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Exercise Series 6

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1. Exercise

Consider a 0-1-Knapsack Problem with following object data and total capacity of 20 (m^3):

Object:	A	B	C	D
Value (CHF):	25	21	30	8
Volume (m^3):	10	7	5	4

Solve this problem with the Branch-and-Bound Method discussed in the course.

Use a *depth-first search* strategy for choosing the next node to be processed (as in the example in the lecture notes), and after branching, continue with the successor node obtained by *rounding up* the fractional variable.

Label the branching tree analogously to the notation used in the course.

2. Exercise

Consider the following ILP:

$$\begin{aligned} &\max \ 5x_1 + 6x_2 \\ &3x_1 + 6x_2 \le 18 \\ &9x_1 + 6x_2 \le 27 \\ &x_1, x_2 \ge 0, \text{ integer} \end{aligned}$$

Solve this ILP using the Branch-and-Bound Method discussed in the course. When branching, take always the fractional variable with the *smallest index* for rounding. Use a *depth-first search* strategy for choosing the next node to be processed, and after branching, continue with the successor node obtained by *rounding up* the fractional variable.

Draw the solution space of the considered sub-problems, and solve the associated LP relaxations graphically.

Label the branching tree analogously to the notation used in the course for the Knapsack Problem. Instead of the sets J_r^0, J_r^1 use the sets $J_r^{\le}, J_r^{\ge}$ whose elements are now pairs of the form (i,b). Here, $(i,b) \in J_r^{\le}$ means that in sub-problem *r*, constraint $x_i \le b$ has been added (analogously for $J_r^{\ge}$).

This is not a Knapsack problem because we not have only binary decisions !

Exercise 1

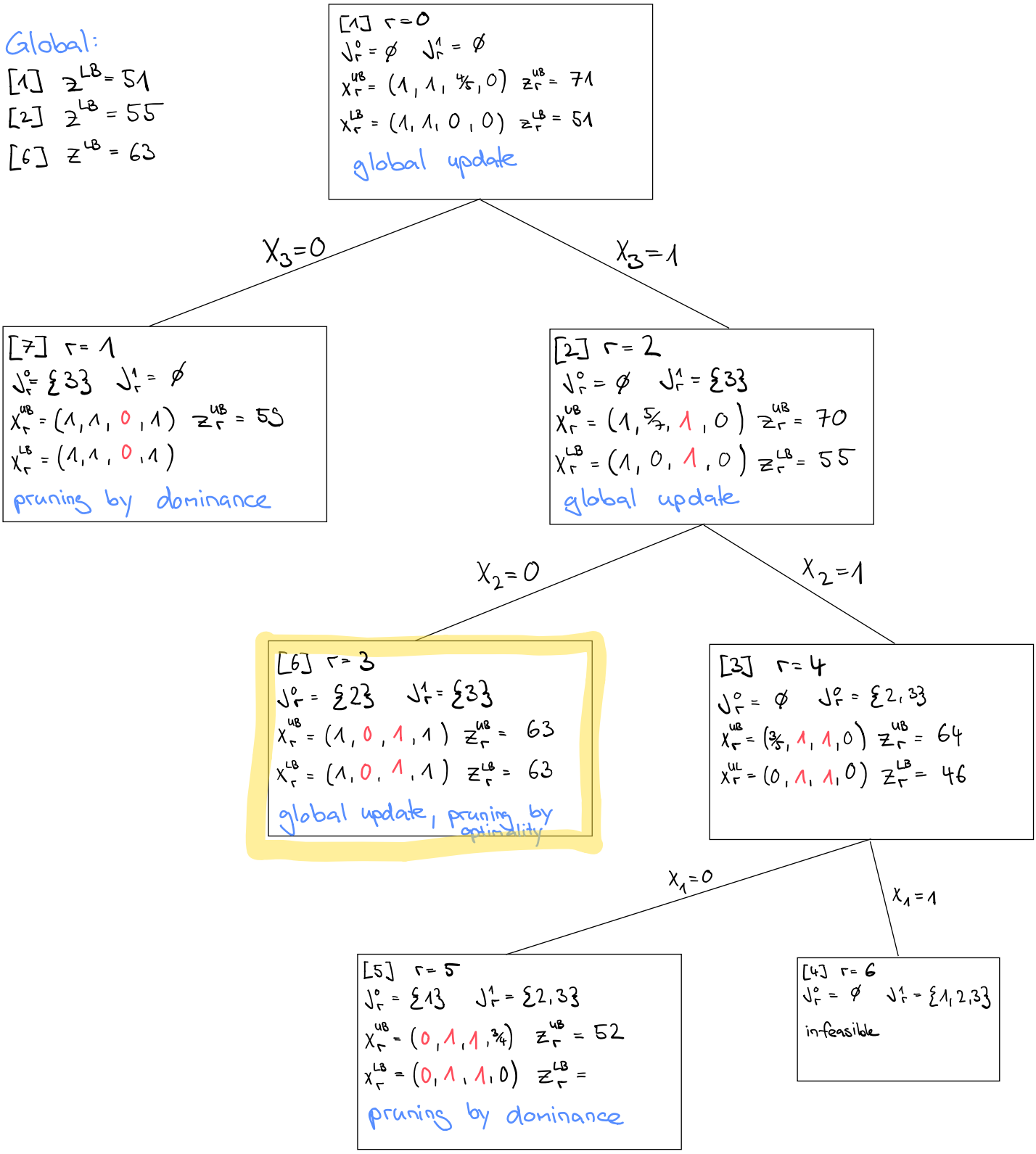
Object	A	B	C	D
c = Value	25	21	30	8
a = Volume	10	7	5	4
$\frac{c_j}{a_j}$	2/5	1/3	1/6	1/2

