Time series graphics

Analysis of Sequential Data

MSE Data Science

Credits

Slides and book openly published by Rob Hyndman:

https://robjhyndman.com/teaching/

https://otexts.com/fpp2/

Customization by Giorgio Corani for the MSE course.

Outline

`ts` objects and `ts` function

A time series is stored in a `ts` object in R:

- a list of numbers
- information about times those numbers were recorded.

Example

Year	Observation
2012	123
2013	39
2014	78
2015	52
2016	110

```
y <- ts(c(123,39,78,52,110), start=2012)
```

`ts` objects and `ts` function

For observations that are more frequent than once per year, add a `frequency` argument.
E.g., monthly data stored as a numerical vector `z`:

```
y <- ts(z, frequency=12, start=c(2003, 1))
```

`ts` objects and `ts` function

`ts(data, frequency, start)`

Type of data	frequency	start example
Annual	1	1995
Quarterly	4	c(1995,2)
Monthly	12	c(1995,9)
Daily	7 *or* 365.25	1 *or* c(1995,234)
Weekly	52.18	c(1995,23)
Hourly	24 *or* 168 *or* 8,766	1
Half-hourly	48 *or* 336 *or* 17,532	1

Australian GDP

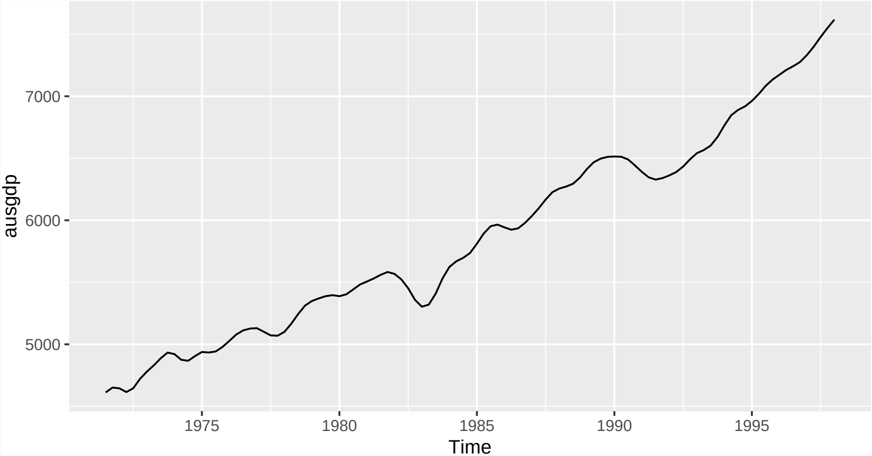
- Class: “ts”, frequency =4
- Print and plotting methods available.

```
ausgdp
```

```
##      Qtr1 Qtr2 Qtr3 Qtr4
## 1971           4612 4651
## 1972 4645 4615 4645 4722
## 1973 4780 4830 4887 4933
## 1974 4921 4875 4867 4905
## 1975 4938 4934 4942 4979
## 1976 5028 5079 5112 5127
## 1977 5130 5101 5072 5069
## 1978 5100 5166 5244 5312
## 1979 5349 5370 5388 5396
```

Australian GDP

```
autoplot(ausgdp)
```



Residential electricity sales

```
elecsales
```

```
## Time Series:
## Start = 1989
## End = 2008
## Frequency = 1
##  [1] 2354.34 2379.71 2318.52 2468.99 2386.09
##  [6] 2569.47 2575.72 2762.72 2844.50 3000.70
## [11] 3108.10 3357.50 3075.70 3180.60 3221.60
## [16] 3176.20 3430.60 3527.48 3637.89 3655.00
```

Class package

```
> library(fpp2)
```

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This loads:

- some data for use in examples and exercises

Class package

```
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```

This loads:

- some data for use in examples and exercises
- **forecast** package (for forecasting functions)
- **ggplot2** package (for graphics functions)
- **fma** package (for lots of time series data)
- **expsmooth** package (for more time series data)

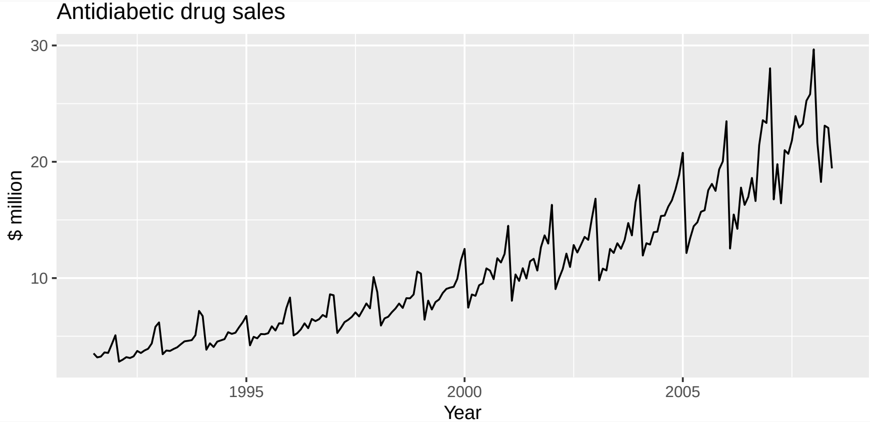
Outline

Time plots

