Unconstrained cutting stock problem with regular and irregular pieces

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Abstract

In this paper we present a study and solution method for the unconstrained cutting stock problem involving rectangular and irregular pieces, whose format is type L. The proposed strategy combines L-shaped and rectangular items in plates in order to minimize the waste. This is a NP-hard combinatorial optimization problem and appears in many industrial processes. To solve this problem, we use non-guillotine cutting patterns which are combinations of guillotine and simple non-guillotine cuts. Modifications are proposed in the AND/OR Graph approach, that is a good strategy proposed in the literature to solve two-dimensional cutting stock problems. To analyze the solutions, instances from the literature and a random generated instances were used. The computational results show a good performance of the developed strategy.

Keywords: unconstrained cutting stock problems, AND/OR Graph approach, combinatorial optimization.

1. Introduction

The cutting stock problems (CSP) are central problems for production planning in several practical situations and have been extensively studied in the literature. Basically, the CSP consist of determining the best way to cut units of material (objects) available in stock in order to produce a set of pieces (items) in quantities and dimensions specified, with the objective to optimize an objective function.

In the literature, there are several studies that consider cutting stock problems involving only rectangular items (two-dimensional cutting stock problem). This is a classical cutting stock problem and the solution method was proposed by Gilmore and Gomory (1965). Due practical application in industries and the challenge they offer to academia, an increasing number of researchers have studied these problems. Despite their apparent simplicity, these problem generally are computationally difficult to solve. Studies and solution methods for these problems can be found in Golden (1976), Hinxman (1980), Dyckhoff and Waescher (1990), Dowsland and Dowsland (1992), Sweeney and Paternoster (1992), Dyckhoff and Finke (1992), Morabito *et al.* (1992), Arenales and Morabito (1995), Bischoff and Waescher (1995), Dyckhoff *et al.* (1997), Arenales *et al.* (1999), Valdés *et al.* (2002), Cintra *et al.* (2007), among others.

The problems with L-shaped items present geometry handling complexity and, although they arise in practical situations, were not found many articles in the literature that consider L-shaped cutting stock problems. Applications involving this problem can be found in the foam mattress cutting, textile industry, manufactures furniture, pallet loading, placement in Integrated Circuit (IC) Layout, among others.

Sun and Liu (1992) proposed a heuristic procedure to minimize the used area for floorplans with L-shaped regions. They extended the concept of cut line and defined eight types of cut lines which were used to decompose floorplans with L-shaped regions. Xu et al. (1998) presented an approach extending the Sequence Pair (SP) approach for rectangular block placement with arbitrarily sized and shaped rectilinear blocks. The properties of L-shaped blocks were examined first, and then arbitrarily shaped rectilinear blocks were decomposed into a set of L-shaped blocks.

Kang *et al.* (1998) proposed a method to represent arbitrary shaped rectilinear blocks based on the Sequence Pair (SP) structure. Every rectilinear piece was divided into a set of rectangular sub-pieces and each sub-piece was individually handled in the sequence pair as a unit piece. They proved that always exists a feasible SP of convex rectilinear pieces and vice versa. To solve the problem, they defined three necessary and sufficient operations on the SP, each of them incrementally changes a feasible SP and the resulting SP remains feasible.

Pang *et al.* (2001) present an algorithm to produce floor plans for rectilinear shaped modules based on the O-tree representation. Although the packing has lower complexity time, the overall approach is less flexible and more restricted. Each rectilinear shaped piece is partitioned into a set of sub-L-shaped-blocks and all the pieces must be compacted.

Ma *et al*. (2001) proposed an algorithm to handle the abutment constraint with non-slicing structure. The method considers L-shaped and T-shaped bock partitioning into a collection of rectangles blocks with additional abutment constraint. The rotation and reflection of the block are allowed. According to the authors, the algorithm is promising.

Roberts (1984) proposes heuristic procedure to solve a real cutting-stock problem in a furniture manufacture. In this problem, rectangular and L-shaped items should be combined to be cut minimizing the waste, minimizing the number of offcuts produced and optimizing the saw utilization. To solve this problem, the author reduced it to a series of one-dimensional problems, since the plate is cut in band with dimension very close of one side of the items and thus only one L-shaped or rectangular item is allocated in the width of each plate band. With this strategy, the solutions obtained were satisfactory, however, this strategy is specific to solve the problems of the company.