sorted →

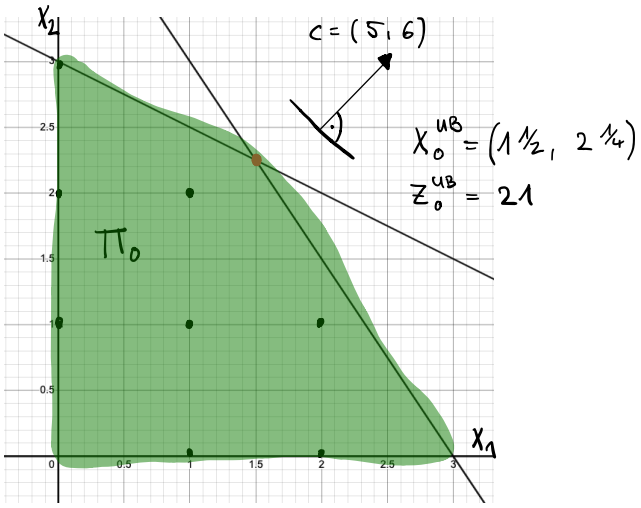
Object	C	B	A	D
c = Value	30	21	25	8
a = Volume	5	7	10	4

capacity b = 20

Global:
[1] $z^{LB} = 51$
[2] $z^{LB} = 55$
[6] $z^{LB} = 63$

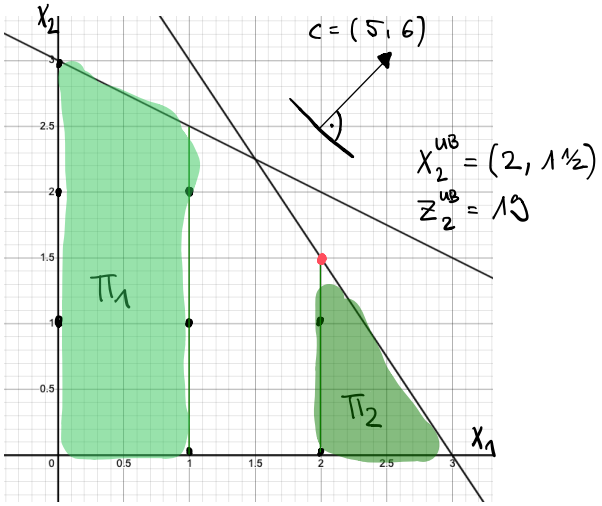


Exercise 2