```
autoplot(melsyd[,"Economy.Class"])
```

Time plots

```
autoplot(a10) + ylab("$ million") + xlab("Year") +
         ggtitle("Antidiabetic drug sales")
```



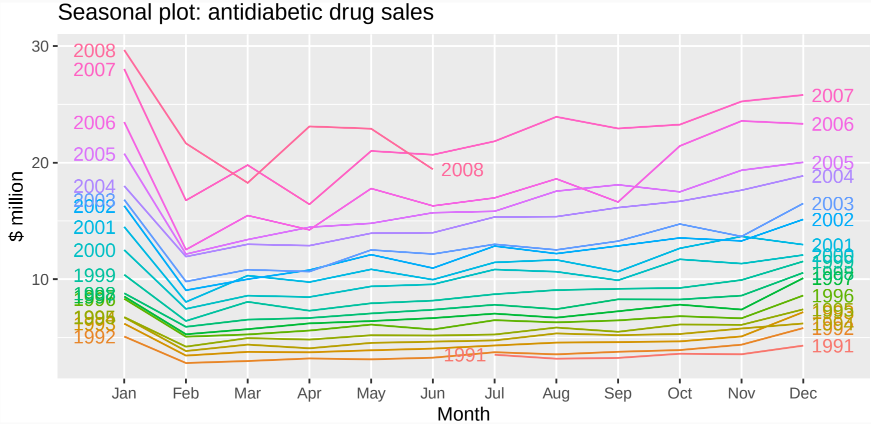
Your turn

- Create plots of the following time series: `dole`, `bricksq`, `lynx`, `goog`
- Use `help()` to find out about the data in each series.
- For the last plot, modify the axis labels and title.

Outline

Seasonal plots

```r
ggseasonplot(a10, year.labels=TRUE, year.labels.left=TRUE) +
  ylab("$ million") +
  ggtitle("Seasonal plot: antidiabetic drug sales")
```

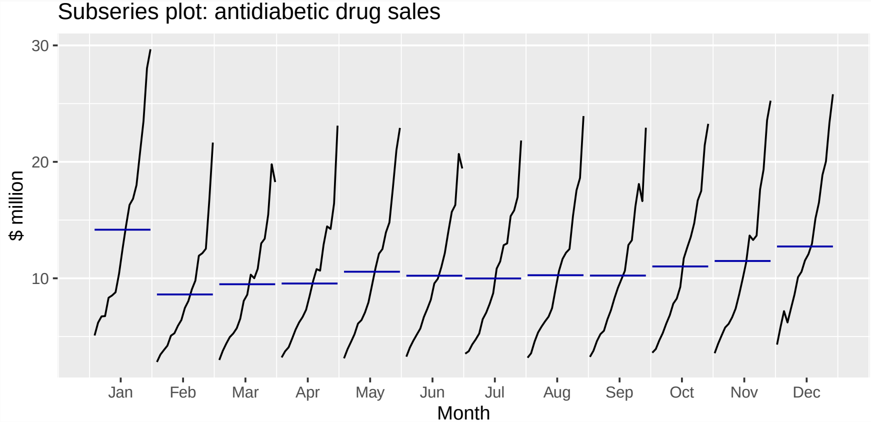


Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
- In R: `ggseasonplot()`

Seasonal subseries plots

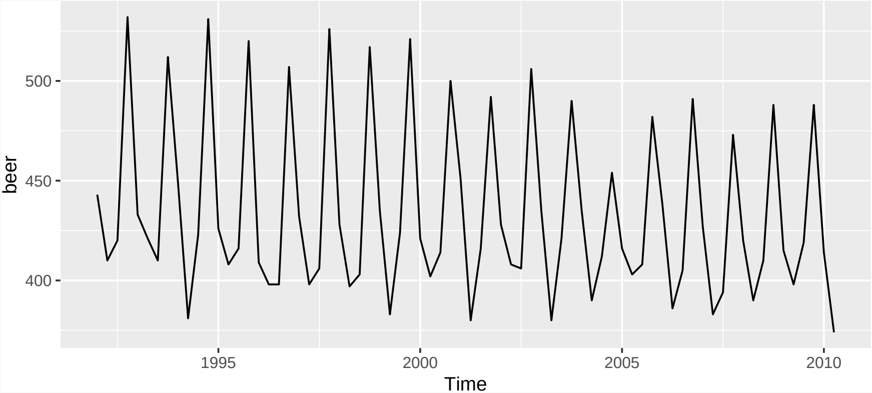
```r
ggsubseriesplot(a10) + ylab("$ million") +
  ggtitle("Subseries plot: antidiabetic drug sales")
```



- Data for each season collected together in time plot as

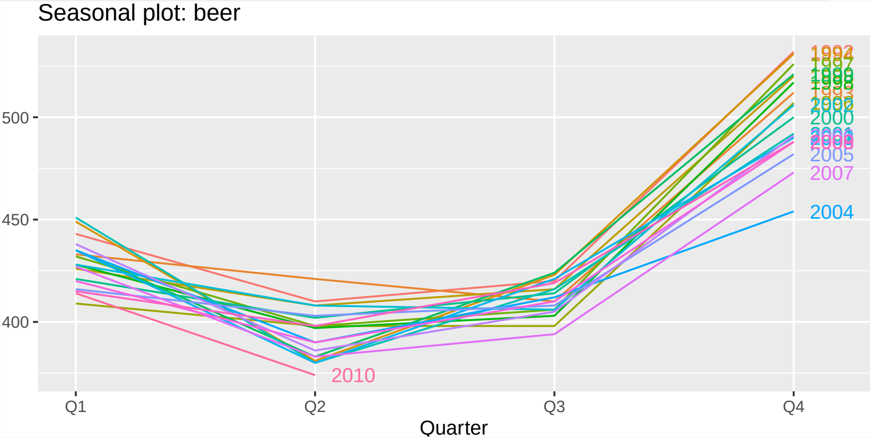
Quarterly Australian Beer Production

