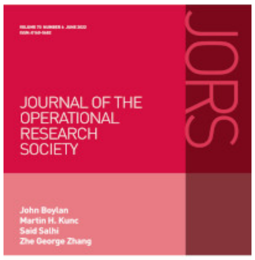

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ORIGINAL ARTICLE

Minimizing saw cycles on the cutting stock problem with processing times depending on the cutting pattern

F. K. Lemos[a], A. C. Cherri[a], S. A. de Araujo[a] and H. H. Yanasse[b]

[a]São Paulo State University (UNESP), São Paulo, Brasil; [b]Federal University of São Paulo (UNIFESP), São Paulo, Brasil

ABSTRACT
In this paper the cutting stock problem minimizing saw cycles is considered. In this context where objects can be cut simultaneously the trade-off between raw material and machine utilization costs is of major interest. Different processing times for each cutting pattern are considered, according to the number of items on it, different from previous works of the literature. The optimization aims to minimize both machine utilization costs due to the saw cycles and raw material usage costs. An integer mathematical formulation is proposed, and a solution method based on a column generation procedure is used. Computational tests were performed to assess the impact on costs for illustrative and real industry instances. A set of instances were also randomly generated to analyse the behaviour of the outcome of the problem in face of different technical parameters.

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Cutting; saw cycles; machine utilization; column generation; integer programming

1. Introduction

The cutting stock problem (CSP) aims to assign small items into larger objects, ensuring the demand attendance and optimizing some objective function, for instance, related to material usage (Gilmore & Gomory, 1961, 1963). It is one of the oldest and most inspiring problems of operations research, due to a large number of real-world applications in industries like clothing, paper, furniture and metallurgical (Bezerra et al., 2020), and even in non-industrial environments like budgeting, project scheduling and staffing (Edwards et al., 2021; Marzouk & Kamoun, 2021). Although many direct applications are found in practice, a lot of variants are addressed in the literature, according to the requirements of different industrial settings (Melega et al., 2018). It can be part of more complex problems (Leao et al., 2020), such as scheduling, resource allocation (Qin et al., 2019) or lot-sizing (Campello et al., 2020; Christofoletti et al., 2021). In addition, the integration with the production planner's needs and concerns has recently gained focus in the literature (Wuttke & Heese, 2018).

Classic CSP assumes that objects are cut one by one in the production process. Nevertheless, some machines allow cutting simultaneously more than one object at a time. A saw cycle is defined as the processing time, which includes setup and machine cutting time, to cut a set of objects simultaneously (the set can contain a single object) according to a cutting pattern. Cutting several objects simultaneously gains relevance in high-machine-cost environments (Yanasse, 2008). In these cases, in addition to material costs, expenditures related to machine time must also be considered.

This problem is intrinsically related to cutting and packing problems, but it can arise in other production systems (Marvizadeh & Choobineh, 2013). The minimization of saw cycles is related to good machine utilization, a concern presented in the literature for some decades, with setup minimization problems (Allahverdi et al., 1999; Araujo et al., 2014; Cui et al., 2015; Henn & Wäscher, 2013; Ma et al., 2019; Pan & Ruiz, 2012; Vanderbeck, 2000) and minimization of the number of different cutting patterns (Alves et al., 2009; Diegel et al., 2006; Mobasher & Ekici, 2013; Moretti & Salles Neto, 2008; Yanasse & Limeira, 2006).

Cutting machines may become a bottleneck in high-demand contexts, imposing a trade-off between increased machine capacity utilization and raw material waste (Toscano et al., 2017). Using the maximum capacity of the machine in all cutting patterns is a common practice in industry. However this practice leads to an optimal solution only for high-demand contexts (Yanasse, 2008). For lower demand, good machine utilization and raw material usage may be conflicting objectives (Toscano et al., 2017). In these cases, a trade-off between productivity and minimization of raw material utilization must be considered (Rangel & Figueiredo, 2008). This condition is observed in furniture (Vanzela et al., 2017), mechanic (Simões et al., 2017), textile (Degraeve et al., 2002; Degraeve & Vandebroek, 1998), and vulcanization (Trigos & Lopez, 2017) industries.

CONTACT F. K. Lemos felipeklemos@gmail.com Av. Eng. Luís Edmundo Carrijo Coube, 14-01 - Vargem Limpa, Bauru - SP 17033-360 - Brazil.

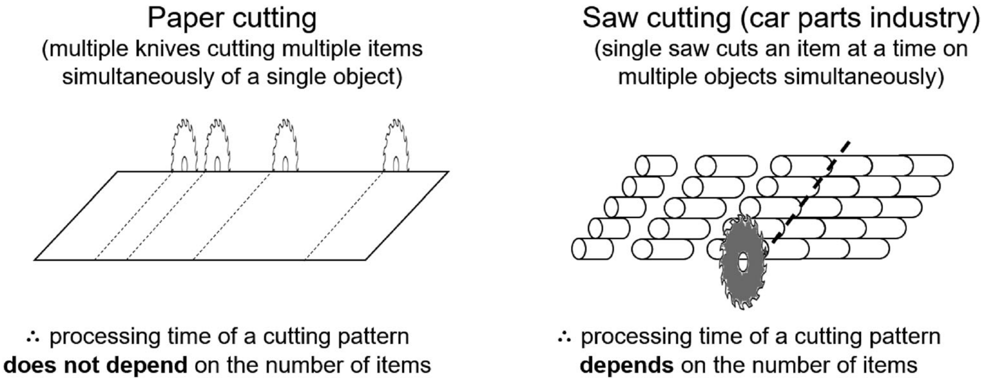

Figure 1. Difference between dependent and independent processing time according to the number of items cut in a cutting pattern.

In this paper, we approach the minimization of saw cycles problem in industrial settings where processing times are dependent on the cutting patterns. We consider setup costs and unitary cutting costs for each cut required to obtain an item (or items cut simultaneously) in the cutting pattern. This problem arises in the metal-mechanic industry when dealing with titanium plates where machine cutting times are elevated and each item demands a long individual time to be cut, in addition to the setup.

To the best of our knowledge, previous articles on the saw cycle minimization problem have considered equal processing times for different cutting patterns (Degraeve et al., 2002; Toscano et al., 2017; Trigos & Lopez, 2017; Vanzela et al., 2017; Yanasse, 2008).

Yanasse (2008) approached the problem of minimization of saw cycles considering equal processing times, defining it and proposing linear models and constructive heuristics as solution method. Malaguti et al. (2014) addresses a two-dimensional cutting operation with three stages in which objects can be stacked at a time from one to a maximum of the machine (which is analogous to a saw cycle). The decision of the number of objects in a cycle is discretized on the decision variables. Similarly, Degraeve et al. (2002) applies this kind of formulation to the textile industry.

Vanzela et al. (2017) integrated the saw cycle minimization problem with lot sizing in a multi-period setting. The capacity of each period is the number of saw cycles. Toscano et al. (2017) proposes a price-and-cut heuristic to approach the two-dimensional cutting problem with saw cycles, in which the problem is reformulated dynamically through adding constraints and variables that preserve a minimum frequency of a given cutting pattern used.

This paper contributes to the literature by (i) considering processing times as being dependent on the number of items in the cutting pattern, a feature that arises in practice; (ii) proposing a mathematical formulation for the problem; (iii) proposing a solution method based on a column generation procedure, and (iv) showing the influence of industrial parameters on the outcome of the problem, conducted by extensive computational tests.

The paper is organized in six sections. In this introduction, the problem, the literature and the contribution of the study are described. In Section 2 we detail the production process and manufacturing environment that motivates the study. In Section 3, a mathematical formulation of the problem is proposed. In Section 4, a solution method based on a column generation procedure is proposed. In Section 5, the computational test results for real and random instances are presented. In Section 6, we summarize the main findings and propose additional studies that can be explored.

2. Motivation and description of the production process

The factory that motivates the present approach to the problem is a metal-mechanic manufacturer that supplies car parts to assembly lines of agriculture machines. It deals with a great variety but relatively low volume of each item.

The process studied is the first step of a flowshop focused on machining. Raw material bought in bigger units must be cut in smaller parts, as any classic cutting problem. However, as the factory deals with hard materials, as titanium and special iron, saw cutting is used to cut several objects together in a single process.

It is relevant to differ this production environment from paper cut machines, object of study of several cutting problems (Leao et al., 2017). Figure 1 highlights the main differences. Paper cut uses multiple knives to cut multiple items simultaneously of a single object. Hence, the processing times of an object do not depend on the number of items in a cutting pattern. On the other hand, in the saw cutting a single saw cuts an item type at a time on multiple objects simultaneously. That is why the processing time of a cutting pattern depends on the number of items on it. This gets even more relevant, considering long processing times.

