DEMG 6090 Homework 3

Adriana Scanteianu

October 2, 2023

PSET2 Part B

Problem 1

Demonstrate that if survival probabilities and the absolute number of births are constant over time, then the conventional IMR is equal to the refined IMR.

$$IMR_{conventional}(T) = \frac{D_0(T)}{B(t)} = \frac{{}_sD_0(T) + {}_pD_0(T)}{B(T)}$$

$$IMR_{refined}(T) = \frac{{}_sD_0(T)}{B(T)} + \frac{B(T) - {}_sD_0(T)}{B(T)} * \frac{{}_pD_0(T)}{B(T-1) - {}_sD_0(T-1)}$$

When survival probabilities and the absolute number of births are constant over time, we have that $B(T) = B(T-1)$ and ${}_sp_0(T) = {}_sp_0(T-1)$.

Note, the above means:

$${}_sp_0(T) = 1 - {}_sq_0(T) = {}_sp_0(T-1) = 1 - {}_sq_0(T-1)$$

$${}_sq_0(T) = \frac{{}_sD_0(T)}{B(T)} = {}_sq_0(T-1) = \frac{{}_sD_0(T-1)}{B(T-1)}$$

Since $B(T) = B(T-1)$, we have

$${}_sD_0(T) = {}_sD_0(T-1)$$

Therefore, we have:

$$IMR_{refined}(T) = \frac{{}_sD_0(T)}{B(T)} + \frac{B(T) - {}_sD_0(T)}{B(T)} * \frac{{}_pD_0(T)}{B(T-1) - {}_sD_0(T-1)} =$$

$$= \frac{{}_sD_0(T)}{B(T)} + \frac{\cancel{B(T) - {}_sD_0(T)}}{B(T)} * \frac{{}_pD_0(T)}{\cancel{B(T) - {}_sD_0(T)}} =$$

$$= \frac{{}_sD_0(T) + {}_pD_0(T)}{B(T)} = IMR_{conventional}(T)$$

Problem 2

In the former Soviet Union, the tradition is to compute the IMR as follows:

$$IMR_{Soviet}(T) = \frac{{}_sD_0(T)}{B(T)} + \frac{{}_pD_0(T)}{B(T-1)}$$

Demonstrate that if again survival probabilities and the absolute number of births are constant over time, then the Soviet IMR is equal to the refined IMR.

$$IMR_{Soviet}(T) = \frac{{}_sD_0(T)}{B(T)} + \frac{{}_pD_0(T)}{B(T-1)} =$$

$$= \frac{{}_sD_0(T)}{B(T)} + \frac{{}_pD_0(T)}{B(T)} =$$

Using Problem 1,

$$= \frac{{}_sD_0(T) + {}_pD_0(T)}{B(T)} = IMR_{conventional}(T) = IMR_{refined}(T)$$

PSET3 Part A, Problems 1-9

Problem 1

What was life expectancy at birth? (give units)

$e_0 = 81.333$ years

Problem 2

What was life expectancy at age 35?

$e_{35} = 47.529$ years

Problem 3

Give a verbal interpretation of the number you presented in (2).

A female who has reached age 35 is expected to live 47.529 additional years.

Problem 4

What was the probability of surviving from birth to age 25?

Probability of surviving from age x to age y = $\frac{l_y}{l_x} = \frac{l_{25}}{l_0} = \frac{98{,}891}{100{,}000} = 0.98891$

Problem 5

What was the probability that a female who survived to age 25 would die before age 50?

Probability that a person surviving to age x will die before age y = $\frac{l_x - l_y}{l_x} = 1 - \frac{l_y}{l_x} = 1 - \frac{l_{50}}{l_{25}} = 0.032301$

Problem 6

How many years could a newborn female expect to live in the age interval 15-65? So how many years of potential life in this interval were lost due to mortality?

Number of years that a newborn can expect to live between age x and age y $= \frac{T_x - T_y}{l_0} = \frac{T_{15} - T_{65}}{l_0} = 48.185509$ years, i.e. $50 - 48.185509 = 1.814491$ years of life lost due to mortality.

Problem 7

How many years, on average, were lived in the age interval 1-4 by females who died in that interval?

Average years lived between 1 and 4 by females who died in that interval $= {}_4a_1 =$ 1.513824 years.

Problem 8

What was the probability that a female newborn would experience her death in the age interval 65-69?
Probability that a newborn would experience death between age x and age y $= \frac{(l_x - l_y)}{l_0} = \frac{(l_{65} - l_{70})}{l_0} = 0.049344616$

Problem 9

Assume that a woman had a daughter when she was 30. Assuming that mortality conditions will stay constant and that survival probabilities for both individuals are independent, what is the probability that both mother and daughter will still be alive 30 years later?
This is equivalent to calculating the probability that a newborn will survive to the age of 30, multiplied by the probability that a person surviving to age 30 will survive to age 60. Probability the daughter will survive to age 30 $= \frac{l_{30}}{l_0} = 0.98590349$
Probability the mother will survive from age 30 to age 60 = $= \frac{l_{60}}{l_{30}} = 0.926102731$
Multiplied together, the probability that the mother and daughter will still be alive 30 years after the daughter's birth $= 0.913047915$