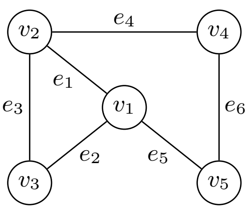
Exercise Set 5

Problem 1

a) Represent graphically the undirected graph associated with the following adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

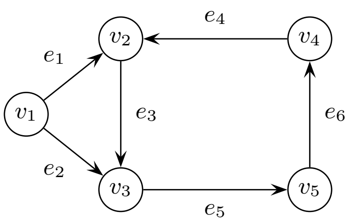
b) Give the incidence matrix, the adjacency matrix and the incidence function of the following graph:



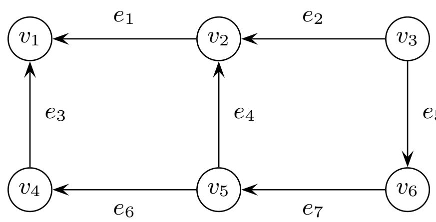
Problem 2

a) Give the incidence matrix and the adjacency matrix of the following graphs:

i)



ii)



b) Represent graphically the directed graphs associated with the following incidence matrices:

i) $$\begin{pmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

ii) $$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

c) Represent graphically the directed graph given by the following incidence function:

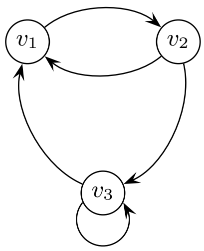
$$\begin{aligned} V &= \{v_1, v_2, v_3, v_4\} \\ E &= \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\} \end{aligned}$$

ψ	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
$u(e)$	v_1	v_3	v_1	v_2	v_1	v_3	v_2	v_2
$v(e)$	v_3	v_1	v_3	v_1	v_2	v_2	v_3	v_2

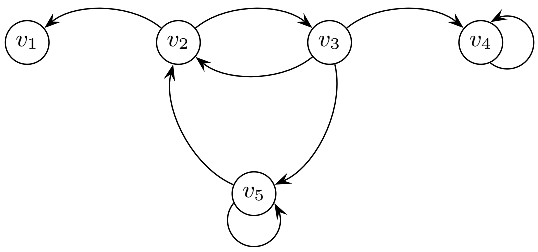
Problem 3

Determine the strongly connected components of the following digraphs by using a marking algorithm:

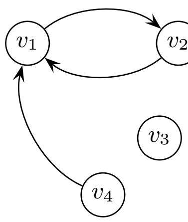
a)



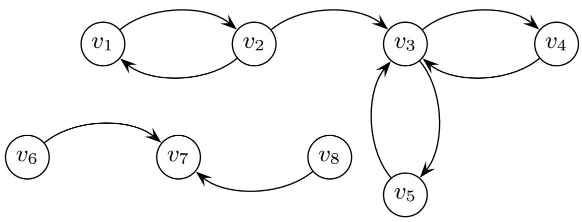
b)



c)

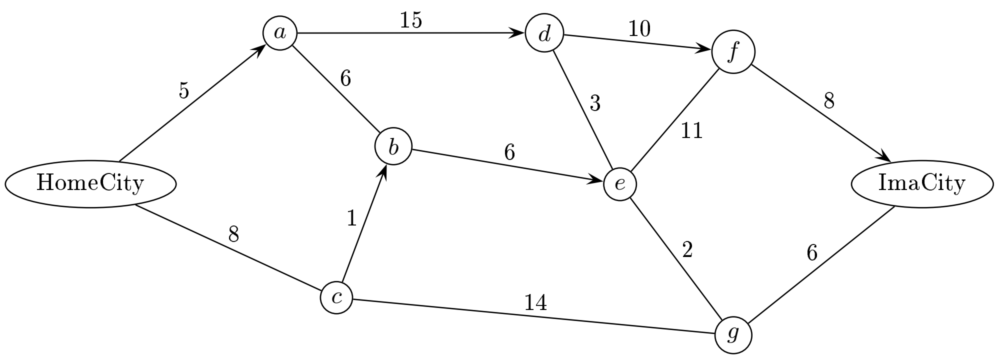


d)



Problem 4

Anne lives in HomeCity and works at ImaCity. She would like to determine what is the fastest itinerary between these two locations.

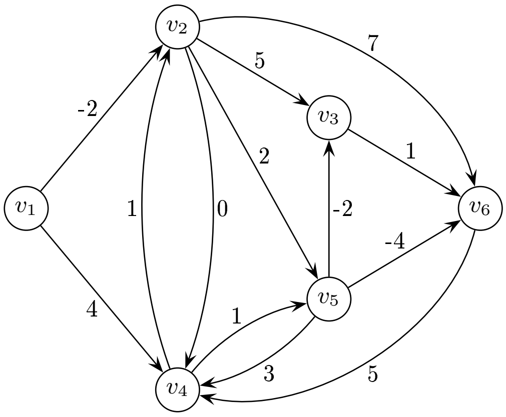


Vertices represent crossroads, the arcs are one-way roads, and the egdes are two-way roads. Values beside arcs and edges are amount of time in minutes from a crossroads to the next one. There is also a 3 minutes waiting time at each crossroads except at Homecity and at ImaCity.

Model this problem as a shortest path problem in a network. Determine the optimal path from HomeCity to ImaCity. How long is the travel ?

Problem 5

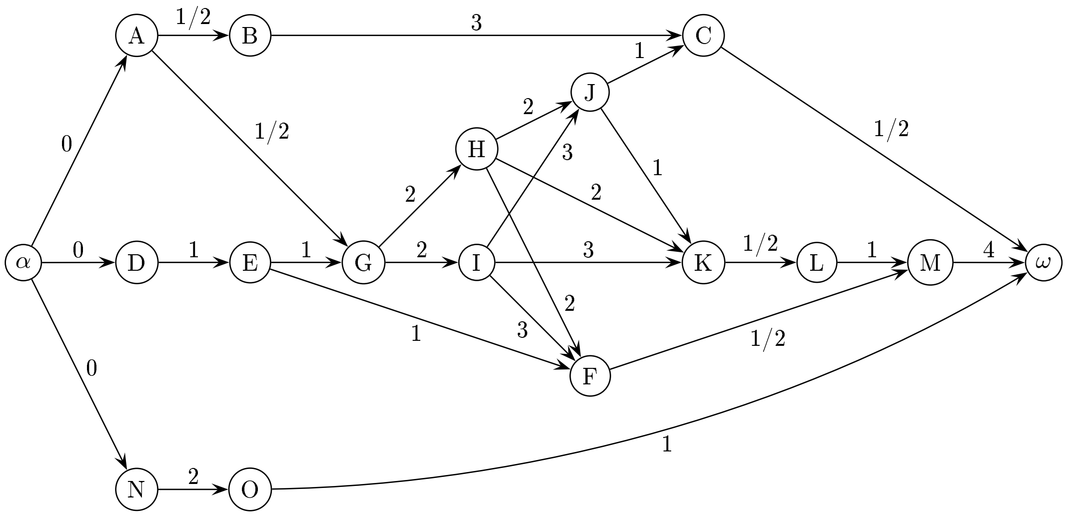
We consider the following network $R = (V,E,c)$:



Determine a shortest path from vertex v_1 to v_6.

Problem 6

We consider the acyclic graph below.



a) Determine the shortest paths from α to the other vertices.
b) Determine the longest paths from α to the other vertices.