```r
beer <- window(ausbeer,start=1992)
autoplot(beer)
```



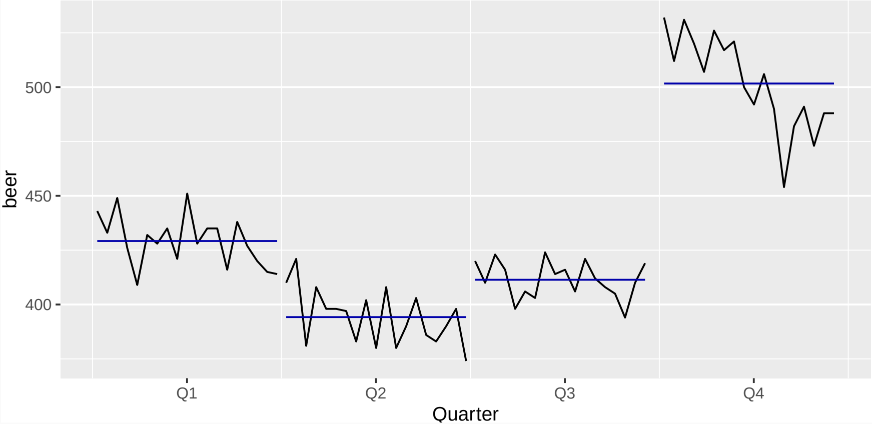
Quarterly Australian Beer Production

```
ggseasonplot(beer,year.labels=TRUE)
```



Quarterly Australian Beer Production

```
ggsubseriesplot(beer)
```



Your turn

The `arrivals` data set comprises quarterly international arrivals (in thousands) to Australia from Japan, New Zealand, UK and the US.

- Use `autoplot()` and `ggseasonplot()` to compare the differences between the arrivals from these four countries.
- `ggseasonplot()` should be applied to each column separately
- Can you identify any unusual observations?

Outline

Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

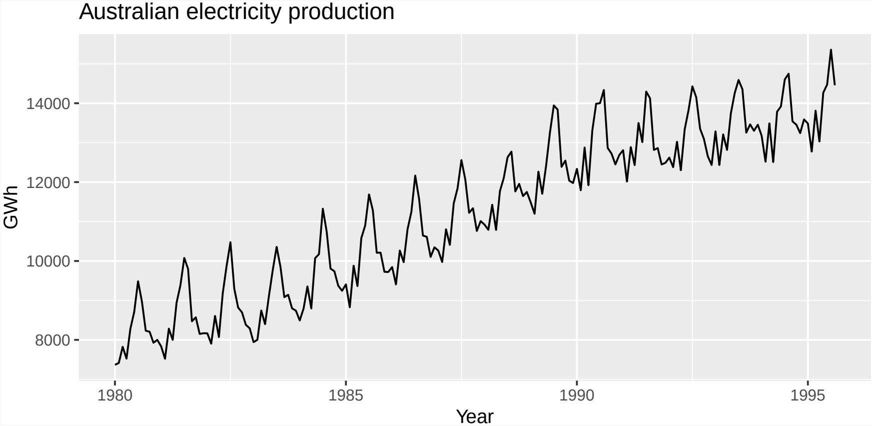
Time series components

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

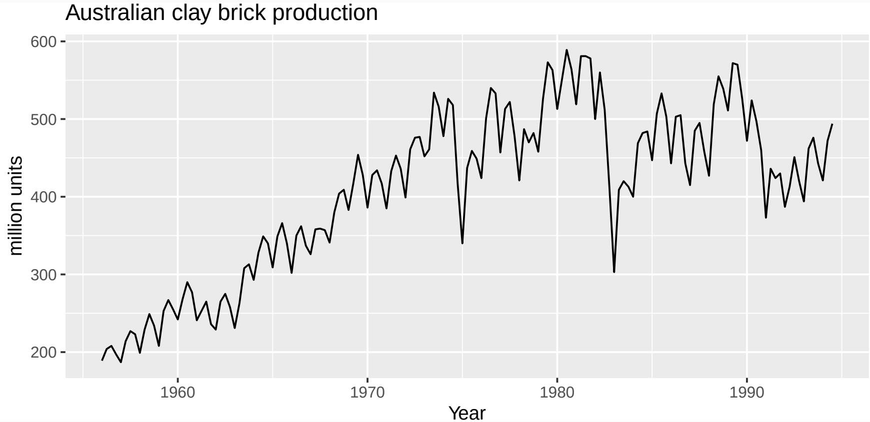
Time series patterns

