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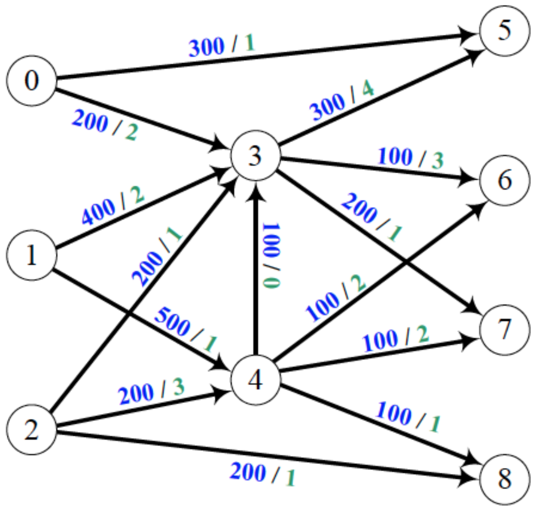
Exercise 12

Task 1

In order to cover the peak in demand at noon, an electricity company can connect three pump-storage power plants (nodes 0,1,2) to the network and easily sell all the power produced via substations (nodes 5 to 8). Sales are limited by the transport capacities (blue values, in MW) of the transition network given below, and the power transmission has a certain cost for each edge (green values, in arbitrary units per MW).



What is the maximum amount of power the company can sell, and how much power should each power plant produce?

What is the minimum transport cost for the maximum flow?

Task 2

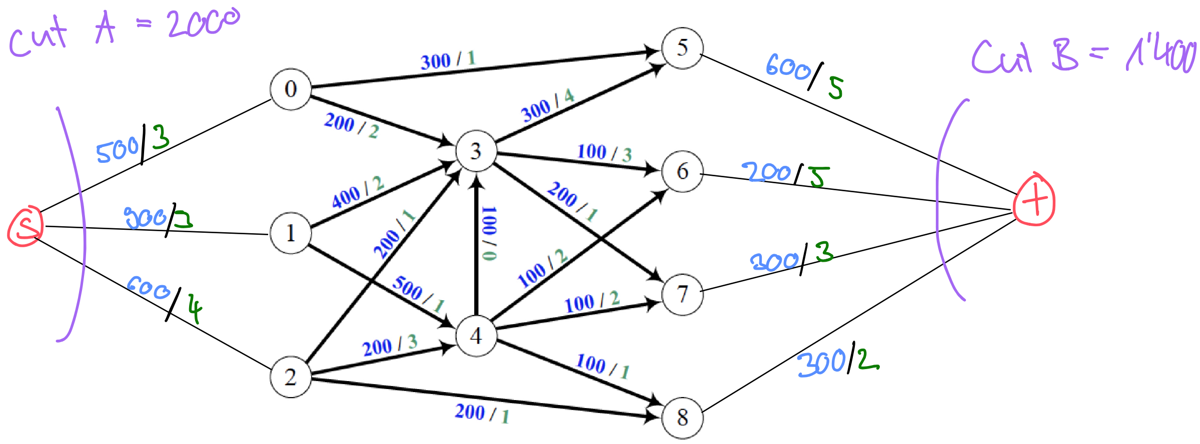
The locomotive depots D1 and D2 contain 18 and 12 locomotives respectively. The three stations S1, S2, S3 require 11, 10 and 9 of these locomotives respectively. The distances between the depots and the stations are given in the following table:

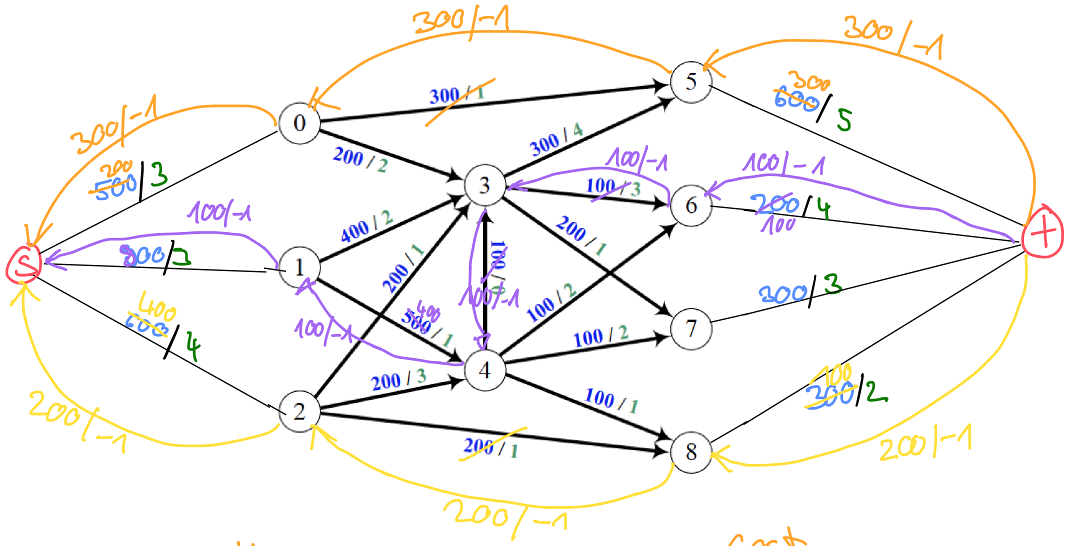
	S1	S2	S3
D1	50 km	40 km	90 km
D2	70 km	80 km	90 km

