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ORIGINAL ARTICLE

Integrated lot-sizing and cutting stock problem applied to the mattress industry

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ABSTRACT

In many productive processes, two important problems arise in the production planning: the lot-sizing problem and the cutting stock problem. Generally, companies deal with these problems separately but, by considering them in an integrated way, better results can be obtained. In this paper, the integrated lot-sizing and three-dimensional cutting stock problem applied to the mattress industry is investigated, aiming at reducing costs and waste. A mathematical model of mixed integer programming was proposed and solved with an optimisation package. Computational tests based on data collected at a mattress factory were carried out, allowing the comparison of the solutions proposed by the model and the solutions adopted by the factory. Additional tests were performed with random data in order to evaluate the behaviour of the model for different cases. The results indicate that the model performs well, reducing the objective function costs for different data sets. Based on the results, some interesting options can be explored by the industry; for example, by increasing the number of cutting patterns up to a certain level, the number of possible combinations for cutting is increased, resulting in better use of the material and a consequent reduction in costs.

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Lot-sizing problem; three-dimensional cutting stock problem; integrated problem; mattress industry

1. Introduction

With the rapid advance of technology in an increasingly globalised world, competition between companies has also grown as consumers have become more demanding in terms of diversity, quality, price and speed of delivery. In the Brazilian mattress sector, competition among companies is related to both the technology and the management tools involved. Therefore, in order to be more competitive, the companies need to reduce costs and waste while increasing their productivity. In general, they do this by investing in more sophisticated machines and acquiring new technologies. A less common approach, but probably a good survival strategy in a highly competitive market, is to focus on solving management and manufacturing problems using mathematical models and optimisation techniques as tools for decision-making.

Two important problems that arise in the production planning of many industries are the Lot-Sizing Problem (LSP), which consists of deciding the quantity of each product that must be produced in each period of a planning horizon, and the Cutting Stock Problem (CSP), which consists of cutting larger objects, available in stock, into smaller pieces (items) in order to satisfy a demand for items and minimise, for example, the total material loss.

Usually, the companies consider these problems separately, first solving the LSP and then the CSP. However, with the technological advances and greater understanding of these problems, industries have been looking for integrated mathematical modelling and solution methods that better represent their activities. The integration of the LSP and the CSP allows for better analysis of the trade-off between storage costs, machine setup costs and costs related to the cutting process.

In recent years, the Integrated Lot-Sizing and Cutting Stock Problem (ILSCSP) has attracted the interest of both the academic community and industry. Although it is difficult to solve, its great improvement potential and its wide ranging of application make this subject extremely relevant. A recent literature review can be found in Melega, de Araujo, and Jans (2018). In this paper, the authors propose a general formulation for the ILSCSP, considering two types of integration: the integration across time periods and the integration between production levels. Melega et al. (2018) also provided information about the types of industry where the model had been used. Following these classification criteria, this paper can be classified as a three-dimensional problem and it deals with both types of integration (across time periods and between

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production levels) applied to a mattress industry and restricted to two production levels (Level 1 and Level 2), since the model has decision variables related to the purchase and cutting of objects in a discrete and multi-period planning horizon (*L1/L2/-/M*).

According to the recent literature review proposed by Melega et al. (2018), the integration between these two levels (1 and 2) can be found in 13 papers from the literature. These papers are presented in Section 2. Apart from an updated version of a previous paper, no new papers were found. To the best of our knowledge, there is no published paper on integrated lot-sizing and three-dimensional cutting stock problem applied in the mattress industry. Thus, the focus of this paper is to propose and solve a mathematical model for such a problem.

Therefore, the main objective of the present study is to propose and solve (via optimisation package) a mathematical model for the ILSCSP in a small Brazilian mattress industry, obtaining computational results based on real data, besides evaluating the performance of the proposed model regarding the reduction of costs and waste in the company. The proposed mathematical model is innovative in the sense that it extends the literature by considering, in a three-dimensional setting, linked inventory balance constraints for items and objects, as well as, safety stock for items and objects. Moreover, an equally innovative value for the "big M" in the setup constraints is proposed.

This paper is organised as follows. In Section 2, a literature review on related papers is presented. Section 3 contains the description of the productive process in a small mattress industry. In Section 4, a mathematical model that considers the integrated lot-sizing and cutting stock decisions is presented. Subsequently, it follows for the implementation and solution of the model with instances based on real data in GAMS software, whose results are discussed in Section 5. Finally, Section 6 closes this paper bringing the conclusions and future proposals.

2. Literature review

In this section we review the papers from literature that consider the ILSCSP and more specifically the integration between Level 1 and Level 2 in a multi-period context (*L1/L2/-/M* according to Melega et al. (2018)), i.e., papers on which the mathematical models have decisions variables related to the production (inventory, purchase or setup) and cutting of objects over a planning horizon. The integration between these two levels appears in different applications found in the metal, furniture and, mainly, the paper industry.

Reinders (1992) models a capacity constraint in the cutting stock problem and overtime can be used, i.e., if necessary, additional time is available to the cutting process. The production planning of tree trunks is done at Level 1, followed by a crosscutting process to produce the logs and a sawing process to produce the boards, which constitute the final demand in the production planning. The crosscutting process and the sawing process correspond to cutting processes belonging to Level 2 of the classification and are operated sequentially.

Hendry, Fok, and Shek (1996) study the applicability of the integrated problem to the production planning in a copper smelting industry consisting, basically, of melting pieces of copper in a furnace, producing bars of specific diameters which are cut to produce the final items. The solution method presented by the authors has two stages: in the first it is assumed a maximum number of bars of all diameters that can be produced per period, taking into account the capacity of the furnace and, thus, solves a one-dimensional cutting stock problem; the solution to this problem provides the number of bars of each diameter that should be produced in each period of the planning horizon.

Correia, Oliveira, and Ferreira (2004) consider an application at a paper mill and discuss the general ideas of the constraints and objective function without presenting a formal mathematical model. The authors propose a heuristic solution method based on three stages. The first stage consists of enumerating all cutting patterns and auxiliary coils based on a fixed width of the master coils and the demanded items. This set of cutting patterns and auxiliary coils is subjected to a selection process in which unnecessary patterns and coils are eliminated. In the second stage of this heuristic, the cut patterns accepted in the first stage are used as columns in a linear programming model for solving the problem (solved by the simplex algorithm). A post-optimisation analysis is made on the solution found for the feasibility of those constraints involving integer and/or binary variables.

Trkman and Gradisar (2007) propose a mathematical model which consists of satisfying orders set in consecutive time periods, with usable leftovers. Constraints guarantee that the leftovers return to the warehouse and are available for cutting in future time periods, either if they are longer than the size limit or if the costs of returning the stock length to the warehouse are cheaper than the trim loss costs. For each period, a constraint ensures that a new order must be satisfied without backorder or inventory, by either using new objects available in that time period or by objects/leftovers from stock.