```r
autoplot(window(elec, start=1980)) +
  ggtitle("Australian electricity production") +
  xlab("Year") + ylab("GWh")
```



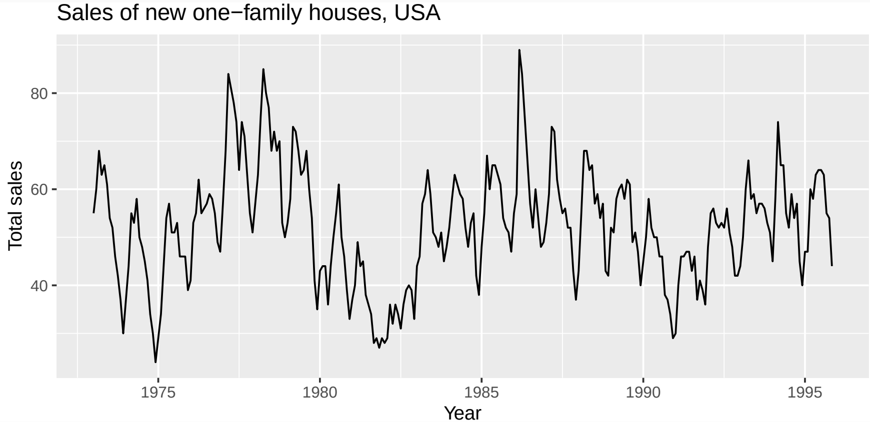
Time series patterns

```r
autoplot(bricksq) +
  ggtitle("Australian clay brick production") +
  xlab("Year") + ylab("million units")
```



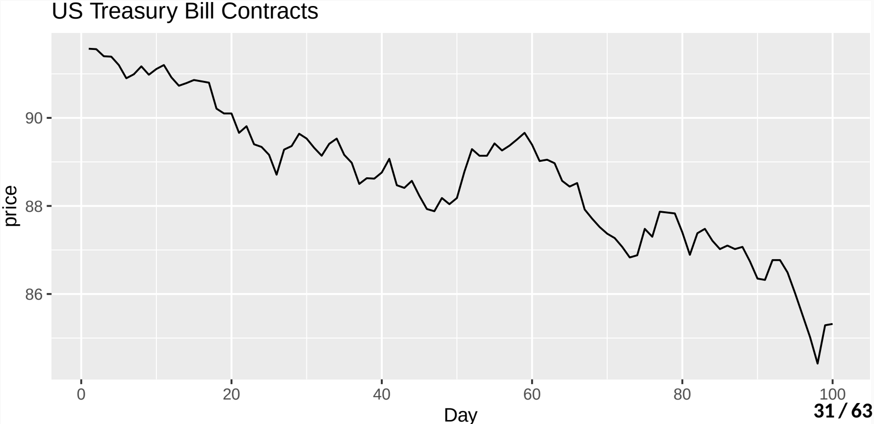
Time series patterns

```r
autoplot(hsales) +
  ggtitle("Sales of new one-family houses, USA") +
  xlab("Year") + ylab("Total sales")
```



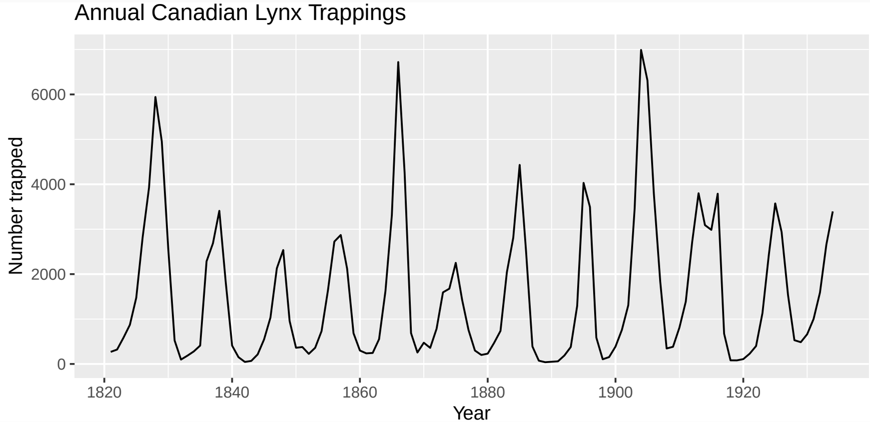
Time series patterns