How must the locomotives be distributed so that the sum of the total distances travelled is as small as possible? Try to solve the problem with the mincost-maxflow algorithm.

Task 1

a) First we reduce the distribution problem to a Max-Flow problem



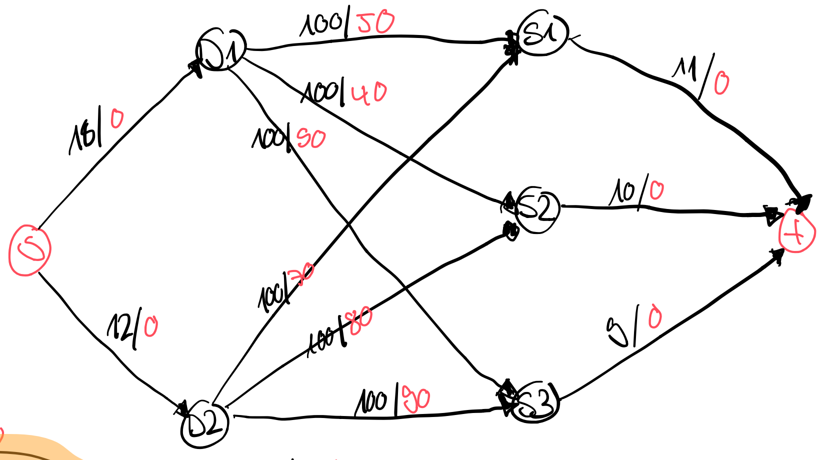


capacity	path	cost
300	s → 0 → 5 → t	1
200	s → 2 → 8 → t	1
100	s → 1 → 4 → 3 → 6 → t	1

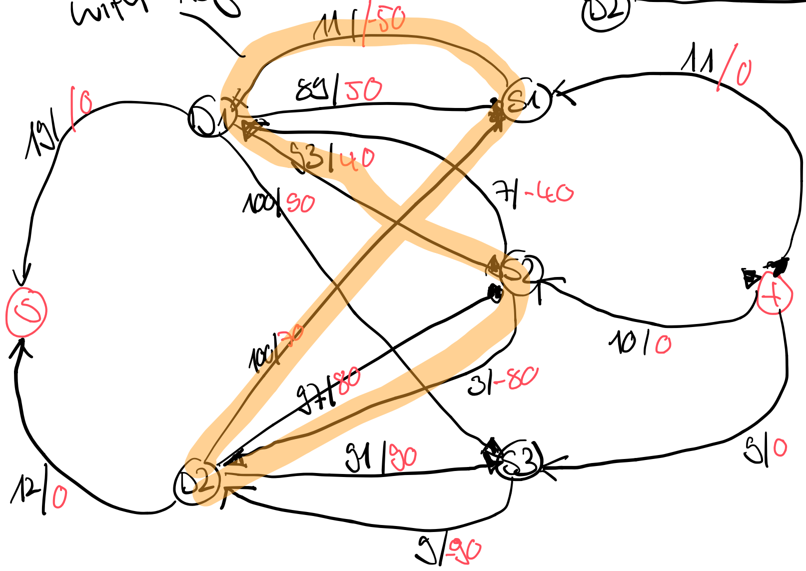
...