Poltroniere, Poldi, Toledo, and Arenales (2007) and Poltroniere, de Araujo, and Poldi (2016) study an application on a paper industry, and propose a mathematical model which considers the integration of the LSP and the one-dimensional CSP in a planning horizon divided into periods. Planning decisions of jumbos manufacturing and the cutting stage are interdependent problems. In Poltroniere et al. (2007), the authors propose two heuristics based on the Lagrangian relaxation of the linking constraints, in which the cutting patterns are generated by heuristic approaches. The resulting problem, in turn, can be decomposed into two separable problems: the lot-sizing problem with capacity and setup and, the cutting stock problem, which are solved by specific heuristics. The heuristics differ each other by the order in which the problems are solved. In general, the best results for the objective function, gap and running time are found by the cutting-lot heuristic. Poltroniere et al. (2016) propose two reformulations which are solved both heuristically and using an optimisation package. Attempting to get lower bounds for the problem, relaxed versions of the models also have been solved.

In Malik, Qiu, and Taplin (2009), the number of pieces cut from an object according to a cutting pattern is a decision variable, whereas the number of objects cut according to a cutting pattern is an input parameter. A constraint guarantees that the number of objects produced over the whole planning horizon is equal to the number of objects cut in the cutting process and there is no inventory of objects. A capacity constraint limits the number of objects produced in each period. The authors develop an Excel-based Genetic procedure to search for the optimal solution for the model. The strategy has found solutions which are likely close to the global optimum and compared with a decomposition approach, a high reduction in the total costs and improvements in the customer service levels are observed.

Silva, Alvelos, and Valério de Carvalho (2014) propose two integer programming models to optimise a production process in a furniture industry. The proposed models allow the inventory of items and leftovers, which can be used in subsequent periods. The first model is an extension of the model proposed in previous research. The second model is based on the model (called one-cut) proposed on the literature for the one-dimensional cutting stock problem, where each decision variables corresponds to a single cutting operation in a single object. Computational results are presented using real data from a furniture industry.

Poldi and de Araujo (2016) propose an arc-flow reformulation and a heuristic procedure, which consists in a column generation with rolling horizon strategies. The heuristic approach shows to be much faster than the integer programming. The authors also consider the number of objects available in each period as a decision variable and the obtained results show to be even better than the previous one.

Agostinho, Cherri, de Araujo, and Nascimento (2016) and Viegas, Vieira, Henriques, and Sousa (2016) assume that the production level of objects is already decided, as a parameter, and the inventory balance constraints at Level 1 just model the planning of objects in stock. In cases where leftovers are allowed, either they are added to the stock for the next days, i.e., the usable leftovers of one day are considered as new stock objects in the following days (Viegas et al., 2016) or the inventory balance constraint for the leftovers is modelled (Agostinho et al., 2016). These models do not take into account the setup for producing objects. Consequently, there is no setup cost associated with production at Level 1 and no capacity constraint at Level 1 is modelled. Viegas et al. (2016) propose an algorithm based on known heuristics of cutting stock literature, such as first-fit decreasing and best-fit decreasing approaches in order to minimise the stock growth. The results show that the proposed algorithms are able to keep the stock size low and to generate a small number of stock pieces.

Leão, Furlan, and Toledo (2016) propose integrated models where at Level 1 a complete capacitated lot-sizing problem with setup is modelled. The authors investigate Dantzig-Wolfe decompositions with column generation techniques to obtain upper and lower bounds for the integrated problem. The authors observe that the column generation heuristic for the problem without applying decompositions presented better feasible solutions with low computational effort for instances of the literature.

Campello, Oliveira, Ayres, and Ghidini (2017), Campello, Ghidini, Ayres, and Oliveira (2019) present a formulation which is quite similar to the one proposed by Poltroniere et al. (2007) and the authors consider a multi-objective optimisation approach to analyse the trade-offs between the LSP and CSP by analysing the costs variations of these problems simultaneously. Several costs variations of the lot-sizing and cutting stock problem are analysed simultaneously. For this, the authors proposed a multiobjective approach in order to compare, detect and analyse the trade-offs and correlations existing among the costs and their decision variables in the integrated problem. The computational results showed that increasing LSP total costs directly imply in decreasing CSP total costs, and vice versa.

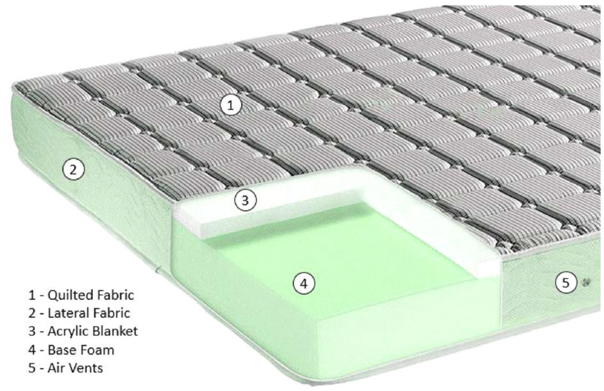

Figure 1. Structure of a classic foam mattress.

3. The mattress production process in a small factory

The mattress production process described in this section is based on information collected in a small factory located in the state of São Paulo, Brazil, which produces both spring mattresses and foam mattresses. The focus of this paper is on foam mattresses made of polyurethane, with different sizes and densities. The materials used in a classic foam mattress are shown in Figure 1.

The production environment is composed of two levels. The first production level (LSP) corresponds to the purchasing planning of foam blocks (objects), which must be ordered from suppliers. The foam blocks can be purchased from different suppliers and can vary slightly in size depending on the supplier. The foam blocks are produced in four different densities (D15, D23, D26, D33), and foams with higher densities are indicated for orthopaedic purposes, as they are more rigid. There is an independent demand for the foam blocks and the company maintains a safety stock of D23 foam blocks (best seller). The second production level corresponds to the three-dimensional CSP, in which objects are cut into smaller parts (items) of foams through the use of cutting patterns. These items can be directly considered as final products or can be combined with other components to produce the mattresses.

The foam mattress manufacturing process, shown in Figure 2, is divided into four main sectors (Cutting; Gluing; Sewing and Dispatching) and into a total of five work stations (Foam Cutting; Cutting of Quilted Fabrics, Lateral Fabric and Acrylic Blanket; Gluing; Sewing; Packaging and Delivery), starting the process in two parallel production lines ((i) Foam Cutting and (ii) Cutting of Quilted Fabrics, Lateral Fabric and Acrylic Blanket) in the Cutting sector. In one of the lines, the three-dimensional foam blocks are cut in the Foam Cutting station generating three-dimensional rectangular foam items as needed. The second line consists of cutting

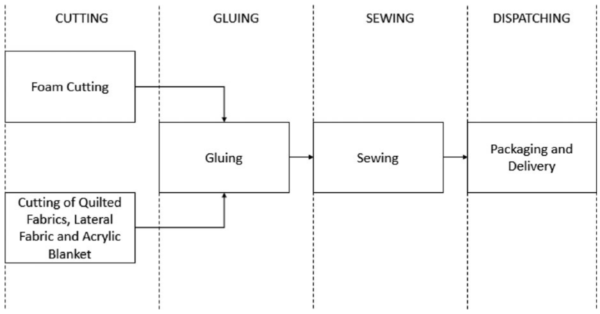

Figure 2. Foam mattress manufacturing process.

quilted fabrics, lateral fabrics and acrylic blankets. After the completion of the processes in these two lines, the semi-finished products are glued in the Gluing station. Top and bottom quilted fabrics and lateral fabric are sewn in the Sewing station. Finally, the foam mattresses go to the last station where they are packed and stored for future deliveries.

