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The cutting stock problem with multiple manufacturing modes applied to a construction industry

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This paper addresses the problem of multiple manufacturing modes integrated into the cutting stock problem, based on a real-life application of the concrete pole manufacturing. The main aim is to propose, formulate and test this integrated problem, which can be applied to the construction industry and other contexts of multiple manufacturing modes with cutting processes. The motivation for this proposal is the construction industry, in which reinforced concrete structures can be reinforced by various combinations of one-dimensional steel bars of varying thicknesses and lengths. An integer programming mathematical formulation is proposed aiming to minimise the total cost and to meet a demand of final products with different possible configurations. A column generation procedure is used as the solution method together with a heuristic procedure to find an integer solution. Computational results were performed with practical instances in order to assess the value of the approach and with a set of random generated instances in order to explore the influence of parameters on the results. Some managerial insights are presented.

Keywords: cutting stock problems; integer programming; multiple modes; construction industry; integrated problems; column generation

1. Introduction

Cutting stock problems (CSP) aim to minimise the amount of objects to cut a set of weakly heterogeneous demanded items without overlapping (Wäscher, Haußner, and Schumann 2007). It is one of the most well-known combinatorial optimisation problems (Bischoff and Wäscher 1995), which has kindled intense practical and theoretical interest due to the diversity of corresponding real problems and their complexity (Erjavec, Gradišar, and Trkman 2009; Gomes et al. 2013). Immersion in real situations has motivated research with different objectives and constraints, taking into account production, marketing, technological and distribution aspects (Tomat and Gradišar 2017). In fact, several real-world operational conditions have been addressed recently (Foerster and Wascher 2000; Neidlein, Scholz, and Wäscher 2016; Wang et al. 2019), challenging researchers and practitioners to consider the production system as a whole (Arbib, Marinelli, and Ventura 2016).

This paper addresses one of these operational issues, motivated by the real-life problem of a concrete pole factory application, which has a final product manufactured in a number of different ways. The construction industry, indeed, is significantly important for economic growth and it faces strong market competition (Chen et al. 2018). As any reinforced concrete structure, such as beams or cornerstones, a concrete pole is a combination of concrete and a steel armour. The latter is built as a set of one-dimensional steel bars, which links the situation to the cutting stock problem since large stocked objects are cut as part of the production process of smaller pieces to satisfy the demand of the armour (Martin et al. 2019).

Nevertheless, the pole armour must be stronger (more steel reinforcement) at the bottom than at the top of it and specifications are given, such as the total amount of steel on a longitudinal section. As a consequence, the same product may be manufactured with different combinations of steel bars, varying in lengths and thicknesses, without impairing its quality. The minimum total cost to manufacture an specific demand can reduce wasteful processes, material, and activities, which consume resources but do not add value to the final deliverable (Meng 2019).

The aim of this paper is to explore the integration of the cutting stock problem considering the environment of alternative modes of manufacturing. The main contributions are (i) proposing an unexplored integration of these problems; (ii) an explicit mathematical formulation (1D-CSP-AM) for this problem, (iii) adopting a straightforward solution method from

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the literature, based on a column generation procedure; (iv) extensive computational tests, including real data and random generated data, to analyse the influence of parameters on the difficulty of the instances and different managerial contexts; and (v) analyzing the value of integration by comparing the solutions of the integrated model to the sequential solution method, which is used in practice and does not fully reflect the integrated aspects. We evaluated how different parameters such as the number of different lengths of items, the number of different thicknesses, the number of different alternative modes, and the length of items and costs of objects have an impact on the value of the integration.

This paper is organised into seven sections, including this introduction, which poses the problem, its motivation and the objectives of the study. Section 2 details the production process that motivated the problem to make clear what the trade-offs of multiple manufacturing modes are in a cutting process environment. Section 3 discusses a literature review related to the problems that will be integrated within the cutting stock problem, especially the problem of product substitutions and multi-mode scheduling tasks. Section 4 proposes the mathematical formulation, while Section 5 describes the solution method based on a column generation procedure. Section 6 shows the computational results for real and random instances. Section 7 discusses the main findings of the results and proposes future research.

2. Production process and trade-offs

Concrete poles are construction products that are produced, as any reinforced concrete structure, by depositing concrete on a steel armour. While concrete is responsible for the toughness and resistance to axial efforts, it is not a material that assures shear force resistance. This is why a steel armour is needed. The relative weight of steel on the total cost of the bill of materials is far greater than the concrete.

For concrete poles, the steel armour is divided into two main parts: the longitudinal armour, constituted by the steel bars along the pole length and the transversal armour, which corresponds to a certain number of stirrups. The first one usually corresponds to more than 90% of the cost and raises the question studied in this paper.

The structural requirements for a certain specification of shear force results in a total amount of area of steel in each cross section along the product. Figure 1 illustrates the reasoning behind the decision process: a certain effort projected on the poles is represented as a shear force diagram, which requires different steel section areas along the pole. Thus, different combinations of lengths and thicknesses of steel bars can meet technical requirements.

It would be trivial to decide the least expensive combination when analyzing the manufacturing of a single pole and raw material costs incurred. However, given a certain demand for multiple products, each with a set of possible configurations using different thicknesses of steel objects, the overall cost is not always minimised by choosing the best individual configuration of each product. Indeed, not even for a single type of pole in multiple quantities is it true that the less costly individual option will optimise the total cost as leftovers may be better combined.

Figure 2 explores this trade-off with the example of a large turnover product, a small pole used for residential electricity metres, 600 cm long and a requirement of bearing 150 daN. Two alternative modes are compared, as an example (even other combinations are possible), in which mode A uses less material of greater thickness and mode B uses more length of a thinner thickness.

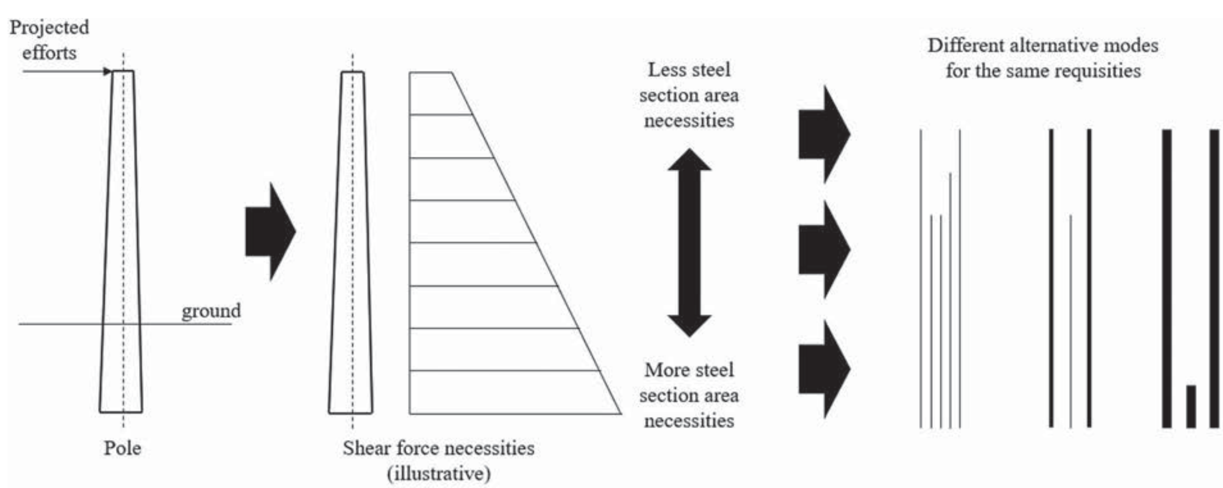

Figure 1. Alternative modes for a single product.

	Mode A	Mode B
Thickness of raw material:	1/2"	3/8"
Unit cost (objects with 1,200 cm)	$ 40.00	$ 30.00
Items required	4 pieces of 600 cm	4 pieces of 600 cm 2 pieces of 280 cm
Unit cost (without further use of leftovers)	2 bars of 1,200 cm Total cost = **$ 80**	3 bars of 1,200 cm Total cost = **$ 90**
Unit cost (considering total use of leftovers)	$\frac{4 \times 600}{1200} \times 40 = \$\,\mathbf{80}$	$\frac{4 \times 600 + 2 \times 280}{1200} \times 30 = \$\,\mathbf{74}$
5 units lot	1 unit	4 units
	Total cost = $ 380 Unit cost = $ 76	

Figure 2. Comparison of costs having alternative modes for the same product.

Considering the assumption that there is no possibility of using leftovers, one should choose mode A, as it has a lower unit cost. Assuming that all the leftovers can be used, one should opt for mode B, once the cost of the material effectively used per product is lower. Finally, considering a lot containing five products to be produced, the most economic combination is to produce 4 poles of mode B and 1 pole of mode A, demanding 2 bars with a thickness of 1/2″ and 10 bars with a thickness of 3/8″.