It is worth mentioning that we are not considering different unitary processing times according to the number of objects on a saw cycle. This is due to the characteristics of the practical application considered. Incremental time spent to cut a single object or a full cycle was insignificant in the factory studied.

The characteristic of great variety with a low volume of each item can make empirical solutions of imposing full cycles more ineffective. For greater demands, one could simply impose full cycles and get a solution close to optimal.

3. Mathematical formulation

This section describes the proposed mathematical formulation for the problem of minimizing the total costs of a cutting process that allows saw cycles where multiple one-dimensional objects can be cut simultaneously.

Let us denote X_{pk}, the number of cutting patterns of type p $(p = 1, \dots, NP)$ cutting simultaneously k objects on a saw cycle where $(k = 1, \dots, NK)$ and NK is the maximum number of objects that can be cut simultaneously in the machine. NP is the number of different cutting patterns. Each cutting pattern is characterized by a_{ip}, which denotes the number of items i in an object cut according to cutting pattern p.

Each object has a unitary cost denoted by θ. Therefore, the cost of raw material in a saw cycle is given by the number of objects k cut in it and its unitary cost θ. If, in a saw cycle, k objects are cut, the total cost of the objects used is $\theta \sum_{k=1}^{NK} k(\sum_{p=1}^{NP} X_{pk})$.

The machine cost associated with a saw cycle is determined by a setup cost (st) and a unitary cost (γ^{un}) for each item that composes the cutting pattern. If a cutting pattern p is composed by $\sum_{i=1}^{NI} a_{ip}$ items then the machine cost to cut this cutting pattern p is given by (1). NI is the number of different sizes of items to be cut. The cost of the total machine utilization is given by $\sum_{k=1}^{NK} \sum_{p=1}^{NP} \gamma_p X_{pk}$.

$$\gamma_p = st + \sum_{i=1}^{NI} \gamma^{un} a_{ip} \tag{1}$$

The objective function that minimizes the total cost of raw material and the machine utilization costs is presented in (2).

Constraints (3) ensure that the total demand is met for all items i. Each item i $(i = 1, \dots, NI)$ has length l_i and demand b_i. At each saw cycle, k times this amount of items i are produced, once k objects are cut simultaneously at a time. The domain of the decision variables is presented in (4).

The one-dimensional saw cycle problem is formulated as (2)–(4).

$$\min \ \theta \sum_{k=1}^{NK} k \left(\sum_{p=1}^{NP} X_{pk} \right) + \sum_{k=1}^{NK} \sum_{p=1}^{NP} \gamma_p X_{pk} \tag{2}$$

$$s.t.: \ \sum_{k=1}^{NK} k \left(\sum_{p=1}^{NP} a_{ip} X_{pk} \right) \geq b_i, \quad i = 1, \dots, NI, \tag{3}$$

$$X_{pk} \in \mathbb{Z}^+, \quad p = 1, \dots, NP, \quad k = 1, \dots, NK. \tag{4}$$

The model (2)–(4) presents the complexity of a potential large number of columns (variables X_{pk}), due to the combination of items arranging different cutting patterns (a_{ip}). Moreover, the parameter γ_p must reflect the fixed cost of the machine time used during the manufacture of the cutting pattern p, which includes its setup time (st) and the processing time of each item (γ^{un}). In order to address these issues, column generation is proposed as a solution method.

4. Solution method

Considering the large number of potential variables (Zhen et al., 2021) in light of the multiple possibilities of cutting patterns a column generation method based on Gilmore and Gomory (1961, 1963) was proposed. New cutting patterns and its parameters for (2)–(4) were generated as needed when we minimize the reduced cost based on the dual values of the restricted master problem (2)–(4) with the integrality conditions (4) relaxed.

Cutting patterns are generated until all the reduced costs of decision variables X_{pk} are nonnegative. Considering π_i the dual values associated to constraint i of (3) and α_i a decision variable that represents the amount of items i in the cutting pattern p (i.e., a_{ip}), and γ_p the cost of the machine time to process the cutting pattern p, the reduced cost associated with X_{pk}, denoted by $\bar{c}_{pk}$, is as follows:

$$\bar{c}_{pk} = \theta \cdot k + \gamma_p - k \left(\sum_{i=1}^{NI} \alpha_i \pi_i \right) \tag{5}$$

Because there is the possibility that different numbers of objects can be cut simultaneously, the subproblem must generate new columns while there is any reduced cost negative for any amount k. So, the proposed formulation for the subproblem minimizes the minimum $\bar{c}_{pk}$ for all k.

The processing time of the cutting pattern γ_p defines the cost of the machine utilization spent to manufacture that saw cycle. It is a sum of a fixed setup cost st and a unitary cost for each item on the cutting pattern γ^{un}. Thus, the term γ_p in the reduced cost is represented by (1), which totals this processing time according to the number of items assigned to the cutting pattern.

It is possible to prove that the most negative value for the same cutting pattern generated will always take place when $k = NK$. Because st, θ, γ_{un}, and all α_i are non-negative values, a cutting pattern will deliver a negative reduced cost only when

$$k \sum_{i=1}^{NI} \alpha_i \pi_i \geq \theta \cdot k + (st + \sum_{i=1}^{NI} \alpha_i \gamma^{un}) \quad (6)$$

Particularly, since $st + \sum_{i=1}^{NI} \alpha_i \gamma^{un}$ is a non-negative term, the right side of the inequation can only be increased by it. Thus, if the left side of the inequality is greater of equal to the right side, it must also be greater of equal to the raw material cost (θk) in particular. So, it is also true that

$$k \sum_{i=1}^{NI} \alpha_i \pi_i \geq \theta \cdot k \Rightarrow k\left(\left(\sum_{i=1}^{NI} \alpha_i \pi_i\right) - \theta\right) \geq 0 \quad (7)$$

Therefore, for a given cutting pattern, a larger k leads to a more valuable column. The subproblem to generate columns for (2)–(4) is presented in (8)–(10).

$$\min \theta \cdot NK + \left(st + \sum_{i=1}^{NI} \alpha_i \gamma^{un}\right) - NK \sum_{i=1}^{NI} \alpha_i \pi_i \quad (8)$$

$$s.t.: \sum_{i=1}^{NI} \alpha_i l_i \leq L \quad (9)$$

$$\alpha_i \in \mathbb{Z}^+, \quad i = 1, \dots, NI. \quad (10)$$

After the column generation procedure, a heuristic solution is obtained by running the model (2)–(4) in its integer version, using all the columns generated during the column generation procedure. This heuristic procedure is used in many cases in the literature (Bertoli et al., 2020; de Lara Andrade et al., 2021; Lemos et al., 2021) when final gaps are lower than the application requires.

The heuristic, therefore, consists on performing the column generation procedure until there is no more valuable column to be added. When no more cutting patterns deliver a negative reduced cost, a restricted version of the model (2)–(4) is run with the set of columns generated. The method is not exact, because it is possible that the optimal solution of the integer problem uses a column with positive reduced cost in the final solution of the relaxed problem. Nevertheless, the value of the linear solution is a valid lower bound.

It is important to stress that the subproblem (8)–(10) (not the main model (2)–(4)) gives the main contribution in terms of formulating the production process described. While the main model (2)–(4) is similar to previous ones in the literature (Degraeve et al., 2002; Malaguti et al., 2014) (with the addition of the machine time cost term on the objective function), the subproblem (8)–(10) allows computing cutting patterns with different processing times, considering setup times (st) and unitary processing times per cut (γ^{un}).

Table 1. Data of the items to be cut.

Item	1	2	3	4	5	6	7	8
l_i	130	134	176	183	191	210	219	299
b_i	11	35	18	44	46	14	28	21

5. Computational results

The model proposed, as well as the solution method described, was tested in three different environments: (i) an illustrative instance is generated to show the relevance of an integrated approach with a small example; (ii) an industrial example, with collected values, is compared with an empirical approach and with CSP imposing full saw cycles; (iii) a set of random instances was generated to explore the performance of the model and its bounds.

All the experiments were executed on a 16GB RAM memory computer with a 2.2 GHz, hexacore, Intel Core i7-8750H processor, using CPLEX 12.8 as solver and OPL language to code the method.

5.1. Illustrative instance

The instance presented in this section was generated with the aim of illustrating the relevance of the integrated approach for the problem. In Table 1 the data for the demanded items are presented.

