The multiperiod cutting stock problem with usable leftover

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Abstract
This paper aims to integrate two variations of the one-dimensional cutting stock problem: the cutting stock problem with usable leftovers and the multiperiod cutting stock problem. Also, it tries to find a solution to the problem resulting from this integration. The cutting stock problem with usable leftovers consists of cutting large objects into smaller items in order to meet a known demand, minimizing the waste and allowing generate retails that will be used to cut future items. In the multiperiod cutting stock problem demands occur in a finite period of time, being possible to cut in advance a stored object to produce items that have known demands in a future period of time. A mathematical model is proposed for the multiperiod cutting stock problem with usable leftover and is being solved using simplex method with column generation. Some preliminary computational results are presented with a simulation where each period is considered individually.

Keywords: multiperiod cutting stock problem; usable leftovers; linear and integer optimization.

1 Introduction

The cutting stock problem (CSP) consists of cutting a set of objects available in stock in order to produce a set of demanded items for clients or to add stock, seeking to optimize an objective function. These problems appear in the production lines of several industries as paper, aluminum, steel, glass and wood, among other. In the sixties several important works relating to CSP were published, highlighting the papers Gilmore and Gomory (1961) and (1963).

A particularity of CSP consists in usable leftover of cutting patterns. This problem was cited by Brown (1971) and is known in the literature as cutting stock problem with usable leftovers (CSPUL). In this problem, a set of demanded items must be produced by cutting either standard objects or retails available in stock. Similar to the CSP, the demands of items and the objects in stock are known. The demands must be met by cutting the available objects and retails, with the possibility to generate limited quantities of predefined retails for stock.

Roodman (1986) considered the use of leftovers in the CSP to solve a one-dimensional problem with several types of objects in stock. After the cutting process, the leftovers generated, since larger than a certain length, were stored as retails to be used in the future. Scheithauer (1991) modified the problem proposed by Gilmore and Gomory (1963) to consider the usable leftovers. The strategy used by this author consists of including extra items (retails) in the problem without any demand to be met. Gradisar *et al.* (1997) presented a mathematical model to solve the CSPUL, however, a heuristic procedure was used to solve the problem. Gradisar *et al* (1999) developed a strategy to solve the CSPUL that combines linear programming and heuristic procedure.

Abuabara and Morabito (2009) used the mathematical model proposed by Gradisar et al. (1997) to solve the CSPUL in a Brazilian company that builds agricultural aircrafts. Cherri et al. (2009) proposed modifications on constructive and residual heuristics of the literature for solving the CSPUL. Cui and Yang (2010) proposed an extension for the model in Scheithauer (1991) that limits the objects in stock by controlling the quantity of retails generated in the cutting patterns. Cherri et al. (2013) modified the heuristics developed in Cherri et al. (2009). In addition to the waste minimization, the usage of stocked retails was prioritized during the cutting process. Cherri et al. (2014) presented in a survey the existing papers that investigate the one-dimensional CSPUL.

Arenales et al. (2015) proposed a mathematical model to solve the CSPUL with the objective of minimizing the waste of material. By this model, retails have length and quantities previously defined and can be generated for stock just to reduce the waste. This model was solved using the column generation technique proposed in Gilmore and Gomory (1963) and optimal continuous solutions were obtained.

Poldi and Arenales (2010) studied another variation of CSP, that is the multiperiod cutting stock problem (MCSP). In this problem, the demand of items is required in different periods of a finite planning horizon, being possible to anticipate or not their production. The objects not used in a period are stored for the next period together with the new acquired objects. Poldi and de Araujo (2016) extended the research presented in Poldi and Arenales (2010) and generalized the mathematical model proposed by Gilmore and Gomory (1963) and the arc flow model (Valério de Carvalho, 1999, 2002) to solve the MCSP. In these models, the objects in stock were considered as both parameters and variables.

The integrations of the CSPUL and MCSP were not found in the literature and will be studied in this paper. Thus, decisions about generating retails or anticipating the production of items must be taken. A mathematical model to represent this problem was proposed. Implementations are being developed and must provide good solutions for the problem. By convenience, we used CSPULM to refer to cutting stock problem with usable leftovers multiperiod.