3. Related literature

The main issue of this paper is related to multiple manufacturing modes for the same product in a cutting process environment. This section presents some literature related to this condition, especially the problem of product substitutions and the multi-mode scheduling problem.

Product substitution is a flexibility instrument that can be applied within supply chain management (Lang and Domschke 2010). It addresses decision environments, where a product can be used to meet the demand of a similar one (Chopra and Meindl 2007). It can be further divided into customer-driven and firm-driven (Hale, Pyke, and Rudi 2000). The latter is more closely related to this paper, in which the supplier decides on using a substitute. It can also be classified according to its substitution mechanism (assortment-based, inventory-based or price-based) and the direction of substitutability (Shin et al. 2015).

Applications of the product substitution problem are found in multiple situations from pharmaceutical product supply chains (Zahiri, Jula, and Tavakkoli-Moghaddam 2018), semiconductors (Gallego, Katircioglu, and Ramachandran 2006), glass (Taşkın and Ünal 2009), steel manufacturing (Balakrishnan and Geunes 2003; Denton and Gupta 2004) and online retailing (Saberi et al. 2017); to specific decision making processes, such as shelf-space optimisation (Hübner and Schaal 2017), pricing decisions (Ma et al. 2018) and procurement optimisation (Pak 2016). Pan (2018) compares 12 mathematical models, according to the demand characteristic, objective function, ending inventory and decision variables. For surveys on the subject of product substitution, the paper by Shin et al. (2015) can be consulted.

Although the product substitution feature proposes that different products can meet the same demand, the civil construction industry shows a situation where different combinations of raw materials can result in the same unique product. Moreover, the integration of multi-level processing (in this case, the cutting phase and the manufacturing one) on a single-period decision environment is rare (Melega, Araujo, and Jans 2018). In particular, a lack of integrated approaches with cutting problems was observed. Early approaches considered assortment problems in cutting decisions, where bigger items could be cut to meet the demand of smaller ones (Pentico 1988; Sadowski 1959; Wolfson 1965), but without any integration on further process stages or impact on substitution processes.

Multi-mode scheduling problems, on the other hand, deal with alternative ways to perform an activity (Ballestín, Barrios, and Valls 2013; Zhang 2016) by distinct combinations of production means, such as machines and labour assignment, resulting in different times or costs (Zhang and Tam 2006). It is more common in the project scheduling area, although there are industrial applications, such as combinations of multiple machine classes available to execute an order (Riise, Mannino, and Lamorgese 2016) or assignments of different transportation modes to a supply chain (Fan, Schwartz, and Voß 2017). Other applications are computer processing scheduling (Bianco et al. 1997; Drozdowski 1996), maintenance, cleaning and quality auditing (Abdelmaguid, Shalaby, and Awwad 2014), workforce assignment (Chou 2013) and health services (Riise, Mannino, and Lamorgese 2016).

Different formulations can be proposed for the multi-mode scheduling problems according to renewability, constraints and decision environments of the resources (Beşikci, Bilge, and Ulusoy 2015; Cheng et al. 2015; Riise, Mannino, and Lamorgese 2016). The problem is NP-hard (Blazewicz, Lenstra, and Kan 1983; Jarboui et al. 2008) and, depending on the constraints considered, its feasibility can already be NP-complete (Kolisch 1995; Kolisch and Drexl 1997). Some research can be referred to on the subject (Brucker et al. 1999; Hartmann and Briskorn 2010; Weglarz et al. 2011) and also on solution methods for it (Van Peteghem and Vanhoucke 2014). Nevertheless, there is still a gap in the literature concerning integration with cutting and packing aspects.

4. Mathematical formulation

This section describes the one-dimensional cutting stock problem with alternative manufacturing modes (1D-CSP-AM). Let $J(j \in J,\ j = 1, \ldots, NJ)$ represent a set of final products with demand d_j and $K(k \in K,\ k = 1, \ldots, NK)$ a set of thickness options of one-dimensional raw materials, that can be used to manufacture J, using items of length $i(i \in I,\ i = 1, \ldots, NI)$ cut from objects of length L and cost θ_k depending on thickness k. Let $M_j(m \in M_j)$ be a set of alternative modes to meet the demand of a product j. Each mode m is characterised by a combination of one-dimensional items with thickness k and length l_i in quantity b_{ikm}. In other words, a given product j may be built in M_j different ways, each one of them requiring b_{ikm} items of length l_i of thickness k.

Let $P(p \in P)$ be a set of cutting patterns for the objects of thickness k, each one producing an amount a_{ip} of items of length l_i of this thickness. A cutting pattern is a possible way of cutting an object into small items, so that the sum of their lengths respects the total length of the object.

The objective of this formulation is to meet the demand of all final products of J, choosing the most convenient combination of configurations in order to minimise the total raw material cost. It is important to stress that any one of the configurations meets the quality and technical specification of the resulting armour of the final product. The number of objects of thickness k cut according to pattern p is an integer decision variable represented by X_{pk}. The variables Z_{jm} indicate the amount of final products j produced according to the mode $m(m \in M_j)$.

The One-dimensional Cutting Stock Problem with Alternative Modes (1D-CSP-AM) is formulated in (1)–(5).

$$\min \quad \sum_{p \in P} \sum_{k \in K} \theta_k X_{pk} \tag{1}$$

$$s.t.: \quad \sum_{m \in M_j} Z_{jm} \geq d_j, \qquad j \in J, \tag{2}$$

$$\sum_{p \in P} a_{ip} X_{pk} \geq \sum_{j \in J} \sum_{m \in M_j} b_{ikm} Z_{jm}, \qquad i \in I,\ k \in K, \tag{3}$$

$$Z_{jm} \in \mathbb{Z}^+, \qquad j \in J,\ m \in M_j, \tag{4}$$

$$X_{pk} \in \mathbb{Z}^+, \qquad p \in P,\ k \in K. \tag{5}$$

In the mathematical model (1)–(5), the objective function (1) minimises the total cost by cutting the set of objects of different thicknesses. Constraints (2) assure that the demand of each product j is met, with all possible modes. In (3), the demand of final products j is related to the respective demands of items i to be cut according to the modes Z_{jm} chosen. Therefore, the total of items i cut among all the set P must meet the demand required by amounts Z_{jm}. Decision variables are integer, as stated in (4) and (5).

5. Solution method

All the Z_{jm} variables, in polynomial number, are enumerated, according to the technological data, and included in the initial model, before starting the column generation. Taking into account the large amount of variables X_{pk}, due to multiple possible cutting patterns, it is not reasonable to solve the mathematical model (1)–(5) with all the variables explicitly enumerated.

To deal with this difficulty, the column generation method based on Gilmore and Gomory (1961, 1963) can be applied in order to generate the coefficients of variables X_{pk} ('columns') for the optimal solution of the relaxed linear model.

To perform the column generation process, the integrality conditions (4) and (5) are relaxed and the restricted master problem (1)–(5) is initialised with homogeneous cutting patterns. In these initial columns, there is only one item type with the maximum amount of it as possible. They assure the existence of a viable integer solution after the column generation procedure. At each iteration, the dual values referring to constraints (3) are used to compute the objective function of a subproblem that minimises the relative cost associated to a new set of X_{pk}.

Considering that π_{ik} are the dual values associated to constraints (3) and α_i is a decision variable that represents the amount of items i in the column generated, the reduced cost associated with X_{pk}, denoted by $\overline{c}_{pk}$, is presented by (6).

$$\overline{c}_{pk} = \theta_k - \sum_{i \in I} \pi_{ik} \alpha_{ip}. \tag{6}$$

For each thickness k, a subproblem must be solved and new columns are generated. These columns must be added to the restricted master problem while the relative cost is attractive, that is, they improve the solution of the restricted master problem.

The subproblem to generate columns for (1)–(5) for each thickness k is presented in (7)–(9).

$$\min \quad \theta_k - \sum_{i \in I} \pi_{ik} \alpha_i \tag{7}$$

$$s.t.: \quad \sum_{i \in I} \alpha_i l_i \le L, \tag{8}$$

$$\alpha_i \in \mathbb{Z}^+, \quad i \in I. \tag{9}$$

In the mathematical model (7)–(9), the objective function (7) minimises the relative cost of raw material with thickness k. Constraints (8) assure that the length of the objects (L) is respected, limiting the items assigned in the cutting pattern. Finally, (9) defines the domain of the decision variables.

After the column generation procedure, a heuristic solution is obtained by running the mathematical model (1)–(5) in its integer version, using the initial columns and the columns generated by the formulation (7)–(9), which is called 'integer' run. The value of the solution of the linear relaxation of the formulation after the column generation procedure is a valid lower bound and it is used to compute the quality of the integer solution.

