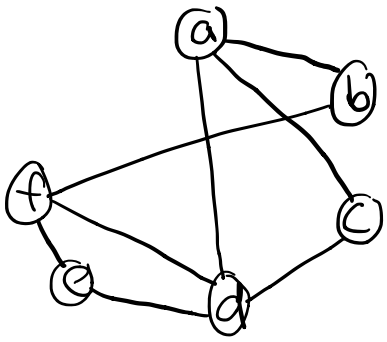
Exercise 10

Task 1



a) Given the adjacency list:

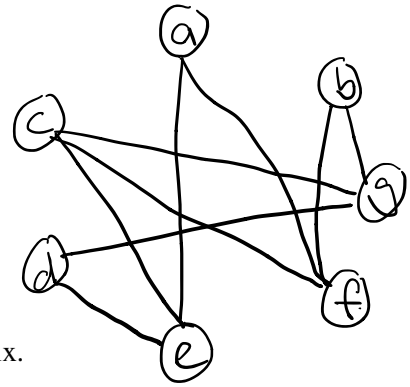
a	b	c	d	
b	a	f		
c	a	d		
d	a	c	e	f
e	d	f		
f	b	d	e	

Draw the graph for the above adjacency list.

b) Given the adjacency matrix:

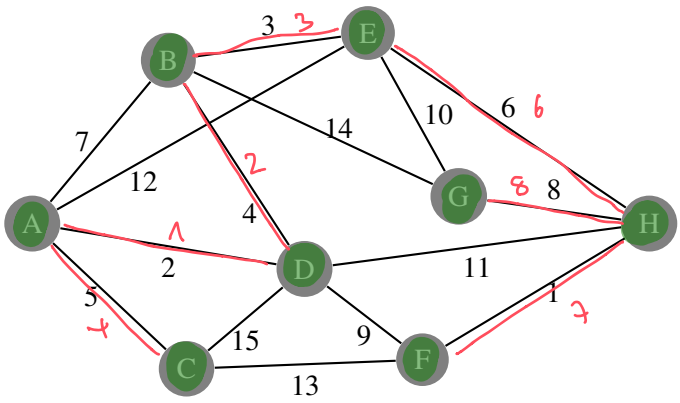
	a	b	c	d	e	f	g
a	0	0	0	0	1	1	0
b	0	0	0	0	0	1	1
c	0	0	0	0	1	1	1
d	0	0	0	0	1	0	1
e	1	0	1	1	0	0	0
f	1	1	1	0	0	0	0
g	0	1	1	1	0	0	0

Draw the graph for the above adjacency matrix.



Task 2

An electricity company wants to construct power lines to connect all cities (vertices) in the graph below. It wants to determine a cost-optimal choice of connections, where the construction costs of the connections are given by the edge weights in the graph below.



In which order are the connections built if Prim's algorithm is used, starting from city A?

AD, BD, BE, AC, EH, FH, GH ✓

Task 3

Given the following table with distances between 9 cities.

	Brugg	Basel	Bern	Chur	Geneva	Lugano	Lucerne	St. Gallen	Zurich
Brugg		58	101	149	250	237	84	114	33
Basel	58		93	198	243	258	94	163	83
Bern	101	93		237	156	271	109	202	122
Chur	149	198	237		386	142	140	102	118
Geneva	250	243	156	386		428	263	357	276
Lugano	237	258	271	142	428		165	243	206
Lucerne	84	94	109	140	263	165		139	55
St. Gallen	114	163	202	102	357	243	139		84
Zurich	33	83	122	118	276	206	55	84	

a) Draw the graph which is given by the above weighted adjacency matrix.

b) You are planning a fibre-optic network designed to connect the cities. Calculate the minimum length of cables you need to lay.

c) Calculate the shortest distance from Zurich to all other cities with Dijkstra's algorithm. What do you find?

Task 4

Given the following table with the distances (in km) of flight routes between 6 airports.