The remainder of the present paper is organized as follows: in Section 2 the CSPULM together with the proposed mathematical model and the strategy solution are defined. As the proposed model is still being implemented, in Section 3 a strategy to consider the CSPUL simulating periods is presented. In Section 4 some computational tests using an approaching by period are presented. With these tests, we aim to show that our approach using CSPULM is promising. Conclusions are presented in Section 5.

2 Problem definition and mathematical modeling

In a multiperiod problem a finite planning horizon is divided into T periods, $t = 1, \ldots, T$. In each period t of the planning horizon, objects are available in stock and the demand of items has to be met. The multiperiod cutting stock problem with usable leftovers (MCSPUL) consists of cutting a set of standard objects (objects bought on the market) or non-standard objects (retails generated in previous cuts) available in stock in each period of the planning horizon, in order to produce a set of demanded items with the objective of minimize the production costs. In this problem, in each period t, new retails can be generated in quantities and lengths previously defined. Furthermore, as the demand of items is known in the planning horizon, it is possible to anticipate or not their production.

By the definition of the MCSPUL, there is a decision to be made: to generate retails or anticipate the demand. The mathematical model proposed to solve this problem was based in Arenales *et al.* (2015). The following data were used in the model:

- S: number of types of standard objects. We denote object type s, $s = 1, \ldots, S$;
- R: number of types of retails in stock. We denote leftover type k, $k =1, ..., R$;
- T: number of periods of time,
- m: number of types of demanded items;
- e_{st}: number of objects type s available in stock in period t, $s = 1, \ldots, S$; $t = 1, ..., T$;
- er_{kt}: number of retails type k available in stock in period t, $k = 1, ..., R$; $t = 1, ..., T$;
- d_{it}: demand for item type i in period t, $i = 1, \ldots, m$, $t = 1, ..., T$;
- J_{st}: set of cutting patterns for object type s in period t, $s = 1, ..., S$; $t = 1, ..., T$;
- $J_{st}(k)$: set of cutting patterns for object type s generating a retail type k in period t, $k = 1, \ldots, R$, $s = 1, \ldots, S$, t = $1, ..., T$;
- Jr_{kt}: set of cutting patterns for retail type k in period t, $k = 1, ..., R$, $t = 1, ..., T$;
- a_{ijst}: number of items type i in cutting pattern j for object type s in period t, $i = 1, ..., m$, $j \in J_{st}$, $s = 1, ..., S$, $t = 1, ..., T$;
- a_{ijskt}: number of items type i in cutting pattern j for object type s generating a retail type k in period t, $i = 1, ..., m$, $j \in J_{st}(k)$, $s = 1, ..., S$, $k = 1, ..., R$, $t = 1, ...,T$;
- ar_{ijkt}: number of items type i in cutting pattern j for retail type k in period t, $i = 1, ..., m$, $j \in Jr_{kt}$, $s = 1, ..., S$, $t = 1, ..., T$;
- U_{kt}: maximum number allowed for retails type k in period t, $k = 1, ..., R$, $t = 1, ...,T$;

Parameters:

- c_{jst}: waste of cutting object type s according to pattern j in period t, $j \in J_{st}$, $s = 1, ..., S$, $t = 1, ..., T$;
- c_{jskt}: waste of cutting object type s according to pattern j when generating a retail type k in period t, $j \in J_{st}(k)$, $s = 1, ..., S$, $k = 1, ..., R$, $t = 1, ...,T$;
- cr_{jkt}: waste of cutting retail type k according to pattern j in period t, $j \in Jr_{kt}$, $k = 1, ..., R$; $t = 1, ..., T$;
- py_{it}: cost to stock the item type i at the end of the period t, $i = 1, ..., m$, $t = 1, ..., T$;
- pw_{st}: cost to stock the object type s at the end of the period t, $s = 1, ..., S$; $t = 1, ...,T$;
- pz_{kt}: cost to stock the retail type k at the end of the period t, $k = 1, ..., R$, $t = 1, ...,T$;

Variables:

