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Lab 1: Signal and Spectrum

Issue: 8th November 2022

Submission: 15th November 2022

Task 1: Introduction into Matlab

Matlab is a professional multi-paradigm numerical computing environment for simulation, modeling and RAD (Rapid Development) based on matrix calculations, similar to R. It comes with a huge variety of different toolboxes, e.g. for signal processing, image processing or deep learning as well as a graphical UI framework.

Programs written in Matlab are stored in so-called m-files, i.e. text files containing Matlab commands. You can write your own m-files containing Matlab commands. These can be directly called from the Matlab command line or other m-files by just entering the name of the m-file (without .m) as a command.

In addition, Matlab supports so-called life scripts, which are similar Jupyter notebooks. To open a life script type `edit` *`lifescriptname`* on the Matlab command line.

As the programs you use in these labs comprise their own UI you need to know only few Matlab commands:

- To call a m-file called prog.m you just type prog on the matlab command line
- To get help for a command `prog` just type `help prog`
- Type `doc` to get help on all available toolboxes
- `Edit prog` opens the m-file prog.m in the editor.
- `help` gives you an overview on all available toolboxes
- `s = [1, 4, 2, 8];` creates a variable s with vector [1, 4, 2, 8]
- `s(2)` selects element 2 of vector `s`. **Attention: Vectors in Matlab start with index 1!**
- `s = [1:5];` creates a variable s with the vector `[1,2,3,4,5]`
- `s` without `;` lists the content of s
- `plot(s)` plots the values of `s` as a line graph
- `0.5*s` multiplies all values of `s` with `0.5`

Task:

Start Matlab and generate the following signals s and plot them:

1. s = 1, 2, 3, ..., 200
2. s = 1, 3, 5, 7, ..., 99
3. s = 100 zeros, except for s(50) = 1
4. s = 50 zeros followed by 50 ones, followed by 50 zeros
5. s = cos(2π*k/N), k=1, ..., 100; N = 50

see file task1_introduction_into_Matlab.mlx

Task 2: Aliasing

Call the program `aliasing`. You should see and hear a sinusoidal signal with 1000 Hz frequency that has been sampled with 4 kHz. You can now increase the signal frequency with the slider or by typing into the text field beneath. In the graphics you see the analog signal in red and the sampled values of the sampled signal in blue.

Answer the following questions:

1. What do you hear when you decrease the signal frequency stepwise down to 0 Hz?

   The sound becomes deeper and duller. When signal freq. = 0 then no sound can be heard.

2. What do you hear when you increase the signal frequency stepwise up to 2000 Hz?

   The sound becomes higher and more pointed.

3. What do you hear when you increase the signal frequency above 2000 Hz? (above)

   It becomes deeper and duller again and sounds actually the same as in point 1 at the end.

4. What frequency do you hear approximately when you chose a signal frequency of 3800 Hz, 3600Hz, 3000Hz? What general formula describes this behavior?

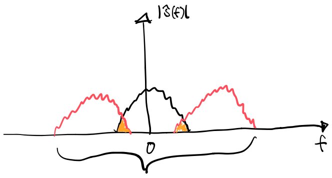
   Approximately: 3800 Hz ≈ 200 Hz
   3600 Hz ≈ 400 Hz
   3000 Hz ≈ 1000 Hz

   The sampling theorem describes this behaviour: $\boxed{f_{s_{min}} > 2 * f_h}$

   f_h: highest frequency in analog signal

5. Try to explain this behavior from what you know about the spectrum of a sampled signal.

   Sampling rate must be high enough (see sampling theorem above) otherwise aliasing occurs.
   This can lead to overlaps in the frequency spectrum.



After reconstruction we will see a signal that was previously not present.
This is due to the overlaps (marked orange)

Task 3: Complex Exponential Functions

Start the live script ComplexExponentialFunction.mlx by typing `edit ComplexExponentialFunction` on the matlab command line. Goto the Mathlab tab „Live Editor“. There you can run the whole script or only sections of it. The corresponding results are displayed within the script or at the side of it (for a better overview view at the side is preferable).

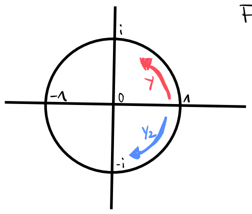
When you change the slider control in the script the corresponding section is rerun automatically.