Such items must be cut on objects of size 1000 ($L = 1000$), having the possibility of cutting at most 7 objects of the same cutting pattern simultaneously. The cost of each object is the same as the setup of a saw cycle and also is equal to the cost of machine time for each cut ($\theta = 1$, $st = 1$ and $\gamma^{un} = 1$).

Optimizing the cut process with the classic cutting stock problem, it is possible to achieve a lower waste of raw material, using 42 objects (which is the ceiling of the linear bound). However, the number of objects demanded for each cutting pattern, as predicted by Yanasse (2008), are not multiples of 7, generating 10 cycles in which 54 cuts are made, with a total machine cost of 64. The detailed solution is in Table 2.

During 10 saw cycles, it could be possible to cut 70 objects, whereas only 42 are cut. A possible empirical solution would be optimizing the problem imposing the use of cycles of 7 simultaneous objects,

Table 2. Solution using the classic cutting stock problem.

Items on cutting patterns (a_{ip})								Machine cost (γ_p)	Amounts (X_{pk})	Total cost
1	2	3	4	5	6	7	8			
0	0	0	1	0	0	1	2	5	$k=4 : X_{pk} = 1$	9
0	2	0	4	0	0	0	0	7	$k=6 : X_{pk} = 1$	13
0	0	0	0	3	2	0	0	6	$k=3 : X_{pk} = 1$	9
0	1	0	1	2	0	0	1	6	$k=7 : X_{pk} = 1$	13
									$k=6 : X_{pk} = 1$	12
6	0	0	0	0	0	1	0	8	$k=1 : X_{pk} = 1$	9
1	2	0	0	2	0	1	0	7	$k=5 : X_{pk} = 1$	12
2	0	0	3	1	0	0	0	7	$k=1 : X_{pk} = 1$	8
0	0	2	0	0	1	2	0	6	$k=7 : X_{pk} = 1$	13
									$k=2 : X_{pk} = 1$	8
Raw material utilization: 98.8% (42 objects)										**106**
Saw cycles occupation: 60.0% (10 cycles)										

Table 3. Solution using the cutting stock problem and imposing full cycles ($k=7$).

Items on cutting patterns (a_{ip})								Machine cost (γ_p)	Amounts (X_{pk})	Total cost
1	2	3	4	5	6	7	8			
7	0	0	0	0	0	0	0	8	$k=7 : X_{pk} = 1$	15
0	7	0	0	0	0	0	0	8	$k=7 : X_{pk} = 1$	15
0	0	5	0	0	0	0	0	6	$k=7 : X_{pk} = 1$	13
0	0	0	5	0	0	0	0	6	$k=7 : X_{pk} = 2$	26
0	5	0	0	5	0	0	0	6	$k=7 : X_{pk} = 2$	26
0	0	0	0	0	4	0	0	5	$k=7 : X_{pk} = 1$	12
0	0	0	0	0	0	4	0	5	$k=7 : X_{pk} = 1$	12
0	0	0	0	0	0	0	3	4	$k=7 : X_{pk} = 1$	11
Raw material utilization: 59.3% (70 objects)										**130**
Saw cycles occupation: 100% (10 cycles)										

Table 4. Solution of integrated cutting stock problem and saw cycle minimization ($\theta = 1$, $st = 1$ and $\gamma^{un} = 1$).

Items on cutting patterns (a_{ip})								Machine cost (γ_p)	Amounts (X_{pk})	Total cost
1	2	3	4	5	6	7	8			
0	0	0	0	0	0	0	3	4	$k=5 : X_{pk} = 1$	9
1	0	2	0	0	0	1	1	6	$k=7 : X_{pk} = 1$	13
0	0	0	0	3	2	0	0	6	$k=7 : X_{pk} = 1$	13
0	0	1	0	2	0	2	0	6	$k=7 : X_{pk} = 1$	13
1	2	0	0	2	0	1	0	7	$k=7 : X_{pk} = 1$	14
0	2	0	4	0	0	0	0	7	$k=6 : X_{pk} = 1$	13
									$k=5 : X_{pk} = 1$	12
Raw material utilization: 94.3% (44 objects)										**87**
Saw cycles occupation: 89.8% (7 cycles)										

Table 5. Solution of the model with different γ^{un} ($\theta = 1$, $st = 1$ and $\gamma^{un} = 10$).

Items on cutting patterns (a_{ip})								Machine cost (γ_p)	Amounts (X_{pk})	Total cost
1	2	3	4	5	6	7	8			
7	0	0	0	0	0	0	0	8	$k=2 : X_{pk} = 1$	10
0	7	0	0	0	0	0	0	8	$k=5 : X_{pk} = 1$	13
0	0	5	0	0	0	0	0	6	$k=4 : X_{pk} = 1$	10
0	0	0	5	0	0	0	0	6	$k=7 : X_{pk} = 1$	13
									$k=2 : X_{pk} = 1$	8
0	0	0	0	5	0	0	0	6	$k=7 : X_{pk} = 1$	13
									$k=3 : X_{pk} = 1$	9
0	0	0	0	0	4	0	0	5	$k=4 : X_{pk} = 1$	9
0	0	0	0	0	0	4	0	5	$k=7 : X_{pk} = 1$	12
0	0	0	0	0	0	0	3	4	$k=7 : X_{pk} = 1$	11
Raw material utilization: 86.4% (48 objects)										**108**
Saw cycles occupation: 68.6% (10 cycles)										

so that the quality of the optimization could be adapted to the system, using full productivity. In this case, the solution is shown in Table 3.

According to Yanasse (2008), this solution would be reasonable for high-demand situations, which is not the case. Using this strategy, the good utilization of time is penalized with greater material waste.

Integrating both problems using the approach described in Section 3, we optimized the balance between both objectives. The solution is shown in Table 4.

This example also shows that empirical approaches, like imposing full cycles or ignoring that simultaneous objects can be cut, can lead to suboptimal solutions. It is observed that both objectives raw material utilization and machine utilization can be a trade-off, which is influenced by the structure of the problem data.

Suppose next that a change on γ^{un} in an industrial setting where the time to cut each item is longer because of the difficulty of cutting ($\gamma^{un} = 10$). The new integrated solution changes to the one presented in Table 5.

It is possible to verify that the change of an industrial technical parameter changes the cutting plan, having practical repercussions on the shop floor. Ignoring the parameter γ^{un} can lead to wrong decisions, even for small instances such as this one.

5.2. Industrial instances

The data presented in this subsection were collected from a metal mechanic factory, in a cutting process of steel and titanium tubes with 1.25 inches of diameter, which supplies a machining process for car part pieces. These objects can be cut in at most 7 units simultaneously in the same saw cycle.

Demand data and the lengths of the items were collected for a horizon of two shifts of 8 h. In Table 6 the data of the 23 items to be produced are presented. The example was developed with titanium items, that is the set with the most relevant machine costs.

The objects used are all identical in length (L), 1,200 cm. The unitary cost was collected with the financial department, with a value of \$122.78 per object in local monetary units. The machine utilization cost was estimated using the fees charged for outsourcing services, which is \$77.04 per hour. It was also considered to have a medium setup of 12.5 min and a medium machine time for one cut 1.67 min. The costs were st = \$16.05 and γ^{un} = \$2.14.

Table 6. Data of items to be cut.

Item (i)	Length (l_i) (cm)	Demand (b_i) (units)
1	593	37
2	590	29
3	526	38
4	517	35
5	506	21
6	494	41
7	486	34
8	464	42
9	447	43
10	420	27
11	415	33
12	401	31
13	400	38
14	391	38
15	382	47
16	357	28
17	355	43
18	353	45
19	318	39
20	293	38
21	265	31
22	265	45
23	246	30

These data were used in the approach described in Section 3. Column generation procedure time was 5.5 s and the search for an integer solution with columns generated was limited to 3,600 s.

The solution presented a plan with 49 saw cycles, of which 33 were completed (7 objects) and 16 not (3 cycles with 6 objects, 4 cycles with 5 objects, 3 cycles with 3 objects, 2 cycles with 2 objects, and 4 cycles with 1 object). The machine time utilization was 83.4%. The total number of objects was 286, which represents an occupation of 99.34%. The makespan (total time spent to produce all the orders) of the solution was 14.2 h, with a total cost of \$36,209.69. The gap over the relaxed solution was 0.25%.

To establish a comparison with the *modus operandi* of the factory studied, the empirical method used on the shop floor was simulated, which consists of filling cutting patterns from the biggest length items to the smaller ones and always using full cycles (7 objects). In Table 7, the solution of the model is compared with the empirical approach, with the CSP imposing cycles of 7 objects and with CSP relaxing integrality constraint of X_{pk} and partial saw cycles rounded so that the demand is met.

