Line balancing with heterogeneous products: a two-step formulation to minimize number of stages in the whole system of lines

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Abstract

The paper addresses the Mixed-model Assembly Line Balancing Problem on an audio amplifier factory with objective of minimizing total number of stages. Two characteristics make it different from classic problems: several assembly lines are optimized as a whole; and products are heterogeneous, so converting precedence diagrams in a joint one is unrealistic. A two-step formulation is proposed: first, cycle times are minimized for each product and all possible number of stages; and a global optimization (with data from first model), where configurations of the lines are chosen to minimize the total number of stages. Data from 24 products, 4 lines and 337 tasks were collected on the factory. Results were obtained in 26.5 seconds and promoted a reduction of 44.8% of the stages. The paper impacts literature exploring particularly the optimization of a whole sector, without necessity of joint precedence graphs; and also the company, reducing meaningfully the number of stages used.

Keywords: Line balancing; Mixed-model assembly line; Two-step formulation; Mixed integer linear problem.

1 Introduction

The Assembly Line Balancing Problem (ALBP) consists of the assignment of tasks necessary to assemble a certain product to a set of workstations of that line [1], aiming to optimize one or more objectives (such as to minimize cycle time or number of workstations), subject to precedence constraints and other particular line restrictions [2]. It is a classical problem from operations research literature, since [3] when it was first studied.

ALBP was reviewed in [4], where it is categorized according to the mix of products manufactured in single-model lines (one model), mixed-model lines (several products mixed) and multi-model lines (several products, one at a time, separated by setup times). More recently, [5] reviews the problem, classifying in eight types, by the combination of numbers of models (single-model or multi-model); nature of task times (deterministic or probabilistic); and type of assembly line (straight-type or U-type).

In this paper, we address a MALBP, on an audio amplifier factory. The company produces 24 products in 4 assembly lines, having a total of 29 stages before this study, with an 83% balancing efficiency, as defined in [6]. The products are not homogeneous, so it is difficult or unrealistic to convert several precedence diagrams in a joint diagram, as seen in [7]. Products vary from 4 to 27 tasks (average of 14). The objective is to minimize the number of stages in the whole system of lines, since the company, currently, has a scenario of demand scarcity. Using typology in [5], it's a multi-model, deterministic task times and straight-type assembly line (MM_D_S).

The objective of this paper is to propose a two-step formulation as an exact solution to MALBP with two specific characteristics: (i) the sector is composed of more than one line to be optimized as a whole; and (ii) the products are heterogeneous enough to be unrealistic to convert precedence diagrams in a joint one.

Classical MALBP problems were fully explored in literature, as it can be seen in [5], although it is pointed out that much less work is done on multi-model than single-model lines. Several particular conditions for the MALBP are studied by other papers. Heterogeneous workforce is addressed by [8], considering a context of disabled workers. Multi-manned assembly lines, the ones where several tasks are performed in each stage, are approached in [9] using simulated annealing to optimize cycle time. The same problem is addressed in [10], where a mathematical formulation is proposed with a metaheuristic and neighborhood search approach. Two-sided lines are seen in [11], where both sides of the line can be used to produce high-volume large-sized standardized products.

A particular characteristic close to the problem addressed here is parallelism. It is defined by [12] when two or more stages are set in parallel to allow better cycle times. In [13] it can be seen parallel lines; while in [14] parallel workstations performing the same task set; and in [15] parallel two sided lines. Neither cases reflect the situation considered here, where the focus is not the possibility of performing a task in more than one stage; or using better resources sharing among lines. The focus of the paper is optimizing a set of independent lines that must attend to production schedule.

2 Problem description and mathematical formulation

This paper proposes an optimization method, for the minimization of the number of stages on a set of parallel lines through a two-step formulation: firstly, it is optimized the cycle time (C) for each product and a certain configuration (number of stages in the line), subject to its precedence network and uniqueness constraints; secondly, with all optimum cycle times for each product and each possible configuration, it is possible to optimize which configurations must be chosen in each line.

2.1. First step: optimizing cycle times

Let I be a set of tasks of a product ($i = 1, \ldots, n$) to be performed on an assembly line; and K, a set of stages ($k = 1, \ldots, m$). Let a_{ij} be a binary parameter which denotes precedence between tasks i and j, being 1 when there is precedence and 0, otherwise. Let u_i, also be a binary parameter, which assumes value 1 if task i must be performed isolated on a stage (intermediate quality tests, in general) and 0, otherwise. Finally, cycle time of each task is given by t_i.