```
autoplot(ustreas) +
  ggtitle("US Treasury Bill Contracts") +
  xlab("Day") + ylab("price")
```



Time series patterns

```
autoplot(lynx) +
  ggtitle("Annual Canadian Lynx Trappings") +
  xlab("Year") + ylab("Number trapped")
```



Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

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Seasonal or cyclic?

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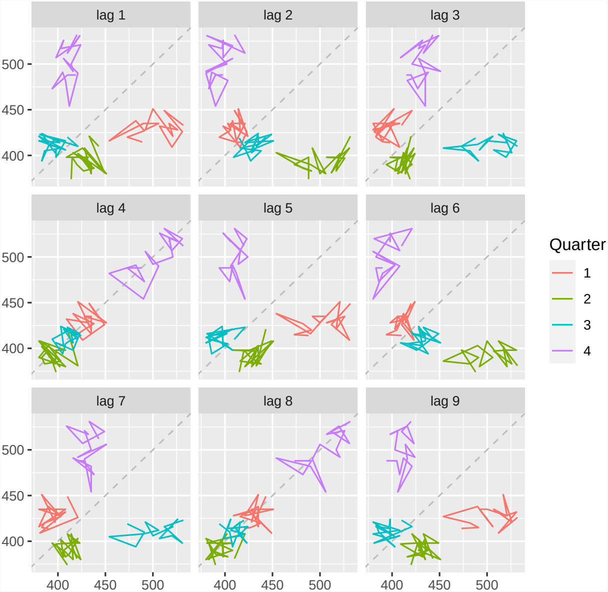
The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

Outline

(this slide to be disregarded)



Autocorrelation

- **correlation** measures the extent of a linear relationship between two variables
- **autocorrelation** measures the linear relationship between lagged values of a time series.

We use the notation:

- r_k: correlation between y_t and y_{t-k}

For instance:

- r_2: correlation between y_t and y_{t-2}

Autocorrelation

$$r_k = \frac{\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

- T is the lenght of the time series
- the denominator of r_k is the variance of y_t

Autocorrelation

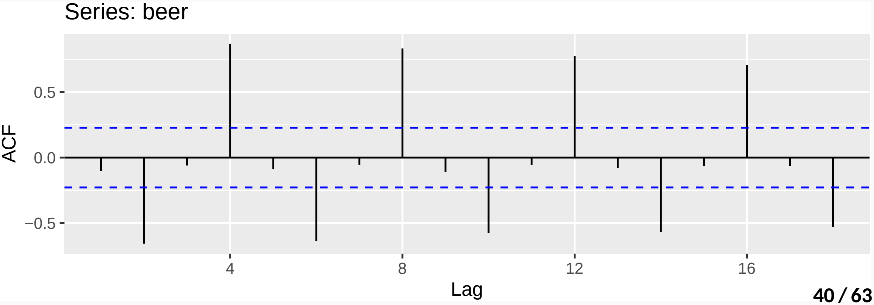
- $(y_t > \bar{y}$ and $y_{t-k} > \bar{y}) \rightarrow r_k$ >0
- $(y_t < \bar{y}$ and $y_{t-k} < \bar{y}) \rightarrow r_k$ >0
- $(y_t < \bar{y}$ and $y_{t-k} > \bar{y}) \rightarrow r_k$ <0
- $(y_t > \bar{y}$ and $y_{t-k} < \bar{y}) \rightarrow r_k$ <0

Autocorrelation

Results for first 9 lags for beer data:

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
-0.102	-0.657	-0.060	0.869	-0.089	-0.635	-0.054	0.832	-0.108

```
ggAcf(beer)
```

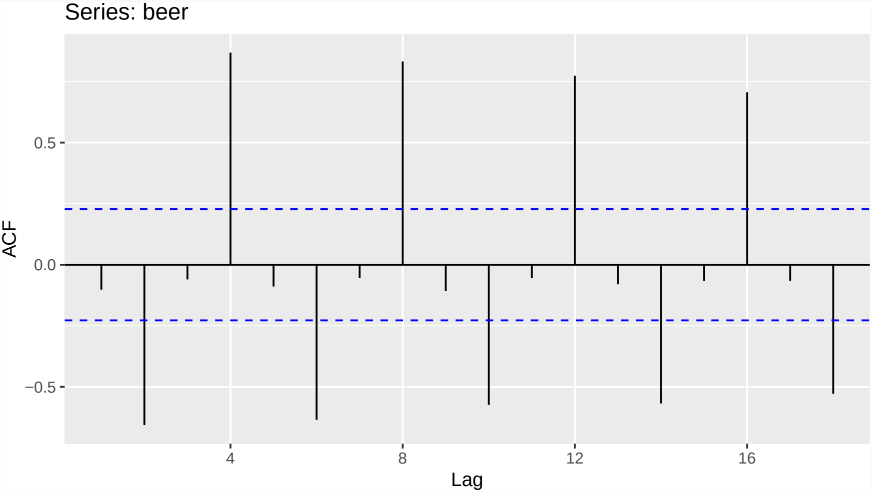


Autocorrelation

- r_4 higher than for the other lags. This is due to **the seasonal pattern in the data**: the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- r_2 is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.
- Together, the autocorrelations at lags 1, 2, $\ldots$, make up the *autocorrelation* or ACF.
- The plot is known as a **correlogram**

ACF