By definition, both the empirical method and CSP with the imposition of 7 objects have full machine utilization (once both run only full cycles). Nevertheless, this constraint leads to the overproduction of the items, which makes the usage of raw material worse. The final cost of the optimization of saw cycles represented gains of 8.4% over CSP with 7 objects, 23.7% over the empirical method with close computational CPU times and 0.12% over CSP with X_{pk} rounded.

5.3. Random instances

In this section, the influence of technical parameters in the outputs of the instances is assessed, when submitted to the approach described in Section 3. A

Table 7. Comparison between proposed method, empirical method, and CSP with 7 objects imposed.

Output	Proposed model	Empirical method	CSP with full cycles	CSP rounding results
Saw cycles (units)	49	54	**45**	51
Objects (units)	286	378	315	**285**
Objects occupation (%)	99.34%	75.2%	90.2%	**99.3%**
Execution time (hours)	14.2	15.4	**13.0**	14.8
Machine utilization (%)	83.4%	**100%**	**100%**	80.1%
Solution cost (\$)	**36,209.69**	47,596.40	39,678.29	36,252.49

set of 1,710 instances was designed, combining different scenarios. Distinct parameters were used for the item size (l_i), the number of simultaneous objects cut (NK), demand size (b_i), costs (θ) of raw material, setup (st), and unitary cut cost (γ^{un}). Most of the ranges tested were based on empirical data of the factory, which motivated the study.

Instances were generated with 30 items types ($NI = 30$) to be cut from objects with size 1,000 ($L = 1{,}000$). Three scenarios of sizes were tested: "small" ($l_i \in U[100, 300]$), "big" ($l_i \in U[300, 700]$) and "varied" ($l_i \in U[100, 700]$). The values were determined arbitrarily, based on empirical data of the factory that motivated the study, although they are close to the classes defined by Lodi et al. (2002).

Three scenarios with different numbers of maximum simultaneous objects were tested: $NK = 3$, $NK = 9$, and $NK = 15$. The values were also inspired by examples of real factories, and they were meant to represent cases with few, intermediate, and many objects at a time. In cases where $NK < 3$, the problem gets very close to the regular CSP, while in cases where $NK > 15$, there are almost no full saw cycles present in the solutions.

The scenarios considered for demand were: "low" ($b_i \in U[10, 50]$) and "high" ($b_i \in U[50, 100]$). Greater demands return to the case in which good solutions are obtained by using the CSP and rounding up the amounts obtained according to a multiple of NK. The cases of low and high demands were established according to empirical data.

Finally, 19 scenarios of relative costs were tested, using cost of objects (θ), setup (st), and unitary cutting costs (γ^{un}) with values 1, 10, and 100 and eliminating equivalent proportions (for example, cost 1 : 1 : 1 and 10 : 10 : 10 are the same scenario, except for a constant). In Table 8 the scenario reduction is illustrated.

The combination of these scenarios generated 342 types of instances; for each type five instances were created, summing up 1,710 instances. The CPU time was limited to 60 s after column generation to test the performance of the practical industrial conditions.

Table 8. Relative cost scenarios.

	Costs				
Scenario	θ	st	γ^{un}	Proportion	Elimination
1	1	1	1	1:1:1	
2	1	1	10	1:1:10	
3	1	1	100	1:1:100	
4	1	10	1	1:10:1	
5	1	10	10	1:10:10	
6	1	10	100	1:10:100	
7	1	100	1	1:100:1	
8	1	100	10	1:100:10	
9	1	100	100	1:100:100	
10	10	1	1	10:1:1	
11	10	1	10	10:1:10	
12	10	1	100	10:1:100	
13	10	10	1	10:10:1	
14	10	10	10	1:1:1	Equivalent to 1
15	10	10	100	1:1:10	Equivalent to 2
16	10	100	1	10:100:1	
17	10	100	10	1:10:1	Equivalent to 4
18	10	100	100	1:10:10	Equivalent to 5
19	10	1	1	10:1:1	
20	100	1	10	100:1:10	
21	100	1	100	100:1:100	
22	100	10	1	100:10:1	
23	100	10	10	10:1:1	Equivalent to 10
24	100	10	100	10:1:10	Equivalent to 11
25	100	100	1	100:100:1	
26	100	100	10	10:10:1	Equivalent to 13
27	100	100	100	1:1:1	Equivalent to 1

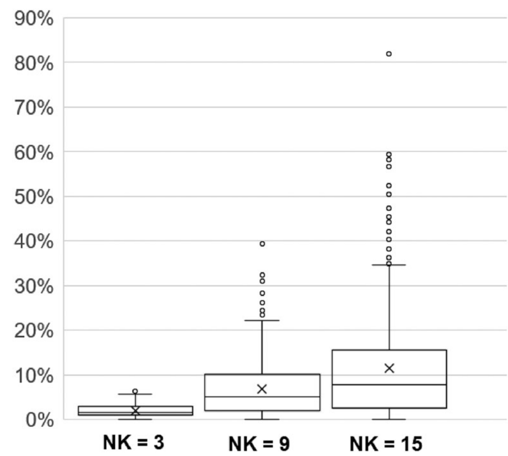

Figure 2. Box-plot of gap according to NK.

Table 9. Results according to the maximum number of objects cut simultaneously on a saw cycle (NK).

	Maximum of objects on a saw cycle (NK)				
Result	3	9	15	Average	p-value
gap (%)	2.0 (0.0 to 6.6)	6.8 (0.0 to 39.5)	11.4 (0.0 to 81.9)	6.7 (0.0 to 81.9)	**< 0.001**
Objects occupation (%)	94.5 (72.6 to 99.8)	93.4 (72.3 to 99.4)	92.5 (73.0 to 99.6)	93.5 (72.3 to 99.8)	**< 0.001**
Saw cycle occupation (%)	97.5 (82.9 to 100.0)	88.6 (50.9 to 97.6)	80.7 (29.5 to 94.1)	88.9 (29.5 to 100.0)	**< 0.001**
Machine costs (%)	61.2 (0.8 to 99.5)	50.2 (0.3 to 98.7)	45.5 (0.2 to 98.2)	52.3 (0.2 to 99.5)	**< 0.001**
Bin costs (%)	38.8 (0.5 to 99.2)	49.8 (1.3 to 99.7)	54.5 (1.8 to 99.8)	47.7 (0.5 to 99.8)	**< 0.001**
CPU time (s)	38.5 (1.0 to 97.7)	42.9 (0.8 to 101.5)	43.2 (1.1 to 112.5)	41.5 (0.8 to 112.5)	**0.023**
Cutting patterns	50.0 (12 to 85)	49.9 (14 to 91)	49.3 (9 to 83)	49.7 (9 to 91)	0.681

Table 10. Results according to the size of the items (l_i).

	Size of the items (l_i)				
Result	"Small" $U[100, 300]$	"Big" $U[300, 700]$	"Varied" $U[100, 700]$	Average	p-value
gap (%)	10.2 (0.4 to 81.9)	4.9 (0.0 to 30.8)	5.2 (0.0 to 35.3)	6.7 (0.0 to 81.9)	**< 0.001**
Objects occupation (%)	95.8 (76.0 to 99.8)	88.6 (72.3 to 97.9)	96.1 (76.4 to 99.6)	93.5 (72.3 to 99.8)	**< 0.001**
Saw cycle occupation (%)	86.1 (29.5 to 100.0)	90.6 (64.6 to 99.5)	90.1 (53.7 to 99.9)	88.9 (29.5 to 100.0)	**< 0.001**
Machine costs (%)	56.4 (0.6 to 99.5)	49.3 (0.2 to 99.0)	51.2 (0.3 to 99.3)	52.3 (0.2 to 99.5)	**0.004**
Bin costs (%)	43.6 (0.5 to 99.4)	50.7 (1.0 to 99.8)	48.8 (0.7 to 99.7)	47.7 (0.5 to 99.8)	**0.004**
CPU time (s)	71.7 (6.0 to 112.5)	13.1 (0.8 to 87.4)	39.8 (1.0 to 100.4)	41.5 (0.8 to 112.5)	**< 0.001**
Cutting patterns	62.4 (45 to 85)	36.1 (9 to 74)	50.7 (19 to 91)	49.7 (9 to 91)	**< 0.001**