- x_{jst}: number of objects type s cut according to pattern j in period t, $s = 1, ..., S$, $j \in J_{st}$, $t = 1, ..., T$;
- x_{jskt}: number of objects type s cut according to pattern j and generating a retail type k in period t, $s = 1, ..., S$, $j \in J_{st}(k)$, $k = 1, ..., R$, $t = 1, ..., T$;
- xr_{jkt}: number of retail type k cut according to pattern j in period t, $s = 1, ..., S$, $j \in Jr_{kt}$, $t = 1, ..., T$;
- y_{it}: number of items type i anticipated for the period t, $i = 1, ..., m$, $t = 1, ..., T$;
- w_{st}: number of objects type s not used in period t, and available in period $t + 1$, $s = 1, ..., S$; $t = 1, ..., T$;
- z_{kt}: number of retails type k not used in period t, and available in period $t + 1$, $k = 1, ..., R$, $t = 1, ..., T$;

Mathematical Model:

Minimizing

$$\sum_{t=1}^{T}\left(\sum_{s=1}^{S}\sum_{j\in J_{st}} c_{jst}x_{jst} + \sum_{s=1}^{S}\sum_{k=1}^{R}\sum_{j\in J_{st}(k)} c_{jskt}x_{jskt} + \sum_{k=1}^{R}\sum_{j\in Jr_{kt}} cr_{jkt}xr_{jkt} + \sum_{i=1}^{m} py_{it}y_{it} + \sum_{s=1}^{S} pw_{st}w_{st} + \sum_{k=1}^{R} pz_{kt}z_{kt}\right) \quad (1)$$

Subject to

$$\sum_{s=1}^{S}\sum_{j\in J_{st}} a_{ijst}x_{jst} + \sum_{s=1}^{S}\sum_{k=1}^{R}\sum_{j\in J_{st}(k)} a_{ijskt}x_{jskt} + \sum_{k=1}^{R}\sum_{j\in Jr_{kt}} ar_{ijkt}xr_{jkt} + y_{t-1} - y_t = d_{it},\ i=1, ..., m, t=1,...,T \quad (2)$$

$$\sum_{j\in J_{st}} x_{jst} + \sum_{k=1}^{R}\sum_{j\in J_{st}(k)} x_{jskt} - w_{t-1} + w_t \le e_{st},\ s = 1, ..., S,\ t = 1,...,T \quad (3)$$

$$\sum_{j\in Jr_{kt}} xr_{jkt} - z_{t-1} + z_t \le er_{kt},\ k= 1, ..., R,\ t = 1,...,T \quad (4)$$

$$\sum_{s=1}^{S}\sum_{j\in J_{st}(k)} x_{jskt} - \sum_{j\in Jr_{kt}} xr_{jkt} \le U_{kt} - er_{kt},\ k=1,..., R,\ t=1,...,T \quad (5)$$

$x_{jst} \ge 0$, $s = 1, ..., S$, $t = 1, ..., T$, $j \in J_{st}$ and integer; (6)

$x_{jskt} \ge 0$, $s = 1, ..., S$, $k = 1, ..., R$, $t = 1, ..., T$, $j \in J_{st}(k)$ and integer;

$xr_{jkt} \ge 0$, $k = 1, ..., R$, $t = 1, ..., T$, $j \in Jr_{kt}$ and integer;

$y_{it} \ge 0$, $i = 1, ..., m$, $t = 1, ..., T$ and integer;

$w_{st} \ge 0$, $s = 1, ..., S$; $t = 1, ...,T$ and integer;

$z_{kt} \ge 0$, $k = 1, ..., R$; $t = 1, ...,T$ and integer

In the model (1)-(6), the objective function (1) minimizes the waste and inventory costs. Constraint (2) refers to demand of item in each period. Constraints (3) and (4) guarantee that the quantity of standard objects or retails used during the cutting process is not higher than the available quantity in stock in each period. Constraint (5) limits the quantity of retails that can be generated for each type during the cutting process in each period. Constraint (6) refers to integrality and non-negativity of the variables used.