```
ggAcf(beer)
```

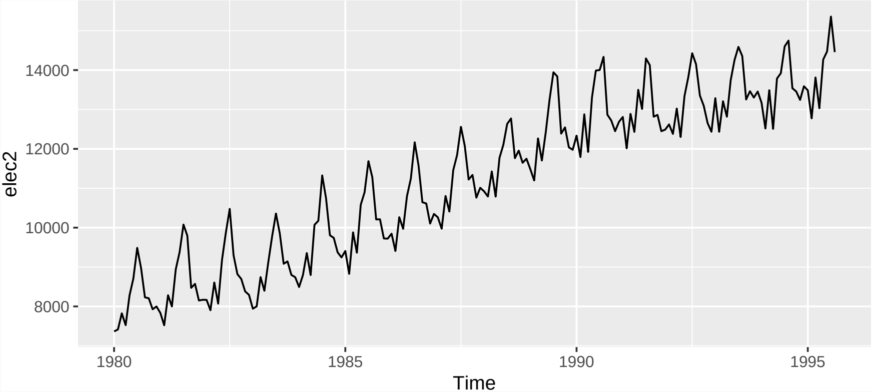


Trend and seasonality in ACF plots

- When data have a trend, the autocorrelations for small lags tend to be large and positive.
- When data are seasonal, the autocorrelations will be larger at the seasonal lags (i.e., at multiples of the seasonal frequency)
- When data are trended and seasonal, you see a combination of these effects.

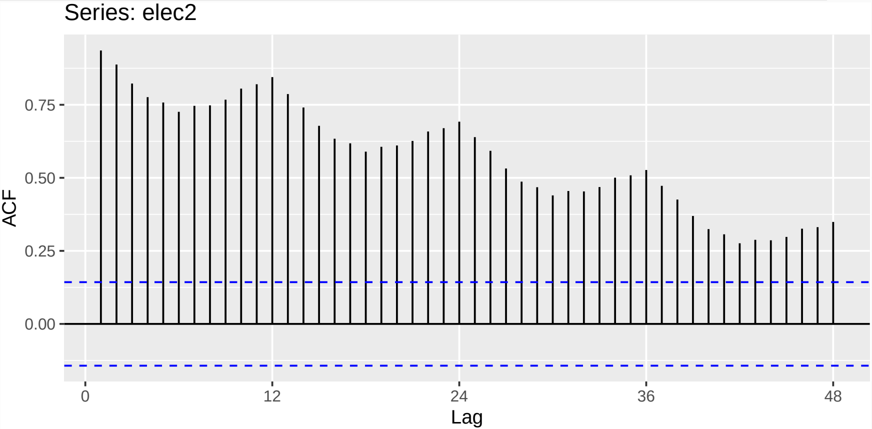
Aus monthly electricity production

```
elec2 <- window(elec, start=1980)
autoplot(elec2)
```



Aus monthly electricity production

```
ggAcf(elec2, lag.max=48)
```



Aus monthly electricity production

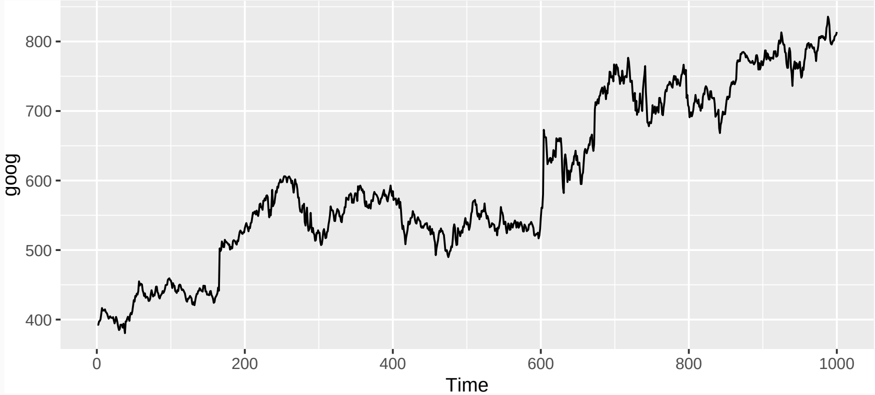
Time plot shows clear trend and seasonality.

The same features are reflected in the ACF.

- The slowly decaying ACF indicates trend.
- The ACF peaks at lags 12, 24, 36, . . . , indicate seasonality of length 12.

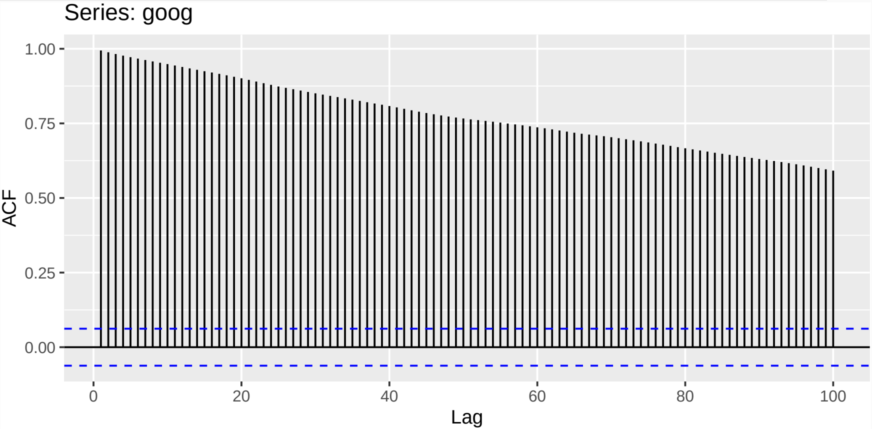
Google stock price