	A	B	C	D	E	F
A		200	580		250	1200
B	200		500	820		
C	580	500		230	150	1100
D		820	230		380	
E	250		150	380		
F	1200		1100			

a) Draw the graph which is given by the above weighted adjacency matrix.

b) Starting from A, find all airports in the network using breadth-first and depth-first search. Give the airports in the sequence they are encountered if you process the neighbors of each vertex in alphabetical order.

c) Find a set of flight routes that connects all cities and has minimum total travel distance.

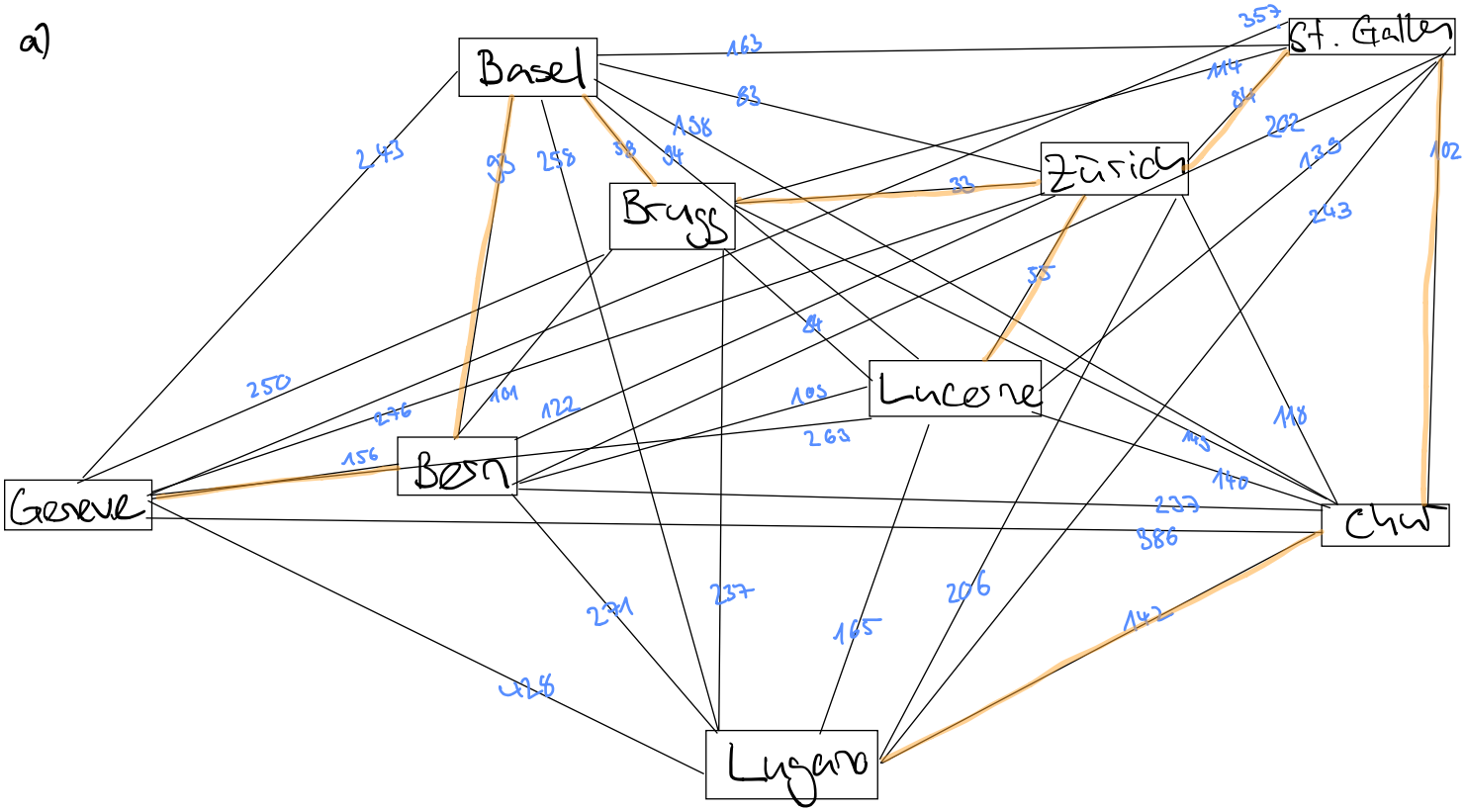
d) Calculate the shortest distance from A to all other airports with Dijkstra's algorithm.

e) Calculate the shortest distance between all airports with the Floyd-Warshall algorithm.

f) Does the set of flight routes found in c) necessarily minimize the total distance flown by the airline, without further assumptions about the occupancy rate of the flight routes?