In spite of the simplicity of the procedure, tight gaps were obtained in reasonable computational times along the computational experiments, as can be seen further on. Other column generation-based methods for different problems in the literature consider this approach when the final gap is lower than a specific threshold (Bertoli, Kilby, and Urli 2019; Flores-Quiroz, Pinto, and Zhang 2019; Liang et al. 2018). For random instances, in addition to this procedure, we also tested what was called 'rounded' run, in which the model was solved after column generation with the variables X_{pk} relaxed in their integrality constraints. The final result was obtained rounding up the values of X_{pk}.

6. Computational results

The proposed model, as well as the described solution method, were tested for three different situations: (i) an illustrative instance is generated, in order to show the relevance of an integrated approach; (ii) a practical instance is solved, considering real data, and the results show its advantage when compared with the empirical approach used by the factory that motivated the study; and (iii) a set of random instances was generated in order to explore the performance of the model and its bounds.

All the experiments were performed on a 16 Gb RAM memory computer with i7 processor CPLEX 12.8 was used to solve the master problem and subproblems in each iteration of the model and OPL language was used to code the method.

Table 1. Products, alternative modes, lengths and amounts of items required.

Product (j)	Manufacturing mode (m)	Thickness (k)	Required items (b_{ikm})
$j = 1$	$m = 1$	$k = 1$	4×700 mm
		$k = 2$	2×300 mm
		$k = 3$	$\varnothing$
		$k = 4$	$\varnothing$
	$m = 2$	$k = 1$	$\varnothing$
		$k = 2$	6×700 mm
		$k = 3$	2×400 mm
		$k = 4$	$\varnothing$
	$m = 3$	$k = 1$	$\varnothing$
		$k = 2$	$\varnothing$
		$k = 3$	6×700 mm
		$k = 4$	8×600 mm
	$m = 4$	$k = 1$	4×700 mm
		$k = 2$	$\varnothing$
		$k = 3$	$\varnothing$
		$k = 4$	2×700 mm
$j = 2$	$m = 1$	$k = 1$	4×700 mm
		$k = 2$	2×600 mm
		$k = 3$	$\varnothing$
		$k = 4$	$\varnothing$
	$m = 2$	$k = 1$	$\varnothing$
		$k = 2$	8×400 mm
		$k = 3$	6×600 mm
		$k = 4$	$\varnothing$
	$m = 3$	$k = 1$	$\varnothing$
		$k = 2$	$\varnothing$
		$k = 3$	16×300 mm
		$k = 4$	12×400 mm
	$m = 4$	$k = 1$	8×300 mm
		$k = 2$	$\varnothing$
		$k = 3$	$\varnothing$
		$k = 4$	8×700 mm

Table 2. Information about objects.

Thickness	1	2	3	4
Unit cost (θ_k)	\$ 40	\$ 25	\$ 20	\$ 10

6.1. Illustrative instance

This small size instance has an illustrative character in order to explore details of the solution and make clear the advantage of an integrated approach. Consider a request card of 2 distinct products ($NJ = 2$), each one with a demand of 100 units ($d_1 = d_2 = 100$), where it is possible to produce each one in 4 alternative modes ($|M_1| = |M_2| = 4$). These modes are shown in Table 1, with their respective amounts b_{ikm} required of items with length l_i of thickness k.

All objects have the same length ($L = 1000$ mm). However, their costs are distinct (θ_k) due to physical property differences (such as thickness, for example). Table 2 shows the real data.

The unit cost of each mode can be calculated for each final product. There is an assumption, in this case, of the future use of all the leftovers. Table 3 presents these costs, highlighting the best one for each product through this approach.

In a 'sequential' approach, the best alternative (in terms of raw material unit cost) is chosen first among the available modes. For posterior classic CSP optimisation, product 1 would be manufactured on mode 2 and for product 2 mode 1 would be chosen. However, this would result in a demand of 400 items of 700 mm length with thickness 1; 200 items of 600 mm length with thickness 2; 600 items of 700 mm with thickness 2; and 200 items of 400 mm with thickness 3. Once

Table 3. Unit cost of each alternative mode, considering 100% of usage on cutting.

Manufacturing modes (m)	Products (j): $j = 1$	$j = 2$
$m = 1$	$\frac{40 \cdot 4 \cdot 700 + 25 \cdot 2 \cdot 300}{1000} = \$127$	$\frac{40 \cdot 4 \cdot 700 + 25 \cdot 2 \cdot 600}{1000} = \mathbf{\$142}$
$m = 2$	$\frac{25 \cdot 6 \cdot 700 + 20 \cdot 2 \cdot 400}{1000} = \mathbf{\$121}$	$\frac{25 \cdot 8 \cdot 400 + 20 \cdot 6 \cdot 600}{1000} = \$152$
$m = 3$	$\frac{20 \cdot 6 \cdot 700 + 10 \cdot 8 \cdot 600}{1000} = \$132$	$\frac{20 \cdot 16 \cdot 300 + 10 \cdot 12 \cdot 400}{1000} = \$144$
$m = 4$	$\frac{40 \cdot 4 \cdot 700 + 10 \cdot 2 \cdot 700}{1000} = \$126$	$\frac{40 \cdot 8 \cdot 300 + 10 \cdot 8 \cdot 700}{1000} = \$152$

Table 4. Problem solution on a 'sequential' approach.

	Cutting pattern					
p	$l_i = 700$	$l_i = 600$	$l_i = 400$	Thickness (k)	Amount (X_{pk})	Cost
1	1	0	0	1	400	$40 \cdot 400 = \$16,000$
				2	600	$25 \cdot 600 = \$15,000$
2	0	1	0	2	200	$25 \cdot 200 = \$5000$
3	0	0	2	3	100	$20 \cdot 100 = \$2000$
		Total			**1300**	**\$ 38,000**

objects have a length of 1000 mm, 400 objects of thickness 1, 800 objects of thickness 2 and 100 objects of thickness 3 would be used. The total cost for this approach is \$ 38,000, as can be seen in Table 4.

Integrating both problems, using mathematical model (1)–(5), the optimisation considers the combination of alternative modes to be more suitable for global optimisation. By using the solution method presented in Section 5, the total cost is \$ 28,275 (26% better than the 'sequential' approach). The computational time was 0.27 seconds, including the column generation and the gap was 0.07% relative to the linear lower bound. Table 5 presents details of the item sizes and thicknesses for each of the modes available and for the cutting patterns used. The amounts defined by the model are shown for Z_{jm} (modes chosen to meet the total demand of the product) and X_{pk} (objects cut to meet the demanded items of the modes chosen) and they are drilled-down into amounts of each item and thickness below, summing up the same amount on both sides.

The results show evidence that the improved solution is obtained from a diversity of alternative modes, which is particularly clear for product 2, in which all the configurations are used in some amount. Moreover, it is interesting to notice that the best unit costs are not even the most used for each product. In fact, product 1 was concentrated in the most expensive unit cost (Table 3). Therefore, the integration of the problem can potentially be far from the empirical common sense of a sequential decision approach.

6.2. Real industrial instance

The instance presented in the previous section shows the potential gain of integrating the cutting stock problem with multiple manufacturing modes in an illustrative example. In this section, a real instance is presented, collected from a concrete pole factory. It is based on an order received to attend an allotment, totalising 570 poles (final products) of 5 different types, besides 285 support pieces and 28 cross-arms (also final products), totalling seven different products ($NJ = 7$).

The longitudinal armour of these products is built using five different options of steel thickness, detailed in Table 6. All the objects have the same length ($L = 12$ metres) and the cost is determined by its thickness.

Considering the operational procedure to use objects of only two different thicknesses (one on the main armour and another for the auxiliary armour, with varying lengths according to structural demands), initially there would be 25 alternative modes to produce each pole. However, some combinations are technically eliminated: (i) the thickness of the main armour must be bigger than the auxiliary one; (ii) an armour with more than 10 pieces is not allowed, because the concrete process is jeopardised; and (iii) there is no auxiliary armour when the main one is enough to meet the structural demand.

The data of the seven ordered products, with their respective amounts and alternative manufacturing modes for the longitudinal armour, are shown in detail in Table A1 of the Appendix.

Table 5. Problem solution through the proposed model.