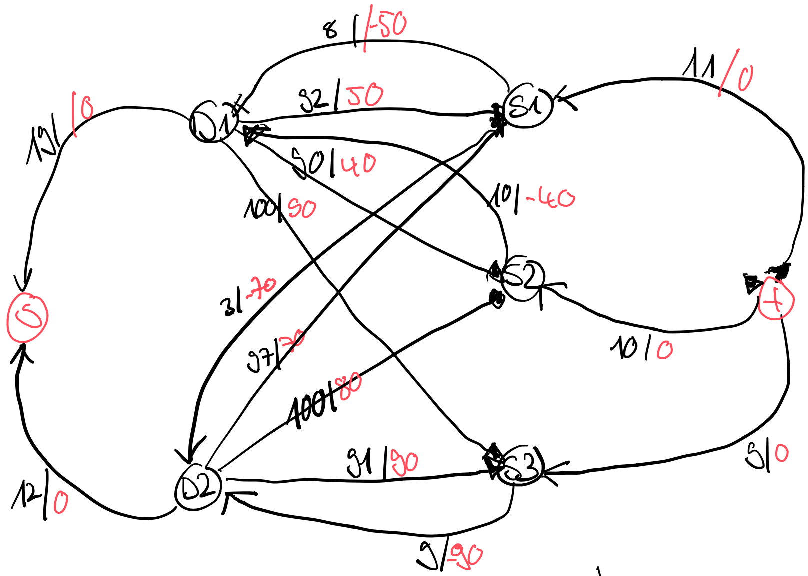
Task 2

	S1	S2	S3
D1	50	40	90
D2	70	80	80



one cycle with negative cost





cost

capacity	path	cost
11	$s \to D1 \to S1 \to t$	$11 \cdot 50 = 550$
9	$s \to D2 \to S3 \to t$	$9 \cdot 90 = 810$
3	$s \to D2 \to S2 \to t$	$3 \cdot 80 = 240$
7	$s \to D1 \to S2 \to t$	$7 \cdot 40 = 280$

we have one cycle with negative cost

3	$D2 \to S1 \to D1 \to S2 \to D2$	$3 \cdot -20 = -60$

1'820 km

Task 3

Consider the implementations in R of a hill-climbing algorithm, a simulated annealing algorithm and a tabu-search algorithm for minimizing the (continuous) Rosenbrock function (`http://en.wikipedia.org/wiki/Rosenbrock_function`). Try to understand the algorithms and experiment with the adjustment options.

Task 4

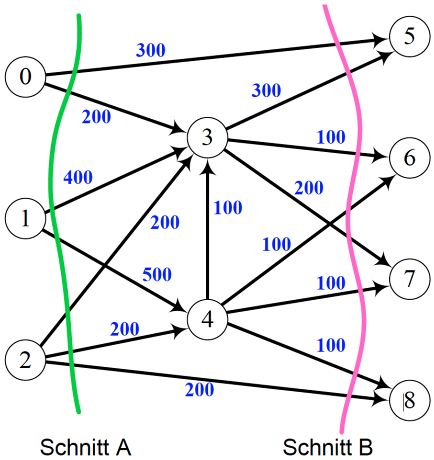
Try to come up with a new heuristic for solving the TSP. What would be possible modification steps to obtain adjacent TSP solutions?

→ Start with a random tour and then explore the neighbourhood which is defined through move-set (2-change).
We could then for example use Tabu search.

Solutions to Exercise 12

Solution to Task 1

Before we apply the mincost-maxflow algorithm, we consider two easy cuts to get some feeling for what we can hope to achieve:
Cut A : 300 + 200 + 400 + 500 + 200 + 200 + 200 = 2000
Cut B : 300 + 300 + 100 + 100 + 200 + 100 + 100 + 200 = 1400
This tells us that a flow of at most 1400 can be achieved. Because the edges around nodes 3 and 4 have enough capacity, the max flow is supposedly 1400.



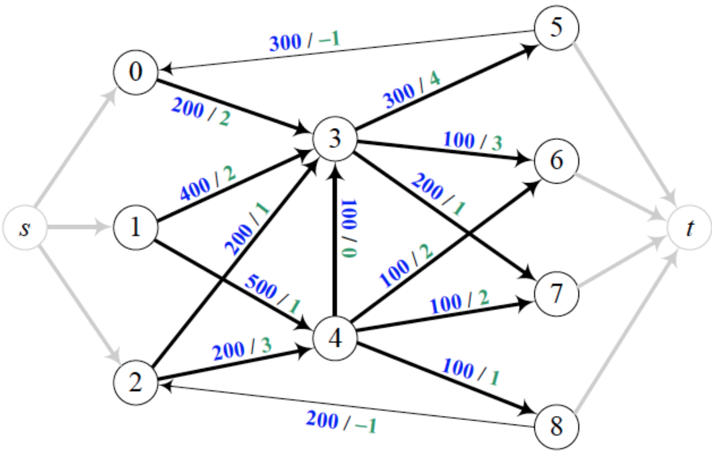
For the actual flow computation, we extend the graph to form an s-t-network. The capacities from s and to t are unlimited and the costs of these edges are 0.