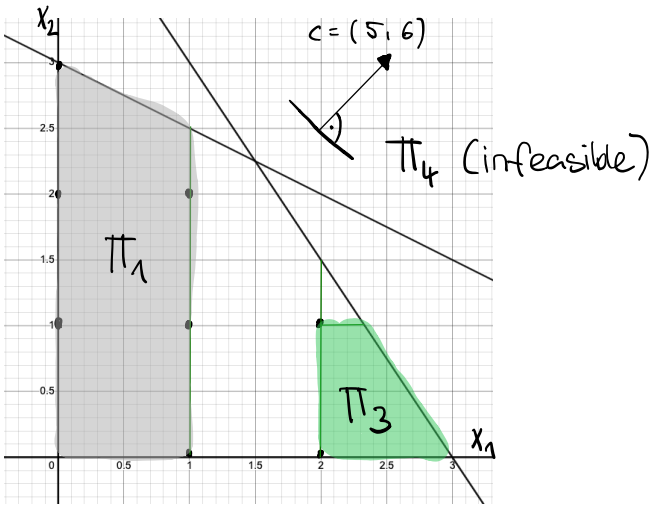
$x_0^{UB} = (1\frac{1}{2}, 2\frac{1}{4})$

$z_0^{UB} = 21$

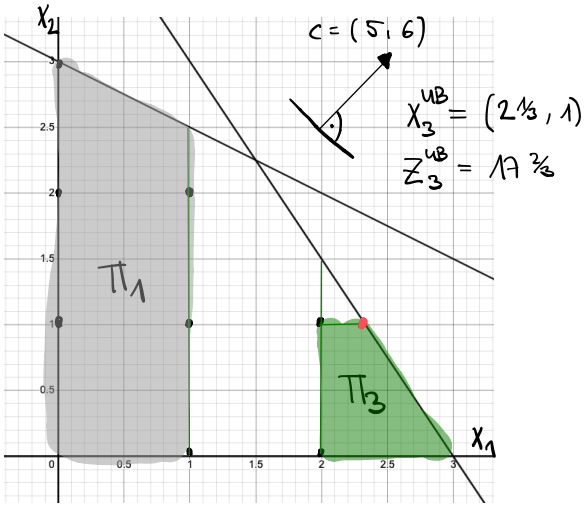


$x_2^{UB} = (2, 1\frac{1}{2})$

$z_2^{UB} = 19$

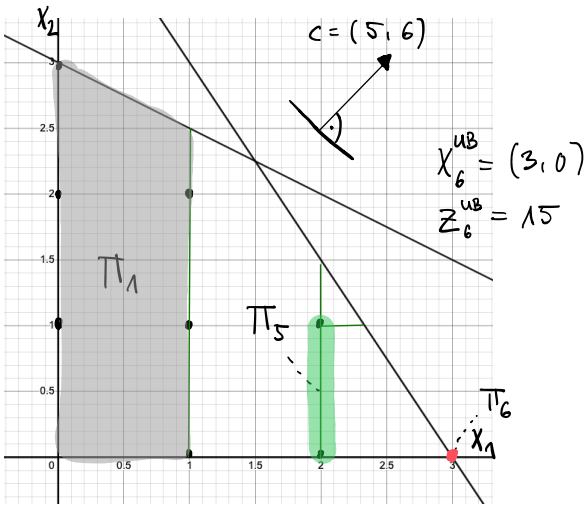


π_4 (infeasible)