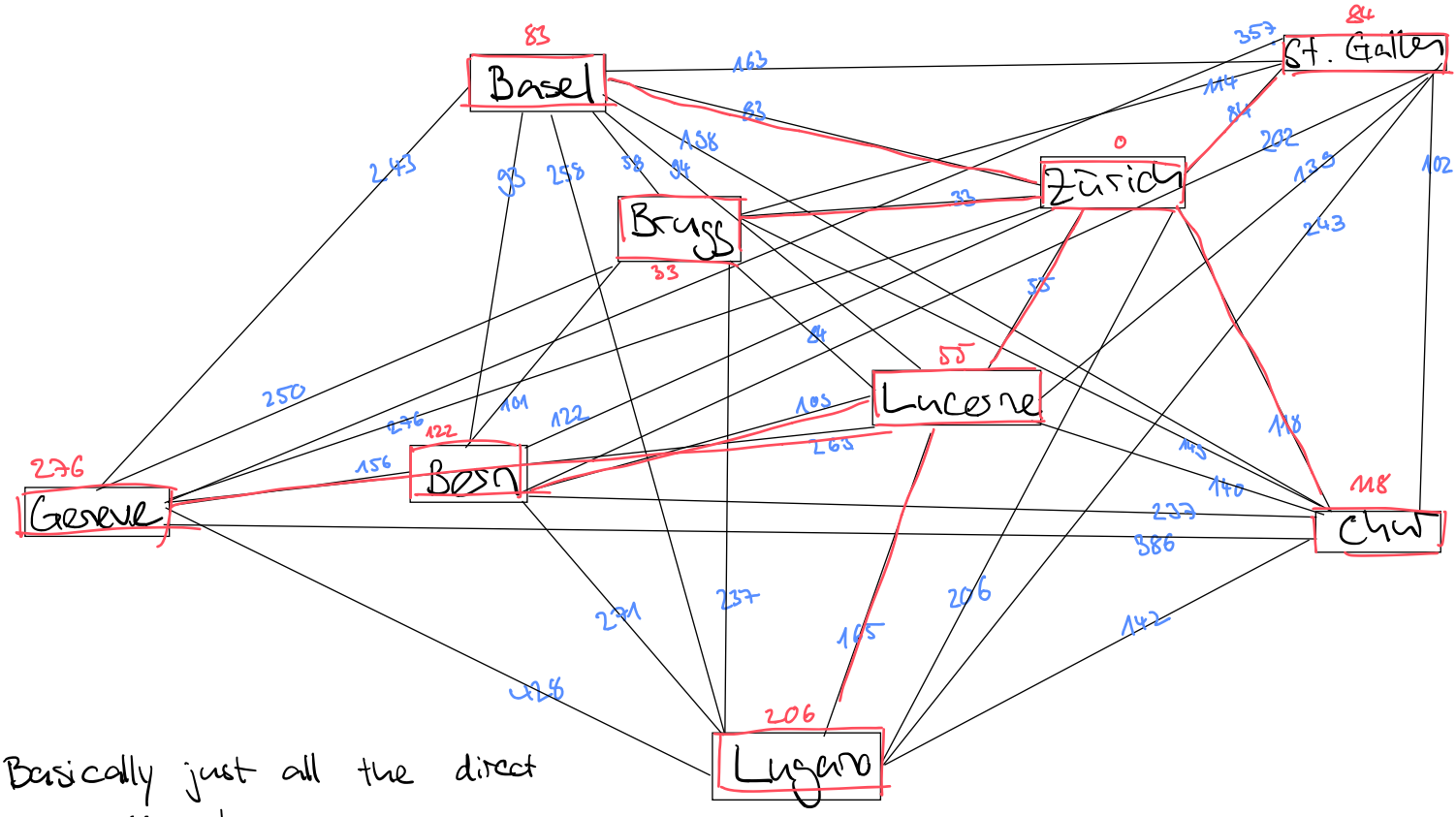
Task 3

a)