There are many possible ways to choose the augmenting paths. E.g.:

$s \to 0 \to 5 \to t$ with capacity 300 and cost 1.
$s \to 2 \to 8 \to t$ with capacity 200 and cost 1.

Intermediate residual network:

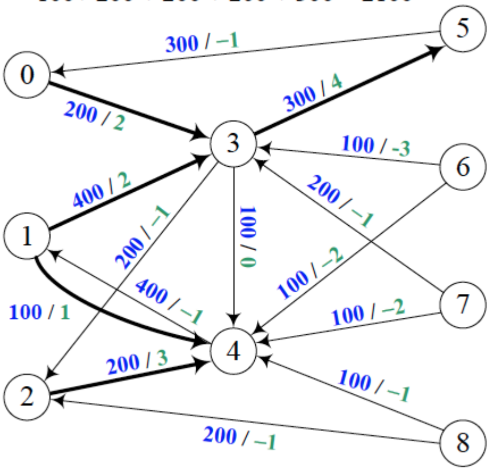
$f_1 = 300 + 200$
$k_1 = 300 + 200$



$s \to 1 \to 4 \to 3 \to 6 \to t$ with capacity 100 and cost 4.
$s \to 1 \to 4 \to 6 \to t$ with capacity 100 and cost 3.
$s \to 1 \to 4 \to 7 \to t$ with capacity 100 and cost 3.
$s \to 1 \to 4 \to 8 \to t$ with capacity 100 and cost 2.
$s \to 2 \to 3 \to 7 \to t$ with capacity 200 and cost 2.

Intermediate residual network:

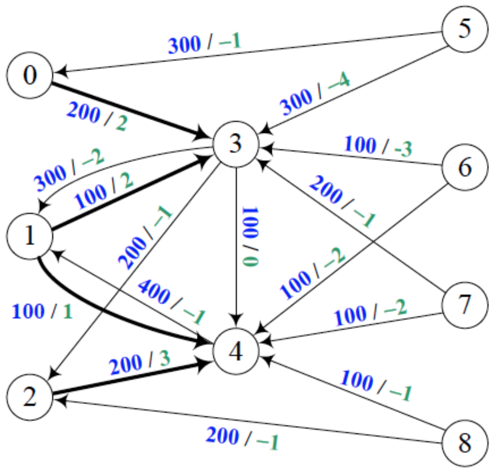
$f_2 = f_1 + 400 + 200 = 1100$
$k_2 = k_1 + 400 + 200 +$
$100 + 200 + 200 + 200 + 300 = 2100$



$s \to 1 \to 3 \to 5 \to t$ with capacity 300 and cost 6.

Final residual network:

$$f_3 = f_2 + 300 = 1400$$
$$k_2 = k_2 + 600 + 1200 = 3900$$



For this choice of augmenting paths, no cycles with negative costs arise which would have to be resolved. Thus the value of the maximum flow is indeed 1400, with a minimum cost of 3900.

In the final flow, the following edges are used:

$s \to 0$ with a flow of 300
$s \to 1$ with a flow of 700
$s \to 2$ with a flow of 400
(These values correspond to the power production at each of the power plants.)

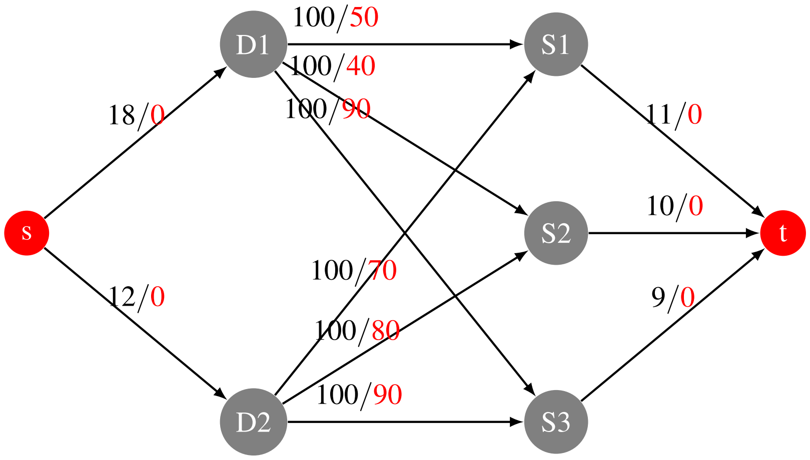
$0 \to 5$ with a flow of 300
$1 \to 3$ with a flow of 300
$1 \to 4$ with a flow of 400
$2 \to 3$ with a flow of 200
$2 \to 8$ with a flow of 200

