Solution exercice 1

From the equation

$$n_t = w^t n_0$$

if we are given n_0 and n_t, we can solve for t, which gives

$$t = \frac{\log\left(\frac{n_t}{n_0}\right)}{\log(w)}$$

We have $w = 1.011$, $n_0 = 7.4 \times 10^9$, and $n_t = 2 \times 7.4 \times 10^9$. So $n_t / n_0 = 2$ and

$$t = \frac{\log(2)}{\log(1.011)} = 63.35$$

Solution exercice 2

The number of *E. coli* in a cubic meter is 10^{18} individuals, whereby the number that can be put into the observable universe is $4 \times 10^{80} \times 10^{18}$. Hence $n_t = 4 \times 10^{80} \times 10^{18}$ and we have $n_0 = 1$. So the number of demographic periods to fill up the volume of the universe is

$$t = \frac{\log\left(\frac{n_t}{n_0}\right)}{\log\left(w\right)} = \frac{\log(4 \times 10^{98})}{\log(2)} \approx 328$$

Finally, 328 generations of 20 minutes each is about 109 hours which is 4.5 days.

Solution exercice 3

The fitness of an individual for this situation is

$$w(n) = s + \frac{f}{1 + \gamma n}$$

The (non-trivial) population equilibrium must satisfy

$$w(n^*) = 1$$

, which can rearranged to $1 + \gamma n^* = s(1 + \gamma n^*) + f$ and solving for n^* gives

$$n^* = \frac{f - 1 + s}{\gamma(1 - s)}.$$

This equilibrium is an increasing function in s. The carrying capacity is thus increased relative to the case where there is no survival. This can be seen by noting that the numerator is always increasing in s, while the denominator is decreasing in s, which implies that the ratio is necessarily increasing in s.