Items		Modes available								Cutting patterns							
k	l_i	$j=1$				$j=2$											
		1	2	3	4	1	2	3	4	1	2	3	4	5	6	7	8
	300								8				1				
$k=1$	400																
$\theta_1=40$	600																
	700	4			4	4							1				
	300	2															
$k=2$	400						8				2				1		
$\theta_2=25$	600					2									1		
	700		6														
	300							16				3		1			1
$k=3$	400		2														
$\theta_3=20$	600						6										1
	700			6										1			
	300																
$k=4$	400							12								1	
$\theta_4=10$	600			8												1	
	700				2				8	1							
		Modes chosen (Z_{jm})								Total objects cut (X_{pk})							
		0	0	99	1	19	5	66	10	82	1	144	80	594	38	792	30
	300								80				80				
$k=1$	400																
	600																
	700				4	76							80				
	300																
$k=2$	400						40				2				38		
	600					38									38		
	700																
	300							1056				432		594			3
$k=3$	400																
	600						30										3
	700			594										594			
	300																
$k=4$	400							792								792	
	600			792												792	
	700				2				80	82							
		Total demand								Total cost = $28,275							
		$d_1=100$				$d_2=100$				$820	$25	$2880	$3200	$11,880	$950	$7920	$600

The instance was solved by the mathematical model (1)–(5), using the column generation method described with a final step that considers the columns generated by (7)–(9) and integrality constraints. The solution obtained presented a production plan with a total raw material cost of $ 54,271.31, using 4257 objects. The total computational time was 4.88 seconds, with 4.85 seconds used to generate cutting patterns and 0.03 seconds in the final optimisation. Moreover, 17 columns were generated and 12 columns were actually used in the final solution. The gap was 0.009% relative to the linear relaxation. Tables 7 and 8 show, respectively, raw material consumption by type and modes chosen by product.

Table 8 shows some products are made in more than one mode to optimise the total cost. Besides, even for products with all the production concentrated in a single mode, the choice was not restricted to the less expensive unit configuration.

To establish a comparison with the studied factory *modus operandi*, a simulation was made of the present procedure, according to the cutting programme used there. The procedure was called the 'sequential' solution, detailed in Section 6.1 (choosing the less expensive unit cost and optimising with CSP then). The solution used the same set of four raw materials,

Table 6. Information about objects for real instance.

Thickness (k)	Unit cost (θ_k)
1 (16 mm or 5/8″)	\$ 66.95
2 (12.5 mm or 1/2″)	\$ 40.86
3 (10 mm or 3/8″)	\$ 26.80
4 (8 mm or 5/16″)	\$ 17.97
5 (6.4 mm or 1/4″)	\$ 8.63

Table 7. Cutting programme with the proposed model.

Thickness (k)	Objects (X_{pk})	Cost
1 (16 mm or 5/8″)	0	$\varnothing$
2 (12,5 mm or 1/2″)	0	$\varnothing$
3 (10 mm or 3/8″)	600	$26.80 \cdot 600 = \$16,080.00$
4 (8 mm or 5/16″)	710	$17.97 \cdot 710 = \$12,758.70$
5 (6,4 mm or 1/4″)	2947	$8.63 \cdot 2947 = \$25,432.61$
Total	**4257**	**\$ 54,271.31**

Table 8. Modes chosen to satisfy demand through the proposed approach.

Products (j)	Manufacturing mode (m)	Amount (Z_{jm})
$j = 1$	$m = 6$	$Z_{16} = 285$
$j = 2$	$m = 5$	$Z_{25} = 1$
	$m = 6$	$Z_{26} = 57$
	$m = 7$	$Z_{27} = 78$
$j = 3$	$m = 5$	$Z_{35} = 52$
$j = 4$	$m = 5$	$Z_{45} = 69$
$j = 5$	$m = 4$	$Z_{54} = 28$
$j = 6$	$m = 3$	$Z_{63} = 201$
	$m = 4$	$Z_{64} = 84$
$j = 7$	$m = 4$	$Z_{74} = 28$

Table 9. Comparative between optimised solution and present factory procedure.

Indicator		Proposed model	Present procedure
Total cost (\$)		**\$ 54,271.31**	\$ 64,232.01
Objects (units)		4257	**3321**
Thickness usage	1	0	0
	2	0	224
	3	600	484
	4	710	2094
	5	2947	519

but in different amounts. The total cost was \$ 64,232.01, that is, 15.5% higher than the integrated model. The amount of objects used was 3321, detailed per type in Table 9, where a comparison between both solutions is made.

Table 9 illustrates the root of the economy, which was driven by substituting more expensive raw material for cheaper ones, leading to a greater usage, with less leftovers in this case. Although the total number of steel bars found by the model is bigger, the total cost is minimised by choosing the cheapest combination. However, it is important to reinforce that quality is guaranteed, once all the alternative modes lead to the same structural properties. The use of less thick materials is compensated by a bigger amount of them.

6.3. Random generated instances

In order to verify the behaviour of the formulation (1)–(5), with its respective column generation procedure (7)–(9), a set of 1215 random instances was generated to explore different scenarios of the parameters of the model. All the instances were generated with 30 final products ($NJ = 30$), objects of length 1200 ($L = 1200$). Instances were generated with 20, 30 and 40 different lengths of items (NI).

The number of different thicknesses (NK) was considered in three scenarios: 2, 5 and 8. The number of different alternative modes per product ($|M_j|$) was also generated in three scenarios: 5, 10 and 15. The length of the items (l_i) was generated either with 'small' lengths ($l_i \in U[120, 360]$), 'large' lengths ($l_i \in U[360, 840]$) and 'varied' lengths ($l_i \in U[120, 840]$). Finally, costs of objects were considered either 'identical' ($\theta_k = 1$), 'homogeneous' ($\theta_k \in U[1, 10]$) and 'heterogeneous' ($\theta_k \in U[1, 100]$). The notation U here stands for the uniform distribution in a discrete set.

Demands of final products were always generated among 1 and 10 units ($d_j \in U[1, 10]$). Amounts required of an item i of a certain thickness k in an alternative mode $m(b_{ikm})$ were also generated randomly, considering 98% of chances of being

Table 10. Results according to the amount of different items (NI).

	Different lengths of items (NI)			
Result	20	30	40	p-Value
gap_r(%)	4.80 (0.00 – 24.13)	5.16 (0.00 – 16.66)	5.10 (0.00 – 24.48)	.421
t_r(s)	29.64 (0.96 – 370.50)	73.92 (1.06 – 615.86)	170.60 (4.68 – 2562.78)	**< .001**
gap_i(%)	0.71 (0.00 – 5.61)	0.63 (0.00 – 4.64)	0.47 (0.00 – 2.08)	**< .001**
t_i(s)	44.96 (1.03 – 430.59)	98.14 (1.17 – 676.16)	199.04 (4.72 – 2623.15)	**< .001**

Table 11. Results according to the number of different thicknesses of raw materials (NK).

NI	NK	gap_r (%)	t_r (s)	gap_i (%)	t_i (s)
20	2	4.61 (0.00 – 21.93)	8.87 (0.96 – 28.15)	0.47 (0.00 – 2.94)	16.90 (1.03 – 88.22)
	5	4.87 (0.00 – 19.68)	21.06 (2.57 – 66.70)	0.75 (0.00 – 4.06)	34.39 (2.60 – 126.80)
	8	4.90 (0.04 – 24.13)	59.00 (6.05 – 370.50)	0.92 (0.01 – 5.61)	83.59 (6.15 – 430.59)
	Average	4.80 (0.00 – 24.13)	29.64 (0.96 – 370.50)	0.71 (0.00 – 5.61)	44.96 (1.03 – 430.59)
	p-value	0.842	**< 0.001**	**< 0.001**	**< 0.001**
30	2	5.49 (0.00 – 16.66)	14.54 (1.06 – 113.15)	0.51 (0.00 – 4.64)	25.79 (1.17 – 120.43)
	5	4.89 (0.00 – 14.09)	54.80 (7.20 – 155.85)	0.69 (0.00 – 3.24)	80.70 (7.33 – 215.95)
	8	5.09 (0.06 – 14.10)	152.44 (17.07 – 615.86)	0.68 (0.03 – 2.73)	187.92 (17.18 – 676.16)
	Average	5.16 (0.00 – 16.66)	73.92 (1.06 – 615.86)	0.63 (0.00 – 4.64)	98.14 (1.17 – 676.16)
	p-value	0.480	**< 0.001**	**0.011**	**< 0.001**
40	2	5.61 (0.00 – 24.48)	23.88 (4.68 – 61.24)	0.43 (0.00 – 1.85)	37.92 (4.72 – 116.08)
	5	4.80 (0.05 – 13.76)	127.26 (14.78 – 485.88)	0.52 (0.02 – 2.01)	157.92 (14.90 – 546.25)
	8	4.89 (0.29 – 12.82)	360.67 (33.68 – 2562.78)	0.47 (0.02 – 2.08)	401.27 (33.84 – 2623.15)
	Average	5.10 (0.00 – 24.48)	170.60 (4.68 – 2562.78)	0.47 (0.00 – 2.08)	199.04 (4.72 – 2623.15)
	p-value	0.193	**< 0.001**	0.150	**< 0.001**

null ($b_{ikm} = 0$) and 2% following a uniform distribution between 2 and 8 units ($b_{ikm} \in U[2, 8]$). The threshold of 98% was inspired by the matrix of the industrial example, where 98% of elements b_{ikm} were equal to 0.