Decision variables considered are cycle time C, already mentioned, and also x_{ik}, a binary variable which assumes value 1 when task i is assigned to stage k and 0, otherwise. The mathematical formulation that minimizes C on assembly line is given by (1.1)-(1.8).

$$\text{Min} \quad C \tag{1.1}$$

$$\text{s.a.} \quad \sum_{k=1}^{m} x_{ik} = 1 \qquad i \in \{1, \dots, n\} \tag{1.2}$$

$$\sum_{i=1}^{n} t_i x_{ik} \leq C \qquad k \in \{1, \dots, m\} \tag{1.3}$$

$$x_{jk} \leq \left(1 - a_{ij}\right) + \sum_{l=1}^{k} a_{ij} x_{il} \qquad i \in \{1, \dots, n\}, j \in \{1, \dots, n\}, k \in \{1, \dots, m\} \tag{1.4}$$

$$\sum_{i=1}^{n} x_{ik} \geq 1 \qquad k \in \{1, \dots, m\} \tag{1.5}$$

$$\sum_{i=1}^{n} u_i x_{ik} \leq 1 \qquad k \in \{1, \dots, m\} \tag{1.6}$$

$$\sum_{i=1}^{n} x_{ik} \leq 1 + n \times \left(1 - \sum_{i=1}^{n} u_i x_{ik}\right) \quad k \in \{1, \dots, m\} \tag{1.7}$$

$$x_{ik} \in \{0,1\}, \quad i \in \{1, \dots, n\}, k \in \{1, \dots, m\}, \quad C \in \mathbb{R}^{+} \tag{1.8}$$

The objective function (1.1) consists of minimizing cycle time of the line, considering m stages available. Constraint (1.2) assures that each task is assigned to one, and only one, stage. Constraint (1.3) assures that the total time assigned to a certain stage is inferior to cycle time. On (1.4) precedence constraints are imposed. Constraint (1.5) imposes the use of all stages (important to second step formulation), requiring at least one task on each stage. Constraints (1.6) and (1.7) assure tasks which require isolation to be alone on a stage. Finally, on (1.8), it is expressed the decision variables domain.

2.2. Second step: optimizing the total number of stages

Running formulation (1.1)-(1.8) for each product and each possible line configuration (configuration meaning number of stages on assembly line), it is possible to obtain a set of cycle times optimized for a product and a configuration. With these data, it is proposed a model that aims to minimize the number of stages on the set of lines available.

Let an assembly sector with M assembly lines, in which it is possible to produce each one of P products of the portfolio of a factory. Each line can be configured in K different formats (k = 1, …, K), according to the number of stages k. The demand of product p is given by D_p; the minimum and maximum number of stages on line m; besides t_{pk}, the optimized cycle time of a product k with a line configured with k stages (obtained in the first step). The total time available to schedule all demands is given by T.

Decision variables are x_{pmk}, the number of units of product p to be assembled in line m with configuration k. Let E_{mk} be a binary set that assumes value 1 if line m is configured with k stages and 0, otherwise. Finally, U_{pm} is a binary set with value 1 if the product p is scheduled to line m and 0, otherwise. Mathematical model is exposed in (2.1)-(2.10).

$$\text{Min} \quad ET = \sum_{k=1}^{K}\sum_{m=1}^{M} E_{mk}k \tag{2.1}$$

$$\text{s.a.} \quad \sum_{m=1}^{M}\sum_{k=1}^{K} x_{pmk} = D_p \qquad p \in \{1,\dots,P\} \tag{2.2}$$

$$\sum_{k=1}^{K} E_{mk} = 1 \qquad m \in \{1,\dots,M\} \tag{2.3}$$

$$\sum_{p=1}^{P} x_{pmk} \le E_{mk}\sum_{p=1}^{P} D_p \qquad m \in \{1,\dots,M\}, k \in \{1,\dots,K\} \tag{2.4}$$

$$\sum_{k=1}^{K}\sum_{p=1}^{P} x_{pmk}t_{pk} \le T \qquad m \in \{1,\dots,M\} \tag{2.5}$$

