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Mathematical model to the irregular strip packing problem with continuous rotations

Luiz Henrique Cherri[1]
Instituto de Ciências Matemáticas e de Computação, USP, São Carlos, SP
Adriana Cristina Cherri[2]
Departamento de Matemática, Faculdade de Ciências, UNESP, Bauru, SP
Edilaine Martins Soler[3]
Departamento de Matemática, Faculdade de Ciências, UNESP, Bauru, SP

1 Introduction

In the irregular strip packing problems a set of convex and non-convex two-dimensional pieces must be cut from a board with fixed height and infinite length aimming to reduce the length of the board used to cut all the pieces. These problems arise in many industries as garment manufacturing, metal mechanic, furniture, footwear, among others.

An important characteristic observed in real applications, mainly in the leather and metal industry, is the possibility of rotating the pieces by any angle to be placed in the board. Mixed integer non-linear models to irregular strip packing problems with continuous rotations were proposed by some authors as [4], [2], [3] and [5].

In this study, a mathematical formulation for the irregular strip packing problem with continuous rotations of the pieces is proposed and solved by exact methods proposed in the free academic solvers COUENNE, SCIP and BARON. The formulation has linear and quadratic constraints and can handle with non-convex pieces.

2 Mathematical model considerations

To mathematically formulate the irregular strip packing problems with continuous rotation, the polygonal pieces are represented by a set of points ordered in the counter-clockwise direction and a reference point is used to indicate the locus of the piece on the board. Also, the non-convex pieces are decomposed into a set of convex parts.

An additional difficult to formulate models for these problems is geometric consideration needed. First, it is necessary to define when two pieces allocated in a board are overlapping, touching or are separated. Furthermore, it is necessary to ensure that the

[1]lhcherri@icmc.usp.br
[2]adriana@fc.unesp.br
[3]edilaine@fc.unesp.br

pieces are completely allocated into the board. Thus, to formulate these conditions the D-function [1] is used. Given a point and an oriented line, this function indicates in which side of the line the point is. To ensure that the pieces can be rotated at any angle, a two-dimensional rotation matrix was also used to reformulate the D-function.

Due to the characteristics of the problem, a set of symmetry breaking constraints was proposed and two variants of the model were analyzed, differing on the way that the non-overlap constraints are addressed.

3 Conclusions

Concerning this problem, the proposed model is the first in the literature to solve it using direct trigonometry to deal with the non-overlap between non-convex pieces.

Due to the simplicity on the pieces representation and non-overlap constraints formulation, the model becomes simple, making it adaptable to other variants of irregular cutting and packing problem.

Computational experiments were performed using instances from the literature considering convex and non-convex pieces and showed that the proposed method finds numerically accurate solutions in competitive computational times.

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