The major concern at the factory is related to the final cost of the product and, according to data collected, the foam costs represent about 65% of the final cost of the product. In the mathematical model, which will be described in the next section, only the foam cutting process will be considered, disregarding the other stages of the productive process, due to the foam cost and the fact that the cutting process is the bottleneck of the production process, since it limits the amount of products that the system can generate as a whole. Thus, the focus of this study is to optimise the foam cutting process in order to obtain a good combination of cutting patterns and reduce material waste, which results in lower production costs.

The factory catalogue considered in this study contains seven types of foam mattresses, given by: two models of crib mattresses – cm^1 (1.28 m × 0.58 m × 0.12 m) and cm^2 (1.28 m × 0.68 m × 0.12 m), two models of single mattresses – sm^1 (1.88 m × 0.78 m × 0.14 m) and sm^2 (1.88 m × 0.88 m × 0.14 m), a double mattress – dm (1.88 m × 1.38 m × 0.14 m), a queen size mattress – qm (1.98 m × 1.58 m × 0.2 m) and a king size mattress – km (1.98 m × 1,86 m × 0.2 m). The foam mattresses can be produced in four different densities, resulting in 28 distinct final products.

The decisions about the ILSCSP are taken based on customer demand, respecting the delivery deadlines (after the order) of up to 10 days for mattresses with usual measures (the seven types described above) and up to 20 days for non-standard mattresses, but always leaving some safety stock. The safety stock of final products corresponds to: 50 single mattresses of 1.88 m × 0.78 m × 0.14 m, 50 single mattresses of 1.88 m × 0.88 m × 0.14 m and 20 double mattresses of 1.88 m × 1.38 m × 0.14 m; all corresponding to density D23.

4. Mathematical modelling

In this section, a mathematical model for the ILSCSP is proposed. The model is based on the description of the productive process of the mattress industry presented in Section 3. It is worth noticing that the foam blocks (objects) are bought from different suppliers, so the blocks vary slightly in size depending on the supplier. For simplicity, a single type of stock object with a standard size of $2.76\,\mathrm{m} \times 1.88\,\mathrm{m} \times 1.15\,\mathrm{m}$ will be considered for all suppliers, differentiated only by the density.

The proposed model consists of a generalisation of the model by Gilmore and Gomory (1961, 1963) for the CSP, to deal with the multi-period CSP (Poldi & de Araujo, 2016), and in a generalisation of the model proposed by Trigeiro, Thomas, and Mcclain (1989) for the LSP. The following data are considered for the mathematical modelling:

Sets:
$t = 1, \ldots, T$: periods in the planning horizon;
$k = 1, \ldots, K$: types of objects available in stock (different densities);
$j = 1, \ldots, N$: cutting patterns;
$i = 1, \ldots, m$: item types.

Parameters:
T: number of planning horizon periods;
K: number of object types available in stock;
N: number of cutting patterns;
m: number of item types;
d_{ikt}: demand of the item (foam) type i with density k in period t, $i = 1, \ldots, m$, $k = 1, \ldots, K, t = 1, \ldots, T$;
a_{ij}: number of units of item type i in the cutting pattern j, $i = 1, \ldots, m$, $j = 1, \ldots, N$;
pc_{kt}: purchase cost of object with density k in period t, $k = 1, \ldots, K$, $t = 1, \ldots, T$;
hc^o_{kt}: holding cost of object with density k in period t, $k = 1, \ldots, K$, $t = 1, \ldots, T$;
d^o_{kt}: independent demand of object with density k in period t, $k = 1, \ldots, K$, $t = 1, \ldots, T$;
sc_{kjt}: cutting machine setup cost for cutting an object with density k according to the cutting pattern j in period t, $k = 1, \ldots, K$, $j = 1, \ldots, N$, $t = 1, \ldots, T$;
cc_{kjt}: cost to cut an object with density k according to the cutting pattern j in period t, $k = 1, \ldots, K$, $j = 1, \ldots, N$, $t = 1, \ldots, T$;
st_j: setup time for cutting an object according to the cutting pattern j, $j = 1, \ldots, N$;
pt_{kj}: production time required for cutting an object with density k according to the cutting pattern j, $k = 1, \ldots, K$, $j = 1, \ldots, N$;
Cap_t: cutting machine capacity (in time units) in period t, $t = 1, \ldots, T$;
hc_{ikt}: holding cost of item type i with density k in period t, $i = 1, \ldots, m$, $k = 1, \ldots, K$, $t = 1, \ldots, T$;
M'_{kjt}: large number calculated based on the density k, cutting pattern j and period t, $k = 1, \ldots, K$, $j = 1, \ldots, N$, $t = 1, \ldots, T$;
ES_{ikt}: safety stock for item type i with density k in period t, $i = 1, \ldots, m$, $k = 1, \ldots, K$, $t = 1, \ldots, T$;
ES^o_{kt}: safety stock for object with density k in period t, $k = 1, \ldots, K$, $t = 1, \ldots, T$.

Decision variables:
y_{kt}: purchased quantity of objects with density k in period t;
s_{kt}: stock of object k at the end of period t;
w_{jt}: binary variable indicating the setup or not of the machine to use the cutting pattern j in period t;
x_{kjt}: number of objects with density k cut according to the cutting pattern j in period t;
r_{ikt}: stock of items type i with density k at the end of period t

Mathematical model:

$$\text{Min} \sum_{t=1}^{T} \left(\sum_{k=1}^{K} (pc_{kt} y_{kt} + hc^o_{kt} s_{kt}) + \sum_{k=1}^{K} \sum_{j=1}^{N} (sc_{kjt} w_{jt} + cc_{kjt} x_{kjt}) + \sum_{i=1}^{m} \sum_{k=1}^{K} hc_{ikt} r_{ikt} \right) \tag{1}$$

Subject to:

$$\sum_{j=1}^{N} a_{ij} x_{kjt} + r_{ik,t-1} - r_{ikt} = d_{ikt}, i = 1, \ldots, m, \quad k = 1, \ldots, K, t = 1, \ldots, T \tag{2}$$

$$s_{k,t-1} + y_{kt} = \sum_{j=1}^{N} x_{kjt} + d^o_{kt} + s_{kt}, \quad k = 1, \ldots, K, t = 1, \ldots, T \tag{3}$$

$$x_{kjt} \le M'_{kjt} w_{jt}, \; j = 1, \ldots, N, k = 1, \ldots, K, t = 1, \ldots, T \tag{4}$$

$$\sum_{k=1}^{K} \sum_{j=1}^{N} (st_j w_{jt} + pt_{kj} x_{kjt}) \le \text{Cap}_t, \; t = 1, \ldots, T \tag{5}$$

$$r_{ikt} \ge ES_{ikt}, \; i = 1, \ldots, m, k = 1, \ldots, K, t = 1, \ldots, T \tag{6}$$

$$s_{kt} \ge ES^o_{kt}, \; k = 1, \ldots, K, t = 1, \ldots, T \tag{7}$$

$$y_{kt}, s_{kt}, r_{ikt} \ge 0, w_{jt} \text{ binary}, x_{kjt} \ge 0 \text{ and integer}, \quad i = 1, \ldots, m, j = 1, \ldots, N, k = 1, \ldots, K, t = 1, \ldots, T \tag{8}$$

