Exercice 1

The pedigree of half-sibs:

$$r_{\text{sib}} = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 0 = \frac{1}{4}$$

Suppose the common parent is the mother so that the siblings have different fathers. Then

- With probability 1/2 the gene in A comes from its mother. Then, the maternal gene in B is identical-by-descent with probability 1/2.
- With probability 1/2 the gene in A comes from its father. The paternal gene in B cannot be identical-by-descent because the gene comes from a different father.

Exercice 1

General rule for calculating relatedness:

- Find the most recent common ancestor/ancestors.
- For each common ancestor, calculate the probability that a given allele copy resides in both individuals (A and B), which is given by 0.5^k, where k is the number of generations (meiotic divisions)
- If A and B have more than one common ancestors then add the probabilities that A and B share gene copies that are identical-by-descent through both ancestors.

Exercice 2

We have $s_{\mathrm{J}}(x, x_{\mathrm{r}}) = s_{\mathrm{b}} + x_{\mathrm{r}} - x^2$ with $w(x, x_{\mathrm{r}}) = k s_{\mathrm{J}}(x, x_{\mathrm{r}})$.
Hence, fitness is

$$w(x, x_{\mathrm{r}}) = k\left(s_{\mathrm{b}} + x_{\mathrm{r}} - x^2\right)$$

and

$$\underbrace{\frac{\partial w(x, x_{\mathrm{r}})}{\partial x}|_{x=x_{\mathrm{r}}=y}}_{-c} = -k2y \qquad \text{and} \qquad \underbrace{\frac{\partial w(x, x_{\mathrm{r}})}{\partial x_{\mathrm{r}}}|_{x=x_{\mathrm{r}}=y}}_{b} = k$$

$$S(y) = \frac{\partial w(x, x_{\mathrm{r}})}{\partial x}|_{x=x_{\mathrm{r}}=y} + r\frac{\partial w(x, x_{\mathrm{r}})}{\partial x_{\mathrm{r}}}|_{x=x_{\mathrm{r}}=y} = k\left(r - 2y\right)$$

Exercice 2

$$S(y) = k\,(r - 2y)$$

- If $y = 0$, then $S(0) = kr > 0$. Hence, selection will favor the evolution of "helping" and the trait increases over evolutionary time.

- If $y = 1$, then $S(1) = k\,(r - 2) < 0$. Hence, selection will disfavor helping and the trait decreases over evolutionary time.

That means there is a unique convergence stable strategy solving $S(y^*) = 0$ and given by

$$y^* = \frac{r}{2}$$

We then have $y^* = 1/4$ (full-sibs), $y^* = 1/8$ (half-sibs), $y^* = 1/16$ (cousin) and $y^* = 0$ (non-relative).