Usable leftovers in the multiperiod cutting stock problem: a new approach

Douglas Nogueira do Nascimento
Programa de Pós-graduação em Engenharia de Produção, FEB, UNESP, Bauru SP, Brasil.
douglasnn@fc.unesp.br

Silvio Alexandre de Araujo
Departamento de Matemática Aplicada, IBILCE, UNESP, São José do Rio Preto, SP, Brasil.
saraujo@ibilce.unesp.br

Adriana Cristina Cherri
Departamento de Matemática, FC, UNESP, Bauru SP, Brasil.
adriana@fc.unesp.br

Abstract

This paper addresses the multiperiod cutting stock problem with usable leftovers (MCSPUL), which differentiates from the classic cutting stock problem (CSP) by considering two variations of the one-dimensional cutting stock problem: the multiperiod cutting stock problem and the cutting stock problem with usable leftovers. The objective of this problem is to minimize de cost of cutting items from objects available in stock, allowing the production of items that have known demands in a future period of time and allowing the generation of retails that will be used to cut future items. These retails are not considered waste in the current period. A new mathematical model for the MCSPUL is proposed to integrate these two variations, making the decision of generate retails or anticipate the demand. Some preliminary computational tests were performed, and results for integer solutions obtained from a heuristic procedure are presented.

Keywords: Multiperiod cutting stock problem; usable leftovers; mathematical model; heuristic procedure; linear and integer optimization.

1 Introduction

Cutting Stock Problems (CSP) consists of cutting large objects available in stock into a set of smaller items with specified quantities and sizes by optimizing an objective function, such as minimizing the total waste or minimizing the cost of the objects cut. Many papers in the literature have studied the CSP and have proposed solution methods for this problem.

A variation of CSP that often appears in practice consists of considering the generation of usable leftovers of cut objects. These leftovers are not considered as waste, and they are kept in stock to meet future demands. This variation is called Cutting Stock Problem with Usable Leftovers (CSPUL). It was first mentioned by Brown (1971), and a considerable number of articles has been published on CSPUL since then. Cherri et al. (2014) presented in a survey the existing papers that investigate the one-dimensional CSPUL.

The first mathematical model for CSPUL was proposed by Scheithauer (1991), which modified the column generation technique proposed by Gilmore and Gomory (1963) to consider leftovers. This model considers fictitious items that can be cut in the future, being the same as leftovers with special utility values. Gradisar et al. (1997) proposed a model for the CSPUL with two objective functions: minimizing the number of items whose demand are not satisfied and minimizing the total trim loss. This study was applied in a clothing industry, with the possibility of unused pieces of clothes being returned to stock. Cherri et al. (2009) adapted classic heuristic procedures, as greedy and residual procedures, to solve the one-dimensional CSPUL. Abuabara and Morabito (2009) studied the one-dimensional CSPUL in a Brazilian aeronautical company. The authors proposed a mathematical model that is a mixed integer problem (MIP) adaptation of Gradisar et al. (1997) model. Cui and Yang (2010) extended the Scheithauer (1991) model to include upper bounds on the number of leftovers, and the number of objects in stock is limited. Cherri et al. (2013) proposed a heuristic procedure based on Cherri et al. (2009), considering that retails had priority in the cutting process compared to standard objects. Arenales et al. (2015) proposed a mathematical model to solve the CSPUL with the objective of minimizing the waste of material. By this model, retails have length and limited quantities previously defined and can be generated for stock to reduce the waste. The problem was solved using the column generation technique and optimal continuous solutions were presented. Tomat and Gradisar (2017) analyzed the possibility of generating leftovers considering consecutive demands. The method proposed aims to find the best quantities of leftovers for the stock and the ideal length for the new leftovers generated.

Another variation of CSP is the Multiperiod Cutting Stock Problem (MCSP), that considers required demand of items in different periods of a finite planning horizon. The production of these items can be anticipated or not, depending on the storage costs of the items. Also, the objects not used in a period are stored for the next period.

Poldi and Arenales (2010) proposed a linear integer optimization model that considers the storage costs of objects and items in the objective function. The simplex method with column generation was used to solve the linear relaxation, and two approaches were developed to round the solution. Poldi and de Araujo (2016) extended the research presented in Poldi and Arenales (2010) generalizing the mathematical model proposed by Gilmore and Gomory (1963) and the arc flow model (Valério de Carvalho, 1999, 2002) to solve the MCSP.