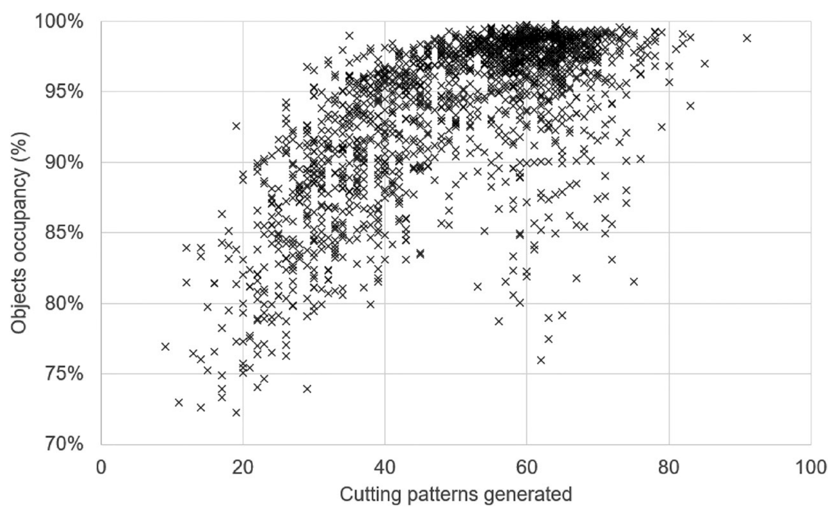

Figure 3. Dispersion of number of cutting patterns and objects occupation.

Table 11. Results according to the demand of the items (b_i).

	Demand of the items (b_i)			
Result	"Lower" $U[10, 50]$	"Higher" $U[50, 100]$	Average	p-value
gap (%)	9.9 (0.0 to 81.9)	3.6 (0.0 to 22.6)	6.7 (0.0 to 81.9)	**< 0.001**
Objects occupation (%)	92.5 (72.3 to 99.5)	94.4 (72.6 to 99.8)	93.5 (72.3 to 99.8)	**< 0.001**
Saw cycle occupation (%)	84.8 (29.5 to 100.0)	93.0 (53.0 to 100.0)	88.9 (29.5 to 100.0)	**< 0.001**
Machine costs (%)	52.8 (0.2 to 99.5)	51.9 (0.2 to 99.5)	52.3 (0.2 to 99.5)	0.614
Material costs (%)	47.2 (0.5 to 99.8)	48.1 (0.5 to 99.8)	47.7 (0.5 to 99.8)	0.614
CPU time (s)	39.0 (0.9 to 110.0)	44.1 (0.8 to 112.5)	41.5 (0.8 to 112.5)	**0.001**
Cutting patterns	49.9 (9 to 85)	49.5 (11 to 91)	49.7 (9 to 91)	0.573

The average gap verified was 6.75%, varying from 0.01% to 81.89%. Considering the time to generate the cutting patterns and to find an integer solution, CPU time varied from 0.8 to 112.5 s, with an average of 41.5 s, and 49.7 cutting patterns were generated on average, ranging from 9 to 91. The occupation of the objects (raw material usage) was 93.5% on average, with a minimum of 72.3% and a maximum of 99.8%. The occupation of saw cycles (number of objects cut compared to its full capacity), on the other hand, was 88.9% on average, with a 29.5% minimum and 100% maximum. The proportion of costs in the objective function was 52.3% of machine costs to 47.7% of material costs on average: the maximum of machine costs on an instance was 99.5%, whereas the maximum of material cost was 99.8%.

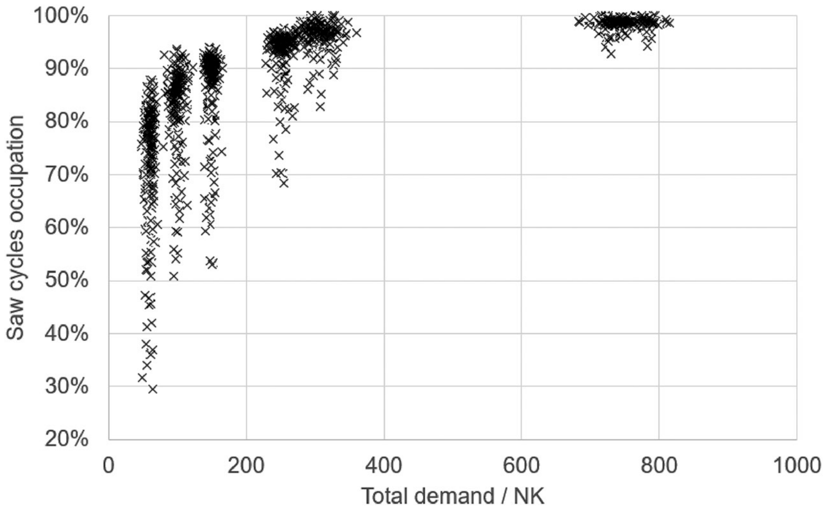

Figure 4. Dispersion of saw cycle occupation as a function of the relation of $\sum b_i/NK$.

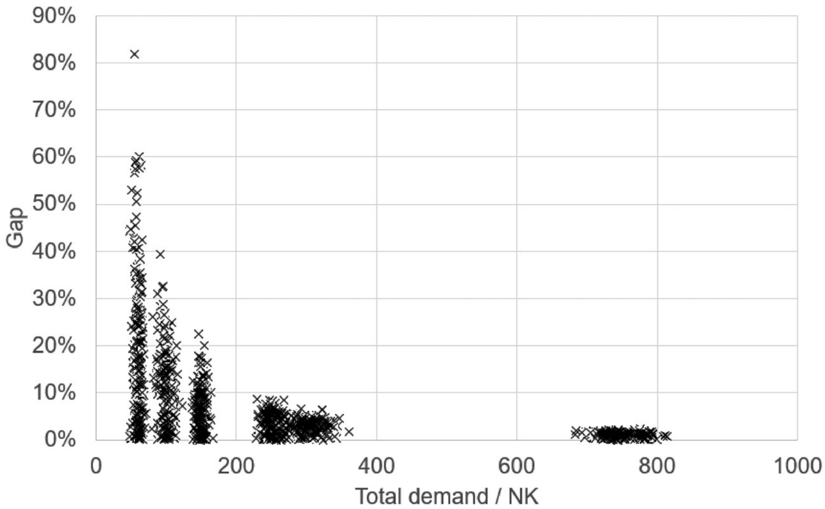

Figure 5. Dispersion of total demand and gap.

Table 12. Results according to the proportion of costs (θ: γ^{un}: st).

$\theta : st : \gamma^{un}$	Gap (%)	Objects occupation (%)	Saw cycles occupation (%)	Bin costs (%)	Machine costs (%)	CPU time (s)	Cutting patterns (units)
1:1:1	5.1	94.0	90.1	62.1	37.9	38.5	49.2
1:1:10	8.9	92.4	92.0	21.8	78.2	33.0	50.6
1:1:100	11.9	92.4	91.7	3.0	97.0	30.4	51.5
1:10:1	8.1	93.2	91.1	35.5	64.5	39.7	49.7
1:10:10	10.0	93.0	92.0	17.0	83.0	34.7	51.2
1:10:100	11.8	92.4	91.7	2.9	97.1	33.3	50.9
1:100:1	11.7	93.6	91.7	7.1	92.9	43.7	51.3
1:100:10	12.3	92.1	91.8	5.8	94.2	42.3	48.8
1:100:100	12.3	92.5	91.7	2.1	97.9	40.8	50.2
10:1:1	1.7	94.8	84.9	92.9	7.1	50.1	50.2
10:1:10	4.5	93.5	89.7	67.8	32.2	40.7	49.3
10:1:100	9.0	92.6	91.7	22.3	77.7	33.7	51.1
10:10:1	3.1	93.8	87.1	81.4	18.6	46.2	46.7
10:100:1	7.4	93.7	90.6	40.3	59.7	44.1	49.6
100:1:1	0.7	95.0	80.4	99.2	0.8	50.7	50.6
100:1:10	1.6	94.3	84.1	94.3	5.7	46.6	48.1
100:1:100	4.5	93.9	89.7	68.4	31.6	44.9	49.9
100:10:1	1.0	94.7	81.1	97.5	2.5	50.2	48.8
100:100:1	2.7	94.2	86.7	84.5	15.5	45.6	47.3
Average	6.7	93.5	88.9	47.7	52.3	41.5	49.7
p-value	**< 0.001**	**0.001**	**< 0.001**	**< 0.001**	**< 0.001**	**< 0.001**	0.780