Combining these scenarios, 243 sets of parameters were proposed, for which 5 different instances were randomly generated, summing up 1215 instances. Data was submitted to the 1D-CSP-AM model in two different procedures after column generation: (i) running the model with the integrality constraints of the variables X_{pk} relaxed and after rounding up the results of each X_{pk} (which will be called ‘rounded’ run); and (ii) running the model requiring that all the variables generated during the column generation procedure are integer for 60 seconds (which will be called ‘integer’ run). In both cases, gaps were calculated using the relaxed solution of the formulation (1)–(5) as the lower bound (LB) after all the columns were generated. The solutions obtained either on the ‘rounded’ run (UB_r) or on the ‘integer’ run (UB_i) are used to calculate the gap of its respective runs (gap_r and gap_i) as in (10). CPU time on the ‘integer’ run will be denoted as t_i and on the ‘rounded’ run t_r.

$$gap_i = \frac{UB_i - LB}{LB} \quad gap_r = \frac{UB_r - LB}{LB}. \tag{10}$$

Results presented an average gap of 5.02% in 91.39 seconds (including column generation) on the ‘rounded’ run, ranging from 0.00% to 24.48% in a range of 0.96 seconds to 2562.78 seconds. In the ‘integer’ run, gaps were 0.61% on average within 114.05 seconds, ranging from 0.00% to 5.61% in 1.03 seconds to 2623.15 seconds. Table 10 summarises the results for different lengths of items (NI). As expected, the results were improved on the integer run, even limiting time after column generation to 60 seconds.

This table, as well as further ones will present this comparison in the different scenarios of parameters tested, through analysis of variance, with a significance level (p-value) of 5%. Then, for p-values less than 0.05, there is a statistical difference among the results for each subset of instances, highlighted in bold letters. Gaps are statistically significant different for CPU times (both) and gaps on the ‘integer’ run.

Table 11 shows the average (and the range in brackets) results according to the number of different raw materials (NK). It can be observed that the CPU time presented a statistically significant difference either for the rounded version or for the integer one for all the NI analysed. This allows the inference of a relevance of this parameter in the column generation

Table 12. Results according to the number of different alternative modes per product ($|M_j|$).

NI	$\lvert M_j \rvert$	gap_r (%)	t_r (s)	gap_i (%)	t_i (s)
20	5	3.99	27.46	0.55	46.09
		(0.00 – 11.71)	(1.17 – 132.77)	(0.00 – 2.94)	(1.19 – 192.86)
	10	4.74	34.67	0.71	49.68
		(0.00 – 13.44)	(0.96 – 370.50)	(0.00 – 2.61)	(1.03 – 430.59)
	15	5.66	26.79	0.88	39.11
		(0.00 – 24.13)	(2.19 – 158.81)	(0.00 – 5.61)	(2.35 – 180.52)
	Average	4.80	29.64	0.71	44.96
		(0.00 – 24.13)	(0.96 – 370.50)	(0.00 – 5.61)	(1.03 – 430.59)
	p-value	**0.007**	0.219	**< 0.001**	0.322
30	5	4.30	69.54	0.50	94.44
		(0.00 – 13.46)	(2.49 – 615.86)	(0.00 – 1.81)	(2.52 – 676.16)
	10	5.37	77.74	0.71	102.45
		(0.00 – 16.66)	(1.06 – 369.99)	(0.00 – 4.64)	(1.17 – 430.24)
	15	5.80	74.49	0.69	97.52
		(0.00 – 16.61)	(1.79 – 391.98)	(0.00 – 3.24)	(1.85 – 452.19)
	Average	5.16	73.92	0.63	98.14
		(0.00 – 16.66)	(1.06 – 615.86)	(0.00 – 4.64)	(1.17 – 676.16)
	p-value	**0.009**	0.761	**0.003**	0.839
40	5	4.37	154.48	0.38	183.00
		(0.00 – 12.10)	(4.68 – 2192.12)	(0.00 – 1.59)	(4.72 – 2252.40)
	10	5.16	186.12	0.50	213.36
		(0.05 – 15.31)	(9.95 – 2562.78)	(0.03 – 1.71)	(10.06 – 2623.15)
	15	5.76	171.21	0.54	200.75
		(0.00 – 24.48)	(10.11 – 1076.02)	(0.03 – 2.08)	(10.29 – 1136.54)
	Average	5.10	170.60	0.47	199.04
		(0.00 – 24.48)	(4.68 – 2562.78)	(0.00 – 2.08)	(4.72 – 2623.15)
	p-value	**0.015**	0.614	**0.001**	0.669

Table 13. Results according to the length of items (l_i).

NI	l_i	gap_r (%)	t_r (s)	gap_i (%)	t_i (s)
20	Small	9.84	56.33	1.24	89.96
		(4.07 – 24.13)	(7.87 – 370.50)	(0.12 – 5.61)	(9.06 – 430.59)
	Large	0.82	9.74	0.26	10.34
		(0.00 – 3.22)	(0.96 – 27.82)	(0.00 – 0.92)	(1.03 – 65.41)
	Variated	3.73	22.86	0.65	34.58
		(0.25 – 9.02)	(4.00 – 65.52)	(0.05 – 2.24)	(4.19 – 120.26)
	Average	4.80	29.64	0.71	44.96
		(0.00 – 24.13)	(0.96 – 370.50)	(0.00 – 5.61)	(1.03 – 430.59)
	p-value	**< 0.001**	**< 0.001**	**< 0.001**	**< 0.001**
30	Small	10.05	134.47	1.11	178.53
		(6.05 – 16.66)	(11.21 – 615.86)	(0.13 – 4.64)	(12.11 – 676.16)
	Large	1.21	22.74	0.26	23.99
		(0.00 – 4.20)	(1.06 – 115.07)	(0.00 – 1.03)	(1.17 – 115.83)
	Variated	4.20	64.57	0.53	91.89
		(0.42 – 10.79)	(5.55 – 301.39)	(0.04 – 1.79)	(5.73 – 361.73)
	Average	5.16	73.92	0.63	98.14
		(0.00 – 16.66)	(1.06 – 615.86)	(0.00 – 4.64)	(1.17 – 676.16)
	p-value	**< 0.001**	**< 0.001**	**< 0.001**	**< 0.001**
40	Small	9.80	338.66	0.85	385.62
		(5.56 – 24.48)	(14.85 – 2562.78)	(0.11 – 2.08)	(14.94 – 2623.15)
	Large	1.41	42.72	0.22	44.90
		(0.00 – 4.31)	(4.68 – 186.33)	(0.00 – 1.29)	(4.72 – 193.72)
	Variated	4.09	130.43	0.36	166.59
		(0.14 – 10.49)	(9.95 – 584.24)	(0.03 – 1.14)	(10.06 – 644.51)
	Average	5.10	170.60	0.47	199.04
		(0.00 – 24.48)	(4.68 – 2562.78)	(0.00 – 2.08)	(4.72 – 2623.15)
	p-value	**< 0.001**	**< 0.001**	**< 0.001**	**< 0.001**

process (common to both solution methods). Gaps, in general, were not impacted by NK (except for the ‘integer’ run with 30 items). This implies that the influence of NK is moderated by the number of items NI.

Table 12 shows results according to the number of different alternative modes per product ($|M_j|$). A statistically significant difference can be observed for gaps, but not for CPU times, which is valid for all NI. As expected, the number of possible modes per product affects the quality of the solution in reasonable CPU time.

No pattern of relation between CPU times and the number of different alternative modes per product ($|M_j|$) was seen. Since (i) processing times are mostly affected by the column generation procedure, (ii) the number of alternative modes has little impact on the number of integer variables and (iii) there is no impact on the number of constraints of the main model, it is possible to infer that this parameter has little impact on the number of valuable combinations among the items.

Table 13 shows the results according to the length of items (l_i). All the outputs were significantly affected by this parameter for all the sizes of NI. Again, a statistically significant difference can be observed for the gaps and CPU times, either for the rounded version or for the integer one. Results show that the prevalence of small items led to a solution of more CPU time and with less final quality. In fact, the existence of only large items had the opposite effect, which reinforces this inference. It is interesting to notice that the difference of rounded and integer methods was greater on instances with smaller items.

Finally, Table 14 shows results according to the costs of raw materials (θ_k). In general, for this parameter, neither the gap, nor CPU time differ statistically for both solution methods (except for the CPU time of the rounded method on instances with 40 items). The inference with these results is that this parameter has little effect on the difficulty of the instances.

Tables 10–14 show a perspective of the influence of each parameter isolated. In order to analyse the interactions of the parameters and also the comparative intensity of different factors, Tables 15 and 16 show, respectively, the average results of gaps and CPU times for the ‘integer’ run. As the sense of the results of the ‘rounded’ run are similar, the analogous tables are shown in Appendix Tables A2 and A3. The background colour for each result gives the comparative intensity between the set of results on a grey scale.

Table 15 (as well as A2) shows a clear concentration of bigger gaps when the prevalence of small items is greater. The inference is that this parameter may be the most intense interference on the instance difficulty.

Table 14. Results according to the costs of raw materials (θ_k).