```
autoplot(goog)
```



Google stock price

```r
ggAcf(goog, lag.max=100)
```



Your turn

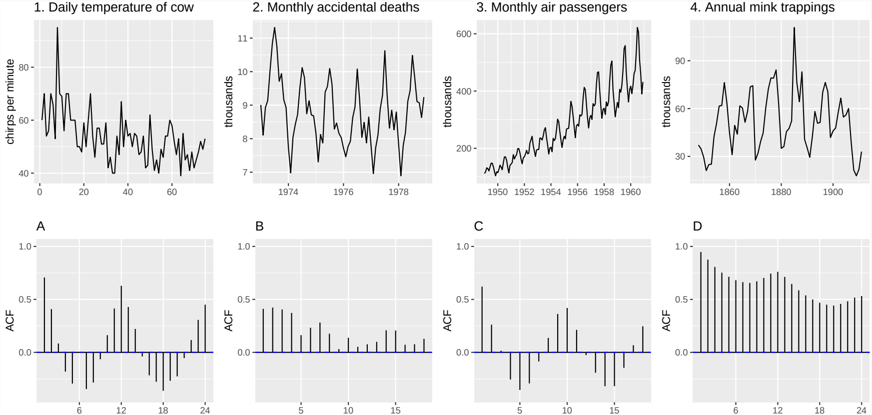
We have introduced the following graphics functions:

- `gglagplot`
- `ggAcf`

Explore the following time series using these functions. Can you spot any seasonality, cyclicity and trend? What do you learn about the series?

- `hsales`
- `usdeaths`
- `bricksq`
- `sunspotarea`
- `gasoline`

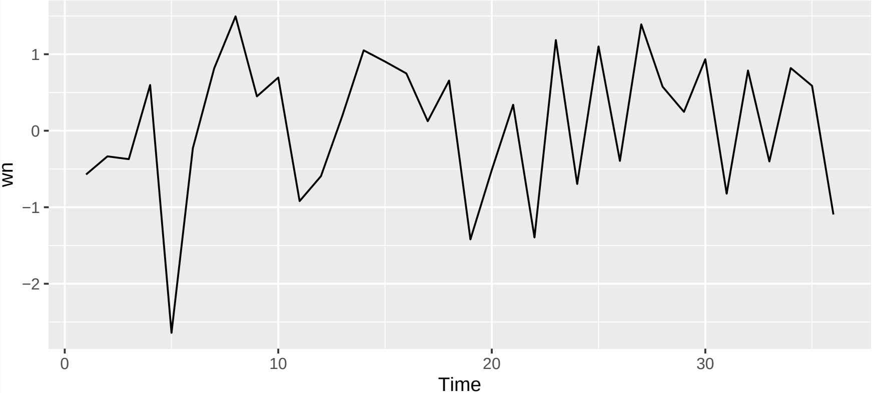
Which is which?



Outline

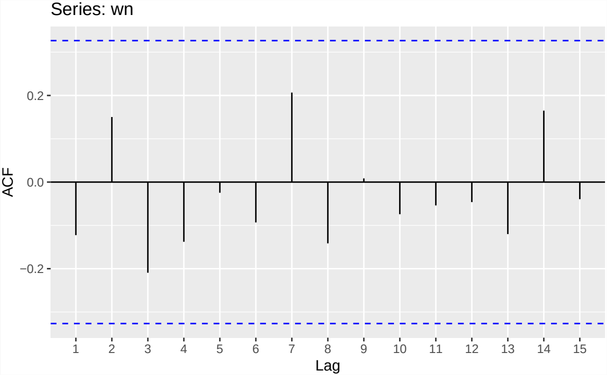
Example: White noise