Lins *et al.* (2003) present an L-approach for packing (l, w) rectangles into larger rectangular and L-shaped pieces. This problem has applications for non-guillotine cutting and pallet/container loading. To solve this problem, the authors propose a recursive partition of a rectangular or L-shaped piece into two pieces, each of which is again a rectangular or L-shaped. According to the authors, the approach was able to find the optimal solution for all the tested instances

To solve the two-dimensional cutting stock problem with rectangular and L-shaped items, we propose modifications in the AND/OR Graph approach (Morabito, 1989) that is a flexible strategy to solve problems with two dimensions. This problem presents a great complexity in its solution method, because the geometry of the items is also involved in the problem resolution. When L-shaped items are included in the problem, the difficult to solve it is even greater because the cuts realized in the plate are not guillotined. Some computational experiments were realized with instances from the literature.

The remaining of the paper is organized as follows. In Section 2, we present the two-dimensional cutting stock problem with rectangular and L-shaped items. In the Section 3, we describe the AND/OR Graph approach that was modified to solve the problem. Section 4 presents some computational experiments and analyze of the obtained solutions. The Section 5 is dedicated to conclusions and perspectives for future works.

2. The two-dimensional cutting stock problem with rectangular and L-shaped items

The unconstrained cutting stock problem with rectangular and L-shaped items consists of cutting a rectangular plate (L, W), where L is the plate's length and W its width, in order to produce rectangular items with dimensions (ℓ_i, w_i), $i = 1, \ldots, m$, where ℓ_i is the item's length and w_i is the width and L-shaped items with dimensions $(\ell_{i1}, w_{i1}, \ell_{i2}, w_{i2})$, where ℓ_{i1} and ℓ_{i2} are under and upper item's length and w_{i1} and w_{i2} represent the left and right item's width, respectively. Since the problem is unconstrained, there is no quantity of items to be produced. The problem to be solved is to determine a cutting pattern that minimizes the waste of the plate without overlapped items.

It is possible to limit the quantity of items type to be produced. In this case we say that the problem is constrained and, due the constraint of items to be produced, the problem is more difficult to be solved.

There are two forms of items allocation when the problem involves L-shaped items. The figure 1 and 2 show these possibilities.

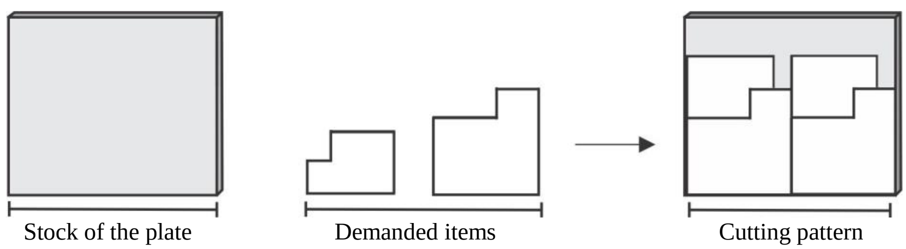


Figure 1: Cutting stock problem with L-shaped items – fitted items.

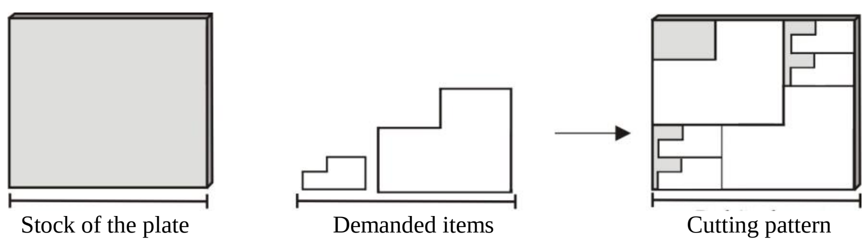


Figure 2: Cutting stock problem with L-shaped items – items not fitted.

Solutions like in the Figure 1 can generate better solutions to the CSP, however, there is a greater difficult to cut the items. This situation can be impracticable in some practical applications. Allocation as in Figure 2 facilitates the cuts of the plate, however, generates larger wastes. In this work, we will use allocations of L-shaped items as in Figure 2.

Our problem will also combine rectangular and L-shaped items and thus is not possible realize only guillotine cuts in the generation of the items. The cuts must combine guillotine and non-guillotine cuts (Morabito, 1989). The Figure 3 shows an allocation involving these two type of items.

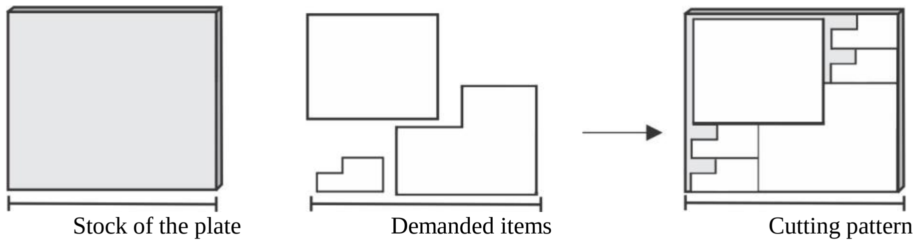


Figure 3: Cutting stock problem with rectangular and L-shaped items.