b) 33 + 55 + 58 + 84 + 93 + 102 + 142 + 156 = 723

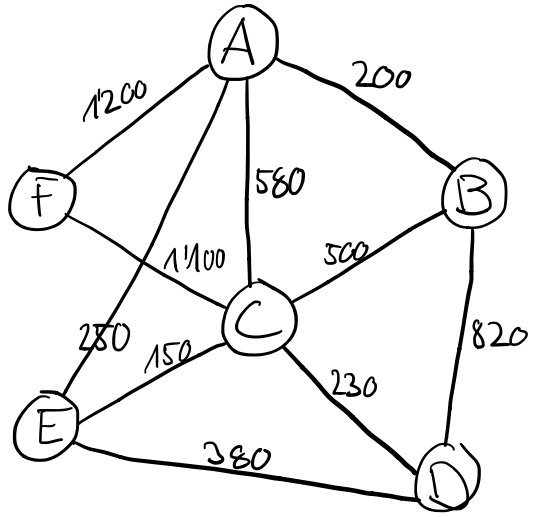
c)



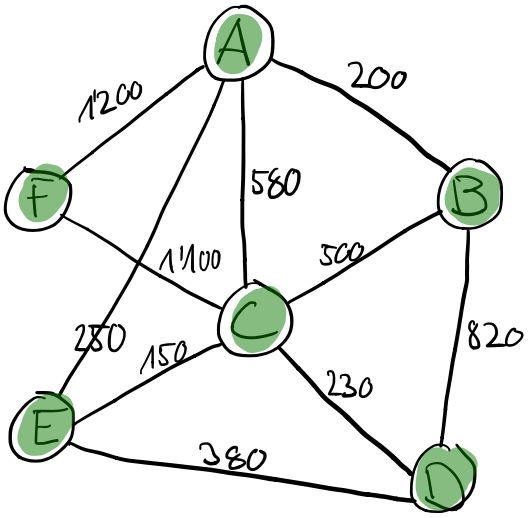
Basically just all the direct connections!

Task 4

a)