The objective function (1) minimises the purchase costs and holding costs of objects, the setup costs and cutting costs of objects, in addition to the holding costs for items, i.e., the overall costs at each level. The constraints (2) make sure that the precise demand for items is met, while constraints (3) are the inventory balance constraints of objects and are part of the set of constraints of the LSP at Level 1 (purchase of objects). The constraints (3) also connect Level 1 (purchase of objects) with Level 2 (cut objects into items), ensuring the purchasing of enough objects in the cutting process and integrating the LSP and CSP. The objects in stock not used in a period t became available in period $t+1$ with a "penalty," i.e., the holding cost hc^{o}_{kt}. The constraints (4) and (5) are related to the production of foam items in the cutting process. The constraint (4) forces a pattern setup in the cutting machine, whenever an object is cut according to a new cutting pattern (setup takes place only when the cutting pattern is changed) and the parameter $M^{'}_{kjt}$, used in the model and present in these constraints, can be defined as:

$$M'_{kjt} = \min \begin{cases} M_{kjt} = \max_{i/a_{ij}>0} \left\{ \left\lceil \dfrac{\sum_{l=t}^{T} d_{ikl} + \mathrm{ES}_{ikt}}{a_{ij}} \right\rceil \right\} \\ M_{kjt} = \left\lfloor \dfrac{\mathrm{Cap}_t - st_j}{pt_{kj}} \right\rfloor \end{cases} \tag{9}$$

In brief, this definition limits the production as the minimum between: the capacity of the cutting machine minus the setup time and the demand plus the safety stock of items, thus guaranteeing a stronger linear relaxation. The constraints (5) refer to the capacity limit, taking into account the setup time for the cutting pattern and the production time to cut the object according to the cutting pattern, while the constraints (6) and (7) take into account the safety stocks for items and objects, respectively. Finally, the constraints (8) are the constraints of non-negativity and integrality for the model. In the literature related to the LSP, it is quite common to consider the variables related to production and stock as continuous variables because in many applications the high values associated with these variables make the fact of considering them continuous rather than integers has a negligible effect on the solution of the problem. Thus, these variables will also be considered in this way, following this approach from the literature.

Various research papers have used valid inequalities and alternative formulations to model the classical lot sizing problem, in order to strengthen its linear relaxation. Two important reformulations have been proposed. The first one deals with the reformulation of the problem as the Shortest Path problem in which a redefinition of the variables proposed by Eppen and Martin (1987) is used. A second one consists of a reformulation based on the Simple Plant Location problem studied in Krarup and Bilde (1977). Various theoretical and computational results concerning such reformulations have been published in the literature (for example, Denizel and Süral (2006), Denizel, Altekin, Süral, and Stadtler (2008) and de Araujo, de Reyck, Degraeve, Fragkos, and Jans (2015)).

We have tried to reformulate constraints (2) and (4) using the ideas from the literature. However, since constraints (2) are based on cutting patterns, with several items on each cutting pattern, instead of individual items as in the other papers from the literature, we could not find an equivalent reformulation that improves the computational results. Alternatively, we have reformulated constraint (3) using the ideas of facility location reformulation but, since there are no setups for objects, such reformulation does not improve the linear relaxation and the computational results are quite similar comparing to the proposed mathematical model.

It is worth noticing that we have already proposed an innovative and very tight "big M" on constraints (4) and the computational results presented in Section 5 show that the formulation (1)–(8) is quite strong and gaps are quite low, at least, for the tested instances.

5. Results and discussions

In this section, the data collected at the mattress factory is presented, as well as a comparison of the solutions proposed by the model and the solutions adopted by the factory, as well as additional computational tests, in order to verify the applicability of the results and the performance of the model considered in this paper.

5.1. Numerical data

The parameters, described in Section 4, were collected at the mattress factory and are shown in this section:

- Four periods in the planning horizon ($T = 4$), each period consisting of an interval of two weeks, covering a total planning horizon of two months;
- Four types of objects available in stock, i.e., four types of foam blocks of different densities (D15, D23, D26, D33);
- Five different cutting patterns ($N = 5$) for the first set of tests and ten different cutting patterns ($N = 10$) for the second set of tests. These are the same cutting patterns used by the factory in practice for the three-dimensional cut, since the

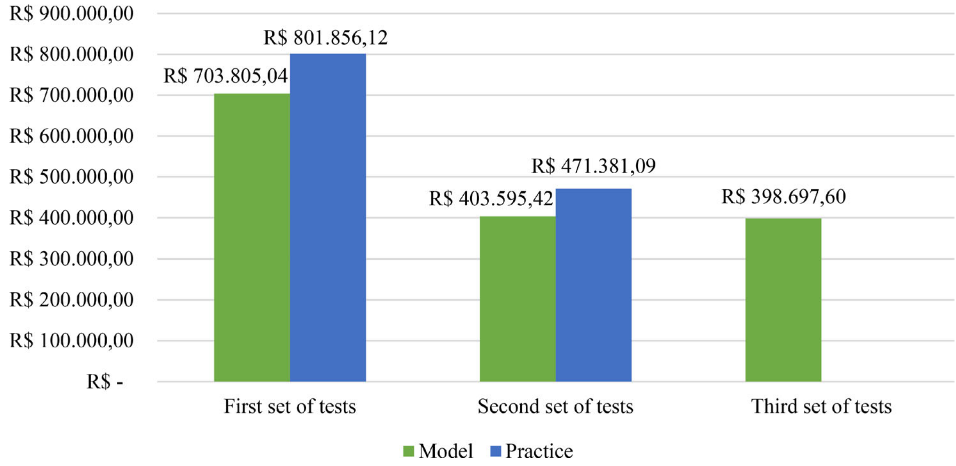

Figure 3. Objective function costs.

intention is to find solutions easily applicable in practice. Moreover, in intend to compare the model solution to the solution adopted by the factory.

- A third set of tests with fifteen different cutting patterns ($N = 15$) is considered, being formed by the ten previous cutting patterns and by five new cutting patterns from a heuristic that generates three-dimensional cutting patterns, which is based on the first two algorithms presented in the online supplement of Zhu and Lim (2012). Since the goal was to generate three-dimensional cutting patterns for the integrated problem, the first algorithm generates "blocks" by grouping mattresses of the same type (same dimensions). Starting with a single mattress, it increases one by one while the dimensions of this block are no larger than the dimensions of the foam or the demand of the mattress has been attended. In the end, a large set of blocks of mattresses of the same type (homogeneous blocks) is generated, each type having its own set. Afterward, in the second algorithm, homogeneous blocks of different types are grouped, generating heterogeneous combinations of blocks from then, following the same previous logic. The five new cutting patterns were chosen in a random manner, considering only those that presented a combination of two items being produced in the cutting pattern, since the industry uses cutting patterns with a maximum of two different items, and thus, the comparisons would be closer to reality. It is worth noticing that we have tried to add more cutting patterns, but the problem become quite difficult to be solved exactly and the reduction in the objective function was not expressive;
- Seven item types (two models of crib mattresses, two models of single mattresses, one double mattress, one queen mattress and one king mattress), representing the factory's product portfolio;
- An available cutting capacity of 9600 minutes for each period, taking into account the machines operating eight hours a day and five days a week for two weeks.