The integrations of the MCSP and CSPUL were not found in the literature and will be studied in this paper. Thus, decisions about generating retails or anticipating the production of items must be taken. A mathematical model to represent this problem was proposed. A simplex method with column generation was used to solve the linear relaxation of the model and a heuristic procedure was used to find the integer solution. Preliminary tests were performed and provided good solutions for the problem. By convenience, we used MCSPUL to refer to multiperiod cutting stock problem with usable leftovers.

Recently, Melega et al. (2018) proposed a classification of the literature related to the integration between the lotsizing and cutting stock problem. A deterministic mathematical model, that considers multiple dimensions of integration and comprises several aspects found in practice was proposed. This model is used as a framework to classify the current literature in this field. The main classification of the literature is organized around two types of integration. In a planning horizon which consists of multiple periods, the inventory provides a link between the periods. This integration across time periods constitutes the first type of integration. The proposed model also considers the production of different types of items at three different levels: objects are fabricated or purchased (Level 1) and next cut into pieces (Level 2) which are then assembled into final products (Level 3).

The present paper is organized as follows: in Section 2 the MCSPUL is defined together with the proposed mathematical model and the rounding heuristic. In Section 3 computational tests varying the maximum number allowed for retails are presented. With these tests, we aim to show that usable leftovers approach can improve the solutions in multiperiod cutting stock problems. Conclusions are presented in Section 4.

2 Problem definition and solution method

To understand the MCSPUL approach, consider a finite planning horizon, divided into T periods. These periods can be, for example, work shifts, days or weeks. Also, consider that there are S types of standard objects in stock in a quantity e_{st} for each period t of the planning horizon, $s = 1, ..., S$, $t = 1, ..., T$. Besides, R types of retails can be generated in period t, being available in stock for the period t+1.

In the MCSPUL there are three categories of objects that can be cut: 1) standard objects, with length L_s; 2) retails, with length L_r; and 3) standard objects generating retails, with length L_s-L_r. Since each type of retail can be generated from all types of standard objects, the maximum number of different types of objects available for cutting is $S+R+S*R$.

For each period t, M items type i must be cut to attend the demand d_{it}, $i = 1, ..., M$, $t = 1, ..., T$. The production of these items can be anticipated for certain period t, at the cost of storing the item until the period that the item was demanded. This cost represents the space occupied by that item in the stock. On the other hand, the anticipation can allow better combinations of items, which decreases the waste of material.

The MCSPUL consists of producing the demanded items by cutting the objects and retails available in stock in each period of the planning horizon, minimizing the waste of material and the cost of storing objects, retails and items. By the definition of the MCSPUL, there is a decision to be made: to generate retails or anticipate the demand.

The mathematical model proposed to solve the MCSPUL was based on the model presented in Arenales *et al.* (2015). The following data were used in the model:

- S: number of types of standard objects. We denote object type s, $s = 1, …, S$;
- R: number of types of retails in stock. We denote leftover type k, $k =1, ..., R$;
- T: number of periods of time,
- m: number of types of demanded items;
- e_{st}: number of objects type s available in stock in period t, $s = 1, …, S$; $t = 1, ..., T$;
- er_{kt}: number of retails type k available in stock in period t, $k = 1, ..., R$; $t = 1, ..., T$;
- d_{it}: demand for item type i in period t, $i = 1, …, m$, $t = 1, ..., T$;
- J_{st}: set of cutting patterns for object type s in period t, $s = 1, ..., S$; $t = 1, ..., T$;
- $J_{st}(k)$: set of cutting patterns for object type s generating a retail type k in period t, $k = 1, …, R$, $s = 1, …, S$, $t = 1, ..., T$;
- Jr_{kt}: set of cutting patterns for retail type k in period t, $k = 1, ..., R$, $t = 1, ..., T$;
- a_{ijst}: number of items type i in cutting pattern j for object type s in period t, $i = 1, ..., m$, $j \in J_{st}$, $s = 1, ..., S$, $t = 1, ..., T$;
- a_{ijskt}: number of items type i in cutting pattern j for object type s generating a retail type k in period t, $i = 1, ..., m$, $j \in J_{st}(k)$, $s = 1, ..., S$, $k = 1, ..., R$, $t = 1, ...,T$;
- ar_{ijkt}: number of items type i in cutting pattern j for retail type k in period t, $i = 1, ..., m$, $j \in Jr_{kt}$, $s = 1, ..., S$, $t = 1, ..., T$;
- U_{kt}: maximum number allowed for retails type k in period t, $k = 1, ..., R$, $t = 1, ...,T$;

Parameters:

