Exercises: Adaptive dynamics, part I

(1) Suppose that the fecundity of a single focal individual with phenotype x in a population otherwise monomorphic in y is given by the quadratic function $f(x) = 1 - x^2$. Further, lets assume that the fitness of the individual is proportional to its fecundity (namely, fitness is $w(x, y) = k \cdot f(x)$ for some $k > 0$)[1]. For this model:

(i) Plot $f(x)$ as a function of x.

(ii) Aided by the plot you created in (i), can you deduce the singular strategy and determine whether it is uninvadable? Calculate the singular strategy and check if it is uninvadable mathematically.

(iii) What type of selection is acting on the evolving phenotype?

(2) Consider a population in which there is a trade-off between the time one allocates to gathering resources for the production of offspring and to producing these offspring. Suppose that each individual has one unit of time to allocate between these two activities. An evolving trait x represents the fraction of this unit of time that an individual allocates to the production of offspring. The fraction of time allocated to gathering resources is therefore $1 - x$. Suppose further that the fecundity of an individual with phenotype x in a population otherwise monomorphic in y is given by the function $f(x) = x(1 - x)$ (see [2] for connection with economics), and that the fitness of the individual is proportional to its fecundity (namely, fitness is $w(x, y) = k \cdot f(x)$ for some constant $k > 0$). For this model:

(i) Plot $f(x)$ as a function of x.

(ii) Aided by the plot you created in (i), can you deduce the singular strategy and determine whether it is uninvadable? Calculate the singular strategy and check if it is uninvadable mathematically.

(iii) What type of selection is acting on the evolving phenotype?

[1] Recall that k captures density-dependent competition and is generally a function of the resident population y but the effect of y does not matter so we can treat k as a constant

[2] This fecundity function can be thought of as an example of a Cobb-Douglas production function from economics. Here, a single good (the offspring) depends on two factors of production, namely, on time allocated to producing offspring and on time allocated to gathering resources.