On minimizing saw cycles and raw material costs for the Cutting Stock Problem with variable processing times depending on cutting pattern setting

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Abstract

Machine utilization and productivity are concerns inherent in production processes in general and, particularly, in materials cutting. Some industrial processes allow simultaneous objects cutting, arising the trade-off between raw material waste and machine time cost. In this work, we propose a mathematical model that combines the standard objective of minimizing the number of rolls used with machine time fixed costs and stock costs; in a production environment in which processing times of cutting patterns are dependent of the number of items on it. A solution method is proposed using column generation. Computational results are presented for a real industrial instance.

Keywords: cutting and packing; cutting stock problem; saw cycles; machine utilization; column generation.

1 Introduction

Cutting and packing problems seek the assignment of small items on big objects respecting geometric constraints[7]. It is one of the most famous set of combinatorial optimization problems [6]. Particularly, Cutting Stock Problems (CSP) are related to a set of items with big demands, having a size homogeneity [20].

Once those problems are related with real industrial situations, there is a motivation for studying objectives and constraints that capture technological and entrepreneurial features of production, market and distribution [16].

This study, in particular, looks at cutting processes where several objects can be cut simultaneously, where productivity and machine utilization is affected by the decision of so called saw cycles

[17, 21]. This condition is observed in furniture [19], mechanic [15], textile [4, 3] and vulcanization [18] industries. Using full capacity of the machine is a common solution on industry [17], but it is equivalent to optimal only on high demand conditions [21]. Otherwise, productivity and raw material use can be conflicting [17], depending on its relative costs [15].

Approaches for the minimization of saw cycles problem consider equal processing times for different cutting patterns. This work takes into accounting different processing times for different cutting patterns, according to its configuration. Its objective is, therefore, approaching the problem of minimizing saw cycles, considering fix costs associated to machine usage and variable costs associated to stocking items; on production environments in which processing times depend on the cutting pattern setting.

2 Literature

Good machine utilization is a concern presented on literature for some decades, with setup minimization problems [1, 14] and minimization of the number of different cutting patterns [5, 22, 12, 2, 11]. However, the explicit enunciation of the saw cycles problem dates back to the 90ths [21]. Cutting machines, in particular, are used to become bottlenecks in high demand contexts [17], generating a trade-off between better utilization and raw material waste.

A saw cycle is defined as the processing time that includes setups and executing times of a set of cutting patterns simultaneously, condition that gains more relevance in high machine costs environments [21]. The problem is intrinsically related to cutting and packing problems, however, it can be found in lot production systems [9].

Mathematical formulations were proposed on literature, either by the representation of the amount of saw cycles of a given cutting pattern in a decision variable [19, 21, 10]; or by the discretization of the amount of objects cut simultaneously for a given cutting pattern in an index [15, 4, 8].

Since it is a hard solution problem and, therefore, with its optimization constrained to small instances, literature presents heuristic and metaheuristic approaches for the saw cycle minimization problem [17, 21, 8, 13].

However, most of approaches consider equal processing times for different cutting patterns, independent of the amount of items on it. This feature is true for some processes, as paper cutting; but it is not real for other cases, as the own saw, tool that gives the name to this problem.

3 Mathematical model

This section presents the proposed mathematical model for the Cutting Stock Problem integrated to the minimization of saw cycles, considering variable processing times. Therefore, cutting patterns with more items on them take more time to be cut.

Let I ($i \in I$, $i = 1$, ..., NI) be a set of unidimensional items to be cut, with length l_i and demand b_i. Stocks are allowed until e_i units beyond b_i at an unitary cost ce_i. Let P ($p \in P$, $p = 1$, ..., NP) be a set of cutting patterns of an object with length L with unlimited availability, at an unitary cost θ, with a_{ip} items i. Each cutting pattern contains γ_p items, which means, γ_p cuts are necessary to execute it. This parameter is proportional, therefore, to the cut machine occupancy.