This model can be solved from modifications in the simplex method with column generation, proposed by Gilmore and Gomory (1963) that is an efficient strategy to solve linear problems. As the integrality conditions of the variables (constraint (6)) make the solution of the model (1)-(6) difficult to find at the optimality, this conditions are relaxed and continuous solutions can be found to the problem. Afterwards, rounding heuristic can be applied to find an integer solution.

We are currently implementing this solution method using language C with CPLEX software, that is a mathematical optimization tool for solving mixed linear optimization problems, among others. The current version of the program solves the model considering each period separately, according to what is described in the next section. A heuristic procedure is also presented.

3 Simulation by periods and heuristic procedure

The simulation of time periods can be used to address the day by day production of a company. In the simulation with the CSPUL, in each period (that can be measured in days, hours, etc) the demands are met (lot for lot production) and information about the retails that were generated during the cutting process of the current period is transferred to the next period. Standard objects are always available and their updating is not necessary. In each period, the retails generated in the previous period are included into the stock. However, there is no inventory of items (Cherri et al., 2013). Using this simulation, it is possible evaluate the advantages of the generation of retails during the planning horizon.

Although this situation considers periods of time, it cannot be classified as a multiperiod problem since the item types and their demands are unknown for the future periods. Also, in a multiperiod problem the inventory of the standard objects must be analyzed and replaced if necessary. For a multiperiod problem see Poldi and Arenales (2010).

The simulation by periods was performed considering the model (1)-(6). As the integrality constraints (6) were relaxed, a relax-and-fix heuristic was used to obtain integer solution for the problem. According to Cherri (2013), the relax-and-fix heuristic solve the problem by stages and, in each stage, a sub-problem of the original problem is solved exactly. Therefore, the integer variables of the problem are divided in three groups: fixed variables, not fixed integer variables (free) and linearly relaxed integer variables. In each step of the method, we have a sub-problem and each integer variable of the problem must belong to one of these groups. After each step, a solution is obtained for the sub-problem. So, free integer variables are fixed with the value of the found solution. From the relaxed set of integer variables, a new subset of variables (a partition) is chosen to be composed of free integer variables. This procedure is realized until all integer variables of the original problem have been fixed.

Basically, the developed procedure consists in:

Step 1: Generate a solution using the mathematical model (1)-(6), with the relaxed integrality constraints;
Step 2: Set as integer the variables related to the frequencies of the cutting patterns that provide waste lower than or equal to 2% of the length of the cut object. If there are no cutting patterns that meet this criteria, the frequency of the variable of the four cutting pattern with lower waste is set as integer.
Step 3: Solve the problem again, using the mathematical model (1)-(6);
Step 4: Set as integer the frequencies of the selected cutting patterns in step 2 using the integer value obtained in the solution of step 3. The procedure does not require that all selected cutting patterns in step 2 are in the solution of step 3, because in some iteration might not be possible to find a feasible solution. The only requirement is that the solution of step 3 contains at least one of the cutting patterns selected in step 2;
Step 5: Return to step 2 and repeat this procedure until all cutting pattern have integer values for the frequencies.

In order to improve the performance of the heuristic procedure, as the leftover in the last cutting pattern is generally very large, it is stored since it is as long as or longer than the smallest allowed retail. When leftover like this is available in stock, the procedure ensures that it will be the first object to be cut. With the integer solution, it is possible make the analysis of the solution using a simulation by periods.

4 Computational tests

Aiming to show that our approach using CSPULM is promising MCSPUL, we have conducted a simulation with 5 periods of time (Cherri et al., 2013). With these tests, we have tried to simulate the daily production of a company. Model (1)-(6) and the heuristic procedure were implemented using the OPL interface of the software CPLEX 12.6. The experiments were executed on an Intel Core 2.20 GHz, 8 GB RAM computer.

The generated problems consider one type of standard object with length 1200 and availability enough to meet the items demand of all period. Three types of retails can be generated with lengths 400, 500 and 650. Fifteen types of items are generated in all periods. The lengths of the items were generated considering two size categories: medium (M) and large (L). For each category, the length was generated in the following intervals: medium items (M): [140, 400] and large items (L): [300, 700].