$3 \to 5$ with a flow of 300
$3 \to 7$ with a flow of 200
$4 \to 3$ with a flow of 100
$4 \to 6$ with a flow of 100
$4 \to 7$ with a flow of 100
$4 \to 8$ with a flow of 100

$5 \to t$ with a flow of 600
$6 \to t$ with a flow of 200
$7 \to t$ with a flow of 300
$8 \to t$ with a flow of 300

Solution to Task 2

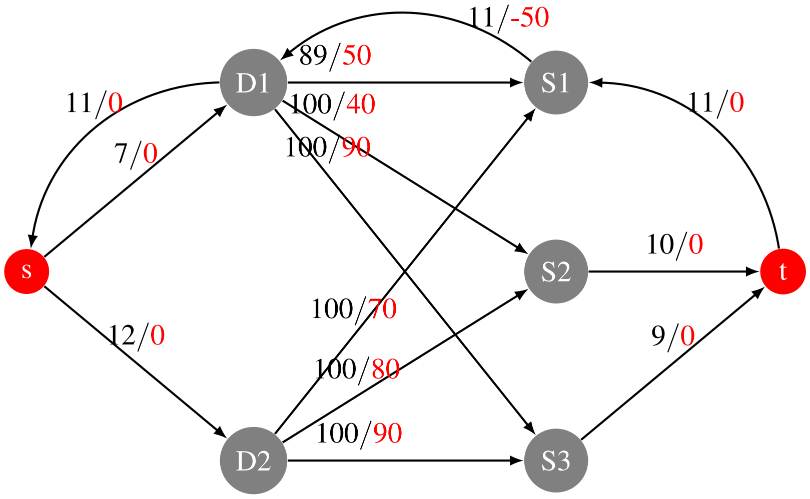
The railway capacities unrestricted. In the following we set these capacities arbitrarily to 100; this is sufficiently large so as not to imply any actual restriction. With this in mind, the situation can be modeled by the following s-t-network:



A mincost maxflow in this network corresponds to a cost-optimal assignment of the locomotives to stations. (Here it is quite obvious that a maxflow of 30 is possible, i.e. all locomotives can be distributed to stations as desired. The interesting part here is the cost minimization.)

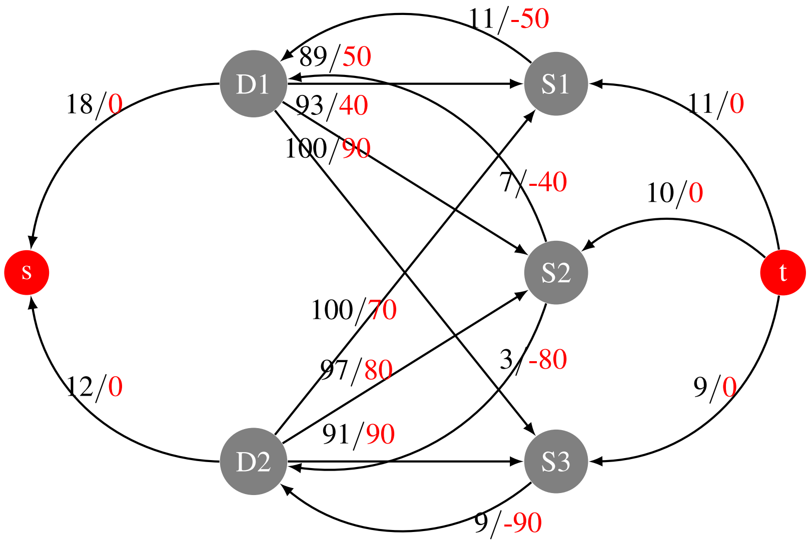
As our first augmenting path, we can choose e.g.
$s \to$ D1 $\to$ S1 $\to t$ with capacity 11 and cost $11 \cdot (0 + 50 + 0) = 550$.