All other parameters are shown in Appendix in Tables A1–A16.

5.2. Computational results

In this section, the solutions proposed by the model and the solutions adopted by the factory for the first and second sets of tests (with 5 and 10 cutting patterns, respectively) will be presented, as well as the solutions found by the model for the third set of tests (with 15 cutting patterns). The objective of considering 15 cutting patterns is to analyse the behaviour of the proposed model, making a comparison with the 10 cutting pattern solutions currently adopted by the factory, demonstrating how much the costs of the objective function could be reduced by increasing the number of cutting patterns. The model (1) - (9) was implemented in the GAMS algebraic modelling language and solved by CPLEX (version 12.6.3.0) optimisation solver. All computational tests were carried out on an i7 computer with 3.6 GHz, 16 GB RAM and Windows 10 Pro operating system.

In Tables 1–5, the solutions proposed by the mathematical model for the decision variables and the solutions adopted by the factory are shown. Figure 3 shows the values obtained for the objective function in the first two sets of tests and for the computational and practical cases, as well as the computational values for the third set of tests. It is worth noticing that all solutions found for the three sets of tests were optimal (null CPLEX gaps), with CPLEX computational times of 0.08 s, 0.39 s and 7.3 s for the first, second and third sets of tests, respectively.

Table 1. Purchased quantity of objects (y_{kt}) – All test sets.

	Periods							
	1		2		3		4	
Test sets	Model	Practice	Model	Practice	Model	Practice	Model	Practice
First	309	338	39	48	47	56	44	49
Second	90	97	39	48	66	73	44	50
Third	88	–	39	–	66	–	44	–

Table 2. Stock of objects (s_{kt}) – All test sets.

	Periods							
	1		2		3		4	
Test sets	Model	Practice	Model	Practice	Model	Practice	Model	Practice
First	2	12	2	11	2	6	2	3
Second	2	10	2	11	2	10	2	2
Third	2	–	2	–	2	–	2	–

Table 1 presents the solutions proposed by the mathematical model and the solutions adopted by the factory for y_{kt}, i.e., the purchased quantity of objects for all densities in each period, for the first and second sets of tests, and also presents the mathematical model results for the third set of tests. It is possible to see that the optimal solutions proposed by the mathematical model are always smaller than the solutions adopted by the industry in the first two sets of tests. It can be seen that, considering the whole planning horizon, there was a decrease in both solutions when passing from the first to the second set of tests, and a small decrease when passing from the second set of tests to the third one. Such behaviour can be explained by the fact that a higher number of different cutting patterns are available to be used, resulting in a higher number of possible combinations for cutting, better material use, less waste and therefore less need to purchase objects.

Table 2 contains the solutions proposed by the mathematical model and the solutions adopted by the factory for s_{kt}, i.e., the stock of objects considering all the densities in each period. Looking at the results of this table, the optimal solutions proposed by the mathematical model are always smaller or equal to the solutions adopted by the industry in the two sets of tests. Also, the solutions proposed by the mathematical model are equal to safety stocks for objects (Table A15) in the three sets of tests.

Table 3 shows the sum of the solutions proposed by the model and the solutions adopted by the factory for w_{jt} (the binary variable indicating the setup or not of the machine to use the cutting pattern j in period t). Looking at mathematical model, it is known that w_{jt} is related to the variable x_{kjt} through constraints (4), so if production occurs, $w_{jt} = 1$, otherwise, $w_{jt} = 0$. In addition, some results proposed by the mathematical model and adopted by the factory are the same in the first two sets of tests. However, there are more setups in practice, especially in period 2 of the second set, since there is no production in this period in the mathematical model solution. Note that there are almost no setups in period 4.

Table 4 presents the solutions proposed by the model and the solutions adopted by the factory for x_{kjt}, the number of objects of all densities cut according to all cutting patterns in each period. Table 4 shows that, on average, the optimal solutions proposed by the mathematical model are smaller than the solutions adopted by the factory in the first two sets of tests. In addition, on average, there was a decrease in both solutions when passing from the first to the second set of tests, with a small decrease also occurring, on average, in the solutions proposed by the model between the second and the third set of tests. This is due to more cutting patterns being used, leading to a higher number of possible cutting combinations, better material utilisation, less waste and, therefore, fewer objects being cut.

Table 5 presents the solutions proposed by the model and the solutions adopted by the industry for r_{ikt}, stock of all items types with all densities in each period. Analysing the results of this table, the optimal solutions proposed by the mathematical model are smaller than the solutions adopted by the factory in the first two sets of tests. In addition, there were decreases in both solutions, on average, from the first to the second set of tests and in the solutions proposed by the mathematical model when passing from the second to the third set of tests. More cutting patterns will be used, resulting in a higher possible cutting combinations and, possibly, fewer items are cut unnecessarily and stored.

Figure 3 shows the values obtained for the objective function costs in the first two sets of tests and for the computational and practical cases, besides the computational values for the third set of tests.

When comparing the values obtained for the objective function, the differences become more

Table 3. Binary variable of setup or not of the machine (w_{jt}) – All test sets.

	Periods							
	1		2		3		4	
Test sets	Model	Practice	Model	Practice	Model	Practice	Model	Practice
First	5	5	0	0	1	3	0	1
Second	7	9	0	10	3	8	0	0
Third	8	–	0	–	3	–	0	–

Table 4. Number of cut objects (x_{kjt}) – All test sets.

	Periods							
	1		2		3		4	
Test sets	Model	Practice	Model	Practice	Model	Practice	Model	Practice
First	267	326	0	0	8	39	0	11
Second	48	60	0	72	27	64	0	0
Third	46	–	0	–	27	–	0	–

Table 5. Stock of items (r_{ikt}) – All test sets.

	Periods							
	1		2		3		4	
Test sets	Model	Practice	Model	Practice	Model	Practice	Model	Practice
First	1606	2111	1320	1825	1128	1899	764	1612
Second	611	673	325	1468	589	2262	225	1898
Third	597	–	311	–	583	–	219	–

Table 6. Instance class parameters.