The item demand was generated considering three categories: low (L), medium (M), and high (H). For each category, the demand was generated in the following intervals: low demand (L): [1, 10]; medium demand (M): [10, 50] and high demand (H): [50, 300]. The maximum number of generated retails of each type was: $U_k = 0$, $U_k = 2$, $U_k = 4$, $U_k = 8$, $k = 1, 2, 3$. That is, when $U_k = 8$, at most 8 retails can be generated to stock per period. In the tests, we also analyze the solution for the classical cutting stock problem ($U_k = 0$ $er_k = 0$, $k = 1, ..., 3$);

Combining the length and demand categories, six instance classes were randomly generated. For example, the class [L,H] represents a class with large items and high demand. These are the values that we have established for the tests, but other values also could have been used.

Table 1 shows the average waste for the 5 periods using the relax-and-fix heuristic. In all classes, in the first period the stock is composed only of standard objects, that is, $er_k = 0$, $k = 1, ..., 3$. From the second period, the retails generated in the first period will be in stock and must be cut.

Table 1: Average waste for 5 periods.

Class	*U = 0*	*U = 2*	*U = 4*	*U = 8*
[M,L]	594.80	154.80	154.80	124.80
[M,M]	1328.00	638.00	542.00	558.00
[M,H]	2186.00	1674.40	1626.00	1606.00
[L,L]	1947.20	1447.20	1337.20	1247.20
[L,M]	15319.40	14020.80	13267.60	12267.60
[L,H]	122924.00	112584.00	111884.00	111484.00

Considering a simulation by periods with a lot for lot production, it is possible to observe by the Table 1 that the waste decreases for larger values of *U*. This is an expected situation because the possibility to generate retails increases the diversity of objects in stock for the next period. Thereby, there will be more possibilities to generate the cutting patterns. Furthermore, the retails are generated during the cutting processes just to reduce the waste. This justify the larger waste for $U = 0$.

Figure 1 depicts the percentage waste reduction for the integer solutions in terms of U related to the classical CSP (U = 0).

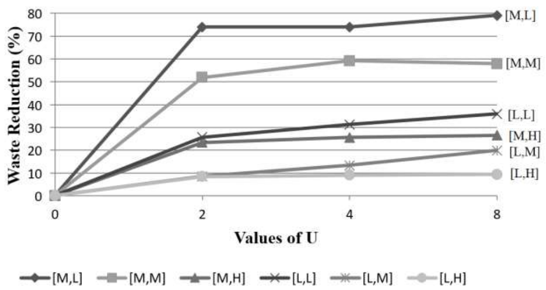

Figure 1: Percentage of waste reduction for each instances classes.

By Figure 1, it is possible to observe that the percentage of waste reduction for each instances class is very significant. For medium items there are more possibilities of combinations in a cutting pattern compared to large items. Due this characteristic, for the classes [ML] and [MM] together with the possibility of generate retail, that also increases the quality of the cutting process (Table 1), the percentage of waste reduction was over 58%.

5 Conclusions

In this paper, we present the multiperiod cutting stock problem with usable leftovers. In this problem, in each period, new retails can be generated in quantities and length previously defined and, as the demand of items is known in the planning horizon, it is possible to anticipate forward or not their production. Thus, there is a decision that is to generate retails x anticipate the demand. A mathematical model was proposed to represent this problem and the solution method is being developed.

To show that our approach is promising, we present solutions simulating periods of time, that is, for each period we have a cutting stock problem to be solved whose stock depends on previous generated retails together with the standard objects. Different from the multiperiod problem, for the simulation the demand of items is not known previously and is not possible to anticipate it during the periods. The results obtained for this problem showed clearly, in terms of wastes, that generate retails is better than consider situations that do not allow the generation of retails.

As future reaserch, the implementations for the solution method to solve the multiperiod model will be finished and computational tests will be performed to show the performance of the proposed model. Compared to the results presented in Section 4, the expectative is to obtain better solutions. Since the whole demand will be known in a period of time, it will be possible to anticipate de production of demanded items or generate retails. This choice will be made during the cutting process according to the reduction on the productions costs.

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