After this step, we have the following residual network:



For the next three paths, we can choose e.g.
$s \to$ D1 $\to$ S2 $\to t$ with capacity 7 and cost $7 \cdot 40 = 280$.
$s \to$ D2 $\to$ S2 $\to t$ with capacity 3 and cost $3 \cdot 80 = 240$.
$s \to$ D2 $\to$ S3 $\to t$ with capacity 9 and cost $9 \cdot 90 = 810$.

After these steps, the residual network is:



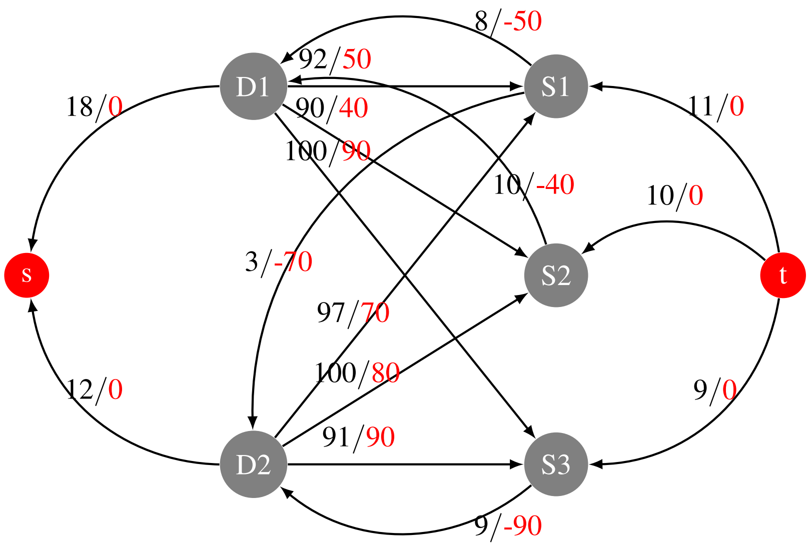
As there are no more paths from s to t, we have found a maximum flow (of value 30), with a cost of $550 + 280 + 240 + 810 = 1880$. To check whether we can improve on this cost, we search for cycles of negative cost.

The only cyle of negative cost is D2 $\overset{70}{\to}$ S1 $\overset{-50}{\to}$ D1 $\overset{40}{\to}$ S2 $\overset{-80}{\to}$ D2. It has a total cost of -20, and the maximum capacity in this cycle is 3. Therefore, we can save a cost of $3 \cdot 20 = 60$ by resolving this cycle, i.e. by rerouting 3 units of flow from D1 $\overset{50}{\to}$ S1 and D2 $\overset{80}{\to}$ S2 to D2 $\overset{70}{\to}$ S1 and D1 $\overset{40}{\to}$ S2. (Note that this step does not change the total flow from s to t!)

Accordingly, we note

D2 $\to$ S1 $\to$ D1 $\to$ S2 $\to$ D2 with cost $3 \cdot (70 + (-50) + 40 + (-80)) = -60$.

The residual network after this step is:



Since no additional cycles with negative cost are present, we have found a maximum flow with minimum cost and are finished. (Admittedly, the fact that no other cycles are present is not quite so easy to see from the drawing.)

Adding up the costs we have calculated previously, we obtain a total cost of $550 + 280 + 240 + 810 + (-60) = 1820$. That is, the sum of the distances traveled is 1820 km in the optimal solution.

The usage of the individual edges can be determined in two ways:

1) By adding up the capacities along the paths:
$s \to$ D1 $\to$ S1 $\to t$ with capacity 11.
$s \to$ D1 $\to$ S2 $\to t$ with capacity 7.
$s \to$ D2 $\to$ S2 $\to t$ with capacity 3.
$s \to$ D2 $\to$ S3 $\to t$ with capacity 9.
D2 $\to$ S1 $\to$ D1 $\to$ S2 $\to$ D2 with capacity 3.

Thus:
D1 $\to$ S1 with capacity $11 - 3 = 8$
D1 $\to$ S2 with capacity $7 + 3 = 10$
D1 $\to$ S3 with capacity $0 = 0$
D2 $\to$ S1 with capacity $0 + 3 = 3$
D2 $\to$ S2 with capacity $3 - 3 = 0$
D2 $\to$ S3 with capacity $9 = 9$

In this way we see again the redistribution of flow caused by the resolution of the negative cost cycle.

2) More conveniently, we can read off these values from the backwards edges in the residual graph. (Option 1 above is recomputing what we already computed when updating the residual graph throughout the algorithm!)

Solution to Task 3

See R solutions.

Solution to Task 4

See discussion in class.