Class	Demand of objects	Demand of items	Capacity	Number of patterns
1	Low	Low	9600 min	5
2	Low	Low	9600 min	10
3	Low	Low	9600 min	15
4	Low	Low	14,400 min	5
5	Low	Low	14,400 min	10
6	Low	Low	14,400 min	15
7	Low	Medium	9600 min	5
8	Low	Medium	9600 min	10
9	Low	Medium	9600 min	15
10	Low	Medium	14,400 min	5
11	Low	Medium	14,400 min	10
12	Low	Medium	14,400 min	15
13	Medium	Low	9600 min	5
14	Medium	Low	9600 min	10
15	Medium	Low	9600 min	15
16	Medium	Low	14,400 min	5
17	Medium	Low	14,400 min	10
18	Medium	Low	14,400 min	15
19	Medium	Medium	9600 min	5
20	Medium	Medium	9600 min	10
21	Medium	Medium	9600 min	15
22	Medium	Medium	14,400 min	5
23	Medium	Medium	14,400 min	10
24	Medium	Medium	14,400 min	15
25	Low	High	9600 min	5
26	Low	High	9600 min	10
27	Low	High	9600 min	15
28	Low	High	14,400 min	5
29	Low	High	14,400 min	10
30	Low	High	14,400 min	15
31	Medium	High	9600 min	5
32	Medium	High	9600 min	10
33	Medium	High	9600 min	15
34	Medium	High	14,400 min	5
35	Medium	High	14,400 min	10
36	Medium	High	14,400 min	15

evident, as can be seen in Figure 3. The purpose of considering three sets of tests with different numbers of cutting patterns is to calculate precisely how much the results can be improved by increasing the number of patterns from 5 to 10, and then from 10 to 15. Looking at the costs of the objective function in Figure 3, there was a reduction of 42.66% in the costs for the mathematical model case and a reduction of 41.21% in the costs for the practical case, when passing from 5 cutting patterns (first set of tests) to 10 cutting patterns (second set of tests), as well as a reduction of 1.21% in costs for the mathematical model when passing from 10 cutting patterns (second set of tests) to 15 cutting patterns (third set of tests), thus verifying the consistency, since there will be a higher number of possible combinations for the foam cutting, resulting in less waste and better use of materials, and a reduction in costs. When comparing the results from the mathematical model and from the practice, there was a decrease of 12.23% and 14.38% for the first and second sets of tests, respectively, in relation to the values provided by the objective function, thus evidencing the good performance of the mathematical model for this problem. In addition, when analysing the contribution of each one of the five portions of the objective function to the total cost, the first portion of the objective function ($pc_{kt}y_{kt}$), related to the purchase of objects, is quite representative, with an average value of approximately 96%, of the total costs.

5.3. Additional computational tests

Other computational tests were performed in order to analyse the behaviour of the proposed model in

Table 7. Results of additional computational tests.

Class	Number of optimal solutions	Objective function costs (R$)	Relative gap % (CPLEX)	Nodes	CPLEX Time (s)
1	10	898,922.02	0.00	0.00	0.08
2	10	485,159.41	0.00	11,525.90	2.96
3	6	461,300.32	0.10	785,559.80	417.06
4	10	898,922.02	0.00	0.00	0.07
5	10	485,159.41	0.00	5431.80	1.49
6	6	461,300.63	0.12	1,272,493.40	492.45
7	10	2,644,961.58	0.00	54.80	0.08
8	5	1,297,806.70	0.05	3,017,351.70	618.80
9	0	1,214,166.11	0.19	2,682,812.60	1000.00
10	10	2,637,512.10	0.00	0.00	0.08
11	6	1,297,567.52	0.03	2,618,354.00	417.50
12	0	1,214,080.08	0.16	3,062,555.50	1000.00
13	10	1,033,360.77	0.00	0.00	0.07
14	10	618,884.60	0.00	17,892.30	3.56
15	2	584,263.37	0.14	1,963,716.60	816.65
16	10	1,033,360.77	0.00	0.00	0.06
17	10	618,884.60	0.00	42,250.00	9.24
18	4	584,258.38	0.13	1,572,255.40	630.25
19	10	2,780,774.93	0.00	16.50	0.07
20	5	1,434,472.37	0.05	2,532,110.60	515.45
21	0	1,350,823.16	0.11	2,465,190.70	1000.00
22	10	2,774,185.24	0.00	0.00	0.05
23	6	1,434,249.84	0.04	2,565,229.90	443.50
24	1	1,350,762.69	0.15	2,709,945.40	973.94
25	0 (10)	–	–	–	–
26	0 (10)	–	–	–	–
27	0 (10)	–	–	–	–
28	7 (3)	4,346,312.57	0.00	493.86	0.11
29	4	2,113,244.51	0.03	3,289,096.50	606.88
30	0	1,977,614.02	0.21	3,990,954.00	1000.00
31	0 (10)	–	–	–	–
32	0 (10)	–	–	–	–
33	0 (10)	–	–	–	–
34	6 (4)	4,535,794.74	0.00	415.00	0.13
35	8	2,231,550.96	0.01	2,101,772.90	345.54
36	1	2,095,645.28	0.15	3,471,711.80	900.88
Average	5.19	1,563,176.69	0.06	1,339,306.37	373.23

() the numbers in parentheses represent the number of instances in which CPLEX prove infeasibility within the time limit.

Section 4 for different cases. A random data generator was used to create 36 classes of problems, each one with 10 instances, totalling 360 instances. However, 67 generated instances were infeasible and after deleting these instances we came out to 293 instances. The details on how these classes were built and the results obtained are shown below.

In order to perform the computational tests, some parameters were fixed, such as:

- Cutting machine capacity: $\text{Cap}_t = 9600\,\text{min}$ and 14,400 min;
- Number of cutting patterns: $N = 5$, 10 and 15 (being equal to the cutting patterns used in the last two sections).
- Other parameters were randomly generated, with a uniform distribution, in the following intervals to define the classes of problems:
- Demand of objects:
  - Medium: $d^o_{kt} \in [6, 9]$ (corresponding to approximately 33% and 50% of the variation range of real data in Section 4.1);
  - Low: $d^o_{kt} \in [1, 5]$ (corresponding to approximately 0% and 25% of the variation range of real data in Section 4.1);
- Demand of items:
  - High: $d_{ikt} \in [76, 134]$ (corresponding to approximately 51% and 89% of the variation range of real data in Section 4.1);
  - Medium: $d_{ikt} \in [50, 75]$ (corresponding to approximately 33% and 50% of the variation range of real data in Section 4.1);
  - Low: $d_{ikt} \in [0, 37]$ (corresponding to approximately 0% and 25% of the variation range of real data in Section 4.1).

Thus, 36 classes of problems were generated, each one containing 10 instances. All these classes of problems were defined according to Table 6.

For the other parameters of the model (not mentioned above), the same values shown in Section 4.1 were used. The proposed model was implemented and solved through CPLEX solver in the GAMS software for the 36 classes, with a default time limit of 1000 s, using 10 equal seeds (one for each instance) for all classes of problems, in order to enable a better analysis and comparison between results. Table 7 shows the averages of the 36 classes, referring to: the number of optimal solutions obtained by CPLEX solver considering the time limit; the total costs of the objective function of the model; the relative gap %age (given by CPLEX); the number of branch-and-cut tree nodes required to obtain such solutions and CPLEX computational time.