In the next section, we present a brief description of the AND/OR Graph approach (Morabito, 1989) and the alterations realized to solve our problem.

3. AND/OR Graph approach to solve the CSP with rectangular and L-shaped items

The AND/OR Graph approach to solve two-dimensional CSP was proposed by Morabito (1989). This approach consists of representing the cutting patterns as a complete path in a graph and enumerating it implicitly with the objective to find an optimal solution.

An AND/OR Graph can be defined to represent all possible cutting patterns of a plate, in that the nodes represent rectangles and arcs represent the cuts. An AND-arc (cut) establishes a relation between a node N (rectangle or L-shaped) with two others nodes N_1 and N_2. These nodes are called N successors and N ancestor of N_1 and N_2. The Figure 4 presents two OR-arcs emerging of the root node, generating two possibilities of cutting patterns (vertical and horizontal).

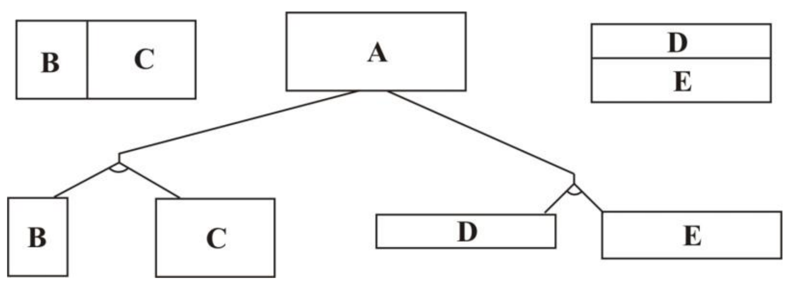


Figure 4: AND/OR Graph ramification.

The generation of the cutting patterns verify all possibility of cuts (OR-arcs) and one of these is to reproduce the own rectangle (called 0-cut). In this case, no other cut will be realized and the ramification of the graph is finished. Without loss of generality, are associated with the final nodes one or more identical items (for example, the first arc on the left emerging of the root in the Figure 5). The initial node is represented by the plate (L, W) and the final nodes are generated by the 0-cut.

The cuts can be restricted to a finite set, called "discretization set" that is formed by the nonnegative linear combinations of the items size (Herz (1972) and Morabito e Arenales (1996)).

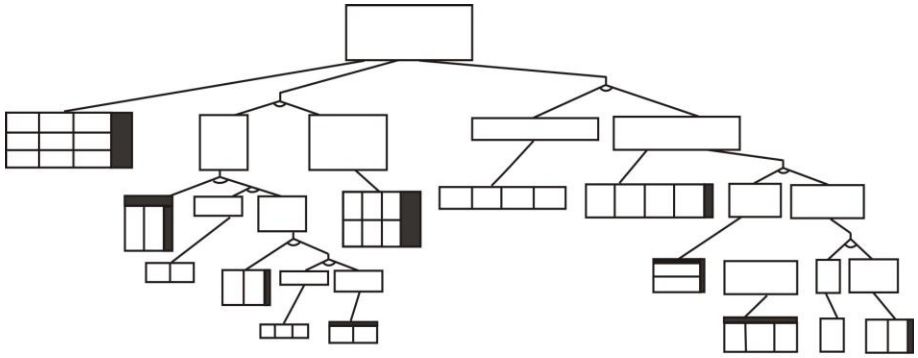
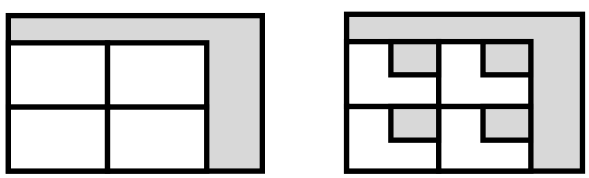


Figure 5: AND/OR Graph representing cutting patterns.

The Figure 5 shows three OR-arcs emerging from root node. They indicate different alternatives to obtain a cutting pattern. Following a sequence of AND-arcs (cuts), a cutting pattern is well defined. This sequence is a "complete path" and each cutting pattern has an associated complete path.

A same node can belong to different sequences, i.e., rectangles with the same sizes can be obtained by different cut sequences, that characterize a cycle. For computational simplicity, we doubled these nodes (that is, different nodes can represent identical rectangles obtained by different cuts) and we use the tree structure that is represented by a connected graph without cycles. The utility value of a cutting pattern is the sum of the final nodes utility value that are associated to a complete path. These utility values are implicitly enumerated and used as bound for others ramifications. This strategy can avoid not promising paths by discarding some node expansions and without losing the optimal solution.