- c_{jst}: waste of cutting object type s according to pattern j in period t, $j \in J_{st}$, $s = 1, ..., S$, $t = 1, ..., T$;
- c_{jskt}: waste of cutting object type s according to pattern j when generating a retail type k in period t, $j \in J_{st}(k)$, $s = 1, ..., S$, $k = 1, ..., R$, $t = 1, ...,T$;
- cr_{jkt}: waste of cutting retail type k according to pattern j in period t, $j \in Jr_{kt}$, $k = 1, ..., R$; $t = 1, ..., T$;
- py_{it}: cost to stock the item type i at the end of the period t, $i = 1, ..., m$, $t = 1, ..., T$;

- pw_{st}: cost to stock the object type s at the end of the period t, $s = 1, ..., S$; $t = 1, ..., T$;
- pz_{kt}: cost to stock the retail type k at the end of the period t, $k = 1, ..., R$, $t = 1, ..., T$;

Variables:

- x_{jst}: number of objects type s cut according to pattern j in period t, $s = 1, ..., S$, $j \in J_{st}$, $t = 1, ..., T$;
- x_{jskt}: number of objects type s cut according to pattern j and generating a retail type k in period t, $s = 1, ..., S$, $j \in J_{st}(k)$, $k = 1, ..., R$, $t = 1, ..., T$;
- xr_{jkt}: number of retail type k cut according to pattern j in period t, $s = 1, ..., S$, $j \in Jr_{kt}$, $t = 1, ..., T$;
- y_{it}: number of items type i anticipated for the period t, $i = 1, ..., m$, $t = 1, ..., T$;
- w_{st}: number of objects type s not used in period t, and available in period $t + 1$, $s = 1, ..., S$; $t = 1, ..., T$;
- z_{kt}: number of retails type k not used in period t, and available in period $t + 1$, $k = 1, ..., R$, $t = 1, ..., T$;

Mathematical Model:

Minimizing

$$\sum_{t=1}^{T}\left(\sum_{s=1}^{S}\sum_{j\in J_{st}} c_{jst}x_{jst} + \sum_{s=1}^{S}\sum_{k=1}^{R}\sum_{j\in J_{st}(k)} c_{jskt}x_{jskt} + \sum_{k=1}^{R}\sum_{j\in Jr_{kt}} cr_{jkt}xr_{jkt} + \sum_{i=1}^{m} py_{it}y_{it} + \sum_{s=1}^{S} pw_{st}w_{st} + \sum_{k=1}^{R} pz_{kt}z_{kt}\right) \quad (1)$$

Subject to

$$\sum_{s=1}^{S}\sum_{j\in J_{st}} a_{ijst}x_{jst} + \sum_{s=1}^{S}\sum_{k=1}^{R}\sum_{j\in J_{st}(k)} a_{ijskt}x_{jskt} + \sum_{k=1}^{R}\sum_{j\in Jr_{kt}} ar_{ijkt}xr_{jkt} + y_{i,t-1} - y_{it} = d_{it},\ i=1,..., m, t=1,...,T \quad (2)$$

$$\sum_{j\in J_{st}} x_{jst} + \sum_{k=1}^{R}\sum_{j\in J_{st}(k)} x_{jskt} - w_{s,t-1} + w_{st} \le e_{st},\ s = 1, ..., S, t = 1,...,T \quad (3)$$

$$\sum_{j\in Jr_{kt}} xr_{jkt} - z_{k,t-1} + z_{kt} \le er_{kt},\ k = 1, ..., R, t = 1,...,T \quad (4)$$

$$\sum_{s=1}^{S}\sum_{j\in J_{st}(k)} x_{jskt} - \sum_{j\in Jr_{kt}} xr_{jkt} \le U_{kt} - er_{kt},\ k=1,..., R, t=1,...,T \quad (5)$$

$$x_{jst} \ge 0,\ w_{st} \ge 0,\ s = 1, ..., S, t = 1, ..., T,\ j \in J_{st} \text{ and integer;} \quad (6)$$

$$x_{jskt} \ge 0,\ s = 1, ..., S, k = 1, ..., R, t = 1, ..., T,\ j \in J_{st}(k) \text{ and integer;}$$

$$xr_{jkt} \ge 0,\ z_{kt} \ge 0,\ k = 1, ..., R, t = 1, ..., T,\ j \in Jr_{kt} \text{ and integer;}$$

$$y_{it} \ge 0,\ i = 1, ..., m, t = 1, ..., T \text{ and integer.}$$

In the model (1)-(6), the objective function (1) minimizes the total waste of cutting all objects (standard and retails) in all periods and the cost of storing items and objects. Constraint (2) ensures that the demand is met. Constraints (3) and (4) ensures that the quantity of standard objects and retails used during the cutting process in each period does not exceed availability. Constraint (5) limits the quantity of each type of retail that can be generated during the cutting process in each period. And (6) is the integrality and non-negativity constraint of the variables.