When the relative gap %age (CPLEX) is less than or equal to 0.0001, the solution was considered to be optimal. The relative gap %age (CPLEX) is provided by CPLEX solver when solving the model, being calculated by the following equation:

$$\text{Relative gap } \% \text{ (CPLEX)} = \frac{100 \times (\text{MIP Solution} - \text{Best Possible})}{\text{MIP Solution}}$$

Tables 6 and 7 show that:

- For classes 1 to 3, 4 to 6, 7 to 9, and so on, i.e., classes with the same types of demand for objects and items and the same capacity, the costs of the objective function always decrease with the increase in the number of cutting patterns. In addition, the number of optimal solutions also decreases with the increasing number of cutting patterns in the vast majority of cases. However, both the relative gap and the computational time always increase with an increase in the number of cutting patterns and this behaviour is also clearly shown in relation to the number of nodes, except for classes 7 to 9 and 19 to 21. These facts can be explained by the increasing amount of data in the problem by using more cutting patterns, leading to more computational difficulty, as well as more difficulty in finding optimal solutions, increasing the relative gap, the number of nodes and the computational time. On the other hand, the objective function costs decrease as more cutting patterns are used, since there will be a higher number of possible combinations for the foam cutting, resulting in less waste and better use of material;
- For classes 1 to 6, 7 to 12, 13 to 18, and so on, i.e., classes with the same types of demand for objects and items, but with variations in the capacity and in the number of cutting patterns, by increasing the capacity of the cutting machine from 9600 min to 14,400 min, the number of optimal solutions remains practically constant for low and medium demand for objects and items, presenting large increases in the number of optimal solutions for the classes with high demand for items (classes 28-30 and 34-36). It can also be seen that, by increasing capacity, the costs of the objective function tend to decrease in general, showing significant differences for the classes with high demand for items, since these are infeasible with a capacity of 9600 min but become optimal or integer with the increase in capacity. In addition, the number of nodes and the computational time are directly proportional to each other, with few exceptions;
- For classes with the same types of object demand, capacity and number of cutting patterns, varying the demand for items (classes 1, 7 and 25; 2, 8 and 26; 3, 9 and 27, among others), it can be observed that, by increasing the demand for items, the costs of the objective function, relative gap, number of nodes and computational time are also increased for almost all classes, and classes 25, 26, 27, 31, 32 and 33 become infeasible with high demand for items and a capacity of 9600 min. However, for classes with the same types of object demand, capacity and number of cutting patterns, the number of optimal solutions has an inverse behaviour, since, in general, it decreases with the increasing demand for items (going from a low item demand to a medium item demand and from a medium item demand to a high item demand), with the exception of classes 17, 23 and 35;
- For classes with the same types of item demand, capacity and number of cutting patterns, varying the demands of objects (classes 1 and 13, 2 and 14, 3 and 15, among others), it is verified that, by increasing the demand for objects, the costs of the objective function, relative gap, number of nodes and computational time are also increased for most classes, and classes 25, 26, 27, 31, 32 and 33 are infeasible with high demands for items and capacity of 9600 min, regardless of the object demand. However, for classes with the same types of item demand, capacity and number of cutting patterns, the number of optimal solutions presents different behaviours, since it increases, decreases or remains unchanged when moving from low object demand to medium object demand;
- The optimality of the solutions changes more with the variation of the items demand than with the variation of the objects demand, since, for the same conditions of demand, capacity and number of cutting patterns, the increase of the items demand ends up reducing considerably the number of optimal solutions, whereas the increase of the objects demand generates smaller alterations in the number of optimal solutions;
- In general, CPLEX found optimality in an expressive number of 10 instances for the great majority of classes, and the average obtained was 5.19;
- The average relative gap %age (CPLEX) was low, with an intermediate average CPLEX computational time, which confirms the good performance of the model for a diverse range of classes.

6. Conclusions and future proposals

In this paper, a mathematical model was proposed to determine the main decisions in the production process of a mattress factory, capturing the interdependencies of the decisions of the lot-sizing and the cutting stock problems, as well as promoting a new approach to the decision making process and an improvement in production scheduling. This paper helps to fill a gap in the literature by addressing the integrated lot-sizing and the three-dimensional cutting stock problem.

The tests to validate the model were based on the product list and on the information provided by the factory. Comparisons were made between the solutions proposed by the model and the solutions adopted by the factory, concluding with additional computational tests, in order to verify the applicability of the results and the performance of the model considered in this paper.

In the first section of the results, the application of the model showed a satisfactory performance, resulting in significant decreases in the costs by increasing the number of cutting patterns. This fact can be explained by the higher number of possible combinations for the foam cutting, resulting in less waste and better material use. Besides that, it was found that if the industry used the model proposed in this paper, the costs for the first and second sets of tests could be reduced by 12.23% and 14.38%, respectively.

In the second section of the results, the additional computational tests with random data proved the good performance of the model for different cases, with a low average relative gap %age (CPLEX) and an intermediate average CPLEX computational time, besides CPLEX solver, in general, to have obtained the optimality in an expressive number of the 10 instances for each one of the 36 classes

The model fulfils what was proposed, acting as a good tool in reducing costs for such problem, which is very important for companies, as they need to continually look for improvements in their processes and to eliminate waste.

For future research, as a direct extension we intend to consider several dimensions of objects purchased from the suppliers. Moreover, we intend to adapt this research to deal with bigger mattress industries and it might be necessary to consider a bigger set of cutting patterns. In this situation, according to some preliminary computational tests, the proposed mathematical model will not be able to solve the problem and heuristics/metaheuristics can be applied to solve the problems. It is also interesting to analyse how to improve the reformulations for the integrated problem considering cutting patterns on the inventory balance constraint.

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Appendix

This appendix describes in detail the other parameters from Section 5.1. Some values had to be estimated for the five new cutting patterns of the third set of tests and will be explained in more detail later.

Table A1. Demands of objects (d^o_{kt}) – All test sets.

		Periods			
		1	2	3	4
Objects	D15	18	16	14	10
	D23	10	14	16	18
	D26	10	8	6	12
	D33	2	1	3	4

Table A2. Demands of items (d_{ikt}) – All test sets.

		Periods			
Objects	Items	1	2	3	4
D15	cm[1]	3	10	2	6
	cm[2]	6	2	10	3
	sm[1]	80	60	90	150
	sm[2]	40	100	60	70
	dm	10	20	40	30
	qm	0	0	0	0
	km	0	0	0	0
D23	cm[1]	0	0	0	0
	cm[2]	0	0	0	0
	sm[1]	9	14	23	15
	sm[2]	9	12	17	12
	dm	10	8	12	10
	qm	0	0	0	0
	km	0	0	0	0
D26	cm[1]	0	0	0	0
	cm[2]	0	0	0	0
	sm[1]	7	12	25	15
	sm[2]	7	4	5	12
	dm	10	8	11	9
	qm	2	4	6	2
	km	2	4	6	2
D33	cm[1]	0	0	0	0
	cm[2]	0	0	0	0
	sm[1]	5	3	2	8
	sm[2]	3	6	0	4
	dm	10	12	8	6
	qm	3	4	2	6
	km	2	3	1	4

Table A3. Number of units of the item in the cutting pattern (a_{ij}).