$$\sum_{k=1}^{K} E_{mk}k \le KMAX_m \qquad m \in \{1,\dots,M\} \tag{2.6}$$

$$\sum_{k=1}^{K} E_{mk}k \ge KMIN_m \qquad m \in \{1,\dots,M\} \tag{2.7}$$

$$\sum_{p=1}^{M} U_{pm} = 1 \qquad p \in \{1,\dots,P\} \tag{2.8}$$

$$\sum_{k=1}^{K} x_{pmk} \le U_{pm}D_p \qquad p \in \{1,\dots,P\}, m \in \{1,\dots,M\} \tag{2.9}$$

$$x_{pm} \in \mathbb{Z}^+, E_{mk} \in \{0,1\}, \quad p \in \{1,\dots,P\}, k \in \{1,\dots,K\}, m \in \{1,\dots,M\} \tag{2.10}$$

The objective function (2.1) consists of minimizing the total number of stages in the M lines with different configurations to be chosen. Constraint (2.2) imposes that the demand of a product p (D_p) must be manufactured in the sum of the productions of each line. Constraint (2.3) assures that only one configuration must be chosen for each line. Coherence between x_{pmk} and E_{mk} is assured in (2.4), allowing productions to be assigned only to configurations that are chosen. Total time available (T) limits the sum of times assigned to each of the M lines in (2.5). The minimum and maximum number of stages of a line is respected through (2.6) and (2.7). The limitation of manufacturing each product on a single line is translated by (2.8). This last constraint is a requisite of the studied factory, but it is not necessarily mandatory in any system. Constraint (2.9) assures coherence between x_{pmk} and U_{pm}. Finally, decision variables domain is defined in (2.10).

3 Data collection and results

The assembly sector of the company is composed of 4 assembly lines (M = 4), where 24 products are assembled (P = 24). The set of tasks of each product was described and measured, totalizing 337 tasks.
A time study was performed during four months, from February to May (2015). Shooting techniques were used, in order to measure times spent without pressure on workers. Data were analyzed with other

methodologies (Single-Minute Exchange of Dice, SMED and Methods-Time Measurement, MTM) and standard times were calculated. The demand used was obtained by the sales forecast.
Models were implemented on OPL (Optimization Programming Language) using CPLEX 12.6.2 as optimization software. Computational tests were run on a 12 Gb RAM memory PC with Intel i7 processor.
Figure 1 illustrates the results of the first step model (1.1)-(1.8) for two different products (p = 1 and p = 7), having its t_{pk} (optimized cycle times of product p with configuration k) shown graphically.