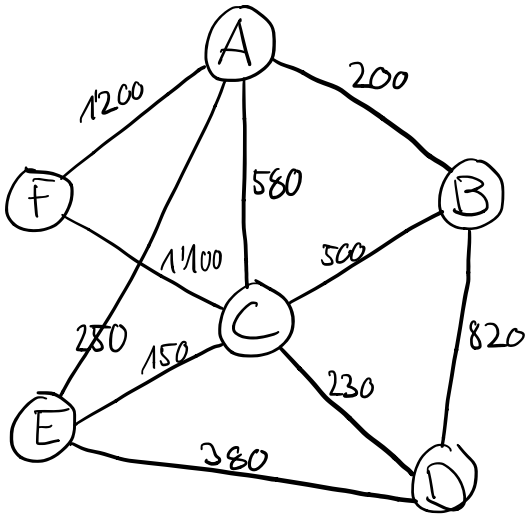
b) BFS



~~A B C E F D~~

A B C E F D

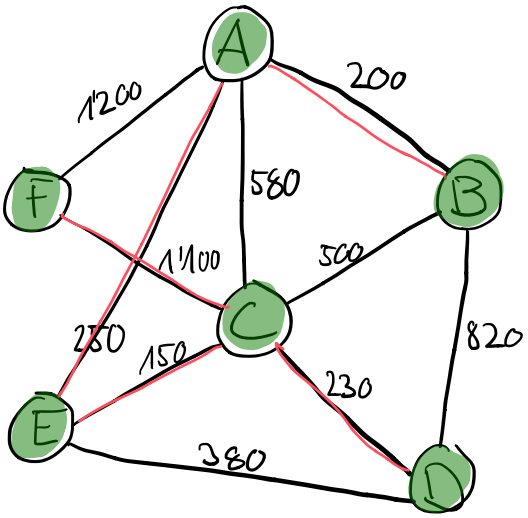
DFS



F
~~E~~
~~D~~
C
B
A

E D F C B A

c) Prim

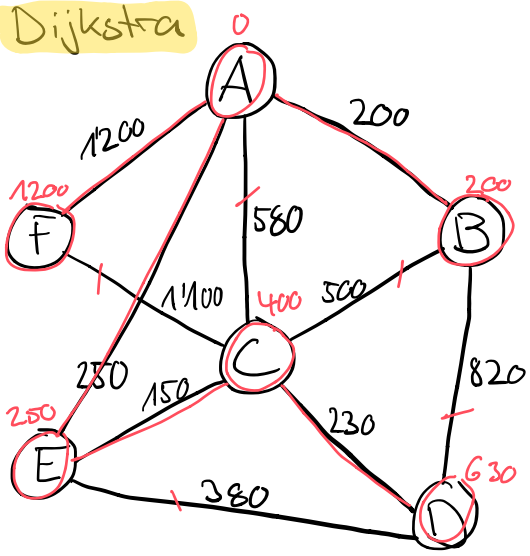


Weight of minimal spanning tree

200 + 250 + 150 + 230 + 1'100 = 1'930

a)

Dijkstra



A = 0
B = 200
C = 400
D = 630
E = 250
F = 1200

c) Floyd - Warshall

$\min(m_{ij}, m_{ik} + m_{kj})$

k=1:

	A	B	C	D	E	F
A	0	200	580	–	250	1200
B	200	0	500	820	450	1'400
C	580	500	0	230	150	1100
D	-	820	230	0	380	–
E	250	450	150	380	0	1'450
F	1200	1'400	1100	–	1'450	0

k=2:

	A	B	C	D	E	F
A	–	200	580	1'020	250	1200
B	200	0	500	820	450	1'400
C	580	500	0	230	150	1100
D	1'020	820	230	0	380	2'220
E	250	450	150	380	0	1'450
F	1200	1'400	1100	2220	1'450	0

k=3:

	A	B	C	D	E	F
A	0	200	580	810	250	1200
B	200	0	500	730	450	1'400
C	580	500	0	230	150	1100
D	810	730	230	0	380	1'330
E	250	450	150	380	0	1'250
F	1200	1'400	1100	1'330	1'250	0

k=4:

	A	B	C	D	E	F
A	0	200	580	810	250	1200
B	200	0	500	820	450	1'400
C	580	500	0	230	150	1100
D	810	820	230	0	380	1'330
E	250	450	150	380	0	1'250
F	1200	1'400	1100	1'330	1'250	0

k=5:

	A	B	C	D	E	F
A	0	200	580	630	250	1200
B	200	0	500	820	450	1'400
C	580	500	0	230	150	1100
D	630	820	230	0	380	1'330
E	250	450	150	380	0	1'250
F	1200	1'400	1100	1'330	1'250	0

k=6:

	A	B	C	D	E	F
A	0	200	580	630	250	1200
B	200	0	500	820	450	1'400
C	580	500	0	230	150	1100
D	630	820	230	0	380	1'330
E	250	450	150	380	0	1'250
F	1200	1'400	1100	1'330	1'250	0

d) No, because A to F is 1'500 in the spanning tree. There is a better route.

Task 5

The following table gives the transport cost per container (in CHF) for the cheapest direct shipping lines between the ports A, B, C, D, E, F:

	A	B	C	D	E	F
A		60	160	-	700	600
B	60		220	140	-	800
C	160	220		20	900	500
D	-	140	20		800	300
E	700	-	900	800		40
F	600	800	500	300	40	

Using the Floyd-Warshall algorithm, determine the matrix of minimal transport costs per container between any two ports if only stop-overs in ports A, D and F are allowed.