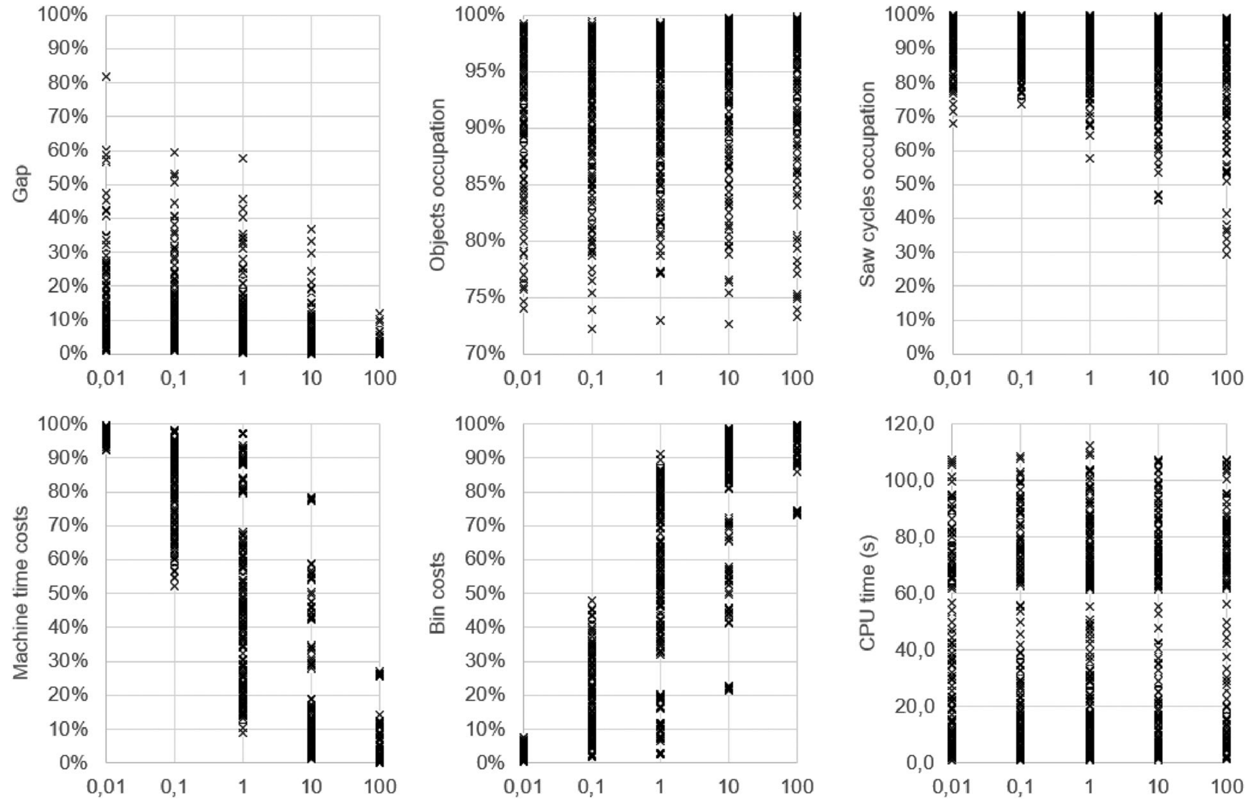

Figure 6. Dispersion of each output according to the ratio $\theta : \gamma^{un}$.

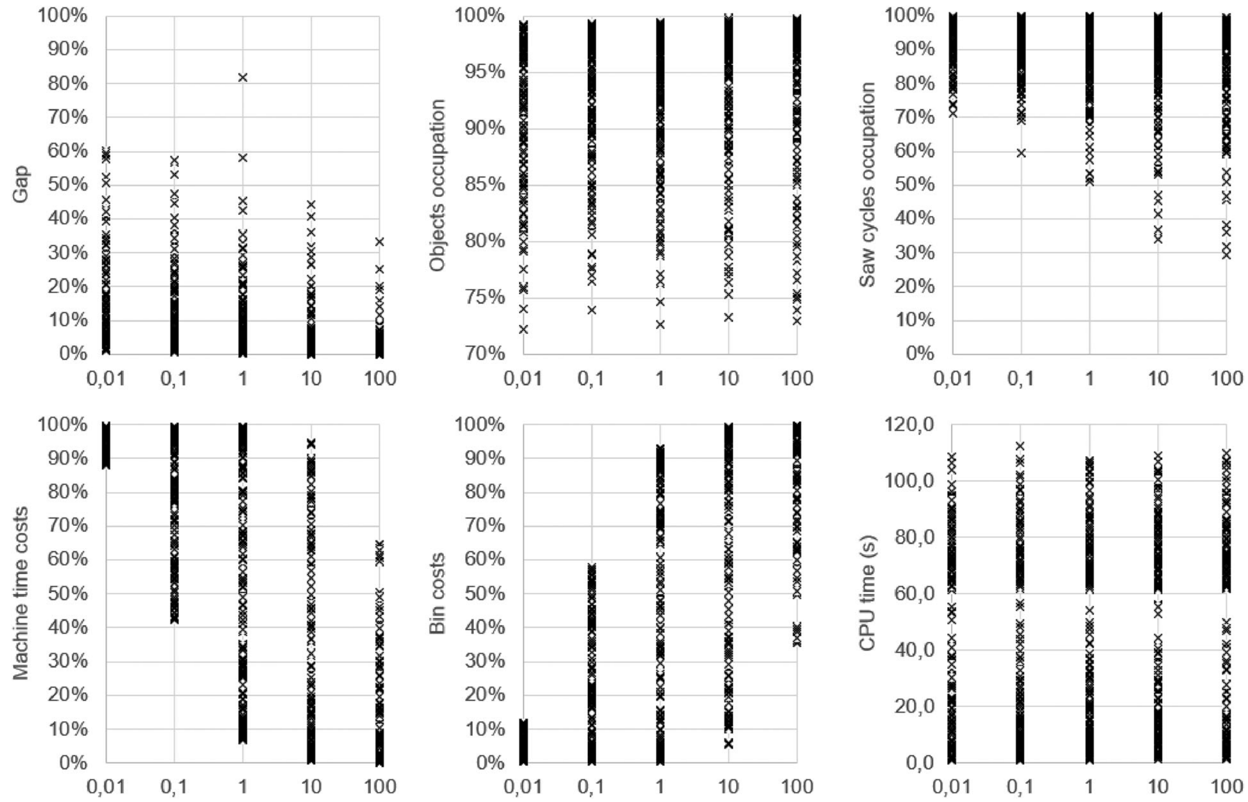

Figure 7. Dispersion of each output according to the ratio $\theta : st$.

The influence of each parameter tested in the outputs was analysed using analysis of variance with a significance level of 0.05.

In Table 9 the results for the maximum of objects that can be cut simultaneously on a saw cycle (*NK*) are shown. All the outputs (except for the number of cutting patterns) were impacted by these parameters ($p<$ 0.05). The difficulty of the instance, either measuring by the gap or by the CPU time, increased with *NK*. Indeed, it is possible to infer that larger