```r
wn <- ts(rnorm(36))
autoplot(wn)
```



Example: White noise

r_1	-0.12
r_2	0.15
r_3	-0.21
r_4	-0.14
r_5	-0.02
r_6	-0.09
r_7	0.21
r_8	-0.14
r_9	0.01
r_{10}	-0.07



Sample autocorrelations for white noise series.

We expect each autocorrelation to be close to zero.

Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically $\text{N}(0,1/T)$.

Sampling distribution of autocorrelations

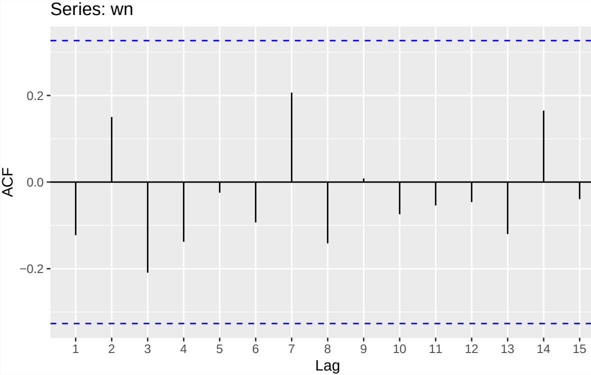
Sampling distribution of r_k for white noise data is asymptotically N(0,$1/T$).

- 95% of all r_k for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the ***critical values***.

Autocorrelation

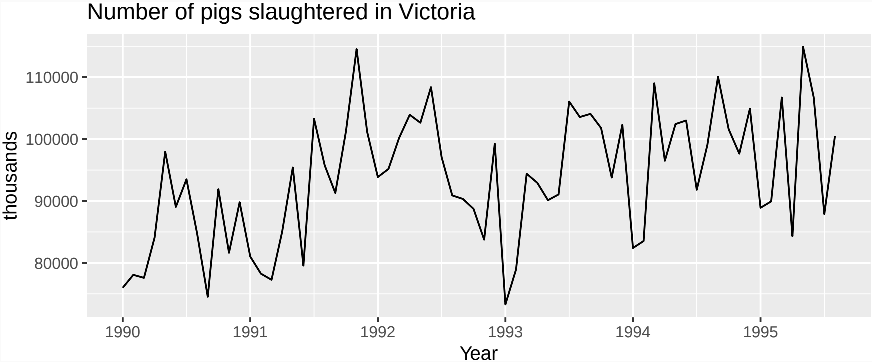
Example:

T = 36 and so critical values at $\pm 1.96/\sqrt{36} = \pm 0.327$. All autocorrelation coefficients lie within these limits, confirming that the data are white noise. (More precisely, the data cannot be distinguished from white noise.)



Example: Pigs slaughtered

```r
pigs2 <- window(pigs, start=1990)
autoplot(pigs2) +
  xlab("Year") + ylab("thousands") +
  ggtitle("Number of pigs slaughtered in Victoria")
```



Example: Pigs slaughtered

```
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```

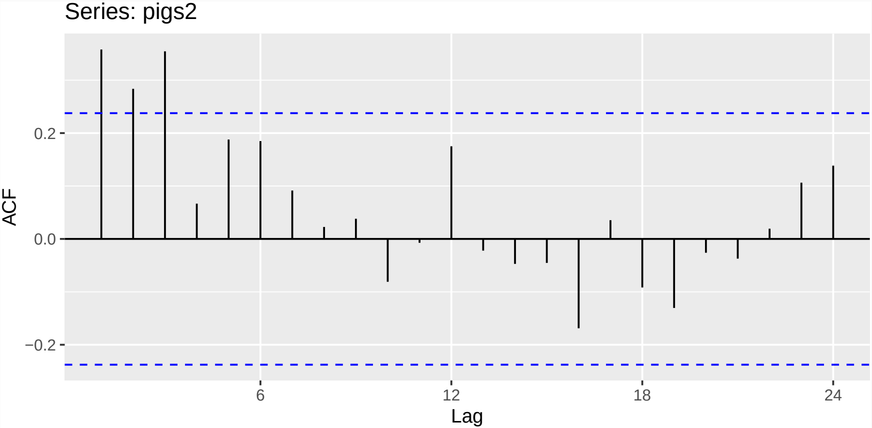

Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 1990 through August 1995. (Source: Australian Bureau of Statistics.)

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- Difficult to detect pattern in time plot.
- ACF shows some significant autocorrelation at lags 1, 2, and 3.
- r_{12} relatively large although not significant. This may indicate some slight seasonality.

Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 1990 through August 1995. (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows some significant autocorrelation at lags 1, 2, and 3.
- r_{12} relatively large although not significant. This may indicate some slight seasonality.

These show the series is **not a white noise series**.

Your turn

You can compute the daily changes in the Google stock price using

```
dgoog <- diff(goog)
```

Does `dgoog` look like white noise?

Explanation

The Google stocks can be modelled by the random walk model $y_{t+1} = y_t + \epsilon_t$

where $\epsilon_t \sim N(0, \sigma^2)$
ϵ_t is i.i.d.: hence ϵ_t is independent from ϵ_{t-1}, ϵ_{t-2}.

By differencing:
$y_{t+1} - y_t = \epsilon_t$,
which is indeed a white noise time series.