Task 6 (*optional)

a) Implement a program that for a given weighted adjacency matrix computes a minimum spanning tree using Prim's algorithm or study one of the implementations of Prim's algorithm available from Moodle. Try it out on the example from Task 2.

b) Adapt an implementation of Prim's algorithm to an implementation of Dijkstra's algorithm, that, given a weighted adjacency matrix and a starting vertex, computes a shortest paths tree. Alternatively study the implementation of Dijkstra's algorithm on Moodle.

c) Implement a program that for a given weighted adjacency matrix computes the shortest pairwise distances using the Floyd-Warshall algorithm or study one of the implementations of Floyd-Warshall's algorithm available from Moodle. Try it out on the examples from Task 4. Extend the program so it can also be used to compute all shortest paths using only certain vertices. Re-solve Task 5 with your program.

Task 5

Using only A

A	B	C	D	E	F
0	60	160	–	700	600
60	0	220	140	760	660
160	220	0	20	860	500
–	140	20	0	800	300
700	760	860	800	0	40
600	660	500	300	40	0

Using A and D

A	B	C	D	E	F
0	60	160	–	700	600
60	0	160	140	760	440
160	160	0	20	820	320
–	140	20	0	800	300
700	760	820	800	0	40
600	440	320	300	40	0

Using A, D and F

A	B	C	D	E	F
0	60	160	500	640	600
60	0	160	140	480	440
160	160	0	20	360	320
300	140	20	0	340	300
640	480	360	340	0	40
600	440	320	300	40	0

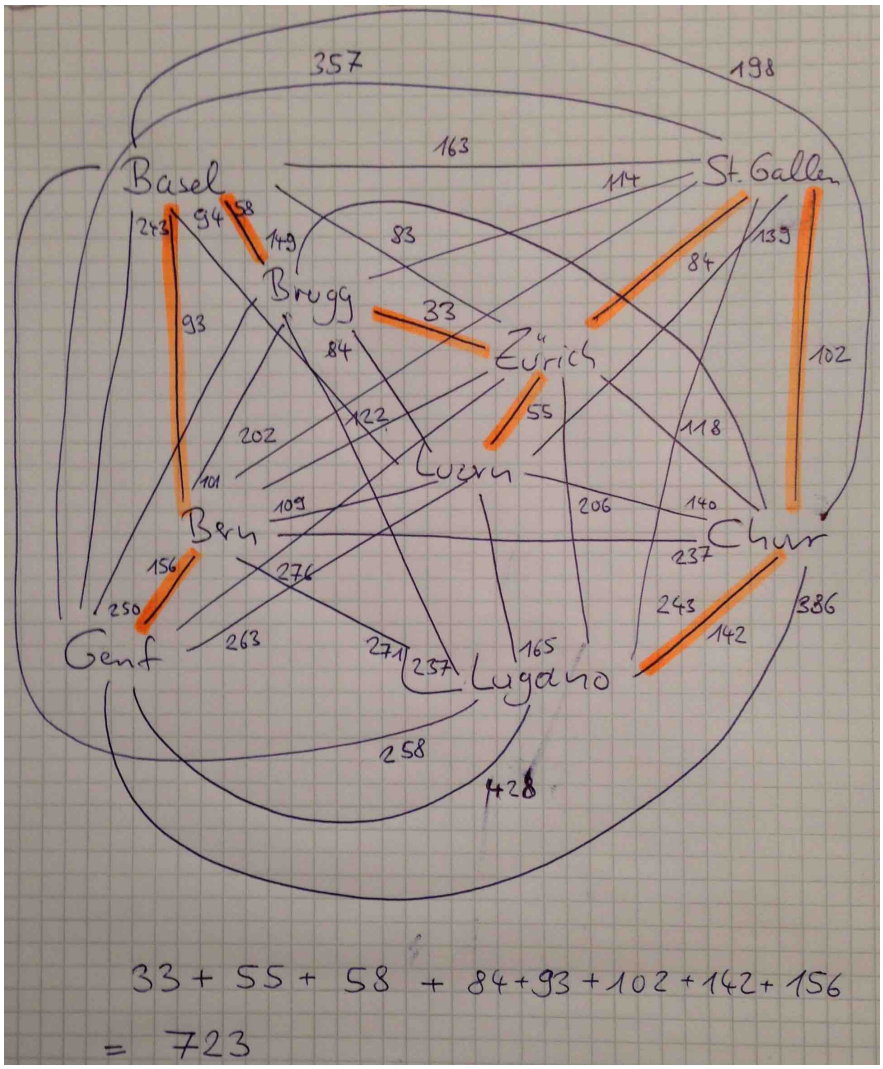
Solutions to Exercise 10

Solution to Task 2

The edges are constructed in the order AD, BD, BE, AC, EH, FH, GH.

