbb{R} \\
& & & & x_2 & & & \leq & 0 \\
& & & & & & x_3 & \geq & 0
\end{array}
$$

b)

$$
\begin{array}{llrcrcr}
\text{Min} & z = & 4x_1 & + & 2x_2 & & \\
\text{s.t.} & & -x_1 & + & 2x_2 & \geq & -5 \\
& & x_1 & + & x_2 & \geq & 0 \\
& & x_1 & & & \geq & 0 \\
& & & & x_2 & \in & \mathbb{R}
\end{array}
$$

c)

$$
\begin{array}{llrcrcrcr}
\text{Max} & z = & 3x_1 & - & 5x_2 & & & & \\
\text{s.t.} & & x_1 & - & x_2 & + & x_3 & \leq & 4 \\
& & & & 2x_2 & + & x_3 & = & 2 \\
& & & & x_2 & , & x_3 & \geq & 0
\end{array}
$$

d)

$$
\begin{array}{llrcrcrcr}
\text{Min} & z = & 3x_1 & & & - & x_3 & & \\
\text{s.t.} & & 2x_1 & - & x_2 & + & 2x_3 & \geq & 5 \\
& & & & 3x_2 & + & 4x_3 & = & 12 \\
& & x_1 & & & & & \leq & 6 \\
& & & & x_2 & , & x_3 & \geq & 0
\end{array}
$$

e)

$$
\begin{array}{llrcrcrcr}
\text{Max} & z = & & & 5x_2 & + & x_3 & & \\
\text{s.t.} & & -x_1 & + & 4x_2 & + & 2x_3 & \leq & 8 \\
& & 2x_1 & & & + & x_3 & = & 6 \\
& & x_1 & , & x_2 & & & \geq & 0 \\
& & & & & & x_3 & \geq & -3
\end{array}
$$

# Problem 3

A steel factory would like to produce 1000 identical items. Each of them needs 1, 0.6, and 0.3 kg of metal M1, M2, and M3 respectively. These metals are present in different alloys that the factory purchases in the market. The price and the composition (in %) of the alloys are given in the table below:

| | Alloy 1 | Alloy 2 | Alloy 3 |
|---|---|---|---|
| M1 | 10% | 40% | 10% |
| M2 | 30% | 60% | 60% |
| M3 | 60% | 0% | 30% |
| Price per ton (kFrs) | 3 | 1 | 4 |

a) The factory would like to minimize its costs. Formulate this problem as a LP (PLP).

b) Determine its dual problem (DLP).

c) Solve PLP with the dual simplex algorithm (phase II).

d) What is the optimal solution ?

# Problem 4

The purpose of this exercise is to show that it exists some LPs having no feasible solutions and whose duals have no feasible solutions neither.

We consider the following LP :

$$
\begin{array}{llrcrcr}
\text{Max} & z = & 3x_1 & - & 2x_2 & & \\
\text{s.t.} & & x_1 & - & x_2 & \leq & 2 \\
 & & -x_1 & + & x_2 & \leq & -3 \\
 & & x_1 & , & x_2 & \geq & 0
\end{array}
$$

a) Show that this LP has no feasible solutions.

b) Formulate its dual problem.

c) Show that its dual has no feasible solutions neither.