NI	θ_k	gap_r (%)	t_r (s)	gap_i (%)	t_i (s)
20	Identical	5.07	26.80	0.74	34.93
		(0.00 – 21.93)	(1.17 – 132.77)	(0.00 – 3.42)	(1.19 – 192.86)
	Homogeneous	5.00	31.29	0.71	47.30
		(0.00 – 24.13)	(1.90 – 370.50)	(0.00 – 5.61)	(2.30 – 430.59)
	Heterogeneous	4.31	30.84	0.68	52.65
		(0.00 – 17.44)	(0.96 – 357.45)	(0.00 – 4.06)	(1.03 – 417.58)
	Average	4.80	29.64	0.71	44.96
		(0.00 – 24.13)	(0.96 – 370.50)	(0.00 – 5.61)	(1.03 – 430.59)
	p-value	0.296	0.615	0.757	**0.039**
30	Identical	5.34	74.88	0.67	94.56
		(0.00 – 16.66)	(2.65 – 374.31)	(0.00 – 3.24)	(2.69 – 434.60)
	Homogeneous	5.06	76.72	0.54	99.27
		(0.00 – 14.45)	(1.79 – 615.86)	(0.00 – 2.22)	(1.85 – 676.16)
	Heterogeneous	5.07	70.18	0.68	100.58
		(0.00 – 16.61)	(1.06 – 391.98)	(0.00 – 4.64)	(1.17 – 452.19)
	Average	5.16	73.92	0.63	98.14
		(0.00 – 16.66)	(1.06 – 615.86)	(0.00 – 4.64)	(1.17 – 676.16)
	p-value	0.820	0.834	0.064	0.898
40	Identical	5.32	194.27	0.52	214.27
		(0.00 – 24.48)	(4.68 – 2562.78)	(0.02 – 2.01)	(4.72 – 2623.15)
	Homogeneous	4.84	162.83	0.44	191.84
		(0.00 – 15.16)	(7.58 – 950.72)	(0.00 – 1.95)	(7.62 – 1011.04)
	Heterogeneous	5.13	154.71	0.46	191.00
		(0.14 – 14.85)	(5.41 – 1436.73)	(0.02 – 2.08)	(5.54 – 1497.15)
	Average	5.10	170.60	0.47	199.04
		(0.00 – 24.48)	(4.68 – 2562.78)	(0.00 – 2.08)	(4.72 – 2623.15)
	p-value	0.615	0.427	0.221	0.740

Table 15. Summary of average gaps on 'integer' run according to the scenario of parameters.

			Large items size			Variated items size			Small items size		
NK	NM	NI	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs
2	5	20	0.13%	0.22%	0.19%	0.28%	0.25%	0.58%	0.86%	0.31%	0.84%
		30	0.2%	0.16%	0.17%	0.45%	0.71%	0.5%	0.77%	0.76%	0.87%
		40	0.14%	0.11%	0.31%	0.4%	0.38%	0.65%	0.58%	0.86%	1.02%
	10	20	0.29%	0.07%	0.19%	0.26%	0.23%	0.39%	0.72%	0.43%	0.86%
		30	0.07%	0.38%	0.35%	0.83%	0.39%	0.55%	1.62%	0.72%	0.89%
		40	0.21%	0.33%	0.32%	0.7%	0.32%	0.7%	0.81%	0.41%	0.85%
	15	20	0.24%	0.1%	0.24%	0.27%	0.3%	0.34%	0.61%	0.5%	0.44%
		30	0.3%	0.17%	0.67%	0.29%	0.44%	0.45%	0.77%	0.66%	0.75%
		40	0.24%	0.12%	0.34%	0.44%	0.49%	0.44%	0.69%	0.37%	0.93%
5	5	20	0.16%	0.25%	0.29%	0.55%	0.61%	0.53%	1.07%	0.96%	0.92%
		30	0.26%	0.23%	0.24%	0.65%	0.42%	0.83%	1.01%	1.19%	1.43%
		40	0.26%	0.29%	0.42%	0.65%	1.07%	0.78%	1.56%	1.68%	2.05%
	10	20	0.22%	0.25%	0.23%	0.39%	0.44%	0.3%	1.06%	0.95%	1.2%
		30	0.41%	0.25%	0.29%	0.54%	0.54%	0.7%	1.38%	1.08%	1.31%
		40	0.13%	0.32%	0.38%	0.88%	0.57%	0.64%	1.42%	1.13%	1.7%
	15	20	0.15%	0.21%	0.17%	0.37%	0.2%	0.35%	0.63%	0.7%	0.97%
		30	0.24%	0.22%	0.26%	0.42%	0.54%	0.31%	0.91%	0.91%	1.09%
		40	0.15%	0.2%	0.33%	0.45%	0.47%	0.45%	0.93%	1.12%	1.29%
8	5	20	0.15%	0.18%	0.28%	0.57%	0.73%	0.42%	1.15%	1.05%	1.37%
		30	0.46%	0.32%	0.36%	0.85%	0.75%	0.74%	1.99%	1.34%	1.55%
		40	0.39%	0.26%	0.47%	1.27%	0.93%	0.9%	1.63%	3.25%	1.36%
	10	20	0.15%	0.25%	0.16%	0.37%	0.25%	0.41%	1.24%	1.16%	1.04%
		30	0.39%	0.33%	0.3%	0.67%	0.55%	0.6%	1.56%	1.23%	1.12%
		40	0.22%	0.14%	0.3%	0.75%	0.49%	0.78%	1.19%	1.35%	1.47%
	15	20	0.12%	0.13%	0.1%	0.3%	0.2%	0.26%	0.82%	0.78%	0.71%
		30	0.17%	0.13%	0.17%	0.23%	0.33%	0.26%	0.79%	1.04%	0.98%
		40	0.23%	0.21%	0.2%	0.43%	0.36%	0.37%	1.28%	1.01%	1.16%

Table 16 (as well as A3), on the other hand, has a clearer influence of the number different thicknesses of objects (*NK*) increasing with CPU times. The combination of greater *NK* and the presence of small items results in greater CPU times. As most of the generated columns and the number of subproblems are linear with *NK*, it is a logical consequence.

Table 16. Summary of average CPU time on 'integer' run according to the scenario of parameters.

			Large items size			Variated items size			Small items size		
NK	NM	NI	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs
2	5	20	7.53	4.28	2.58	9.89	5.52	6.67	49.60	25.92	11.23
		30	2.76	3.69	3.27	44.18	21.71	9.41	57.43	36.53	15.06
		40	18.44	4.22	5.14	23.50	14.33	9.91	27.25	15.73	20.60
	10	20	3.14	16.86	4.58	26.18	41.43	28.42	65.86	34.12	19.99
		30	5.87	8.41	9.58	31.18	18.13	10.03	48.33	39.52	71.61
		40	3.70	15.63	4.10	61.24	14.17	15.10	65.13	15.52	18.54
	15	20	9.19	9.39	6.23	66.15	30.30	34.51	77.73	55.35	40.29
		30	17.95	25.13	21.51	52.90	24.76	23.95	82.14	34.13	21.85
		40	13.43	17.00	17.41	64.82	38.89	15.01	87.39	79.59	56.78
5	5	20	6.37	5.35	6.47	35.03	28.59	17.22	97.16	95.64	79.51
		30	5.98	8.89	8.07	16.91	22.05	22.38	75.31	58.83	40.56
		40	7.18	8.68	9.35	28.62	43.75	19.78	73.67	49.15	57.94
	10	20	31.28	24.91	32.46	100.88	57.99	49.68	137.67	149.67	159.55
		30	13.22	19.25	17.27	74.36	96.99	79.09	161.09	131.23	166.28
		40	23.86	15.40	12.32	92.19	52.22	68.64	141.65	150.78	119.03
	15	20	39.31	40.17	49.02	135.09	104.22	128.19	224.92	283.08	328.63
		30	26.54	25.28	28.95	174.57	152.73	124.50	297.12	257.08	284.01
		40	47.43	46.87	35.46	171.97	176.97	134.72	319.59	290.92	336.54
8	5	20	19.02	18.65	13.79	83.00	107.14	52.41	147.96	157.42	150.57
		30	20.14	19.86	15.92	56.47	40.91	49.81	280.21	249.35	155.65
		40	17.72	15.18	20.64	70.47	53.47	40.62	139.77	162.32	98.52
	10	20	39.34	42.04	33.85	146.19	109.47	162.83	307.16	422.85	301.52
		30	36.44	40.21	45.16	236.71	286.97	131.78	302.28	322.19	363.04
		40	37.87	67.67	43.28	147.47	127.05	214.71	375.42	359.57	370.77
	15	20	70.51	76.37	46.22	321.42	376.99	267.37	591.04	534.49	994.82
		30	61.20	89.83	92.95	239.74	491.28	280.69	916.83	746.82	1.166.35
		40	94.43	84.20	120.38	275.31	315.71	275.06	678.42	772.15	853.79

Table 17. Summary of savings with the model over sequential (empirical) method.