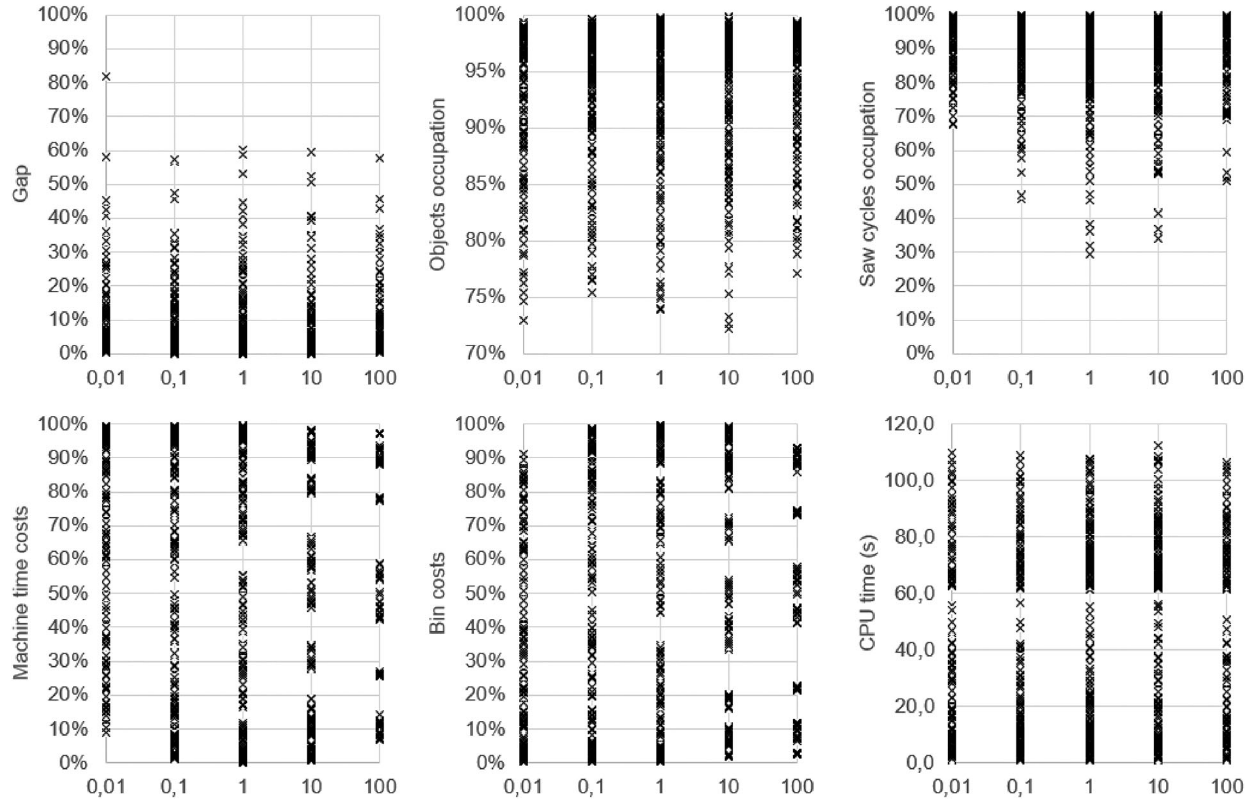

Figure 8. Dispersion of each output according to the ratio $st : \gamma^{un}$.

values of NK make it more difficult to balance good cutting patterns and full saw cycles. The box-plot of Figure 2 shows that the size of the gap as well as the variability increases with NK. The objects and saw cycle occupations empower this inference, as both decrease with NK. The proportion of machine costs over total costs was more important with smaller cycles, which also could be a trivial hypothesis because bigger cycles allow fewer machine costs.

In Table 10 the results according to the size of the items (l_i) are shown. All the outputs were impacted significantly by these parameters ($p< 0.05$). For big items, the difficulty of the instance was lower, either by the gap or by the CPU time. Instances with varied sizes were intermediate, and those with smaller items showed the worst gaps and CPU times.

The occupation of the objects had worse results for instances with big items. A possible reason is the smaller number of possible combinations of different cutting patterns, because of the items' size. Correlation data reinforces this conclusion, once the number of cutting patterns generated has a relative positive correlation with object occupation ($\rho = 0.669$). In spite of not being a strong linear correlation, dispersion on Figure 3 shows a non-linear tendency of this hypothesis. On the other hand, the occupation of the saw cycles was worse in instances with smaller items. Here, the diversity of cutting patterns using different combinations of these small items probably does not meet with enough demand to occupy full cycles.

The results for the parameter of the demand (b_i) are analysed on Table 11. Here, gaps, objects occupation, saw cycles occupation, and CPU time were statistically different ($p < 0.05$). The number of cutting patterns and the participation of machine costs and material costs were not influenced. Higher demands lead to tighter gaps, more material occupation, and more saw cycle occupation.

According to the hypothesis of Yanasse (2008), bigger demands approximate the saw cycle problem to a cutting stock problem where cutting patterns are constrained to a multiplicity NK. Indeed, the dispersion of Figure 4 allows the affirmation that the relation of the demand compared to the multiplicity of the problem is more relevant than the size of the demand. Higher relation leads to the better occupation of the cycles. As a consequence, the problem gets closer to a cutting stock problem and gaps are tighter, as can be verified in Figure 5.

Finally, results according to the proportion of costs are shown in Table 12. All the outputs were statistically influenced by this set of parameters, except for the number of cutting patterns.

To analyse the proportion between the cost of the objects (θ) and the machine cost of each item produced (γ^{un}), in Figure 6 the dispersion of each output is compared to the ratio θ:γ^{un}. With increase in this ratio, it is observed that the gaps decrease, the object occupation increases, and the saw cycle occupation decreases. All these observations are expected because the dilemma

Table 13. Summary of savings with the model over rounded approach.

Demand	$\theta : st : \gamma^{un}$	Small items sizes $NK = 3$	Small items sizes $NK = 9$	Small items sizes $NK = 15$	Mixed items sizes $NK = 3$	Mixed items sizes $NK = 9$	Mixed items sizes $NK = 15$	Large items sizes $NK = 3$	Large items sizes $NK = 9$	Large items sizes $NK = 15$
	1:1:1	12.6%	19.5%	21.3%	3.7%	6.9%	5.1%	1.8%	2.8%	3.6%
	1:1:10	19.1%	35.4%	41.5%	7.9%	16.4%	20.2%	3.0%	6.4%	7.2%
	1:1:100	19.0%	38.5%	45.3%	7.0%	21.0%	24.3%	2.6%	9.2%	13.9%
	1:10:1	16.1%	25.3%	30.6%	5.6%	11.9%	15.3%	2.2%	5.9%	5.2%
	1:10:10	18.5%	32.9%	43.5%	7.5%	14.8%	19.2%	2.3%	8.1%	11.6%
	1:10:100	18.8%	37.4%	48.3%	6.9%	19.1%	24.0%	2.9%	8.3%	12.8%
	1:100:1	15.5%	32.9%	41.3%	6.2%	18.4%	19.8%	2.5%	8.8%	12.1%
	1:100:10	15.4%	36.9%	43.9%	5.8%	16.2%	21.8%	2.6%	6.2%	11.6%
	1:100:100	18.0%	38.3%	50.3%	8.3%	18.8%	22.8%	2.9%	8.7%	13.2%
Low	10:1:1	7.0%	6.5%	5.3%	2.8%	3.4%	2.5%	1.1%	1.5%	1.9%
	10:1:10	14.5%	19.5%	20.0%	3.3%	4.8%	6.7%	1.4%	2.0%	2.8%
	10:1:100	18.9%	35.3%	45.9%	6.5%	13.8%	24.5%	2.6%	6.3%	5.8%
	10:10:1	9.9%	10.3%	8.9%	3.6%	4.6%	4.4%	1.0%	1.9%	2.5%
	10:100:1	14.6%	22.1%	24.9%	4.2%	9.6%	14.7%	2.1%	5.1%	6.4%
	100:1:1	6.3%	5.6%	5.9%	1.9%	2.9%	2.5%	0.7%	1.4%	1.1%
	100:1:10	6.6%	5.6%	5.7%	2.5%	2.3%	2.3%	1.2%	2.1%	1.7%
	100:1:100	13.5%	16.3%	17.8%	5.1%	5.2%	6.2%	1.4%	2.8%	2.9%
	100:10:1	6.3%	5.7%	5.5%	2.2%	2.1%	2.2%	1.0%	1.8%	1.6%
	100:100:1	8.5%	7.8%	7.5%	3.3%	3.6%	3.5%	1.2%	2.2%	1.7%
	1:1:1	7.0%	8.6%	9.0%	1.9%	2.8%	3.1%	0.8%	1.2%	1.7%
	1:1:10	7.9%	17.4%	22.0%	2.8%	5.9%	10.5%	1.4%	2.5%	4.1%
	1:1:100	8.2%	19.3%	26.5%	3.3%	10.8%	12.9%	1.2%	4.1%	4.8%
	1:10:1	6.7%	11.2%	16.1%	2.0%	5.2%	6.7%	0.8%	2.8%	2.3%
	1:10:10	7.8%	17.6%	26.3%	2.8%	7.9%	9.7%	1.0%	3.0%	4.6%
	1:10:100	8.2%	21.3%	30.7%	3.3%	8.5%	13.1%	1.1%	4.0%	5.3%
	1:100:1	7.4%	15.3%	21.7%	2.9%	7.8%	10.0%	1.3%	3.5%	6.0%
	1:100:10	7.9%	17.8%	23.3%	2.6%	6.5%	11.3%	1.3%	3.1%	5.0%
	1:100:100	8.8%	19.7%	28.7%	3.7%	9.0%	15.2%	1.1%	4.8%	6.0%
High	10:1:1	3.4%	2.3%	2.6%	1.2%	1.0%	1.2%	0.5%	0.7%	0.8%
	10:1:10	6.2%	7.8%	9.1%	2.2%	2.5%	2.4%	0.7%	1.0%	1.4%
	10:1:100	7.9%	20.5%	22.7%	3.0%	8.2%	9.6%	1.2%	3.7%	4.1%
	10:10:1	3.9%	3.6%	3.3%	1.3%	1.4%	1.5%	0.5%	0.9%	0.9%
	10:100:1	7.0%	10.6%	12.2%	2.3%	4.7%	6.9%	1.0%	2.0%	2.9%
	100:1:1	2.3%	2.2%	2.1%	1.1%	1.0%	1.2%	0.5%	0.7%	0.9%
	100:1:10	3.1%	2.5%	1.4%	1.1%	0.8%	1.1%	0.5%	0.6%	0.9%
	100:1:100	6.0%	9.3%	7.6%	2.3%	2.8%	3.4%	0.6%	1.1%	1.2%
	100:10:1	2.6%	2.1%	1.9%	1.1%	1.2%	1.1%	0.3%	0.6%	0.9%
	100:100:1	3.5%	3.1%	2.1%	1.1%	1.5%	1.3%	0.7%	0.8%	0.9%

between machine costs and raw material costs is unbalanced in favour of the raw material. CPU time increased discretely, although outliers are at the lowest ratio. The proportion of material costs and machine costs on the final solution trivially follows the ratio.