$\omega_0(k) = (2\pi/N)k$

Answer the following questions;

1. What is the difference between the two functions $y = Ae^{i\omega_0 n}$ and $y_2 = Ae^{-i\omega_0 n}$ drawn in the z-plane?

When looking at the unit circle they follow it in the opposite direction. Furthermore, y is part of IDFT while y_2 is part of DFT equation.



2. What is the normalized sampling period of the two signals (number of samples after which the signal repeats itself)?

The signal repeats after N=80.

3. What signal results when you take the mean of the two functions $y = Ae^{i\omega_0 n}$ and $y_2 = Ae^{-i\omega_0 n}$?

My assumption is that the signal is approximately 0 (real and imaginary part) for N= 40, 80, 120 etc.

Following check in ComplexExponentialFunction.mlx confirmed assumption

```
Y= A*exp(1i*omega0*n)
Y2= A*exp(-1i*omega0*n)
new_Y = [Y, Y2]
mean_ = mean(new_Y)
real(mean_)  % ~ 0.0123       } for N=80
imag(mean_)  % ~ -2.4287e-18  }
```

For other values for N the imaginary part is always approximately 0 while the real part is between 0 and 1 (0< real part <1)

Task 4: Signal and Spectrum

Start the program `signalandspectrum`. You can generate different signals there and study their DFT spectra, i.e.:

- Periodic signals comprising multiple frequency components with and without noise
- Sinusoidal signals where the frequency linearly increases over time (sweep signal)
- Impulse signals
- Impulse train signals
- Your own signals. You can record your own signal

In the graphic on the top right a window of the generated signal is plotted. The chosen segment is used to calculate the spectrum depicted in the graphic in the bottom right (if "in dB"-checkbox is selected, spectrum is shown in dB).

Answer the following questions:

1. What are the frequency components of the multi-sin-signal and what are their amplitudes? Try to figure it out in the time domain and in the frequency domain. In which domain is it easier?

The frequency components are 400 (f_0), 800 (f_1), 1200 (f_2), 1600 (f_3) and 2000 (f_4). The amplitude of each part is per default 0.5 but can be configured with the slides. The amplitude is additive and therefore becomes bigger with each part. Furthermore, there is also a unique component, the white noise. The frequency and amplitude is random for the white noise.

The amplitude is easier to find in the time domain while the frequency is easier to determine in the frequency domain.

2. How does the DFT spectrum change, when the window is shifted?

It remains basically the same but the signal on the borders of window can change such that also the DFT spectrum slightly changes.

3. Which signal component has a significantly different spectrum than the other components? What is the reason?

It is the white noise. The spectrum looks randomly due to the properties of white noise.
series with no autocorrelation and therefore predicting would also not be possible

4. What is the difference between the linear spectrum and the dB-spectrum? What could be an advantage of the dB-spectrum? (generate a signal with two sin-components: one with amplitude 1 and the other with an amplitude that is only one step above zero)

Amplitudes in the dB-spectrum are normalized to a reference amplitude. It has a logarithmic scale. The properties of the dB-spectrum help to better recognize the second sin component with a very low amplitude compared to the linear scale.
It is linear, periodical, convolutional and multiplicative for the frequency and time domain.

5. How does the spectrum of white noise look like?

It looks as expected very random and has no patterns. If there would be any pattern we would need to check if it is really white noise. The max values of the amplitude are also randomly with each generation.

6. How does the spectrum of an impulse look like? How does it change, depending where the impuls lies?

As long as the impulse lies within the window the DFT spectrum is a constant value above 0. If the impulse is not within the window then it is constant 0.

7. Describe the spectrum of an impulse train?

The spectrum of an impulse train has a peak every 1000 Hz. The peak has an amplitude of 0.125. other than that the amplitude is 0.

8. What is the spectrum of a rectangle signal? How does it change when the window is shifted?

As long as the whole rectangle lies within the window the spectrum remains the same.

When the rectangle is outside of the window the spectrum has a constant value of 0.

9. Try to determine the fundamental frequency of your voice.

I got a peak around 150 Hz. This is the frequency after a couple of recordings. The obtained 150 Hz look reasonable because that is typical for males.