Test sets	Cutting patterns	Items						
		cm[1]	cm[2]	sm[1]	sm[2]	dm	qm	km
First	1	3	0	4	0	0	0	0
	2	0	2	0	4	0	0	0
	3	0	0	0	0	16	0	0
	4	0	0	0	0	0	5	0
	5	0	0	0	0	0	0	5
Second	1	3	0	4	0	0	0	0
	2	0	2	0	4	0	0	0
	3	0	0	0	0	16	0	0
	4	0	0	0	0	0	5	0
	5	0	0	0	0	0	0	5
	6	0	0	24	0	0	0	0
	7	0	0	0	24	0	0	0
	8	36	18	0	0	0	0	0
	9	2	0	0	4	0	0	0
	10	0	3	4	0	0	0	0
Third	1	3	0	4	0	0	0	0
	2	0	2	0	4	0	0	0
	3	0	0	0	0	16	0	0
	4	0	0	0	0	0	5	0
	5	0	0	0	0	0	0	5
	6	0	0	24	0	0	0	0
	7	0	0	0	24	0	0	0
	8	36	18	0	0	0	0	0
	9	2	0	0	4	0	0	0
	10	0	3	4	0	0	0	0
	11	6	0	19	0	0	0	0
	12	6	0	0	0	8	0	0
	13	10	18	0	0	0	0	0
	14	0	0	0	5	0	5	0
	15	0	0	0	3	0	0	3

Table A4. Object purchase costs (pc_{kt}, in R$) – All test sets.

		Periods			
		1	2	3	4
Objects	D15	1261.10	1268.20	1273.50	1279.70
	D23	1670.00	1679.40	1686.40	1694.60
	D26	1934.00	1944.90	1953.00	1962.50
	D33	2581.30	2595.80	2606.60	2619.30

Table A5. Machine setup costs (sc_{kjt}, in R$) – All test sets.

		Periods			
		1	2	3	4
Objects	D15	20.00	24.00	26.00	30.00
	D23	26.00	31.20	33.80	39.00
	D26	30.00	36.00	39.00	45.00
	D33	40.00	48.00	52.00	60.00

Table A6. Cutting costs of objects (cc_{kjt}, in R$) – All test sets.

		Periods			
		1	2	3	4
Objects	D15	40.00	48.00	52.00	60.00
	D23	52.00	62.40	67.60	78.00
	D26	60.00	72.00	78.00	90.00
	D33	80.00	96.00	104.00	120.00

Table A7. Setup times (st_j, in minutes) – First set of tests.

Cutting patterns				
1	2	3	4	5
3	4	2	2	2

Table A8. Setup times (st_j, in minutes) – Second set of tests.

Cutting patterns									
1	2	3	4	5	6	7	8	9	10
3	4	2	2	2	2	2	4	3	3

Table A9. Setup times (st_j, in minutes) – Third set of tests.

Cutting patterns														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	4	2	2	2	2	2	4	3	3	3	4	4	5	5

The setup times for the five new cutting patterns of the third set of tests, shown in Table A9, had to be estimated based on the dimensions of the items produced in each pattern.

Table A10. Production times (pt_{kj}, in minutes) – First set of tests.

		Cutting patterns				
		1	2	3	4	5
Objects	D15	11.8	12.3	63	34	40
	D23	13	13.6	69.4	37.3	44
	D26	13.6	14.2	73	39.2	46.2
	D33	15	15.7	80.2	43.1	50.8

Table A11. Production times (pt_{kj}, in minutes) – Second set of tests.

		Cutting patterns									
		1	2	3	4	5	6	7	8	9	10
Objects	D15	11.8	12.3	63	34	40	53.5	60.4	55.3	12	12.3
	D23	13	13.6	69.4	37.3	44	58.9	66.4	60.8	13.2	13.5
	D26	13.6	14.2	73	39.2	46.2	61.8	69.7	63.8	13.9	14.2
	D33	15	15.7	80.2	43.1	50.8	68	76.7	70	15.2	15.6

Table A12. Production times (pt_{kj}, in minutes) – Third set of tests.

		Cutting patterns														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Objects	D15	11.8	12.3	63	34	40	53.5	60.4	55.3	12	12.3	48.2	37.4	30.1	46.6	31.5
	D23	13	13.6	69.4	37.3	44	58.9	66.4	60.8	13.2	13.5	53	41.1	33.1	51.2	34.7
	D26	13.6	14.2	73	39.2	46.2	61.8	69.7	63.8	13.9	14.2	55.6	43.2	34.8	53.8	36.4
	D33	15	15.7	80.2	43.1	50.8	68	76.7	70	15.2	15.6	61.2	47.5	38.2	59.2	40.1

The production times for the first and second test sets are shown in Tables A10 and 11, respectively, and are calculated taking into consideration the dimensions of the items that will be cut in each pattern, as well as their quantities and, as the density increases, the material becomes harder, resulting in a longer time to perform the cutting operation. Again the production times for the five new cutting patterns of the third set of tests, shown in Table A12, were estimated.

Table A13. Holding costs of objects (hc^{o}_{kt}, in R$) – All test sets.

		Periods			
		1	2	3	4
Objects	D15	10.00	12.00	13.00	15.00
	D23	13.00	15.60	16.90	19.50
	D26	15.00	18.00	19.50	22.50
	D33	20.00	24.00	26.00	30.00

Table A14. Holding costs of items (hc_{ikt}, in R$) – All test sets.

		Periods			
Objects	Items	1	2	3	4
D15	cm^1	0.20	0.24	0.26	0.30
	cm^2	0.30	0.36	0.39	0.45
	sm^1	1.00	1.20	1.30	1.50
	sm^2	1.20	1.44	1.56	1.80
	dm	2.00	2.40	2.60	3.00
	qm	3.00	3.60	3.90	4.50
	km	4.00	4.80	5.20	6.00
D23	cm^1	0.26	0.31	0.34	0.39
	cm^2	0.39	0.47	0.51	0.59
	sm^1	1.30	1.56	1.69	1.95
	sm^2	1.56	1.87	2.03	2.34
	dm	2.60	3.12	3.38	3.90
	qm	3.90	4.68	5.07	5.85
	km	5.20	6.24	6.76	7.80
D26	cm^1	0.30	0.36	0.39	0.45
	cm^2	0.45	0.54	0.58	0.67
	sm^1	1.50	1.80	1.95	2.25
	sm^2	1.80	2.16	2.34	2.70
	dm	3.00	3.60	3.90	4.50
	qm	4.50	5.40	5.85	6.75
	km	6.00	7.20	7.80	9.00
D33	cm^1	0.40	0.48	0.52	0.60
	cm^2	0.60	0.72	0.78	0.90
	sm^1	2.00	2.40	2.60	3.00
	sm^2	2.40	2.88	3.12	3.60
	dm	4.00	4.80	5.20	6.00
	qm	6.00	7.20	7.80	9.00
	km	8.00	9.60	10.40	12.00

The holding costs of objects increase with density, since objects with higher densities are more expensive. The holding costs of items are calculated based on the dimensions of each item and the space occupied in the warehouse by each one of them, and these costs also increase with density. Cutting of some items in advance can increase the holding costs of items (hc_{ikt}) but, on the other hand, it can allow a better combination of items in the patterns, which minimises total waste.

Related to safety stock for objects, just a number of objects with density D23 are kept in stock that is presented in Table A15, which is the same for all three sets of tests.

Table A15. Safety stocks for objects (ES^{o}_{kt}) – All test sets.

		Periods			
		1	2	3	4
Objects	D23	2	2	2	2

For items, just some of them with density D23 have safety stock and are presented in Table A16, which are the same for the three sets of tests.

Table A16. Safety stocks for items (ES_{ikt}) – All test sets.

		Periods			
Objects	Items	1	2	3	4
D23	sm^1	50	50	50	50
	sm^2	50	50	50	50
	dm	20	20	20	20