This model was solved using the simplex method with column generation (Gilmore and Gomory (1963)), which is an efficient strategy to solve linear problems. Because of the integrality conditions (constraint (6)) and the exponential number of variables, it is difficult to find the optimal solution in the model (1) – (6). Therefore, these conditions are relaxed and continuous solutions for the MCSPUL can be found.

In real-world applications, it is impossible consider continuous solutions for cutting stock problems. Thus, heuristic procedures were used to find an integer solution. This heuristic was detailed in Wäscher and Gau (1996) and consists in:

- Finding an optimal solution for the model (1)-(6) with the integrality conditions relaxed using the simplex method with column generation;
- Use all the cutting patterns generated during the previous step to solve the model (1)-(6) considering the integrality conditions. These cutting patterns include that ones used to obtain the optimal solution together with those in the optimal solution.

3 Computational tests

To evaluate the performance of the proposed approach, the model (1)-(6) was coded in C++ programming language using CPLEX software, version 12.7. The computational tests were run on a computer Intel Core i5, 1.6 GHz, 6 GB RAM.

The tests considered a planning horizon with T=3 periods and m=10 types of demanded items. The length of items (l_i) was randomly generated in the interval [100, 350]. The upper bound for this interval is the half of the length of the larger retail that can be generated (L_k=700). The demand (d_{it}) was randomly generated in the interval [50, 400]. The storage cost of item i (py_{it}) was pyl_i, with py = 0.01 and 0.05.

There was one type of standard object (S=1) with length L=1500 in stock and availability (e_{st}) was large enough to meet the demands. For these tests, the cost to stock standard objects (pw) was considered 0. Three types of retails were considered with lengths (L_k) 500, 600 and 700. The availabilities of all types of retails were e_{kt}=0, for the initial period. The maximum quantity of each type of retails in each period varied as U_{kt}=0, 1and 3. The storage cost of retail k (pz_{kt}) was pzL_k, with pz = 0.01 and 0.05. The average computation time for each instance was approximately 90 seconds.

Table 1 shows the average cost of 3 instances randomly generated, for different combinations of items storage cost, retails storage cost and maximum number allowed for retails (U) using the proposed method. In all tests, the stock in the first period is composed only of standard objects. From the second period, the retails generated in the first period are stored and can be cut.

Table 1: Average cost for the integer solution.

	pz=0			*pz=0.01*			*pz=0.05*		
	U = 0	*U = 1*	*U = 3*	*U = 0*	*U = 1*	*U = 3*	*U = 0*	*U = 1*	*U = 3*
py=0	1704.00	637.33	604.00	1704.00	648.00	620.00	1704.00	690.67	684.00
py=0.01	1729.36	934.59	896.42	1729.36	943.20	911.12	1729.36	1007.47	953.49
py=0.05	1757.97	1072.50	1022.23	1757.97	1098.03	1058.25	1757.97	1359.03	1180.10

In Table 1, for each value of py, pz and U, we have the average cost for the integer solution obtained from the heuristic procedure described in section 2. It is possible to observe that the total costs decrease for larger values of U and considering the variations of the storage costs. This situation is expected because the diversity of objects in stock increases with the generation of retails. Together with the possibility of anticipating the demand of items, generation of retails allows that good cutting patterns be generated. This justifies the higher costs for $U = 0$.

Higher values for the storage costs of items and retails (*py* and *pz*) reflects in a substantial increase of the total cost and loss of material in the integer solutions. This occurs because these high storage costs inhibit anticipation of items, decreasing the possibility of combining items in the objects.

4 Conclusions

In this paper, we propose a mathematical model to represent the multiperiod cutting stock problem with usable leftovers. This model considers a planning horizon in which the demand of items is known, and it is possible to anticipate or not their production. Also, new retails can be generated in quantities and length previously defined, being available in stock to be cut in the next periods. Although the retails sizes were fixed for the computational tests in this paper, a different approach could be used, by establishing a lower and an upper bound for the size of the new retail generated. In this approach, the waste of the cutting patterns would be added to the retail length.

To verify the performance of the mathematical model to MCSPUL, we present integer solutions with some generated instances. The results obtained showed that generate retails improves the results for the integer solutions, even for high storage costs. This occurs because the savings provided by the generation of retails in terms of cutting costs compensates the cost of storing these retails.

The next step of this research is the implementation of other heuristics procedures, as residual heuristics. Also, capacity constraints will be included in the mathematical model for the MCSPUL.

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