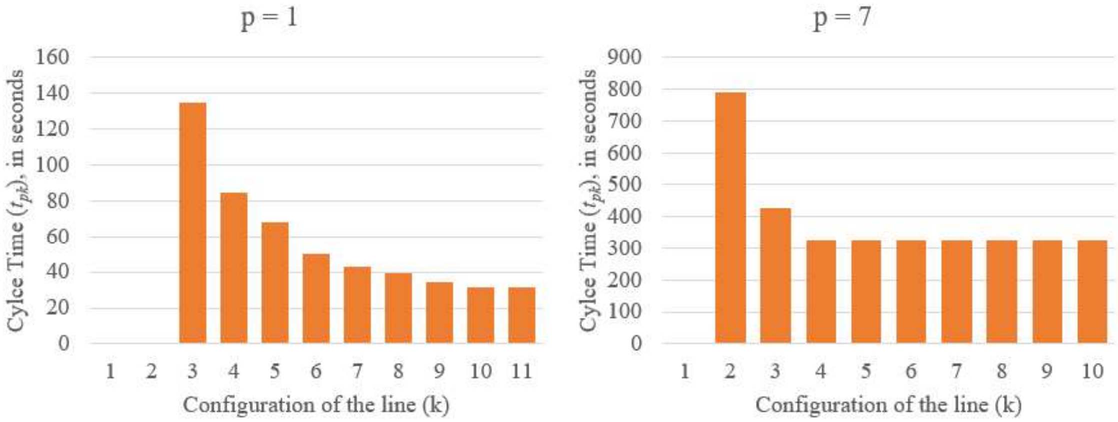

Figure 1 – Results of first step model (1.1)-(1.8) for products p = 1 and p = 7

It is intuitive that the cycle times optimized of a certain product is always non-crescent with the number of stages used, once it is a constraint that gets gradually looser. On both examples illustrated, it is possible to observe that there are products in which cycle times decreases successively (product 1), while in others global optimum is obtained quickly (product 7).
Model (1.1)-(1.8) was run for 264 instances, including the 24 products with configurations from 1 to 11 stages. Even though there were products with more than 11 tasks, the maximum number of stages of the biggest line was 11. Since problems were small, all 264 instances got optimum value quickly, taking 26.22 seconds to run all data set.
With these data available, it was possible to run model (2.1)-(2.10). Time horizon considered for demand and available time was one week, since forecast was more accurate and setup times were proportionally small, as explained earlier. The total time available (T = 123,552 seconds) was calculated using the 44-hour weekly journey with a 78% efficiency coefficient.
The computational experiment was made by the same computer, taking 0.28 seconds to run.
Figure 2 shows, illustratively, results of the model in terms of time assigned on each line for each product on assembly lines. Horizontal line show total time available (T). The optimum solution required 3 stages on line 1, 4 stages on line 2, 5 stages on line 3, 4 stages on line 4, totalizing 16 stages in the whole sector. The utilization (excluding 22% of inefficiency already mentioned) of the lines varied from 94.5% to 98.5%, with an average of 96.7%.

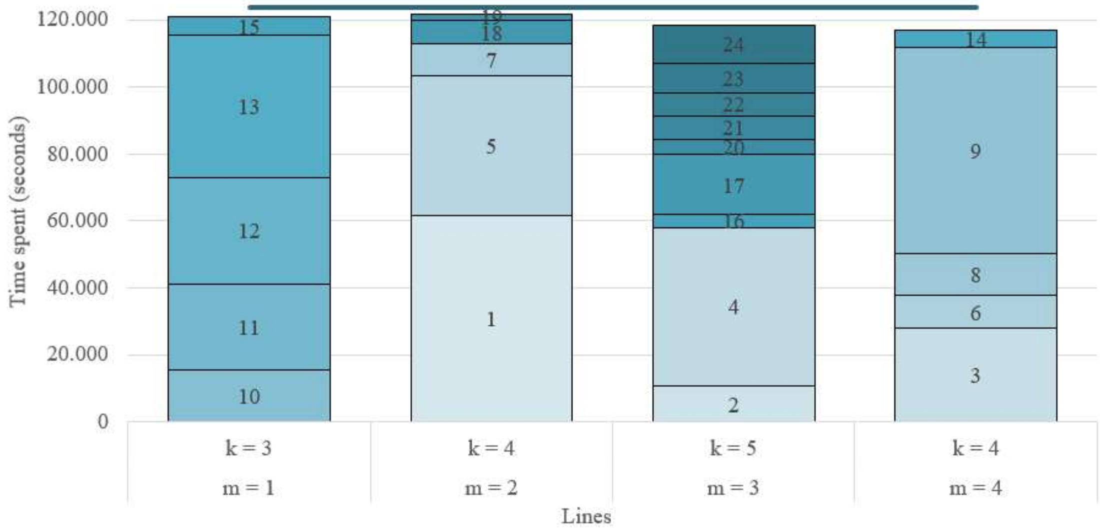

Figure 2 – Assignment of products on each line (m) with configurations chosen (k) and total time spent on each line

Results were compared to company's reality. The previous configuration totalized 29 stages and utilization was 29 with 83% utilization as shown on Table 1.

Table 1 – Final results of line balancing

M	Before study	After line balancing
1	11	3
2	4	4
3	6	5
4	8	4
Total stages	29	16
Utilization	83%	96.7%

4 Conclusions

This paper presented a two-step mathematical formulation to minimize the total number of stages on a system of mixed-model lines on an audio amplifier factory. Results were favorable, with a decrease of more than 40% of the stages, compared to the actual configuration. So, the method impacts the factory immediately, since it allows assembling the same demand with reduced costs.

It also impacts MALBP literature, once it considers two particular conditions: (i) heterogeneous products; and (ii) a set of flexible lines that must be optimized as a whole. The first condition is relevant, once joint precedence diagrams in this case could be unrealistic. With this method, this simplification is avoided. The second condition, although very simple, is often seen in reality.

Data collected in the company presented configured a small instance, so the problem was solved on reasonable computational time (26.4 second for both formulations). Bigger instances could face computational limitations, so future extensions can study heuristic methods.

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