Similar conclusions are found when analysing the ratio between the cost of the objects (θ) and the machine cost of each setup of a saw cycle (st), as can be seen in Figure 7. A slightly distinct observation concerning the previous analysis is the evolution of object occupation, which is less severe than the ratio between θ and γ^{un} because high setup costs can also drive to higher occupation of objects.

Finally, Figure 8 reveals the same relation when comparing the ratio machine cost of setup of the cycle (st) and machine cost related to each item cut (γ^{un}). Here, no clear trends are verified for gaps, CPU times, and the portion of machine utilization and material cost on the final solution. The superiority of setup costs can impact on better occupation of the objects. In addition, saw cycle occupations are greater when these costs are similar.

5.4. Comparison with a rounded approach

In addition to analysing outputs related to the instance difficulty and the balance between costs, the following comparison intends to highlight the technical situations in which the proposed approach provides improved solutions when compared to a simplified approach. The 1,710 random generated instances were also solved using the classic CSP imposing full saw cycles, relaxing integrality constraint of X_{pk}. Non-integer values were considered as partial saw cycles so that the demand is met. For example, if $X_{pk} = 4.5$ on a saw cycle were $NK = 5$, the simplified method computed four full cycles plus a cycle with only three objects.

Table 14. Summary of comparison of the proposed method with Yanasse (2008).

Parameters		Average improvement			Instances improved		
		H1	H2	H3	H1	H2	H3
Items sizes (l_i)	Small	35.8%	2.0%	15.0%	100%	80%	100%
	Variated	15.6%	1.5%	7.8%	100%	97%	100%
	Big	10.1%	1.2%	6.1%	100%	95%	100%
Maximum simultaneous objects in a cycle (*NK*)	3	7.3%	0.5%	5.5%	100%	92%	100%
	9	22.3%	1.5%	10.6%	100%	91%	100%
	15	32.0%	2.7%	12.8%	100%	90%	100%
Demand (b_i)	low	26.3%	2.4%	12.6%	100%	93%	100%
	high	14.7%	0.8%	6.7%	100%	89%	100%
Relative costs ($\theta : st : \gamma^{un}$)	1 : 1 : 1	21.3%	1.1%	7.2%	100%	88%	100%
	1 : 1 : 10	19.6%	1.5%	7.8%	100%	93%	100%
	1 : 1 : 100	19.1%	1.4%	8.6%	100%	84%	100%
	1 : 10 : 1	19.3%	0.7%	14.0%	100%	88%	100%
	1 : 10 : 10	19.0%	1.2%	8.4%	100%	92%	100%
	1 : 10 : 100	18.0%	1.7%	4.3%	100%	89%	100%
	1 : 100 : 1	17.1%	0.2%	22.0%	100%	76%	100%
	1 : 100 : 10	17.2%	0.2%	18.3%	100%	81%	100%
	1 : 100 : 100	18.1%	1.2%	9.3%	100%	88%	100%
	10 : 1 : 1	23.0%	2.2%	5.9%	100%	98%	100%
	10 : 1 : 10	21.5%	1.5%	7.6%	100%	89%	100%
	10 : 1 : 100	19.8%	1.5%	12.5%	100%	92%	100%
	10 : 10 : 1	22.4%	2.3%	8.0%	100%	98%	100%
	10 : 100 : 1	19.7%	0.8%	16.0%	100%	87%	100%
	100 : 1 : 1	23.6%	3.3%	5.8%	100%	100%	100%
	100 : 1 : 10	22.9%	2.4%	5.3%	100%	100%	100%
	100 : 1 : 100	22.0%	1.3%	7.7%	100%	89%	100%
	100 : 10 : 1	23.9%	3.2%	6.0%	100%	100%	100%
	100 : 100 : 1	22.5%	2.0%	8.0%	100%	96%	100%
Average results		20.5%	1.6%	9.6%	100%	91%	100%

In Table 13 the value of the integrated approach is summarized in terms of savings (in percentage) of the total cost using the solution method proposed when compared to this rounded approach. To represent the impact of each parameter on the results, the background grey scale for each one of the cells gives the comparative intensity between the set of results.

The results point out an average savings of 8.6%, ranging from 0.1% to 54.9%. In all instances some improvement were observed, as expected. All four technical characteristics have a statistically relevant difference in this savings ($p < 0.001$ for all of them). Indeed, with Table 13 it is possible to infer that the proposed model has more relevance in terms of savings when compared to a simpler straightforward method when (i) demands are low, (ii) there is a large presence of small items, (iii) *NK* is high, and (iv) θ is low.

5.5. Comparison with heuristics from the literature

In this section, in Table 14, we compare solutions obtained using the proposed method with heuristics H1, H2, and H3 proposed in Yanasse (2008). It can be observed that results were superior most of the times.

The solution in heuristic H1 is obtained by rounding up to the nearest integer the solution of CSP divided by *NK*. The solution in heuristic H2 is obtained from the solution of CSP with demands divided by *NK* and rounded up to the nearest integer. The solution in post-optimized to eliminate surplus production. The solution of H3 is obtained by rounding down the solution of CSP to the nearest multiple of *NK* and optimizing the weighed cost of cutting the residual demand.

On average, the model improved 20.5% the solution delivered by the heuristic H1, 1.6% the heuristic H2 and 9.6% the heuristic H3. The proposed method was superior in all the instances when compared with H1 and H3. Heuristic H2 was superior in some instances, particularly when items were small and setup costs are much superior than material costs and unitary machine cost related to each item cut.

6. Conclusions

In this paper we present an integrated approach for the one-dimensional cutting stock problem with saw cycles considering distinct processing times according to the number of items in the cutting pattern.

A mathematical model for this problem was proposed, considering both machine costs and raw materials costs, meeting the demands of each item. A solution method was proposed based on a column generation procedure. We showed that a full cycle column in the restricted master problem has always the smallest relative cost compared to partial cycles column of the same cutting pattern.

In addition to showing the importance of the integrated approach on a detailed example and potential gains on real industry examples, we conducted random computational experiments to explore the impact of the parameters on the outputs related to the difficulty of the instance (gaps, CPU time and number of cutting patterns generated) and the industrial performance of the solution (proportion of machine and material costs, objects utilization, and saw cycles occupation). Finally, the proposed method was compared to a simplified approach to assess the cases in which potential savings are high.

The impact of each parameter on the outputs can lead to future work on different models or solution approaches for specific industrial environments, according to the characteristics of demand, size of items, the multiplicity of cut, and relative costs. Further work could explore tailored solution methods to address these scenarios. In addition, integration to other decision challenges can be addressed: multi-period decision environments (Poldi & de Araujo, 2016), multiple object sizes, and usable leftovers (Arenales et al., 2015).

The model does not consider different unitary processing times (γ_p) according to the number of objects on a saw cycle, because the incremental time spent to cut a single object or a full cycle was insignificant in the factory studied. But this extension can be easily addressed by replacing parameter γ_p with a γ_{pk} according to the number of objects k on a cycle, when this incremental time is significant.

Finally, stock costs of items surplussing the demand were not considered, since they are not relevant compared to the other costs in the factory focused. These costs may be considered in future works to address the situation where they are relevant.

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ORCID

F. K. Lemos http://orcid.org/0000-0002-0488-7839
A. C. Cherri http://orcid.org/0000-0003-1934-4682
S. A. de Araujo http://orcid.org/0000-0002-4762-2048
H. H. Yanasse http://orcid.org/0000-0002-6946-9670

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