The objective of this model is to meet the demand of I, without exceeding the maximum stock amount permitted, minimizing total cost: raw material use, machine time utilization and stocks generated. Let X_{pk} be a set of integer decision variables that represent the amount of patterns p cut with k objects simultaneously ($k \in K$, $k = 1$, ..., NK). The maximum amount NK of objects is limited by the machine physical capacity. The model is presented in (1)-(4).

$$\min \quad \sum_{p=1}^{NP}\sum_{k=1}^{NK} X_{pk}\left(\theta k + \gamma_p\right) \quad + \quad \sum_{i=1}^{NI} ce_i \left(\left(\sum_{p=1}^{NP}\sum_{k=1}^{NK} a_{ip}X_{pk}k\right) - b_i\right) \tag{1}$$

$$subject\ \ to: \quad \sum_{p=1}^{NP}\sum_{k=1}^{NK} a_{ip}X_{pk}k \geq b_i, \qquad i = 1, ..., NI, \tag{2}$$

$$\sum_{p=1}^{NP}\sum_{k=1}^{NK} a_{ip}X_{pk}k \leq b_i + e_i, \qquad i = 1, ..., NI, \tag{3}$$

$$X_{pk} \in \mathbb{Z}^{+}, \qquad p = 1, ..., NP, \ \ k = 1, ..., NK. \tag{4}$$

The objective function (1) minimizes the total cost: (i) raw material, (ii) machine time fixed cost and (iii) stocks generated. Constraints (2) assure total demand is met for all items i, while (3) limits this amount produced to the maximum amount allowed. Finally, (4) defines the decision variable X_{pk} domain.

4 Solution method

Since the potential number of cutting patterns is huge in a real problem, a column generation method is proposed. The subproblem associated to the cutting pattern generation must minimize reduced costs associated to variables X_{pk}, until the best one is still positive.

Let π_i^1 be the dual values associated to constraints (1), π_i^2 the dual values associated to constraints (2), α_i the amount of items i in the pattern involved by the generated column and γ_p time to process the pattern. The objective associated to the problem is:

$$\overline{c_{pk}} = c_{pk} - \pi^T \mathbf{a_{pk}} \tag{5}$$

$$\overline{c_{pk}} = \theta \times k + \gamma_p + k\left(\sum_{i=1}^{NI} \alpha_i \left(ce_i - \pi_i^1 - \pi_i^2\right)\right) \tag{6}$$

In the sub-problem associated to the column generation of (7)-(15), the parameter M is a value big enough to assure the disjunction of constraint (9). The parameter st is the cost of setup of a saw cycle, while γ^{un} is the cost of each cut of a set of objects simultaneously on a cycle. Considering the decision variable FO_k as the value of (6) for each amount k of objects cut simultaneously; FO_{min} the smaller value founded for all the possible values of k; and the binary variable W_k, which takes value 1 when the cutting pattern has its smaller value of FO_k with k, and 0 otherwise.

$$
\begin{aligned}
\min \quad & FO_{min} && && (7)\\
subject\ \ to: \quad & FO_k = \theta k + \gamma + k \sum_{i=1}^{NI} \alpha_i \left(ce_i - \pi_i^1 - \pi_i^2 \right), && k = 1, ..., NK, && (8)\\
& FO_{min} \geq FO_k - (1 - W_k) M, && k = 1, ..., NK, && (9)\\
& \sum_{k=1}^{NK} W_k = 1 && && (10)\\
& \sum_{i=1}^{NI} \alpha_i l_i \leq L && && (11)\\
& \gamma = st + \sum_{i=1}^{NI} \alpha_i \gamma^{un} && && (12)\\
& W_k \in \{0, 1\}, FO_k \in \mathbb{R}, && k = 1, ..., NK, && (13)\\
& \alpha_i, \in \mathbb{Z}^+, && i = 1, ..., NI, && (14)\\
& FO_{min}, \gamma \in \mathbb{R}. && && (15)
\end{aligned}
$$

It is important to notice that, in this subproblem, each cutting pattern has a set of possible reduced costs, depending on the amount k of objects cut simultaneously. The column generation procedure ends only when the smaller reduced cost of a cutting pattern is positive. This is reflected on (7), in which the smaller relative cost among all possible k is minimized. This set is expressed on (8). Constraints (9) assure variable FO_{min} represents, indeed, the smaller value of FO_k, choosing one, and only one through (10). In (11) the limits of the object is respected. The fixed cost associated to machine time (γ) is defined in (12), being the sum of the setup time and the unitary cost for each cut necessary to perform it. In (13)-(15) the domain of decision variables is defined.