Solution to Task 3

a)



b) $33 + 55 + 58 + 84 + 93 + 102 + 142 + 156 = 723$.

c) The shortest distances from Zurich to all other cities are already given in the last column of the distance matrix. The input data already gives the *shortest* distances between cities.

Solution to Task 4

a) Arrange the nodes e.g. sorted alphabetically in a circle and draw in the edges.

b) Depth-first search: E D F C B A, breadth-first search: A B C E F D.

c) The weight of a minimal spanning tree is $150 + 200 + 230 + 250 + 1100 = 1930$.

d) l(A) = 0, l(B) 200, l(E) = 250, l(C) = 400, l(D) = 630, l(F) = 1200

e)

	A	B	C	D	E	F
A	0	200	580	-	250	1200
B	200	0	500	820	-	-
C	580	500	0	230	150	1100
D	-	820	230	0	380	-
E	250	-	150	380	0	-
F	1200	-	1100	-	-	0

	A	B	C	D	E	F
A	0	200	580	-	250	1200
B	200	0	500	820	450	1400
C	580	500	0	230	150	1100
D	-	820	230	0	380	-
E	250	450	150	380	0	1450
F	1200	1400	1100	-	1450	0

	A	B	C	D	E	F
A	0	200	580	1020	250	1200
B	200	0	500	820	450	1400
C	580	500	0	230	150	1100
D	1020	820	230	0	380	2220
E	250	450	150	380	0	1450
F	1200	1400	1100	2220	1450	0

	A	B	C	D	E	F
A	0	200	580	810	250	1200
B	200	0	500	730	450	1400
C	580	500	0	230	150	1100
D	810	730	230	0	380	1330
E	250	450	150	380	0	1250
F	1200	1400	1100	1330	1250	0

	A	B	C	D	E	F
A	0	200	580	810	250	1200
B	200	0	500	730	450	1400
C	580	500	0	230	150	1100
D	810	730	230	0	380	1330
E	250	450	150	380	0	1250
F	1200	1400	1100	1330	1250	0

	A	B	C	D	E	F
A	0	200	400	630	250	1200
B	200	0	500	730	450	1400
C	400	500	0	230	150	1100
D	630	730	230	0	380	1330
E	250	450	150	380	0	1250
F	1200	1400	1100	1330	1250	0

	A	B	C	D	E	F
A	0	200	400	630	250	1200
B	200	0	500	730	450	1400
C	400	500	0	230	150	1100
D	630	730	230	0	380	1330
E	250	450	150	380	0	1250
F	1200	1400	1100	1330	1250	0

f) No, because e.g. in the minimal spanning tree determined in c) to travel distance from A to F is $250+150+1100 = 1500$. If this flight route is used much more often than the others, then the direct path from A to F should be inserted (and possibly another edge dropped).

Solution to Task 5

The intermediate matrices are as follows:

Using only A:

	A	B	C	D	E	F
A		60	160	-	700	600
B	60		220	140	**760**	**660**
C	160	220		20	**860**	500
D	-	140	20		800	300
E	700	**760**	**860**	800		40
F	600	**660**	500	300	40	

Using A and D:

	A	B	C	D	E	F
A		60	160	-	700	600
B	60		**160**	140	760	**440**
C	160	**160**		20	**820**	**320**
D	-	140	20		800	300
E	700	760	**820**	800		40
F	600	**440**	**320**	300	40	

Using A, D, and F:

	A	B	C	D	E	F
A		60	160	**900**	**640**	600
B	60		160	140	**480**	440
C	160	160		20	**360**	320
D	**900**	140	20		**340**	300
E	**640**	**480**	**360**	**340**		40
F	600	440	320	300	40	

Solution to Task 6

See R scripts on moodle.