			Large items size			Variated items size			Small items size		
NK	NM	NI	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs
2	5	20	2%	2.12%	4.15%	0.55%	2.41%	0.26%	0.44%	0.95%	0.38%
		30	3.21%	2.82%	3.94%	0.67%	1.22%	0.69%	1.26%	1.38%	0.45%
		40	2.66%	1.29%	1.04%	0.99%	1.3%	0.95%	0.42%	1.04%	0.95%
	10	20	4.02%	4.65%	1.71%	5.03%	0.67%	0.33%	0.66%	1.14%	0%
		30	4.26%	3.03%	4.1%	1.15%	0.89%	0.89%	0.96%	0.67%	0.71%
		40	3.5%	1.14%	1.83%	0.94%	0.55%	0.54%	1.14%	2.23%	0.84%
	15	20	2.85%	2.6%	4.96%	0.49%	0.28%	0.38%	0.54%	0.18%	0.42%
		30	2.34%	3.64%	2.36%	1.3%	0.55%	0.27%	0.81%	0.38%	0.43%
		40	2.62%	3.03%	2.64%	0.87%	0.65%	0.26%	1.44%	1.58%	0.24%
5	5	20	5%	5.64%	4.74%	3.22%	1.14%	0.92%	1.82%	1.53%	1.56%
		30	3.84%	6.63%	11.17%	2.54%	4.86%	2.07%	2.89%	2.54%	1.44%
		40	5.02%	6.41%	4.86%	2.58%	2.42%	2.08%	2.84%	3.63%	2.6%
	10	20	3.54%	4.24%	4.69%	3.08%	3.88%	2.35%	0.84%	0.91%	0.43%
		30	2.97%	6.85%	6.93%	2.38%	2.72%	1.03%	2.16%	1.68%	1.02%
		40	6.6%	10.48%	1.35%	2.21%	1.87%	0.8%	1.57%	0.93%	0.57%
	15	20	4.78%	4.51%	4.65%	2.08%	2.38%	2.76%	0.43%	0.47%	0.52%
		30	5.14%	3.13%	3.7%	0.76%	1.46%	1.1%	1.3%	0.85%	0.41%
		40	8.28%	4.17%	5.81%	1.37%	2.55%	1.83%	1.5%	1.37%	0.85%
8	5	20	5.22%	5.97%	4.09%	2.56%	1.5%	3.84%	1.5%	1.58%	0.81%
		30	5.89%	8.77%	4.9%	3.75%	3.85%	1.33%	2.46%	2.06%	1.86%
		40	6.35%	5.99%	4.7%	6.04%	2.98%	3.39%	1.87%	2.85%	3.01%
	10	20	6.87%	3.87%	2.86%	2.69%	4.16%	2.87%	0.5%	0.34%	0.34%
		30	6.03%	3.37%	7.88%	1.43%	1.59%	3.86%	1.47%	1.38%	1%
		40	8.46%	5.75%	7.25%	2.82%	4.34%	1.9%	1.56%	1.66%	1.17%
	15	20	4.44%	3.19%	2.84%	1.21%	2.03%	1.42%	0.38%	0.17%	0.12%
		30	4.15%	6.08%	3.75%	3.38%	2.68%	2.9%	0.47%	0.5%	0.16%
		40	7.41%	5.68%	8.24%	1.57%	1.49%	3.48%	0.33%	0.52%	0.41%

6.3.1. Comparison with sequential approach and managerial implications

After exploring the characterisation of the instances according to its parameters, the following comparison intends to bring managerial insights for practitioners. The 1215 random generated instances were also solved using the sequential method, presented in Section 6.2, which mimics an empirical industrial approach.

Table 17 summarises the value of the integrated approach in terms of savings (in percentage) of the total cost using the solution method proposed when compared to this empirical approach of choosing the best mode first and then solving the cutting stock problems of each material for the resulting demand.

Numerical studies point out average savings on raw material costs were 2.56%, ranging from 0.00% to 18.08%. Only 11.77% of the instances (143 of 1215) have no improvement in the results and no result worsened, as expected. According to the numerical studies, four technical characteristics of the demand can impact costs with greater potential: (i) the presence of larger items (p-value < 0.001); (ii) fewer number of different item sizes (*NI*, p-value $= 0.001$); (iii) larger number of different thicknesses (*NK*, p-value < 0.001); and, in a weaker manner, (iv) larger number of alternative modes per product ($|M_j|$, p-value $= 0.032$). The relative cost among different thicknesses (θ_k) has not shown a statistically significant difference among groups (p-value $= 0.121$).

7. Conclusion

This paper presents an integrated approach for the Cutting Stock Problem with alternative manufacturing modes, a situation in which the same final product can be made with a different bill of materials. In particular, the construction industry motivated the study, as reinforced concrete armours can be built with several combinations of lengths and thicknesses of their items.

An integrated mathematical model was proposed to minimise the total cost of raw materials that meets the demand of final products, considering that they can be manufactured in several different alternative modes. A solution method based on a column generation procedure was developed.

Computational experiments were performed in three parts. First, an illustrative example, which showed the relevance of the integrated approach was presented. Next, a real instance was used to show that the diversity and combination of modes resulted in a 8.1% cost reduction, which is relevant on a low gross margin business. Finally, random instances were generated so that the influence of parameters could be tested. By these tests, the cost of raw materials showed no impact over the final result, while the number of different alternative modes per product, the size of the items and the number of different thicknesses had a statistically significant impact on the gap than the CPU time, mainly the two first ones. Moreover, a sequential approach was performed to compare the solutions of the integrated model to the sequential solution method. This approach showed how different parameters of the proposed model had an impact on the value of integration.

Further work could explore the limits of these good results and the application in environments with more modes per product, such as health care systems, project scheduling, computational processor programming and other different manufacturing systems. Furthermore, different object sizes per raw material are another feature that could be considered. Other future work can be explored, such as the possibility of usable leftovers (Arenales et al. 2015) or a multi-period approach (Poldi and de Araujo 2016). In addition, these extensions can make the problem more difficult, and branch-and-price approaches could be developed to find good quality solutions in applications such as those listed and others.

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Appendix. Detailed data for products ordered on the real instance and their alternative manufacturing modes

Table A1. Products ordered on the real instance and their alternative manufacturing modes.