Since FO_k is a linear function of k, FO_{min} will always be equal either to FO_1 or FO_{NK} in this problem structure. Also, FO_{min} is certainly positive when $FO_{min} = FO_1$. Hence, a simplified version of the subproblem is presented in (16)-(20), considering the minimization of the relative cost when $k = NK$. The objective function and constraints remain analogous to (7)-(15).

$$
\begin{aligned}
\min \quad & \theta \cdot NK + \gamma + NK \sum_{i=1}^{NI} \alpha_i \left(ce_i - \pi_i^1 - \pi_i^2 \right) && && (16)\\
subject\ \ to: \quad & \sum_{i=1}^{NI} \alpha_i l_i \leq L && && (17)\\
& \gamma = st + \sum_{i=1}^{NI} \alpha_i \gamma^{un} && && (18)\\
& \alpha_i, \in \mathbb{Z}^+, && i = 1, ..., NI, && (19)\\
& \gamma \in \mathbb{R}^+. && && (20)
\end{aligned}
$$

5 Computational results

This section present computational results obtained with the solution method proposed on a real instance, based on a metal-mechanic factory that manufacture components for rural tractors. The results are compared with the approach of shop-floor. Experiments were run on a 8 Gb RAM computer with i7 processor, using CPLEX as solver and OPL language for programming.

The steel pipes cut have a 5/4" diameter and they follow for a machining process then. This objects have 6.000 mm length and up to 7 units can be cut in together on the same cycle. This factory has products in its portfolio that are classified in "make-to-order", "make-to-stock with high demand" and "make-to-stock with low demand". The first ones have forbidden stocks ($e_i = 0$), while the other two have a certain flexibility, bigger on the higher demands, where the stock costs are also lower.

Demand and lengths of the 25 items to be cut were collected directly with project area, as well as stock costs with the financial area. Each unit of the objects has a cost of \$ 122.78. The machine fixed cost was estimated using outsourcing services fees (\$ 385.20 per hour), resulting on an unitary cost of cutting of \$ 2.14 (considering a 20 seconds time per cut). Setup time was estimated 10 times this value (2.5 minutes), according to data collected. Therefore, st = \$ 21.40.

Data was submitted to model (1)-(4), using the column generation method in (7)-(15). The columns were generated while relative costs of negative and, finally, after this process, columns generated were used to execute the integer version of the model, limiting processing time to 10 minutes.

The solution obtained presented a 666 cycles plan, using 4,662 objects (98.46% of full capacity of the machine, with 7 objects in all cycles). In this solution, 3 cycles had only 1 object, 3 cycles had 2 objects, 4 cycles had 4 objects, 5 cycles had 5 objects, 17 cycles had 6 objects and 634 cycles were complete (7 objects). It represents a total of 16,664 time unities (11.6 shifts). For a 68,840 units demand among all 25 items, only 1.272 stock units were produced. Not considering them, raw material usage was 96.60%, while with them it is 98.21%. Total solution cost is \$ 597,519.00, which represents a gap of 0.2% over the relaxed solution of the problem (lower bound).

To establish a parallel with the factory *modus operandi*, it was made an interview with the area workers and their reasoning was simulated computationally. According to them, the logic step for the cycles produced follows: (i) items are ordered from the greater demands to the lower ones; (ii), for each items, an homogeneous pattern is cut, eventually completed with the biggest one not produced; (iii) all patterns are run in full cycles (7 objects) until demand is met or overcome. Adjusts are made on shop-floor, nevertheless the comparison in reasonable once the discipline to accomplish this steps is equally unlikely and subjected to failures.

Using this steps, the solution presented a 659 cycles plan, using 4,613 objects (100% of full capacity, by definition of the procedure). It represents a total of 16,610 time units for the problem (11.5 shifts). For the same 68,840 units demand, 1,300 units of surplus items were produced, 430 of those being items "make-to-order". Not considering stock items, the raw material use is 97.63%. Considering them, this figure grows to 99.41%. The false efficiency impression on shop-floor comes from this low amount of wastes, however most of the losses are in the shape of stock items. Total cost, in this case, is \$ 615,516.08.

Besides costs gains, intangible ones can be listed: less material movements, easier identification and stock utilization and less invested capital in raw materials (with doubts in its usage). Besides, a modest computational effort (10 minutes) was used, being possibly shorter that time spent by a human being organizing the procedure described.

6 Conclusions and perspectives

The objective of this paper was to explore the integration of the unidimensional Cutting Stock Problem with the minimization of saw cycles. An integer model was proposed, using column generation as solution method. It has been submitted to an industrial instance, in order to prove potential gains on this integration for real cases. This study contributed to literature once it considers dependent processing times, according to the number of items on a cutting pattern, which is suitable for many industrial processes. Besides, it considers maximum limits for the stock generation and costs associated to them.

Future works can explore random generated instances in order to verify the impact of parameters on the difficulty of the problem and solution behavior.

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