To obtain a lower bound, we use a homogeneous solution that is a trivial solution for a node. In a homogeneous solution, only identical items are cut from the plate. In the problem with rectangular and L-shaped items, the homogeneous solutions were calculated as in the Figure 6.



(a) (b)

Figure 6: Homogeneous solution obtained by rectangular items (a) and L-shaped items (b).

The upper bound is obtained by the minimum between the area of the plate and the area of the demanded items allocated in the plate.

To solve the proposed problem, the cuts are generated according to the plate to be cut. For rectangular plates, rectangular plates/items are obtained by guillotined cuts and also by a cut called "step cut", in that the cut in a rectangular plate generates a new rectangle and a L-shaped plate/item. For L-shaped plates, the cuts realized generate two new rectangles or a new rectangle and an L-shaped plate/item. The Figure 7 shows these cut types.

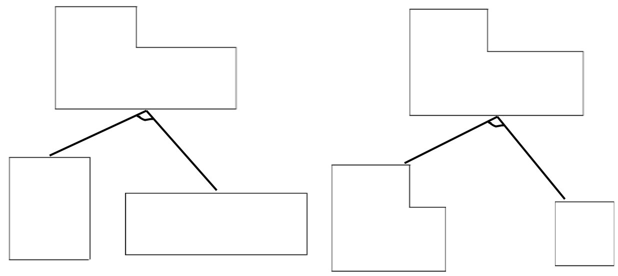


Figure 7: Cut in an intermediated L-shaped plate.

An important modification realized in the AND/OR Graph approach was in the discretization set to generate L-shaped plate/item. The discretization set is formed by the linear combination of the items' size (for rectangular items) added of the larger item's length and the larger item's width (for L-shaped items).

In the next section, we present some computational results obtained using the proposed strategy.

4. Computational experiments

To verify the performance of the procedure described in Section 3, we perform computational experiments using instances from the literature and randomly generated instances. However, the obtained solutions are not compared because our work has the objective of minimize the waste in a limited plate and in the works from the literature the objective is to minimize the total area of the plate (the plates are not limited).

The Table 1 presents the information about the experiments that were performed with the proposed strategy. The algorithm for this strategy was implemented in C++ language programming.

Table 1: Instance dates and Solutions

#	Instance	Plate dimension	Total items	Rectangular items	L-shaped items	Waste (%)
1	ami33LT	1200 x 1000	32	30	2	4.80
2	apteLT	7000 x 7000	9	8	1	2.44
3	Nakatake_test1	310 x 310	35	25	10	4.78
		594 x 255				18.96
4	Xu_instance1	5500 x 6000	28	7	21	3.67
		6314 x 5922				14.98
5	L_20_20	250 x 220	20	16	4	8.45
6	L_20_50	250 x 220	20	10	10	4.80
7	L_20_100	250 x 240	20	0	20	3.45

The instances #1, #2, #3 and #4, of the Table 1, are derivate of VLSI (*Very Large Scale Integration*) circuits (Nakatake *et al*., 1996 and Xu *et al.*, 1998), where the objective is to minimize the area plate used. The instances #1 and #2 have originally rectangular, L-shaped and T-shaped items. To solve the problems using our strategy, the T-shaped items were removed from the examples.

In the instance #3, solved by Nakatake *et al.* (1996), the items were allocated allowing the combination of rectangular and L-shaped items (plate with dimensions (594×255)). The waste given by the authors was 13%. As the proposed algorithm in this work does not allow the fitting of L-shape items, the waste was 18.96%.

Tests realized with the instance # 4 in the plate (6314×5922), proposed by Xu *et al.* (1998) showed a waste of 14.98% using the procedure proposed in this work. Combining items like blocks, rectangular or not, Xu *et al.* (1998) obtained a solution with 5.20% of waste.

The instances #5, #6 and #7 were obtained by the generator given by the Professor Miguel Gomes of the *Faculdade de Engenharia da Universidade do Porto*, Portugal.

In this instance generator, it is possible to control the shape (rectangle, L-shaped or T-shaped items), the total number of items (20) and the percentage of each item type (approximately). Only three examples were generated and, in our analyze, presented good solutions for the problem.

5. Conclusions

In this work, we address the two-dimensional cutting stock problems with rectangular and L-shaped items. To solve this problem, we make modifications in the AND/OR Graph approach that is an efficient strategy propose in the literature to solve problems with two dimensions. The computational tests presented satisfactory solutions and were realized with adapted instances from the literature and random generated instances. These solutions were not compared with the literature due the particularity of each problem.

As continuity of this work, we pretend to consider the fit of the L-shaped in the homogeneous solution (Figure 1). Other suggestion of continuity for this work is to consider T-shaped items. In this case will be necessary rethink in the generation of the “discretization set” for the AND/OR Graph approach.

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