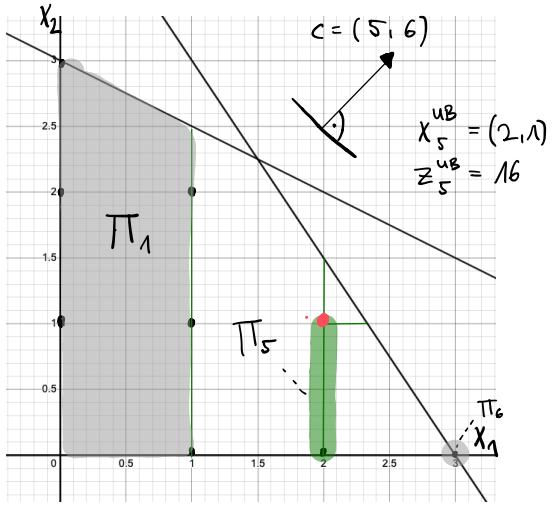
$x_3^{UB} = (2\frac{1}{3}, 1)$

$z_3^{UB} = 17\frac{2}{3}$



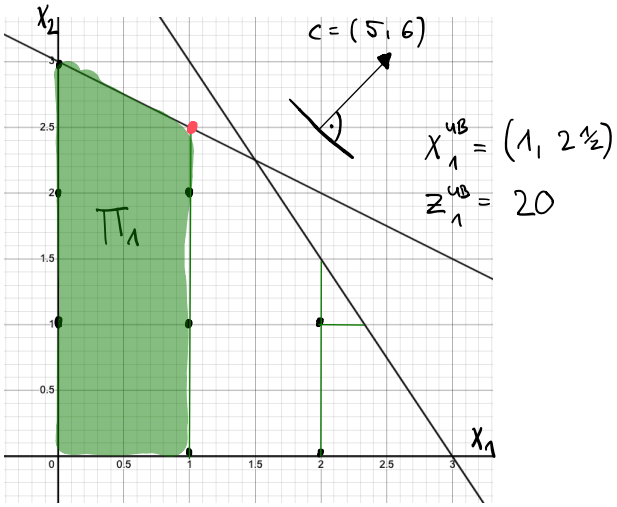
$x_6^{UB} = (3, 0)$

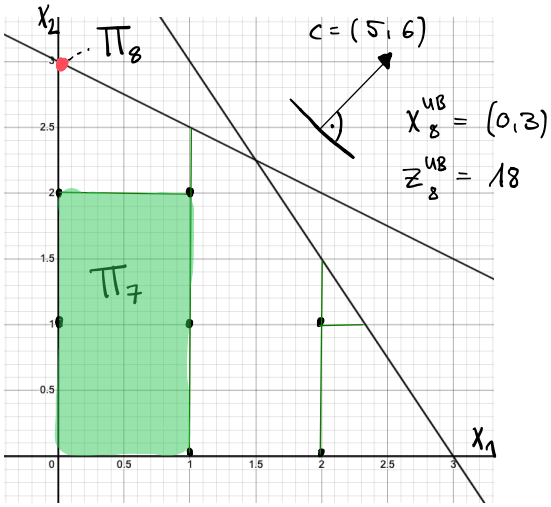
$z_6^{UB} = 15$

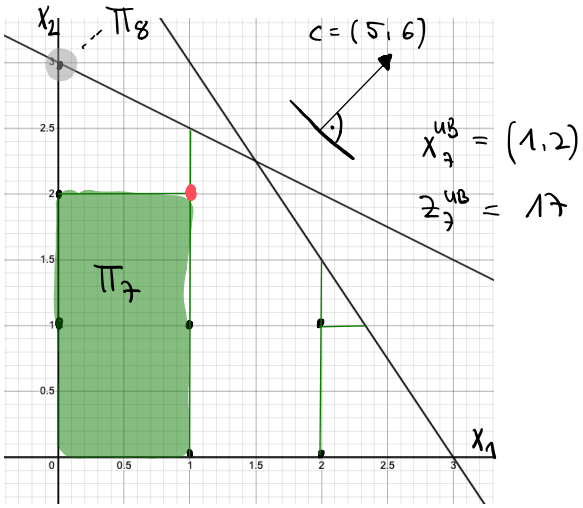


$x_5^{UB} = (2, 1)$

$z_5^{UB} = 16$


x_2
$c = (5, 6)$
π_1
x_1
$x_1^{ub} = (1, 2\frac{1}{2})$
$z_1^{ub} = 20$


x_2
π_8
$c = (5, 6)$
π_7
x_1
$x_8^{ub} = (0, 3)$
$z_8^{ub} = 18$


x_2
π_8
$c = (5, 6)$
π_7
x_1
$x_7^{ub} = (1, 2)$
$z_7^{ub} = 17$

Global:

[5]: $z^{LB} = 15$

[6]: $z^{LB} = 16$

[8]: $z^{LB} = 18$