Product(j)	Demand (d_j)	Modes (m)	Requisites (b_{ikm})
7 m × 150 daN ($j = 1$)	285 units	1	4 × 700 cm (Ø = 16 mm)
		2	4 × 700 cm (Ø = 12.5 mm)
		3	4 × 700 cm (Ø = 10 mm)
		4	4 × 700 cm (Ø = 8 mm) + 4 × 150 cm (Ø = 8 mm)
		5	4 × 700 cm (Ø = 8 mm) + 2 × 400 cm (Ø = 6.4 mm) +2 × 250 cm (Ø = 6.4 mm)
		6	6 × 700 cm (Ø = 6.4 mm) + 4 × 450 cm (Ø = 6.4 mm) +4 × 400 cm (Ø = 6.4 mm) + 4 × 300 cm (Ø = 6.4 mm)
10 m × 150 daN ($j = 2$)	136 units	1	4 × 1000 cm (Ø = 16 mm)
		2	4 × 1000 cm (Ø = 12.5 mm)
		3	4 × 1000 cm (Ø = 10 mm) + 2 × 400 cm (Ø = 10 mm)
		4	4 × 1000 cm (Ø = 10 mm) + 2 × 400 cm (Ø = 8 mm)
		5	4 × 1000 cm (Ø = 10 mm) + 2 × 400 cm (Ø = 6.4 mm) +2 × 100 cm (Ø = 6.4 mm)
		6	4 × 1000 cm (Ø = 8 mm) + 2 × 650 cm (Ø = 8 mm) +2 × 350 cm (Ø = 8 mm)
		7	4 × 1000 cm (Ø = 8 mm) + 2 × 800 cm (Ø = 6.4 mm) +2 × 700 cm (Ø = 6.4 mm) + 2 × 650 cm(Ø = 6.4 mm)
10 m × 300 daN ($j = 3$)	52 units	1	4 × 1000 cm (Ø = 16 mm)
		2	4 × 1000 cm (Ø = 12.5 mm)
		3	4 × 1000 cm (Ø = 10 mm) + 2 × 700 cm (Ø = 10 mm)
		4	4 × 1000 cm (Ø = 10 mm) + 2 × 650 cm (Ø = 8 mm) +2 × 350 cm (Ø = 8 mm)
		5	4 × 1000 cm (Ø = 10 mm) + 2 × 800 cm (Ø = 6.4 mm) +2 × 700 cm (Ø = 6.4 mm) + 2 × 550 cm(Ø = 6.4 mm)
		6	6 × 1000 cm (Ø = 8 mm) + 2 × 650 cm (Ø = 8 mm) +2 × 350 cm (Ø = 8 mm)
11 m × 300 daN ($j = 4$)	69 units	1	4 × 1100 cm (Ø = 16 mm)
		2	4 × 1100 cm (Ø = 12.5 mm)
		3	4 × 1100 cm (Ø = 10 mm) + 2 × 800 cm (Ø = 10 mm) +2 × 250 cm (Ø = 10 mm)
		4	4 × 1100 cm (Ø = 10 mm) + 2 × 700 cm (Ø = 8 mm) +2 × 450 cm (Ø = 8 mm)
		5	4 × 1100 cm (Ø = 10 mm) + 2 × 800 cm (Ø = 6.4 mm) +2 × 650 cm (Ø = 6.4 mm) + 4 × 500 cm(Ø = 6.4 mm)
		6	6 × 1100 cm (Ø = 8 mm) + 2 × 700 cm (Ø = 8 mm) +2 × 450 cm (Ø = 8 mm)
12 m × 600 daN ($j = 5$)	28 units	1	4 × 1200 cm (Ø = 16 mm) + 2 × 450 cm (Ø = 16 mm)
		2	4 × 1200 cm (Ø = 16 mm) + 2 × 700 cm (Ø = 12.5 mm)
		3	4 × 1200 cm (Ø = 12.5 mm) +2 × 1100 cm (Ø = 12.5 mm) +2 × 800 cm (Ø = 12.5 mm) +2 × 300 cm (Ø = 12.5 mm)
		4	4 × 1100 cm (Ø = 10 mm) + 2 × 700 cm (Ø = 8 mm) +2 × 450 cm (Ø = 8 mm)
Support pieces ($j = 6$)	285 units	1	4 × 100 cm (Ø = 16 mm)
		2	4 × 100 cm (Ø = 12.5 mm)
		3	4 × 100 cm (Ø = 10 mm)
		4	8 × 100 cm (Ø = 8 mm)
		5	4 × 100 cm (Ø = 8 mm) + 4 × 100 cm (Ø = 6.4 mm)
		6	12 × 100 cm (Ø = 6.4 mm)
Cross-arms ($j = 7$)	28 units	1	4 × 200 cm (Ø = 16 mm)
		2	4 × 200 cm (Ø = 12.5 mm)
		3	4 × 200 cm (Ø = 10 mm)
		4	8 × 200 cm (Ø = 8 mm)
		5	4 × 200 cm (Ø = 8 mm) + 4 × 200 cm (Ø = 6.4 mm)
		6	12 × 200 cm (Ø = 6.4 mm)

Table A2. Summary of average gaps on ‘rounded’ run according to the scenario of parameters.

			Large items size			Variated items size			Small items size		
NK	NM	NI	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs
2	5	20	0.36%	0.47%	0.33%	2.78%	2.01%	2.85%	6.18%	9.42%	8.42%
		30	0.24%	0.38%	0.84%	3.7%	4.82%	3.61%	7.4%	9.05%	11.92%
		40	0.41%	0.27%	0.87%	3.96%	4.95%	4.56%	8.9%	11.34%	14.56%
	10	20	0.45%	0.53%	0.61%	4.11%	3.62%	3.89%	8.97%	8.8%	7.86%
		30	0.6%	0.98%	0.92%	3.4%	2.85%	5.25%	9.62%	12.21%	12.87%
		40	0.44%	0.62%	0.32%	6.13%	6.4%	7.2%	13.27%	13%	13.27%
	15	20	1.14%	0.26%	0.8%	3.47%	4.24%	3.94%	10.1%	8.51%	9.66%
		30	1.05%	0.48%	1.63%	5.47%	4.5%	4.35%	10.65%	10.24%	11.8%
		40	0.8%	0.85%	0.89%	6.12%	5.81%	5.32%	11.43%	11.46%	16.4%
5	5	20	0.41%	0.87%	0.89%	3.16%	3.39%	3.47%	7.97%	8.51%	7.82%
		30	0.73%	1.27%	0.55%	3.79%	3.53%	4.73%	7.45%	10.52%	10.17%
		40	0.45%	0.4%	0.97%	3.43%	4.13%	3.23%	13.55%	12.17%	14.05%
	10	20	1.12%	1%	1.05%	3.21%	2.45%	2.8%	8.29%	8.62%	9.99%
		30	1.22%	1.81%	0.8%	4.03%	4.12%	5.17%	9.74%	9.52%	10.05%
		40	1.35%	1.83%	0.71%	4.57%	2.89%	4.34%	9.87%	12.12%	9.3%
	15	20	1.52%	1.27%	1.23%	3.3%	3.48%	3.23%	8.23%	7.83%	8.11%
		30	1.17%	1.11%	1.11%	4.19%	3.07%	3.7%	10.09%	9.39%	9.68%
		40	1.15%	1.41%	1.34%	3.68%	5.63%	4.55%	10.49%	8.72%	10.96%
8	5	20	0.71%	0.95%	1.26%	2.68%	4.51%	3.07%	8.05%	8.65%	8.57%
		30	1.46%	1.58%	1.47%	3.93%	3.42%	3.99%	9.47%	9.06%	8.76%
		40	1.58%	0.65%	1.66%	4.12%	4.32%	4.47%	9.61%	14.46%	9.79%
	10	20	1.55%	1.42%	1.94%	2.24%	2.82%	3.94%	8.37%	8.14%	8.26%
		30	2.14%	1.5%	3.04%	4.69%	4.9%	4.19%	10.29%	9.77%	9.36%
		40	1.64%	1.56%	1.59%	5.1%	3.85%	5.22%	10.39%	9.29%	10.2%
	15	20	1.57%	1.77%	1.73%	3.1%	2.9%	2.89%	8.83%	7.35%	7.45%
		30	1.66%	2.38%	2.52%	3.6%	3.69%	3.98%	9.16%	10.34%	8.46%
		40	2.05%	2.14%	3.11%	4.44%	3.47%	4.22%	10.09%	8.51%	10.62%

Table A3. Summary of average CPU time on ‘rounded’ run according to the scenario of parameters.

			Large items size			Variated items size			Small items size		
NK	NM	NI	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs	Heterogeneous objects costs	Homogeneous objects costs	Identical objects costs
2	5	20	7.31	4.07	2.54	7.04	5.38	6.50	13.42	11.69	11.09
		30	2.68	3.58	3.17	7.89	9.09	9.23	9.29	12.35	14.44
		40	6.26	4.05	4.98	10.73	13.26	9.47	14.93	15.37	19.71
	10	20	3.09	4.81	4.53	12.87	17.12	16.21	17.62	19.93	19.79
		30	5.73	8.19	9.41	18.74	17.69	9.92	23.94	24.08	53.64
		40	3.62	3.54	4.03	16.70	13.89	14.91	16.93	13.96	17.57
	15	20	9.07	9.30	6.18	29.11	22.88	34.14	29.13	30.82	39.89
		30	17.56	24.78	21.09	17.51	24.46	23.63	32.82	18.34	20.88
		40	13.19	16.72	17.15	15.38	14.69	14.85	39.05	47.71	54.42
5	5	20	6.27	5.30	6.38	17.32	17.23	17.04	37.10	47.27	47.59
		30	5.84	8.78	7.95	15.21	20.04	21.54	34.84	41.88	30.65
		40	7.04	8.54	9.21	15.54	19.40	19.40	38.38	31.01	31.80
	10	20	28.25	24.62	32.19	40.88	44.81	49.06	83.24	98.48	100.95
		30	13.07	19.05	17.14	37.89	48.81	64.90	100.89	83.00	106.11
		40	23.65	15.20	12.19	62.02	49.31	56.30	81.50	97.10	89.08
	15	20	37.72	39.89	48.71	95.68	79.39	91.85	164.77	222.86	275.24
		30	26.40	24.70	28.77	114.40	115.06	115.74	236.92	196.88	233.25
		40	47.07	46.48	35.27	121.78	126.72	121.27	259.37	242.23	287.59
8	5	20	18.78	18.34	13.62	46.12	47.03	40.07	87.86	97.33	101.69
		30	19.92	19.62	15.78	39.88	30.49	47.19	220.09	189.23	95.53
		40	17.46	15.00	20.47	45.70	39.71	39.60	79.67	109.86	76.85
	10	20	38.86	41.62	33.36	97.96	81.43	115.03	246.98	362.58	241.35
		30	35.27	39.97	44.88	184.87	226.64	98.31	242.09	261.98	302.81
		40	37.64	67.14	43.00	105.33	87.14	154.53	315.17	299.31	310.53
	15	20	69.89	52.16	46.02	261.24	316.81	218.85	530.75	474.20	934.50
		30	57.41	83.94	92.41	179.50	431.05	238.83	856.47	686.48	1.105.98
		40	81.89	80.59	119.13	215.05	255.46	226.34	618.04	711.75	793.40